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The Forward Take-off of a Helicopter

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Summary.—A theoretical analysis is given of the accelerated motion of a single-rotor helicopter for estimation of the forward take-off performance. The motion is considered in stages during which either the disc attitude to the horizontal or the flight speed is constant. Equations are derived for the motion along and normal to the flight path and solutions are given assuming constant mean values for the aerodynamic forces on the rotor and fuselage. The equations of motion for constant disc attitude have a simple solution for motion from rest (and for special initial conditions) giving a straight flight path, and a general solution giving a curved flight path. The performance at constant speed is considered for a general climb away case and also for climb away approaching steady flight conditions with the thrust approximately equal to the aircraft weight. For application of the theory, charts of the various solutions are given covering a representative range of the variables.

1. *Introduction.*—A helicopter should ideally be capable of ascending vertically from the ground in all conditions; in practice, however, it is often necessary to gain speed, normally while airborne within the ground cushion, before climbing away, either because of inadequate vertical climb performance or to provide a greater degree of safety in the event of power failure.

A theory of the accelerated motion of a single-rotor helicopter is developed in this report for use in the forward take-off case. The assumptions about the helicopter and rotor are the same as made by Squire in R. & M. 1730[1] for analysis of the steady climb performance. A main point of difference from the steady flight analysis, however, is that whereas in the latter it may be assumed that the rotor thrust T is approximately equal to the helicopter weight W, in the accelerated motion it is necessary to include the possibility that $T > W$.

In practice, even if no control adjustments are made, T will vary to some extent during the take-off, owing to speed and rotor incidence changes and variations of ground effect with height and speed. In the analysis, however, it is assumed that the take-off is considered in stages during which it is sufficiently accurate to use a constant mean value for T and also for the other aerodynamic forces acting on the helicopter; in addition, during each stage, either the disc attitude to the horizontal, α, or the flight speed, V, is assumed constant.

A list of symbols is given at the end of the report.

* A.A.E.E. Report Res/276, received 3rd July, 1953.

2. *Forces on the Helicopter.*—The motion is considered in general with the helicopter airborne and it is assumed that it is flying with velocity V at a flight-path angle γ to the horizontal. The rotor thrust T acts normal to the disc which is at an angle α to the horizontal; the transverse force H is parallel to the disc. The other forces on the helicopter are the weight W, and the body drag D which is assumed to act along the direction of flight. The force system is illustrated in Fig. 1.

For estimation of the aerodynamic forces it is assumed that the formulae for steady conditions can be used. Thus for given speed and power conditions (including a specified rotor speed) the thrust may be determined (not taking ground effect into consideration) at all except very low speeds, from the momentum thrust equation and the rotor work equation (Ref. 2)

$$T = 2\pi\rho_0 e^2 R^2 v_i (V_i^2 + v_i^2 + 2V_i v_i \sin i)^{1/2} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

where

eR is the effective rotor radius, allowing for tip losses,

v_i is the equivalent induced flow velocity $(v\sqrt{\sigma})$,

i is the rotor incidence to the flight path,

and

$$EP - P_R = Tu_i = T(v_i + V_i \sin i) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

where

EP is the effective power at the rotor

$$P_R = \tfrac{1}{8}\rho_0 C_D bcR\Omega_i^3 R^3 (1 + \mu^2).$$

For the range of speeds arising in take-off it is normally possible to neglect μ^2 in P_R or to use an approximate mean value.

To simplify the application of (1) and (2) a chart derived from these equations is given in Fig. 2, with $T/2\pi\rho_0 e^2 R^2 V_i^2$ as a function of $(EP\sqrt{\sigma} - P_R) / 2\pi\rho_0 e^2 R^2 V_i^3$ and i; the value of i at a point of the take-off path is $(\alpha + \gamma)$. The blade pitch θ for the thrust T follows from the blade element theory formula,

$$T = \tfrac{1}{6}\rho_0 abc\,\Omega_i^2 R^3 \left(\theta - \frac{3}{2}\frac{u_i}{\Omega_i R}\right). \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

At very low speeds the momentum theory is inaccurate and the empirical charts in Ref. 3 may be used; the thrust is there given as a function of the speed, power and disc incidence in a form similar to Fig. 2. The empirical curve of rotor coefficients for vertical flight also in Ref. 3 can be used for estimating the static thrust in motion from rest. The helicopter starts moving in the direction of the resultant force, so that the initial value of the flight path angle, γ_0, is given by

$$\tan \gamma_0 = \frac{T \cos \alpha - W}{T \sin \alpha}.$$

The force H is small and in most cases at low speed can be neglected. Where necessary it can be estimated with sufficient accuracy from the approximate formula

$$H = \tfrac{1}{4}\rho_0 C_D bcR\,\Omega_i RV_i \cos i.$$

If D_0 is the body drag at 100 ft/sec at sea level

then

$$D = D_0 \left(\frac{V_i}{100}\right)^2.$$

The above method of estimating the forces can be used throughout the motion if the analysis is made on a differential step-by-step basis in which V and γ are determined along the flight path, Because of the numerical complexity this is not convenient for general use, however, and the analysis in following paragraphs is developed for finite intervals, either at constant disc attitude α_m, or at constant speed V_m, mean values being taken for the aerodynamic forces T_m, H_m and D_m; methods of evaluating the mean forces are now outlined.

The mean forces during a stage of the take-off with constant disc attitude are considered first for the special case of a straight flight path (section 3.1.1). For a straight path the resultant force normal to the path must be zero, thus

$$T \cos i + H \sin i - W \cos \gamma = 0.$$

With $H = H_0 \cos i$, this may be written in the form

$$\frac{T}{2\pi\rho_0 e^2 R^2 V_i^2} = \frac{W \cos (i - \alpha_m) - H_0 \cos i \sin i}{2\pi\rho_0 e^2 R^2 V_i^2 \cos i} \quad \ldots \quad (4)$$

This relation and the data in Fig. 2 are sufficient to determine T_m and i_m for a given value of α_m, and the mean speed; choice of the mean speed to find T_m is justified by the fact (it can be shown from Fig. 2) that, for constant disc incidence, T varies only slightly with V_i. For convenience in use the data is presented in Fig. 3 with $T/2\pi\rho_0 e^2 R^2 V_i^2$ as a function of i, from Fig. 2 for various values of the power term, and from (4) for various values of $W/2\pi\rho_0 e^2 R^2 V_i^2$ and α_m, the H_0 term in (4) being negligibly small for the range of variables considered; T_m and i_m can be found from the intersection of the curves for the appropriate power and weight conditions. H and D are functions of V_i and the mean values H_m and D_m are simply determined, while the blade pitch θ_m follows from (2) and (3).

