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Summary.—The design of axial-flow turbines has been hampered in the past by a lack of comprehensive data regarding pressure losses and gas deflections through rows of turbine blades. In the present report much of the available information relating to this subject is studied and analysed to determine magnitudes of gas pressure losses and deflections in a wide variety of blade rows and also to determine the separate influences of variables such as blade shape, blade spacing, gas Mach number, Reynolds number, incidence, etc. Of particular importance are the effects of secondary flows on the aerodynamic performance of a blade row and special attention is paid to 'secondary losses', which form the difference between the total losses occurring in an actual turbine blade row and the smaller two-dimensional flow losses which are usually measured in a blade cascade tunnel. Effects of blade tip clearance are also studied.

Resulting from this analysis a number of empirical guiding rules and charts have been derived from which approximate values of the overall pressure losses and gas deflections in a range of blade rows can be deduced.

A particularly significant feature brought to light is that the secondary losses can in many instances be large, the loss being generally found to be great when the blading has low reaction.

1. *Introduction.*—In early years of gas-turbine development absence of reliable data relating to the nature of the flow and the magnitudes of the pressure losses in the blade rows of turbine stages formed a stumbling block in the path of the design of high-efficiency turbines.

Inaccurate data or erroneous assumptions will inevitably lead to a poor compromise between the efficiency of a turbine and the other important factors such as size, weight, cost and mechanical reliability.

During recent years the performances of several turbines have been accurately calibrated on experimental test rigs by the National Gas Turbine Establishment and collaborating firms. Further experimental work has accumulated from other sources such as cascade tunnels, detail explorations of the flow through nozzle rows, and earlier work carried out in the course of steam-turbine development (*e.g.*, experiments conducted by the Steam Nozzle Research Committee).

Much of this experimental evidence has been examined in detail and an attempt has been made to correlate the various experimental results. This has been done with the prime purpose of determining the magnitudes of the pressure losses and the gas efflux angles from rows of turbine blades and relating them to those aerodynamic and geometric variables which exercise a dominant influence.

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2. *Limitations.*—The greatest difficulty that arises in making an analysis which is simple to comprehend and apply is contending with the large number of variables which play a role, large or small, in the overall performance of a turbine stage. In order to derive and present data appertaining to any aspect of the aerodynamic operation of a blade row it becomes necessary to select, either by theoretical reasoning or by a broad examination of existing statistical evidence, only those variables which exercise a dominant influence, and to discard the remainder. Any laws which may then be deduced are necessarily approximate. The number of variables that are selected to define the performance of a row of blades will depend upon, (*a*) the accuracy to which it is required to predict the behaviour of the gas when it flows through a row of blades, and (*b*) the number of variables concerning which there is adequate experimental data.

In the present analysis the aim has been to derive basic data for predicting mean total pressure loss in a blade row with an error of less than $\pm$ 15 per cent and mean gas efflux angle with an error of less than $\pm\ 0 \cdot 02\ \cos^{-1}$ (opening/pitch). These tolerances will enable a prediction of efficiency and flow through a turbine at its design speed and pressure ratio to be made to within $\pm$ 2 per cent of the true value.

However, it should be appreciated that this is possible only when the turbine is designed to conform with the type of blading which is surveyed by the analysis. The range of blade shapes for which statistical data exists is by no means complete. For example, the range of blade sections that have been used up to the present time for low reaction stages having high gas deflections have shown losses which are many times the loss which is necessitated by consideration of skin friction alone. Future research may lead to reductions of these high losses and perhaps to the introduction of further variables which at the present time have been overlooked or ignored.

3. *General Remarks Concerning the Pressure Losses in a Blade Row.*—The overall pressure loss occurring in a blade row may be conveniently subdivided into a number of component losses, each component loss being influenced by some of the variables defining the aerodynamic form of the gas flow and by some of the variables defining the geometric form of the blade row. The component losses which are most frequently considered are :—

(*a*) *Profile loss*, being that loss due to skin friction or separation which will take place with a uniform two-dimensional flow across a cascade of blades

(*b*) *Secondary loss*, which results from non-uniformity of the three-dimensional flow through a row of blades (in particular, losses due to interaction between the blade ends and the boundary layer on the annulus walls)

(*c*) *Tip clearance loss*, or losses due to leakage of gas round shroud bands

(*d*) *Annulus loss*, being the skin-friction loss on the end walls of a row of blades.

Carter[1] points out that (*b*) and (*c*) are closely related theoretically but for analysis purposes it is convenient to keep them separated. On the other hand, since both (*b*) and (*d*) are principally associated with the boundary layers on the annulus walls the authors have adopted the practice of considering the annulus loss as part of the secondary loss. An additional annulus loss would only be added if the length of annulus wall between adjacent blade rows was sufficiently large to make the extra skin-friction loss appreciable.

Wherever convenient, pressure losses are referred to in terms of a loss coefficient, Y, defined as :—

$$Y = \frac{\text{Loss of total-head pressure}}{\text{Total pressure at blade outlet} - \text{static pressure at blade outlet}} .$$

However, it is found that some of the component losses may be correlated better over a wide range of blading by defining loss in terms of a drag coefficient, C_D, based on vector mean velocity. The relationship between C_D and Y is quoted in Appendix I.

This system for expressing loss seems better suited to analysis than the blade velocity coefficients adopted in the past by steam-turbine engineers. It also lends itself more readily to the application of aerodynamic research data acquired from other fields of investigation, and *vice versa*. Furthermore, it may more easily enable the characteristic problems associated with the axial-flow turbine to be linked ultimately with those of its partner the axial compressor.

3.1. *Nomenclature*.—A list of symbols is given in Appendix I. Figs. 1 and 2 illustrate the system adopted for defining the geometry of a blade row and the gas angles relative to a blade row. It is to be noted that the system for defining gas angles is an extension of the system previously established for axial compressors. A consequence of this is that the values of gas outlet angles on turbine blade rows are invariably negative. In the ensuing analysis the various algebraic functions introduced which involve outlet angle are framed on this supposition. However, to avoid confusion it must be stated that wherever the magnitude of an outlet angle is discussed in the text the negative sign has been ignored. Thus, the phrase ' high outlet angles ' refer to angles having a high numerical value and an ' increasing outlet angle ' is an angle whose numerical value increases.

4. *Two-Dimensional Flow through Rows of Turbine Blades*.—Cascade tests[2,3,4,5,6,7,8] have been made on a variety of blade sections to determine profile losses and gas outlet angles over a fairly wide range of incidence, Mach number, and Reynolds number. The blade sections which have been tested comprise:

(*a*) RAF 27 and C.7 aerofoil[3,4,6,7] sections on circular-arc (C.50) and parabolic-arc (P.40) camber-lines having $t/c = 10$ per cent and 20 per cent

(*b*) ' Conventional ' sections[5,8] having $t/c \simeq 15$ per cent to 25 per cent.

The term ' Conventional ' was introduced in Ref. 9 and is somewhat vague. It embraces a large proportion of blade sections at present in use which, for ease of manufacture, are composed of a number (often three[27]) of circular-arcs and straight lines. Broadly speaking it defines turbine blade sections which approximate to a T.6 section[9,14] on a parabolic camber-line, the point of maximum camber being about 40 per cent to 43 per cent of the chord from the leading edge.

For two-dimensional flow the variables in the gas stream likely to effect performance are incidence, Mach number, Reynolds number, and turbulence. The variables defining the geometrical shape of the blade are camber-line shape, stagger angle, base profile shape, thickness/chord ratio, and pitch/chord ratio. The amount of information relating to base profile shape and turbulence is very restricted, particularly turbulence.

4.1. *Profile Losses at Low Mach Number (less than* $0 \cdot 5$*), High Reynolds Number (approximately* 2×10^5*) and small Incidence*.—(*a*) *Nozzle blades* ($\beta_1 = 0$ deg).—Fig. 3a (reproduced from Ref. 9) compares profile-loss coefficients, Y_p, for conventional nozzle sections and sections composed of RAF 27 sections on circular-arc and parabolic-arc camber-lines. Loss varies with s/c and α_2 but appears to be little affected by the variations in base profile shape and thickness/chord ratio. The mean acceleration imparted to the gas flow through nozzle rows is large and Hargest[10] (1950) shows that on conventional nozzle blade profiles the regions of retarded flow are small, so that there is little danger of marked separation of the flow from the upper surface of this type of blade. However, curvature on the upper surface of the profile between the blade throat and trailing edge may lead to larger losses at high outlet Mach number. This point will be discussed in section 4.5. From test data on high reaction blades a family of curves of nozzle blade profile loss has been drawn up as shown in Fig. 4a. These loss values are typical of the types of blade enumerated in section 4.

(*b*) *Blades having* $\beta_1 > 0$ *deg.*—As the mean acceleration of the flow through a high-deflection blade row is reduced (*i.e.*, as the ratio of β_1 to $-\alpha_2$ increases or the stagger angle is reduced) it may be expected that profile form and thickness/chord ratio may become more critical since local pressure gradients on the blade upper surfaces opposing the motion of the gas will become more pronounced. The increasing severity of opposing pressure gradients is illustrated in Fig. 5 in which the pressure distributions on conventional blades of 15 deg and 55 deg inlet angle and 60 deg outlet angle are compared.

Very little systematic work to determine optimum profile shapes has yet been accomplished. Optimum shape will be largely related to the form of the pressure distribution round the blade, particularly on the convex upper surface, since this will govern the behaviour of the boundary layer.

Opinion as to the best form of pressure distribution to achieve differs widely. A. W. Goldstein[11] (1949) suggests that the suction pressure over the upper surface of the blade should be constant over as large an arc of the surface as possible with a final opposing gradient near the trailing edge, this final pressure gradient being as sharp as possible without causing separation. Other opinion favours a peak suction point as far forward towards the leading edge as possible with a linear opposing pressure gradient over the larger part of the blade upper surface. This latter form is one which frequently occurs on compressor blades. Unfortunately there is little experimental evidence to support either theory, although a cascade tested by Eckert[12] (1949) in an interferometer tunnel had a distribution of the first type and at a small pitch/chord ratio showed very little separation (the gas inlet and outlet angles being approximately 56 deg and -75 deg respectively). However, no loss measurements were made on this cascade so that no reliable conclusion may be drawn. The whole problem of optimum shape will be further complicated at high Mach numbers when local shock-waves appear in the passage. It may be that optimum shapes for high and low Mach number will eventually be found to differ.

Returning to existing test results Fig. 3b compares the losses in rows operating under impulse conditions of thick conventional blades (having nearly constant passage area through the row) and 10 per cent thick blades composed of RAF 27 aerofoil sections on circular-arc camber-lines. The losses on the conventional blades are very much higher than those of the aerofoil sections. This might possibly be attributable to differences in t/c although other differences in section profile, camber-line shape, scale, and turbulence must also influence the results.

Some definite evidence on the effect of t/c (other factors being constant) is published in Refs. 6 and 7. Blades composed of a C.7 aerofoil section on a parabolic (P.40) camber-line with $\beta_1 = 30$ deg, $\cos^{-1} o/s = 60$ deg, $s/c = 0{\cdot}625$, and $t/c = 10$ per cent and 20 per cent were tested in the same tunnel. Minimum losses of $0{\cdot}028$ and $0{\cdot}04$ were obtained on the 10 per cent thick and 20 per cent thick blades respectively. On nozzle blades, however, the effect of t/c appears to be very small. The available evidence suggests, therefore, that t/c has an increasing effect as the reaction of a blade row is decreased (or as the ratio β_1/α_2 increases). Tentatively it is suggested that profile loss roughly varies proportionally to $(t/c)^{-\beta_1/\alpha_2}$ for conventional blades. A family of curves of profile losses typical of impulse blades of conventional form and having $t/c = 20$ per cent is shown in Fig. 4b. $t/c = 20$ per cent has been chosen since it is representative of values frequently encountered in practice on impulse blade sections and is also comparable with the values of t/c on blades for which test results are available.

The above statements contradict the old theory that impulse blades should be designed for constant passage area. Impulse blades designed for constant passage area usually have large thickness/chord ratio. However, the available data on impulse blades is so scant that it is not advisable to be emphatic on this point. Indeed, there is some contrary evidence that for very high deflection (120 deg or more) an optimum blade shape may have a fairly thick section (*e.g.*, blade tested in Ref. 12). For this reason it is not advisable to apply the suggested correction for t/c over a wider range than 15 per cent $< t/c <$ 25 per cent on high-deflection near-impulse blades.

Typical profile losses on conventional blades intermediate between nozzle and impulse blades may be interpolated in the following manner:—

(*a*) Determine the value of β_1/α_2, α_2, s/c and t/c for the blade considered

(*b*) From Fig. 4a find $Y_{p(\beta_1=0)}$ for a 20 per cent thick blade having same value of α_2 and s/c

(*c*) From Fig. 4b find $Y_{p(\beta_1=-\alpha_2)}$ for a 20 per cent thick blade having same value of α_2 and s/c

(*d*) Then required value of Y_p is:—

$$Y_p \simeq \left\{ Y_{p(\beta_1=0)} + \left(\frac{\beta_1}{\alpha_2}\right)^2 \left[Y_{p(\beta_1=\alpha_2)} - Y_{p(\beta_1=0)} \right] \right\} \left(\frac{t/c}{0 \cdot 2}\right) . \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The curves in Fig. 4 for $Y_{p(\beta_1=0)}$ and $Y_{p(\beta_1=-\alpha_2)}$ differ slightly from and supersede earlier curves given by the author in Ref. 13 (1949).

4.2. *Variation of Profile Loss with Incidence: and Values of Stalling Incidence.*—Speaking broadly, blades in which the mean acceleration of the gas flow is large have a wide range of incidence over which the profile losses are low whereas low-reaction blades have a smaller incidence range. Many blades, such as the impulse blade represented in Fig. 6, show a narrow incidence range of very low loss. These very low losses probably indicate a large degree of laminar or unseparated flow on the blade. In a turbine stage, however, where turbulence will be very large due to wakes from preceding blade rows it is improbable that such low losses will be achieved (with the possible exception of a first-stage nozzle row). For this reason these narrow ranges of low loss are generally ignored.

The stalling incidence (i_s) is defined as the incidence at which the profile loss is equal to twice the minimum loss. It has been found that the positive stalling incidence on turbine blades can be correlated satisfactorily with α_2, s/c and β_1/α_2. The method adopted was to determine first the variation of i_s and α_2 with s/c for a wide variety of blades. This variation (using $s/c = 0 \cdot 75$ as a datum) is illustrated in Fig. 7a. By this means the stalling incidences of blades of all pitch/chord ratios could be corrected to $s/c = 0 \cdot 75$ and the resulting values of $i_{s(s/c=0 \cdot 75)}$ are plotted in Fig. 7b, using α_2 and β_1/α_2 as parameters. The family of curves in Fig. 7 will enable the positive stalling incidence of an arbitrary conventional turbine blade to be determined to within about $\pm$ 3 deg, which is sufficient for most practical purposes.

It is observed that blades having a high positive stalling incidence generally have a high negative stalling incidence, and *vice versa*. It is possible, therefore, to represent approximately the relative profile loss of any turbine blade $\left(\frac{\text{profile loss}}{\text{profile loss when } i = 0 \text{ deg}}\right)$ as a unique function of relative incidence (i/i_s). This is shown in Fig. 8a. The scatter of the points is large but for the purposes of performance calculation to incidences down to $i/i_s = -2 \cdot 0$ a single mean curve is sufficiently accurate.

4.3. *Gas Efflux Angles at Low Mach Number and High Reynolds Number.*—(*a*) *Zero Incidence.*—Steam-turbine investigators found that the gas outlet angle could be closely related to $\cos^{-1}(o/s)$. The blades for which this relation existed generally had a straight upper surface to the blade profile between the throat and the trailing edge. Recent experimental evidence confirms this finding on such blading but also indicates that curvature of the upper surface between the throat and trailing edge tends to increase (numerically) the gas outlet angle.

