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# Some Remarks on Multhopp's Subsonic Lifting-Surface Theory

*By*

K. W. MANGLER, Dr.sc.nat., A.F.R.Ae.S.,
and
B. F. R. SPENCER, B.Sc.



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# Some Remarks on Multhopp's Lifting-Surface Theory


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*Summary.*—This note contains an alternative method to that proposed by Multhopp[1] in his subsonic lifting-surface theory for dealing with the spanwise integration of the downwash.

The method consists of arranging the series form of the influence function near the inducing section so that the logarithmic term may be integrated, instead of introducing an artificial correction function as Multhopp does.

Multhopp's scheme for the solution of the system of linear equations is retained, and there is no increase in the amount of computor work involved.


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1. *Introduction.*—In Multhopp's method[1] for calculating the lift distribution of a wing at subsonic speeds the form of the chordwise lift distribution is assumed and then interpolation functions of trigonometrical form are used in setting up the downwash equations, which determine the spanwise distribution of the load and the local aerodynamic centre. Multhopp himself pointed out that the integrand for this spanwise integration contains a logarithmic singularity, so that a direct application of the trigonometrical series becomes questionable. He therefore introduced a correction term allowing for this singularity. In the present note, an alternative method is suggested which allows for the integration of this function with a logarithmic singularity in a more straightforward manner.

2. *The Spanwise Integration in Multhopp's Downwash Equation.*—Written in non-dimensional quantities, the downwash equation which relates the loading $l$ of the lifting surface to the incidence $\alpha$ (geometrical incidence plus camber and twist) reads:

$$\alpha(\xi,\eta) = -\frac{1}{2\pi}\int_{-1}^{+1}\frac{\gamma(\eta')i(\xi,\eta,\eta')}{(\eta-\eta')^2}\,d\eta' - \frac{1}{2\pi}\int_{-1}^{+1}\frac{\mu(\eta')j(\xi,\eta,\eta')}{(\eta-\eta')^2}\,d\eta' \quad \ldots \quad \ldots \quad (1)$$

(compare Multhopp[1] equation (60)). Here the 'principal value' of the integral must be taken at $\eta' = \eta$. The functions

$$\gamma = \frac{C_L c}{2b} \text{ and } \mu = \frac{C_m c}{2b} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

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* R.A.E. Tech. Note Aero. 2181, received 30th January, 1953.

denote the non-dimensional lift per unit span (circulation) and the pitching moment per unit span respectively and

$$\xi = \frac{2x}{b} \quad \eta = \frac{2y}{b}$$

are non-dimensional co-ordinates in the chordwise and spanwise directions, referred to the halfspan $b/2$ ($c$ = local chord). The influence functions $i$ and $j$ which depend on the assumptions on the form of the chordwise load distribution (the first producing lift but no moment about the quarter-chord point, the latter no lift, but only a moment) have been tabulated by Multhopp as functions of $\xi$ and $\eta' - \eta$ .

Near the point $\eta' = \eta$ the functions $i$ and $j$ may be written as (compare Multhopp[1], equation (75)):

$$\left.\begin{aligned} i(\xi,\eta,\eta') &= K_1 \left(\frac{b}{2c_v}\right)^2 (\eta - \eta')^2 \log |\eta - \eta'| + i^*(\xi,\eta,\eta') \\ j(\xi,\eta,\eta') &= \bar{K}_1 \left(\frac{b}{2c_v}\right)^2 (\eta - \eta')^2 \log |\eta - \eta'| + j^*(\xi,\eta,\eta') \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3)$$

where $i^*$ and $j^*$ and their first and second derivatives with respect to $\eta'$ are regular and bounded for all values of $\eta' - \eta$ so that these terms can be expanded as a Fourier polynomial. $K_1$ and $\bar{K}_1$ depend on the choice of $\xi$ and thus are constant for the purpose of integration.

The inclusion of an arbitrary constant in the logarithmic term does not improve the approximation, since its contribution to the downwash integral is zero, as will be indicated later on (equation (14)).