In the general case of motion at constant disc attitude (with a curved flight path), determination of T_m from Fig. 2 requires knowledge of the mean disc incidence, i_m. As noted above T varies only slightly with V_i but it also follows from Figs. 2 and 3 that it varies appreciably for large changes of i, in an approximately linear manner. Since α is constant, i_m can be determined from the mean value of γ and an approximate method of finding γ_m is outlined in section 3.1.2.

In motion at constant speed, the disc attitude is not fixed but must be varied in a way to keep the resultant force along the flight path zero; the equation of force along the path is

$$T \sin i - H \cos i - D - W \sin \gamma \doteq 0 .$$

This may be written in the form

$$\frac{T}{2\pi\rho_0 e^2 R^2 V_i^2} = \frac{H_0 \cos^2 i + D + W \sin \gamma}{2\pi\rho_0 e^2 R^2 V_i^2 \sin i} .$$

$T/2\pi\rho_0 e^2 R^2 V_i^2$ is presented in Fig. 4 as a function of i for various values of $(D + W \sin \gamma)/2\pi\rho_0 e^2 R^2 V_i^2$ (the H_0 term is again negligibly small), together with the power curves from Fig. 3. The mean value of γ for determining T_m and i_m from this chart can be found directly because the solution for the motion in section 3.2 is developed in terms of γ as a parameter.

The ground effect on the thrust at a given power in hovering flight may be found from the empirical curves in Ref. 4; there is no published data available on ground effect in forward flight on a helicopter but at low speed the hovering data should be sufficiently accurate. It is necessary to make an assumption about the mean height for which the correction should be made.

3. *Theory of Take-off Flight.*—3.1. *Motion with constant disc attitude.*—Take-off flight is considered in still air with reference to co-ordinate axes fixed with reference to the ground, the x-axis horizontal forwards and the y-axis vertical upwards; distance along the flight path

is specified by s. With the forces outlined above, the equation of motion along the flight path is

$$\frac{T}{W}\sin(\alpha+\gamma)-\frac{H}{W}\cos(\alpha+\gamma)-\frac{D}{W}=\frac{V}{g}\frac{dV}{ds}+\sin\gamma\,.$$

The equation of motion normal to the flight path is

$$\frac{T}{W}\cos(\alpha+\gamma)+\frac{H}{W}\sin(\alpha+\gamma)=\frac{V}{g}\frac{d\gamma}{dt}+\cos\gamma.$$

These equations can be integrated by step-by-step methods, the forces being estimated in the way outlined in section 2. For general use, however, simpler solutions than those requiring numerical integrations are desirable, and further development of the theory is restricted to finding approximate solutions applicable to a finite stage of take-off, in the first place with the disc attitude assumed constant.

The first equation may be written if $\cos\gamma\neq 0$,

$$\left(\frac{T}{W}\sin\alpha-\frac{H}{W}\cos\alpha\right)+\left(\frac{T}{W}\cos\alpha+\frac{H}{W}\sin\alpha\right)\frac{dy}{dx}-\frac{D}{W}\frac{ds}{dx}=\frac{V}{g}\frac{dV}{dx}+\frac{dy}{dx}.$$

The disc attitude is assumed to have the constant value α_m and if mean values T_m, H_m, D_m are taken for the aerodynamic forces this equation may be integrated giving,

$$\left[\frac{T_m}{W}\sin\alpha_m-\frac{H_m}{W}\cos\alpha_m-\frac{D_m}{W}\frac{s}{x}\right]x+\left[\frac{T_m}{W}\cos\alpha_m+\frac{H_m}{W}\sin\alpha_m-1\right]y=\left(\frac{V^2}{2g}-\frac{V_0^2}{2g}\right)$$

where V_0 is the velocity when $x=y=s=0$.

For all but the steepest take-off paths it appears a reasonable approximation to take $s/x=1$ in the last term in the first bracket ; when the path is relatively steep, the flight speed must be low and D_m/W small, so that even in this case the error will be small.

The equation may therefore be written

$$Bx+Ay=\frac{V^2-V_0^2}{2f} \qquad \text{(5)}$$

where

$$\left.\begin{aligned}A&=\frac{\dfrac{T_m}{W}\cos\alpha_m+\dfrac{H_m}{W}\sin\alpha_m-1}{\dfrac{T_m}{W}\sin\alpha_m-\dfrac{H_m}{W}\cos\alpha_m}\\[2ex] B&=1-\frac{D_m/W}{\dfrac{T_m}{W}\sin\alpha_m-\dfrac{H_m}{W}\cos\alpha_m}\\[2ex] f&=g\left\{\frac{T_m}{W}\sin\alpha_m-\frac{H_m}{W}\cos\alpha_m\right\}\end{aligned}\right\} \qquad \text{(6)}$$

Similarly for constant disc attitude and assuming mean values of the forces, the equation of motion normal to the flight path may be written

$$A - \tan \gamma = \frac{V^2}{f} \frac{d\gamma}{dx} \qquad (7)$$

3.1.1. *Special solution of equations of motion* ($\tan \gamma = A$).—It is evident that a special solution of (7) is $\tan \gamma = A =$ constant. This solution applies to the case where the resultant force is along the flight path direction at the point considered. In particular it applies to motion from rest, when, with the assumption of constant mean forces, the aircraft moves off in the direction of the resultant force. It applies equally to backward and forward take-off.

Since the flight path is a straight line

$$\frac{y}{x} = \tan \gamma = A .$$

Hence, with (5)

$$\left.\begin{aligned} x &= \frac{1}{(A^2 + B)} \left\{ \frac{V^2}{2f} - \frac{V_0^2}{2f} \right\} \\ y &= \frac{A}{(A^2 + B)} \left\{ \frac{V^2}{2f} - \frac{V_0^2}{2f} \right\} \end{aligned}\right\} \qquad (8)$$

For $A = 0$, *i.e.*, in horizontal flight, there is a relationship between T and α.

$$\frac{T}{W} \cos \alpha + \frac{H}{W} \sin \alpha = 1 ,$$

Normally $(T/W) \cos \alpha \gg (H/W) \sin \alpha$, and so for a selected mean value of T, $\cos \alpha = W/T$ approximately. Hence with constant mean values for H and D,

$$x = \frac{V^2 - V_0^2}{2g \left[\left\{ \left(\frac{T_m}{W} \right)^2 - 1 \right\}^{1/2} - \frac{H_m}{T} - \frac{D_m}{W} \right]} . \qquad (9)$$

It is sometimes necessary for a helicopter take-off to begin with a ground run, and this type of motion is also conveniently considered here. In a ground run, the resultant force in the vertical direction is less than the weight, so that

$$T \cos \alpha + H \sin \alpha < W .$$

The equation of forward motion is

$$\frac{T}{W} \sin \alpha - \frac{H}{W} \cos \alpha - \frac{D}{W} - \left(1 - \frac{T}{W} \cos \alpha - \frac{H}{W} \sin \alpha \right) \mu_f = \frac{V}{g} \frac{dV}{dx}$$

where μ_f is the coefficient of forward friction.