The relationship between α_2 and $\cos^{-1}(o/s)$ for 'straight-backed' blades deduced from results published by Bridle[5] (1949) is shown in Fig. 9a. This is supported by independent evidence quoted by Emmert[15] (1950), shown by dotted lines in Fig. 9a.

Available data suggests that the increase in gas outlet angle due to curvature of the blade upper surface between the throat and the trailing edge may be approximately related to the ratio s/e where s = blade pitch and e = mean radius of curvature of the upper surface of the blade section between the blade throat and blade trailing edge (*see* Fig. 1). Gas outlet angle is found to fit the relationship:—

$$\alpha_2 = \alpha_2^* - 4(s/e) \qquad \qquad (2)$$

where α_2^* is the outlet angle corresponding to a 'straight-backed' blade (Fig. 9a).

(*b*) *Variation of Outlet Angle with Incidence*.—The variation of gas outlet angle with incidence is only slight. At positive incidences the outlet angle tends to decrease slightly as the loss increases. This slight decrease of angle presumably results from a thickening of the boundary layer on the blade upper surface which will accompany the increase of loss. This trend is demonstrated in Fig. 8b where change of outlet angle (using the outlet angle at the incidence giving minimum loss as a datum) is plotted against relative profile loss viz. : $\left(\frac{\text{loss at any incidence}}{\text{minimum loss}}\right)$. The gas outlet angle from a row of blades tends to decrease by about 2 deg between zero incidence and positive stalling incidence.

At negative incidences there is no clearly defined trend in the change of outlet angle; for performance prediction it may be assumed to remain constant without introducing excessive error.

4.4. *Effect of Reynolds Number on Profile Loss and Gas Efflux Angle*.—Very little detailed study of the effect of Re on loss in turbine blade rows has yet been made. If discussion is confined to two-dimensional flow then the representative scalar length selected to define the Reynolds number is the blade chord and the representative velocity, density, and viscosity is chosen as the blade-outlet mean value. As may be anticipated from work in other fields, *e.g.*, Goldstein (Ed.)[16] (1938), cascade tests (Fig. 10) show that the profile losses increase as the Reynolds number is reduced. The losses frequently increase more rapidly with decreasing Reynolds number when Re is less than about 1×10^5 than at higher values of Re. Below $Re = 1 \times 10^5$ the loss increase appears to be more severe on low-reaction blades having high thickness/chord ratios than on high-reaction nozzle blades, although experimental results do not extend to sufficiently low values of Re to be sure on this point. The general effect of Re may be illustrated by plotting relative loss (defined as $\frac{\text{loss}}{\text{loss at } Re = 2 \times 10^5}$) against Re (Fig. 11a). For performance prediction the effect of Reynolds number down to $Re = 5 \times 10^4$ for all types of blade, including compressor blading, may very approximately be represented in this way by a single curve.

The increase of loss with decreasing Re will be accompanied by an increase in the thickness of the boundary layer or separated flow region on the upper surface of the blade at the trailing edge. This in turn may lead to a slight reduction in mean gas efflux angle from the cascade. This trend is indicated by the experimental results plotted in Fig. 11b. The decrease of angle with decreasing Re appears less on high reaction or nozzle blades than on low reaction high deflection blades. The variation of angle as Re is reduced may correlate better with the increase of loss associated with decreasing Re. Thus, blades in which the increase of loss with reduction in Re is only slight may be expected to show only a small corresponding decrease in outlet angle, and *vice versa*.

The effect of Re on overall turbine performance is discussed briefly in section 8.

4.5. *Effect of Mach Number on Profile Loss and Outlet Angle*.—On nozzle rows having convergent flow passages and high gas efflux angles outlet Mach numbers considerably in excess of unity may be achieved without any very serious increase in loss or severe deviations in outlet angle. This is illustrated by many steam-turbine nozzle tests (*e.g.*, Refs. 17 to 23) and is confirmed to a limited extent by cascade tests on gas-turbine nozzle rows, published by Bridle[5] (1949). Up to the present time gas-turbine designers have not employed nozzle Mach numbers appreciably in excess of unity for the following reasons: (*a*) the nozzle outlet Mach number of turbines operating at high temperature and designed for stage velocity ratios of about 0·5 or more (to achieve good efficiency) are limited by the maximum peripheral speeds at which it is safe to run the rotor rows, this limitation generally restricting the nozzle Mach number to about $1 \cdot 0$ or less, (*b*) fear of loss of efficiency due to interaction between the nozzle shock-waves and the following rotor row. On the other hand turbines employing high supersonic velocities at outlet from the nozzle rows might be designed advantageously in low-temperature refrigerating units where high Mach numbers are achieved with relatively low absolute gas velocities.

An influence of Mach number on loss (at constant incidence) is first noticeable when the local velocities at the peak suction point on the blade exceed sonic velocity, as shown by Hargest[10] (1950); the loss then increasing slightly due to thickening of the boundary layer through the small shock-waves that arise in the blade passage. The 'critical' outlet Mach number at which these shock-waves first appear in the passage is in the region of $0 \cdot 6$ on 20 per cent thick impulse blades and about $0 \cdot 9$ on nozzle blades. As the outlet Mach number is increased above the critical value to unity the profile loss may either continue to rise or it may fall. The processes underlying the variation in loss above the critical Mach number are not yet fully comprehended but it may depend largely upon the blade profile shape and to a limited extent upon the turbulence.

Statistical analysis of many cascade tests suggests that when M_2 approaches unity the profile loss may be influenced considerably by the curvature of the upper surface of the blade between the throat and the trailing edge (defined by the ratio s/e; *see* Fig. 1). The nature of the trend is illustrated by some test results plotted in Fig. 12a which show that for large curvatures of the blade tail (*viz.*, high values of s/e) the profile-loss coefficient as measured in a cascade tunnel at unit outlet Mach number may be as much as four times the loss at low Mach number. Some unpublished work at the N.G.T.E. has shown that this high loss is associated with laminar separation from the back of the blade; the separation being initially triggered by shock-waves in the blade passage when the outlet Mach number exceeds its critical value. If the boundary layer is forced into a turbulent condition (*e.g.*, by a transition wire on the leading portion of the blade upper surface) the separation can be avoided and normal profile-loss coefficients at $M_2 = 1 \cdot 0$ obtained. Curvature of the tail tends to move the point of peak suction on the upper surface towards the trailing edge in comparison to a straight-backed blade. Thus with an inlet gas stream of constant turbulence it may be expected that a laminar boundary layer will persist longer on a curved than on a straight-backed blade. Furthermore, since a shock-wave is more likely to lead to flow separation if the boundary layer is laminar than if it is turbulent (Liepmann[24] (1946)) then the probability of a laminar separation above the critical Mach number becomes greater as the curvature of the tail increases.

It is conceivable that the trend illustrated in Fig. 12a may be partially counteracted by increasing the turbulence in the inlet gas stream, this causing an earlier transition on the upper surface. Thus, it is possible that a blade operating in the highly turbulent gas stream within a turbine may not react to curvature so severely as the cascade tests shown in Fig. 12a indicate. Nevertheless, it is evident that a good insurance against high profile loss at high outlet Mach numbers is to design the blade with little or no curvature on the upper surface between the blade throat and the trailing edge.

The gas outlet angle is also influenced by Mach number. If the pressure loss when $M_2 = 1 \cdot 0$ in a blade row is small or if little loss occurs in the gas downstream of the throat then when the Mach number downstream of the row is unity the flow angle is given closely by $\cos^{-1} o/s$. It is evident that this must be approximately true since when $M_2 = 1 \cdot 0$ the flow area downstream must be almost equal to the blade throat area. It is worth noting at this point that if a blade row has end walls which diverge (*i.e.*, 'flare') or converge between the throat section and the downstream reference flow plane then when $M_2 = 1 \cdot 0$ the gas outlet angle will be given approximately by $\cos^{-1}$ (throat area/outlet annulus area). However, if appreciable pressure losses occur downstream of the blade throat (due, for example, to separation of the flow from the back of the blade in the vicinity of the blade throat) then the resulting decrease in density will necessitate a flow area downstream of the blade when $M_2 = 1 \cdot 0$ greater than the throat area, and hence an outlet angle less than $\cos^{-1} o/s$.

It was shown in Fig. 12a that curvature on the blade upper surface can lead to high profile losses when M_2 approaches unity. Fig. 12b demonstrates the nature of the deviation of α_2 from $\cos^{-1} o/s$ which accompanies these losses when $M_2 = 1 \cdot 0$. Thus it may be fairly stated that on blade rows which exhibit a low loss when $M_2 = 1 \cdot 0$ the outlet angle at this Mach number will be nearly equal to $\cos^{-1} o/s$ but high losses may lead to substantially lower outlet gas angles.

Comparison of Figs. 12b and 9a show that an appreciable change in outlet angle may occur between $M_2 = 0 \cdot 5$ and $M_2 = 1 \cdot 0$, particularly on straight-backed blades designed for low outlet angles. The manner in which the angle varies between $M_2 = 0 \cdot 5$ and $M_2 = 1 \cdot 0$ depends upon the blade design and no definite trend has yet been deduced. However, for the purpose of turbine performance prediction a roughly linear variation of α_2 with M_2 in the range $0 \cdot 5 < M_2 < 1 \cdot 0$ will lead to little significant error.

It is to be emphasised that the above observations are confined to blades operating at zero incidence or close thereto. On low-reaction cascades working at high positive incidences the critical Mach numbers are low and the increase of loss above the critical Mach number is much more pronounced (due to early flow separation on the upper surface, well forward of the blade throat). Under these conditions the outlet angle often increases with Mach number by several degrees in spite of the separation and the accompanying increase of loss.

The influence of Mach number on outlet angle at outlet Mach numbers in excess of unity has not been widely investigated, such high Mach numbers not being of significant interest to gas turbine design at the present time. Hauser, Plohr and Sander[25] (1950) show that the outlet flow angle from a cascade of turbine blades at Mach numbers greater than unity lies roughly half-way between the values which would be estimated by assumptions of (i) isentropic expansion of gas flow downstream of the blade throat and (ii) non-isentropic expansion downstream of the blade trailing edges with constant tangential velocity, it being assumed that α_2 and M_2 in the plane of the trailing edges of the blades is $\cos^{-1} o/s$ and $1 \cdot 0$ respectively.

4.6. *Effect of Trailing-edge Thickness on Blade Loss.*—A theoretical study, which agreed well with some experimental data, of the effect of trailing-edge thickness on blade loss has been made by Reeman and Simonis[26] (1943). Fig. 13 reproduces some theoretical results for blades in terms of the loss coefficient Y.

Turbine tests have shown a marked reduction in efficiency as a result of increasing trailing-edge thickness on either nozzle or rotor blades. A W2/700 turbine in which the ratio t_e/s (t_e = trailing-edge thickness) on the rotor blades was increased from $0 \cdot 018$ to $0 \cdot 075$ showed a reduction in efficiency of about 3 per cent. This experimental result was equivalent to an increase of total rotor blade loss coefficient of about 30 per cent and is of the same order as the calculated increase if the calculation is based on the estimated *total* loss coefficient for the row.

5. *Three-dimensional Flow through Turbine Blades.*—5.1. *Mean Gas Efflux Angles.*—Three-dimensional flow traverses behind turbine nozzle blades have been made by several investigators[27, 28, 29]. A typical example published by Johnston[27] (1951) is shown in Fig. 14. The gas outlet angle diverges markedly from the mid-blade height values near the outer and inner walls but the momentum mean efflux angle correlates well with the predicted two-dimensional value at the mean diameter. Similar distributions of gas outlet angle have been found by other investigators on nozzle rows and also on rotor rows (*e.g.*, Ainley[9] (1948)). On the whole at low Mach numbers the predicted two-dimensional values of outlet angle for the mid-diameter blade section agree reasonably well with the mean gas outlet angle. When the outlet Mach number is unity the mean outlet angle is usually fairly close to $\cos^{-1}$ (throat area/downstream annulus area).

5.2. *Pressure Losses in Nozzle Rows.*—In addition to the profile loss which has been discussed in sections 4.1, 4.2, 4.4 and 4.5 losses are created by secondary flows and it has been demonstrated by Carter[1, 28] (1948, 1945) that these additional losses are mainly confined to the ends of the blades. Assuming that there is no radial clearance at the blade ends then the secondary losses arise as a result of the presence of boundary layers on the end walls. Fig. 15a represents ideally the flow through a row of blades and shows the variation of loss along the span of the blade.

There will be a uniform loss along the blade (assuming a hub ratio of nearly 1·0) corresponding to the two dimensional profile loss, $\bar{\omega}_p$. Also there will be concentrations of secondary loss confined to lengths Δh at the blade extremities and the average value of this loss over the short lengths Δh is represented by $\bar{\omega}_s$. The mean total-head loss over the whole blade will then be given by $\bar{\omega}_p + 2\bar{\omega}_s \Delta h/h$. The term $2\bar{\omega}_s \Delta h/h$ constitutes the secondary loss to be added to the profile loss to give the mean total loss across the blade. Now the length Δh in which the end losses are confined will depend largely upon the thickness, δh, of the wall boundary layers at the outlet from the row. This thickness may be influenced by the inlet velocity distribution, the angle the gas is turned through, and the acceleration imparted to the gas in passing through the row. If on a blade (of constant section) the wall boundary layers remain fixed in size while the blade span h is varied then the mean secondary loss ($2\bar{\omega}_s \Delta h/h$) will vary inversely as the span, or, if the chord is maintained constant, it will vary inversely as the aspect ratio. On the other hand if the wall boundary layers and blade span remain fixed in size while the chord dimension is varied then the magnitude of the secondary loss will not be influenced by the aspect ratio. This is demonstrated in Fig. 15b which has been derived from the results of a series of reaction tests on some steam turbine nozzle blades published by Kraft[30] (1949). Many of these nozzle blades had roughly similar profile form, outlet angle, and pitch/chord ratio but differed widely in span and chord. Although little information is available of the actual thicknesses of the wall boundary layers it is very probable, since all the tests were made on the same apparatus, that the thicknesses were nearly identical on all tests. The tests show clearly a rapid increase of loss as the blade height is reduced (chord remaining constant) particularly when the height is less than about 1 in. It is noteworthy that the rapid increase of loss when the blade height is reduced to less than 1 in. accords with the finding of Guy[43] (1939). On the other hand, when the height is fixed the loss is scarcely affected* by large changes in blade chord.

Thus it is apparent that to correlate secondary losses on different blades it is necessary to know the form of the wall boundary-layer and its size relative to the blade height.

To the time of writing only a very simplified study has been made on the theoretical evaluation of secondary losses. Carter[1] (1948) shows that for cascades of blades of small deflection and small values of blade pitch/blade height the drag coefficient, C_{Ds}, equivalent to secondary loss is

$$C_{Ds} = \tfrac{1}{4}C_L^2[1 - (h'/h)]/(s/c) \qquad \text{(3)}$$

where C_L is lift coefficient based on vector mean velocity

h is blade span

h' is distance between the vortices created at the blade ends downstream of the blades

Now the term $[1 - (h'/h)]$ is probably proportional to the boundary-layer thickness, the thickness of the boundary layer at the blade outlet possibly being more significant in this respect than the thickness at the blade inlet. In particular if the relative boundary-layer thickness, $\delta h/h$, remains constant then so also may $[1 - (h'/h)]$. Furthermore, although the theory is only strictly applicable to blade rows of small camber it provides a possible theoretical basis for an empirical law in which secondary loss is expressed as

$$C_{Ds} = \lambda C_L^2/(s/c) \qquad \text{(4)}$$

where λ is a factor, to be determined by experiment, which may depend, to large or small degree, on the inlet and outlet velocity distribution, angle through which the gas is turned, wall boundary-layer thickness, and possibly hub ratio†.

* The slight increase of loss shown on Fig. 5b as aspect ratio is reduced (height fixed) is probably accountable to the increase in frictional loss on the end walls as the length of the blade passage is increased.

