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The Effect of Heat Transfer on Interactions Involving Laminar Boundary Layers

By

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The Effect of Heat Transfer on Interactions Involving Laminar Boundary Layers

- By -

K. N. C. Bray

Summary

A theory is devloped, from Ref. 1, to allow for heat transfer effects, in interactions between laminar boundary layers and shock waves, and on boundary layer separation.

As a simple example of the use of this theory, the effect of wall temperature on the Mach number change to cause separation of a laminar boundary layer has been found, for large Reynolds numbers.

Suggestions are made for further calculations, based on the method.

1. Introduction

Refs. 1, 2 and 3, describe a theory, due to Crocco and Lees, which deals with the interaction between a region of viscous flow, such as a boundary layer or wake, and an inviscid outer flow. This theory has given good results in calculations involving interactions between shock waves and laminar or turbulent boundary layers, and in the calculation of the base pressure behind a body.

As originally presented, the theory is limited to cases where no heat transfer takes place between the fluid and a solid surface adjacent to it. However, considerable heat transfer often takes place in high speed flight; there appears to be a need for a theoretical estimate of the effects of heat transfer on such phenomena as shock wave boundary layer interactions, and boundary layer separation. This report describes how the theory of Crocco and Lees can be generalised to deal with such problems, in the laminar case.

2. Boundary Layer Functions with Heat Transfer

In Ref. 2 the properties of any laminar boundary layer on an insulated surface are described by three functions of a quantity $\mathcal{K}$, which is the ratio of the mean velocity in the boundary layer to the free stream velocity (u_e). These functions are: $\psi(\mathcal{K})$, which depends on the mean temperature in the boundary layer; $C(\mathcal{K})$, the mixing rate between the boundary layer and the free stream; and $\sigma(\mathcal{K})$, a skin friction parameter. These functions were evaluated from the incompressible boundary layer solutions of Falkner and Skan $(u_e \sim x^m)$, with compressibility taken into account through Stewartson's transformation.

Cohen and Reshotko, Ref. 4, have recently solved the equations derived by Stewartson for a laminar compressible boundary layer with pressure gradient and heat transfer, for cases with $u_e \sim x^m$ and

constant/

constant wall temperature. In the present report, these exact solutions have been used to evaluate the boundary layer functions ψ, C and σ, which are defined in such a way that the governing equations are unchanged from the zero heat transfer case (see Ref. 3). These equations may be solved by the method described in Ref. 2.

The boundary layer solutions of Ref. 4, assume a Prandtl number of unity and a linear viscosity temperature law. Both these approximations are known to lead to errors at high Mach numbers and large heat transfer rates. A further difficulty arises from the fact that the thicknesses of the thermal and velocity boundary layers are different, in cases with velocity gradient and heat transfer. In these calculations the thickness of the velocity layer, defined as the point at which $u/u_e = 0.95$, has been chosen as the edge of the viscous region, since this is the quantity required in the integrated momentum equation. However, it must be admitted that this procedure is somewhat arbitrary.

The assumption that the quantities $\mathcal{H}$, ψ, C and σ, derived from Ref. 4, are applicable to velocity distributions other than $u_e \sim x^m$ is also open to question. These functions must be recalculated from other exact solutions, to check that no significant differences occur.

Despite these approximations, it is believed that the method suggested here will give useful results, at moderate Mach numbers and heat transfer rates. There is no other method, known to the author, which can given even a qualitative indication of the effect of heat transfer on interaction problems.

The quantities $\mathcal{H}$, ψ, C and σ are given in Table I, for a number of different values of the ratio T_W/T_e^o where T_W = wall static temperature and T_e^o = free stream stagnation temperature. It is found that the temperature function ψ is the most affected by wall temperature, as would be expected. The mixing rate function C is almost independent of wall temperature.

From the values of Table I, there appears to be a possibility that a laminar boundary layer on a strongly heated surface may have a critical point (see Refs. 1 and 2), between the flat plate condition and separation. In an interaction with a shock wave, such a boundary layer would behave like a turbulent layer, and the 'upstream influence' would be small. Further calculations are needed to determine whether this condition is likely to be met, in the practical range of wall temperatures and Mach numbers.