Hence with constant mean values for T, H, α and D,

$$x = \frac{V^2 - V_0^2}{2g \left[\frac{T_m}{W} (\sin \alpha_m + \mu_f \cos \alpha_m) - \frac{H_m}{W} (\cos \alpha_m - \mu_f \sin \alpha_m) - \frac{D_m}{W} - \mu_f \right]} . \qquad (10)$$

3.1.2. *General solution.*—In the general case where $\tan \alpha \neq A$, it is convenient to put $Q = V^2/2f$ and $dQ/dx = q$, so that by differentiation of (5), $q = A \tan \gamma + B$; equation (7) becomes

$$(A^2 + B - q)(A^2 + B^2 - 2Bq + q^2) = 2A^2 Qq \frac{dq}{dQ}.$$

Integrating this equation and substituting for q gives

$$\frac{Q}{S} = G(\gamma, A, B), \text{ which is charted in Fig. 5,}$$

$$G(\gamma, A, B) = \left\{\frac{1 + \tan 2\gamma}{(A - \tan \gamma)^2}\right\}^{(A^2+B)/(A^2+1)} \exp\left\{\frac{2A(B-1)}{A^2+1}\gamma\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (11)$$

and
$$S = \frac{Q_0}{G(\gamma_0, A, B)} = \frac{V_0^2}{2fG(\gamma_0, A, B)}.$$

It is useful to note also that

$$\frac{Q}{S} = \frac{V^2}{V_0^2} G(\gamma_0, A, B)$$

Distance along the path is obtained by integrating numerically $1/q$ ($\doteq dx/dq$) as a function of Q/S, giving

$$\frac{x}{S} = \int \frac{1}{q}\, d\left(\frac{Q}{S}\right) = \mathscr{X}\left(\frac{Q}{S}, A, B\right) \text{ say.}$$

$\mathscr{X}(Q/S, A, B)$ is charted in Fig. 6 with the zero value at the value of Q/S corresponding to $\gamma = 0$; this value, Q_0/S, can be determined from Fig. 5. Starting from a non-zero value of γ_0, the initial value, Q_0/S, can be found from the same figure, and then

$$\frac{x}{S} = \mathscr{X}\left(\frac{Q}{S}, A, B\right) - \mathscr{X}\left(\frac{Q_0}{S}, A, B\right);$$

also from (5)

$$\frac{y}{S} = \mathscr{Y}\left(\frac{Q}{S}, A, B\right) - \mathscr{Y}\left(\frac{Q_0}{S}, A, B\right)$$

where

$$\mathscr{Y}\left(\frac{Q}{S}, A, B\right) = \frac{1}{A}\left\{\frac{Q}{S} - B\,\mathscr{X}\left(\frac{Q}{S}, A, B\right)\right\} \text{ and is charted in Fig. 7.}$$

It was pointed out in section 2 that the mean value γ_m of the flight path angle is required in connection with the estimation of the mean rotor thrust in a speed interval from V_0 to V_1 say. An approximate estimate of the angle γ_1 corresponding to V_1, can be made from Fig. 5 using the values of A, B and S for the initial conditions; a first approximation to γ_m follows. Further approximation can be made by using A, B and S for the mean conditions to re-estimate γ_1, from Fig. 5.

3.1.3. *Solution for $B = 1$.*—The general solution takes a simpler form when $B = 1$; this occurs if the body drag is negligibly small compared to the resultant horizontal component of the rotor forces. For $B = 1$,

$$G(\gamma, A, 1) = \frac{1 + \tan^2 \gamma}{(A - \tan \gamma)^2}.$$

$J(\gamma, R_F/W, D/W)$ is plotted in Fig. 8 as a function of γ for a range of values of R_F/W and D/W; the constant has been adjusted to make J zero when $\gamma = 0$.

The theory does not give a simple integral for y and this has been found by integrating $\tan \gamma$ numerically as a function of gx/V^2. Then

$$\frac{gy}{V^2} = K\left(\gamma, \frac{R_F}{W}, \frac{D}{W}\right) - K\left(\gamma_0, \frac{R_F}{W}, \frac{D}{W}\right)$$

where

$$K\left(\gamma, \frac{R_F}{W}, \frac{D}{W}\right) = \int \tan \gamma \, d\left(\frac{gx}{V^2}\right).$$

$K(\gamma, R_F/W, D/W)$ is plotted in Fig. 9 with the zero values at $\gamma = 0$ for a range of values of R_F/W and for $D/W = 0{\cdot}01$.

3.3. *Climb Away at Constant Speed Approaching Steady Climb Condition.*—The final stage of a take-off is the transition to the steady climb condition; for steady climb at a given power and speed V_s the ordinary momentum theory may be used to find the steady flight values, γ_s, α_s, and T_s. The equations of motion at constant speed in section 3.2 apply, and it follows from (11) that

$$\frac{R_s^2}{W^2} = 1 + \frac{D^2}{W^2} + \frac{2D}{W} \sin \gamma_s \, . \qquad (12)$$

Also if it is assumed that $V = V_s$ and $T = T_s$ during the transition to steady climb, then as in (11)

$$\frac{V_s^2}{g} \frac{d\gamma}{dx} \cos \gamma = \left\{\frac{R_s^2}{W^2} - \left(\frac{D}{W} + \sin \gamma\right)^2\right\}^{1/2} - \cos \gamma \, ,$$

and so from (12),

$$\frac{V_s^2}{g} \frac{d\gamma}{dx} = \left\{1 + \frac{2D}{W \cos^2 \gamma}\left(\sin \gamma_s - \sin \gamma\right)\right\}^{1/2} - 1 \, .$$

Since D/W is small and $\sin \gamma$ approaches $\sin \gamma_s$, it is a sufficient approximation to expand in series form retaining the first-power term only; then integrating,

$$\frac{D}{W} \cdot \frac{gx}{V^2} = L(\gamma, \gamma_s) - L(\gamma_0, \gamma_s) \, .$$

$L(\gamma, \gamma_s) = \sin \gamma_s . \gamma - \cos \gamma + \cos \gamma_s \log \dfrac{\cot \frac{1}{2}\gamma_s - \tan \frac{1}{2}\gamma}{\tan \frac{1}{2}\gamma_s - \tan \frac{1}{2}\gamma}$, is plotted in Fig. 10 against γ for a range of values of γ_s.

