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Summary.—Expressions are developed on the basis of an unsteady flow analysis for the yawing derivatives for a delta wing with small dihedral at small incidence flying at supersonic speeds. The assumptions of the linearised theory of flow are made throughout ; only first-order terms in the rate of turn are considered.

The terms dependent on the dihedral alone are continuous and decrease numerically with rising Mach number. The remaining terms are discontinuous at a Mach number at which a leading edge becomes supersonic ; in particular the rolling-moment component due to incidence changes sign ; the other derivatives may do likewise in certain circumstances.

The approximate theory developed in the paper breaks down as a leading edge nears the Mach wave from the vertex of the wing. The yawing amplitude for which the results quoted present reasonable approximations decreases rapidly as this condition is approached ; in particular the contributions of the leading-edge suction become undefined.

Earlier results based on strip theory are greater numerically than those derived in the present paper by significant amounts that increase with Mach number and aspect ratio. The two theories agree for vanishingly small aspect ratios.

1. *Introduction*.—The purpose of this paper is to determine the forces acting on a delta wing at supersonic speeds when undergoing a pure yawing motion. The type of motion considered is of a wing travelling in a wide circle about an axis parallel to its plane of symmetry, and the derivatives found are those corresponding to a vanishingly small frequency parameter. In a subsonic stability analysis it is usual to assume that the aircraft is performing an oscillatory motion rather than a continuous turn, so that the wake may be assumed to stream out straight behind the aircraft ; however, since in supersonic flow the wing is outside the region of influence of the wake, either motion will lead to the same result in deriving the yawing derivatives.

To simplify the problem the two halves of the wing are taken to be flat and infinitely thin and the incidence and dihedral small enough to allow second-order terms in them to be neglected. In addition the rate of turn is assumed to be small enough to ensure that its square is negligible and that the leading edges are either both subsonic or both supersonic over their whole length. Within the framework of these linearising assumptions the solution found is thought to be exact. Assumptions of the type that arise in strip theory are not made.

The aerodynamic forces due to the yawing spring from three sources associated with the facts that, firstly, the aerofoil has velocities normal to its plane, that, secondly, it has velocities in its own plane, a complication which does not arise in rolling or pitching, and that, thirdly, in the quasi-subsonic condition there is an interaction modifying the leading-edge suction forces.

In the quasi-subsonic case a solution to the unsteady potential equation, correct to the required order of approximation, is synthesised by means of suitable transformations from solutions to the steady equation which are obtained by an extension of the method of cone fields introduced by Stewart (Ref. 1). In the definitely supersonic case the problem reduces to the integration of a distribution of sonic sources, the strengths of which are functions of time.

2. NOTATION

V		Free-stream velocity
M		Free-stream Mach number
a		Free-stream velocity of sound
ρ		Free-stream density
p		Excess pressure
β	=	$(M^2 - 1)^{1/2}$
λ	=	$\beta \tan \gamma$
k^2	=	$1 - k'^2 = 1 - \lambda^2, \lambda < 1$
α		Angle of incidence
ψ		Angle of yaw
δ		Angle of dihedral
γ		Semi-vertex angle
r		Rate of yawing
c		Maximum chord
b		Wing span
S		Wing area
K, E		Complete elliptic integrals of 1st and 2nd kind of modulus k
L, N		Rolling and yawing moments
Y		Side force
l_r	=	$\frac{4}{\rho V S b^2} \frac{\partial L}{\partial r}$, non-dimensional rolling-moment derivative
n_r	=	$\frac{4}{\rho V S b^2} \frac{\partial N}{\partial r}$, non-dimensional yawing-moment derivative
y_r	=	$\frac{1}{\rho V S c} \frac{\partial Y}{\partial r}$, non-dimensional side-force derivative
t		Time
x,y,z		Rectangular cartesian coordinates moving uniformly and rectilinearly at speed V relative to the undisturbed air in the positive x-direction ; z-axis downwards parallel to the plane of symmetry of the wing. Fig. 7a refers.
s^2	=	$x^2 - \beta^2 y^2 - \beta^2 z^2$
ω	=	$\beta \frac{y + iz}{s - x}$
∇h^2	=	$-\beta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
$\text{cn}\,\tau$	=	$2i\omega/(1 - \omega^2)$
u,v,w		Induced velocity components in the x,y,z-directions
U_x, V_x, W_x; U_y, V_y, W_y; U_z, V_z, W_z	=	Analytical functions of ω, and therefore of τ, associated with u,v,w

3. *Results.*—(*a*) *General.*—The non-dimensional derivatives quoted in subsections (*b*) and (*c*) below are referred to wind axes with the centre of gravity at the vertex. As rarely more than half the theoretical suction force has been realised in practice for the delta wing at incidence, those parts of the yawing moment and side-force derivatives due to suction are shown separately.

The results in both cases determine the derivatives for the centre of gravity aft of the vertex in conjunction with the corresponding sideslip derivatives (Ref. 2). The primed derivatives given below are referred to a centre at a fraction h of a chord aft of the vertex:—

$$l_r' = l_r + h \cot \gamma \, . \, l_v$$

$$n_r' = n_r + h \cot^2 \gamma \, . \, y_r + h \cot \gamma \, . \, n_v + h^2 \cot^2 \gamma \, . \, y_v$$

$$y_r' = y_r + h y_v \, .$$

In Fig. 1 the derivative l_r is shown graphically as a function of Mach number for various aspect ratios. In Figs. 2 and 3 the derivatives n_r and y_r are similarly plotted with leading-edge suction neglected; the contributions of the latter are shown in Figs. 4 and 5.

(*b*) *The Quasi-subsonic Condition* ($\lambda < 1$).—(i) Derivatives neglecting leading-edge suction:—

$$l_r = -\frac{\left\{\begin{array}{l} \frac{1}{8}\pi\alpha \cot \gamma\{(2 - 5k^2 - k^4)E - 2(1 - 2k^2)k'^2K\} \\ + \frac{1}{8}\pi\alpha \tan \gamma\{(4 - 7k^2 - 2k^4)E - (4 - 5k^2)k'^2K\} \\ + \frac{1}{8}\pi\alpha \tan^3 \gamma\{2k'^2E - (2 - 3k^2)K\} - \frac{1}{6}\delta k^2 E\{(1 + 7k^2)E - (1 + 3k^2)k'^2K\} \end{array}\right\}}{k^2E\{(1 + k^2)E - k'^2K\}}$$

$$n_r = -\alpha l_r - \tfrac{3}{4} \cot^2 \gamma \, . \, y_r$$

$$y_r = -\tfrac{2}{3}\delta \, . \, \frac{\left\{\begin{array}{l} \pi\alpha\{(2 - 5k^2 - k^4)E - 2(1 - 2k^2)k'^2K\} \\ + \pi\alpha \tan^2 \gamma\{(4 - 7k^2 - 2k^4)E - (4 - 5k^2)k'^2K\} \\ + \pi\alpha \tan^4 \gamma\{2k'^2E - (2 - 3k^2)K\} - 2\delta \tan \gamma \, . \, k^2E\{(1 + 5k^2)E \\ \qquad - (1 + 2k^2)k'^2K\} \end{array}\right\}}{\pi k^2 E\{(1 + k^2)E - k'^2K\}} \, .$$

(ii) Contribution of leading-edge suction to the derivatives:—

$$y_r = +\alpha \frac{\left\{\begin{array}{l} \pi\alpha k'^2\{(5 - k^2)E - 5k'^2K\} + \pi\alpha k'^2 \tan^2 \gamma\{(10 + k^2)E - (10 - 7k^2)K\} \\ + \pi\alpha \tan^4 \gamma\{(5 - 3k^2)E - (5 - 7k^2)K\} - 8\delta k^4 \tan \gamma \, . \, E(2E - k'^2K) \end{array}\right\}}{6k \tan \gamma \, . \, E^2\{(1 + k^2)E - k'^2K\}}$$

$$n_r = -\tfrac{3}{4} \operatorname{cosec}^2 \gamma \, . \, y_r \, .$$

(*c*) *The Definitely Supersonic Condition* ($\lambda > 1$).

$$l_r = \frac{\alpha \tan \gamma}{12\lambda^5}\{\lambda^2(4\lambda^2 - 7) - (4\lambda^2 + 3) \tan^2 \gamma\} + \frac{\delta}{2\lambda}$$

$$n_r = -\alpha l_r - \tfrac{3}{4} \cot^2 \gamma \, . \, y_r$$

$$y_r = -\frac{2\alpha\delta}{3\pi\lambda^4(\lambda^2-1)^2}\left\{\frac{\lambda^4\sec^{-1}\lambda}{(\lambda^2-1)^{\frac{1}{2}}}\{(2\lambda^2-5)\tan^4\gamma-(2\lambda^4-9\lambda^2+13)\tan^2\gamma+\lambda^2-4\}\right.$$

$$\left.+(2\lambda^4-\lambda^2+2)\tan^4\gamma-\lambda^2(2\lambda^4-3\lambda^2-5)\tan^2\gamma-\lambda^4(2\lambda^2-5)\right\}$$

$$+\frac{4\delta^2\tan\gamma}{3\pi(\lambda^2-1)}\left\{1+\frac{(2\lambda^2-3)\sec^{-1}\lambda}{(\lambda^2-1)^{\frac{1}{2}}}\right\}.$$

(*d*) *Comparison with Strip Theory.*—In default of an unsteady flow analysis attempts have been made to evaluate the yawing derivatives on the basis of strip theory. The latter is based on the premise that the induced flow over the surface of a wing at incidence but without dihedral, yawing in its own plane, is unchanged when expressed in terms of a coordinate system fixed in the wing, on the grounds that the kinematic boundary condition at the aerofoil is unchanged by the yawing. One might be tempted to accept this premise by the fact that it is true for a quasi-subsonic delta wing in sideslip, although it is no longer true in the definitely supersonic case. However for the yawing wing it is false in both cases.

Such solutions are tentatively put forward in Refs. 7, 8 and 9 for the quasi-subsonic case; the authors are, of course, aware of the shortcomings of the approach which they clearly point out in their reports. In comparison with the results of subsection 3(*b*) these solutions for l_r and y_r referred to wind axes centred at the apex for zero dihedral are:—

$$l_r = \tfrac{1}{4}\pi\alpha\cot\gamma\left\{\frac{\sec^2\gamma}{E}+\frac{k^2\tan^2\gamma}{(1+k^2)E-k'^2K}\right\}$$

and

$$y_r = \tfrac{1}{3}\pi\alpha^2\tan\gamma\left\{\frac{M^2\sec^2\gamma}{kE^2}-\frac{2k^3}{E\{(1+k^2)E-k'^2K\}}\right\}.$$

In the above l_r is the result of a normal pressure distribution over the wing and y_r of leading-edge suction.

These derivatives are in general in excess of those evaluated in this report as shown by the curves of percentage excess for varying aspect ratio and Mach number in Fig. 6. There is agreement, however, for very small aspect ratios.

In the reports quoted the derivatives for the definitely supersonic case are not evaulated by any method, but in Ref. 7 the belief is expressed that there is a reversal of the sign of l_r on passing from one case to the other; this is borne out by the present report (Fig. 1 refers).