Now we introduce

$$\eta' = \cos\vartheta' , \quad \eta = \cos\vartheta$$

and write the identity

$$\gamma(\eta') \,.\, i = \gamma(\eta) \frac{\sin\vartheta'}{\sin\vartheta} (i - i^*) + \left(\gamma(\eta') - \gamma(\eta) \frac{\sin\vartheta'}{\sin\vartheta}\right) i + \gamma(\eta) \frac{\sin\vartheta'}{\sin\vartheta} i^* \quad \ldots \quad (4)$$

and a similar expression for $\mu \,.\, j$ . Here the third term and its first and second derivatives are bounded, since this holds for $i^*$. In the second term the logarithmic singularity, contained in $i$ , is cancelled by the zero of $\gamma(\eta') - \gamma(\eta)\frac{\sin\vartheta'}{\sin\vartheta}$. Thus Multhopp's method for evaluating the spanwise integrals in (1) can easily be applied to the second and third term in (4). For the first term we obtain, using (3) for $i - i^*$:

$$\alpha_1 = -\frac{\gamma(\eta)\, K_1}{8\sin\vartheta} \left(\frac{b}{2c_v}\right)^2 \frac{4}{\pi} \int_{-1}^{+1} \sin\vartheta' \log|\eta' - \eta| \, d\eta' . \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The integral is evaluated in Appendix I and we have

$$\alpha_1 = -\frac{\gamma(\eta)\, K_1}{8\sin\vartheta} \left(\frac{b}{2c_v}\right)^2 (\cos 2\vartheta - \log 4) . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

For the contribution of the second and third terms in (4) to the integral (1) we may write

$$\alpha_2 = -\frac{1}{2\pi}\int_{-1}^{+1}\left[\gamma(\eta')i - \gamma(\eta)\frac{\sin\vartheta'}{\sin\vartheta}K_1\left(\frac{b}{2c_\nu}\right)^2(\eta-\eta')^2\log|\eta-\eta'|\right]\frac{d\eta'}{(\eta-\eta')^2} \quad \cdot\cdot \quad (7)$$

If we use a trigonometrical interpolation formula for the bracket at the $m$ stations ($n = 1, \ldots m$)

$$\eta' = \eta_n' = \cos\vartheta_n' = \cos\left(\frac{\pi}{2} - \frac{n\pi}{m+1}\right) = \sin\frac{n\pi}{m+1} \quad \cdot\cdot \quad \cdot\cdot \quad (8)$$

and calculate $\alpha_2$ for the same set ($\nu = 1, \ldots m$)

$$\eta = \eta_\nu = \sin\frac{\nu\pi}{m+1} \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad (9)$$

of spanwise stations, we have, according to Multhopp[1] (equation (71)):

$$\alpha_{2\nu} = b_{\nu\nu}\gamma_\nu i_{\nu\nu} - \sum_{n=-\frac{(m-1)}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n}\left[\gamma_n i_{\nu n} - \gamma_\nu\frac{\sin\vartheta_n}{\sin\vartheta_\nu}K_1\left(\frac{b}{2c_\nu}\right)^2(\eta_\nu-\eta_n)^2\log|\eta_\nu-\eta_n|\right] \quad \cdot\cdot \quad (10)$$

where the coefficients $b_{\nu\nu}$ and $b_{\nu n}$ are given by Multhopp[1].