Similarly

$$\frac{D}{W} \frac{gy}{V^2} = N(\gamma, \gamma_s) - N(\gamma_0, \gamma_s) \, .$$

$N(\gamma, \gamma_s) = \left\{ \sin \gamma_s \log \left(\dfrac{1}{\sin \gamma_s - \sin \gamma}\right) - \sin \gamma \right\}$ is plotted in Fig. 11 against γ for a range of values of γ_s.

4. *Application of the Theory.*—For convenience in application, the methods of estimating the performance for the various aspects of take-off are now briefly summarised and discussed. It is assumed in the analysis that the take-off can be considered in stages in which either the disc attitude to the horizontal or the flight velocity is constant; in addition, when motion over a range of speed, flight path angle, or height near the ground, is being considered, constant mean values are taken for the rotor thrust T and the drag forces, H and D. These forces can be estimated in the way outlined

in section 2. Estimations of the performance in a stage of take-off starts from known initial conditions, which include the flight path angle, γ_0, and the speed V_0; if the motion is at constant speed, the disc attitude must be varied in a way to keep the resultant force along the flight path zero.

The first type of motion considered at constant disc attitude is the ground run, for which $(T \cos \alpha + H \sin \alpha) < W$; the ground distance to accelerate from speed V_0 to V is given by equation (10). If $(T \cos \alpha + H \sin \alpha) > W$, the helicopter will become airborne; the solution of the motion then depends on A, B and f, which can be estimated from the relations in (6). If it is found that $A = \tan \gamma_0$ then the flight path is a straight line and the special solutions in section 3.1.1 apply; equations (8) give the distances for climbing flight and (9) the distance for horizontal flight ($\gamma_0 = 0$). These solutions also apply to motion from rest, that is for $V_0 = 0$.

If $V_0 > 0$ and $A \neq \tan \gamma_0$, the general solutions in section 3.1.2, for motion at constant disc attitude, apply. These solutions depend on S and Q/S which are defined as follows:

$$S = \frac{V_0^2}{2fG(\gamma_0, A, B)}$$

where $G(\gamma_0, A, B)$ is the function charted in Fig. 5,

and $$\frac{Q}{S} = \frac{V^2}{V_0^2} G(\gamma_0, A, B) .$$

The horizontal and vertical distances x, y along the flight path in accelerating from speed V_0 at flight path angle γ_0 to speed V, are given by

$$\frac{x}{S} = \mathscr{X}\left(\frac{Q}{S}, A, B\right) - \mathscr{X}\left(\frac{Q_0}{S}, A, B\right)$$

$$\frac{y}{S} = \mathscr{Y}\left(\frac{Q}{S}, A, B\right) - \mathscr{Y}\left(\frac{Q_0}{S}, A, B\right).$$

$\mathscr{X}\left(\frac{Q}{S}, A, B\right)$ and $\mathscr{Y}\left(\frac{Q}{S}, A, B\right)$ are given in Figs. 6 and 7.

The flight path angle at speed V can be obtained from Fig. 5. In analysing take-off performance it can normally be assumed that the type of motion represented by this solution obtains only for the limited period of transition to climb away and that thereafter the pilot adjusts the disc attitude to give either a straight path or a constant speed climb, so that the simpler special solutions may be used.

For motion at constant speed, the solutions for x and y are given in terms of γ, D/W and R_F/W, where $R_F^2 = T^2 + H^2$. The horizontal and vertical distances from a flight path angle γ_0 to γ are given by,

$$\frac{gx}{V^2} = J\left(\gamma, \frac{R_F}{W}, \frac{D}{W}\right) - J\left(\gamma_0, \frac{R_F}{W}, \frac{D}{W}\right)$$

$$\frac{gy}{V^2} = K\left(\gamma, \frac{R_F}{W}, \frac{D}{W}\right) - K\left(\gamma_0, \frac{R_F}{W}, \frac{D}{W}\right).$$

$J(\gamma, R_F/W, D/W)$ is given in Fig. 8 and $K(\gamma, R_F/W, D/W)$ in Fig. 9 for a range of values of R_F/W and $D/W = 0{\cdot}01$.

For climb away at constant speed approaching a steady climb condition, the steady flight values γ_s, α_s and T_s may be found from steady-flight momentum theory. The horizontal and vertical distances in the transition, from a flight path angle γ_0 to an angle γ, are given by

$$\frac{D}{W}\frac{gx}{V^2} = L(\gamma, \gamma_s) - L(\gamma_0, \gamma_s)$$

$$\frac{D}{W}\frac{gy}{V^2} = N(\gamma, \gamma_s) - N(\gamma_0, \gamma_s).$$

$L(\gamma, \gamma_s)$ is given in Fig. 10 and $N(\gamma, \gamma_s)$ in Fig. 11 for a range of values of γ_s.

For motion at constant disc attitude, the flight path distances are determined in terms of x/S and y/S ; S is proportional to V_0^2 and so the range of, for example, y/S to cover a given change of height is greater for lower initial speed. To provide completely for estimates from very low initial speeds requires a very wide range of the variables, and for equal accuracy in all conditions several charts would be required covering progressively larger ranges of x/S and y/S. The charts given however, cover a representative range of speeds down to fairly low values only, because it is considered that the overall take-off performance can in general be adequately defined by the performance from the initial speeds covered, together with that from rest, which is given by the straight path solutions in (8) and (9) ; the charts can however be extended to deal with lower initial speeds if required. The solutions for the motion at constant speed are given in terms of gx/V^2 and gy/V^2 and again a number of charts would be required for comparable accuracy at all speeds. Normally, however, climb away is not made at a very low constant speed and a more limited range of the variables should be adequate for most purposes.

The charts given for the different aspects of take-off are given for positive values of γ only and therefore supply solutions only for cases of take-off in which there is no loss of height. The theory can also be used however for analysis of motions in which $\gamma < 0$, including general decelerated landing motions. Extension of the theory to other types of helicopter, including those with multi-rotors or with auxiliary fixed wings, will be considered in connection with the general development of performance theory for these types of aircraft.

Attention is now being given to the development of reduction methods for the take-off performance of a single-rotor helicopter on the basis of the theory in this report.