4. *The Quasi-Subsonic Condition.*—(*a*) *Boundary Conditions and Governing Equation.*—The equation to the wing may be written in the form:—

$$z = f(x,y,t)$$

so that the kinematic boundary condition holding at the wing is:—

$$w = \frac{\partial f}{\partial t} + (u - V)\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y}. \qquad (1)$$

Neglecting terms of order α^2, δ^2 and ψ^2, the equation to the starboard wing after a rotation ψ about the z-axis and a y-wise displacement y_0 of the vertex is:—

$$z = (\delta\psi - \alpha)x - (\delta + \alpha\psi)(y - y_0). \qquad (2)$$

The wing is moving on a circular path relative to still air with speed V and rate of turn r; the co-ordinate system is moving on a straight line with the same speed in the x-direction; at time $t = 0$ the axis of the wing lies in the zx-plane with its vertex at the origin. At time t, therefore, $\psi = rt$ and $y_0 = \frac{1}{2}Vrt^2$. The induced velocities clearly vanish when the incidence and dihedral are both zero, so that u and v are of order α, δ.

It now follows from equations (1) and (2) that the boundary condition at the starboard wing, terms in α^2, $\alpha\delta$, δ^2 and r^2 being neglected, is:—

$$w = V\alpha + r(\delta x - \alpha y) \,, \qquad \qquad (3)$$

with the sign of δ changed for the port wing. To the same order of approximation this condition may be regarded as being met at the projection of the aerofoil on the xy-plane rather than at the aerofoil itself.

Denoting the induced velocity potential by Φ, the velocity vector of any particle of fluid is $\boldsymbol{q} = \text{grad}\ (\Phi - Vx)$ relative to the co-ordinate system. Since the latter is moving uniformly and rectilinearly in relation to the undisturbed air, the usual governing equation:—

$$\frac{\partial^2}{\partial t^2}\Phi + \frac{\partial}{\partial t}\boldsymbol{q}^2 + \boldsymbol{q}\,.\,\text{grad}\ \tfrac{1}{2}\boldsymbol{q}^2 - a^2\,\text{div}\ \boldsymbol{q} = 0$$

must be satisfied. Since Φ is of the order α, δ the latter equation reduces, on neglecting terms in α^2, $\alpha\delta$ and δ^2, to:—

$$\left\{\nabla h^2 + \frac{2M}{a}\frac{\partial^2}{\partial x \partial t} - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right\}\Phi = 0\,. \qquad \qquad (4)$$

For a thin wing lying within the apex Mach conoid the shock wave from its vertex is assumed to be infinitely weak, so that the boundary condition at the Mach conoid is that the induced velocities shall vanish.

(*b*) *The Potential Function at Zero Incidence**.—It follows from the previous subsection that at zero incidence the boundary condition holding over the starboard wing is $w = +rx\delta$ with $w = -rx\delta$ over the port wing. The potential clearly vanishes with r and is therefore of order r. Taking the potential to be $r\delta\phi$ we can, on neglecting second-order terms in r, consider ϕ as being independent of r. The problem therefore reduces to finding the potential corresponding to the wing in the hypothetical steady unyawed state with the true kinematic boundary condition holding over the aerofoil in the neutral position. The corresponding governing equation is $\nabla h^2\phi = 0$.

Since in considering points ahead of the trailing edge the wing may be considered as extending indefinitely, a point (Cx, Cy, Cz) of the system is also a point of a second similar system scaled up by a factor C corresponding to (x,y,z) of the first system. Therefore the potential must be a homogeneous function in x,y,z and from consideration of the boundary conditions it is clearly of degree two.

Any of the first derivatives of the induced velocity components with respect to x,y or z are homogeneous functions of degree zero annihilated by the operator ∇h^2. It therefore follows from the analogue of the general homogeneous solution of degree zero of Laplace's equation in three dimensions published by Donkin in 1857 (Ref. 6) that any of the velocity derivatives

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* This subsection is based on College of Aeronautics Report No. 28 (Ref. 5).

can be expressed as the real part of an analytic function of the complex variable ω defined by:—

$$\omega = \beta\frac{y + iz}{s - x},$$

$$\text{where} \quad s^2 = x^2 - \beta^2 y^2 - \beta^2 z^2 .$$

The conical configuration of the (x,y,z) space is represented by the plane figure in the ω-plane shewn at Fig. 7b.

Take $\partial u/\partial x$, $\partial v/\partial x$, $\partial w/\partial x$ to be the real parts of functions U_x, V_x, W_x of ω respectively.

Since the motion is irrotational, we can write:—

$$\frac{\partial^2 v}{\partial x \partial z} - \frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial z} = \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x^2} = 0 ,$$

hence

$$\left.\begin{aligned} \mathscr{R}\left(\frac{dV_x}{d\omega}\cdot\frac{\partial\omega}{\partial z}\right) &= \mathscr{R}\left(\frac{dW_x}{d\omega}\cdot\frac{\partial\omega}{\partial y}\right) \\ \mathscr{R}\left(\frac{dW_x}{d\omega}\cdot\frac{\partial\omega}{\partial x}\right) &= \mathscr{R}\left(\frac{dU_x}{d\omega}\cdot\frac{\partial\omega}{\partial z}\right) \\ \text{and} \quad \mathscr{R}\left(\frac{dU_x}{d\omega}\cdot\frac{\partial\omega}{\partial y}\right) &= \mathscr{R}\left(\frac{dV_x}{d\omega}\cdot\frac{\partial\omega}{\partial x}\right) \end{aligned}\right\} \quad \ldots \ldots \quad (5)$$

Now $s(\partial\omega/\partial x)$, $s(\partial\omega/\partial y)$, $s(\partial\omega/\partial z)$ are analytic functions of ω, and if a pair of analytic functions have identical real parts they can only differ by a pure imaginary constant, so that equations (5) become:—

$$\left.\begin{aligned} \frac{dV_x}{d\omega}\cdot\frac{\partial\omega}{\partial z} - \frac{dW_x}{d\omega}\cdot\frac{\partial\omega}{\partial y} &= \frac{iX}{s} \\ \frac{dW_x}{d\omega}\cdot\frac{\partial\omega}{\partial x} - \frac{dU_x}{d\omega}\cdot\frac{\partial\omega}{\partial z} &= \frac{iY}{s} \\ \frac{dU_x}{d\omega}\cdot\frac{\partial\omega}{\partial y} - \frac{dV_x}{d\omega}\cdot\frac{\partial\omega}{\partial x} &= \frac{iZ}{s} \end{aligned}\right\} \quad \ldots \ldots \quad (6)$$

where X, Y, Z are real constants. It will be seen from equations (6) that:—

$$X\frac{\partial\omega}{\partial x} + Y\frac{\partial\omega}{\partial y} + Z\frac{\partial\omega}{\partial z} = 0$$

which implies a linear dependence between the derivatives of ω that does not exist. Therefore X, Y and Z are all zero, from which it immediately follows that:—

$$\frac{dU_x}{d\omega}\bigg/\frac{\partial\omega}{\partial x} = \frac{dV_x}{d\omega}\bigg/\frac{\partial\omega}{\partial y} = \frac{dW_x}{d\omega}\bigg/\frac{\partial\omega}{\partial z}$$

or

$$\frac{i\beta\, dU_x}{2\omega} = \frac{i\, dV_x}{1 + \omega^2} = \frac{dW_x}{1 - \omega^2} . \quad \ldots \ldots \quad (7)$$

This last result was first obtained by different means by Stewart (Ref. 1) in a slightly different form. Ref. 2 may be compared.

The two sets of functions U_y, V_y, W_y and U_z, V_z, W_z corresponding to the derivatives with respect to y and z are connected by similar relations. It is clear that the sets of functions of the form U_x, U_y, U_z are also similarly related, so that the nine functions are completely related.

While the boundary conditions at the aerofoil in terms of the complex functions are clear, it is necessary to examine those at the Mach cone more closely. The latter demand that the induced velocities should vanish, for which a sufficient condition is that they should vanish at one point and their first derivatives everywhere on the cone. It will be shewn that this may be taken to be a necessary condition.

With the aid of all the relations of the form (7) it can be shewn that if $\partial u/\partial x$ is defined in a certain region on the Mach cone then the remaining derivatives are also defined in that region. If u is zero in such a region the following equations must be satisfied:—

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

$$z \frac{\partial u}{\partial y} - y \frac{\partial u}{\partial z} = 0$$

so that

$$zx \frac{\partial u}{\partial x} + (y^2 + z^2) \frac{\partial u}{\partial z} = 0 . \qquad \ldots \qquad (8)$$

Now on the Mach cone we may write $\omega = e^{i\theta}$, where θ is real, so that equation (8) reduces to:—

$$\beta \sin \theta \frac{\partial u}{\partial x} - \frac{\partial w}{\partial x} = 0 \qquad \ldots \qquad (9)$$

while equations (7) become:—

$$\beta \sin \theta \, dU_x = \cot \theta \, dV_x = dW_x = F(\theta) \, d\theta, \text{ say.} \qquad \ldots \qquad (10)$$

Hence

$$\beta \sin \theta \frac{d}{d\theta} \cdot \frac{\partial u}{\partial x} = \frac{d}{d\theta} \cdot \frac{\partial w}{\partial x} = \mathscr{R} F(\theta) .$$

It will now be seen on differentiating equation (9) with respect to θ that $\partial u/\partial x$ and, therefore, $\partial v/\partial x$ and $\partial w/\partial x$ vanish. The other velocity derivatives may also be shewn to vanish.

Since the velocity derivatives vanish in any region on the Mach cone where they are defined, it may be assumed from the physical nature of the present problem that they are either zero or infinite over the whole cone. Now the induced velocity potential is an odd function of z, so that $\partial u/\partial x$ will be zero on the Mach cone at $z = 0$; therefore all the velocity derivatives will vanish on the Mach cone.

A further transformation will now be introduced defined by:—

$$\text{cn}\, \tau = 2i\omega/(1 - \omega^2)$$

where cn τ is the Jacobian elliptic function of modulus k in Glaisher's notation. The interior of the Mach cone is represented in the τ-plane by the interior of the rectangle with vertices $\pm 2iK'$, $K \pm 2iK'$; the imaginary axis between $\pm 2iK'$ corresponds to the Mach cone and the parallel line between $K \pm 2iK'$ to the aerofoil. (See Fig. 7c.) A point in the ω-plane inside the unit circle (Mach cone) corresponds to a given point inside this rectangle in the τ-plane; it also corresponds to the image of that point in the point $K + iK'$ and to all points congruent (mod. $4K$, $4iK'$) to that point and its image. The exterior of the unit circle in the ω-plane corresponds to the image of this configuration in the imaginary axis in the τ-plane.