When adding the contribution $\alpha_1$ and omitting the terms dependent on $\mu$ for the time being, we obtain the following system of linear equations, which replaces the integral equation (1):

$$\alpha_\nu = \gamma_\nu\left[b_{\nu\nu}i_{\nu\nu} + \frac{K_1(\log 4 - \cos 2\vartheta_\nu)}{8\sin\vartheta_\nu}\left(\frac{b}{2c_\nu}\right)^2 + \sum_n{}' \, b_{\nu n}\frac{\sin\vartheta_n}{\sin\vartheta_\nu}K_1\left(\frac{b}{2c_\nu}\right)^2(\eta_\nu-\eta_n)^2\log|\eta_\nu-\eta_n|\right] - \sum_n{}' \, b_{\nu n}\gamma_n i_{\nu n}$$

or

$$\alpha_\nu = \gamma_\nu b_{\nu\nu}\bar{i}_{\nu\nu} - \sum_{n=-\frac{(m-1)}{2}}^{\frac{(m-1)}{2}}{}' \, b_{\nu n}i_{\nu n}\gamma_n \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad (12)$$

with

$$\bar{i}_{\nu\nu} = i_{\nu\nu} + \left(\frac{b}{2c_\nu}\right)^2\frac{K_1}{b_{\nu\nu}}\left[\frac{(\log 4 - \cos 2\vartheta_\nu)}{8\sin\vartheta_\nu} + \sum_n{}' \, b_{\nu n}\frac{\sin\vartheta_n}{\sin\vartheta_\nu}(\eta_\nu-\eta_n)^2\log|\eta_\nu-\eta_n|\right]. \quad (13)$$

Substituting the values of $b_{\nu\nu}$ and $b_{\nu n}$ as given by Multhopp, and simplifying, we have,

$$\bar{i}_{\nu\nu} = i_{\nu\nu} + 4K_1\left(\frac{b}{2c_\nu}\right)^2\frac{1}{(m+1)^2}\left[\sum_n{}' \sin^2\vartheta_n\log|\eta_\nu-\eta_n| - \frac{m+1}{8}(\cos 2\vartheta_\nu - \log 4)\right]. \quad (14)$$

If an arbitrary constant is included in the logarithmic term we find that its coefficient in equation (14) becomes zero.

When using two terms for the chordwise lift distribution we obtain a similar expression for $\bar{\jmath}_{\nu\nu}$, where $K_1$ is replaced by the corresponding value for the second term. Thus Multhopp's scheme for the solution of the linear equations can be retained, the only difference being the new expressions for the 'diagonal' terms $\bar{\imath}_{\nu\nu}$ and $\bar{\jmath}_{\nu\nu}$ respectively.

3. *Numerical Values.*—Multhopp's equations (86) are now replaced by a similar set based on equation (14). The values for the cases of one and two chordwise pivotal stations are as follows:—

(*a*) *One chordwise pivotal station* 0·75*c*

$$\bar{\imath}_{\nu\nu} = 1\cdot8847 + 3\cdot9205\left(\frac{b}{2c_\nu}\right)^2 F(\vartheta)$$

(*b*) *Two chordwise pivotal stations* 0·9045*c* and 0·3455*c*

0·9045*c*

$$\bar{\imath}_{\nu\nu}' = 1\cdot9742 + 4\cdot7894\left(\frac{b}{2c_\nu}\right)^2 F(\vartheta)$$

$$\bar{\jmath}_{\nu\nu}' = 0\cdot2859 - 36\cdot9168\left(\frac{b}{2c_\nu}\right)^2 F(\vartheta)$$

0·3455*c*

$$\bar{\imath}_{\nu\nu}'' = 1\cdot4055 + 7\cdot74996\left(\frac{b}{2c_\nu}\right)^2 F(\vartheta)$$

$$\bar{\jmath}_{\nu\nu}'' = 3\cdot1702 + 44\cdot2381\left(\frac{b}{2c_\nu}\right)^2 F(\vartheta)$$

.. .. .. .. (15)

where the dashes have the same significance as in Multhopp's report, and

$$F(\vartheta) = \frac{1}{(m+1)^2}\left[\sum_n{}' \sin^2\vartheta_n \log|\eta_\nu - \eta_n| - \frac{m+1}{8}(\cos 2\vartheta_\nu - \log 4)\right]. \qquad .. \qquad (16)$$

$F(\vartheta)$ has been evaluated for the most used cases of 7 and 15 spanwise stations, the results being given in the following Tables 1 and 2.

