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Wind-tunnel Tests on the Symmetric and Antisymmetric Flutter of Swept-back Wings

By

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Summary.—Wind-tunnel tests to determine the symmetric and antisymmetric flutter characteristics of a swept-back wing are described. The investigation covers the separate experimental treatment of the symmetric and antisymmetric body freedoms over a range of wing sweepback angles. Consideration is also given to the effect on critical flutter speed and frequency of variations in overall centre of gravity position, fuselage pitching moment of inertia, fuselage rolling moment of inertia, fuselage mass and tailplane volume coefficient. The test results indicate that flutter speeds lower than the flutter speed of the wing with the root rigidly fixed may be obtained for a tailless aircraft with slight sweepback under unfavourable inertia conditions of the fuselage; for other sweepback and inertia conditions the flutter speeds are likely to be equal to, or higher than, the fixed root speeds.

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1. *Introduction.*—The theoretical approach to the flutter problems of wings with body freedoms has served to emphasise the need for thorough experimental investigation in this field. The experimental work of which the results are available does not give either a comprehensive picture of the flutter characteristics of a wing for a range of sweepback, or indicate the importance of inertia loading of the fuselage.

Early theoretical consideration of unswept wing flutter by Frazer and Duncan[1] (1931) showed that flutter speeds greater than the fixed-root flutter speed were obtained by allowing the wing the antisymmetric body freedom of roll about the root, and speeds equal to the fixed-root speed were obtained by allowing the wing the symmetric body freedoms of pitch and normal translation. Simple experiments confirmed the theory in the antisymmetric case and subsequent work was largely directed towards symmetric flutter in which it was found that the body freedoms could be disadvantageous under certain conditions that were not covered in Frazer and Duncan's work. In this report a comprehensive experimental investigation of flutter of a model wing with body freedoms is described. Separate experimental treatment of the symmetric and antisymmetric characteristics was undertaken, and the tests cover variations in wing sweepback, wing and fuselage centre of gravity position, fuselage mass, fuselage pitching and rolling moments of inertia, and tailplane volume coefficient.

The test results show that the comparative likelihood of symmetric or antisymmetric flutter having the lower flutter speed is uncertain except for a tailless wing of moderate sweepback with small fuselage pitching moment of inertia, for which symmetric flutter is likely to be the more critical. The parameters that have the greatest effect both on flutter speed and on the type of flutter are angle of sweepback of the wing, fuselage pitching moment of inertia, fuselage

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rolling moment of inertia, and tailplane volume coefficient. The position of the overall centre of gravity is of secondary importance, and the effect of fuselage mass variation is very small.

2. *Model Details.*—2.1. *Wing.*—A half-span model wing of wood and silk construction was used for the tests (Fig. 1). In the standard condition the model span (root to tip) was 36 in. and the taper ratio 3·5 to 1. A single spar of spruce at 30 per cent chord was swept back at an angle of 23 deg, and tapered uniformly from root to tip. Spruce ribs of $\frac{3}{8}$-in. thickness, and 1 in. apart, were glued to the spar parallel to the direction of the airflow. The ribs were fitted with built-in lead weights to give an inertia axis at 43 per cent chord. The leading and trailing edges of the wing were stiffened with paper glued to the ribs, but a small unstiffened section was left between each rib to avoid increasing the torsional stiffness appreciably. The ribs were covered with silk doped with a solution of Vaseline in chloroform. Further details of the wing geometry are given in Fig. 1. Balsa wood blocks were fitted to the wing at the root, and could be removed when larger angles of sweepback were required. Angles of sweepback of the spar from zero (*i.e.*, 23 deg sweep forward from the standard case) to 53 deg were obtained by rotating the wing about the main support point which was located on the spar 3·75 in. inboard of the root chord.

The weights and centre of gravity positions of the wing were obtained for angles of sweepback of the spar: 0, 13, 23, 33 and 43 deg, with the appropriate balsa wood blocks fitted. The pitching moments of inertia of the wing were measured by timing free oscillations of the suspended model about a pitch axis, and the rolling moments of inertia by timing free oscillations about the rolling axis, which corresponded to the centre line of the aircraft. Flexural and torsional stiffness tests were made at sweepback angles of 0 deg and 23 deg. At 0-deg sweepback the contour board through which the loads were applied was fixed normal to the spar, and no special precautions were taken to prevent the loads being applied through more than one rib. Resonance tests were made on the wing in the standard condition (23-deg sweepback) with the root fixed, and the first three normal modes obtained (Fig. 3). Excitation was by spring and eccentric, the modes being measured by observation of the amplitudes of pins fixed to the leading and trailing edges of seven equally spaced chordwise sections. The light construction of the model prevented the use of any of the pick-ups normally used for resonance testing. The stiffness and resonance tests were made before the wind-tunnel tests were undertaken, and it may be noted that the glued joints and silk covering of a wood and silk model often deteriorate with age and use. The normal modes and frequencies given in Table 1 and Fig. 3 therefore were almost certainly subject to slight modification as the wind-tunnel tests proceeded. The effect on flutter speed could not be detected during either the symmetric or antisymmetric series of tests but some months elapsed between each series and during this time the fixed-root flutter speed had fallen by nearly 20 per cent.

2.2. *Symmetric Body Freedom Rig.*—For the symmetric wind-tunnel tests, the wing was attached vertically to a rig which allowed the symmetric body freedoms of pitch and normal translation (Fig. 2). The rig consisted of a body (or fuselage) supported on three vertical light legs having cross-spring bearings at each end. One leg supported the body forward of the wing attachment points and on the centre-line of the body, whilst the other two legs were equally spaced on each side of the body aft of the wing, and were attached to the body through a rigid cross member. The body was thus free to move in the plane formed by the tops of three supporting legs. A drag bar prevented fore-and-aft movement. A flat plate between the body and the wing was faired into the wind-tunnel nozzle so that only the wing was in the airflow; the plate acted as a reflector for symmetric flutter of the half-wing model. Weights and springs were attached to the body to represent inertia conditions of an aircraft fuselage. Full details of the rig are given by Jordan and Smith[2] (1950).

Four rigid tailplanes were used in the tests; all were constructed of spruce and were unswept, untapered, and had the same chord (6 in.). The spans of the tailplanes were (root to tip) 6·0, 6·8, 7·7, 8·6 in. respectively. The tailplane section was RAE 101 with thickness/chord ratio of 10 per cent. Each tailplane was attached to the fuselage in the plane of the wing and at zero incidence. The tail arm was 37 in. in all tests in which a tailplane was fitted.

2.3. *Antisymmetric Body Freedom Rig.*—The wing was attached to a rig which allowed body freedom in roll about the root ; the rig is shown diagrammatically in Fig. 17. The wing, which was mounted vertically in the wind tunnel, was attached at its root end to a rigid framework. The framework was supported by two ball-races, fore and aft of the wing root, giving the wing freedom to roll about an axis parallel to the airflow, and corresponding to the centre-line of an aircraft. A lever arm extended vertically downward from the framework, and at the end of this arm a tube was fitted which projected horizontally to a point outside the tunnel working-section, where it was supported by a long vertical cord. A ball bearing joint formed the junction of the tube and the lever arm. From Fig. 17 it can be seen that by attaching weights to the tube, the rolling moment of inertia of the rig (or fuselage) could be varied, and the rig design enabled such variations to be made during the operation of the wind tunnel.

For stabilizing the model, a pair of springs was attached to the lever arm in such a way that they provided a restoring moment when the wing was displaced in roll. The flutter frequency was obtained from a make and break contact on the lever arm which operated a Veeder counter for a selected length of time. A description of this equipment is given in Appendix I.

3. *Symmetric Wind-tunnel Tests.*—3.1. *Test Programme.*—With the body locked, flutter speed and frequency were investigated at sweepback angles of 0, 13, 23, 33 and 43 deg. In some later tests with tailplane, the fixed-root flutter speeds and frequencies were also obtained at 5 deg and 9 deg sweepback.

With body freedoms in pitch and normal translation, flutter speed and frequency were measured over the same range of sweepback as for fixed root and over a wide range of fuselage inertia parameters. Three positions of the overall centre of gravity of the wing and fuselage were chosen—at the calculated wing aerodynamic centre and 1 in. forward and aft of it. For each centre of gravity position, the fuselage pitching moment of inertia about the centre of gravity and the fuselage mass were varied to cover a range of values extending beyond full-scale practice.

The effect of an added tailplane was examined since calculations have shown that body-freedom flutter would not be expected with an unswept wing aircraft having a conventional tailplane. The present investigation covered a range of tailplane volume coefficients from 0·19 to 0·28 and the inertia parameters and range of sweepback corresponded to the tests without tailplane. Four unswept, untapered tailplanes were used and in each case the overall centre of gravity was at the estimated neutral point for the wing and tailplane. The value of tail arm was kept constant throughout at 37 in., measured between wing and tailplane mean quarter-chord points. The values of tailplane volume coefficient are small by present day standards, but they were chosen to cover the range in which body-freedom flutter could occur.

The tests were made in the Royal Aircraft Establishment 5-ft Open Jet Wind Tunnel using the technique of Jordan and Smith[2] (1950). The procedure is described briefly below.

For any angle of sweepback the required position of the overall centre of gravity is obtained by the addition of weights to the fuselage. For stabilisation of the rig two pairs of springs are attached to the fuselage, forward and aft of the centre of gravity. Under steady oscillating conditions the springs act as negative masses and therefore a correction must be applied to the mass and moment of inertia of the fuselage, appropriate to the spring stiffnesses and frequency of oscillation. If the two pairs of springs are of equal stiffness and are equidistant from the actual centre of gravity, then the position of the effective centre of gravity will be unaltered by any change in frequency. In this condition the pitching moment of inertia is varied by the spreading of equal weights at equal distances from the centre of gravity, and the mass is varied by adding weights at the centre of gravity. Where low values of mass are required stiff springs are used, and for low values of moment of inertia they are widely spaced. If the frequency changes, through change in the mode of flutter, then the equivalent negative masses of the springs also change, thereby giving a discontinuity in the equivalent total fuselage mass and pitching moment of inertia. The range covered by the discontinuity may, however, be reduced by the judicious use of springs of various stiffnesses.

The critical flutter speed is measured when the oscillation is just self-maintained, and the frequency is determined from readings of a Veeder counter operated from a make-and-break contact connected to the fuselage. The counter and a stop watch are started simultaneously and stopped after any desired time by means of an electrical system[1].

Preliminary wind-tunnel tests indicated that the fore-and-aft stiffness of the wing was low, and considerable fore-and-aft oscillation of the wing occurred in the fluttering condition. If allowed to continue, this would undoubtedly have caused early failure of the wing; so a long tethering cord was attached to the wing tip. The tethering cord provided only a negligible stiffness in pitch or normal translation, and a series of tests with and without tethering cord showed that this had no effect on flutter speed and frequency.

The failure of the model spar after seventy flutter tests led to a revision of the test programme, since it was evident that wood and silk models would only withstand a limited amount of tunnel testing. In view of this, the number of cases to be tested was cut considerably, though without restricting the range of the tests. In order to safeguard the model further a wing grab system was installed, and the amplitudes of oscillation and time of fluttering the wing were kept to the minimum for recording flutter speed and frequency. All the test results given were obtained after the spar had been repaired; test results before failure are not given since the repair caused some alteration in the flexural and torsional stiffnesses.