† The system heretofore adopted (Ainley[9, 13] (1948, 1949)) of defining secondary loss as $Y_s = 0{\cdot}04\{1 - (\beta_1/\alpha_2)\}C_{L\tau2}^2$ has now been discarded. This system does not compare well with recent data. The system now adopted has a firmer theoretical basis and furthermore resembles the system adopted in axial-compressor theory. It is hoped that eventually secondary losses in axial compressors and turbines may be related on a truly common basis.

The main difficulty that arises in applying this procedure is determining the boundary-layer thickness. The common practice of defining the limit of the boundary layer as the point at which the velocity is 99 per cent of the free-stream velocity is impracticable since generally it is impossible to define the free-stream velocity in the distributions which frequently occur in turbines. Alternatively, it is desirable to define some length which is representative of the depth of flow in the vicinity of the walls in which the velocity gradients are large. For approximate comparative purposes the boundary-layer thickness is defined by the point on the velocity distribution curve at which $d(V/\bar{V})/d(y/h) = 1/3$, where V = velocity, $\bar{V}$ = mean velocity, y = distance measured along the blade, h = blade span.

The table below presents some experimental values of λ and boundary-layer thickness for four nozzle rows:

TABLE 1

Nozzle row	1	2	3	4
s/c (m.d.)	0·74	0·990	0·59	0·77
α_2	63	59	66	35
Y_{total} (measured)	0·061	0·056	0·069	0·058
$Y_{profile}$	0·028**	0·026**	0·037**	0·027***
$Y_{secondary} = Y_{tot} - Y_{profile}$	0·033	0·030	0·032	0·031
λ	0·0074	0·0078	0·0063	0·022
$\delta h/h$	0·10	0·12	0·14	0·15
i.d./o.d.	0·72	0·72	0·86	0·59

** From Fig. 4.

*** Measured independently by a cascade test.

Secondary loss is defined for present purposes as the difference between the measured mean total loss and the estimated profile loss (*see* section 4.1) at mid-blade height. The definition of secondary loss is somewhat arbitrary. As defined here it will include any losses which might result from local stalling of the blade roots and tips (particularly on rotor rows). However, since such stalling will generally result from three-dimensional distributions of gas velocity and flow angle which differ from the theoretical design distributions it is reasonable to include any consequent stalling losses at the blade ends with secondary losses.

Now it may be anticipated that the size and form of the boundary layers on the annulus walls at outlet from a blade row will be influenced largely by the amount of acceleration imparted to the gas stream as it passes through the row. For this reason the tabulated values of λ, together with some results derived from two 50 per cent reaction turbines in which the gas direction relative to the inlet of the blades was nearly axial, are plotted in Fig. 16a against $(A_2/A_1)^2$; where A_1 = inlet flow area at zero incidence = inlet annulus area $\times$ cos B_1 and A_2 = outlet flow area = outlet annulus area $\times$ cos α_2.

Corrections were made to the turbine results for the effects of tip clearance (*see* section 5.3 later). The value of λ decreases as the acceleration through the nozzle row increases. In the majority of instances the value of $\delta h/h$ at the blade outlet or inlet is not accurately known. It lies generally in the range 0·10 to 0·15, the larger values usually occurring when the acceleration through the blade is only slight.

Since the outlet gas angles from nozzle rows normally lie within the range 55 deg $< \alpha_2 <$ 75 deg (equivalent to $0 \cdot 07 < (A_2/A_1)^2 < 0 \cdot 33$) then the consistency of the experimental results in this region indicates that the secondary losses in nozzle rows might be represented by a single curve of λ against $(A_2/A_1)^2$, as drawn in Fig. 16a. Such a curve should give a reasonable estimate of secondary loss in nozzle rows in which $\delta h/h$ is about 0·1 to 0·15. Rows in which the blade height is very small (say less than 1·0 in.) or in which inlet velocity distribution is comparatively poor may be expected to show higher secondary losses and conversely rows having large blade height or very uniform inlet velocity distributions may be expected to give smaller losses.

By adding values of secondary loss predicted from Fig. 16a to values of profile loss predicted from Fig. 4, an estimate of total loss may be made for a nozzle row. A comparison of the predicted and measured values for the nozzles quoted in Table 1 is made in the following table.

TABLE 2

Row	1	2	3	4
Measured Y_{tot}	0·061	0·056	0·069	0·058
Predicted Y_{tot}	0·057	0·054	0·067	0·0520
Nozzle velocity coefficient ($M_2 = 0·8$) . .	0·978	0·980	0·975	0·979

This table indicates that total loss in a typical nozzle row of a present-day turbine may be estimated with an error within about ± 10 per cent. An error of ± 10 per cent in the mean loss coefficient for each row of a turbine corresponds to an error in calculated turbine efficiency (peak value) of about ± 1 per cent to ± 1½ per cent; which may be regarded as satisfactory for most purposes. Experimental error in measuring turbine performance is frequently of the same magnitude.

The four nozzles detailed in Tables 1 and 2 give velocity coefficients of between 0·975 and 0·98, which compare well with values measured on modern steam-turbine nozzles (*e.g.*, Dollin[39] (1940), Guy[43] (1939)) having the same efflux angle.

A comparison has been made between the losses measured on various types of nozzle blade (mostly bent sheet metal nozzle blades) by the Steam Nozzle Research Committee (Refs. 18 to 23) and the predicted losses for conventional section nozzle blades having similar pitch/chord ratios and gas outlet angles. This is presented in Table 3 below, an outlet Mach number of 0·8 being assumed in all instances.

TABLE 3

	Gas efflux angle	$\cos^{-1}\frac{o}{s}$	$\frac{s}{c}$	$\frac{t}{c}$	$\frac{t_e}{s}$	$\frac{f}{s}$ †	Y_t	Predicted Y_t for conventional nozzle ‡
Profile nozzle (simliar to conventional section nozzle blade) . .	77	77·3	0·75	0·20	0·02	—	0·135	0·106
Thin sheet metal nozzle	72	73	0·32	0·015	0·05	1·08	0·148	0·105
Thick sheet metal nozzle	79*	78·7	0·45	0·064	0·15	0·84	0·260	0·120
Thick sheet metal nozzle, trailing edge chamfered	79	78·7	0·45	0·064	0	0·84	0·166	0·120
Thick sheet metal nozzle, trailing edge chamfered	79*	78·7	0·33	0·048	0	1·47	0·141	0·126
Thin sheet metal nozzle	78·5	79·7	0·45	0·013	0·03	0·48	0·156	0·116
Thick sheet metal nozzle	78·5	83·2	0·57	0·052	0·09	0·53	0·186	0·113
Thick sheet metal nozzle, trailing edge chamfered	82·5	83·2	0·57	0·052	0	0·53	0·165	0·147
Thick sheet metal nozzle, trailing edge chamfered	82·5*	83·2	0·45	0·041	0	0·88	0·135	0·148

* Gas angle is conjectural.
† f is radius of curvature of bent sheet.
‡ All predicted values of Y_t correspond to $t_e/s = 0·025$
For full details of the blading tested Refs. 18 to 23 should be consulted.

In comparing these values of loss coefficient it must be remembered that the possible error of the predictions for conventional blades at these high values of gas outlet angle (being an extrapolation of existing data) will be about $\pm$ 15 per cent and the possible experimental error in determining the loss coefficient in the steam nozzle tests will be about $\pm$ 10 per cent (equivalent to an error of $\pm$ 0·5 per cent in velocity coefficient).

Thus the predicted and measured values of loss for the profiled nozzle having a gas outlet angle of 77 deg seem to be in fair agreement*. The losses on the thin and thick sheet metal blades are greater than would be expected from conventional nozzle blades. This applies particularly to the thick sheet metal blades, much of the high loss on these blades being associated with the very large trailing-edge thickness. On the other hand if the trailing edge of the thick sheet metal blades is chamfered the losses compare well with those expected from profiled blades, particularly when the gas efflux angle is very high (about 80 deg). However, it is not anticipated that this will apply to nozzles having efflux angles less than about 70 deg.

5.3. *Effect of Radial Tip Clearance.*—As the radial tip clearance in a blade row of a turbine is increased the pressure losses in the row increase, resulting in a decrease of turbine efficiency. The gas mass flow and the power output corresponding to a fixed turbine speed and pressure ratio also change. Published quantitative data on the effect of tip clearance is very incomplete and in most instances the details of the turbines to which experimental results refer are not quoted, so that a reliable collective analysis is difficult.

Stodola[17] quotes a value (attributed to Anderhub) for the loss associated with the tip clearance as

$$\zeta_{\text{clearance}} = 6 \cdot 26 k^{1 \cdot 4}/h \quad \ldots \quad (5)$$

where :—

$\zeta = \psi^{-2} - 1$

ψ is velocity coefficient $= \frac{\text{actual outlet velocity}}{\text{theoretical outlet velocity}}$

k is radial clearance

h is blade height corresponding to zero clearance.

This formula is empirical, it is not non-dimensional, and cannot be regarded as satisfactory. In a later paper Stodola[33] (1925) quotes that on a Brown Boveri reaction steam-turbine the drop in efficiency due to tip clearance (clearance on both rotor and stator rows) could be expressed approximately by $\Delta\eta \simeq 3 \cdot 1(k/h)$. Meldahl[32] (1941) found on a single-stage reaction turbine that $\Delta\eta \simeq 3 \cdot 5(k/h)$. Other unpublished data gives $\Delta\eta \simeq 2 \cdot 6(k/h)$ on a 50 per cent reaction turbine.

From a theoretical standpoint Carter[1] (1948) points out the similarity of the induced effects of tip clearance and secondary flow, and a simple theoretical expression may be derived for the drag coefficient on a row due solely to radial clearance at one end as

$$C_{Dk} = \tfrac{1}{2} C_L^{2} (k/h)/(s/c) \,, \quad \ldots \quad (6)$$

if s/h and gas turning angle is small.

It is interesting to determine what effect this drag would have on the efficiency of a 50 per cent reaction turbine. An expression for the efficiency of a 50 per cent reaction turbine is

$$1/\eta = 1 + 2 \operatorname{cosec} 2\alpha_m \,(C_D/C_L) \,. \quad \ldots \quad (7)$$

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* It is of interest to note that the series of nozzles tested by Kraft[30] (1949) (results shown in Fig. 15b) gave a loss coefficient of 0·075 for a nozzle comparable to the S.N.R.C. profiled blade. This is rather lower than the predicted value. However, the predicted value gives a good indication of the average loss measured on high efflux angle nozzle rows, and until such time as the factors leading to very low losses are completely understood a general prediction cannot go any further than this. Furthermore, nozzles of such high outlet angle as 77 deg are at present only of academic interest to gas turbines.

Suppose $C_D = C_{D0} + C_{Dk}$

where C_{D0} is drag coefficient with zero clearance

C_{Dk} is drag coefficient due to clearance

$= \frac{1}{2} C_L^2 (k/h)/(s/c)$.

We then find that

$$[1/(\eta)^2][\partial\eta/\partial(k/h)] = \operatorname{cosec} 2\alpha_m C_L/(s/c) \; . \qquad (8)$$

The values of $\Delta\eta/(k/h)$ given by this expression of reaction turbines having gas outlet angles from the blade rows of 50 deg, 60 deg and 70 deg are approximately 2·0, 2·3, and 2·9 respectively. On the whole, the measured values of loss appear a little higher than the estimated values but since the measurements are probably not very accurate, due to the difficulty of making accurate experimental measurements of the effect of change in clearance, the use of the simple theoretical value for clearance loss seems justifiable until more systematic and accurate data becomes available.

So far it has been assumed that clearance is varied equally on both rotor and stator of a 50 per cent reaction turbine. It may be demonstrated that in a turbine of any degree of reaction the reduction in stage efficiency due to small radial clearance on a stator row alone is

$$\Delta\eta \simeq \frac{\eta^2 \, . \, \frac{1}{4} \sec^3 \alpha_{ms} \, [C_L/(s/c)]_s^2}{(K_p \, . \, \Delta T/U_m^2)(U_m/V_a)^2} \, (k/h)_s \, . \qquad (9)$$

The reduction in stage efficiency due to a small radial clearance on a rotor row alone is:—

$$\Delta\eta \simeq \frac{\eta^2 \, . \, \frac{1}{4} \sec^3 \alpha_{mr} \, [C_L/(s/c)]_r^2}{(K_p \, . \, \Delta T/U_m^2)(U_m/V_a)^2} \, (k/h)_r \qquad (10)$$

ΔT = turbine work temperature drop per stage; suffix s refers to stator row; suffix r refers to rotor row. These expressions show that on low-reaction stages a clearance on the stator row has a slightly greater effect than the same clearance on the rotor row, particularly if the nozzle gas outlet angle is high.

Shroud bands round the blade tips are often used since a smaller flow leakage round the blade tips may sometimes be achieved with an end-tightened shroud band than with a radial tip clearance. The flow leakage round a shroud band will depend largely upon the minimum clearance between the band and the stator casing. The simplest form of shroud band is shown diagrammatically in Fig. 2, in which the minimum clearance is represented by k'. If this is compared with a simple radial clearance in which $k = k'$ then it is probable that the shrouded arrangement will incur less loss; partly due to less interference between the leakage flow and the mainstream flow, and partly due to the fact that the more tortuous path which the flow must take round the shroud will reduce the actual quantity of leakage flow. For this reason it is suggested that for such a shrouded row the losses might be about half of the losses corresponding to an unshrouded row in which $k = k'$. Thus, for a simple shrouded blade row it will be assumed that

$$C_{Dk} = \frac{1}{4}[C_L^2/(s/c)]k'/h \, . \qquad (11)$$

On more complex shrouds (*e.g.*, shrouds incorporating a labyrinth seal) which, for a given minimum clearance, reduce the flow to a fraction of the value corresponding to the simple form considered above it may be anticipated that the losses will be even smaller.

In addition to creating pressure losses a radial clearance also affects the momentum mean gas efflux angle from a row since the clearance space will allow a portion of the gas flow to pass through the row with little or no deflection. Suppose that with zero clearance a row passes a mass flow, W, with a momentum mean efflux angle of α_2. When clearance is introduced suppose that a fraction of the flow, $X(k/h)(\cos \alpha_1/\cos \alpha_2)W$, passes through undeflected whilst the remainder, $[1 - X(k/h)(\cos \alpha_1/\cos \alpha_2]W$, issues from the row at an angle α_2. The mean outlet angle is then given approximately by

$$\alpha_2' = \tan^{-1}\{[1 - X(k/h)(\cos \alpha_1/\cos \alpha_2)] \tan \alpha_2 + X(k/h)(\cos \alpha_1/\cos \alpha_2) \tan \alpha_1\} . \qquad (12)$$

The fraction passing through undeflected is defined in terms of $(k/h)(\cos \alpha_1/\cos \alpha_2)$ since this term represents the ratio of the flow area of the undeflected flow in the clearance space to the throat area of the blade. Some unpublished nozzle cascade tests and turbine tests suggest a value for X of $1 \cdot 35$. This value for X appears large and it may be that reduction in outlet gas angle not only results from the simple mixing process assumed above but also from induced flows created by the vortices and possibly by the vorticity set up by the motion of the blade tips relative to the stationary wall.

It may be expected that the reduction in mean outlet angle will be less with shrouded blades due to the reduction in leakage flow for a given clearance. In such instances it is suggested that the expression derived in the last paragraph might be used for simple shrouding if k is replaced by k' and if it is assumed that $X = 0 \cdot 7$. Alternatively, for complex shrouds, the expression $(k/h)(\cos \alpha_1/\cos \alpha_2)$ may be replaced by (w/W) where w is the estimated leakage flow round the band and W is the total mass flow.

Combining the expressions derived above for increase in loss and reduction in mean gas outlet angle it is possible to calculate the change in power output and mass flow through a stage operated at fixed speed and pressure ratio resulting from an increase in clearance. Calculations show that on a typical 50 per cent reaction stage the mass flow increases slightly and the power output decreases; on a typical impulse stage the power output and mass flow both decrease. The tests by Meldahl[32] (1941) on a reaction stage partially confirm this.