TABLE 1

$m = 7$

| $\nu$ | ±3 | ±2 | ±1 | 0 |
|---|---|---|---|---|
| $F(\vartheta)$ | 0·00125 | 0·00542 | 0·00958 | 0·01130 |

TABLE 2

$m = 15$

| $\nu$ | $\pm 7$ | $\pm 6$ | $\pm 5$ | $\pm 4$ | $\pm 3$ | $\pm 2$ | $\pm 1$ | 0 |
|---|---|---|---|---|---|---|---|---|
| $F(\vartheta)$ | 0·000078 | 0·000375 | 0·000825 | 0·00135 | 0·00188 | 0·00233 | 0·00263 | 0·002735 |

To get some idea of the differences involved in using the new correction, the values of the coefficient of $(b/2c_\nu)^2$ in the $\bar{\imath}_{\nu\nu}'$ term in the two cases have been compared, as follows:

| | | *Multhopp Method* | *Revised Method* |
|---|---|---|---|
| $m = 7$ | | | |
| | $\nu = \pm 3$ Coeff. of $\left(\frac{b}{2c_\nu}\right)^2$ in $\bar{\imath}_{\nu\nu}'$ term | 0·0087 | 0·0060 |
| | $\nu = 0$ ,, ,, ,, ,, | 0·0596 | 0·0541 |
| $m = 15$ | | | |
| | $\nu = \pm 7$ Coeff. of $\left(\frac{b}{2c_\nu}\right)^2$ in $\bar{\imath}_{\nu\nu}'$ term | 0·00058 | 0·00037 |
| | $\nu = 0$ ,, ,, ,, ,, | 0·0152 | 0·0131 |

With corresponding differences in the $\bar{\jmath}_{\nu\nu}'$, $\bar{\imath}_{\nu\nu}''$ and $\bar{\jmath}_{\nu\nu}''$ terms. These show the percentage differences to be considerably greater at the stations nearest the wing tips than at the central wing section.

4. *Examples.*—(i) The following tables show the load distribution on a delta wing (60-deg leading-edge sweep) of aspect ratio 2·31 at unit incidence as obtained by using

(*a*) Multhopp's correction term

(*b*) New correction term.

| (*a*) $\nu$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | 0·7006 | 0·6827 | 0·6326 | 0·5540 | 0·4524 | 0·3348 | 0·2119 | 0·0929 |
| $\mu$ | $\bar{0}$·0719 | $\bar{0}$·0416 | $\bar{0}$·0231 | $\bar{0}$·0117 | $\bar{0}$·0011 | 0·0081 | 0·0106 | 0·0015 |
| $X_{\text{A.C.}}$ | 0·3737 | 0·3109 | 0·2865 | 0·2711 | 0·2524 | 0·2258 | 0·2000 | 0·2339 |

| (*b*) $\nu$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | 0·7045 | 0·6867 | 0·6367 | 0·5588 | 0·4574 | 0·3403 | 0·2184 | 0·1044 |
| $\mu$ | $\bar{0}$·0731 | $\bar{0}$·0424 | $\bar{0}$·0237 | $\bar{0}$·0121 | $\bar{0}$·0012 | 0·0087 | 0·0118 | 0·0040 |
| $X_{\text{A.C.}}$ | 0·3747 | 0·3117 | 0·2872 | 0·2716 | 0·2526 | 0·2244 | 0·1960 | 0·2117 |

Whilst we have seen that the values of the $\bar{i}_{\nu\nu}$ and $\bar{j}_{\nu\nu}$ terms obtained by the two methods differed considerably, their effect on the final lift and $X_{A.C.}$ distributions is very small. It is even smaller on the overall lift slope and aerodynamic centre. We find

$$\frac{d\bar{C}_L}{d\alpha} = 2\cdot422 \text{ (Multhopp)} = 2\cdot445 \text{ (new)}$$

$$-\frac{\partial\bar{C}_M}{\partial\bar{C}_L} = 1\cdot178 \text{ (Multhopp)} = 1\cdot181 \text{ (new)}$$

(measured from apex in terms of mean chord).

