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DORIS E. LEHRIAN, B.Sc.,
of the Aerodynamics Division, N.P.L.

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Calculation of Stability Derivatives for Oscillating Wings

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Summary.—A method of calculating stability derivatives for wings oscillating at low frequencies is developed from the modified vortex-lattice method outlined in R. & M. 2470[1]. Derivatives are obtained for the following wings describing plunging (vertical translational) and pitching oscillations :

(i) Delta wings of aspect ratio $A_r = 1\cdot2$, 2 and 3 with a taper ratio 1/7.

(ii) Arrowhead wing of aspect ratio $1\cdot32$ with a taper ratio 7/18 and angle of sweep of $63\cdot4$ deg at quarter chord.

(iii) Circular plate.

Comparison is made with measured values of the derivatives for the delta wing of aspect ratio $1\cdot2$ and the arrowhead wing oscillating in incompressible flow. Satisfactory agreement is obtained with experiment, and also with values calculated by the Multhopp-Garner method[4].

For the delta wing $A_r = 3$, derivatives are also obtained for particular (small) values of the mean frequency parameter ω_m. These results were calculated by using the $\omega_m \to 0$ solution, which is accurate to first order in frequency, with a correction to allow for the oscillatory wake. Comparison with values from Ref. 2, in which all frequency effects are taken into account, indicates that the present method is adequate for small values of ω_m.

The theory of Ref. 3 for wings oscillating in compressible flow is applied to the delta wing $\bar{A}_r = 3$, and the stability derivatives for pitching oscillations are calculated for $M = 0\cdot745$ and $0\cdot917$. The values of $-m_{\dot{\alpha}}$ are compared with values estimated by the Multhopp-Garner method and with high-speed experimental results, for two positions of the axis of oscillation. For the front axis, the estimated values of $-m_{\dot{\alpha}}$ are in agreement but are higher than the experimental results. For the rear axis, the Multhopp-Garner values are closer to experiment than the values obtained by the vortex-lattice method, and only differ appreciably from the measured values at the higher Mach numbers. It is thought that the accuracy of the vortex-lattice results would be improved by taking more collocation points in the solution.

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1. *Introduction.*—A method is developed from the theory of Ref. 1, by the use of a modification suggested by W. P. Jones, and it is applied in this note to the calculation of stability derivatives for wings of various plan-form. The method of Ref. 1 is satisfactory for values of the frequency parameter ω_m in the flutter range[5], but it does not appear to be suitable for lower values of ω_m since it gives infinite limiting values for $l_{\dot{\alpha}}$ and $-m_{\dot{\alpha}}$ for a finite wing as $\omega_m \to 0$. This difficulty is avoided if the lift distribution is assumed to be a combination of simple functions, such that the corresponding doublet distributions are expressible as polynomials in ω_m. The doublet distribution over the wing and wake is then interpreted so that the main contribution to the

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The oscillatory tables referred to in section 3.3 of this report will be published as R. & M. 2956[10].

downwash is obtained generally in terms of ω_m, with a correction to the downwash to allow for the effect of the oscillatory wake. The Falkner lattice scheme[6] is used for the downwash calculation and this reduces the latter to a simple routine which avoids the use of complex numbers. The solution for $\omega_m \to 0$ is limited to first-order terms in frequency, and the present calculations for particular (small) values of ω_m are based on this solution, with a correction to allow for the higher order frequency effects due to the oscillatory wake. The method is extended in Ref. 2 to take into account all frequency effects, and this can be used as an alternative to the method of Ref. 1 for values of ω_m in the flutter range.

The results given in this note are based on 21 × 6 lattice solutions. The use of 6 chordwise vortices on a highly swept low-aspect-ratio wing may appear to be an inadequate representation of the chordwise loading, especially when the solutions are required for the calculation of pitching-moment derivatives. However, in Ref. 7, this lattice is used to calculate the lift on delta wings of aspect ratio 1·0 to 2·5, and it gives results which agree well with the experimental values. The present results for incompressible flow are also in satisfactory agreement with the measured values and the Multhopp-Garner values, so that the 21 × 6 lattice does seem to give reasonable accuracy for swept wings of low aspect ratio. Solutions based on a finer lattice would give further information on accuracy, but for oscillatory problems the calculation would be rather laborious.

The stability and flutter derivatives for compressible flow can be calculated, as suggested in Ref. 3, by relating the oscillatory motion of a wing in subsonic flow to that of a wing of reduced plan-form in incompressible flow. The present incompressible solutions for the delta wings $A_r = 2$ and $1 \cdot 2$ are used, in conjunction with the transformation formulae given in section 5.2, to calculate the stability derivatives of the delta wing $A_r = 3$ at $M = 0 \cdot 745$ and $0 \cdot 917$. The values of the derivative $-m_{\dot{\alpha}}$ are plotted in Fig. 6 and, for comparison, the values of $-m_{\dot{\alpha}}$ obtained in recent tests at the N.P.L. and the results given by the Multhopp-Garner method[4] are also plotted. It should be noted that the agreement between the vortex-lattice and the Multhopp-Garner values, for the incompressible flow solutions for the delta wings $A_r = 1 \cdot 2$ and 2, is quite good. The discrepancy in the compressible flow results for $-m_{\dot{\alpha}}$ seems to occur because relatively small numbers in the incompressible solution are multiplied by β^{-3} as can be seen in equation (27). The effect of the β^{-3} terms is to magnify inaccuracies occurring in the incompressible solution and so it is apparent that a very reliable method is required for the latter solution. The difference between the theoretical and experimental values may be partly due to the effects of thickness and boundary layer which are not allowed for in the theories, and partly due to wind-tunnel interference effects.

LIST OF SYMBOLS

V	Velocity of flow	
$x = R(y) - \frac{c}{2}\cos\theta$		Definitions of chordwise parameters θ and ξ where $R(y)$ is the mid-chord line
$\quad = R(y) + \frac{c}{2}\xi$		
x_t	Trailing-edge co-ordinate	
y	$= s\eta$, definition of spanwise co-ordinate η	
$c(y)$	Local chord	
c_0	Root chord	
c_m	Mean chord	

s		Semi-span
S		Area of wing
A_r		Aspect ratio $= 4s^2/S$
$p/2\pi$		Frequency
ω	$=$	$2\omega' = pc/V$, local frequency parameter
ω_m	$=$	pc_m/V, mean frequency parameter
$K\,e^{ipt}$		Doublet distribution (discontinuity in velocity potential)
$\Gamma\,e^{ipt}$		Bound vorticity
$E\,e^{ipt}$		Free vorticity
$W\,e^{ipt}$		Induced downward velocity
$z'\,e^{ipt}$		Normal downward displacement
Γ_0	$=$	$2\cot\dfrac{\theta}{2}$
Γ_1	$=$	$-2\sin\theta + \cot\dfrac{\theta}{2} + i\omega'[\sin\theta + \frac{1}{2}\sin 2\theta]$
Γ_n	$=$	$-2\sin n\theta + i\omega^{1}\left[\dfrac{\sin\overline{n+1}\theta}{n+1} - \dfrac{\sin\overline{n-1}\theta}{n-1}\right]$ when $n \geqslant 2$
K_0	$=$	$K_a(\theta) + i\omega' K_b(\theta) + O(\omega^2)$ $\quad 0 \leqslant \theta \leqslant \pi$
	$=$	$K_0(\pi)\,.\,\exp\{-ip(x - x_t)/V\}$ $\quad x \geqslant x_t$
K_1	$=$	$\dfrac{c}{2}[\sin\theta + \frac{1}{2}\sin 2\theta]$
K_n	$=$	$\dfrac{c}{2}\left[\dfrac{\sin\overline{n+1}\theta}{n+1} - \dfrac{\sin\overline{n-1}\theta}{n-1}\right]$ when $n \geqslant 2$

Definition of Derivatives for Translational and Pitching Oscillations

(i) Delta and arrowhead wings :

$$\frac{\text{Lift}}{\rho V^2 S\,e^{ipt}} = (l_z + i\omega_m l_{\dot z})z + (l_\alpha + i\omega_m l_{\dot\alpha})\alpha$$

$$\frac{\text{Pitching moment}}{\rho V^2 S c_m\,e^{ipt}} = (m_z + i\omega_m m_{\dot z})z + (m_\alpha + i\omega_m m_{\dot\alpha})\alpha$$

where $\omega_m = pc_m/V$, and $c_m z$ and α are the amplitudes of the translational and angular displacements of the oscillation.

(ii) Circular plate :

$$\frac{\text{Lift}}{\rho V^2 S\, e^{ipt}} = (l_z + i\omega_r l_{\dot z})z + (l_\alpha + i\omega_r l_{\dot\alpha})\alpha$$

$$\frac{\text{Pitching Moment}}{\rho V^2 S r\, e^{ipt}} = (m_z + i\omega_r m_{\dot z})z + (m_\alpha + i\omega_r m_{\dot\alpha})\alpha$$

where $\omega_r = pr/V$ and r = radius of circle, and where rz and α are the amplitudes of the translational and angular displacements of the oscillation.