5.4. *The Pressure Losses in Rotor Rows.*—Typical total pressure losses in rotor rows have been derived for a wide variety of blade rows from routine performance tests on turbine stages in which measurements were made of overall pressure ratio, total-head isentropic efficiency, mass flow, and rotational speed. The method of deriving the rotor losses was as follows. First an estimate of the mean total-head loss coefficient in the nozzle row was made from the data derived in the preceding sections. Then by trial and error a value for total loss coefficient in the rotor was determined such that the calculated overall efficiency of the turbine at a selected value of overall pressure ratio and speed equalled the experimental efficiency. Having derived a value of total loss coefficient in this way an estimate was made of (*a*) profile loss (as in section 4.1) and (*b*) the blade tip clearance or shroud loss (section 5.3). The remaining secondary loss in the rotor row was then derived as:—

$$\text{Secondary loss} = \frac{\text{Derived total}}{\text{loss}} - \frac{\text{Estimated profile}}{\text{loss}} - \frac{\text{Estimated tip}}{\text{clearance loss}} . \qquad (13)$$

Now the absolute error in this estimated secondary loss is likely to be considerable since it will comprise the separate errors involved in estimating the total loss, the profile loss, and the tip clearance loss. However, providing the secondary losses determined in this way are always used in conjunction with the profile losses and clearance losses specified in sections 4.1 and 5.3 then the final estimate of total loss in a blade row should not be excessive.

The results of an analysis of the performance of a number of turbine stages which was carried out in the manner described is tabulated in the following table together with some relevant details of the rotor blades in each instance.

TABLE 4

Turbine	$\beta_1(m/d)$	$\bar{\alpha}_2$	$s/c(m/d)$	Hub ratio, m	A_2/A_1	λ
1	52	−62	0·45	0·85	0·94	0·025
2	39	−54	0·46	0·78	0·89	0·022
3	40	−63	0·63	0·88	0·73	0·011
4	31	−50	0·68	0·65	0·75	0·021
5	43	−54	0·61	0·85	0·92	0·029
6	44	−68	0·43	0·80	0·66	0·008
7	14	−46	0·77	0·69	0·67	0·011
8	5	−69	0·71	0·64	0·38	0·005
9	34	−53	0·67	0·79	0·81	0·010
10	18	−46	0·77	0·71	0·78	0·012

Note.—A_1 = (annulus area at inlet to blade row) × cos β_1
A_2 = (annulus area at outlet from blade row) × cos $\bar{\alpha}_2$

As was observed for the nozzle rows (section 5.2) the values of λ are apparently related largely to the value of A_2/A_1, the secondary losses being particularly large on the blades of turbines Nos. 1 and 5 which had low-reaction blades of high deflection.

Following the same procedure as with the nozzle blades the values in Table 4 are plotted in Fig. 16b against $(A_2/A_1)^2$. This figure clearly demonstrates the tendency for secondary losses to be high when the acceleration of the gas flow in the blade row is small. It seems also that the effect of area ratio is more important than the magnitude of the gas deflection (*cf.* turbines Nos. 5 and 6). Reeman[4] (1946) also found on cascade tests that secondary losses were little influenced by the magnitude of the deflection; impulse blades of low deflection having secondary losses as high as impulse blades of much larger deflection.

The figures in Table 4 suggest that, in addition to A_2/A_1, the hub ratio influences the secondary loss. For example, turbines Nos. 3 and 4 have roughly similar values for A_2/A_1 but the secondary loss of the row having the smaller hub ratio is very much greater than the other. This effect is probably associated with the low stage reactions (as compared with the mid-blade height) at the roots of rows having a small hub ratio, although the precise nature of the flow at the roots of such stages has not yet been amply investigated.

On account of the apparent effect of hub ratio it has been preferred to plot λ against a parameter $(A_2/A_1)^2/[1 + (\text{I.D./O.D.})]$. This is shown in Fig. 17 in which all available results, including the nozzle data discussed in section 5.2 and some cascade losses published by Reeman[4] (1946), have been plotted. In selecting cascade results from Ref. 4 only blade arrangements of significant interest to turbine design have been chosen. Furthermore, the accuracy with which these cascade losses were measured may have been poor, particularly on rows which gave the flow only a small acceleration ; this may account for much of the scatter displayed by the cascade results.

Traverses at the turbine outlet have only been made in a few instances so that correlation with the parameter $\delta h/h$ has not been possible. On the nozzle blades which were traversed the values of $\delta h/h$ were between 0·10 and 0·15 whereas in the few instances when the outlet distributions were measured on the low-reaction rotor blades larger values of $\delta h/h$ (about 0·15 to 0·20) were recorded. It is thus probable that the higher secondary losses on these blades are partly accountable to thicker boundary layers on the end walls. Nevertheless, it is also very probable that comparatively thick boundary layers are unavoidable in the low-reaction blades.

A typical outlet distribution of velocity from a low-reaction turbine stage is shown in Fig. 19. The major thickening of the boundary layer occurs at the blade root; at the tip the flow is accelerated through the radial tip clearance space. A major fraction of the secondary loss on such blade rows therefore appears to occur in the vicinity of the blade root, where the local gas accelerations through the row are least.

On high-reaction turbines the velocity distributions are more uniform and the secondary losses are not so great.

The trend of these secondary loss results is significant in that it demonstrates the desirability of providing a marked degree of gas flow acceleration in a blade row, particularly when it is wished to negotiate high deflections. This accords with observations made in the past by steam-turbine investigators who found that the introduction of a small degree of reaction into a turbine stage which had initially been designed as an impulse stage allowed a substantial improvement in efficiency.

The following table illustrates the comparative magnitudes of the various component losses in some typical impulse and nozzle blade sections.

TABLE 5

Blade type	Nozzle			Impulse		
Outlet angle	50	60	70	50	60	
Deflection	50	60	70	100	120	
s/c	0·9	0·8	0·7	0·7	0·6	
Profile-loss coefficient, Y_p	0·021	0·024	0·038	0·074	0·101	
Secondary-loss coefficient, Y_s ..	0·0242	0·0266	0·0314	0·259	0·328	
Clearance-loss coefficient, Y_k ($k/h = 0 \cdot 02$) ..	0·0273	0·0396	0·0603	0·094	0·119	
Total-loss coefficient, Y_t	0·0715	0·0902	0·1297	0·427	0·548	
Blade velocity coefficient ($M_2 = 0 \cdot 8$)	0·974	0·967	0·955	0·877	0·852	
Total-loss coefficient, Y_t	0·0452	0·0506	0·0694	0·333	0·429	Zero tip clearance.
Blade velocity coefficient ($M_2 = 0 \cdot 8$)	0·983	0·980	0·975	0·899	0·877	

On nozzle blades the profile losses are comparable to the losses which would be anticipated due to skin friction associated with a turbulent boundary layer on the entire blade surface together with the loss due to the finite trailing-edge thickness (*cf.* Andrews and Schofield[71] (1950)). The secondary losses compare with the theoretical estimate (equation (3), section 5.2) if $[1 - (h'/h)]$ is approximately equal to $0 \cdot 03$. Now if the ratio of the boundary-layer thickness to the blade height is approximately $0 \cdot 15$ (*see* Table 1) then a value of $[1 - (h'/h)]$ of $0 \cdot 03$ is approximately equal to the ratio of the displacement thickness of the wall boundary layer to half the blade height, assuming a turbulent wall boundary layer. In other words, following Carter's analogy[1] (Carter[1] (1948)) between secondary loss and tip clearance loss, the secondary loss is roughly equivalent to the theoretical loss due to a tip clearance at each end of the blade equal to the wall boundary-layer displacement thickness. It seems plausible to conjecture from this that the measured secondary losses are as low as may be expected for the type of velocity distribution normally encountered in a nozzle row; and future development is unlikely to result in a substantial decrease in nozzle loss, except perhaps by such a method as wall boundary-layer suction which in hot turbines does not appear practicable. On low-reaction blades, however, the magnitude of the profile losses implies that there is a substantial amount of loss due to flow separation; this may eventually be reduced by use of better section profiles. The secondary losses are also very much larger (in terms of equivalent tip clearance) on impulse blades than on nozzle blades and this may indicate that losses are taking place due to separation resulting from secondary flows. Whether such losses may be reduced by use of different blade profiles or whether they are inescapable on high-deflection blades having low reaction is not known. However, it may be significant that a normal value for λ on an axial compressor blade row in which the gas is diffused, is about 0·02; this value is much less than that deduced for an impulse turbine blade (about 0·028). Thus it may be conjectured that either turbine impulse-blade secondary losses can eventually be reduced to give a value of λ at least comparable to the axial compressor value or alternatively the higher value of λ measured on impulse turbine blades is inherently associated with the high gas deflection in the latter and that the earlier deduction that secondary loss is not greatly influenced by deflection is erroneous.

Since the use of high-deflection blades with low reaction is highly advantageous in gas turbines (in relation to size, weight, cost and the use of blade cooling) a future detail study of these problems is desirable.

5.5. *Variation of Secondary and Profile Losses with Incidence in a Turbine.*—The previous analysis of secondary loss relates strictly only to blades operating at incidences in the vicinity of zero. If it is assumed that λ remains constant for all incidences and the profile loss is varied in accordance with the mean curve shown in Fig. 8a then it is found that estimates of total loss at high negative incidence are generally lower rather than those measured. Correlation may be improved either by adjusting the variation of profile loss with incidence or by adjusting the value of λ at incidences other than zero. An adjustment of the former type results in a variation of profile loss with incidence for turbine blade rows as shown in Fig. 18. Thus, by adjusting profile loss with incidence in accordance with the mean curve marked 'turbine' and assuming that λ remains constant as the incidence is varied the total loss at incidences down to $i/i_{\text{stall}} = -2 \cdot 0$ may be predicted with fair accuracy ($\pm$ 15 per cent).

It is worthy of note that the differences between the 'turbine' and 'cascade' variations of profile loss with incidence are qualitatively similar to those that might be anticipated as a consequence of a peaked velocity distribution in a turbine blade row.

6. *Optimum Blade Pitching.*—Secondary and tip clearance losses in a blade row may be expressed in the form of a loss coefficient as

$$\begin{aligned} Y_s + Y_k &= (C_{Ds} + C_{Dk})(\cos^2 \alpha_2/\cos^3 \alpha_m)/(s/c) \\ &= [\lambda + 0 \cdot 5(k/h)][\cos^2 \alpha_2/\cos^3 \alpha_m][C_L/(s/c)]^2 \\ &= 4[\lambda + 0 \cdot 5(k/h)][\cos^2 \alpha_2/\cos \alpha_m][\tan \alpha_1 - \tan \alpha_2]^2 \quad \ldots \quad (14) \end{aligned}$$

Equation (14) shows that secondary and tip clearance losses in a blade row having fixed inlet and outlet gas angles are independent of s/c. This leads to the simple conclusion that the optimum pitching of a row of blades is equal to the pitch giving the minimum profile loss. This conclusion agrees with that reached by Johnston[27] (1951).

Thus, the optimum pitch/chord ratios for nozzle and rotor blades may be determined directly from Fig. 4. These values are plotted in Fig. 20 against gas outlet angle. It is of interest to compare these values with those suggested by other investigators.

The earliest rule for optimum spacing of impulse blades was formulated by Brilling (*see* Ref. 17) who stated that the mean radius of curvature of the blade passage should be twice the passage width. This results in an optimum pitch/chord ratio for impulse blades (designed for constant passage area) of $1/(2 \cdot 5 \sin 2\alpha_2)$. Zweifel[35] (1945), by a theoretical approach, extended this rule to other types of blading and deduced that the optimum spacing was specified by a 'loading coefficient' (equivalent to the lift coefficient based on outlet velocity, $C_{L\,v2}$) of 0·8. Independently, but by a similar theoretical approach, Howell and Carter[36] (1946) deduced that optimum spacing occurred when $C_{L\,v2} = 1\cdot125[6(s/c) - 1]/5(s/c)$. The values of optimum s/c derived by these rules are compared with those derived by the present analysis in Fig. 20. There is appreciable difference between the values given by each of the rules; notably the present analysis does not indicate a tendency for optimum s/c to increase at high gas outlet angles. Some tests by Dowson[38] (1938) on the effect of circumferential pitching on a reaction steam turbine equipped with blades having 65 deg outlet angle and small inlet blade angle (*i.e.*, nearly equivalent to nozzle blades) gave an optimum pitch/chord ratio of roughly 0·6. This is smaller than would be predicted by the present analysis and certainly lends no support to the theory that optimum values of s/c increase at high outlet angles.

The present analysis gives very high values of s/c for low-deflection impulse blades when compared with the previous theories. In applying this data it should be noted that large values of s/c lead to small stalling incidences. Thus values of s/c which are less than the optimum may give slightly higher losses than the minimum but at the same time will give a better working range of incidence. Furthermore, it may be seen from Fig. 4 that considerable variation of s/c is possible without appreciably affecting the loss.

7. *Effect of Annulus Flare.*—Small angles of divergence between the walls at the ends of blades reduce the mean acceleration of the gas in the blade passages. This probably results in a thickening of the wall boundary layers at the outlet from the blade and an increase in secondary loss in accordance with the increase in the parameter $(A_2/A_1)^2/[1 + (\text{I.D./O.D.})]$ as shown in Fig. 17. Large angles of flare, however, may result in a complete separation of the flow from the inner wall, accompanied by high pressure losses and large angular deviations of the flow from the design blade efflux angles in the vicinity of the inner half of the blade height. On nozzle rows having outlet angles in the region of 60 deg to 70 deg a divergence between the walls of about 25 deg can probably be employed without risk of this separation. On low-reaction blade rows or rows having small hub ratios, however, the flare angle should be less than this if possible.

8. *Effect of Reynolds Number on Turbine Efficiency.*—The influence of *Re* on profile loss has been briefly discussed in section 4.4. In addition to profile loss it is possible that *Re* will also influence the magnitude of the boundary-layer thickness on the annular walls and consequently affect the secondary loss. Thus, if secondary loss is roughly proportional to the boundary-layer thickness and the boundary layer on the annulus walls is turbulent (as it may be expected to be) then it might be anticipated that secondary loss will vary approximately as $Re^{-1/5}$. Above $Re = 1\cdot0 \times 10^5$ Fig. 10 shows that profile loss also varies roughly to the same power of *Re* so that is is probable that for a complete turbine stage $(1 - \eta) \propto Re^{-1/5}$ when $Re > 1\cdot0 \times 10^5$. When $Re < 1\cdot0 \times 10^5$ cascade profile losses tend to increase more steeply with decreasing *Re*, in many instances the loss being more nearly proportion to $Re^{-1/2}$, particularly with low-reaction blades. However, the high degree of turbulence that must exist in a turbine stage may have the effect of reducing the 'critical' *Re*, below which the profile loss begins to rise steeply, to values substantially less than the value of about $1\cdot0 \times 10^5$ indicated by cascade tests.

Evidence exists of reaction turbines giving relatively high efficiencies (over 85 per cent) at Reynolds numbers as low as 2×10^4, which implies that the fall of efficiency with decreasing Reynolds number below $Re = 1 \times 10^5$ may not be as severe as cascade tests might suggest. More experimental evidence is required before this may be proved or refuted.

However, in the absence of more explicit data the assumption that turbine aerodynamic efficiency varies with *Re* according to the law $(1 - \eta) \propto Re^{-1/5}$ should prove to be reasonably reliable down to Reynolds numbers of about $3\cdot0 \times 10^4$.

It should be emphasised that this law applies only to aerodynamic efficiency. If the reduction of Re occurs as a consequence of a reduction in rotational speed or, for aircraft units, as a consequence of an increase in altitude then the ratio of mechanical losses in bearings, gears, etc., to the turbine power output will increase; thus the mechanical efficiency will drop. For example, on an aircraft engine operating at constant speed at different altitudes the mechanical efficiency will vary roughly according to the law $(1 - \eta_m) \propto Re^{-1}$. This may have an appreciable effect on the overall turbine efficiency (*i.e.*, $\eta_{\text{aerodynamic}} \times \eta_{\text{mechanical}}$) at low values of Re.