3.2. *Test Results.*—The curves of flutter speed and frequency plotted against angle of sweepback for the fixed root condition are given in Fig. 4. It will be seen that a minimum value of flutter speed occurs at a sweepback angle of approximately 11 deg; this result agrees well with that of Molyneux[3] (1950) who also obtained a minimum at 11 deg with a wing of similar design. He found, however, that the flutter frequency was ' comparatively insensitive to angle of sweepback, but increased slightly with increasing sweepback'. In the present tests a 10 per cent change in frequency was recorded over the sweepback range zero to 43 deg, the curve of frequency against sweepback being of similar shape to the flutter speed curve, but having a minimum at approximately 7 deg. There is, however, a difference in frequency of only 0·05 cycles per second between the maximum and minimum values in the range of sweepback zero to 13 deg. These small differences, which were obtained by stroboscopic methods, however, are not very reliable.

Figures 5 to 9 give the curves of flutter speed and frequency plotted against the total pitching moment of inertia about the overall centre of gravity position of the fuselage and the wing. At each angle of sweepback, the overall centre of gravity position has been varied by 1 in. fore-and-aft of the calculated position of the wing aerodynamic centre; this total shift corresponds to 9·5 per cent of the wing root chord in the standard condition. Two distinct types of flutter can occur for any angle of sweepback and centre of gravity position, and the value of the total pitching moment of inertia determines which type occurs at the lower flutter speed. ' Symmetric body-freedom ' flutter, which occurs at the lower values of moment of inertia, is characterised mainly by body pitching and wing flexure. ' Symmetric disturbed-root ' flutter, which occurs with higher values of moment of inertia, has only small root movements and differs only slightly from fixed-root flutter in mode. As with body-freedom flutter the body or root movement is mainly pitching, but the relative amplitude of the body is very much smaller in disturbed-root flutter than in body-freedom flutter.

Starting from values of pitching moment of inertia in the region of body-freedom flutter, the flutter speed increases, and the frequency decreases slightly, as the moment of inertia is increased, until a transition point is reached. As the moment of inertia is increased beyond the value at the transition point, disturbed-root flutter occurs, and the flutter speed drops asymptotically to the fixed-root value, the frequency being very close to the fixed-root flutter frequency. At the transition, the flutter speed exceeds the corresponding fixed-root flutter speed by one or two per cent, and there is a discontinuity in the frequency curve.

The effect of variation in the overall centre of gravity position may be seen in Fig. 14, in which the values of pitching moments of inertia at transition are given in terms of the ratio of the total pitching moment of inertia to the appropriate pitching moment of inertia of the wing, which varies with sweepback. The region over which body-freedom flutter occurs is seen to contract as the centre of gravity is moved aft, and to expand as it is moved forward.

It was found from preliminary tests that the effects of variation of fuselage mass, as distinct from pitching moment of inertia, were small and within experimental scatter over a wide range of variation, and for this reason no detailed investigation of mass effect has been made in these tests.

The variations of flutter speed and frequency with pitching moment of inertia under various conditions of tailplane volume coefficient is given in Figs. 10 to 13. It will be seen that the general pattern of the curves remains similar to the ' no tailplane ' condition, but that the value of the moment of inertia at the transition varies rapidly with tailplane volume coefficient. This effect can best be seen in Figs. 15 and 16. For values of tailplane volume coefficient up to 0·223 the boundary curve of the transition is similar to, and lies close to the curve for no tailplane. As the tailplane volume coefficient is increased, the area of the graph in which body freedom flutter can occur becomes progressively smaller. With a tailplane volume coefficient of 0·284, which was the largest value used in the tests, the boundary curve lies in the region of small positive values of pitching moment of inertia, and it may be deduced that for tailplanes having volume coefficients in excess of about 0·3 the boundary would be wholly in the region of negative pitching moment of inertia.

Examination of the frequency values given in the curves of Figs. 10 to 13 shows that the presence of a tailplane has no appreciable effect upon the frequency of body-freedom flutter.

3.3. *Discussion of Symmetric Tests.*—3.3.1. *Wing—fixed root.*—It has already been mentioned in section 3.2 that the wind-tunnel test results in the fixed-root condition show good agreement with those of Molyneux[3] as far as flutter speed is concerned. The minimum values of flutter speed in both the present tests and those of Molyneux occur at angles of sweepback that are virtually the same. Molyneux was unable to obtain flutter speed values close to the minimum, whereas in the present tests the flutter speed was measured at sweepback angles of 0, 5, 9 and 13 deg and the results enable the minimum to be obtained with a fair degree of accuracy. This result confirms the validity of the sweepback correction that Molyneux suggested should be made to the torsional stiffness criterion of R. & M. 2154.

The application of Molyneux's modified torsional stiffness criterion[3] to the present wing does not, however, yield a satisfactory result. The criterion relies on the measurement of the wing torsional stiffness in the zero sweepback position. At zero sweepback of the present wing the ribs are at an angle of 23 deg to the line of flight, and the fixing of a contour board for wing loading effectively locks together a number of adjacent ribs. The interpretation of the measured torsional stiffness is therefore difficult, and the flexural axis position obtained from such a test will be affected by the loading condition. If, in spite of this, the criterion is applied, a flutter speed of 80 ft per second is calculated as compared with the measured flutter speed at zero sweepback of 102 ft per second. An explanation of this discrepancy may lie in the nature of the normal modes of the present wing (Fig. 3). For the wings of Ref. 3 the first three modes in ascending frequency were, (*a*) fundamental flexure, (*b*) fundamental torsion, (*c*) flexural overtone. In the case of the present wing, the flexural overtone has a lower frequency than the fundamental torsion mode, and it is highly probable that this has an effect on the flutter speed. The application of the criterion to the present wing, however, does give a flutter speed which is on the 'safe' side of (*i.e.*, less than) the measured value.

3.3.2. *Wing—free root.*—The most important result obtained from the symmetric tests is the variation of sweepback with pitching moment of inertia at the transition. In Fig. 14, the values of total pitching moment of inertia at the transition points have been divided by the wing pitching moments of inertia about the centre of gravity for the appropriate sweepback angle to give a non-dimensional ratio. The wing pitching moment of inertia varies with sweepback, however, and Fig. 16 has been drawn to indicate the effect of sweeping back a wing while keeping the fuselage inertia conditions constant. In Fig. 16 the pitching moment of inertia is expressed as the ratio of fuselage pitching moment of inertia to that of the wing at zero sweepback, and the effect of varying sweepback is shown simply by a vertical line on the diagram.

3.3.3. *Wing—free root. Comparison with delta wing.*—The general shape of the flutter speed and frequency curves of Figs. 5 to 9 is similar to those obtained with a delta wing by Gaukroger, Chapple and Milln[4] (1950), but there are two differences that may be noted. The flutter speed in body-freedom flutter of the delta wing rises to about 20 per cent above the fixed-root flutter speed before the transition point is reached. With the present wing the rise above fixed-root speed is rarely more than 2 or 3 per cent. There is some practical difficulty in testing cases close to the transition point in the disturbed-root flutter region ; the difficulties arise from the need for springs of high stiffness at small angles of sweepback where aerodynamic stability is low. The use of stiff springs gives rise to a large change in effective pitching moment of inertia as the frequency changes, and a negative value of the moment of inertia at the lower frequency may be equivalent to a large positive value at the higher frequency. If values of the pitching moment of inertia between these limits are to be investigated, the springs must be widely spaced at the high frequency and close together at the low frequency. With the present rig, space considerations prevent the former solution and, as already mentioned, stability considerations the latter. At the low frequency, additional masses may be used to increase the moment of inertia of the fuselage, but no such means can be used for reducing the moment of inertia at the high frequencies. For small angles of sweepback, therefore, the transition point is best obtained from the curve of body-freedom flutter speed, which can be extrapolated until it cuts the line corresponding to fixed-root flutter speed. The value of the flutter speed at the transition point has not been obtained in all cases owing to the need to restrict the number of tests and preserve the wing (section 3.1) but there is sufficient evidence from the tests to indicate that the maximum flutter speed is only 2 or 3 per cent above the fixed-root value, and considerably less than in the case of a delta wing.

A second difference between the characteristics of the delta and present wings was in the amount of fuselage motion in disturbed-root flutter. For the delta wing the fuselage motion was sufficiently large for the frequency to be obtained from the make-and-break contact attached to the fuselage, but in the present tests fuselage movement was extremely small and frequencies were measured with a stroboscope. The frequencies measured were invariably equal to the fixed-root flutter frequency, and although there must be some slight frequency difference (which could be measured for the delta wing) stroboscopic methods were not sufficiently accurate to detect it in the present series of tests.

3.3.4. *Wing and tailplane—free root.*—Figs. 15 and 16 show that the presence of a tailplane reduces the range of conditions of angle of sweepback and pitching moment of inertia under which body-freedom flutter occurs. The largest value of tailplane volume coefficient tested was 0·284 but this figure is so much less than the normal value of current design tailplane volume coefficients that the curves of Figs. 14 and 16 are unrepresentative of real aircraft. The average value of tailplane volume coefficient for current aircraft design is about 0·6, and it can readily be seen that the appropriate boundary curve for such a figure would be well inside the region of negative moment of inertia. The margin of safety is, in fact, so great that it is safe to conclude that any conventional aircraft having a tailplane volume coefficient representative of current design will be free from symmetric body-freedom flutter.

3.3.5. *Frequency parameter.*—The range of mean frequency parameter covered in the symmetric tests is from 0·25 to 0·75. For body-freedom flutter the values of frequency parameter lie between 0·25 and 0·5. The value decreases along any given curve of flutter speed as the moment of inertia is increased, since the frequency drop is small compared to the rise in flutter speed. In the case of disturbed-root and fixed-root flutter, the frequency parameter lies between 0·6 and 0·75.

3.3.6. *Value of the tests.*—Although the test results indicate the importance of sweepback and pitching moment of inertia and, by comparison, the lesser significance of centre of gravity position, the limitations of the work should be borne in mind. The only wing parameter that has been varied is angle of sweepback, and though it is unlikely that variations of wing inertia and stiffness distribution, or of wing plan-form, would cause a marked change in the general pattern of the results, it cannot be assumed that quantitative values derived from the tests may be used on other wings. For this reason the wing tests without tailplane should be regarded only as a guide to the flutter characteristics of tailless aircraft. The results may also be of assistance in the selection of degrees of freedom for flutter calculations but only in cases where the model is representative of the aircraft concerned.

The results of the tests with tailplane have a wide application. It has been pointed out that the safety margin between current values of tailplane volume coefficient and the maximum value necessary to prevent symmetric body-freedom flutter within the positive range of total pitching moment of inertia in the present tests is large. It is most unlikely that any variation of wing parameters could reduce this margin appreciably, and therefore the influence of tailplane in the present tests can be extended to aircraft generally. It can therefore be inferred that the number of degrees of freedom that must be considered in wing symmetric flutter calculations for a tailed aircraft may be restricted to those involving only the wing normal modes.

3.3.7. *Conclusions* (*symmetric flutter*).—The following conclusions may be drawn from the symmetric test results :

(*a*) With the wing root fixed the results agree well with those of previous tests and give a minimum flutter speed at a sweepback of 11 deg. The frequency variation is greater than had previously been encountered on similar models.

(*b*) For a wing having symmetric body freedoms and no tailplane, two types of flutter (body-freedom and disturbed-root) may be obtained. The occurrence of either type depends mainly on the angle of sweepback, and the value of the fuselage pitching moment of inertia. The effects of variations in fuselage mass and in overall centre of gravity position are small, compared with the sweepback and pitching moment of inertia effects.

(*c*) The body-freedom flutter speed increases with increase in fuselage pitching moment of inertia up to a maximum which is 2 or 3 per cent above the corresponding fixed-root flutter speed. For moments of inertia above this, the flutter is of the disturbed-root type, and the flutter speed is approximately equal to the fixed-root speed.