The relations between the complex functions now become:

$$\left.\begin{aligned} -\beta^2 \operatorname{nc} \tau \, dU_x &= i\beta \operatorname{ns} \tau \, dU_y &&= \beta \, dU_z \\ = i\beta \operatorname{ns} \tau \, dV_x &= \operatorname{cs} \tau \operatorname{ns} \tau \, dV_y &&= -i \operatorname{cs} \tau \, dV_z \\ = \beta \, dW_x &= -i \operatorname{cs} \tau \, dW_y &&= -\operatorname{cn} \tau \, dW_z \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (11)$$

It now remains to choose one function, say $dW_x/d\tau$, so that the necessary conditions are all met. Certain properties of the velocity derivatives are known, which by the aid of equations (11), can be interpreted in terms of restrictions on the choice of $dW_x/d\tau$ as follows:—

Properties of the velocity derivatives	Requirements to be met by the function $dW_x/d\tau$
(i) By symmetry $\partial w/\partial x = 0$ on the zx-plane	Pure imaginary on the lines OC, AB, A′B′
(ii) Since w is an even function of z, $\partial w/\partial z = 0$ on $z = 0$ at points not on the aerofoil	Real on PQ, P′Q′
(iii) By the boundary conditions $\partial w/\partial x$, $\partial w/\partial y$ are constant over the two halves of the aerofoil	Real on BB′ with no simple poles except possibly at B, C, B′
(iv) By the boundary conditions $\partial w/\partial x = +r\delta$, $y > 0$, and $-r\delta$, $y < 0$, on the aerofoil	A simple pole of residue of $-2ir\delta/\pi$ at C and poles of opposite residue at B, B′
(v) At the aerofoil $\partial w/\partial y = 0$	$\int_0^K \frac{dW_y}{d\tau} d\tau$ to be imaginary
(vi) All the derivatives must vanish on the Mach cone	Real with no poles on the imaginary axis, AA′, and at least simple zeros at P, P′
(vii) The velocity components are single valued	The function must be repeated over the congruent rectangles to which the interior of the Mach cone also corresponds. In addition it must have no branch points or singularities within the rectangle ABB′A′
(viii) On physical grounds the velocity components can have no singularities except at the axis and leading edges of the aerofoil	The function can have no poles except at B, Q, C, Q′, B′. Therefore together with the foregoing, noting that (vi) implies that the function takes conjugate values at image points in the imaginary axis, it must be doubly periodic and meromorphic and, hence, elliptic; and furthermore a rational algebraic function of $\operatorname{sn} \tau$, $\operatorname{cn} \tau$, $\operatorname{dn} \tau$
(ix) The aerodynamic forces are finite	The singularities on BB′ are not to be of too high an order

The necessary function is found to be:—

$$\frac{dW_x}{d\tau} = i \operatorname{sc} \tau \operatorname{nd}^4 \tau \, (A \operatorname{dn}^2 \tau + B \operatorname{cn}^2 \tau) \quad \ldots \quad \ldots \quad \ldots \quad \ldots (12)$$

where A and B are real constants such that $\partial w/\partial x$ and $\partial w/\partial y$ have the correct values on the two halves of the aerofoil. Any other function of this form would lead to an inadmissible singularity of one or more of the complex derivatives at one of the points P, P′, Q, Q′.

This function has a residue of $-iA/k'^3$ at C and therefore:—

$$A = 2\gamma\delta k'^3/\pi \, .$$

From equations (11) and (12) we have:—

$$\frac{\partial w}{\partial y} = -\beta \, \mathscr{R} \int_0^\tau \operatorname{sc}^2 \tau \operatorname{nd}^4 \tau \, (A \operatorname{dn}^2 \tau + B \operatorname{cn}^2 \tau) \, d\tau$$

$$= \frac{-\beta}{3k^2k'^4} \mathscr{R} \Big\{ 3k^2 A [k'^2\tau - 2E(\tau) + k'^2 \operatorname{sc} \tau \operatorname{nd} \tau + 2k^2 \operatorname{cn} \tau \operatorname{sd} \tau]$$

$$- B[k'^2\tau - (1 + k^2)E(\tau) + k^2(1 + k^2) \operatorname{cn} \tau \operatorname{sd} \tau + k^2k'^2 \operatorname{cn} \tau \operatorname{sn} \tau \operatorname{nd}^3 \tau] \Big\}$$

$$= \frac{-\beta}{3k^2k'^4} \{ 3k^2 A (k'^2 K - 2E) - B(k'^2 K - (1 + k^2)E) \} \, , \quad \text{for } \tau = K \, .$$

Therefore in order that $\partial w/\partial y$ may vanish on the aerofoil:—

$$B = \frac{6}{\pi} \gamma \delta k^2 k'^3 \frac{2E - k'^2 K}{(1 + k^2)E - k'^2 K} \, .$$

Also from equations (11) and (12):—

$$\frac{\partial u}{\partial x} = -\frac{1}{\beta} \mathscr{R} \int_0^\tau i \operatorname{sn} \tau \operatorname{nd}^4 \tau \, (A \operatorname{dn}^2 \tau + B \operatorname{cn}^2 \tau) \, d\tau$$

$$= \frac{1}{k'^2 \beta} \mathscr{R} \{ iA \operatorname{cd} \tau + \tfrac{1}{3} iB \operatorname{cd}^3 \tau \} \, .$$

Now on $z = 0$ $\operatorname{cn} \tau = i\beta y(x^2 - \beta^2 y^2)^{-1/2}$ so that $\operatorname{cd} \tau = iy(x^2 \tan^2 \gamma - y^2)^{-1/2}$ on the upper surface of the aerofoil ($z = -0$) with opposite sign on the lower surface.

Hence on the upper surface of the aerofoil:—

$$\frac{\partial u}{\partial x} = -\frac{1}{k'^2 \beta} \left\{ \frac{Ay}{(x^2 \tan^2 \gamma - y^2)^{1/2}} - \frac{By^3}{3(x^2 \tan^2 \gamma - y^2)^{3/2}} \right\}$$

and

$$u = \frac{2}{\pi} \gamma \delta y \left\{ \operatorname{ch}^{-1} \frac{-x \tan \gamma}{|y|} - \frac{2E - k'^2 K}{(1 + k^2)E - k'^2 K} \cdot \frac{k^2 x \tan \gamma}{(x^2 \tan^2 \gamma - y^2)^{1/2}} \right\} . \quad \ldots \quad \ldots (13)$$

with opposite signs on the lower surface. There is no uncertainty with regard to the introduction of a function of y on integrating with respect to x, for on integrating again to obtain the potential the only possible homogeneous addition is of the form $(ax + by)y$, and this cannot vanish at both leading edges as is necessary, the potential being continuous and odd in z.

4(b). *The Potential Function for Zero Dihedral.*—It will be seen from equation (3) that the boundary condition at the aerofoil is $w = \alpha(V - ry)$. Though the solution corresponding to $w \propto y$ has already been found by other means by Robinson (Ref. 4) in his treatment of the delta wing in roll, it is more convenient in this instance to treat the problem as a whole.

The wing is describing a circle in the yawing plane with velocity V at an angular rate r. In the co-ordinate system chosen the axes are not rotating but are moving with velocity V such that at time $t = 0$ the vertex is at the origin and the axis of the wing in the zx-plane; hence at the time t the wing has rotated through an angle $\psi = rt$ while the vertex has undergone a y-wise displacement of approximately $\frac{1}{2}Vrt^2$. For a wing on a rectilinear path the x-wise component of the velocity of propagation along the Mach cone of a disturbance initiated at the apex is $-a\beta^2/M$; therefore to sufficient accuracy the equation to the apex Mach wave for a wing on a circular path is:—

$$x^2 - \beta^2 \left\{y - \tfrac{1}{2}Vr\left(t + \frac{Mx}{a\beta^2}\right)^2\right\}^2 - \beta^2 z^2 = 0 \,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots (14)$$

It may be shown, provided $crM^3/ak^2\beta^3$ is small enough, to allow terms in r^2 to be neglected, that under the transformation:—

$$\left.\begin{aligned} x' &= x + \psi y + \frac{M^3 rxy}{2a\beta^2 k^2} \\ y' &= y - \psi x + \frac{M^3 rx^2 \tan^2 \gamma}{2a\beta^2 k^2} - \tfrac{1}{2}Vrt^2 \\ z' &= z \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (15)$$

the displaced delta wing transforms into a wing of the same semi-vertex angle, unyawed and with its vertex at the origin, while the conoid given by equation (14) becomes the apex Mach cone, $x'^2 - \beta^2 y'^2 - \beta^2 z'^2 = 0$, associated with the steady undisplaced wing.

The limiting case in which a shock begins to form at a leading edge occurs when a characteristic through an outer wing tip in the xy-plane is tangential to that leading edge. This condition arises when $r = ak^2\beta^3/2cM^3$ approximately; under these circumstances the wing is wholly within the apex Mach wave.

From consideration of equations (15) a reasonable criterion for the validity of the approximate transformation appears to be that r should be less than a fifth of the above value. It is believed that if r is greater than this the aerodynamic forces due to yawing can no longer be regarded as linear in r. In practice the restriction on r is not severe; as an example, when the Mach angle exceeds the semi-vertex angle by 3 deg the lateral acceleration in the limiting case is about $25g$ for $M = 1 \cdot 3$ and $300g$ for $M = 3$ for a wing of 10-ft chord.

The governing equation (4) is transformed by (15) into:—

$$\nabla'^2_h \phi + \frac{2k^2 - M^2}{ak^2\beta^2} Mr \frac{\partial}{\partial x'} \left\{\beta^2 y \frac{\partial \phi}{\partial x'} + x \frac{\partial \phi}{\partial y'}\right\} - \frac{Mr}{a\beta^2} \cdot \frac{\partial \phi}{\partial y'} = 0 \quad \ldots \quad \ldots (16)$$

where terms in r^2 are neglected and where ϕ is not explicitly a function of time.

Consider a steady, unyawed wing in the primed space with its vertex at the origin and its corresponding Mach cone. If we find a function of the primed co-ordinates satisfying equation (16) which also satisfies the boundary condition $w = \alpha(V - ry')$ at this hypothetical aerofoil and the necessary conditions at its Mach cone, that function, regarded as a function of x, y, z and t by virtue of equations (15), will be the required velocity potential.

Let ϕ_0 be the velocity potential for a steady, unyawed wing at incidence and define a function ϕ_0^* as follows:—

$$\phi_0^*(x, y, z, t) = \phi_0(x', y', z') \ .$$

Define a further function ϕ_1^* such that the required potential is:—

$$\phi = \phi_0^* + \phi_1^* \ .$$

It is clear that, when $r = 0$, ϕ_1^* vanishes and that therefore is of order r. Since $\nabla' h^2 \phi_0^* = 0$, we obtain on neglecting terms in r^2 by substituting for ϕ in equation (16) the following equation for ϕ_1^*:—

$$\nabla' h^2 \phi_1^* + \frac{2k^2 - M^2}{ak^2\beta^2} Mr \frac{\partial}{\partial x'} \left\{ \beta^2 y' \frac{\partial \phi_0^*}{\partial x'} + x' \frac{\partial \phi_0^*}{\partial y'} \right\} - \frac{Mr}{a\beta^2} \cdot \frac{\partial \phi_0^*}{\partial y'} = 0$$

a solution to which is $\dfrac{2k^2 - M^2}{2ak^2\beta^4} Mrx' \left\{ \beta^2 y' \dfrac{\partial \phi_0^*}{\partial x'} + x' \dfrac{\partial \phi_0^*}{\partial y'} \right\} + \dfrac{Mr}{2a\beta^2} y' \phi_0^*$. Hence, to the same order of accuracy, we may write, when $t = 0$:—

$$\phi = \phi_0^* + \frac{2k^2 - M^2}{2ak^2\beta^4} Mrx \left\{ \beta^2 y \frac{\partial \phi_0}{\partial x} + x \frac{\partial \phi_0}{\partial y} \right\} + \frac{Mr}{2a\beta^2} y \phi_0 + \phi_2 \qquad \ldots \qquad \ldots (17)$$

where ϕ_2 is a function such that $\nabla h^2 \phi_2 = 0$ and such that ϕ satisfies boundary conditions and has no singularities.