(iii) Transformation formulae :

If the definitions above are for a reference axis OY, as shown in Fig. 1, then the derivatives referred to an axis at a distance $h'c_m$ (or $h'r$ in the case of the circular plate) back from OY, are obtained by the usual transformation formulae

$$\begin{aligned} l_z' &= l_z \\ l_\alpha' &= l\alpha - h'l_z \\ -m_z' &= -m_z - h'l_z \\ -m_\alpha' &= -m_\alpha - h'(l_\alpha - m_z) + h'^2 l_z, \quad \text{etc.} \end{aligned}$$

2. *Theory.*—In the general linearized theory given in Refs. 1 and 8 for a thin wing describing simple harmonic oscillations of small amplitude in incompressible inviscid flow, it is shown that the discontinuity in the velocity potential can be represented by a doublet distribution $K\, e^{ipt}$ over the wing and wake, where $K\, e^{ipt}$ is related to the bound vorticity $\Gamma\, e^{ipt}$ and the free vorticity $E\, e^{ipt}$ by the equations

$$\left.\begin{aligned} V\Gamma &= ipK + V\frac{\partial K}{\partial x} \\ VE &= -ipK \end{aligned}\right\} \qquad \ldots \qquad (1)$$

where the time exponential terms are omitted for convenience. Since Γ is a function of the chordwise parameter θ, the doublet distribution over the wing can be expressed as

$$K(\text{wing}) = K(\theta) = \frac{c}{2} e^{-i\omega'\xi} \int_{-1}^{\xi} \Gamma\, e^{i\omega'\xi}\, d\xi \qquad -1 \leqslant \xi \leqslant 1$$

where $\xi = -\cos\theta$. $\qquad \ldots \qquad (2)$

Also $\Gamma = 0$ in the wake, hence the doublet distribution over the wake is

$$\begin{aligned} K(\text{wake}) &= K(\pi)\, e^{-i\omega'(\xi-1)} \qquad & \xi \geqslant 1 \\ &= K(\pi)\, e^{-ip(x-x_t)/V} \qquad & x \geqslant x_t\,. \end{aligned} \qquad \ldots \qquad (3)$$

The downwash $W\, e^{ipt}$ induced at a point (x_1, y_1) on the wing, by the distribution $K\, e^{ipt}$ over the wing and wake is known to be

$$4\pi W = \iint K \frac{\partial^2}{\partial {z_1}^2}\left(\frac{1}{r}\right) dx\, dy \qquad \ldots \qquad (4)$$

where $r^2 = (x - x_1)^2 + (y - y_1)^2 + {z_1}^2$ and $z_1 \to 0$. Then W has to satisfy the tangential flow condition, which for simple harmonic oscillations is

$$W = ipz' + V\frac{\partial z'}{\partial x_1} \qquad \ldots \qquad (5)$$

where $z'\, e^{ipt}$ is the normal downward displacement of the point (x_1, y_1) at time t. The problem is to find a distribution K which will satisfy equations (4) and (5).

2.1. It is assumed that the bound vorticity over the finite wing can be represented by

$$\Gamma = V \Sigma \Sigma \Gamma_n C_{nm} A_m \quad \ldots \quad (6)$$

where C_{nm} are arbitrary coefficients. In Ref. 1, the chordwise distribution Γ_0 is defined in terms of the lift function $C(\omega')$. The latter tends to $[1 + i\omega'(\gamma + \log_e \frac{1}{2}\omega')]$ as $\omega' \rightarrow 0$, so that infinite limiting values are obtained for the derivatives $l_{\dot{a}}$ and $- m_{\dot{a}}$. This difficulty is avoided in the present method by assuming

$$\Gamma_0 = 2 \cot \tfrac{1}{2}\theta \, . \quad \ldots \quad (7)$$

The Γ_n distributions, $n \geqslant 1$, are the same as in Ref. 1 and are defined in the list of symbols. The spanwise function A_m is defined as

$$cA_m = sT_m = s\eta^{m-1} \sqrt{(1 - \eta^2)} \quad \ldots \quad (8)$$

where m takes odd or even values according as symmetrical or antisymmetrical motion of the wing is considered.

The corresponding distribution K over the wing and wake is then

$$K = V \Sigma \Sigma K_n C_{nm} A_m \quad \ldots \quad (9)$$

where K_n corresponds to the chordwise distribution Γ_n. Consider the distribution $K_0(\theta)$ over the wing defined by equations (2) and (7). Since the present method is to be used only for small values of the frequency parameter $\omega = 2\omega'$, it is permissible to expand (2) with respect to ω' and obtain $K_0(\theta)$ in the form

$$\left.\begin{aligned} K(\text{wing}) &= K_0(\theta) = K_a(\theta) + i\omega' K_b(\theta) + O(\omega'^2) \\ \text{where } K_a(\theta) &= c[\theta + \sin\theta] \\ K_b(\theta) &= c\left[-\frac{\theta}{2} - \sin\theta + \frac{\sin 2\theta}{4} + \theta\cos\theta\right] \end{aligned}\right\} \quad \ldots \quad (10)$$

Then, by (3), the distribution K_0 over the wing and wake can be regarded as the sum of two distributions K_0' and K_0'' such that

$$\left.\begin{aligned} K_0' &= K_0(\theta) \text{ over the wing} \qquad 0 \leqslant \theta \leqslant \pi \\ &= K_0(\pi) \text{ over the wake} \qquad x \geqslant x_t \\ K_0'' &= K_0(\pi)\,[e^{-ip(x-x_t)/V} - 1] \\ &\qquad \text{over the wake} \qquad x \geqslant x_t \end{aligned}\right\} \quad \ldots \quad (11)$$

It follows, by (10), that distributions K_a', K_a'' and K_b', K_b'' can be similarly defined in terms of K_a and K_b. It should be noted that there is, in general, a spanwise variation in the local parameter ω' in equation (10), and this is allowed for in the calculation by writing

$$\omega' = \omega_m \left(\frac{c}{2c_m}\right) = \omega_m \, . \, f(|\eta|) \, . \quad \ldots \quad (12)$$

The distributions K_n, for $n \geqslant 1$, are the same as in Ref. 1 and are defined in the List of Symbols. They are independent of the frequency and zero in the wake.

The downwash W induced by the K distribution (9), is obtained from integral (4) in the form

$$W = V\left[\sum_m (W_{0m}' + W_{0m}'')C_{0m} + \sum_{n=1}\sum_m W_{nm}C_{nm}\right] \quad \ldots \quad (13)$$

where W_{0m}', W_{0m}'', and W_{nm} are the normal induced velocities due to the doublet distributions $K_0'A_m$, $K_0''A_m$ and K_nA_m respectively. Furthermore, by (10) and (12), the downwash W_{0m}' will be in the form

$$W_{0m}' = W_{am}' + i\omega_m W_{bm}' + 0(\omega_m^2) \quad \ldots \quad (14)$$

where W_{am}', W_{bm}' . . . . , the downwash corresponding to the doublet distributions $K_a'A_m$, $K_b'A_m f(|\eta|)$ . . . . , are independent of the frequency.

3. *The Calculation of the Downwash.*—Since W_{am}', W_{bm}' .... are integrals of the simple type such as occur in steady-motion problems, they can be evaluated approximately by the ordinary vortex-lattice method. The downwash W_{0m}'', induced by the doublet distribution $K_{0m}''A_m$ over the wake, is evaluated for the general case $\omega_m \to 0$ as shown in section 3.2, or for a particular value of ω_m by the modified vortex-lattice method as indicated in section 3.3. The downwash W_{nm}, for $n \geqslant 1$, is also independent of the frequency and is calculated by the ordinary lattice method.

3.1. The Falkner vortex-lattice method is described in detail in Ref. 6. A steady doublet distribution $K(\theta).A_m(\eta)$, to which corresponds a vorticity distribution $\partial K(\theta)/\partial x.A_m(\eta)$, is replaced by a lattice of rectangular vortices each of width $2s_1$, and strength $cL(k).A_m(\eta_1) = sL(k).T_m(\eta_1)$ as shown in Fig. 1. The chordwise factors $cL(k)$ are chosen on the usual two-dimensional basis, to give the same downwash at selected points on the chord, as the continuous distribution* $cK(\theta)$. The total downwash W_{nm} at a collocation point (x_1, y_1) is then obtained, by summation, by using the downwash factors given in Ref. 9 for a rectangular vortex of unit strength.