(*d*) The flutter frequency with body-freedom flutter is low compared with that of fixed-root and falls slightly with increase of pitching moment of inertia. The frequency of disturbed-root flutter is approximately equal to that of fixed-root. A discontinuity in frequency occurs when the type of flutter changes.

(*e*) The effect of tailplane on wing flutter with body freedoms is to reduce the value of fuselage pitching moment of inertia at which the change from body-freedom to disturbed-root flutter occurs. The flutter speeds and frequencies in both types of flutter are similar to those for the wing alone.

(*f*) No body-freedom flutter occurs for any positive values of fuselage pitching moment of inertia provided the value of tailplane volume coefficient exceeds 0·3. An aircraft having a tailplane volume coefficient comparable in value to that of present day aircraft will not be susceptible to symmetric body-freedom flutter.

4. *Antisymmetric Wind-tunnel Tests.*—4.1. *Test Programme.*—The angle of sweepback of the wing spar was varied from 13 to 53 deg in intervals of 10 deg. At each angle of sweepback the fuselage rolling moment of inertia was varied from large positive values down to zero, and, in some cases, negative values. The latter were obtained by using stabilizing springs of high stiffness, which, under steady oscillating conditions, acted as large negative masses, and thus provided an effectively negative rolling moment of inertia. The maximum positive values of fuselage rolling moment of inertia were higher in relation to wing rolling moment of inertia than normally occur on conventional aircraft.

The tests were made in the Royal Aircraft Establishment 5-ft Open Jet Wind Tunnel. The rolling axis of the rig coincided with the edge of the tunnel nozzle, and the leading edge at the wing root was 12 in. downstream of the nozzle. Whereas in making symmetric flutter tests a flat plate is placed at the wing root to act as a reflector for the half-wing model (section 2.2) the plate was removed in the present tests. It may be argued that, in an open-jet tunnel, with a half-span model having its root at the edge of the air stream, the absence of any reflecting surface is possibly the most satisfactory arrangement aerodynamically for antisymmetric oscillatory tests.

The method of testing was to stabilize the wing in such a way that the effective moment of inertia of the fuselage (including the spring correction) was negative in the fluttering condition. After measuring the critical flutter speed and frequency the tunnel speed was reduced slightly; weights were then added to the loading tube (Fig. 17) and the new critical flutter conditions found. This procedure was repeated until all the rolling moment of inertia values required at one sweepback angle had been obtained. The advantage of this technique is that it enables a set of results to be obtained quickly, and under conditions of temperature and humidity that remain practically constant throughout the time needed to do the test. With wood and silk models (which are often affected by temperature and humidity changes) this is an important consideration, and the quicker method of test reduces the experimental scatter of the results.

4.2. *Test Results.*—The results of the tests are shown in Figs. 18 and 19, in which flutter speed and frequency are plotted against fuselage rolling moment of inertia for each of the five angles of wing sweepback 13, 23, 33, 43 and 53 deg. It will be seen that the flutter speed and frequency curves have the same general characteristics at all sweepback angles. The complete curves are best shown at 53-deg sweepback. Starting at negative values of the fuselage rolling moment of inertia the flutter speed is above, and the frequency below, the corresponding fixed-root values. This form of flutter may be called ' antisymmetric body-freedom flutter,' since it is associated with a greater amplitude in the rolling mode than the second form of antisymmetric flutter which will be described later. As the rolling moment of inertia of the fuselage is increased the flutter speed rises, and the frequency decreases, until the type of flutter changes abruptly and there is a discontinuity in the frequency curve; the frequency is now higher than the corresponding fixed-root frequency. This second form of flutter may be called ' antisymmetric disturbed-root flutter,' and is characterised by having only a small displacement in roll; the wing mode closely resembles the fixed-root flutter mode. When the rolling moment of inertia is increased further, both flutter speed and frequency fall and become asymptotic to the fixed-root values. The amplitude of the wing in the rolling freedom also diminishes such that at high values of the rolling moment of inertia there is no visible rolling amplitude.

The effect of reducing the wing sweepback is to reduce the value of the fuselage rolling moment of inertia at which the transition from disturbed-root to body-freedom flutter occurs. Thus at 53 deg the rolling moment of inertia value at the transition point is small and positive; at 43 deg it is practically zero, and at 33 deg the value is negative. At 23 and 13 deg the transition point has not been obtained, and it may be deduced from the shape of the flutter speed and frequency curves that it lies well inside the region of negative rolling moment of inertia. The general level of flutter speed is lower at the smaller sweepback angles and is due to the change in fixed-root flutter speed with sweepback (section 3.2).

4.3. *Discussion of Antisymmetric Tests.*—4.3.1. *Wing—free root.*—Over the range of sweepback tested the results confirm the conclusions drawn from earlier work of Frazer and Duncan[1], and Houbolt[5], that flutter speeds for the antisymmetric cases are higher than the corresponding fixed-root values. It may be noted, however, that the body-freedom flutter portion of the speed curve at 53 deg sweepback (Fig. 19) is falling rapidly with decreasing rolling moment of inertia. This might indicate that at some value of rolling moment of inertia outside the range of the present tests the flutter speed falls below the fixed-root speed. However the 33-deg sweepback curves (Fig. 18) show the body-freedom flutter speed approaching the fixed-root speed almost asymptotically with decrease of moment of inertia. The same tendency occurs at 43-deg sweepback (Fig. 19) although the two points concerned are so close together that they do not constitute a convincing confirmation. This question, however, would only be of practical importance in cases where body-freedom flutter occurred largely in the region of positive values of rolling moment of inertia. The present tests indicate that, for this to be so, the angle of sweepback would certainly have to be more than 60 deg.

The main limitation of the present series of tests is that no critical flutter conditions were examined at values of rolling moment of inertia close to the transition. The reason is that the use of springs as negative masses gives rise to difficulty when investigating a region of inertia variation in which an abrupt change of flutter frequency occurs. In the present antisymmetric tests the overall shape of the flutter speed curve was flatter than in the earlier symmetric tests, and the range of inertia that could not be investigated was wide in relation to the total range of inertia variation.

In full-scale practice the value of fuselage rolling moment of inertia would generally be about 0·1 of the total wing value, although with highly swept-back wings the ratio would be somewhat greater. For a flying-wing aircraft with no fuselage the ratio would be zero. In Figs. 18 and 19, arrows on the rolling moment of inertia scale indicate the value 0·2 times wing moment of inertia for the appropriate sweepback angle. Values of fuselage moment of inertia between these values and zero represent conditions likely to be encountered in practice.

It is of some interest to make a comparison of the results of the present tests with the theoretical work of Houbolt[5] (1949). Both investigations show that an antisymmetric form of body-freedom flutter may occur, but whereas Houbolt showed that it could occur for real (positive) rolling moments of inertia for sweepback angles between 0 and 45 deg, the wind-tunnel tests indicate a real branch only for sweepback angles greater than 43 deg. The fundamental flexural and torsional modes of the wind-tunnel model (Fig. 3) resembled those assumed by Houbolt but, in addition, the model had a flexural overtone mode with a frequency lying between the two fundamental mode frequencies. Houbolt also assumed two-dimensional flow and made a correction for the effect of sweepback on the aerodynamic forces. Houbolt's investigation also showed that when the angle of sweepback was small, and the rolling moment of inertia of the fuselage zero, body-freedom flutter could occur at a speed slightly below the fixed-root flutter speed. This does not agree with the results of the present tests ; flutter speeds obtained in the wind tunnel were higher for antisymmetric body-freedom flutter than for fixed-root flutter under all the conditions of sweepback and fuselage rolling moment of inertia that were tested.

The assumptions that Houbolt had to make may well account for the discrepancy between the theoretical and experimental investigations, but on the other hand the wind-tunnel results should not be taken as applying to wings of different aerodynamic or structural characteristics.

However, the general indication from both investigations is that antisymmetric body-freedom flutter is unlikely to occur, for practical designs, at speeds below the fixed-root flutter speed.

4.3.2. *Conclusions* (*antisymmetric flutter*).—The following conclusions may be drawn from the antisymmetric test results :

(*a*) Two forms of antisymmetric flutter were obtained, body-freedom and disturbed-root. The occurrence of either form depends mainly on angle of sweepback and fuselage rolling moment of inertia.

(*b*) The body-freedom form of flutter occurs only at large sweepback angles. The flutter speed increases with increasing fuselage rolling moment of inertia, and the flutter frequency decreases.

(*c*) All the body-freedom flutter speeds measured were above, and the frequencies below, the corresponding fixed-root values.

(*d*) The flutter speed and frequency of disturbed-root flutter are both higher than the fixed-root values, and both approach the fixed-root values when the fuselage rolling moment of inertia is increased.

(*e*) It seems unlikely that for any conventional aircraft antisymmetric flutter would occur at speeds below the fixed-root flutter speed.

5. *Comparison of Symmetric and Antisymmetric Characteristics.*—The same model wing was used for both symmetric and antisymmetric tests and a direct comparison of the results of both investigations is therefore possible. It should be noted, however, that in the antisymmetric tests the flutter speeds with fixed root are lower than the corresponding values in the symmetric tests. This reduction was due to ageing of the model over the period of some months which occurred between the two series of tests (section 2.1). During this period the model was used for a large number of flutter and resonance tests which resulted in a stiffness reduction. It is therefore advisable in comparing the symmetric and antisymmetric tests results, to consider the flutter speeds in relation to the appropriate fixed-root values.

In the antisymmetric case, whichever form of flutter occurs, body-freedom or disturbed-root, the flutter speeds are never less than the fixed-root flutter speed. In the symmetric case, disturbed-root flutter gives flutter speeds above the fixed-root flutter speed, but body-freedom flutter can give flutter speeds below it. It may therefore be concluded that both symmetric and antisymmetric flutter must be considered except for those conditions in which symmetric body-freedom flutter occurs at speeds below the fixed-root flutter speed, and in which symmetric flutter will therefore occur before antisymmetric flutter. From the investigation of symmetric flutter, such conditions are provided by a tailless wing of small or moderate sweepback (up to about 30 deg) with small fuselage pitching moment of inertia.

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APPENDIX I

Frequency Counter for Antisymmetric Tests

The aim of the design of the counter used in the antisymmetric tests was to provide a means of counting the number of cycles of an oscillation, over a period of either 5 or 10 seconds, in such a way that the time period would be accurate, and the operation automatic after the mechanism had been started.

The basis of the method is that a half-second pulse, controlled by a clockwork mechanism, operates the drive of a uniselector. One bank of the uniselector forms a counting circuit, which is connected in series through a Veeder counter and the make-and-break contact whose frequency is to be measured. The counting circuit operates for 10 to 20 contacts (which must be of the ' make-before-break ' type) giving 5 or 10 second periods. The uniselector is started by a push button on the driving circuit, and stops operating when the rotating arm reaches its original position. The wiring diagram is shown in Fig. 20. An indicator lamp has been included in the circuit to show that the make-and-break contact is operating correctly. The circuit is driven from 24 volts D.C.

The largest possible error is one cycle in the measurement time, since the clockwork half-second pulse is extremely accurate, and its error may be neglected. At 5 cycles per second, therefore the error may be 4 per cent if a 5-second measurement is made, or 2 per cent in 10 seconds. Since a 10-second count is generally made in flutter tests, the error is small enough to be acceptable.