To determine ϕ_2 it is necessary to investigate the behaviour of $\{\beta^2 y(\partial \phi_0/\partial x) + x(\partial \phi_0/\partial y)\}$ and its derivative with respect to z at the steady, unyawed aerofoil and its Mach cone. The function $\partial \phi_0/\partial z$ is homogeneous of degree zero and is the real part of a complex function W for which $dW/d\tau = V\alpha k'^2 \, \text{nd}^2 \, \tau/E$ (Ref. 1) ; therefore:—

$$\frac{\partial}{\partial z} \left\{ \beta^2 y \frac{\partial \phi_0}{\partial x} + x \frac{\partial \phi_0}{\partial y} \right\} = \mathscr{R} \left\{ \beta^2 y \frac{\partial W}{\partial x} + x \frac{\partial W}{\partial y} \right\}$$

$$= - \frac{V\alpha k'^2}{E} \mathscr{R} \left\{ \text{cn}\, \tau \, \text{nd}^3 \, \tau \left[\frac{\beta^2 y}{\omega} \cdot \frac{\partial \omega}{\partial x} + \frac{x}{\omega} \cdot \frac{\partial \omega}{\partial y} \right] \right\}$$

$$\text{since } \frac{d\tau}{d\omega} = - \frac{\text{cd}\, \tau}{\omega}$$

$$= \frac{V\alpha k'^2}{E} \mathscr{R} \left\{ \text{cn}\, \tau \, \text{nd}^3 \, \tau \left[\frac{s}{y + iz} - \frac{i\beta^2 z}{s - x} \right] \right\} .$$

Now at the aerofoil $z = 0$ and $\text{cn}\, \tau \, \text{nd}^3 \, \tau$ is pure imaginary and at the Mach cone $s = 0$ and $\text{cn}\, \tau \, \text{nd}^3 \, \tau$ is real ; hence the above derivative with respect to z vanishes at the aerofoil and Mach cone. Since $\partial \phi_0/\partial x$ and $\partial \phi_0/\partial y$ are zero over the Mach cone, $\{\beta^2 y(\partial \phi_0/\partial x) + (\partial \phi_0/\partial y)\}$ vanishes and therefore its other derivatives at the Mach cone.

We have from Refs. 1 and 3 that at the upper surface of the aerofoil:—

$$\phi_0 = V\alpha(x^2 \tan^2 \gamma - y^2)^{1/2}/E \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots (18)$$

so that $\{\beta^2 y(\partial \phi_0/\partial x) + x(\partial \phi_0/\partial y)\}$ has an infinity of order a half at the leading edges.

As the derivatives of ϕ_0 vanish at the Mach cone and $\partial\phi_0/\partial z = V\alpha$ at the wing, it now follows from (17) that the derivatives of ϕ_2 must vanish at the Mach cone and that at the aerofoil $\partial\phi_2/\partial z = -\alpha r y(3M^2 - 2)/2\beta^2$. Furthermore ϕ_2 must have a singularity to cancel that introduced by $x\{\beta^2 y(\partial\phi_0/\partial x) + x(\partial\phi_0/\partial y)\}$. Since the latter function is homogeneous of degree two and since the boundary conditions for ϕ_2 are associated with such a function, it is clear that ϕ_2 is also homogeneous of degree two.

Therefore ϕ_2 can be derived from a complex function $d\bar{W}_x/d\tau$ on the lines of the previous subsection. A set of requirements for $d\bar{W}_x/d\tau$ can be written down that is identical with that for $dW_x/d\tau$ except for the change of conditions at the aerofoil. The changes are that $d\bar{W}_x/d\tau$ does not have a pole at the point C, while $\partial w/\partial y$ takes a different value and the singularities at Q, Q′ are of a higher order. By comparison with equation (12) it will be seen that:—

$$d\bar{W}_x/d\tau = i \operatorname{sn} \tau \operatorname{cn} \tau \, (G \operatorname{nd}^4 \tau + H \operatorname{nd}^6 \tau) \qquad \ldots \qquad (19)$$

where G and H are real constants.

From equations (11) and (19) we obtain:—

$$\frac{\partial^2\phi_2}{\partial y \partial z} = -\beta \, \mathscr{R} \int_0^\tau \operatorname{sn}^2 \tau \, (G \operatorname{nd}^4 \tau + H \operatorname{nd}^6 \tau) \, d\tau$$

$$= -\frac{\beta}{15k^2k'^6} \mathscr{R} \Big[\{5k'^2(1 + k^2)G + (3 + 7k^2 - 2k^4)H\}\{E(\tau) - k^2 \operatorname{sn} \tau \operatorname{cd} \tau\}$$

$$- k'^2\{5k'^2G + (3 + k^2)H\}\{\tau + k^2 \operatorname{sn} \tau \operatorname{cn} \tau \operatorname{nd}^3 \tau\} - 3Hk^2k'^4 \operatorname{sn} \tau \operatorname{cn} \tau \operatorname{nd}^5 \tau \Big]$$

$$= -\frac{\beta}{15k^2k'^6} \Big[5k'^2\{(1 + k^2)E - k'^2K\}G + \{(3 + 7k^2 - 2k^4)E - (3 + k^2)k'^2K\}H \Big]$$

when $\tau = K$.

Therefore, since $\partial\phi_2/\partial z = -\alpha r y(3M^2 - 2)/2\beta^2$ at the aerofoil,

$$\{5k'^2G + (5 - k^2)H\}\{(1 + k^2)E - k'^2K\} - Hk'^2\{(2 - k^2)E - 2k'^2K\} = 15k^2k'^6\alpha r(3M^2 - 2)/2\beta^3. \qquad \ldots \qquad (20)$$

Also from equations (11) and (19) we obtain:—

$$\frac{\partial^2\phi_2}{\partial x^2} = -\frac{1}{\beta} \mathscr{R} \int_0^\tau i \operatorname{cn}^2 \tau \operatorname{sn} \tau \, (G \operatorname{nd}^4 \tau + H \operatorname{nd}^6 \tau) \, d\tau$$

$$= \frac{1}{15k'^4\beta} \mathscr{R} \{5(k'^2G + H)i \operatorname{cd}^3 \tau - 3k^2Hi \operatorname{cd}^5 \tau\}$$

$$= \frac{1}{15k'^4\beta} \left\{ \frac{5(k'^2G + H)y^3}{(x^2 \tan^2 \gamma - y^2)^{3/2}} + \frac{3k^2Hy^5}{(x^2 \tan^2 \gamma - y^2)^{5/2}} \right\}$$

on the upper surface of the aerofoil. Hence:—

$$\phi_2 = -\frac{\beta y}{15k'^6} \Big[\{5k'^2G + (5 - k^2)H\}(x^2 \tan^2 \gamma - y^2)^{1/2}$$

$$- k^2Hx^2 \tan^2 \gamma (x^2 \tan^2 \gamma - y^2)^{-1/2} \Big]. \qquad \ldots \qquad (21)$$

In order that the potential may have no singularities, it follows from equations (17), (18) and (21) that:—

$$H = 15k'^4 M^2 \alpha r (2k^2 - M^2)/2k^2\beta^3 E$$

and hence we obtain from equations (17), (20) and (21) that:—

$$\phi = \frac{\alpha}{E}(x^2 \tan^2 \gamma - y^2)^{1/2} \left\{ V - \frac{ry}{2k^2\beta^2} \cdot \frac{k^2(M^4 - 2k^2)E + M^2(3k^2 - 2M^2)(E - k'^2K)}{(1 + k^2)E - k'^2K} \right\}. \quad (22)$$

Neglecting terms in r^2 it follows from the transformation (15) that for $t = 0$:—

$$\frac{\partial \phi}{\partial t} = ry \frac{\partial \phi_0}{\partial x} - rx \frac{\partial \phi_0}{\partial y} = \frac{V \alpha r x y \sec^2 \gamma}{E(x^2 \tan^2 \gamma - y^2)^{1/2}} . \quad \ldots \quad (23)$$

5. *The Definitely Supersonic Condition.*—Suppose that the normal velocity component is know at every point of the xy-plane in the form $w = w(x,y,t)$. If the axes are changed to a set at rest in the fluid, coincident with the moving axes at the time $t = 0$, the new co-ordinates being $\bar{x},y,z$, then $w = w(\bar{x} - Vt, y, t)$. If there are no boundaries other than the xy-plane, the flow in the half space $z \leqslant 0$ may be regarded as being due to a distribution of stationary sonic sources of density $w/2\pi$ over the $\bar{x}y$-plane. The potential at a point $(\bar{x},y,z)$ at rest in the fluid due to a source element at $(\bar{x}_0, y_0, 0)$ is:—

$$\frac{1}{2\pi\bar{R}} w(\bar{x}_0 - Vt + M\bar{R}, y_0, t - \bar{R}/a)\, d\bar{x}_0\, dy_0$$

where $\bar{R}$ is the distance between the two points. Hence the potential at a point (x,y,z) referred to the moving axes is:—

$$\Phi = \iint w(x_0 + MR, y_0, t - R/a) \frac{dx_0\, dy_0}{2\pi R}$$

where $R^2 = (x - x_0)^2 + (y - y_0)^2 + z^2$ and where the integration is over the whole xy-plane of the moving system.

On changing the variable x_0 to $(x_0 + MR)$ this integral transforms, when $M > 1$, into:—

$$\Phi = \iint \left\{ w\left[x_0, y_0, t + \frac{M}{a\beta^2}(x - x_0) + \frac{s_0}{a\beta^2}\right] + w\left[x_0, y_0, t + \frac{M}{a\beta^2}(x - x_0) - \frac{s_0}{a\beta^2}\right] \right\} \frac{dx_0\, dy_0}{2\pi s_0} \quad \ldots (24)$$

where $s_0^2 = (x - x_0)^2 - \beta^2(y - y_0)^2 - \beta^2 z^2$ and where the integration is over that part of the xy-plane for which $s_0^2 \geqslant 0$ and $x_0 \geqslant x$.

In the definitely supersonic case the stream is undisturbed ahead of the leading edges, where, therefore, $w = 0$, while over the aerofoil w is determined by the kinematic boundary conditions ; consequently the potential may be determined at any point ahead of the trailing edge by evaluating integral (24).