9. *Disc Windage Losses.*—A number of experimenters[16, 17, 37] have measured the power required to rotate discs in still gas or fluid and have attempted to correlate the power with theoretical estimates based on skin-friction laws for flow on flat surfaces. A résumé of the theoretical work and some reliable experimental results are quoted by Goldstein (Ed.)[16] (1938) for discs rotating in a stationary fluid which extends to infinity. The results may be expressed approximately (for both faces of a disc) as

$$\text{H.P.} = 0\cdot 09\ \nu^{0\cdot 2}\rho(D)^{4\cdot 6}(N/1000)^{2\cdot 8}. \qquad \text{if } ND^2/\nu > 10^7 \qquad (15)$$

(D is disc diameter, ft; N = r.p.m.; ν is kinematic viscosity, sq ft/sec; ρ is gas density, lb/cu ft).

This expression can only be expected to give a very approximate idea of the disc friction since it is known on the one hand that it can be substantially reduced by the presence of adjacent stationary surfaces (Stodola[17] (1945)) and on the other hand it can be increased by the presence of excrescences on the disc surface (such as bolt heads).

However, as a proportion of the turbine output power the disc friction is normally sufficiently small to be neglected except on very small turbines or turbines operating at very low Reynolds number.

10. *Note on Influence of Outlet Flow Conditions from the L.P. Stage on Overall Expansion Efficiency.*—On low-reaction turbines the axial velocity and swirl often has a form similar to that shown in Fig. 19. Notably there is frequently a marked reduction in velocity near the inner diameter of the turbine annulus which may lead to early flow separation on the inner surface of an axial annular diffusing duct placed downstream of the turbine. Now the outlet velocities are greatest (and consequently the desirability for diffusion in the exhaust duct is also greatest) on low-reaction turbines having high output per stage and on such turbines the reduction in overall expansion efficiency as a result of exhaust duct losses may be considerable. Thus, when speculating upon the number of stages to be employed in a turbine to achieve a required work output this exhaust loss should be given careful consideration. Table 6 below compares the overall expansion efficiencies of a single and two-stage turbine for a hypothetical jet engine in which it is assumed that the exhaust and jet-pipe losses are 20 per cent of the kinetic energy at the turbine outlet.

TABLE 6

	Single stage turbine	Two stage turbine
$K_p \Delta T/\frac{1}{2}U^2$ (per stage)	4·0	2·0
U/V_a (outlet)	1·0	1·8
η (turbine alone)	0·86	0·90
η (overall)	0·825	0·885

It is noteworthy that the two-stage turbine not only has a smaller exhaust loss than the single-stage turbine but also (if blade peripheral speeds are the same in both cases) the aerodynamic efficiency of the turbine alone is improved, thus leading to a large gain in overall efficiency.

11. *Principal Conclusions.*—(*a*) As a result of an examination of available experimental and theoretical work relating to the pressure losses and gas efflux angles in turbine blade rows a number of curves have been derived enabling approximate pressure losses and gas efflux angles to be predicted for a wide range of turbine blading. The analysis applies primarily to 'conventional' blading and caution must be exercised in applying it to types of blading beyond the scope of the analysis.

(*b*) A significant feature which the analysis has brought to light is the high loss (particularly secondary loss) which normally occurs in high-deflection blading having low reaction. Furthermore, secondary loss appears to be strongly influenced by the magnitude of the mean acceleration imparted to a gas as it flows through the blading, the losses increasing rapidly as the mean acceleration (irrespective of the magnitude of the deflection) is reduced.

(*c*) The magnitudes of the losses on low-reaction blades having large deflections are suggestive of much separated flow. This separation occurs with two-dimensional flow and it may further be amplified by secondary flow. Some evidence suggests that a large proportion of the loss is created at the inner diameter of such blade rows. Since the use of low-reaction blading is highly desirable in gas turbines for many applications it is considered that a more exhaustive investigation of the performance of such blading might usefully form an important part of future turbine research.

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APPENDIX I

Nomenclature

α_1	Gas inlet angle
α_2	Gas outlet angle
α_m	Vector mean gas angle
β_1	Blade inlet angle
β_2	Blade outlet angle
γ	Ratio of specific heats
ρ	Gas density
ψ	Blade velocity coefficient
λ	A factor defining secondary loss $= C_{D\,s}\,(s/c)/C_L^2$
$\bar{\omega}$	Loss of total pressure between blade inlet and outlet
η_s	Stage efficiency
a	Distance of point of maximum camber from leading edge
b	Maximum camber
c	Blade chord
e	Mean radius of curvature of upper surface between the throat and the trailing edge
h	Blade height
i	Incidence
k	Radial tip clearance
k'	Minimum shroud clearance
k_p	Specific heat at constant pressure
o	Blade opening
M_2	Mach number at outlet from a blade row
s	Blade pitch
t	Blade thickness
t_e	Trailing-edge thickness
A_1	Inlet flow area = (annulus area at blade inlet) $\times$ cos β_1
A_2	Outlet flow area = (annulus area at blade outlet) $\times$ cos α_2
X	Factor defining amount of gas undeflected by rotor row due to end clearance
C_D	Drag coefficient based on vector mean velocity
$C_{D\,k}$	Drag coefficient due to tip clearance, based on vector mean velocity
C_L	Lift coefficient based on vector mean velocity
$C_{L\,v2}$	Lift coefficient based on outlet velocity
P_1	Total pressure of gas entering blade row
P_2	Total pressure of gas leaving blade row

$P_{2\,stat}$	Static pressure of gas leaving blade row
U_m	Mean turbine rotor blade speed
V_1	Inlet velocity
V_2	Outlet velocity
$V_{a\,1}$	Inlet axial velocity
$V_{a\,2}$	Outlet axial velocity
$V_{W\,1}$	Inlet whirl velocity
$V_{W\,2}$	Outlet whirl velocity
V_m	Vector mean velocity
Y_p	Profile-loss coefficient
Y_s	Secondary-loss coefficient
Y_k	Tip-clearance-loss coefficient
Y_t	Total-loss coefficient

Fundamental relationships :—

(*a*) Vector mean angle:—

$$\tan \alpha_m = \tfrac{1}{2}[\tan \alpha_1 + \tan \alpha_2]$$

(*b*) Lift coefficient:—

$$C_L = 2(s/c)[\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m$$
$$C_{L\,v2} = 2(s/c)[\tan \alpha_1 - \tan \alpha_2] \cos^3 \alpha_m/\cos^2 \alpha_2$$

(*c*) Drag coefficient:—

$$C_D = \bar{\omega}(s/c) \cos \alpha_m/(1/2)\rho \,.\, V_m^2$$
$$C_D = (s/c) \,.\, Y \cos^3 \alpha_m/\cos^2 \alpha_2$$

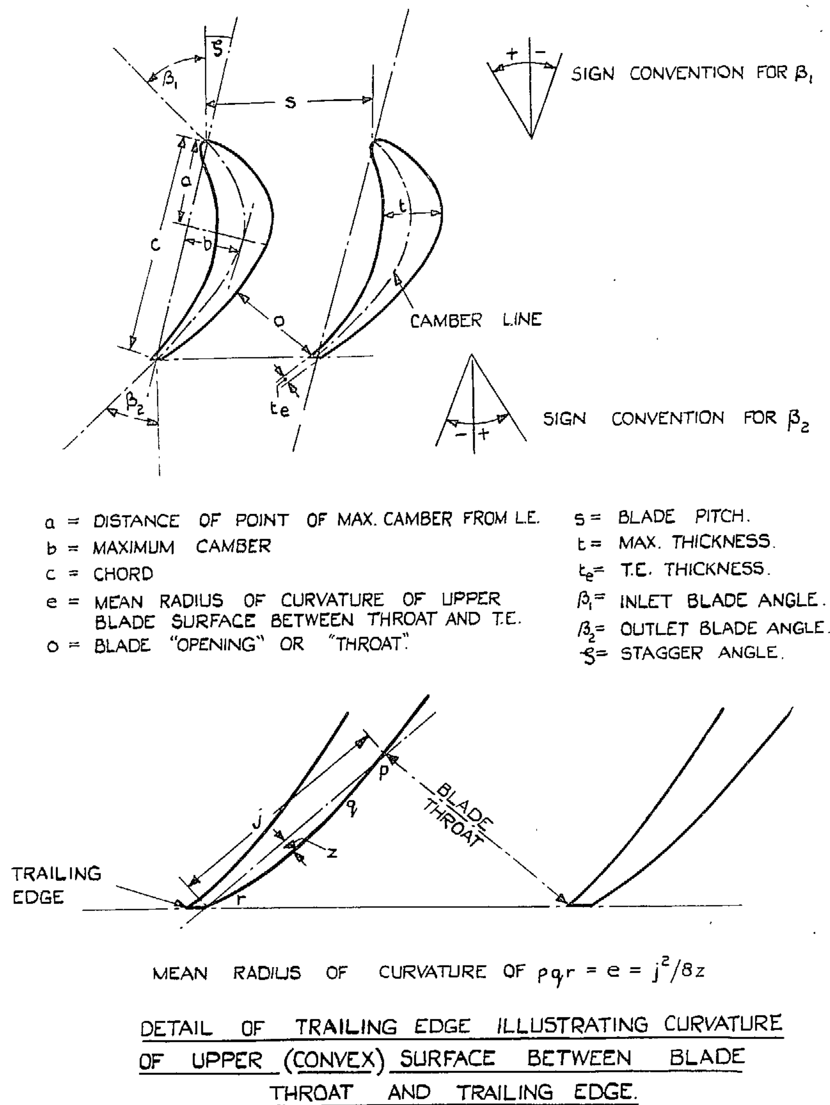

FIG. 1. Turbine blade nomenclature.

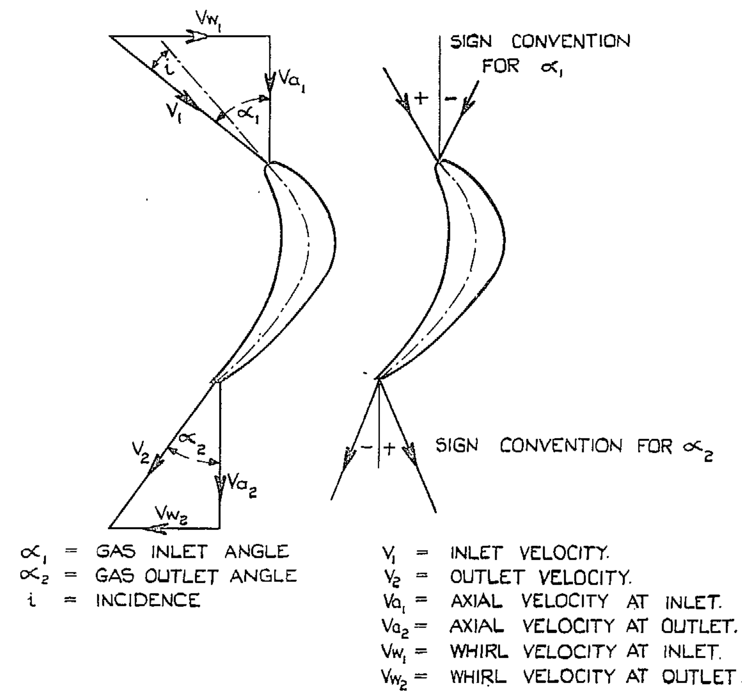

FIG. 2a. Gas angle notation.

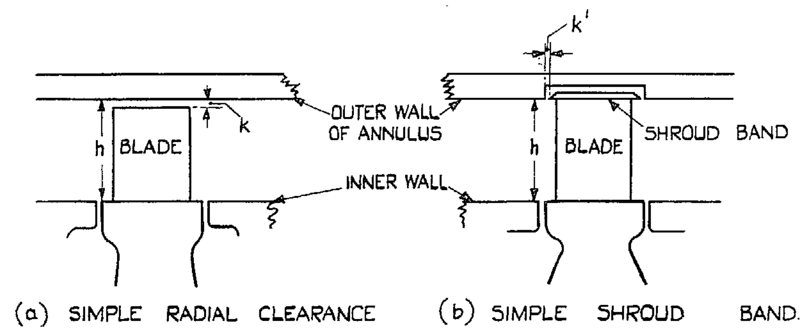

FIG 2b. Diagram illustrating blade end clearances.

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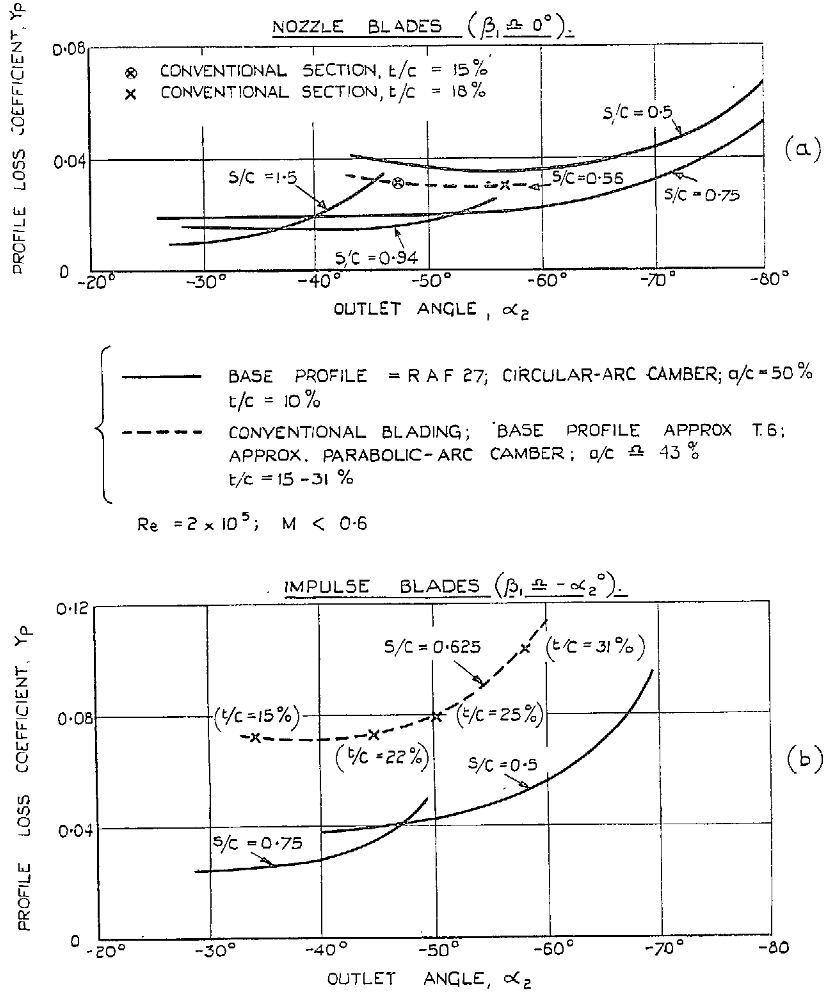


FIG. 3. Profile-loss coefficients for cascades of turbine blades. Incidence ≏ 0 deg.