The results given in this note are based on a 21×6 lattice with 6 collocation points placed on the $\frac{1}{2}$ and 5/6 chord, at the spanwise positions $\eta = 0{\cdot}2$, $0{\cdot}6$ and $0{\cdot}8$. It is usual, in applying this lattice, to use fewer chordwise vortices when the spanwise parameter† $Y \geqslant 10$; for example, with a steady doublet distribution $K = Vc(\theta + \sin\theta)$, that is a vorticity distribution $\partial K/\partial x = 2V \cot \frac{1}{2}\theta$, the lattice is reduced to one vortex at the quarter-chord position. It is thought that the accuracy of solutions for low aspect-ratio wings may be improved by taking two chordwise vortices at one-quarter and three-quarter chord position, when $Y \geqslant 10$, for all the K_n distributions considered. In the case of the delta wing $A_r = 3$, this modification decreases the lift-slope coefficient from $3{\cdot}13$ to $3{\cdot}08$ which agrees better with the value of $3{\cdot}05$ given by the Multhopp-Garner (2 chordwise, $m = 15$ solution) and the experimental value of $3{\cdot}05$.

The chordwise factors $L_a'(k)$, $L_b'(k)$ and $L_1(k)$ corresponding to the two-dimensional doublet distributions K_a', K_b' and K_1, are given in Table 1 for $k = 1, 2 \ldots 6$ with the lattice positions at 1/12, 3/12 .... 11/12 chord, and for $k = 1, 2$ with the lattice at one-quarter and three-quarter chord.

3.2. If a solution is required for $\omega_m \to 0$, the calculation can be kept quite general throughout by considering only first order terms in ω_m and by evaluating W_{0m}'' numerically from equation (15) given below.

Since the point (x_1, y_1) is outside the range of integration, the integral W_{0m}'' obtained from (4), reduces to

$$4\pi W_{0m}'' = -\int_{-s}^{s}\int_{x_t}^{\infty} \frac{K_0'' A_m}{r_1^3}\, dx\, dy$$

where $r_1^2 = (x - x_1)^2 + (y - y_1)^2$ and K_0'' is defined by (10) and (11). If the exponential term is expanded and only first-order terms in p are retained, it may be shown that

$$W_{0m}'' = \frac{1}{4\pi}\int_{-s}^{s}\int_{x_t}^{\infty} i\,\frac{p}{V}\cdot\frac{K_a(\pi)\, A_m\,(x - x_t)}{r_1^3}\, dx\, dy$$

$$= i\omega_m \frac{s^2}{4c_m}\int_{-1}^{1} T_m \int_{x_t}^{\infty}\left[\frac{x - x_1}{r_1^3} - \frac{x_t - x_1}{r_1^3}\right] dx\, d\eta$$

$$= i\omega_m \frac{s^2}{4c_m}\int_{-1}^{1} T_m \left[\frac{r_t - (x_t - x_1)}{(y - y_1)^2}\right] d\eta$$

* The two-dimensional downwash corresponding to the distribution cK_0' is

$$W_0' = W_a' + i\omega' W_b' + O(\omega'^2)$$

where $W_a' = 1$, $W_b' = \cos\theta + \log_e 2|1 + \cos\theta|$

† Y is the spanwise distance from the centre-line of a rectangular vortex to the collocation point (x_1, y_1), in terms of the semi-vortex width s_1.

where $T_m = \eta^{m-1} \sqrt{(1-\eta^2)}$ and $r_t^2 = (x_t - x_1)^2 + (y - y_1)^2$, and where, in general, x_t is a function of the spanwise parameter η. The integral may be written as

$$W_{0m}'' = i\omega_m \frac{s}{4c_m} \int_{-1}^{1} \frac{\eta^{m-1} \sqrt{(1-\eta^2)}}{e + \sqrt{e^2 + (\eta - \eta_1)^2}} d\eta \quad \ldots \quad (15)$$

where $e = \left(\frac{x_t - x_1}{s} \right)$.

The total downwash W at a point (x_1, y_1) is then given by equations (13)—(15), where for the $\omega_m \to 0$ solution, only first-order terms in ω_m are retained. When W is known at a sufficient number of collocation points, the arbitrary coefficients C_{nm} are determined so that W satisfies equation (5) at each point. The downwash equations can be written in matrix notation as

$$[A + i\omega_m B]\{C\} = \{h\} \quad \ldots \quad (16)$$

where the elements A and B of the matrix are real and independent of ω_m; $[A]$ is the matrix corresponding to the $\omega_m = 0$ solution and the column matrix

$$\{C\} = \{C_{01}, C_{11}, C_{21} \ldots . C_{0m}, C_{1m}, C_{2m} \ldots .\}.$$

The column matrix on the right-hand side of (16) is

$$\{h\} = \left\{ \frac{W(x_1, y_1)}{V}, \frac{W(x_2, y_2)}{V}, \frac{W(x_3, y_3)}{V} \ldots . \right\}$$

with the values W/V obtained from equation (5).

3.3. If a solution is required for a particular value of ω_m, frequency effects higher than first-order may also be allowed for in the calculation of W_{0m}'', by using the modified lattice scheme of Ref. 1. The continuous doublet distribution $K_0'' A_m$ is defined by equations (10) to (12) as

$$A_m[K_a(\pi) + i\omega_m K_b(\pi) f(|\eta|) + O(\omega_m^2)][e^{-ip(x-x_t)/V} - 1].$$

This is replaced by doublet strips of width $2s_1$ and strength proportional to $s_1[e^{-ip(x-x_t)/V} - 1]$, extending from $x = x_t$ to $x = \infty$. The downwash W_{0m}'' is calculated by summation by using the tables of Ref. 10, which give the downwash due to a doublet strip of strength $s_1 e^{-ipx/V}$ oscillating at a frequency $p/2\pi$, together with the tables of Ref. 9.

The total downwash W at a point (x_1, y_1) is then obtained from equations (13) and (14) for the particular value ω_m. In this case, the set of downwash equations can be solved directly or by the approximate method suggested in section 4.1 if the value of ω_m is small. The equations in matrix notation are

$$[A + (a + ib)]\{C\} = \{h\} \quad \ldots \quad (17)$$

where A, a and b are real, and the column matrices $\{C\}$ and $\{h\}$ are as defined above for equation (16).

4. *Solution of Equations.*—The equations for the $\omega_m \to 0$ solution are given by (16), and in this case the values of $\{h\}$ are obtained from equation (5) in the form

$$\{h\} = \{h' + i\omega_m h''\} \quad \ldots \quad (18)$$

Let $\{C\} = \{C' + i\omega_m C''\}$

so that equation (16) becomes

$$[A + i\omega_m B]\{C' + i\omega_m C''\} = \{h' + i\omega_m h''\} \quad \ldots \quad (19)$$

Then if terms $O(\omega_m^2)$ are neglected

$$[A]\{C'\} = \{h'\}$$

$$[B]\{C'\} + [A]\{C''\} = \{h''\}$$

and hence the solution is

$$\left.\begin{aligned} \{C'\} &= [A]^{-1}\{h'\} \\ \{C''\} &= [A]^{-1}\{h''\} - [A]^{-1}[B][A]^{-1}\{h'\} \end{aligned}\right\} \quad \ldots \quad (20)$$

where $\{C\} = \{C' + i\omega_m C''\}$ and $[A]^{-1}$ is the inverse matrix of $[A]$.

4.1. The equations for a particular value of ω_m are given by (17). For small values of ω_m, a and b are small compared with A and the following approximate solution is suggested which avoids the solution of complex simultaneous equations and reduces the amount of computing when several values of ω_m are considered.

Let C_1 be a first approximation to C, such that

$$[A]\{C_1\} = \{h\} . \quad \ldots \quad (21)$$

Let $C_2 = C_1 + \varepsilon$ be a second approximation to C, where ε is assumed small compared with C_1, so that the term $[a + ib]\{\varepsilon\}$ can be neglected. Then, substituting in equation (17),

$$[A]\{C_1\} + [a + ib]\{C_1\} + [A]\{\varepsilon\} = \{h\}$$

and, by (21)

$$[A]\{\varepsilon\} = -[a + ib]\,[A]^{-1}\{h\}.$$

Hence

$$\{C_2\} = \{C_1 + \varepsilon\} = [A]^{-1}\{h\} - [A]^{-1}[a + ib]\,[A]^{-1}\{h\} . \quad \ldots \quad (22)$$

This process can be continued if C_2 is not a sufficiently accurate solution.

5. *Application of the Method*.—5.1. *Incompressible Flow*.—The method is used to calculate the aerodynamic coefficients of the following wings, which are describing plunging (vertical translational) and pitching oscillations for $\omega_m \to 0$.

(i) Delta wings of aspect ratio $A_r = 1 \cdot 2$, 2 and 3, with a taper ratio 1/7. Values of the derivatives for the axis position at $0 \cdot 556 c_0$ are given in Table 2(A) and values of the derivative $-m_{\dot{\alpha}}$ are tabulated in Table 2(B) for various axis positions. The latter results are plotted in Figs. 2 and 3.

(ii) Arrowhead wing of aspect ratio $1 \cdot 32$, with a taper ratio 7/18, and angle of sweep of $63 \cdot 4$ deg at quarter-chord. Derivatives are given in Table 3 and the variation of $-m_{\dot{\alpha}}$ with axis position is shown in Fig. 4.