3. Separated Flows

Nothing is known about the behaviour of separated boundary layers under heat transfer conditions. Ref. 4 gives a number of solutions for the case $u_e \sim x^m$ with m chosen so that $(\partial u/\partial y)_{y=o}$ is negative, but these solutions are for boundary layer profiles which are similar for all values of x and we have no evidence that the same solutions are applicable to the practical cases of separation and reattachment. Nor is there much justification for extending the assumption of Ref. 1, that $\mathcal{H}_{sep.}$ = constant, to the heat transfer case. In fact, the separated solutions in Ref. 4 indicate that $\mathcal{H}$ may increase considerably, after separation, especially for flow over cooled walls.

A simple experiment is required to solve these problems, but in the meantime, it is suggested that the solutions of Ref. 4, be used in calculations on separated boundary layers, and that the assumption $\mathcal{H}_{sep.}$ = constant be dropped.

Allowing/

Allowing $\mathcal{K}$ to increase after separation may well improve the agreement between the calculations of Ref. 2, and experimental evidence. It was noted in Ref. 2, that the pressure gradients in the part of the interaction downstream of the shock wave were much too small in these calculations, and it is easy to show that the gradients in this region will be increased if $\mathcal{K}$ is allowed to increase after separation.

4. Separation of Laminar Boundary Layers at Large Reynolds Numbers

As a simple example of the use of this theory, the Mach number change to cause separation of a laminar boundary layer at large Reynolds numbers has been calculated for a number of wall temperatures, by the method of Ref. 3, Section V, in which it is shown that

$$M_s = M_o \, e^{\int_{\mathcal{K}_o}^{\mathcal{K}_s} \frac{d\mathcal{K}}{\psi}}$$

if $R_e \to \infty$ where suffix 'o' indicates conditions ahead of the interaction and suffix 's' indicates conditions at the separation point.

The results of these calculations are shown in Fig. 1. Considerably larger values of M_o/M_s can be expected at finite Reynolds numbers, and it is not suggested that these results are accurate quantitatively particularly at very high and very low temperatures, but it is believed that they can give a qualitative picture of the effect of wall temperature on separation. For example, a considerable amount of cooling $(T_w/T_e^o \simeq 0.25)$ is required to delay separation to the same extent as it is delayed by asymptotic suction.

5. Suggestions for Further Work

Any of the problems suggested in Ref. 1, can also be tackled for laminar boundary layers with heat transfer. The effect of wall temperature on shock wave boundary layer interactions at finite Reynolds numbers would be of interest, and could be found by the method of numerical solution developed in Ref. 2.

Calculations to find under what conditions a laminar boundary layer can possess a critical point, and calculations on separated boundary layers with heat transfer would also be of interest. Experimental confirmation of these last two points could be sought.

6. Conclusions

The theory of Crocco and Lees has been extended to cover the case of laminar boundary layers with heat transfer. The modified theory can be used to calculate the effect of wall temperature on shock wave boundary layer interactions, and to give an estimate of heat transfer effects after separation.

As an example of the use of the theory, the Mach number change to cause separation of a laminar boundary layer at large Reynolds numbers has been calculated, as a function of wall temperature. A comparison has been made between the effects of wall cooling and boundary layer suction, as means of delaying separation.

Suggestions are made for further calculations based on the method.

List of Symbols

C	=	mixing rate function
M_o	=	initial free stream Mach number
M_s	=	free stream Mach number at separation
m	=	index in relation $u_e \sim x^m$
P_R	=	Prandtl number
R_e	=	Reynolds number
T_e^o	=	free stream stagnation temperature
T_w	=	wall temperature
u	=	velocity in x direction
u_e	=	free stream velocity
x	=	co-ordinate along surface
y	=	co-ordinate normal to surface
β	=	$\frac{2m}{m + 1}$
$\mathcal{H}$	=	boundary layer profile parameter
$\mathcal{H}_o$	=	initial value of $\mathcal{H}$
$\mathcal{H}_s$, $\mathcal{H}_{sep.}$	=	value of $\mathcal{H}$ at separation
μ	=	viscosity coefficient
σ	=	skin friction function
ψ	=	temperature function