Using a different approach Mr. H. C. Garner of the National Physical Laboratory, Teddington also tried to improve on Multhopp's original treatment of the logarithmic singularity in the spanwise integration. His results are in complete agreement with our equations (15) (Tables 1 and 2). He also calculated the load on two plan-forms using Multhopp's and the new coefficients. The results are given here with his permission:

(ii) *Arrowhead wing* ($A = 6$, taper ratio $\lambda = 0$, sweepback of quarter-chord line = 45 deg) 15-point solution:

(*a*)

| $\nu$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $\gamma$ | 0·4087 | 0·3715 | 0·2437 | 0·0866 |
| $\mu$ | −0·0346 | 0·0041 | 0·0068 | −0·0003 |
| $X_{A.C.}$ | 0·3806 | 0·2390 | 0·2221 | 0·2535 |

(*b*)

| $\nu$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $\gamma$ | 0·4155 | 0·3790 | 0·2522 | 0·0966 |
| $\mu$ | −0·0358 | 0·0048 | 0·0082 | 0·0003 |
| $X_{A.C.}$ | 0·3822 | 0·2373 | 0·2175 | 0·2469 |

$$\frac{d\bar{C}_L}{d\alpha} = 3\cdot552 \text{ (Multhopp)} \quad 3\cdot648 \text{ (new)}$$

$$-\frac{\partial\bar{C}_M}{\partial\bar{C}_L} = 1\cdot702 \text{ (Multhopp)} \quad 1\cdot711 \text{ (new)}$$

(iii) *Cropped Delta wing* ($A = 3$, taper ratio $\lambda = 1/7$, Leading-edge sweep 45 deg) 7-point solution:

(*a*)

| $\nu$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\gamma$ | 0·6600 | 0·6025 | 0·4539 | 0·2505 |
| $\mu$ | −0·0294 | 0·0068 | 0·0137 | 0·0159 |
| $X_{A.C.}$ | 0·3331 | 0·2387 | 0·2198 | 0·1865 |

| (b) $\nu$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\gamma$ | 0·6683 | 0·6111 | 0·4635 | 0·2580 |
| $\mu$ | −0·0300 | 0·0076 | 0·0158 | 0·0210 |
| $X_{\text{A.C.}}$ | 0·3334 | 0·2376 | 0·2159 | 0·1686 |

$$\frac{d\bar{C}_L}{d\alpha} = 3{\cdot}071 \quad \text{(Multhopp)} \quad 3{\cdot}122 \quad \text{(new)}$$

$$-\frac{\partial \bar{C}_M}{\partial \bar{C}_L} = 0{\cdot}9180 \text{ (Multhopp)} \quad 0{\cdot}9177 \text{ (new)}\,.$$

In all these examples the difference between the old and the new results is fairly small. The $\gamma$-values and the values for the lift slope are more affected than the $\mu$-values and the a.c.-position. Since the functions $F(\vartheta)$ in (15) are multiplied by $(b/c_\nu)^2$ the effect of the new integration method will be most noticeable for wings of a fairly large aspect ratio and for wings with a small taper ratio $\lambda$ (pointed tips). In the latter case the load near the wing tips will be most affected. This general trend is confirmed by the examples. The delta wing of aspect ratio 2·31 shows the smallest difference between the old and the new values, whereas the results for the arrowhead wing ($A = 6$) differ most.

5. *Conclusion.*—The method of correcting for the logarithmic singularity in the spanwise integration as suggested by this note entails solely the modification of Multhopp's equations (86) as shown in equations (15) (in conjunction with Tables 1 and 2).