TABLE 1

Wing inertia details

Spar sweepback (deg)	Wing weight (lb)	I_P (lb in.^{2})	I_R (lb in.^{2})
0	5·26	41·3	—
13	5·10	105·5	1458
23	4·95	176·4	1281
33	4·83	274·8	1042
43	4·67	480·9	766
53	4·67	—	541

I_P = Pitching moment of inertia about the axis through the aerodynamic centre.

I_R = Rolling moment of inertia about the axis through the root attachment points.

Wing stiffnesses

Spar sweepback (deg)	m_θ at 0·7 span measured in line of flight (lb ft/radian)	l_ϕ at 0·7 span (lb ft/radian)
0	11·6	287
23	10·1	149

Wing flexural axis at 23-deg sweepback : 30 per cent chord.

Wing flexural axis at 0-deg sweepback : 44 per cent chord*.

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* Based on measurements at loading section (*see* section 2.1).

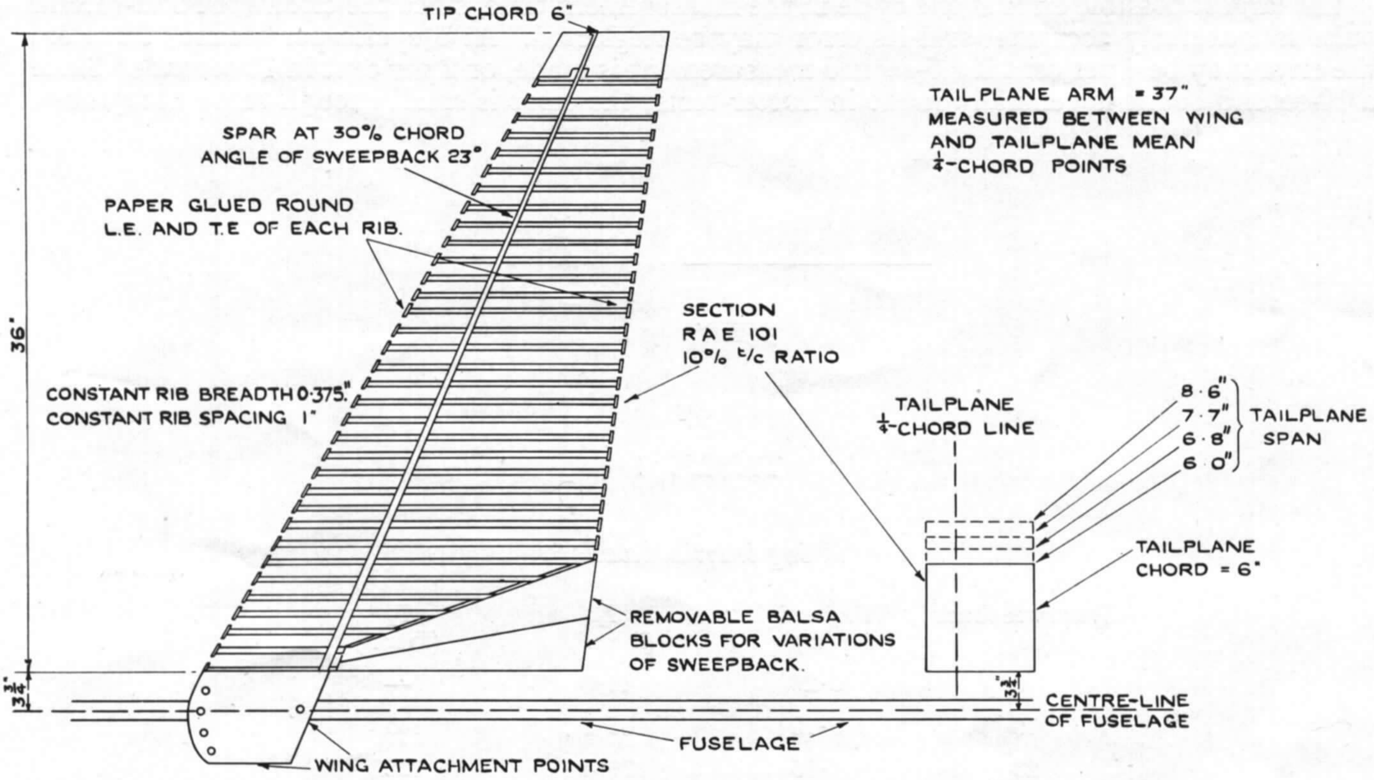

FIG. 1. Wing and tailplane details.

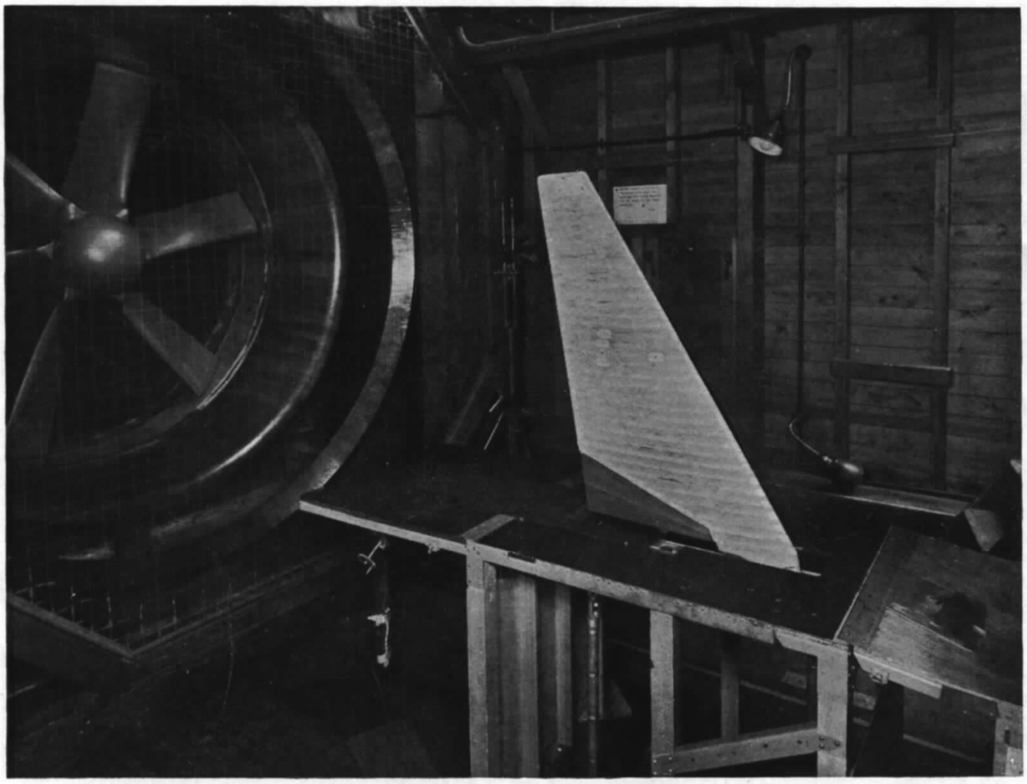

FIG. 2. Model rigged for test in R.A.E. 5-ft Wind Tunnel.

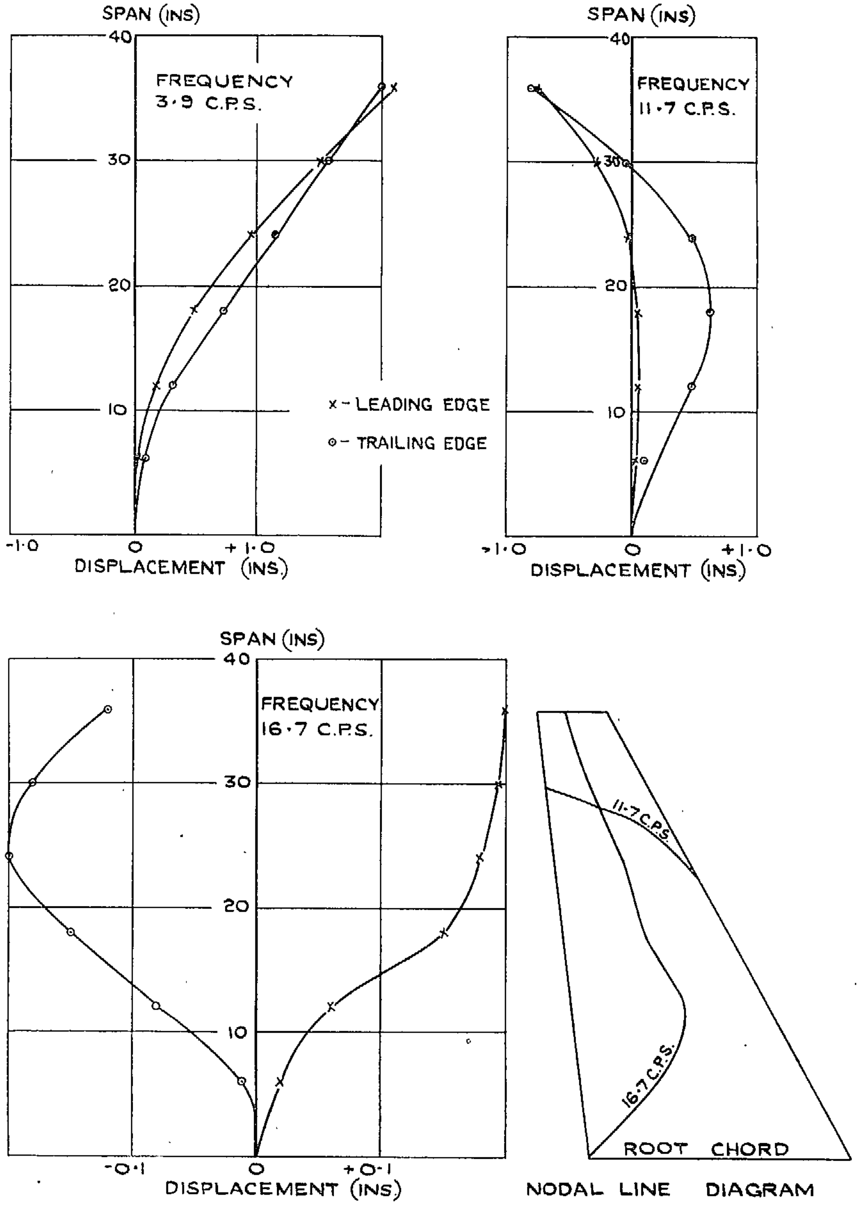

FIG. 3. Wing normal modes at 23-deg sweepback—fixed root.