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REFERENCES

No.	*Author*	*Title, etc.*
1	H. B. Squire	The flight of a helicopter. R. & M. No. 1730. 1936.
2	F. O'Hara	A method of performance reduction for helicopters. R. & M. No. 2770. October, 1947.
3	A. L. Oliver	The low-speed performance of a helicopter. C.P. 122. May, 1952.
4	J. K. Zbrozek	Ground effect on the lifting rotor. R. & M. No. 2347. July, 1937.

LIST OF SYMBOLS

a Slope of blade lift-coefficient curve

$$a_1 = \left(\frac{R_F^2}{W^2} - \frac{D^2}{W^2} - 1\right)$$

$$a_2 = \left(\frac{R_F^2}{W^2} + \frac{D^2}{W^2} - 1\right)$$

$$A = \frac{\frac{T_m}{W}\cos\alpha_m + \frac{H_m}{W}\sin\alpha_m - 1}{\frac{T_m}{W}\sin\alpha_m - \frac{H_m}{W}\cos\alpha_m}$$

b Number of blades

$$b_1 = 2\frac{D}{W}$$

$$b_2 = 2\frac{D}{W}\frac{R_F}{W}$$

$$B = 1 - \frac{D_m/W}{\frac{T_m}{W}\sin\alpha_m - \frac{H_m}{W}\cos\alpha_m}$$

c Rotor blade chord

C_D Blade profile-drag coefficient at the mean effective lift coefficient

D Body drag

D_m Mean body drag

D_0 Body drag at 100 ft/sec at sea level

E Ratio of effective power at rotor to total power

$$f = g\left[\frac{T_m}{W}\sin\alpha_m - \frac{H_m}{W}\cos\alpha_m\right]$$

H Transverse force on rotor

H_m Mean transverse force on rotor

i Rotor disc incidence to the flight path

P Engine power

P_R Power required for rotor torque due to profile drag

$$q = \frac{dQ}{dx}$$

q_0 Initial value of q

$$Q = \frac{V^2}{2f}$$

Q_0		Initial value of Q
R		Rotor radius
R_F		Resultant rotor force
R_s		Resultant rotor force in steady climb
s		Distance along the flight path
S	$=$	$\dfrac{Q_0}{G(\gamma, A, B)}$, $G(\gamma, A, B)$ being defined in (11)
T		Rotor thrust
T_m		Mean value of rotor thrust
T_s		Rotor thrust in steady climb
u		Airflow velocity normal to the rotor
v		Induced velocity at the rotor
v_0	$=$	$\left(\dfrac{W}{2\pi\rho_0 R^2}\right)^{1/2}$
V		Flight speed
V_0		Initial flight speed
V_i	$=$	$V\sqrt{\sigma}$
V_s		Speed in steady climb
x, y		Co-ordinates relative to ground axes, x horizontal forwards, y vertical upwards
W		Helicopter weight
α		Rotor disc attitude to the horizontal, positive downwards
α_m		Mean value of α
α_s		Value of α in steady climb
γ		Flight path angle to the horizontal, positive upwards
γ_0		Initial value of γ
γ_s		Value of γ in steady climb
ε	$=$	$\arcsin\left[\dfrac{D/W + \sin\gamma}{R_F/W}\right]$
θ		Mean blade angle
μ	$=$	$\dfrac{V\cos i}{\Omega R}$
ρ		Air density
σ		Relative air density
Ω		Rotor speed

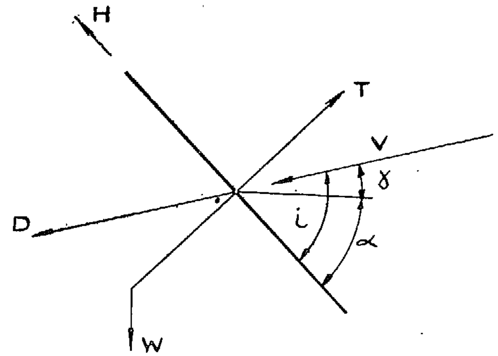

FIG. 1. Forces on helicopter.

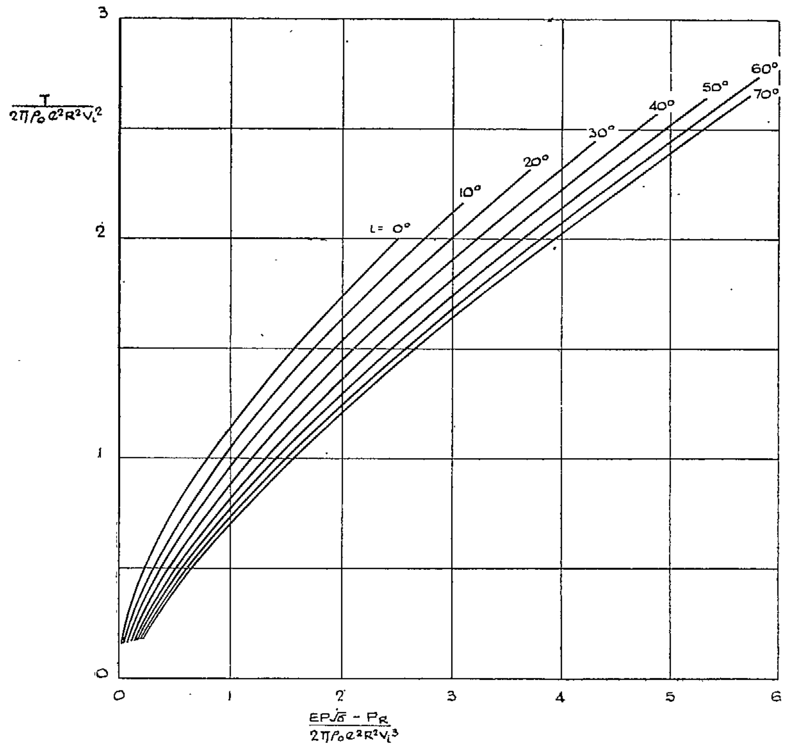

FIG. 2. Rotor thrust.