While for most practical plan-forms the results for the $\gamma$- and $\mu$- distributions as obtained by the present method apparently do not differ very much from the results obtained by Multhopp's original method, it is felt that the use of the new correction is justified on account of its sounder basis and the fact that it does not involve any departure from Multhopp's procedure[1] or any additional work.

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## APPENDIX I

*Evaluation of an Integral*

We have to evaluate the integral in equation (5), namely

$$h(\vartheta) = \frac{4}{\pi}\int_{-1}^{+1} \sin\vartheta' \log|\eta' - \eta|\, d\eta' .$$

Substituting the values

$$\eta = \cos\vartheta\ , \quad \eta' = \cos\vartheta'\ , \quad d\eta' = -\sin\vartheta'\, d\vartheta'$$

we have

$$h(\vartheta) = \frac{4}{\pi}\int_{0}^{\pi} \sin^2\vartheta' \log|\cos\vartheta' - \cos\vartheta|\, d\vartheta' . \qquad \text{(I.1)}$$

Next we differentiate equation (I.1) with respect to $\vartheta$, obtaining

$$\frac{dh}{d\vartheta} = \frac{4}{\pi}\int_{0}^{\pi} \frac{\sin^2\vartheta' \sin\vartheta}{\cos\vartheta' - \cos\vartheta} d\vartheta'$$

which becomes

$$\frac{dh}{d\vartheta} = \frac{2\sin\vartheta}{\pi}\int_{0}^{\pi} \frac{(1 - \cos 2\vartheta')}{(\cos\vartheta' - \cos\vartheta)} d\vartheta' . \qquad \text{(I.2)}$$

According to Glauert[2] we have for $n = 0, 1, 2 \ldots$

$$\int_{0}^{\pi} \frac{\cos n\vartheta'}{\cos\vartheta' - \cos\vartheta} d\vartheta' = + \frac{\pi \sin n\vartheta}{\sin\vartheta} .$$

Substituting these values in equation (I.2) we obtain

$$\frac{dh}{d\vartheta} = -2\sin 2\vartheta$$

giving

$$h(\vartheta) = \cos 2\vartheta + C \qquad \text{(I.3)}$$

where $C$ is a constant.

To evaluate the constant $C$ we substitute the particular value of $\vartheta = \pi/2$ in equation (I.1) which becomes

$$h\left(\frac{\pi}{2}\right) = \frac{4}{\pi}\int_{0}^{\pi} \sin^2\vartheta' \log|\cos\vartheta'|\, d\vartheta'$$

$$= \frac{4}{\pi}\left[h_1\left(\frac{\pi}{2}\right) - \frac{1}{2}\int_{0}^{\pi} \cos 2\vartheta' \log|\cos\vartheta'|\, d\vartheta'\right] \qquad \text{(I.4)}$$

where

$$h_1\left(\frac{\pi}{2}\right) = \frac{1}{2}\int_0^{\pi} \log |\cos \vartheta'| \, d\vartheta' = \int_0^{\pi/2} \log |\cos \vartheta'| \, d\vartheta'$$

$$= \int_0^1 \frac{\log \eta'}{\sqrt{(1-\eta'^2)}} d\eta' = -\frac{\pi}{2} \log 2 \qquad \text{(I.5)}$$

(*see* Ref. 3). We integrate the second term of equation (I.4) by parts and obtain after simplifying:

$$h\left(\frac{\pi}{2}\right) = \frac{4}{\pi}\left\{h_1\left(\frac{\pi}{2}\right) - \frac{1}{2}\int_0^{\pi} \sin^2 \vartheta' \, d\vartheta'\right\}$$

$$= \frac{4}{\pi} h_1\left(\frac{\pi}{2}\right) - 1 = -(1 + \log 4) . \qquad \text{(I.6)}$$

Comparing (I.6) with (I.3) taken at the value of $\vartheta = \pi/2$ we find that $C = -\log 4$, and the complete solution is given by

$$h(\vartheta) = \cos 2\vartheta - \log 4 . \qquad \text{(I.7)}$$



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