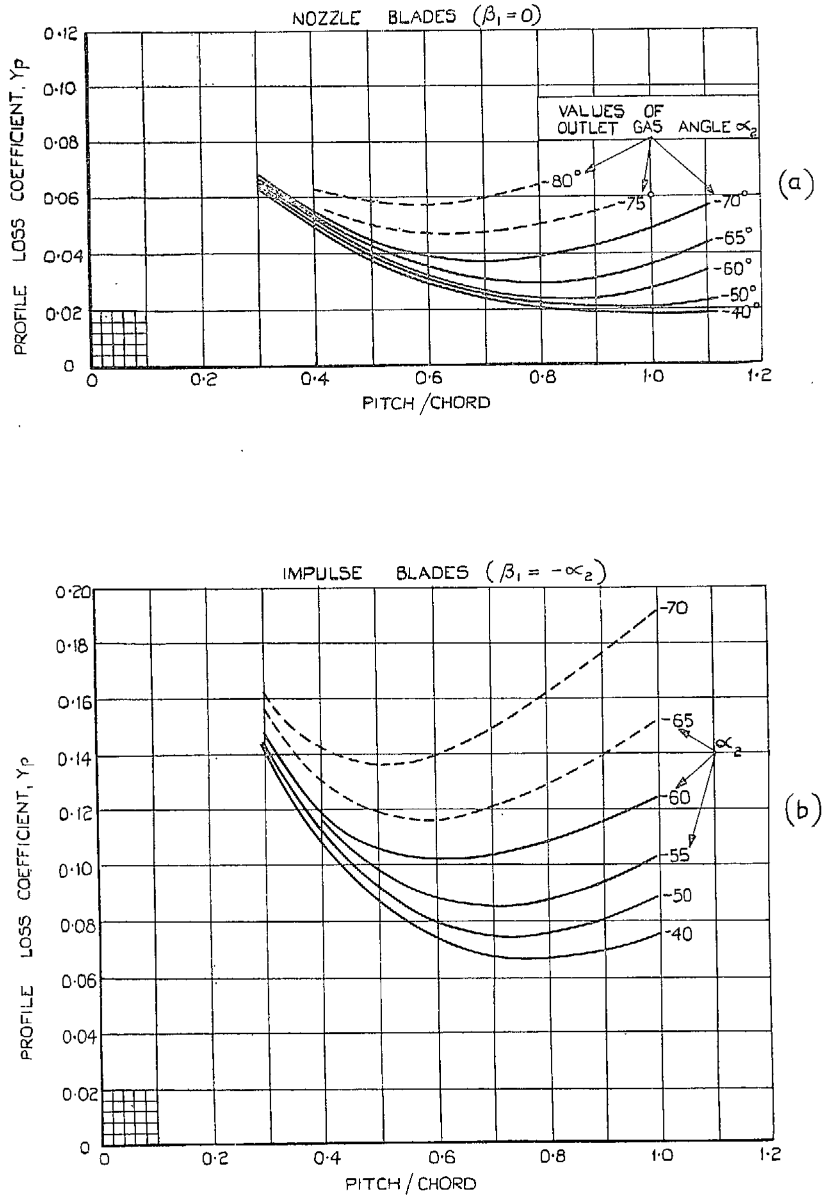


FIG. 4. Profile-loss coefficients for conventional section blades at zero incidence. $t/c = 20$ per cent; $Re = 2 \times 10^5$; $M < 0{\cdot}6$.

B*

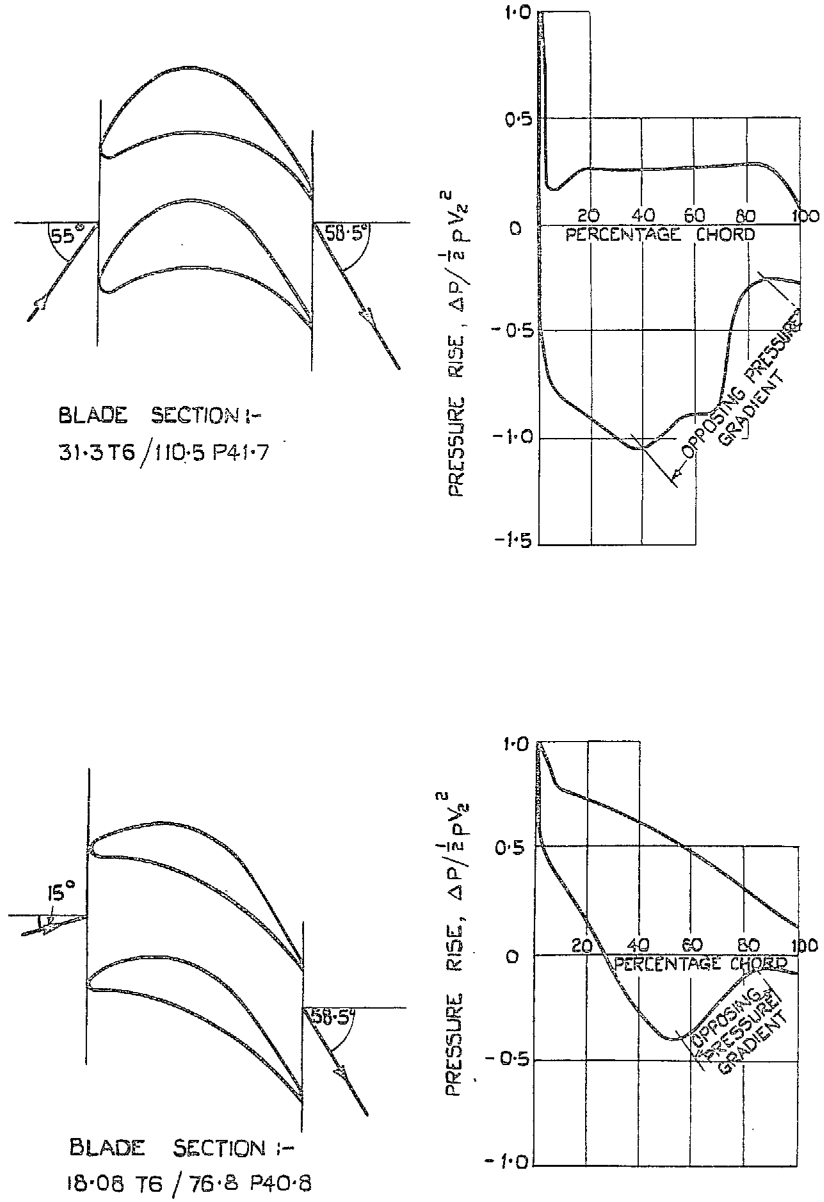


FIG. 5. Theoretical pressure distributions (incompressible flow).

BLADE DETAILS					
	β_1	β_2	$\cos^{-1} o/s$	s/c	t/c
IMPULSE BLADE	45·5	-45·8	50	0·625	22%
REACTION BLADE	18·9	-47·1	50	0·58	15%

$M_2 = 0{\cdot}5$, $Re \simeq 1{\cdot}5 \times 10^5$

BLADE PROFILE T.6

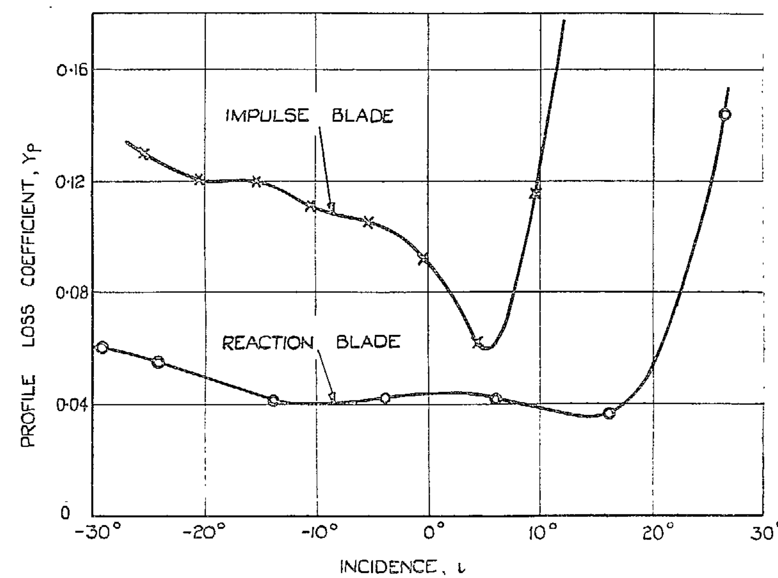


FIG. 6. Variation of profile loss with incidence for typical turbine blades.

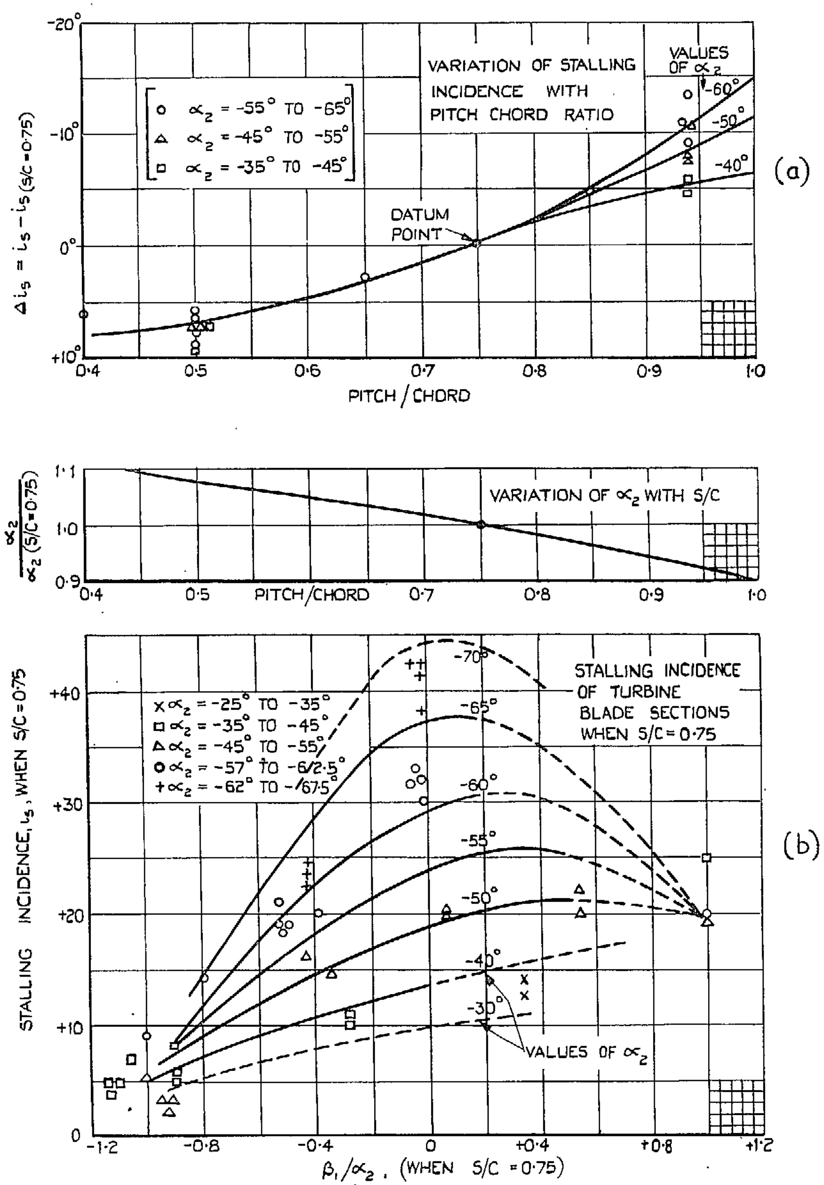


FIG. 7. Positive stalling incidences of cascades of turbine blades. $Re = 2 \times 10^5$; $M < 0{\cdot}6$.

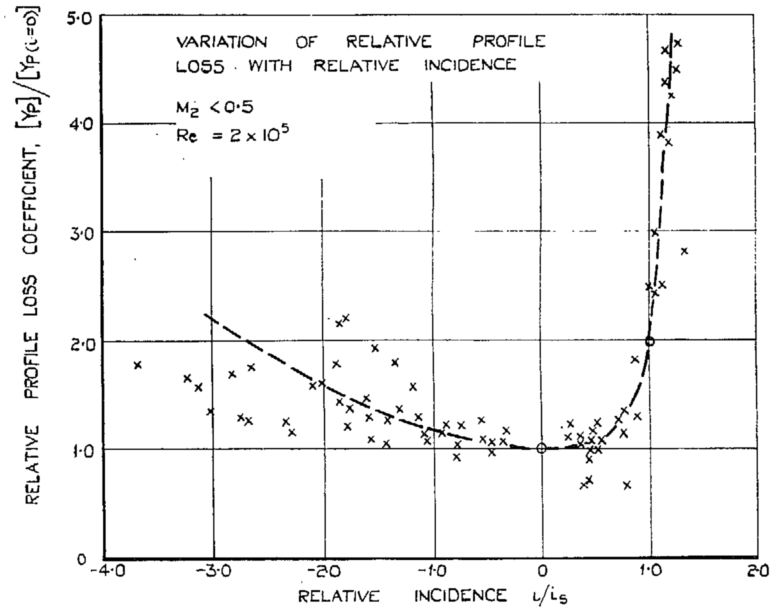


FIG. 8a. Variation of loss and outlet angle with incidence.

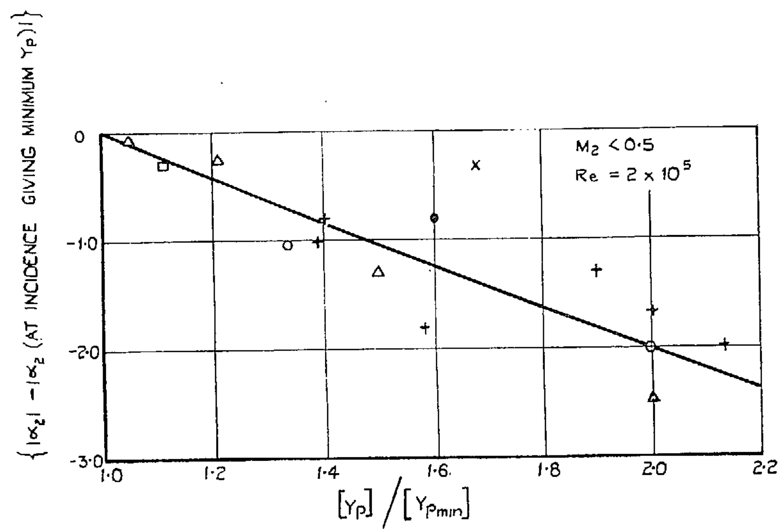
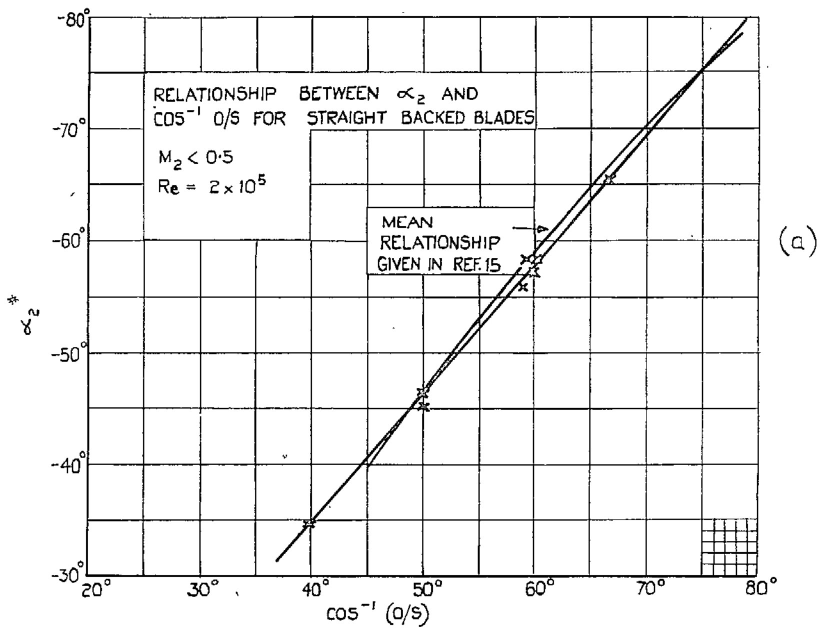


FIG. 8b. Variation of α_2 with relative profile loss at positive incidences.