References/

References

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Table I/

Table I

Boundary Layer Functions with Heat Transfer

	$\frac{T_w}{T_e^o}$	β	$\mathcal{H}$	ψ	C	σ
S	0	-0.3260	0.6497	1.0729	3.3361	0
		-0.3657	0.6444	0.9003	3.4158	0.0761
		-0.3884	0.6528	0.6993	3.3959	0.2188
		-0.3600	0.6673	0.5410	3.1680	0.4134
		-0.3000	0.6789	0.4509	2.9416	0.5778
		-0.1400	0.6914	0.3500	2.5937	0.8382
F.P.		0	0.6984	0.3016	2.4249	1.0000
		0.5	0.7070	0.2178	2.1045	1.3658
		2.0	0.7195	0.1304	1.8838	1.9171
	0.2	-0.10	0.8277	2.1629	2.4436	-0.2547
		-0.2685	0.6649	1.3556	3.1124	-0.0846
S		-0.3088	0.6528	1.1260	3.2649	0
		-0.325	0.6522	1.0014	3.3139	0.0779
		-0.3285	0.6526	0.9411	3.2768	0.1102
		-0.3258	0.6563	0.8601	3.2494	0.1775
		-0.325	0.6582	0.8178	3.2120	0.2210
		-0.3	0.6677	0.7097	3.0832	0.3575
		-0.14	0.6889	0.5259	2.6400	0.7598
F.P.		0	0.6984	0.4578	2.4249	1.0000
		0.5	0.7111	0.3465	2.0178	1.5953
		1.5	0.7241	0.2532	1.7700	2.3669
		2.0	0.7281	0.2273	1.7195	2.6592
	0.6	-0.235	0.6641	1.5526	3.0870	-0.0847
S		-0.246	0.6602	1.3696	3.1873	0
		-0.2483	0.6594	1.2484	3.1318	0.0829
		-0.24	0.6625	1.1468	3.0562	0.1803
		-0.2	0.6738	0.9884	2.9012	0.3929
F.P.		0	0.6984	0.7705	2.4249	1.0000
		0.5	0.7173	0.6173	1.8209	2.0831
		2.0	0.7385	0.4614	1.2785	4.5076
	1.0*	-0.1947	0.6648	1.8673	3.0050	-0.0860
S		-0.198	0.6642	1.6892	3.0374	0
		-0.19	0.6672	1.4908	3.0040	0.1543
		-0.18	0.6692	1.4215	2.9436	0.2255
		-0.16	0.6739	1.3345	2.8679	0.3440
		-0.14	0.6796	1.2698	2.8285	0.4436
		-0.10	0.6877	1.1867	2.7258	0.6187
F.P.		0	0.6984	1.0830	2.4249	1.0000
		0.5	0.7252	0.9060	1.6244	2.6471
		1.0	0.7335	0.8576	1.2012	4.2198
		2.0	0.7377	0.8277	0.7783	7.2904
	2.0	-0.1	0.7058	3.9269	2.7304	-0.3318
		-0.1305	0.6694	2.7579	2.9200	-0.0885
S		-0.1295	0.6683	2.5617	2.9285	0
		-0.1	0.6748	2.1641	2.7996	0.3317
F.P.		0	0.6984	1.8646	2.4249	1.0000
		0.3	0.7228	1.7996	1.4470	2.9481
		0.5	0.7287	1.9033	1.0193	4.5093
		1.0	0.7478	2.2306	0.4904	9.8347
		1.5	0.7980	2.4094	0.3147	18.8896
		2.0	0.8637	2.4687	0.2402	37.2425

S separation point F.P. flat plate point (zero pressure gradient)
* calculated from Hartree, Ref. 5.