(iii) Circular plate, aspect ratio $4/\pi$. Derivatives are given in Table 4 and the variation of $-m_{\dot{\alpha}}$ with axis position is shown in Fig. 5.

For the delta wing $A_r = 3$, derivatives are also obtained for $\omega_m = 0 \cdot 13$ and $0 \cdot 26$. The values are given in Table 2 and Fig. 2.

The results are based on solutions in which the general distributions Γ and K, defined by (6) and (9) were limited to the chordwise terms $n = 0$ and 1, with the $K_0(\theta)$ distribution limited to first-order terms in ω_m, and the spanwise terms $m = 1$, 3 and 5. For $\omega_m \to 0$, all terms $O(\omega_m^2)$ were neglected throughout the solution and the downwash values were calculated by the modified vortex-lattice method described in section 3; the downwash values $W_{nm} = [A + i\omega_m B]$ are tabulated in Tables 5A to 5D for the wings (i) and (ii) and the values $W_{nm} = [A + i\omega_r B]$ are given in Table 6 for wing (iii). The solutions for the particular (small) values of ω_m considered in this note were also based on the first-order method, but higher order frequency effects due to the oscillatory wake were allowed for in the calculation of W_{0m}''. The unknown coefficients

C_{nm} were determined by solving the simultaneous equations obtained from (5) and (13). For plunging and pitching oscillations the displacement $z' e^{ipt}$ is defined as follows :

$$z' = c_m z + x_1 \alpha \text{ for wings (i) and (ii)}$$

where $c_m z$ and α are the amplitudes of the translational and angular displacements :

$$z' = rz + x_1 \alpha \text{ for the circular plate}$$

where r = radius, rz and α are the amplitudes of the translational and angular displacements.

When the coefficients C_{nm} have been determined, the lift distribution $\rho V \Gamma$ defined by (6) is integrated to give the total lift and the pitching moment for the axis OY. The aerodynamic derivatives are then computed according to the definitions given in the List of Symbols.

5.2. *Compressible Flow*.—In Ref. 3, W. P. Jones has shown that stability and flutter derivatives for a finite wing oscillating in compressible flow, can be estimated by considering a wing of related plan-form in incompressible flow. If the original wing of aspect ratio $\bar{A}$ at Mach number $M = \sqrt{(1 - \beta^2)}$, is describing simple harmonic oscillations at a frequency $\bar{p}/2\pi$, then a solution for stability derivatives, to first-order accuracy in frequency, can be obtained by calculating the incompressible flow solution of a wing of reduced aspect ratio $A_r = \beta \bar{A}$ (reduced laterally by the factor β), with a changed mode of oscillation and a frequency $p/2\pi = \bar{p}/2\pi\beta^2$.

If $\bar{l}$, k, w refer to the lift, circulation and downwash of the original wing $\bar{A}$, then

$$\left.\begin{aligned} \bar{l} &= \rho V \Gamma \exp\{i(\lambda_m X + \bar{\omega}_m T)\} \\ k &= K \exp\{i(\lambda_m X + \bar{\omega}_m T)\} \\ w &= \beta W \exp\{i(\lambda_m X + \bar{\omega}_m T)\} \end{aligned}\right\} \quad \ldots \quad (23)$$

where Γ, K, W are the distributions defined in section 2 for the reduced wing A_r at $M = 0$, oscillating at a mean frequency parameter value $\omega_m = pc_m/V \to 0$, and where

$$\bar{\omega}_m = \frac{\bar{p}c_m}{V}, \; \lambda_m = \left(\frac{1 - \beta^2}{\beta^2}\right) \bar{\omega}_m$$

$$\bar{\omega}_m T = \bar{p}t, \quad X = x/c_m .$$

For plunging and pitching oscillations of amplitude $c_m z$ and α, the tangential flow condition for the original wing $\bar{A}$ is

$$w = V[i\bar{\omega}_m(z + X\alpha) + \alpha] \, e^{i\bar{\omega}_m T} . \quad \ldots \quad (24)$$

Hence, by (23), the flow condition for the reduced wing A_r is

$$\begin{aligned} W &= \frac{w}{\beta} \exp\{-i(\lambda_m X + \bar{\omega}_m T)\} \\ &= \frac{V}{\beta} [i\bar{\omega}_m(z + X\alpha) + \alpha] \, e^{-i\lambda_m X} . \end{aligned} \quad \ldots \quad (25)$$

It is shown in Ref. 7 that to first-order accuracy in frequency

$$4\pi W = \iint_{z_1 \to 0} K \frac{\partial^2}{\partial z_1^2} \left(\frac{1}{r}\right) dx \, dy, \text{ where integration is over wing } A_r \text{ and wake.}$$

Thus the downwash W at a point (X_1, Y_1) on the reduced wing A_r in incompressible flow, may be calculated by the method for $\omega_m \to 0$ outlined in section 3. The set of downwash equations may be expressed in matrix notation as

$$[A + i\omega_m B]\{C\} = \left\{\frac{W(X_j, Y_j)}{V}\right\} \quad \ldots \quad (26)$$

where $\omega_m = \bar{\omega}_m/\beta^2$.

The column matrix $\{C\} = \{C_{01}, C_{11}, C_{21} \ldots C_{0m}, C_{1m}, C_{2m} \ldots\}$, where the C_{nm}'s are the arbitrary coefficients assumed in the Γ distribution. The column matrix $\{W(X_j, Y_j)/V\}$ denotes the values of W/V at collocation points (X_j, Y_j) as given by equation (25). If these values are expanded in terms of $\bar{\omega}_m$ and only terms of first order are retained, then equations (26) can be solved by the method given in section 4. The coefficients $\{C\}$ for plunging and pitching oscillations are obtained in the form

$$\{C\} = \{C' + i\bar{\omega}_m C''\} \qquad (27)$$

where

$$\{C'\} = \alpha \, . \, \frac{1}{\beta} [A]^{-1} \{1\}$$

$$\{C''\} = z \, . \, \frac{1}{\beta} [A]^{-1} \{1\} - \alpha \left(\frac{1 - 2\beta^2}{\beta^3}\right) [A]^{-1} \{X_j\}$$

$$- \alpha \, . \, \frac{1}{\beta^3} [A]^{-1} [B] [A]^{-1} \{1\}$$

and $[A]^{-1}$ is the inverse matrix of $[A]$.

The amplitude of the lift distribution for the original wing $\bar{A}$ is, by (23) and (6),

$$= \bar{l} \, e^{-i\bar{\omega}_m T} = \rho V^2 \left[\sum_n \sum_m \Gamma_n \, C_{nm} A_m \right] e^{i\lambda_m X} \qquad (28)$$

where $cA_m = sT_m = s\eta^{m-1} \sqrt{(1 - \eta^2)}$,

semi-span of wing $A_r = s = \beta \bar{s}$ and $\bar{s}$ = semi-span of wing $\bar{A}$.

The stability derivatives for the original wing at Mach number M, are then obtained by integration, only terms of first order in frequency being retained.

In the present report the incompressible flow solutions for the delta wings $A_r = 1 \cdot 2$ and 2 were used to calculate the stability derivatives for pitching oscillations of the delta wing $\bar{A} = 3$ at $M = 0 \cdot 745$ and $0 \cdot 917$. The values are tabulated in Table 7, and the derivative $- m_{\dot{\alpha}}$ is plotted against M in Fig. 6.

6. *Results Quoted for Comparison.*—Low-speed tests on the delta wing $A_r = 1 \cdot 2$ have recently been made at the N.P.L. for axis positions at $0 \cdot 431c_0$ and $0 \cdot 556c_0$ (Ref. 11). Values of the pitching derivatives are given in Table 2 for the $0 \cdot 556c_0$ axis and values of $- m_{\dot{\alpha}}$ are plotted in Fig. 2 for both axis positions. The results quoted are for zero mean incidence and the tests show no amplitude effects. The derivatives were measured for frequency parameter values $\omega_m = 0 \cdot 06$ to $0 \cdot 60$ and were approximately constant over this range.

Tests on the arrowhead wing of aspect ratio $1 \cdot 32$ have recently been made at the N.P.L. Measurements for zero mean incidence, made by Scruton, Woodgate and Alexander[11], indicate that the damping derivative $- m_{\dot{\alpha}}$ for an axis at $0 \cdot 738c_0 \equiv 1 \cdot 063c_m$, decreases approximately linearly from $0 \cdot 196$ to $0 \cdot 135$ for the range $0 \cdot 027 \leqslant \omega_m \leqslant 0 \cdot 2$ and has a nearly constant value $0 \cdot 135$ for the range $0 \cdot 2 \leqslant \omega_m \leqslant 0 \cdot 6$. The value $- m_{\dot{\alpha}} = 0 \cdot 21$ which is plotted in Fig. 4, was obtained by extrapolating these results to $\omega_m = 0$.