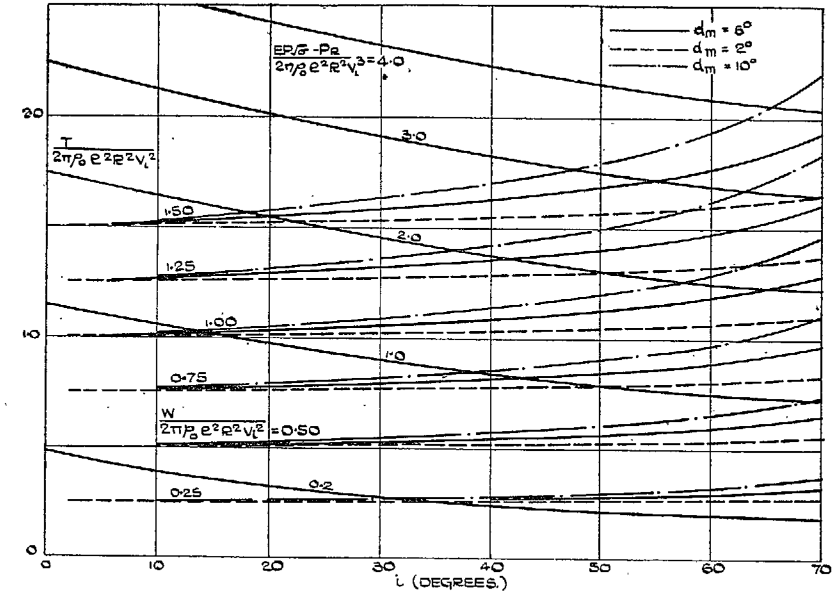

FIG. 3. Rotor thrust for straight flight path.

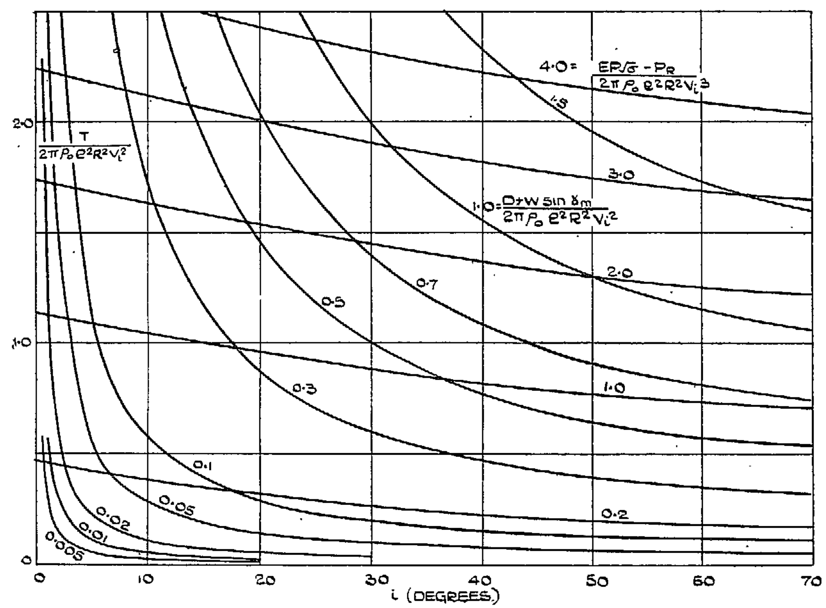

FIG. 4. Rotor thrust for constant speed.

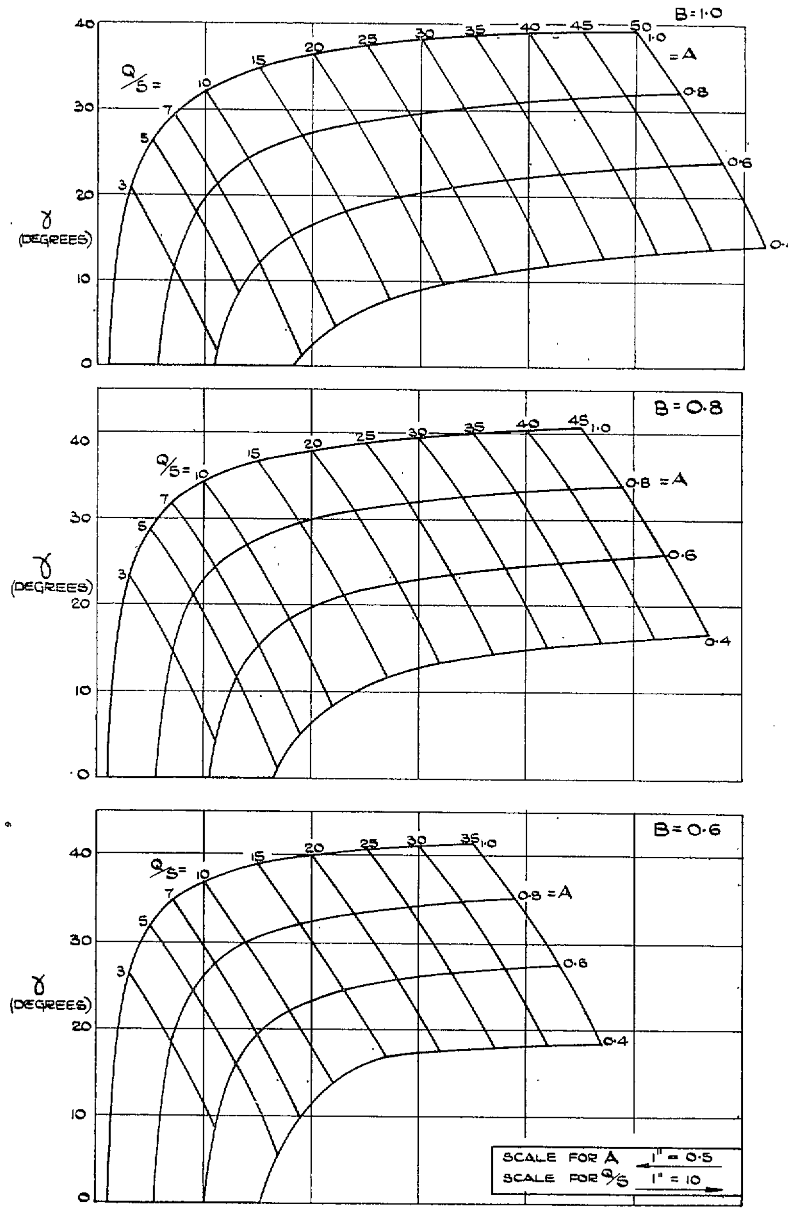

FIG. 5. $Q/S = G(\gamma, A, B)$.

Note. For scale 1″ read 1 division.