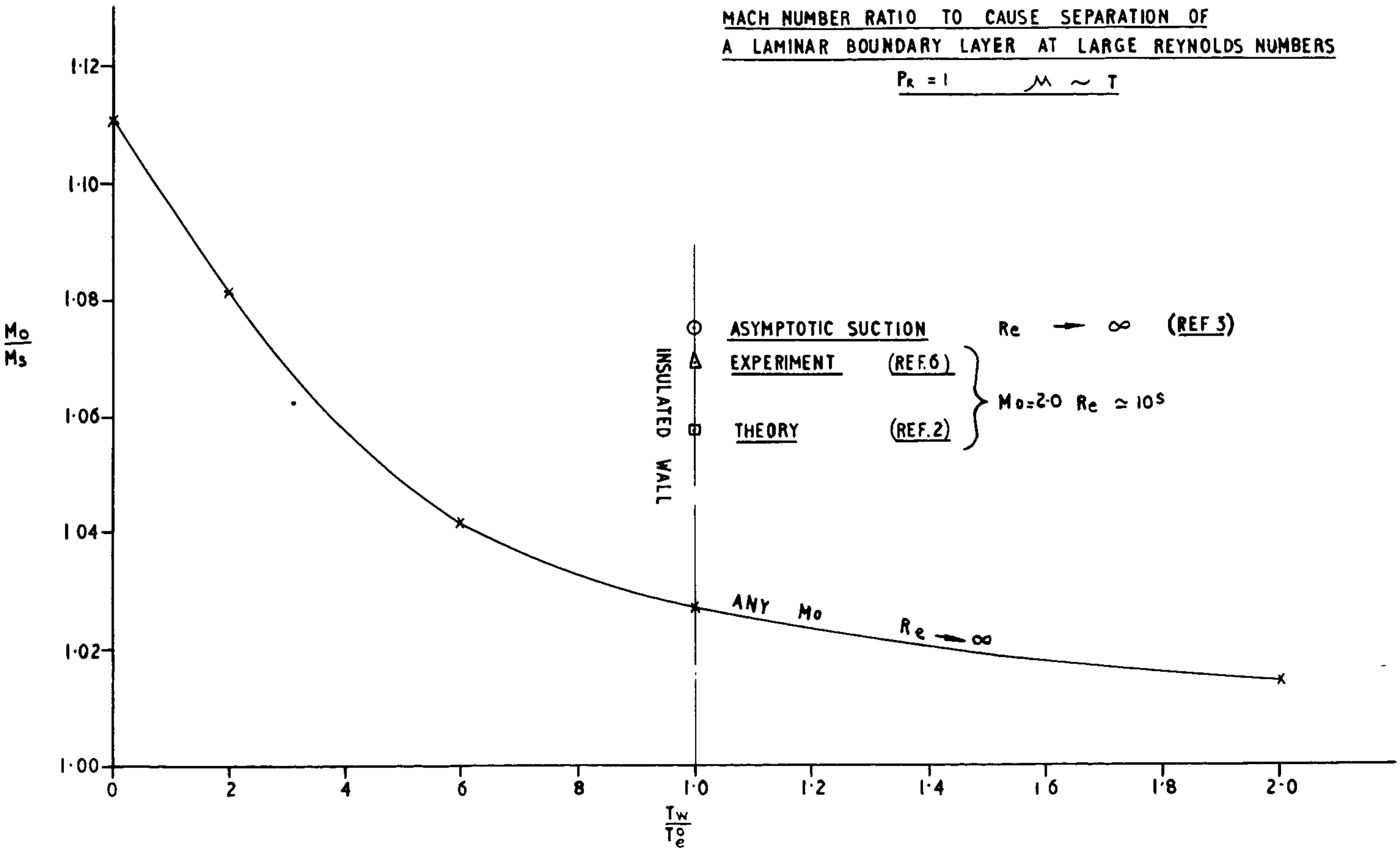
MACH NUMBER RATIO TO CAUSE SEPARATION OF
A LAMINAR BOUNDARY LAYER AT LARGE REYNOLDS NUMBERS
PR = 1 μ ~ T
Mo/Ms
1·12
1·10
1·08
1·06
1·04
1·02
1·00
0
2
4
6
8
1·0
1·2
1·4
1·6
1·8
2·0
Tw/T°e
INSULATED WALL
ASYMPTOTIC SUCTION Re → ∞ (REF 3)
EXPERIMENT (REF.6)
THEORY (REF.2)
Mo=2·0 Re ≃ 10^5
ANY Mo
Re → ∞

FIG. 1

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