Values of $- m_{\dot{\alpha}}$, for the delta wing $\bar{A} = 3$, have recently been measured by Bratt in a high-speed wind tunnel at the N.P.L.; the results given in Figs. 2 and 3 are for $M = 0 \cdot 4$ and those plotted in Fig. 6 are for the subsonic flow range $0 \cdot 4 < M < 0 \cdot 9$. The values quoted for the two axis positions $0 \cdot 431c_0$ and $0 \cdot 556c_0$ are for a mean frequency parameter value $\omega_m = 0 \cdot 07$ and zero mean incidence.

The Multhopp subsonic lifting-surface theory for steady motion is extended, in Ref. 4, to deal with harmonic oscillations of low frequency. Values of $- m_{\dot{\alpha}}$ calculated by this method for the delta wing $A_r = 3$, the arrowhead wing $1 \cdot 32$ and the circular plate in incompressible flow, are plotted in Figs. 3 to 5. The effect of compressibility on the derivatives of the delta wing $\bar{A} = 3$, is also estimated in Ref. 4 and the values of $- m_{\dot{\alpha}}$ are plotted against Mach number in Fig. 6. These results were obtained from solutions based on 2 chordwise and $\overline{m}$ spanwise terms.

7. *Concluding Remarks.*—A comparison of the vortex-lattice results and experimental values indicates that the present method is satisfactory for the calculation of stability derivatives for incompressible flow. Solutions based on a 21×6 lattice with 6 collocation points give reasonable accuracy even for wings of very low aspect ratio. The wings are assumed to be rigid in the present calculations, but the effects of distortion could readily be taken into account ; the method of Ref. 1 has been applied to the case of a delta wing oscillating in elastic modes in Ref. 12.

For compressible flow, the present solutions give values of $-m_{\dot{\alpha}}$ for the delta wing $\bar{A} = 3$, which are not in such good agreement with the measured values. This indicates that a very reliable method is required for the equivalent wing in incompressible flow. It is possible that the accuracy of the vortex-lattice solutions would be improved by taking more collocation points in the calculation, and further information on accuracy might also be provided by solutions based on a finer lattice. The difference between theory and experiment may also be partly due to the effects of thickness, boundary layer and wind-tunnel interference, since these are not allowed for in the calculation.

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TABLE 1

Chordwise Factors of the Rectangular Vortices Representing the Doublet Distributions K'_a, K'_b and K_1

k	$L'_a(k)$	$L'_b(k)$	$L_1(k)$	Position of vortex, on chord
1	$0 \cdot 4512\pi$	$-0 \cdot 1143\pi$	$0 \cdot 1709\pi$	1/12
2	$0 \cdot 2051\pi$	$-0 \cdot 2025\pi$	$0 \cdot 0114\pi$	3/12
3	$0 \cdot 1367\pi$	$-0 \cdot 2537\pi$	$-0 \cdot 0358\pi$	5/12
4	$0 \cdot 0976\pi$	$-0 \cdot 2895\pi$	$-0 \cdot 0553\pi$	7/12
5	$0 \cdot 0684\pi$	$-0 \cdot 3149\pi$	$-0 \cdot 0570\pi$	9/12
6	$0 \cdot 0410\pi$	$-0 \cdot 3251\pi$	$-0 \cdot 0342\pi$	11/12

When the spanwise parameter $Y \geqslant 10$, the 6-step lattice is replaced by a 2-step lattice with the following factors:

k	$L'_a(k)$	$L'_b(k)$	$L_1(k)$	Position of vortex, on chord
1	$0 \cdot 750\pi$	$-0 \cdot 5767\pi$	$0 \cdot 125\pi$	1/4
2	$0 \cdot 250\pi$	$-0 \cdot 9233\pi$	$-0 \cdot 125\pi$	3/4

TABLES 2A and 2B

Aerodynamic Derivatives for a Family of Delta Wings of Taper Ratio 1/7

A *Derivatives for axis position* $hc_0 = 0 \cdot 556c_0$

	A_r	ω_m	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	$-m_z$	$-m_{\dot{z}}$	$-m_\alpha$	$-m_{\dot{\alpha}}$
	1·2	→0	0	0·815	0·815	0·778	0	—0·008	—0·008	0·268
Experiment		0·06 to 0·6			0·850	0·867			0·008	0·265
	2	→0	0	1·191	1·191	0·940	0	—0·043	—0·043	0·313
	3	→0	0	1·539	1·539	0·926	0	—0·083	—0·083	0·346
		0·13	—0·003	1·533	1·537	0·970	—0·002	—0·085	—0·087	0·341
		0·26	—0·015	1·521	1·531	1·021	—0·009	—0·090	—0·096	0·340
From Ref. 2*		0·26	—0·017	1·490	1·499	1·030	—0·008	—0·082	—0·088	0·342

* The method of Ref. 2 takes into account all frequency effects.

B *Variation of the pitching-moment damping coefficient* $-m_{\dot{\alpha}}$ *with axis position*

Axis position		$A_r = 1 \cdot 2$	$A_r = 2$	$A_r = 3$		
h	h'	$\omega_m \to 0$	$\omega_m \to 0$	$\omega_m \to 0$	$= 0 \cdot 13$	$= 0 \cdot 26$
0	—0·750	1·789	2·313	2·623	2·654	2·685
0·1	—0·575	1·401	1·787	1·998	2·023	2·051
0·2	—0·400	1·064	1·334	1·468	1·487	1·510
0·3	—0·225	0·777	0·954	1·032	1·045	1·062
0·4	—0·050	0·539	0·646	0·691	0·697	0·707
0·5	0·125	0·352	0·412	0·443	0·442	0·446
0·6	0·300	0·214	0·251	0·290	0·281	0·277
0·7	0·475	0·126	0·163	0·231	0·215	0·202
0·8	0·650	0·088	0·147	0·267	0·242	0·220
0·9	0·825	0·100	0·205	0·397	0·363	0·330
1·0	1·000	0·162	0·335	0·621	0·578	0·534

A_r = Aspect ratio. hc_0 = Distance back from apex. $h'c_m$ = Distance back from leading edge at mean chord.

TABLES 3A and 3B

Aerodynamic Derivatives for $\omega_m \to 0$, for the Arrowhead Wing of Aspect Ratio 1·32

A *Derivatives for axis through wing apex*

$l_z = 0$
$l_{\dot z} = 0{\cdot}833$
$l_\alpha = 0{\cdot}833$
$l_{\dot\alpha} = 1{\cdot}491$
$-m_z = 0$
$-m_{\dot z} = 0{\cdot}795$
$-m_\alpha = 0{\cdot}795$
$-m_{\dot\alpha} = 1{\cdot}653$

B *Variation of $-m_{\dot\alpha}$ with the axis position $h'c_m$*

h'	$-m_{\dot\alpha}$	h'	$-m_{\dot\alpha}$
0	1·653	1·2	0·110
0·2	1·230	1·4	0·085
0·4	0·872	1·6	0·128
0·6	0·582	1·8	0·237
0·8	0·358	2·0	0·413
1·0	0·200	2·1	0·525

$$h' = \frac{\text{Distance back from apex}}{\text{Mean chord } c_m}$$

TABLES 4A and 4B

Aerodynamic Derivatives for $\omega_r \to 0$, for the Circular Plate

A *Derivatives for mid-chord axis position*

$l_z = 0$
$l_{\dot z} = 0{\cdot}891$
$l_\alpha = 0{\cdot}891$
$l_{\dot\alpha} = 1{\cdot}196$
$-m_z = 0$
$-m_{\dot z} = -0{\cdot}462$
$-m_\alpha = -0{\cdot}462$
$-m_{\dot\alpha} = 0{\cdot}279$

B *Variation of $-m_{\dot\alpha}$ with the position $h'r$*

h'	$-m_{\dot\alpha}$	h'	$-m_{\dot\alpha}$
0	1·904	1·2	0·168
0·2	1·437	1·4	0·128
0·4	1·040	1·6	0·160
0·6	0·715	1·8	0·262
0·8	0·462	2·0	0·436
1·0	0·279		

$$h' = \frac{\text{Distance back from axis } OY}{\text{Radius } r}$$

N.B. For the circular plate, the derivatives are defined in terms of radius r and the frequency parameter $\omega_r = pr/V$

TABLE 5

Values of the Downwash W_{nm} at Collocation Points (ξ, η), as $\omega_m \to 0$: Matrix $[W_{nm}] = [A + i\omega_m B]$

A *Delta wing, $A_r = 1 \cdot 2$, taper ratio 1/7*

Values of A

(ξ, η)	(n, m)					
	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	2·108514	−0·192982	−0·097694	+0·376591	−0·049566	−0·018831
(0 , 0·6)	1·869389	+1·229127	+0·323397	+0·527714	+0·188470	+0·020035
(0 , 0·8)	1·460079	+2·153558	+1·600454	+0·647579	+0·475148	+0·290190
(0·6, 0·2)	1·858905	−0·450832	−0·204099	−0·035192	−0·035352	−0·013248
(0·6, 0·6)	1·821870	+1·042284	+0·090942	−0·034863	−0·023507	−0·032319
(0·6, 0·8)	1·791262	+2·329668	+1·622047	−0·024056	+0·004907	−0·005391