It is believed that the question as to whether the boundary condition at the leading-edge shock wave is satisfied is automatically answered by the fact that, since the potential is derived from a distribution of sonic sources over the entire xy-plane at rest in the fluid, the shock wave does not represent a boundary in the problem treated. It may, however, be easily shown that the results obtained in this section do in fact satisfy the condition of the potential vanishing over the shock wave, terms in r^2 being neglected.

The boundary conditions at the aerofoil are the same as in the quasi-subsonic case, that is $w = V\alpha + r(\delta x - \alpha y)$ over the starboard wing, with the sign of δ changed for the port wing : the function w is dependent on time for the reason that the area over which the boundary condition holds varies. The potential may be written in the form $\Phi = V\alpha\Phi_1 + r\Phi_2$, where $\partial\Phi_1/\partial z = 1$ and $\partial\Phi_2/\partial z = \delta x - \alpha y$ at the aerofoil.

In evaluating Φ_1 the source distribution may be regarded as the sum of two distributions ; a uniform, steady distribution over the area covered by the wing in its initial position and a fluctuating distribution over the small areas swept out by the leading edges during the small time interval 0 to t. The former corresponds to a steady, unyawed wing, therefore contributing nothing to the yawing derivatives, and it will be ignored. As the area of integration for the latter is of order ψ, variations of the integrand of the same order can be ignored in evaluating integral (24): in effect the fluctuating sources can be regarded as being concentrated at the leading edges. The source distribution making a significant contribution to Φ_1 reduces to line sources along the initial positions of the leading edges, the density at a point being proportional to the distance normal to the leading edge swept out by that point.

At a time t the wing has a y-wise displacement $\frac{1}{2}Vrt^2$ and a yaw $\psi = rt$ and the line source density along the starboard leading edge is therefore $\{r/2\pi\}\{\frac{1}{2}Vt^2\cos\gamma + xt\sec\gamma\}$, with the opposite sign for the other wing. Hence:—

$$\Phi_1 = \frac{r}{2\pi}\int_{s_1^2>0}\left\{V\left[\left(t + \frac{M}{a\beta^2}(x - x_0)\right)^2 + \frac{s_1^2}{a^2\beta^4}\right] + 2x_0\sec^2\gamma\left[t + \frac{M}{a\beta^2}(x - x_0)\right]\right\}\frac{dx_0}{s_1}$$

$$- \frac{r}{2\pi}\int_{s_2^2>0}\left\{V\left[\left(t + \frac{M}{a\beta^2}(x - x_0)\right)^2 + \frac{s_2^2}{a^2\beta^4}\right] + 2x_0\sec^2\gamma\left[t + \frac{M}{a\beta^2}(x - x_0)\right]\right\}\frac{dx_0}{s_2}$$

where:—

$$(\lambda^2 - 1)s_1^2 = \{\lambda x + \beta y\}^2 - \{(\lambda^2 - 1)x_0 + x + \lambda\beta y\}^2$$

$$(\lambda^2 - 1)s_2^2 = \{\lambda x - \beta y\}^2 - \{(\lambda^2 - 1)x_0 + x - \lambda\beta y\}^2$$

and where x_0 is restricted in both integrals such that $x \leqslant x_0 \leqslant 0$.

When $t = 0$, we have:—

$$\Phi_1 = \frac{Mr}{2\pi a\beta^4}\left\{\int\{M^2(x - x_0)^2 + 2\beta^2\sec^2\gamma\,.\,x_0(x - x_0) + s_1^2\}\frac{dx_0}{s_1}\right.$$

$$\left. - \int\{M^2(x - x_0)^2 + 2\beta^2\sec^2\gamma\,.\,x_0(x - x_0) + s_2^2\}\frac{dx_0}{s_2}\right\}$$

$$\frac{\partial\Phi_1}{\partial t} = \frac{r}{\pi\beta^2}\left\{\int\{(\lambda^2 - 1)x_0 + M^2x\}\frac{dx_0}{s_1} - \int\{(\lambda^2 - 1)x_0 + M^2x\}\frac{dx_0}{s_2}\right\}.$$

Finally we obtain for $x < \beta y < -x$:—

$$V\alpha\frac{\partial\Phi_1}{\partial x} = -\frac{\alpha r M^2\tan\gamma}{\pi(\lambda^2 - 1)^{5/2}}\left\{x\tan\gamma\{3\sec^2\gamma - 2(\lambda^2 - 1)\}\tan^{-1}\left[y\cot\gamma\left(\frac{\lambda^2 - 1}{x^2 - \beta^2y^2}\right)^{1/2}\right]\right.$$

$$+ y\{(2\lambda^2 + 1)\sec^2\gamma - 2(\lambda^2 - 1)\}\tan^{-1}\left[-x\left(\frac{\lambda^2 - 1}{x^2 - \beta^2y^2}\right)^{1/2}\right]$$

$$\left. + \frac{2M^2}{\beta^2}xy(\lambda^2 - 1)^{3/2}(x^2 - \beta^2y^2)^{-1/2}\right\} \quad \ldots \quad (25)$$

and

$$V\alpha\frac{\partial \Phi_1}{\partial t} = \frac{2\alpha r V}{\pi(\lambda^2-1)^{1/2}}\left\{x \tan^{-1}\left[y \cot \gamma\left(\frac{\lambda^2-1}{x^2-\beta^2y^2}\right)^{1/2}\right] - y \tan \gamma \ \tan^{-1}\left[-x\left(\frac{\lambda^2-1}{x^2-\beta^2y^2}\right)^{1/2}\right]\right\} . \quad (25)$$

For $-\lambda x \geqslant \beta y \geqslant -x$ the integrals reduce to:—

$$V\alpha\frac{\partial \Phi_1}{\partial x} = -\frac{\alpha r M^2 \tan \gamma}{2(\lambda^2-1)^{5/2}}\left\{x \tan \gamma \left\{3 \sec^2 \gamma - 2(\lambda^2-1)\right\} + y\{(2\lambda^2+1)\sec^2\gamma - 2(\lambda^2-1)\}\right\} \quad (26)$$

and

$$V\alpha\frac{\partial \Phi_1}{\partial t} = \frac{\alpha r V}{(\lambda^2-1)^{1/2}}\{x - y \tan \gamma\} . \quad \ldots \quad (26)$$

For $\lambda x \leqslant \beta y \leqslant x$ the terms in x in expressions (26) have the opposite signs.

In evaluating Φ_2, as in subsection 4(b), the wing may be regarded as being fixed in its neutral position: therefore from (24) we obtain:—

$$\Phi_2 = \frac{1}{\pi}\int_{y_0>0}\int (\delta x_0 - \alpha y_0)\frac{dx_0\, dy_0}{s_0} - \frac{1}{\pi}\int_{y_0 \leqslant 0}\int (\delta x_0 + \alpha y_0)\frac{dx_0\, dy_0}{s_0}$$

where the integration is over the aerofoil such that $s_0^2 = (x - x_0)^2 - \beta^2(y - y_0)^2 \geqslant 0$ and $x_0 \geqslant x$. For $-x \geqslant \beta y \geqslant 0$ this transforms into:—

$$\Phi_2 = \frac{2}{\pi}\int_{-1}^{t_0}\int_0^{q_1} + \int_{t_0}^{1}\int_0^{q_0}\frac{\{\beta\delta(1+t^2)+2\alpha t\}q + (\delta x - \alpha y)(1-t^2)}{(1-t^2)^2}\,dq\,dt$$

$$-\frac{2}{\pi}\int_{t_0}^{1}\int_{q_0}^{q_2}\frac{\{\beta\delta(1+t^2)-2\alpha t\}q + (\delta x + \alpha y)(1-t^2)}{(1-t^2)^2}\,dq\,dt$$

where $x_0 = x + \beta q(1+t^2)/(1-t^2)$ and $y_0 = y - 2qt/(1-t^2)$ and where q_0, q_1 and q_2 are the values of q for (x_0, y_0) on the lines $y = 0$, $y = -x \tan \gamma$ and $y = x \tan \gamma$ respectively: t_0 is the value of t for (x_0, y_0) at the origin. Integrating with respect to q and differentiating with respect to x we obtain:—

$$\frac{\partial \Phi_2}{\partial x} = \frac{2}{\pi}\int_{-1}^{t_0}\left\{\frac{2t(\delta + \alpha\tan\gamma)(x\tan\gamma + y)}{\{\lambda(1+t^2)-2t\}^2} - \frac{(\delta x - \alpha y)\tan\gamma}{\lambda(1+t^2)-2t}\right\}dt$$

$$+\frac{2}{\pi}\int_{t_0}^{1}\left\{\frac{2t(\delta + \alpha\tan\gamma)(x\tan\gamma - y)}{\{\lambda(1+t^2)+2t\}^2} - \frac{(\delta x + \alpha y)\tan\gamma}{\lambda(1+t^2)+2t}\right\}dt$$

$$-\frac{2}{\pi}\delta y \log t_0 , \quad \ldots \quad (27)$$

$$= \frac{2}{\pi(\lambda^2-1)^{3/2}}\left\{x\tan\gamma\{(2-\lambda^2)\delta + \alpha\tan\gamma\}\tan^{-1}\left[y\cot\gamma\left(\frac{\lambda^2-1}{x^2-\beta^2y^2}\right)^{1/2}\right]\right.$$

$$\left.+ y\{\delta + \alpha\lambda^2\tan\gamma\}\tan^{-1}\left[-x\left(\frac{\lambda^2-1}{x^2-\beta^2y^2}\right)^{1/2}\right]\right\} + \frac{2\delta y}{\pi}\operatorname{ch}^{-1}\left(\frac{-x}{\beta|y|}\right) . \quad \ldots \quad (28)$$

The same expression serves for the case when $x \leqslant \beta y \leqslant 0$. For $-\lambda x \geqslant \beta y \geqslant -x$, $\partial \Phi_2/\partial x$ is given by putting $t_0 = 1$ in equation (27):—

$$\frac{\partial \Phi_2}{\partial x} = \frac{1}{(\lambda^2 - 1)^{3/2}} \Big\{x \tan \gamma \, \{(2 - \lambda^2)\delta + \alpha \tan \gamma\} + y\{\delta + \alpha \lambda^2 \tan \gamma\}\Big\} . \quad .. \quad .. \; (29)$$

For $\lambda x \leqslant \beta y \leqslant x$ the sign of x in expression (29) is changed.

6. *The Aerodynamic Forces.*—The forces acting on the aerofoil are the result of the excess pressure acting normally to the wing surface and, in the quasi-subsonic case, of a leading-edge suction acting in the plane of the wing perpendicularly to the leading edge.