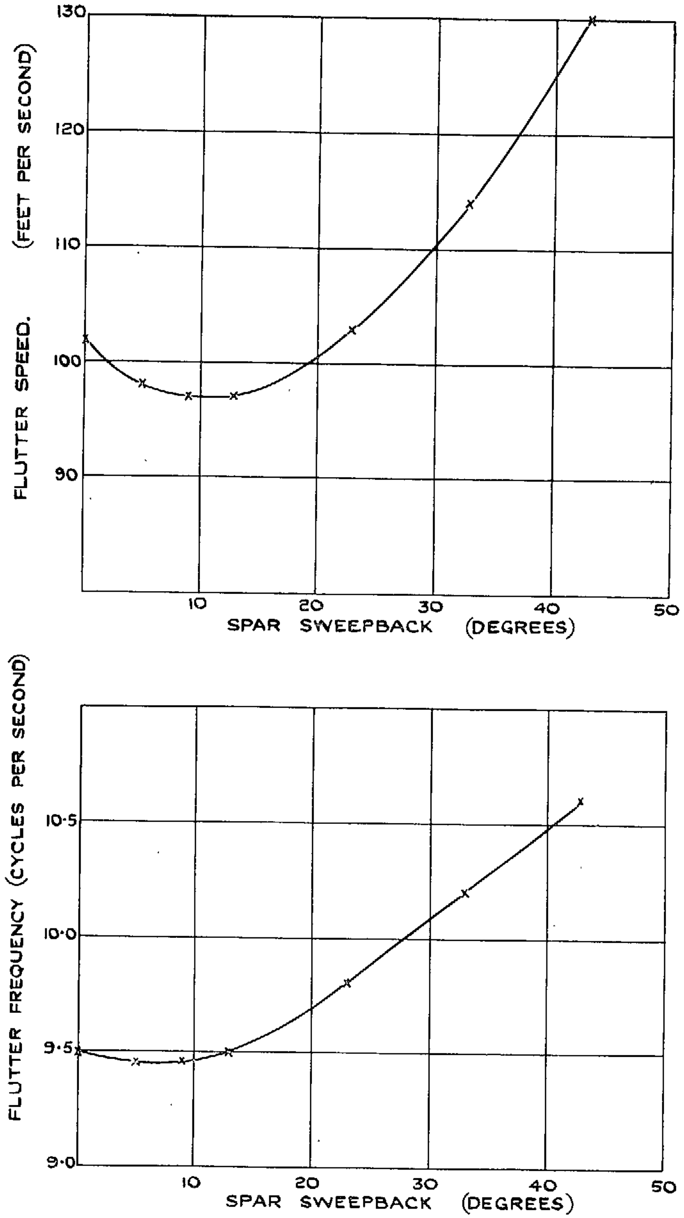

FIG. 4. Variation of fixed-root flutter speed and frequency with sweepback.

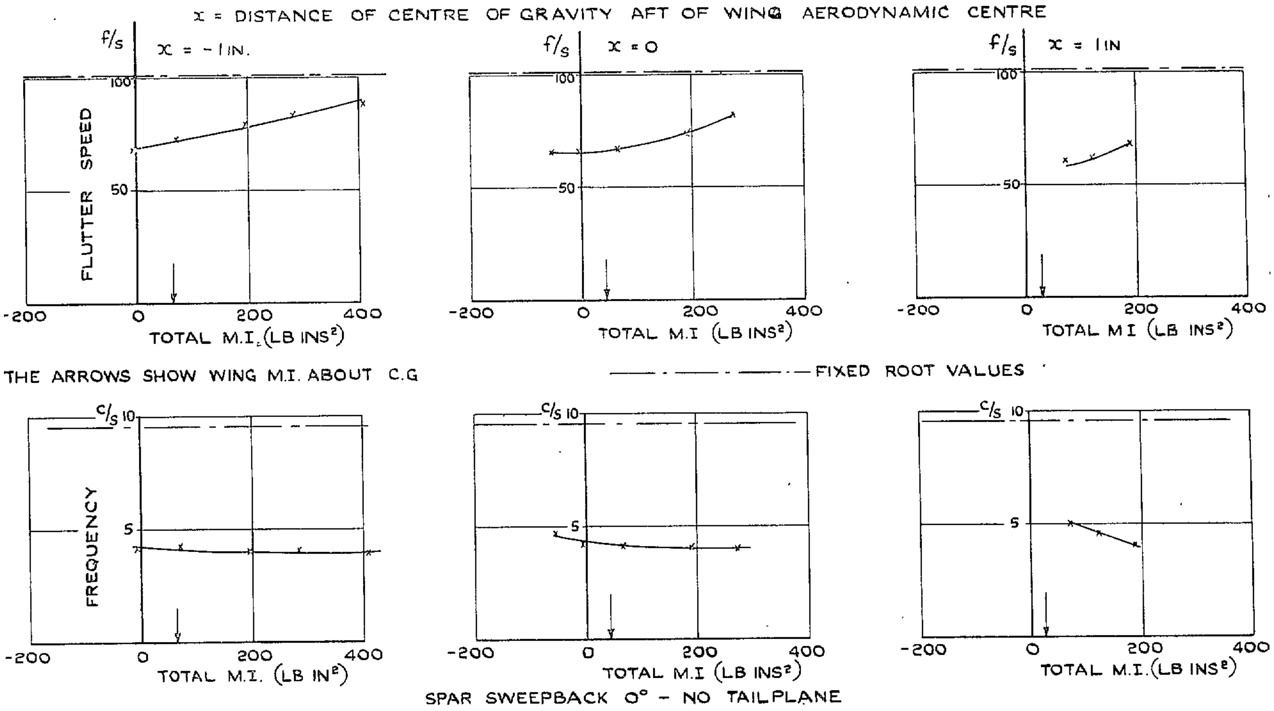

FIG. 5. Flutter speed and frequency plotted against total pitching moment of inertia.

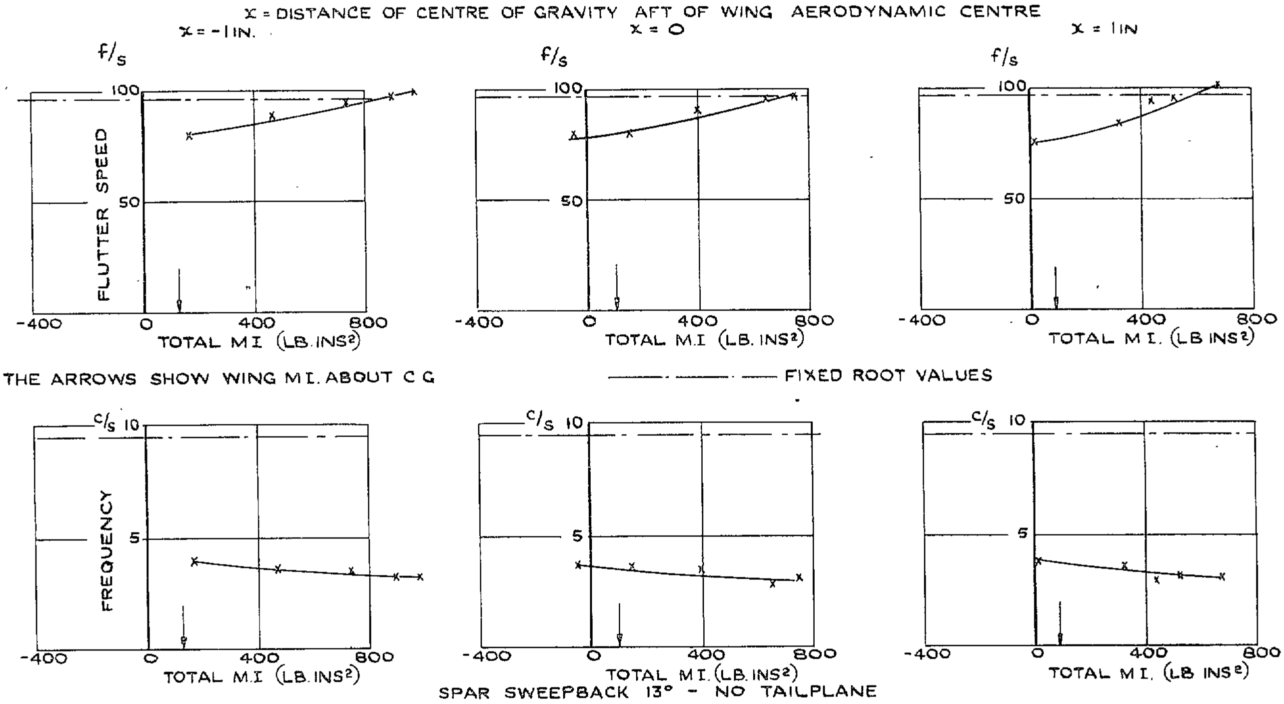

FIG. 6. Flutter speed and frequency plotted against total pitching moment of inertia.

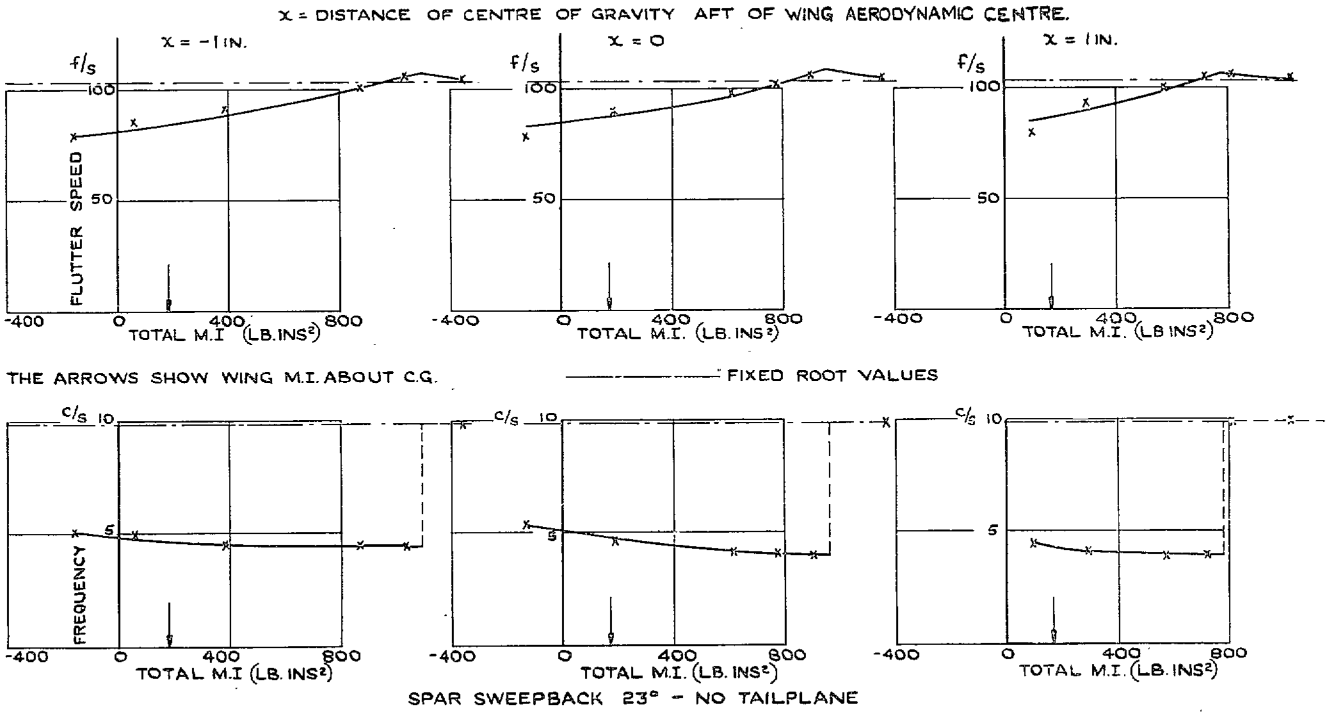

FIG. 7. Flutter speed and frequency plotted against total pitching moment of inertia.