Values of B

(0 , 0·2)	−0·902452	+0·037571	+0·043006	0	0	0
(0 , 0·6)	+0·590266	−0·337046	−0·146324	0	0	0
(0 , 0·8)	+1·189472	−0·092964	−0·226604	0	0	0
(0·6, 0·2)	−1·746289	+0·208218	+0·123541	0	0	0
(0·6, 0·6)	+0·105414	−0·636394	−0·186398	0	0	0
(0·6, 0·8)	+0·822195	−0·538394	−0·537242	0	0	0

Matrix $[A]^{-1}$ = Inverse $[A]$

=

−0·024799	+0·022579	+0·024209	+0·443605	+0·060215	+0·022560
+0·180849	−0·264924	+0·113464	−0·959311	+1·181661	−0·235201
−0·152872	+0·297337	−0·167934	+0·812630	−1·719792	+0·912387
+2·204322	+0·160846	−0·011079	−2·147829	−0·380839	−0·137279
−7·604031	+7·100503	−1·242028	+7·171327	−5·351964	+0·554261
+7·157274	−11·772497	+5·466740	−6·543621	+10·025678	−4·001210

* Values computed using a 21 × 6 lattice, *see* section 5.1.

TABLE 5—*continued*

Values of the Downwash $W_{nm}^{\cdot}$ *at Collocation Points* (ξ, η), *as* $\omega_m \to 0$: *Matrix* $[W_{nm}] = [A + i\omega_m B]$

B *Delta wing,* $A_r = 2$, *taper ratio* 1/7

Values of A

(ξ, η)	(n, m) (0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0, 0·2)	2·246360	−0·198365	−0·109259	+0·491625	−0·039526	−0·017030
(0 , 0·6)	2·075138	+1·276306	+0·333906	+0·673876	+0·247882	+0·047694
(0 , 0·8)	1·841571	+2·347650	+1·720971	+0·789012	+0·572563	+0·360294
(0·6̇, 0·2)	2·058579	−0·418811	−0·193909	−0·073434	−0·033248	−0·010680
(0·6̇, 0·6)	2·100124	+1·143870	+0·142683	−0·097033	−0·042369	−0·037870
(0·6̇, 0·8)	2·105804	+2·495959	+1·730210	−0·102412	−0·038766	−0·032003

Values of B

(ξ, η)	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	−0·577420	+0·087529	+0·065797	0	0	0
(0 , 0·6)	+0·884008	−0·239310	−0·098152	0	0	0
(0 , 0·8)	+1·386536	−0·006636	−0·172537	0	0	0
(0·6̇, 0·2)	−1·541114	+0·251686	+0·145470	0	0	0
(0·6̇, 0·6)	+0·308518	−0·576252	−0·156581	0	0	0
(0·6̇, 0·8)	+1·008446	−0·464492	−0·494361	0	0	0

Matrix $[A]^{-1}$ = Inverse $[A]$

=

+0·004305	+0·031684	+0·012656	+0·382884	+0·031939	+0·021845
+0·115475	−0·173535	+0·095157	−0·904595	+1·097641	−0·245764
−0·100315	+0·169963	−0·095475	+0·770748	−1·585657	+0·850936
+1·735261	−0·009304	+0·010876	−1·656416	−0·160768	−0·071822
−6·373714	+6·373417	−1·414931	+6·006561	−4·988878	+0·859485
+6·033481	−9·951018	+4·771559	−5·662248	+8·538777	−3·783405

TABLE 5—*continued*

Values of the Downwash W_{nm} at Collocation Points (ξ, η), as $\omega_m \to 0$: Matrix $[W_{nm}] = [A + i\omega_m B]$

C *Delta wing, $A_r = 3$, taper ratio 1/7*

Values of A

(ξ, η)	(n, m) (0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	2·458756	−0·195188	−0·115555	+0·645743	−0·025716	−0·013964
(0 , 0·6)	2·414351	+1·374274	+0·365665	+0·869258	+0·324655	+0·082044
(0 , 0·8)	2·330373	+2·610238	+1·882558	+1·003016	+0·712047	+0·456151
(0·$\dot{6}$, 0·2)	2·330578	−0·386435	−0·185827	−0·126059	−0·033164	−0·009490
(0·$\dot{6}$, 0·6)	2·484706	+1·280207	+0·205682	−0·180251	−0·066988	−0·044739
(0·$\dot{6}$, 0·8)	2·555151	+2·742383	+1·889420	−0·205678	−0·099858	−0·068765

Values of B

(ξ, η)	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	−0·103935	+0·163728	+0·098566	0	0	0
(0 , 0·6)	+1·324390	−0·102120	−0·032462	0	0	0
(0 , 0·8)	+1·749240	+0·144602	−0·080946	0	0	0
(0·$\dot{6}$, 0·2)	−1·216082	+0·318155	+0·177002	0	0	0
(0·$\dot{6}$, 0·6)	+0·618644	−0·480253	−0·108708	0	0	0
(0·$\dot{6}$, 0·8)	+1·300771	−0·350863	−0·427588	0	0	0

Matrix $[A]^{-1}$ = Inverse $[A]$

=

+0·023749	+0·030203	+0·006559	+0·328850	+0·014627	+0·019822
+0·055675	−0·083183	+0·061865	−0·843274	+0·999144	−0·233840
−0·059792	+0·058528	−0·023527	+0·727855	−1·446895	+0·766814
+1·363347	−0·090436	+0·021499	−1·282768	−0·033984	−0·042999
−5·266358	+5·472015	−1·338337	+4·938347	−4·428781	+0·920182
+5·029755	−8·262751	+3·943692	−4·746507	+7·167276	−3·269687

TABLE 5—*continued*

Values of the Downwash W_{nm} at Collocation Points (ξ, η), as $\omega_m \to 0$: Matrix $[W_{nm}] = [A + i\omega_m B]$

D *Arrowhead wing, $A_r = 1\cdot32$*

Values of A

(ξ, η)	(n, m)					
	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	2·455413	−0·094812	−0·072411	+0·508700	−0·046908	−0·016548
(0 , 0·6)	1·709472	+1·246541	+0·350006	+0·681309	+0·207564	+0·019803
(0 , 0·8)	1·155726	+1·991932	+1·493441	+0·697997	+0·470449	+0·274126
(0·$\dot{6}$, 0·2)	2·177069	−0·347152	−0·160035	−0·142266	−0·059275	−0·017160
(0·$\dot{6}$, 0·6)	1·880055	+1·131840	+0·152661	−0·071080	−0·085286	−0·074402
(0·$\dot{6}$, 0·8)	1·661872	+2·304480	+1·627161	+0·042384	+0·006498	−0·023228

Values of B

(ξ, η)	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	−0·618615	−0·002877	+0·031718	0	0	0
(0 , 0·6)	+0·840746	−0·326405	−0·149544	0	0	0
(0 , 0·8)	+1·275811	−0·183516	−0·309744	0	0	0
(0·$\dot{6}$, 0·2)	−1·575591	+0·123108	+0·090074	0	0	0
(0·$\dot{6}$, 0·6)	+0·199694	−0·701210	−0·220355	0	0	0
(0·$\dot{6}$, 0·8)	+0·878692	−0·740021	−0·709154	0	0	0

Matrix $[A]^{-1}$ = Inverse $[A]$

=

−0·005644	+0·065950	+0·020567	+0·397716	+0·001075	+0·005702
+0·044716	−0·340813	+0·300989	−0·769294	+1·310483	−0·399569
+0·011434	+0·227656	−0·344788	+0·616710	−1·698750	+1·102595
+1·631330	−0·052604	−0·049933	−1·601837	−0·188179	−0·010150
−6·176317	+7·181906	−1·858422	+6·123068	−5·281307	+0·983731
+6·082440	−11·233294	+6·569049	−5·876145	+9·270489	−4·789940

TABLE 6

Circular Plate: Values of the Downwash W_{nm} at Collocation Points (ξ, η), as $\omega_r = pr/V \to 0$.*
Matrix $[W_{nm}] = [A + i\omega_r B]$

Values of A

(ξ, η)	(n, m)					
	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	1·588024	−0·392250	−0·182390	+0·399294	−0·049289	−0·022845
(0 , 0·6)	1·573808	+0·827946	+0·045484	+0·394206	+0·183869	+0·024514
(0 , 0·8)	1·581476	+1·912974	+1·315291	+0·394538	+0·394107	+0·268244
(0·$\dot{6}$, 0·2)	1·729384	−0·524945	−0·244644	+0·006680	−0·018514	−0·007363
(0·$\dot{6}$, 0·6)	1·772298	+0·927496	+0·008054	−0·011469	+0·031570	+0·006630
(0·$\dot{6}$, 0·8)	1·839798	+2·225071	+1·506252	−0·046641	+0·064554	+0·065947