(*a*) *Excess Pressure.*—To sufficient accuracy the excess pressure is:—

$$p = \rho \left\{ V \frac{\partial \phi}{\partial x} + \tfrac{1}{2}\beta^2 \left(\frac{\partial \phi}{\partial x}\right)^2 - \tfrac{1}{2}\left(\frac{\partial \phi}{\partial y}\right)^2 - \tfrac{1}{2}\left(\frac{\partial \phi}{\partial z}\right)^2 - \frac{\partial \phi}{\partial t} - \frac{M}{a}\frac{\partial \phi}{\partial x}\frac{\partial \phi}{\partial t} \right\} \quad .. \quad .. \; (30)$$

where ρ is the free-stream density. Hence the pressure differential between the upper and lower surfaces of the aerofoil is, where ϕ is the potential on the upper surface:—

$$\Delta p = 2\rho \left\{ V \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial t} \right\} .$$

For the quasi-subsonic case the excess pressure due to the yawing is:—

$$\Delta p = \frac{\rho r V x y}{(x^2 \tan^2 \gamma - y^2)^{1/2}} \cdot \frac{\left\{ \begin{array}{c} \pi\alpha\{(2 - 5k^2 - k^4)E - 2(1 - 2k^2)k'^2 K\} \\ + \pi\alpha \tan^2 \gamma \, \{(4 - 7k^2 - 2k^4)E - (4 - 5k^2)k'^2 K\} \\ + \pi\alpha \tan^4 \gamma \, \{2k'^2 E - (2 - 3k^2)K\} - 4\delta \tan \gamma \, k^4 E(2E - k'^2 K) \end{array} \right\}}{\pi k^2 E\{(1 + k^2)E - k'^2 K\}}$$

$$+ \frac{4}{\pi} \rho r V \delta y \, . \, \mathrm{ch}^{-1} \left[\frac{-x \tan \gamma}{|y|}\right] .$$

For the definitely supersonic case:—

(i) For $x < \beta y < -x$:—

$$\Delta p = -\frac{2\rho r V}{\pi(\lambda^2 - 1)^{5/2}} \Bigg\{ \{\alpha(\lambda^2 - 1)(1 - \tan^2 \gamma) + 3\alpha \sec^4 \gamma$$

$$+ 2\delta \tan \gamma (\lambda^2 - 1)(\lambda^2 - 2)\} \, x \tan^{-1} \left[y \cot \gamma \left(\frac{\lambda^2 - 1}{x^2 - \beta^2 y^2}\right)^{1/2} \right]$$

$$+ \{2\alpha \tan \gamma (\lambda^2 - 1)(\sec^2 \gamma - \lambda^2)$$

$$+ 3\alpha M^2 \tan \gamma \; \sec^2 \gamma - 2\delta(\lambda^2 - 1)\} \, y \tan^{-1} \left[-x \left(\frac{\lambda^2 - 1}{x^2 - \beta^2 y^2}\right)^{1/2} \right] \Bigg\}$$

$$- \frac{2\rho\alpha r V M^4 x y \tan \gamma}{\pi \beta^2 (\lambda^2 - 1)(x^2 - \beta^2 y^2)^{1/2}} + \frac{4\rho\delta r V y}{\pi} \, \mathrm{ch}^{-1} \left[\frac{-x}{\beta \, |y|}\right] .$$

(ii) $-\lambda x > \beta y > -x$:—

$$\Delta p = -\frac{\rho \gamma V}{(\lambda^2 - 1)^{5/2}} \left\{ x\{\alpha(\lambda^2 - 1)(1 - \tan^2\gamma) + 3\alpha \sec^4\gamma + 2\delta \tan\gamma\,(\lambda^2 - 1)(\lambda^2 - 2)\} \right.$$

$$\left. + y\,\{2\alpha \tan\gamma(\lambda^2 - 1)(\sec^2\gamma - \lambda^2) + 3\alpha M^2 \tan\gamma\, \sec^2\gamma - 2\delta(\lambda^2 - 1)\} \right\} .$$

The side-force and rolling and yawing moments are:—

$$Y = 2\int_{-c}^{0}\int_{0}^{-x\tan\gamma} \delta \Delta p\, dy\, dx$$

$$L = 2\int_{-c}^{0}\int_{0}^{-x\tan\gamma} y\Delta p\, dy\, dx$$

$$N = -\alpha L - \tfrac{3}{4}cY .$$

(*b*) *Leading-Edge Suction.*—Owing to the singularity in the induced velocity in the quasi-subsonic case at the leading edge a suction force exists, which may be calculated by considering the rate of change of momentum of the fluid contained at any instant by a small cylinder about a leading edge.

Variations in the flow along a leading edge are small compared to those in a perpendicular plane, so that the flow for the present purpose is locally two-dimensional and the potential can be expressed in terms of a pair of local co-ordinates, ζ the upward normal and ξ the inward perpendicular to the leading edge. Also define local polar co-ordinates σ, θ such that $\xi = \sigma \cos\theta$ and $\zeta = \sigma \sin\theta$.

The suction force per unit length is given by the limiting rate of change of momentum in the ξ-direction of the fluid in the cylinder $\sigma = \sigma_0$ of unit length, as σ_0 tends to zero:—

$$\lim_{\sigma_0 \to 0} \left\{ \int_0^{2\pi} \left\{ p\cos\theta + (\rho + p/a^2)\left(\frac{\partial \phi}{\partial \sigma} + V\sin\gamma\, \cos\theta\right)\frac{\partial \phi}{\partial \xi} \right\} \sigma_0\, d\theta \right.$$

$$\left. + \frac{\partial}{\partial t}\int_0^{2\pi}\int_0^{\sigma_0} (\rho + p/a^2)\,\frac{\partial \phi}{\partial \xi}\,\sigma\, d\sigma\, d\theta \right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots (31)$$

where ρ is the free-stream density and p the excess pressure ; the density in the disturbed stream is approximately $(\rho + p/a^2)$.

In order to evaluate this limit it is necessary to determine the limiting form of the potential as σ_0 tends to zero.

Introduce auxiliary co-ordinates defined as follows:—

$$\xi_1 k \cos\gamma = \xi$$

and

$$\sigma_1 \cos\theta_1 = \xi_1$$

$$\sigma_1 \sin\theta_1 = \zeta .$$

The governing equation (4) reduces to:—

$$\frac{\partial^2 \phi}{\partial \xi_1{}^2} + \frac{\partial^2 \phi}{\partial \zeta^2} = \frac{2M}{ak}\tan\gamma\,\frac{\partial^2 \phi}{\partial \xi_1\,\partial t} + \frac{1}{a^2}\frac{\partial^2 \phi}{\partial t^2} . \qquad \ldots \quad \ldots \quad \ldots (32)$$

Substituting into equation (18) we find that near the point $(x_0, -x_0 \tan\gamma)$ on the upper surface of the aerofoil:—

$$\phi_0 = V\alpha(-2x_0\xi_1 k \tan\gamma)^{1/2}/E + O(\xi_1) ;$$

since this function is a steady potential function it is harmonic in ξ_1, ζ, while $\partial\phi_0/\partial\zeta$ is bounded on the aerofoil. Hence:—

$$\phi_0 = V\alpha\sigma_1^{1/2} \cos \tfrac{1}{2}\theta_1(-2x_0 k \tan\gamma)^{1/2}/E + O(\sigma_1)$$

and from equation (23):—

$$\frac{\partial\phi}{\partial t} = V\alpha r \sec^2\gamma\ \sigma_1^{-1/2} \cos\tfrac{1}{2}\theta_1 \,.\, x_0(-2x_0 k \tan\gamma)^{1/2}/2kE + O(\sigma_1^{1/2})$$

and in addition, using transformation (15), we find that:—

$$\frac{\partial^2\phi}{\partial t^2} = -Vr\frac{\partial\phi_0}{\partial y} = O(\sigma_1^{-1/2}).$$

Therefore, neglecting terms in r^2, equation (32) reduces to:—

$$\frac{\partial^2\phi}{\partial\xi_1^2} + \frac{\partial^2\phi}{\partial\zeta^2} = -\alpha r x_0 M^2 \sec^2\gamma\ \tan\gamma \,.\, \sigma_1^{-3/2} \cos\tfrac{3}{2}\theta_1(-2x_0 k \tan\gamma)^{1/2}/2k^2E + O(\sigma_1^{-1/2}).$$

A solution to this equation is $Q\sigma_1^{1/2}\cos\frac{3}{2}\theta_1$, where:—

$$Q = \alpha r x_0 M^2 \sec^2\gamma\ \tan\gamma\ (-2x_0 k \tan\gamma)^{1/2}/4k^2E .$$

The complete solution will be this function plus an harmonic function, and from the known value of ϕ at the aerofoil it is concluded that:—

$$\phi = \sigma_1^{1/2}\{P\cos\tfrac{1}{2}\theta_1 + Q\cos\tfrac{3}{2}\theta_1\} + O(\sigma_1)$$

where P is chosen so that $\partial\phi/\partial x$, as given by equations (13) and (22), approximates to $-(P+Q)\tan\gamma/2k\sigma_1^{1/2}$ as $\sigma_1 \to 0$. It is found that:—

$$P = (-2x_0 k\tan\gamma)^{1/2}\left\{\frac{\alpha V}{E} + \frac{2}{\pi}r\delta k^2 x_0 \frac{2E - k'^2K}{(1+k^2)E - k'^2K}\right.$$

$$\left. + \frac{\alpha r x_0 \tan\gamma}{4k^2\beta^2E}\,\frac{k^2(M^4 + M^2k^2 - 4k^2)E + M^2(7k^2 - 5M^2)(E - k'^2K)}{(1+k^2)E - k'^2K}\right\} .$$

It will be noted that $\dfrac{\partial\phi}{\partial t} = \dfrac{2ak}{M}\cot\gamma\, Q\sigma_1^{-1/2}\cos\tfrac{1}{2}\theta_1$.

On substituting for p from equation (30) and dropping terms that will vanish in the limit we derive from (31) the following expression for the suction force:—

$$\lim_{\sigma_0\to 0}\rho\left\{\int_0^{2\pi}\left\{-\tfrac{1}{2}\cos\theta\left(k^2\cos^2\gamma\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \left(\frac{\partial\phi}{\partial\zeta}\right)^2\right) + \left(\frac{\partial\phi}{\partial\sigma} - M^2\sin^2\gamma\cos\theta\frac{\partial\phi}{\partial\xi}\right)\frac{\partial\phi}{\partial\xi}\right\}\sigma_0\,d\theta\right.$$

$$\left. - \frac{M\sin\gamma}{a}\cdot\frac{\partial}{\partial t}\int_0^{2\pi}\int_0^{\sigma_0}\left(\frac{\partial\phi}{\partial\xi}\right)^2\sigma\,d\sigma\,d\theta\right\} .$$

On changing the variables to ξ_1, σ_1, θ_1 the expression becomes:—

$$\lim_{\sigma_0 \to 0} \rho \left\{ \int_0^{2\pi} \frac{-\frac{1}{2}k^2 \cos^2\gamma \cos\theta_1 \left\{ \left(\frac{\partial\phi}{\partial\sigma_1}\right)^2 + \left(\frac{\partial\phi}{\sigma_1 \partial\theta_1}\right)^2 \right\} + \left\{ \frac{\partial\phi}{\partial\sigma_1} - M^2 \sin^2\gamma \cos\theta_1 \frac{\partial\phi}{\partial\xi_1} \right\} \frac{\partial\phi}{\partial\xi_1}}{1 - M^2 \sin^2\gamma \cos^2\theta_1} \sigma_2 \, d\theta_1 \right.$$

$$\left. - \frac{2M \tan\gamma}{ak} \int_0^{2\pi} \int_0^{\sigma_2} \frac{\partial\phi}{\partial\xi_1} \cdot \frac{\partial^2\phi}{\partial\xi_1 \partial t} \sigma_1 \, d\sigma_1 \, d\theta_1 \right\}$$

where $\sigma_2^2 = \sigma_0^2/(1 - M^2 \sin^2\gamma \cos^2\theta_1)$.