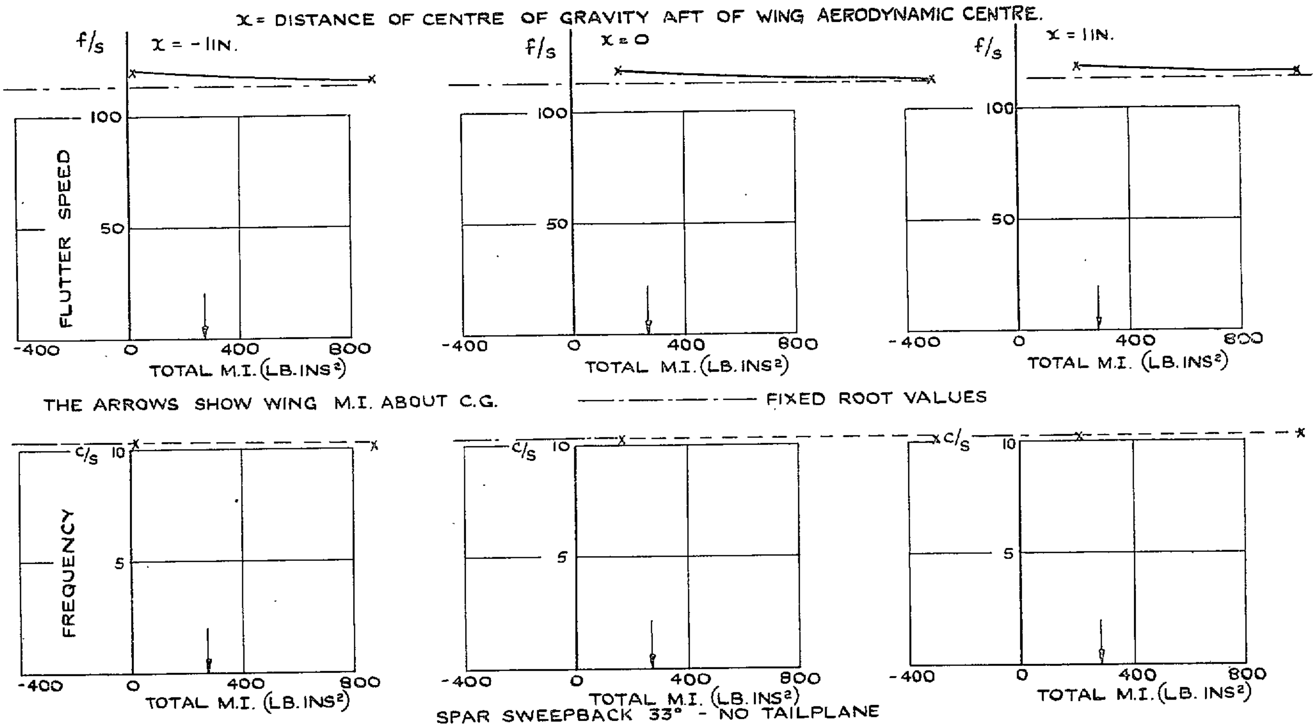

FIG. 8. Flutter speed and frequency plotted against total pitching moment of inertia.

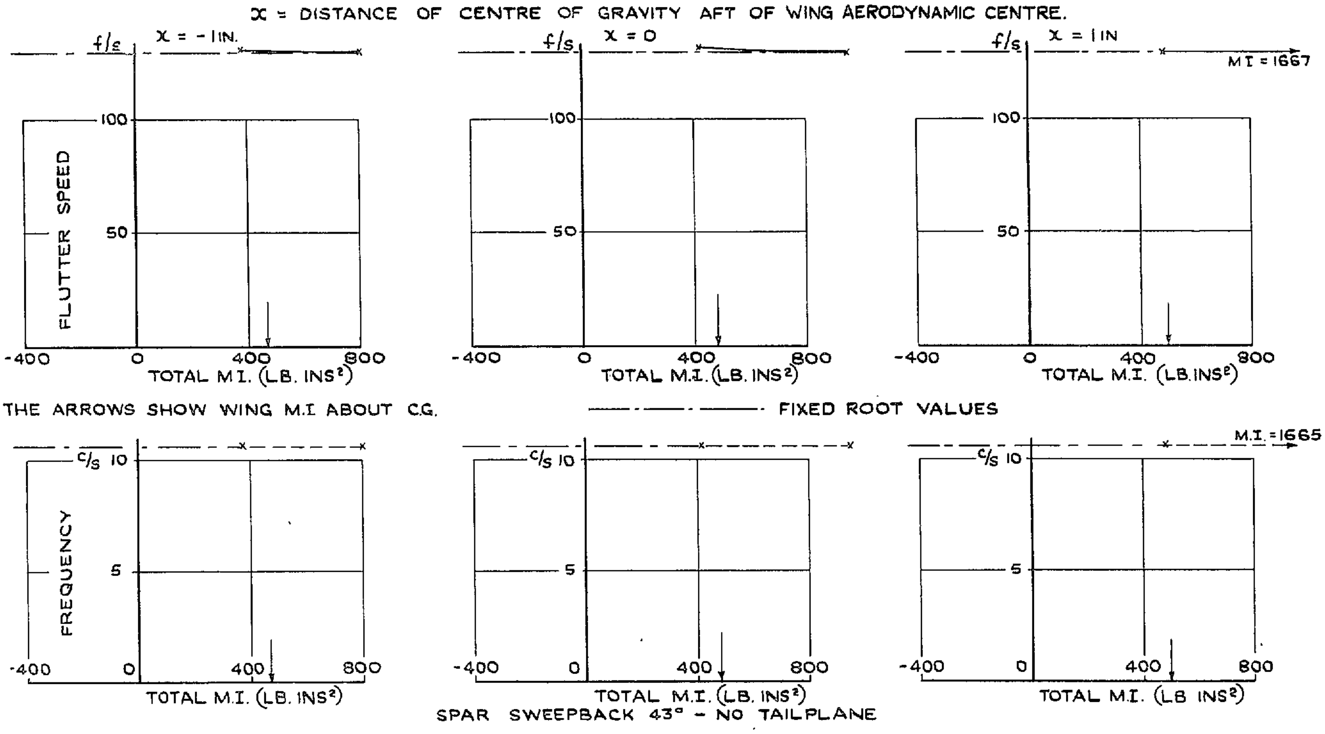

FIG. 9. Flutter speed and frequency plotted against total pitching moment of inertia.

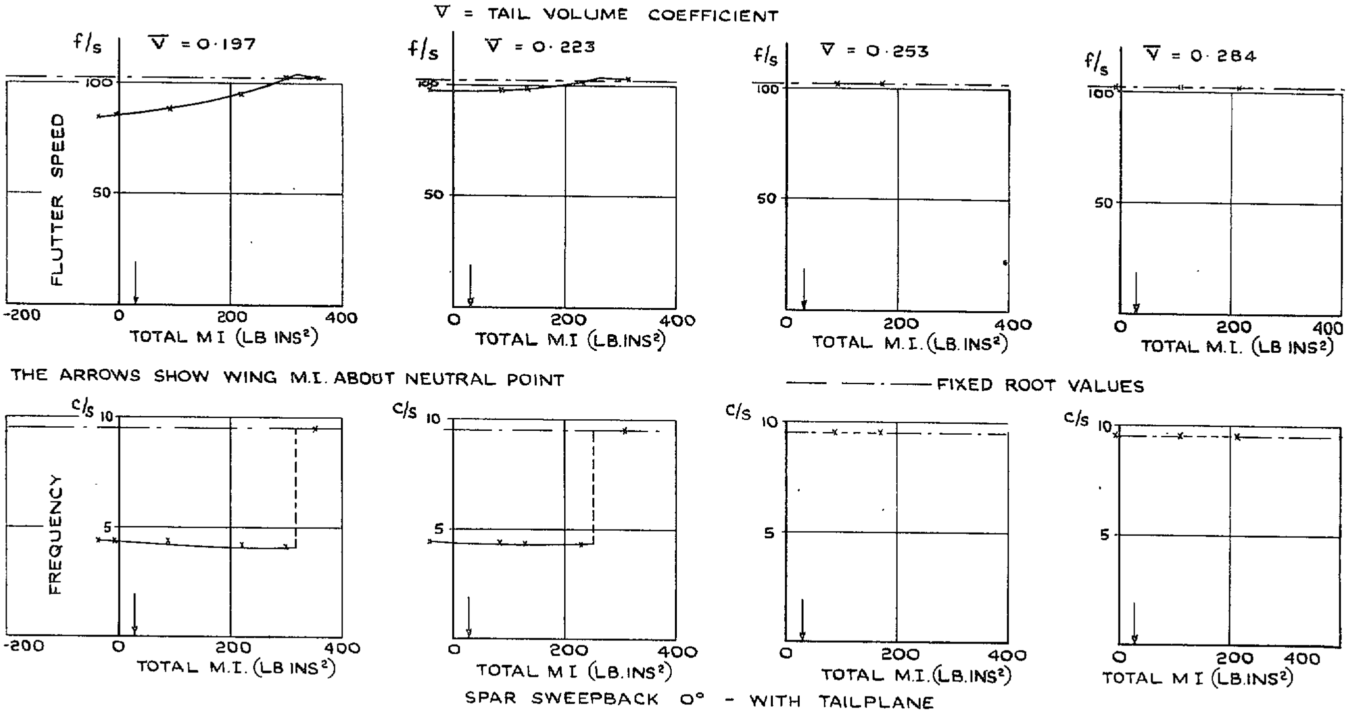

FIG. 10. Flutter speed and frequency plotted against total pitching moment of inertia.

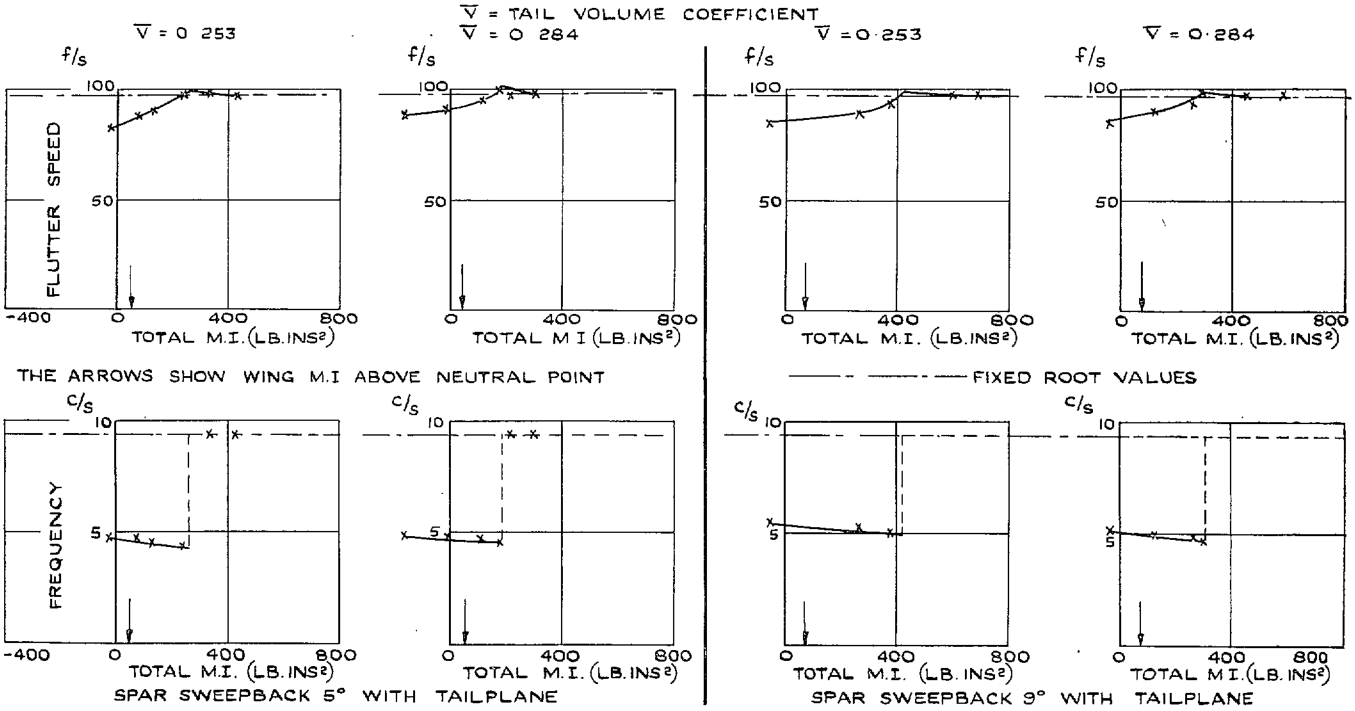

FIG. 11. Flutter speed and frequency plotted against total pitching moment of inertia.