FOR BLADES IN WHICH THE UPPER SURFACE OF THE BLADE PROFILE IS CURVED BETWEEN THE THROAT AND THE TRAILING EDGE THE OUTLET ANGLE IS APPROXIMATELY EQUAL TO $\alpha_2^* - 4(s/e)$, WHEN $M_2 < 0{\cdot}5$ AND $Re = 2 \times 10^5$

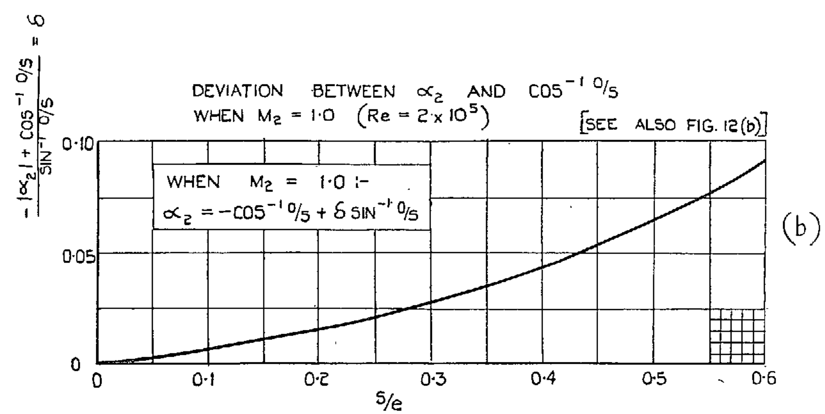
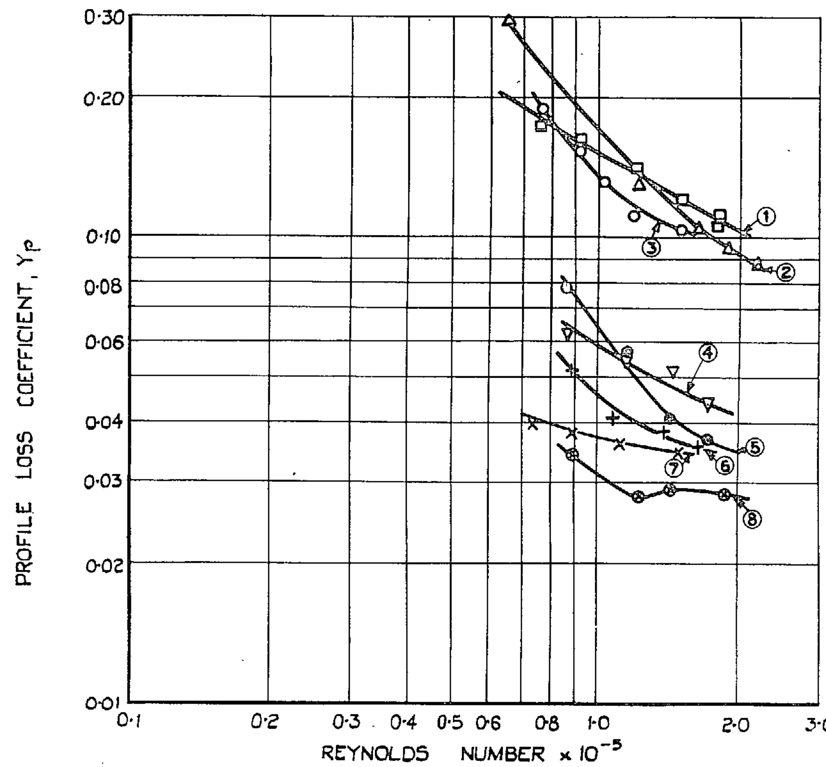


FIG. 9. Gas outlet angle from cascades of turbine blades.



BLADE DETAILS					
BLADE	SECTION	t/c	s/c	β_1	α_2
1	T. 6	31·3 %	0·625	55·4	58
2	T. 6	21·7 %	0·625	45·5	45
3	T. 6	24·6 %	0·625	49·0	50
4	T. 6	23·6 %	0·584	35·5	57
5	C. 7.	20·0 %	0·625	30	62
6	T. 6.	15·0 %	0·584	18·9	46
7	T. 6.	18·1 %	0·551	18·8	57
8	C. 7.	10·0 %	0·625	30	58

FIG. 10. Variation of profile loss with Reynolds number ($i \simeq 0$).

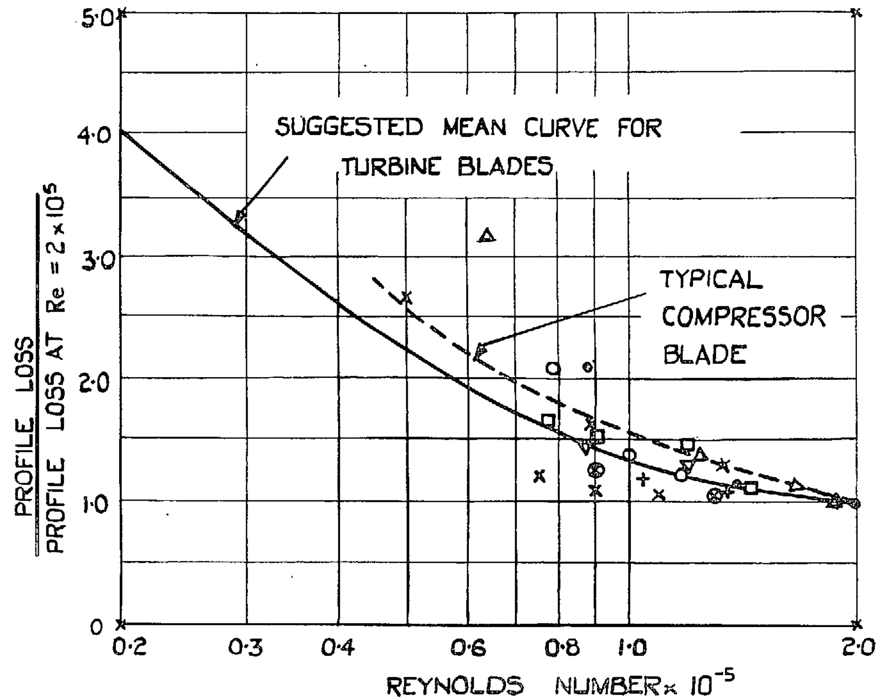


FIG. 11a. Variation of profile loss with Reynolds number.

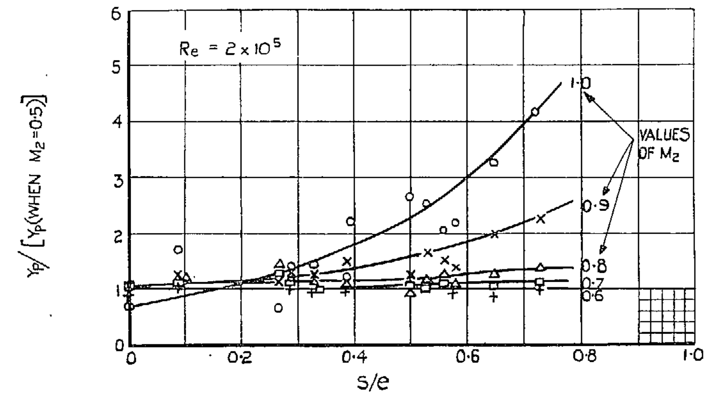


Fig. 12a. Variation of profile loss with Mach number and s/e.

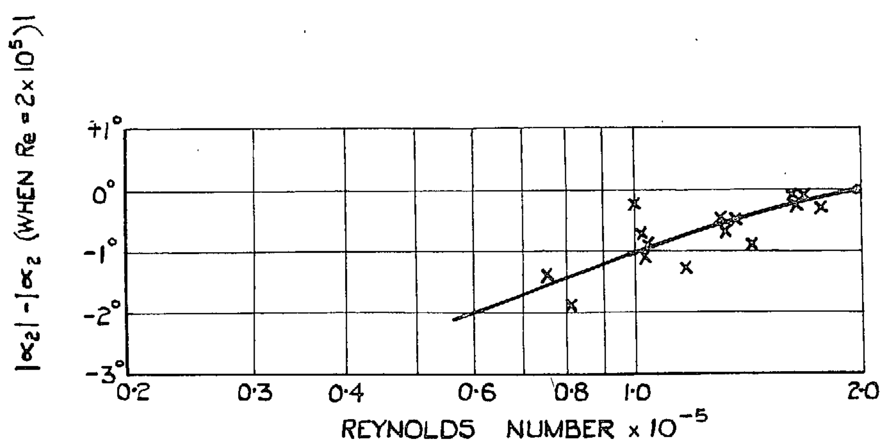


FIG. 11b. Variation of α_2 with Reynolds number.

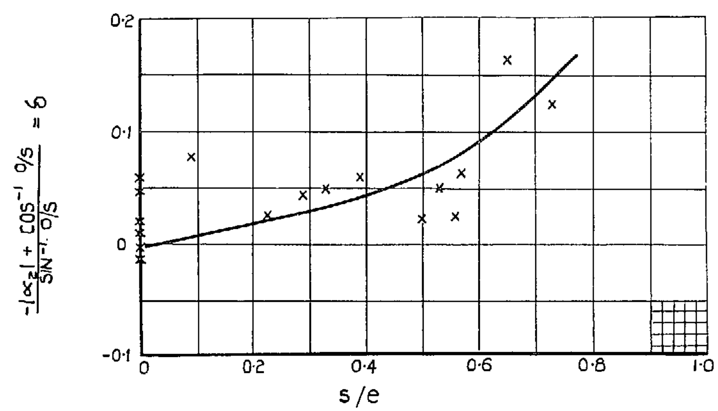


Fig. 12b. Gas outlet angle when $M_2 = 1 \cdot 0$.
($\alpha_2 \doteq - \cos^{-1} o/s + \delta \sin^{-1} o/s$ when $M_2 = 1 \cdot 0$.)

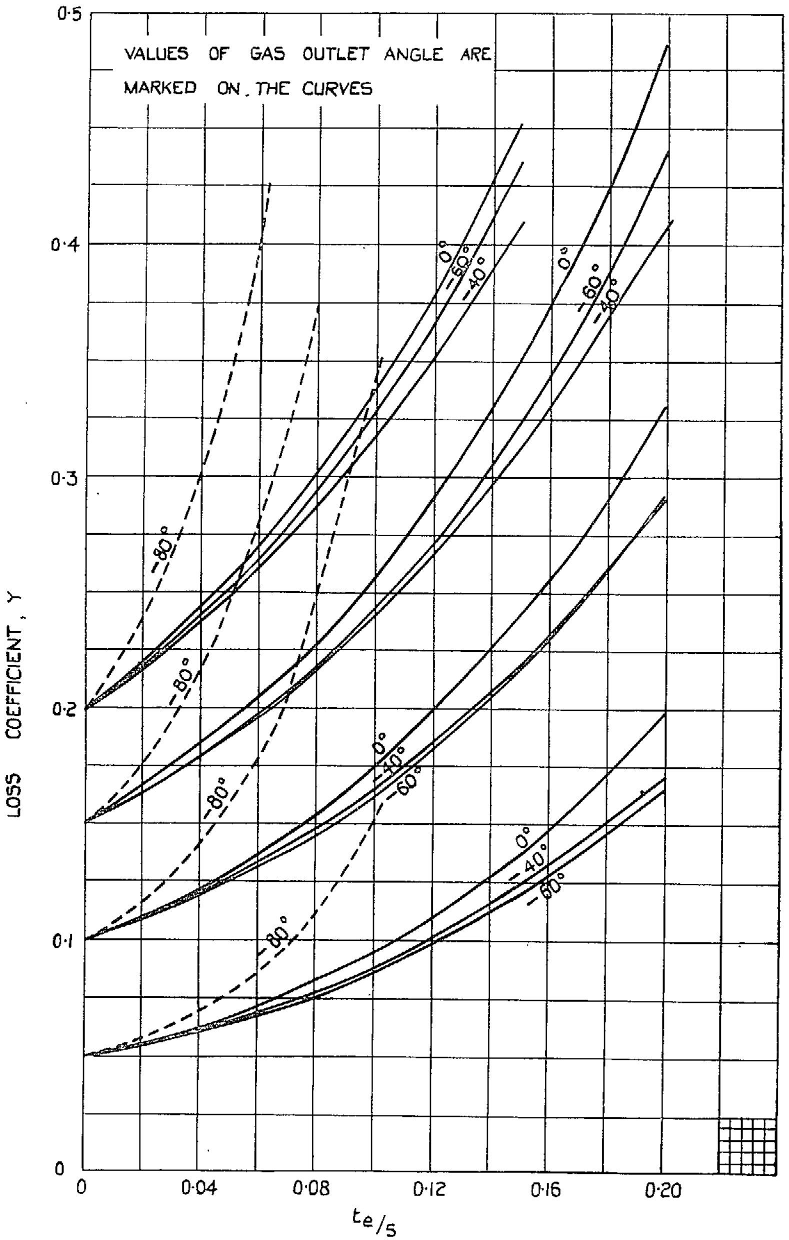


FIG. 13. Effect of trailing-edge thickness on blade loss (calculated).

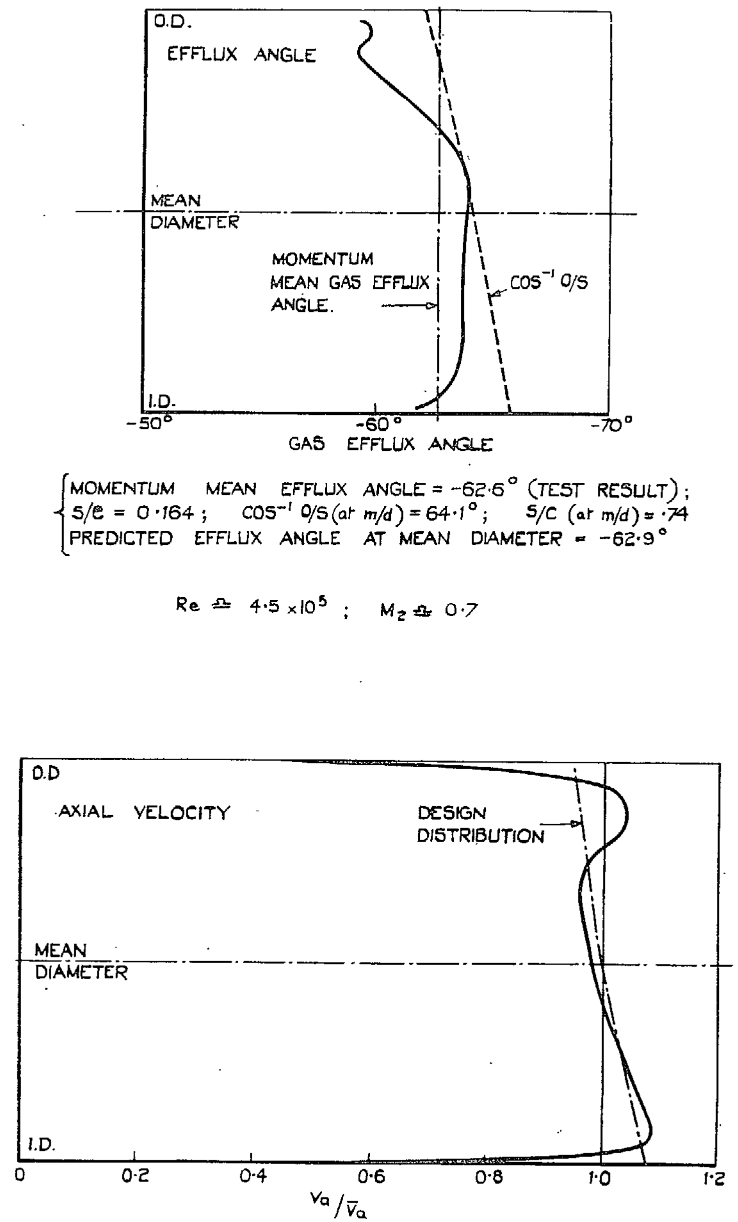


FIG. 14. Variations of efflux angle and axial velocity at outlet from a nozzle row.

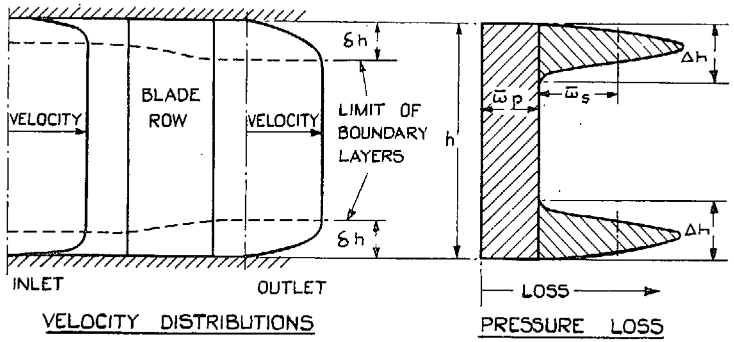


FIG. 15a. Idealised three-dimensional flow through a row of blades.