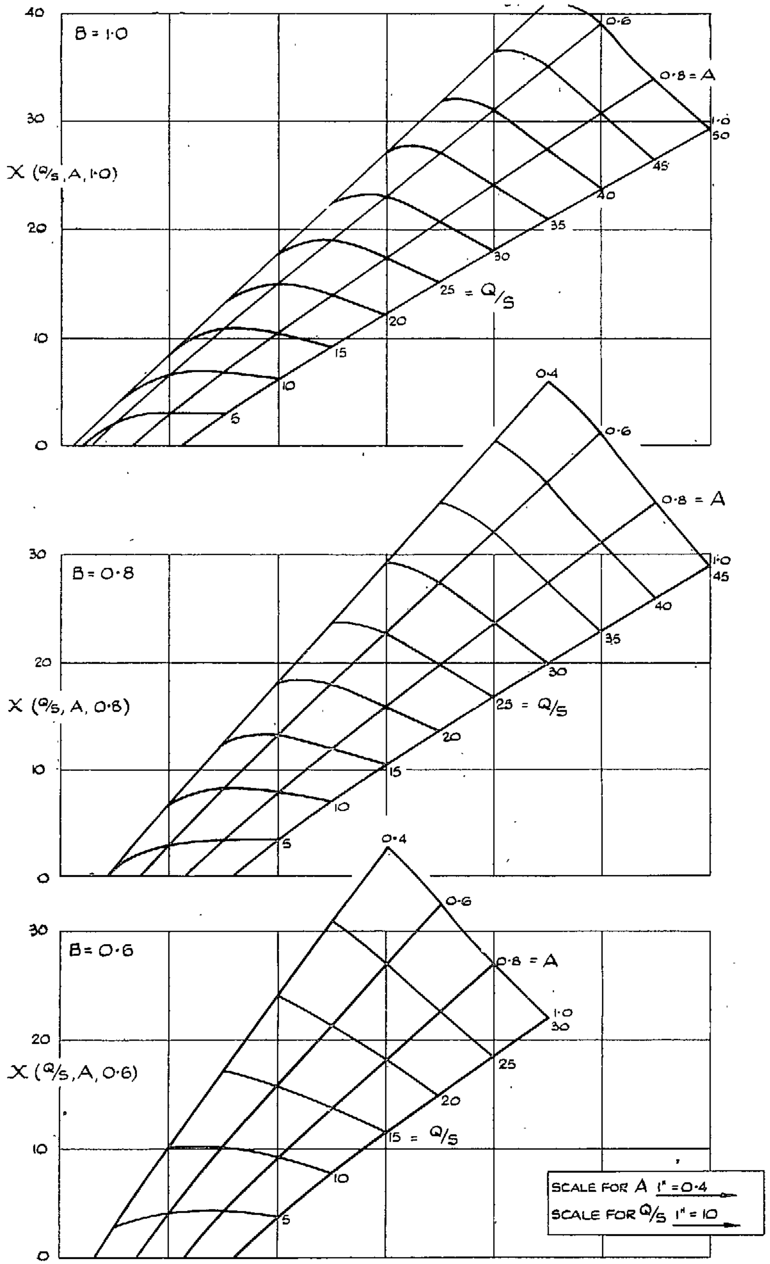

FIG. 6. $\mathscr{X}(Q/S, A, B)$.

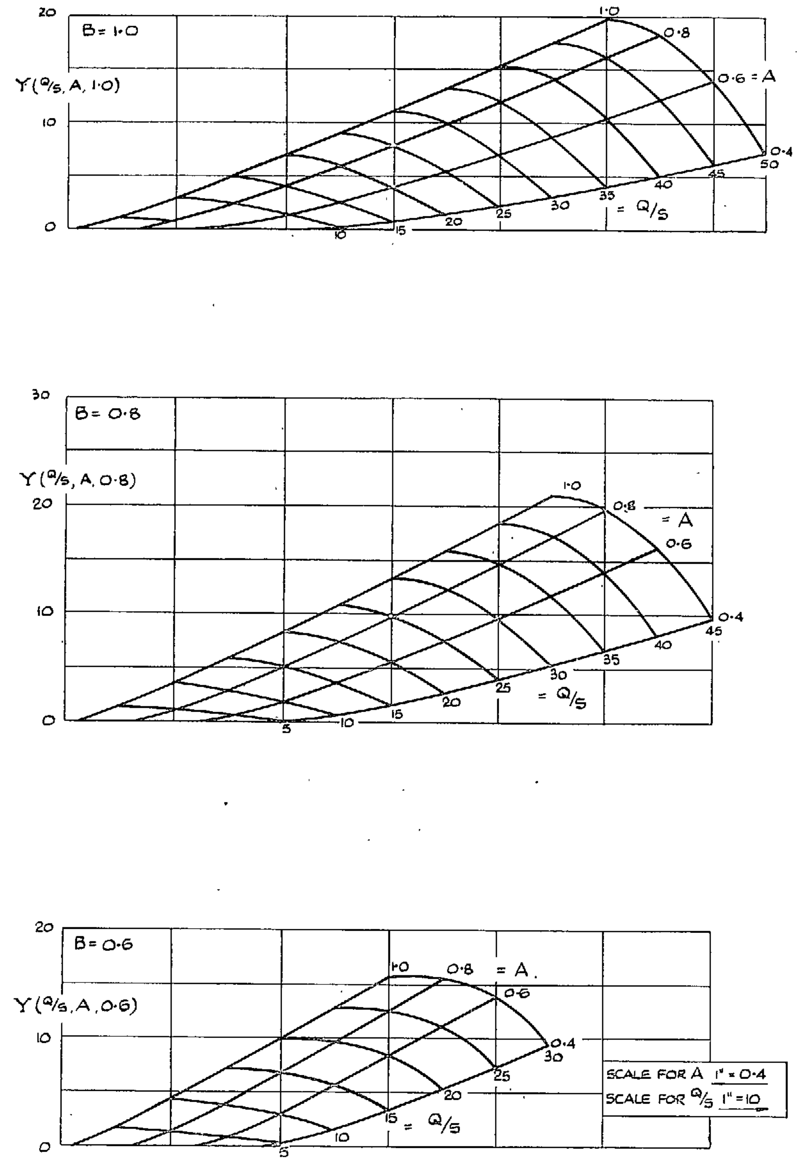

FIG. 7. $\mathscr{Y}(Q/S, A, B)$.

Note. For scale 1″ read 1 division.