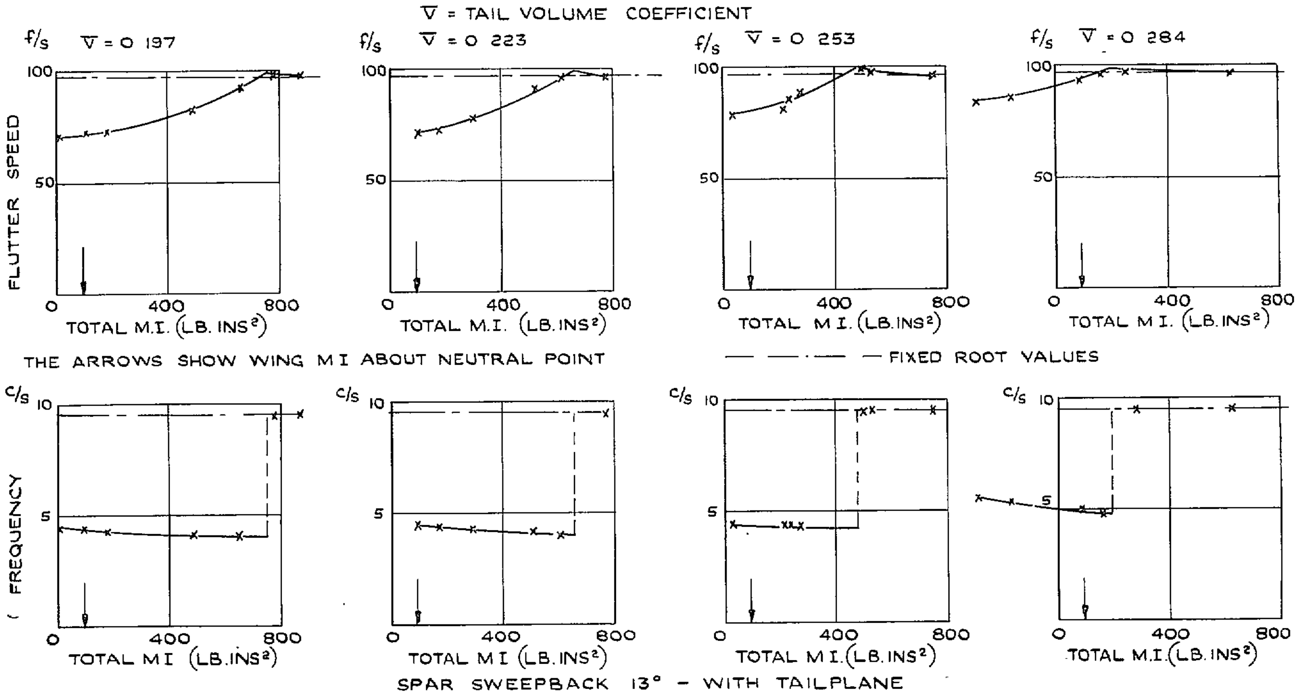

FIG. 12. Flutter speed and frequency plotted against total pitching moment of inertia.

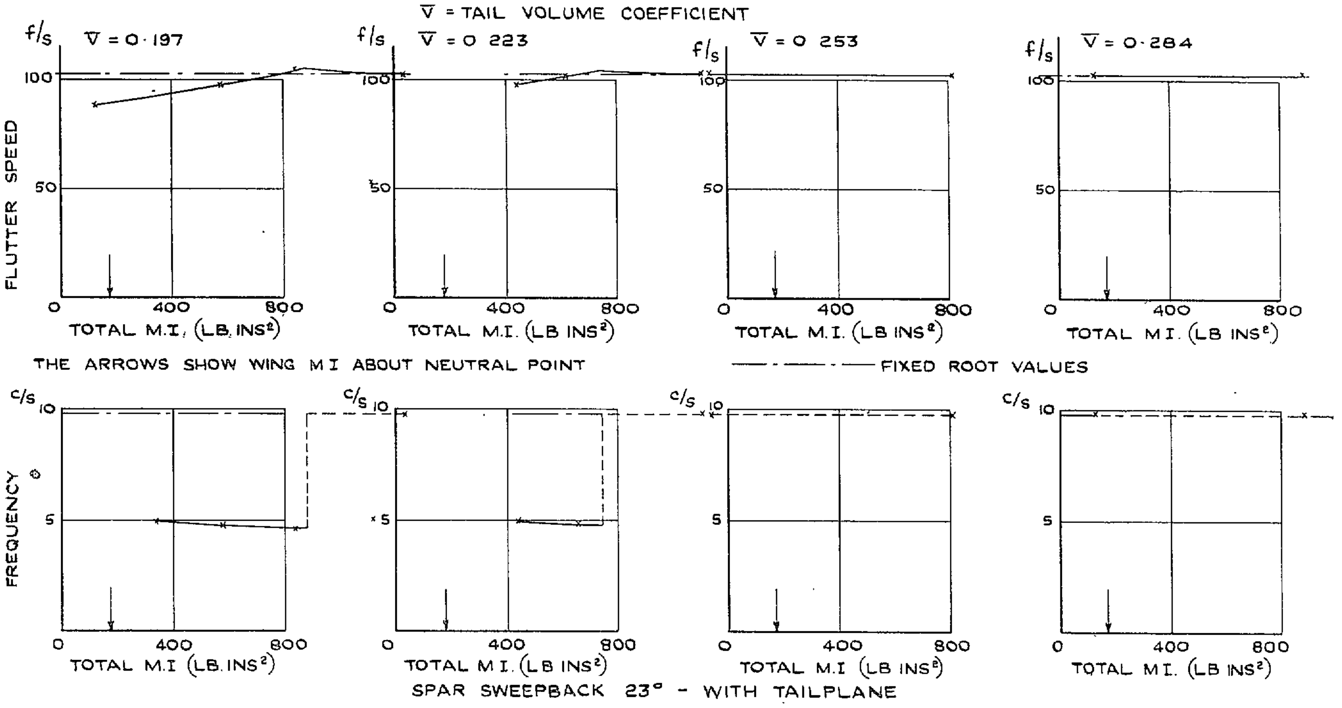

FIG. 13. Flutter speed and frequency plotted against total pitching moment of inertia.

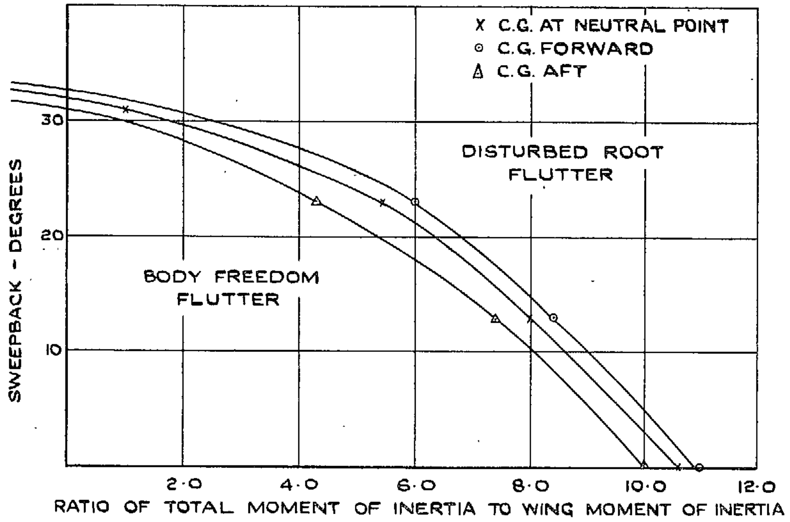

FIG. 14. Centre of gravity position effect on the transition between body-freedom and disturbed-root symmetric flutter

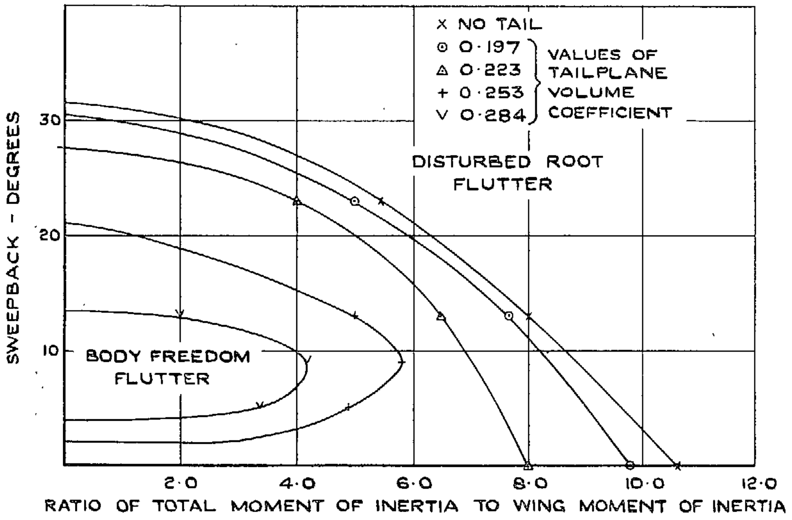

FIG. 15. Tailplane effect on the transition between body-freedom and disturbed-root symmetric flutter.

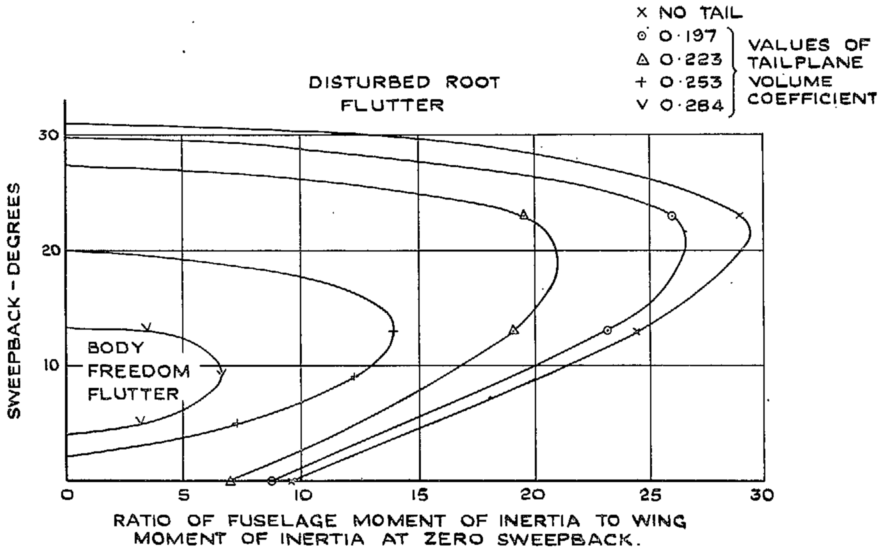

FIG. 16. Tailplane effect on the transition between body-freedom and disturbed-root symmetric flutter.