Values of B

(ξ, η)	(0, 1)	(0, 3)	(0, 5)	(1, 1)	(1, 3)	(1, 5)
(0 , 0·2)	−0·683272	+0·333750	+0·156084	0	0	0
(0 , 0·6)	−0·272188	−0·462896	−0·062111	0	0	0
(0 , 0·8)	+0·234933	−0·663879	−0·569026	0	0	0
(0·$\dot{6}$, 0·2)	−1·771717	+0·653911	+0·308148	0	0	0
(0·$\dot{6}$, 0·6)	−1·113127	−0·946461	−0·080023	0	0	0
(0·$\dot{6}$, 0·8)	−0·314434	−1·440170	−1·101193	0	0	0

Matrix $[A]^{-1}$ = Inverse $[A]$

=

−0·014807	+0·031298	−0·012862	+0·439196	+0·050040	+0·079559
+0·234863	−0·229869	+0·023623	−1·021814	+1·132930	−0·157266
−0·235226	+0·520231	−0·236785	+0·889188	−1·926126	+0·981190
+2·222748	+0·109719	+0·180027	−1·947160	−0·134917	−0·206900
−6·397469	+7·473457	−1·153862	+5·434107	−6·134857	+0·922633
+5·695743	−12·237549	+6·226831	−4·782236	+10·281807	−5·209850

* Values of the downwash computed using a 21 × 6 lattice, *see* section 5.1 : r = radius of circle.

TABLE 7

Effect of Compressibility on the Derivatives of the Delta Wing $\bar{A} = 3$, for Pitching Oscillations as $\bar{\omega}_m \to 0$

M	hc_0	l_α	$l_{\dot{\alpha}}$	$-m_\alpha$	$-m_{\dot{\alpha}}$
0	$0 \cdot 431c_0$	1·539	1·262	0·253	0·604
0·745		1·786	1·014	0·326	0·847
0·917		2·036	0·066	0·425	1·129
0	$0 \cdot 556c_0$	1·539	0·926	−0·083	0·346
0·745		1·786	0·623	−0·065	0·640
0·917		2·036	−0·379	−0·021	1·119

hc_0 = Position of axis measured from apex

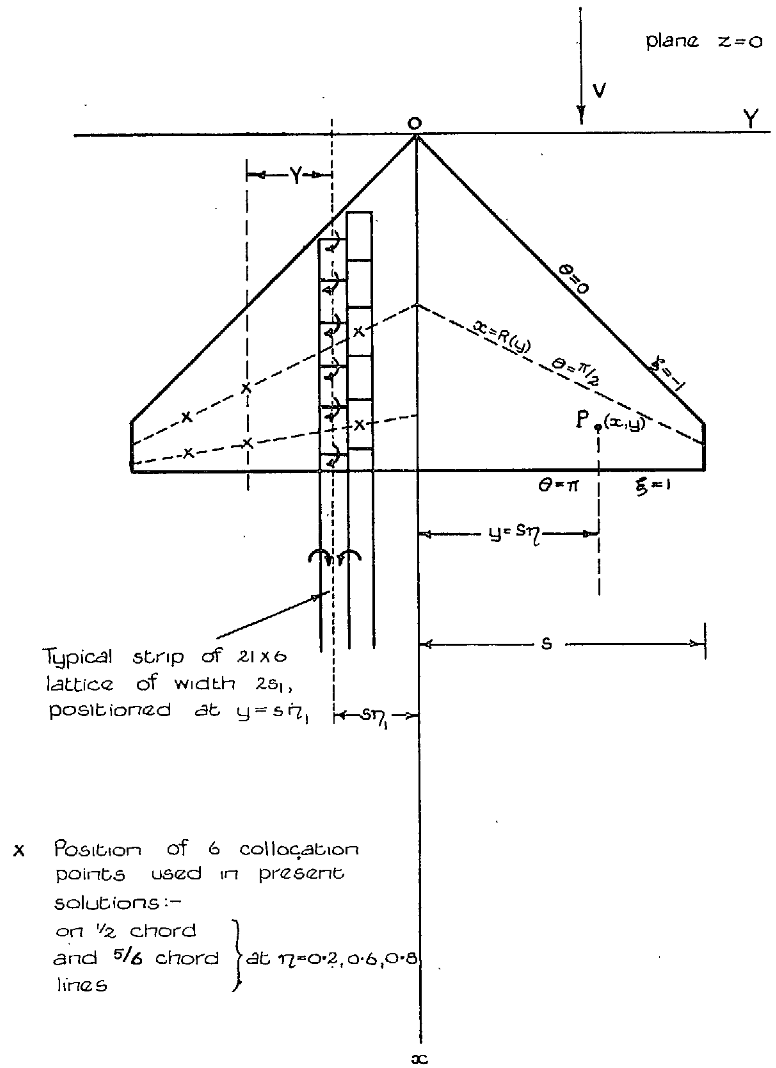

Fig. 1. Layout for 21/6 vortex lattice.

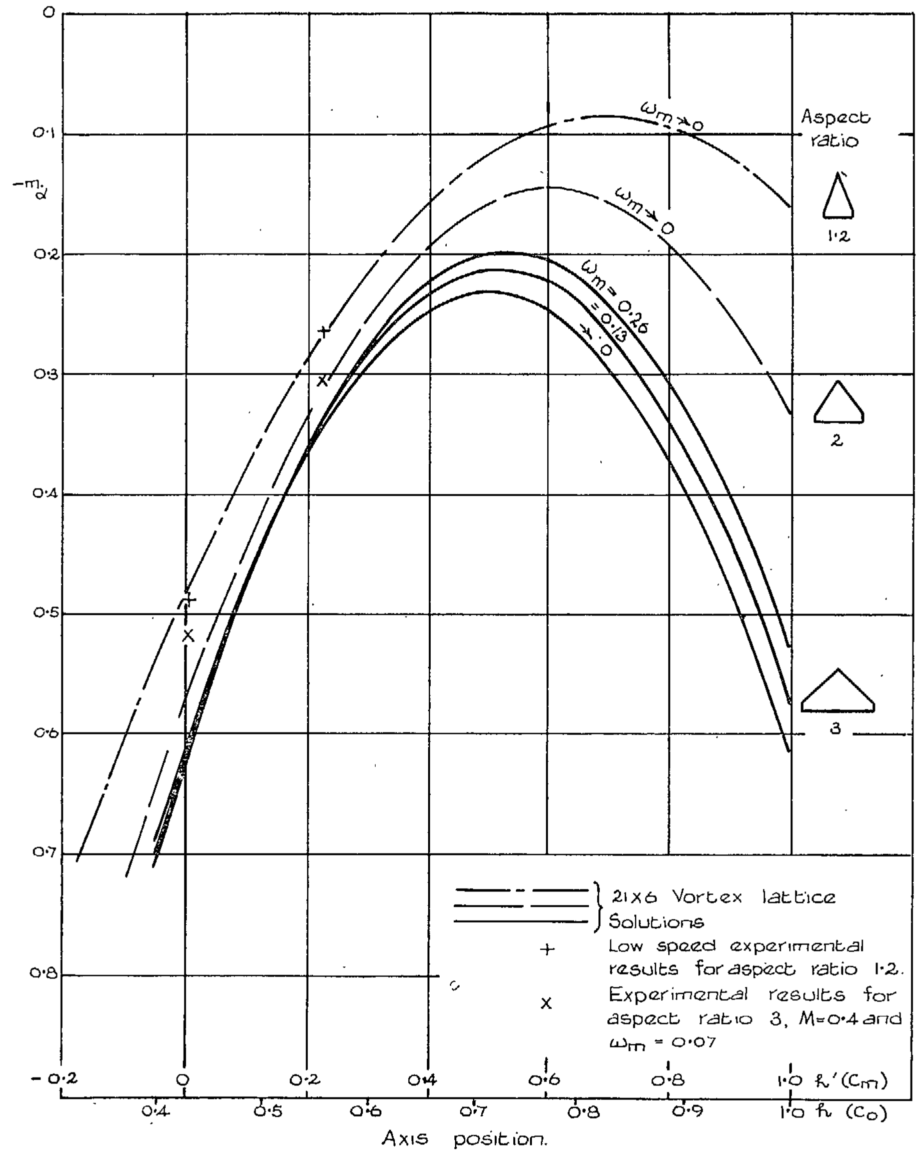

Fig. 2. Variation of $-m_{\dot{\alpha}}$ with axis position—family of delta wings with taper ratio 1/7.