Ignoring terms in Q^2, since they are of order r^2, and terms that are odd functions of $\cos\theta$, since they will eventually vanish, we obtain on substituting for ϕ:—

$$\tfrac{1}{8}\rho \int_0^{2\pi} \left\{ P^2 - 2PQ \frac{\cos 2\theta - M^2 \sin^2\gamma \cos\theta \cos 3\theta}{1 - M^2 \sin^2\gamma \cos^2\theta} \right\} d\theta + \tfrac{1}{2}\rho \int_0^{2\pi} \int_{\sigma_0}^{\sigma_2} PQ \cos 2\theta \frac{d\sigma \, d\theta}{\sigma} = \tfrac{1}{4}\pi\rho P^2 .$$

Only the terms in r in P^2 contribute to the yawing derivatives and the corresponding suction force per unit length of the starboard wing is:—

$$- \frac{\rho\alpha r V x_0^2 \tan\gamma}{4k\beta^2 E^2\{(1 + k^2)E - k'^2 K\}} \{\pi\alpha \tan\gamma \, [k^2(M^4 + M^2k^2 - 4k^2)E + M^2(7k^2 - 5M^2)(E - k'^2K)]$$

$$+ 8\delta\beta^2 k^4 E(2E - k'^2 K)\} ,$$

with the opposite sign for the other wing.

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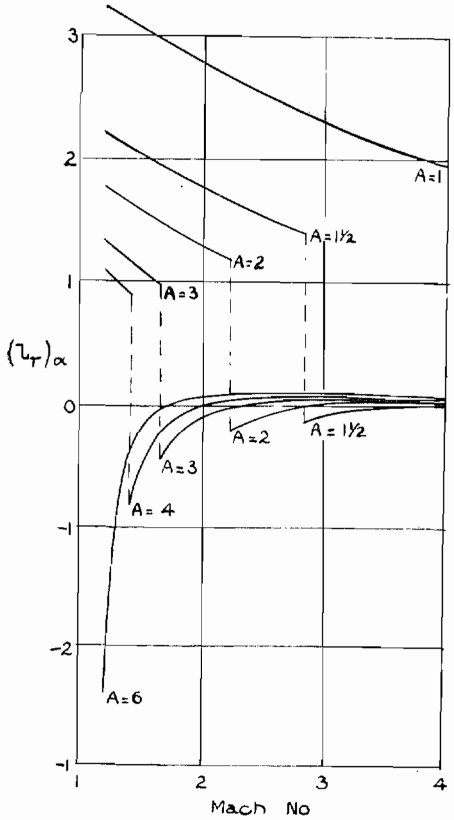


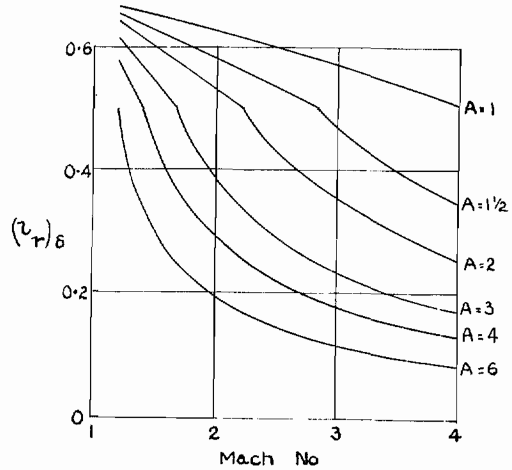


FIG. 1. Rolling-moment derivative l_r.

$$l_r = \alpha(l_r)_\alpha + \delta(l_r)_\delta .$$

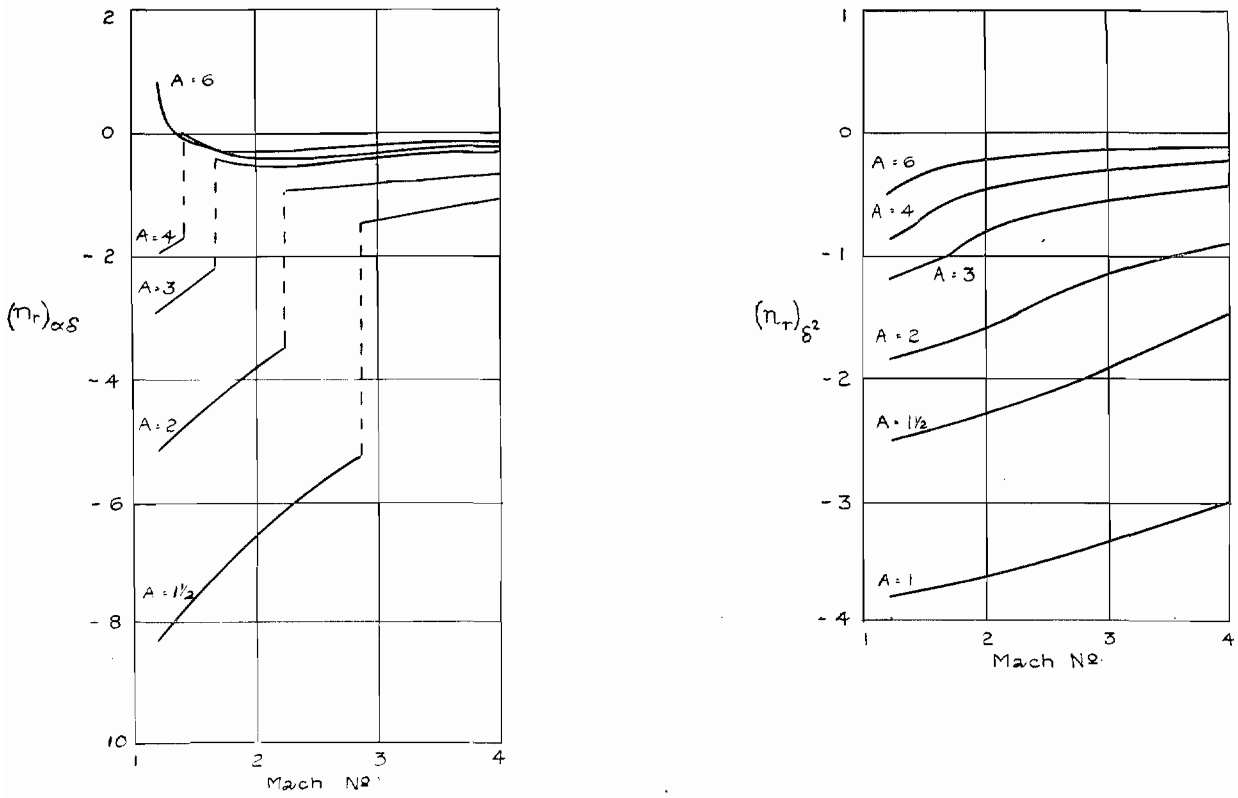
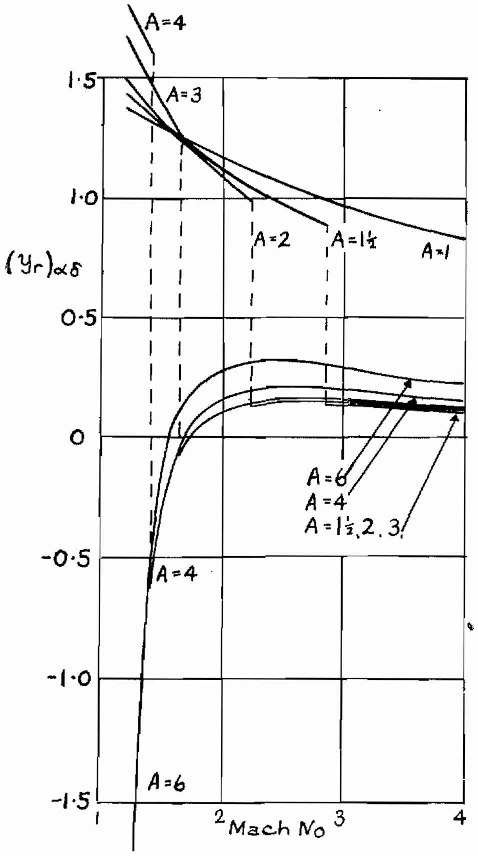


FIG. 2. Yawing-moment derivative n_r. Neglecting leading-edge suction.

$n_r = \alpha^2(n_r)_{\alpha^2} + \alpha\delta(n_r)_{\alpha\delta} + \delta^2(n_r)_{\delta^2}$, where $(n_r)_{\alpha^2} = -(l_r)_\alpha$; *see* Fig. 1.



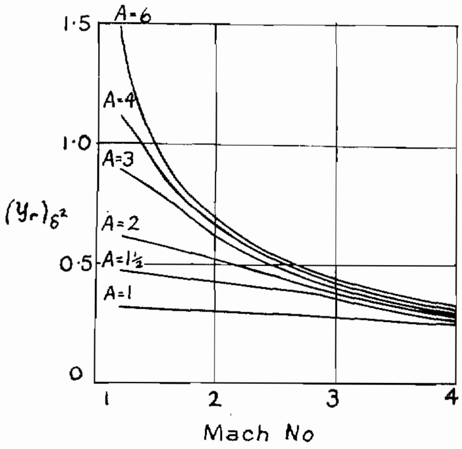


FIG. 3. Side-force derivative y_r. Neglecting leading-edge suction.