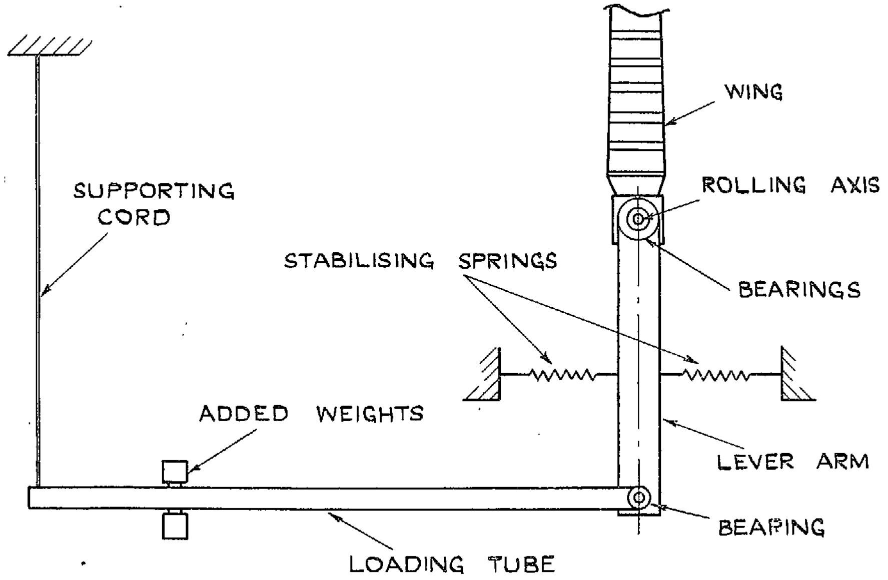

FIG. 17. Arrangement of rolling freedom.

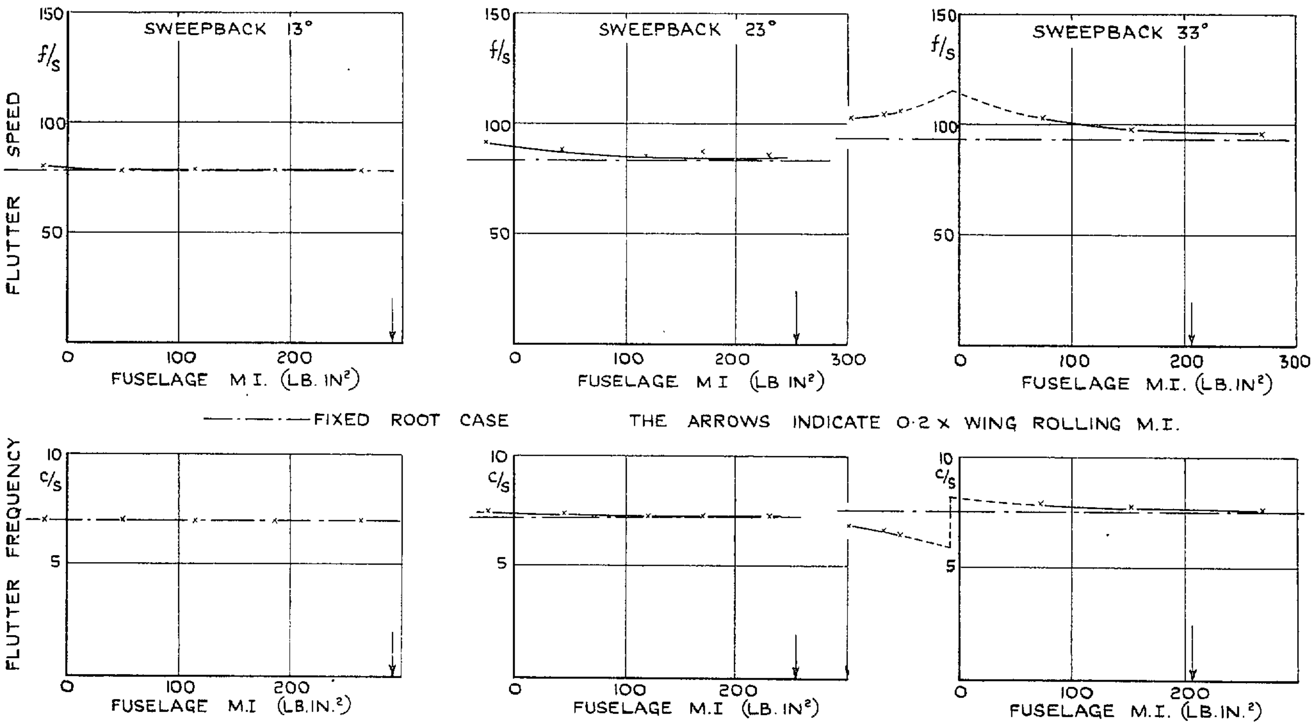

Fig. 18. Flutter speed and frequency plotted against fuselage rolling moment of inertia.

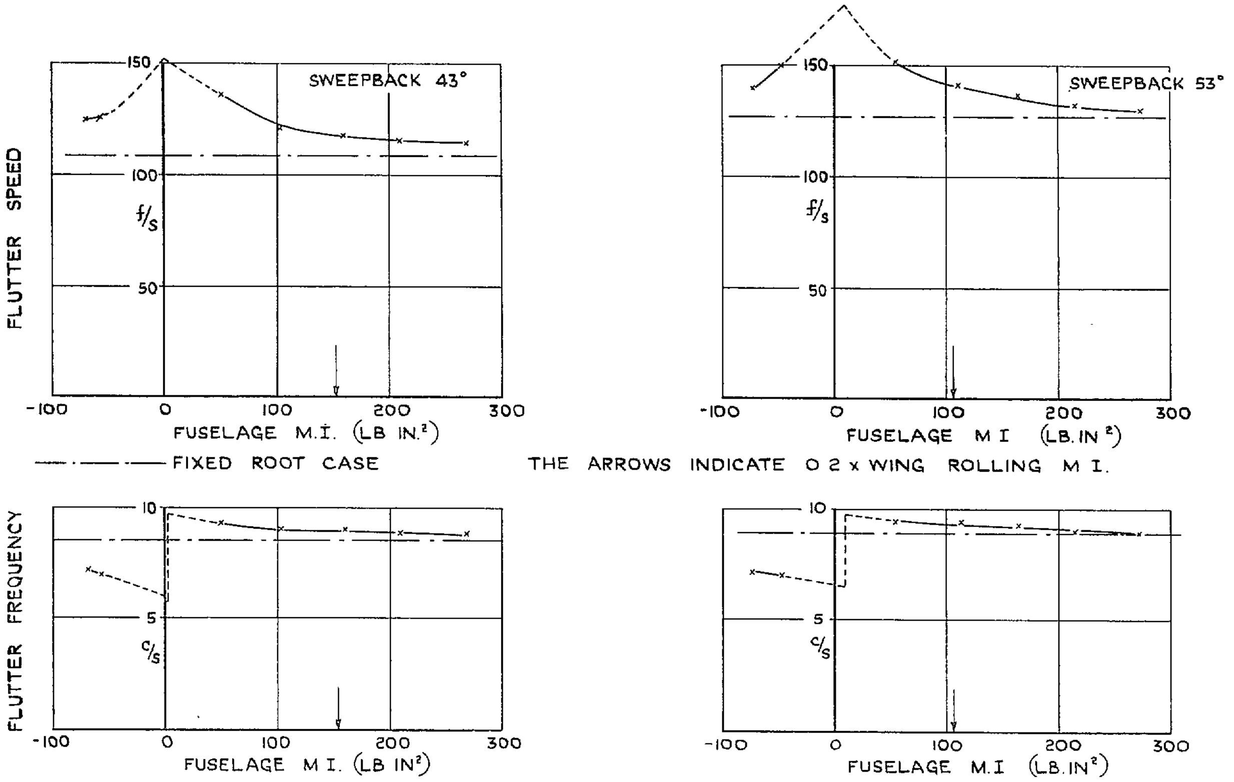

Fig. 19. Flutter speed and frequency plotted against fuselage rolling moment of inertia.

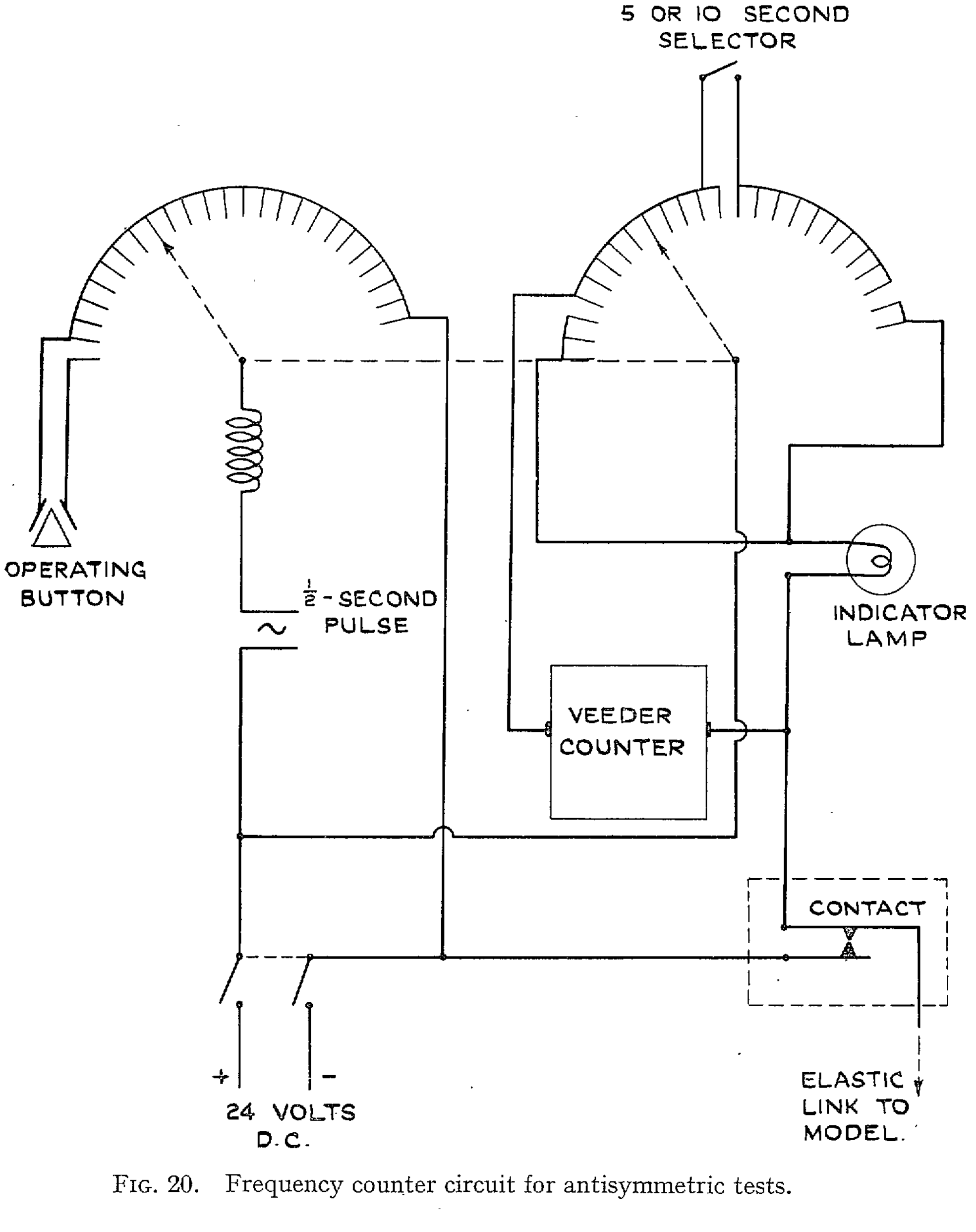

FIG. 20. Frequency counter circuit for antisymmetric tests.

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