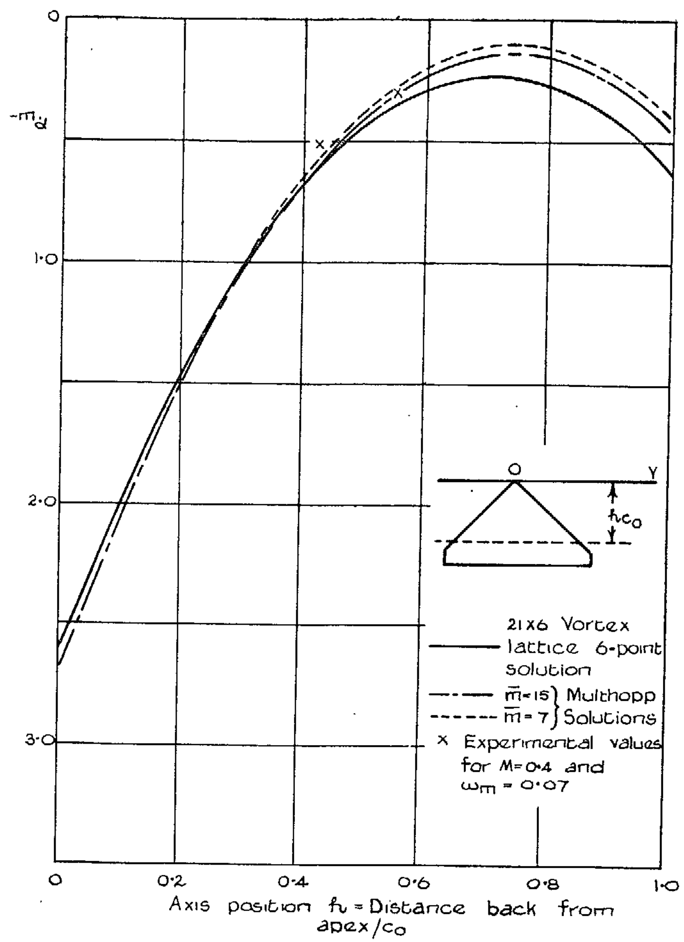

FIG. 3. Values of $-m_{\dot{\alpha}}$ for delta wing of aspect ratio 3 as $\omega_m \rightarrow 0$.

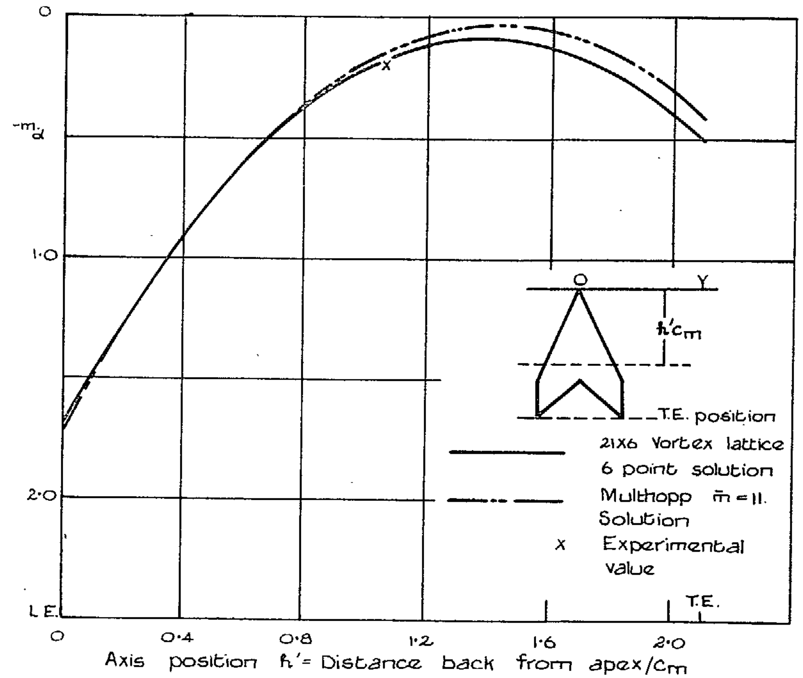

FIG. 4. Values of $-m_{\dot{\alpha}}$ for arrowhead wing of aspect ratio 1·32, as $\omega_m \rightarrow 0$.

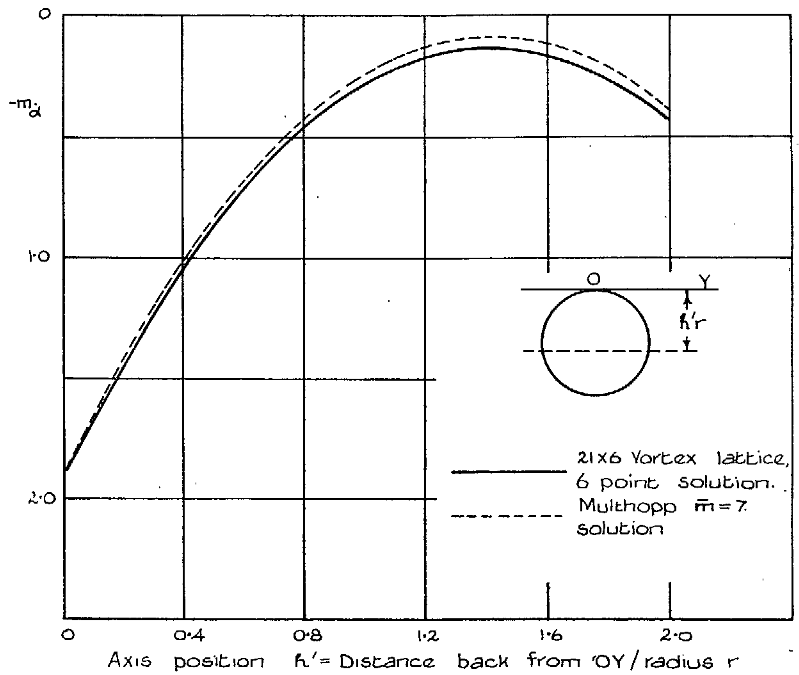

FIG. 5. Values of $-m_{\dot\alpha}$ for circular plate as $\omega_r \to 0$.

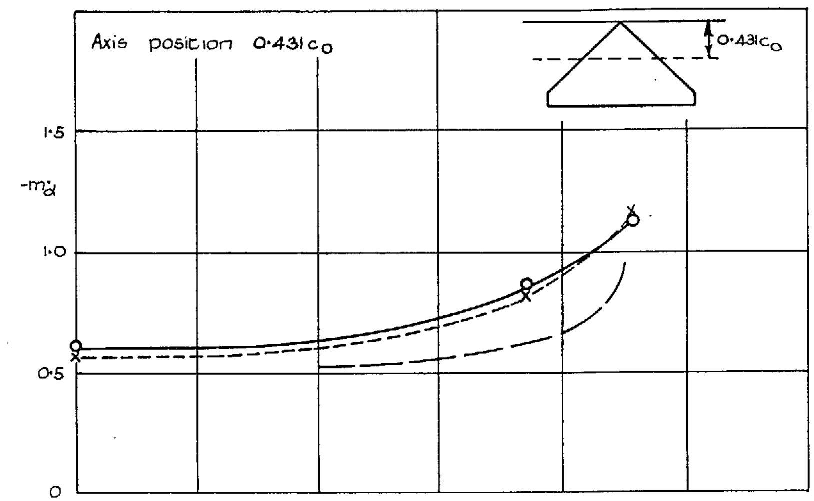

FIG. 6a.

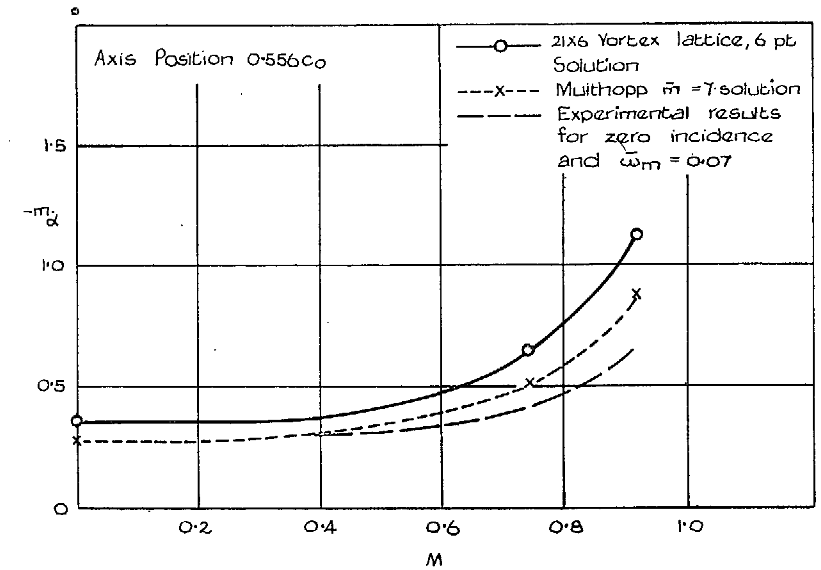

FIG. 6b.

FIGS. 6a and 6b. Effect of compressibility on the derivative $-m_{\dot\alpha}$ for delta wing $\bar{A} = 3$ as $\bar\omega_m \to 0$.

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