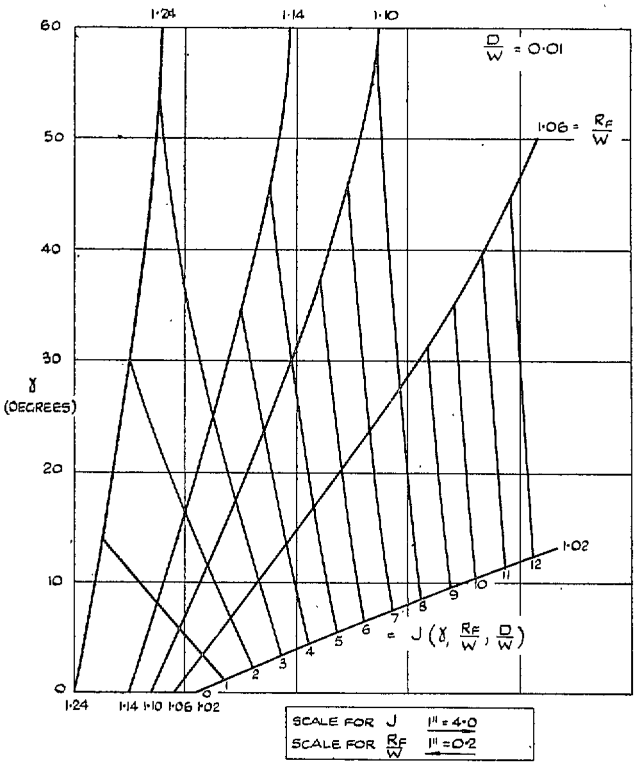

FIG. 8. $J(\gamma, R_F/W, D/W)$.

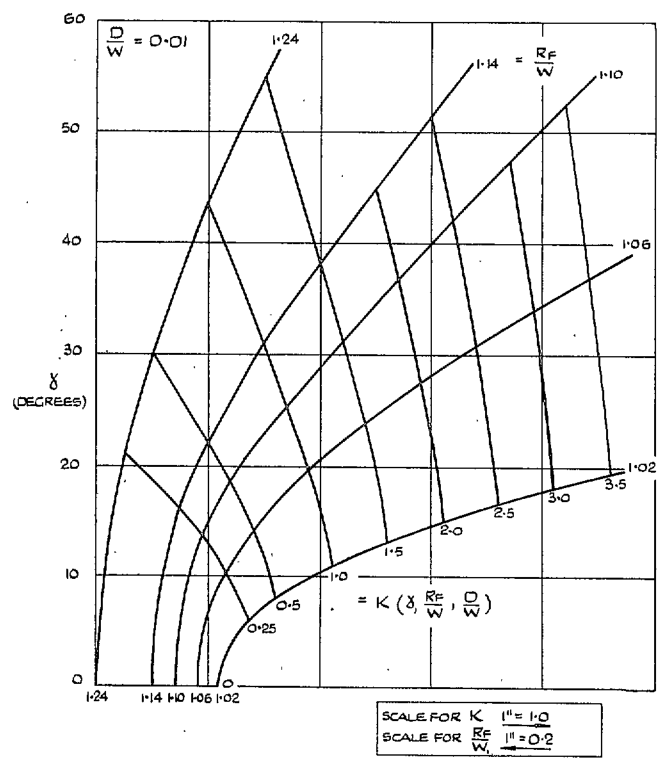

FIG. 9. $K(\gamma, R_F/W, D/W)$.

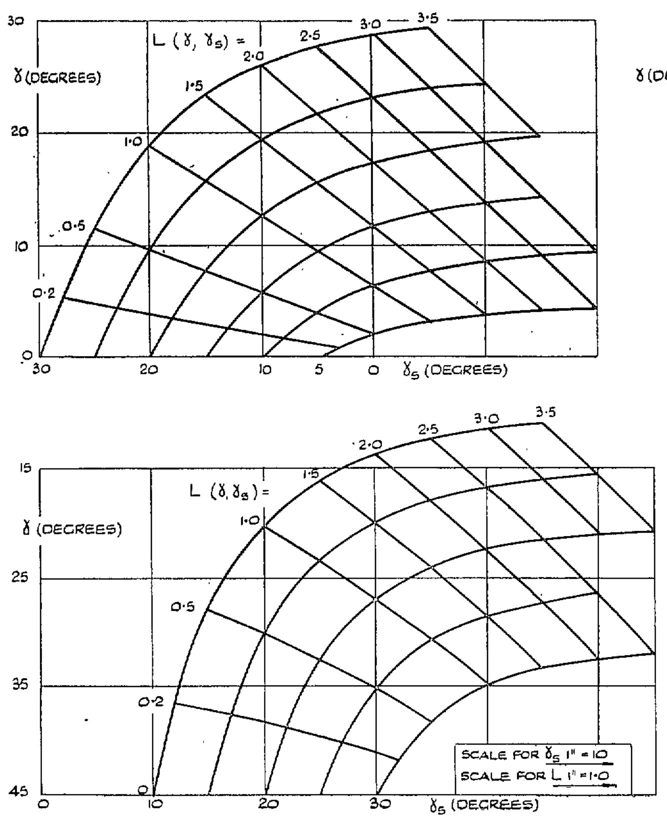

FIG. 10. $L(\gamma, \gamma_s)$.

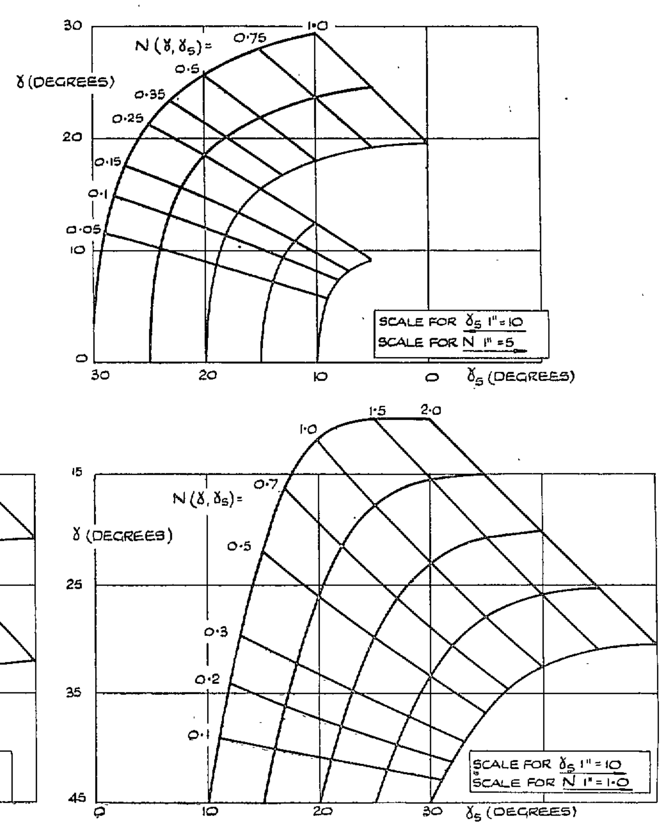

FIG. 11. $N(\gamma, \gamma_s)$.

Note. For scale 1" read 1 division.

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