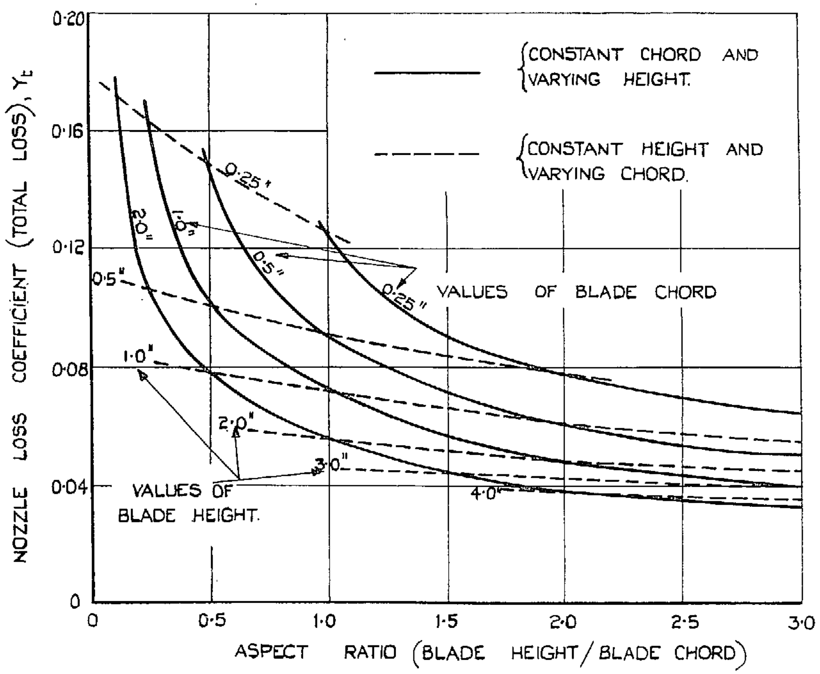


FIG. 15b. Variation of nozzle-loss coefficient with blade aspect ratio. $s/c \simeq 0{\cdot}77$; $\cos^{-1} o/s \simeq 78$ deg; $M_2 = 0{\cdot}8$.

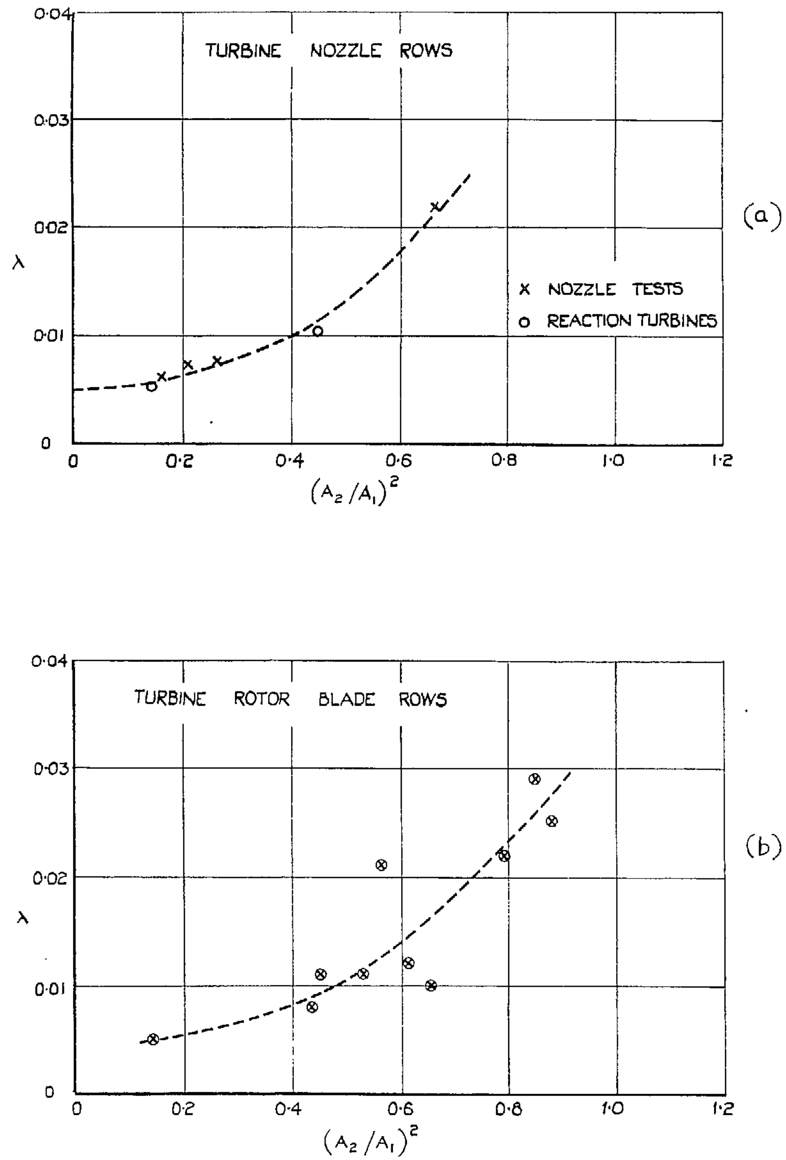


FIG. 16. Secondary losses in turbine blade rows.

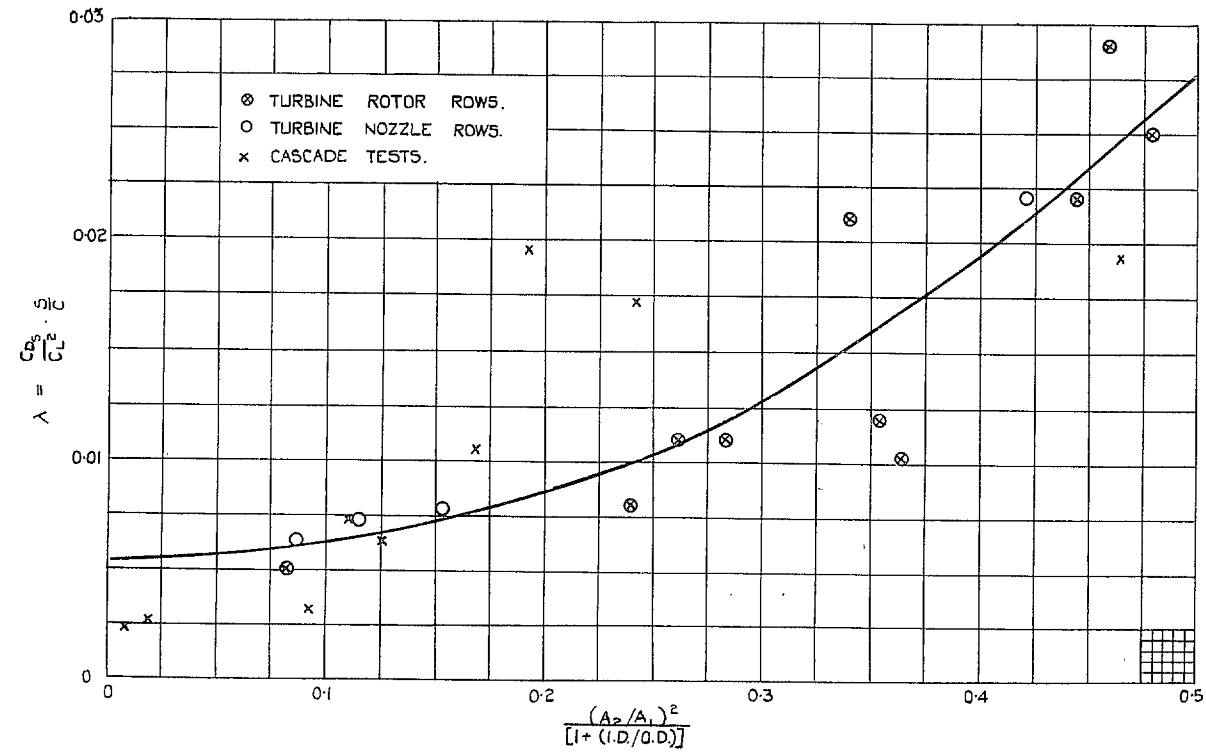


FIG. 17. Secondary losses in turbine blade rows.

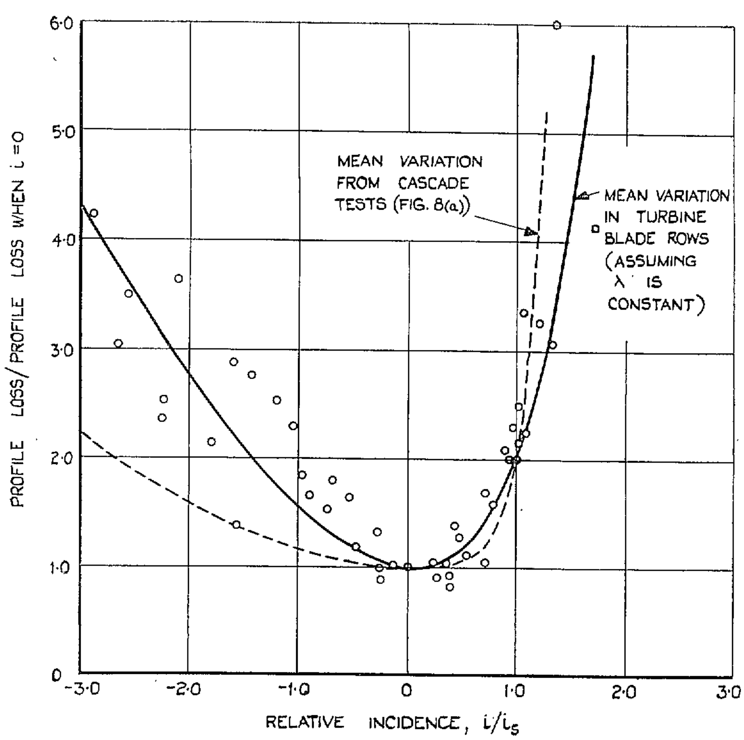


FIG. 18. Variation of profile loss with incidence in turbine rows, (deduced from turbine tests, assuming that λ is constant for a blade row).

(63368) Wt. 18/9296 K.9 6/55 Hw.

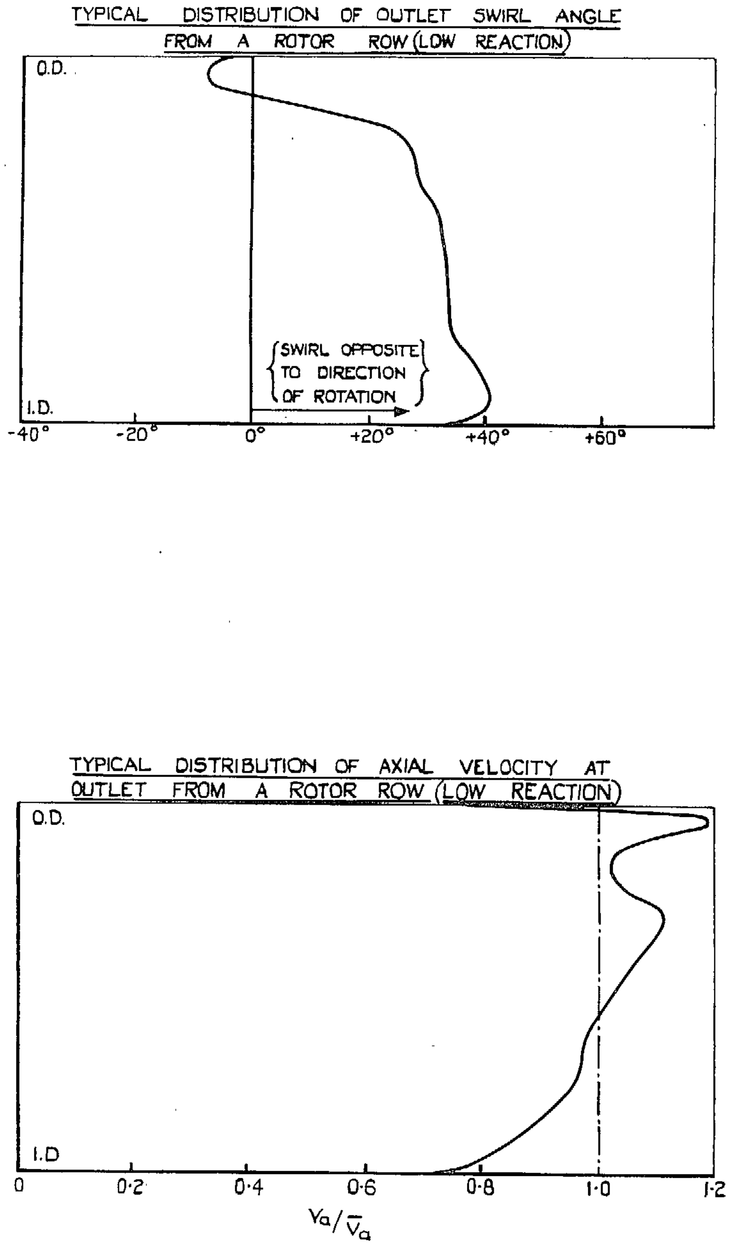


FIG. 19. Swirl angle and velocity at outlet from a turbine (low reaction).

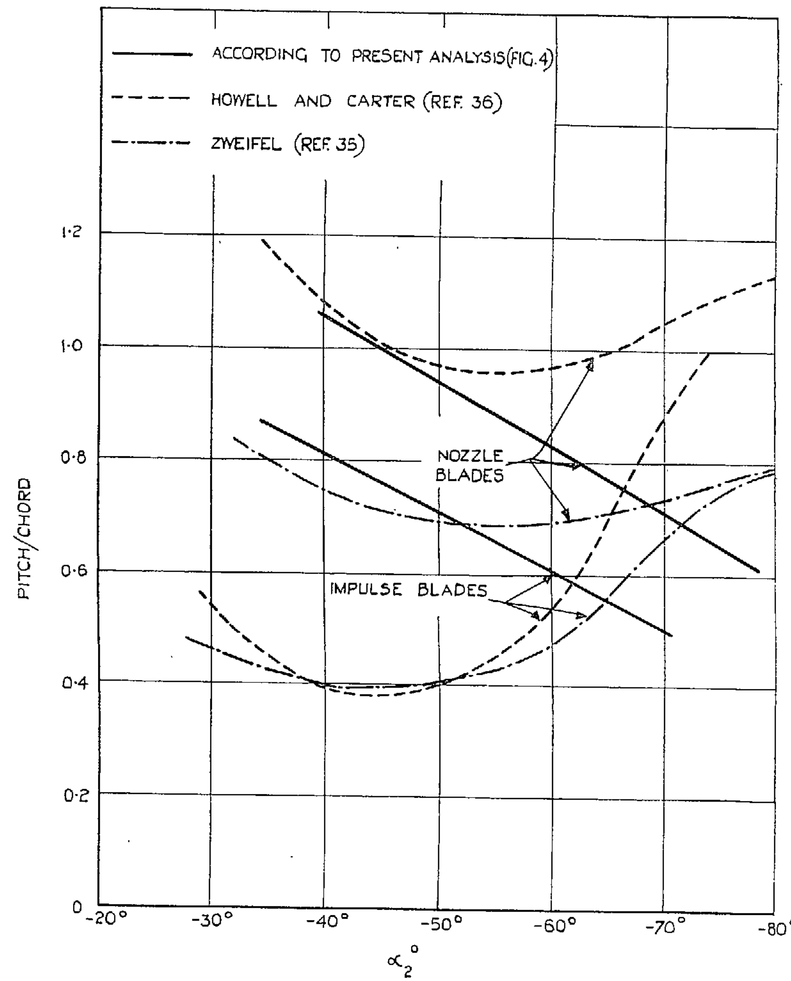


FIG. 20. Pitch/chord ratio giving minimum loss (turbine blades).

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