$$y_r = \alpha\delta(y_r)_{\alpha\delta} + \delta^2(y_r)_{\delta^2}$$

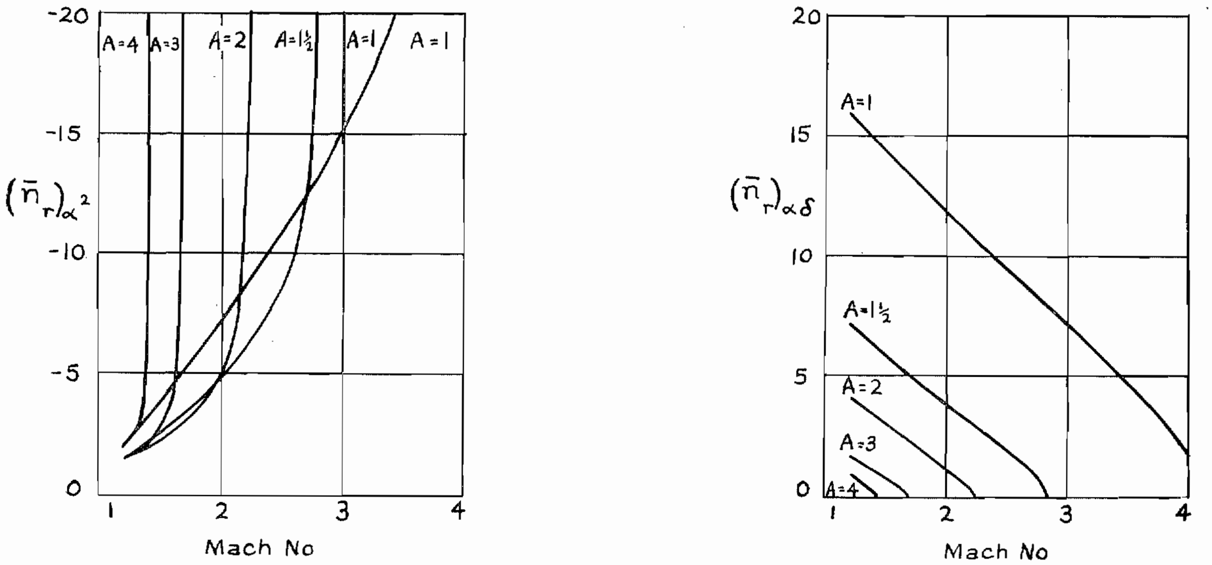


FIG. 4. Contribution of leading-edge suction to yawing-moment derivative $\bar{n}_r$.

$$\bar{n}_r = \alpha^2(\bar{n}_r)_{\alpha^2} + \alpha\delta(\bar{n}_r)_{\alpha\delta}$$

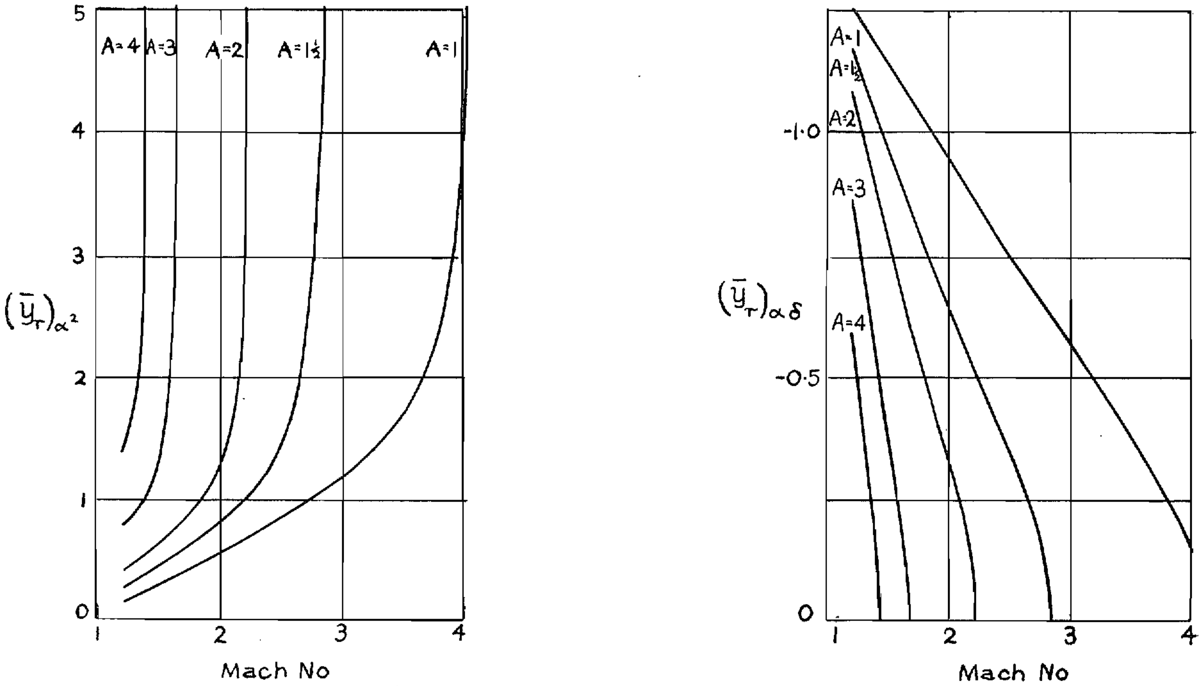
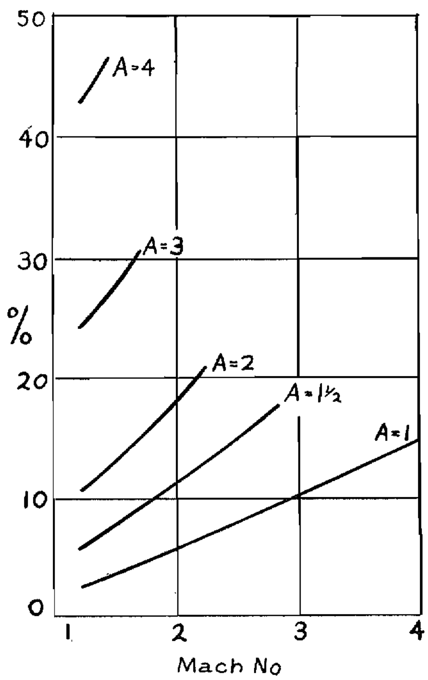
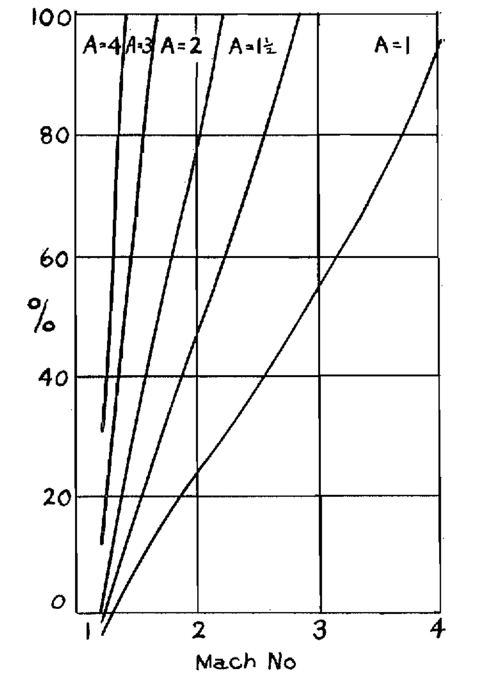


FIG. 5. Contribution of leading-edge suction to side-force derivative $\bar{y}_r$.

$$\bar{y}_r = \alpha^2(\bar{y}_r)_{\alpha^2} + \alpha\delta(\bar{y}_r)_{\alpha\delta}$$



Percentage excess of strip theory over unsteady flow theory result for rolling derivative l_r due to incidence only.



Percentage excess of strip theory over unsteady flow theory result for side-force derivative y_r due to incidence only.

FIG. 6.

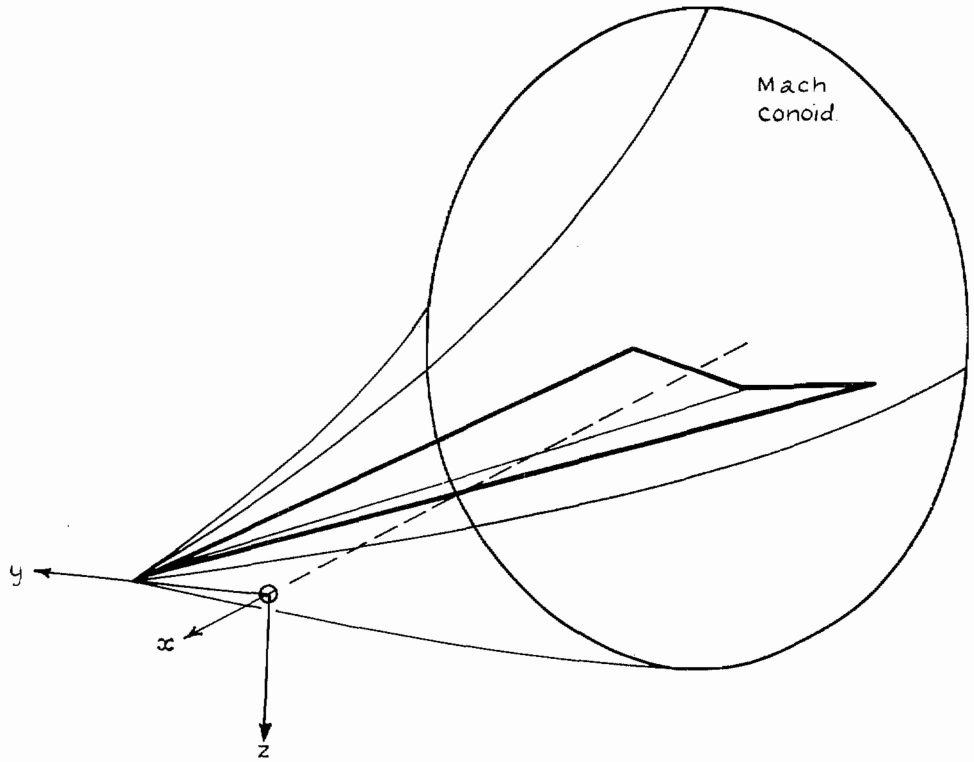


FIG. 7a.

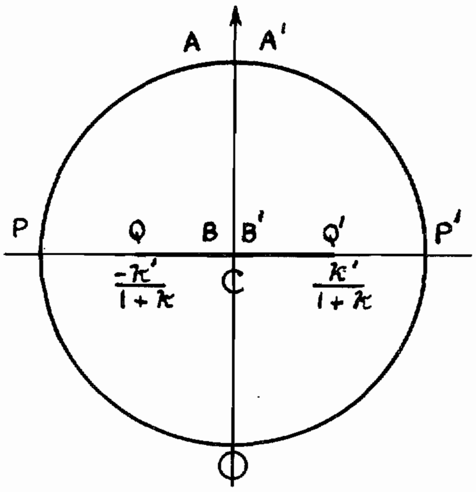


FIG. 7b. ω-plane.

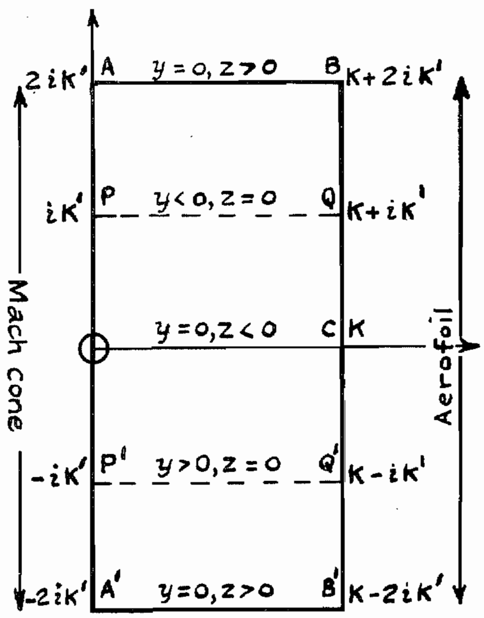


FIG. 7c. τ-plane.

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