R. & M. No. 2084
(5234)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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Reports and Memoranda No. 2084
22nd July 1941

Summary.—A method is developed of calculating the performance of a pair of contra-rotating airscrews, closely analogous to that described in R. & M. 2035[3] for a single airscrew. The assumptions made are considered to be theoretically justifiable if the interference velocities are so small that their squares and products may be neglected. It is hoped to compare calculations by the present method with experimental results.

The equations have been applied by an approximate single radius method to give the difference in blade setting between the front and back airscrews for equal power input; a comparison is also made between the efficiencies of single- and contra-rotating airscrews.

1. *Introduction.*—The present note contains equations for a close contra-rotating pair of airscrews based on the same assumptions as those of R. & M. 1674[1] and 1849[2], together with the following special assumptions. These assumptions appear to be justifiable when the interference velocities are considered as small quantities of the first order of which squares and products may be neglected.

(i) The interference velocities at any blade element may be calculated by considering the velocity fields of the two airscrews independently and adding the effects.

(ii) Either airscrew produces its own interference velocity field which so far as it affects the airscrew itself is exactly the same as if the other airscrew were absent and includes the usual tip loss correction.

(iii) Added to this is the velocity field of the other airscrew. Since the two are rotating in opposite directions, the effect will be periodic and its time average value may be taken to be equal to the average value round a circle having a radius of the blade element.

(iv) In considering the interference of either airscrew on the other, it is necessary to resolve the mean interference velocity into axial and rotational components.

The average value round a circle of the axial component interference velocity varies slowly through the airscrew disc. It is therefore reasonable to assume for the axial component for a *close* contra-rotating pair that the effect of either airscrew (y) on the other (z) is equal to the mean axial component in the plane of the airscrew disc of (y).*

The average value round a circle of the rotational component is zero[3] at any distance in front of the airscrew disc and has a constant value at any distance behind, this value being twice the mean effective value for the airscrew blade sections. It is therefore assumed as regards the rotational component that the effect of the rear airscrew on the forward airscrew is zero; the effect of the forward airscrew on the rear airscrew is equal to twice the mean value of the rotational component in the plane of the disc of the forward airscrew with its direction reversed.

* Varying degrees of closeness might be allowed for empirically by multiplying u_F by $(1 - \mu)$ and u_B by $(1 + \mu)$, where μ is a parameter varying from a small value for a close pair to a value near unity for a distant pair.

2. *Equations of motion* will now be written down on the lines of the above assumptions using as far as possible the ordinary notation (*see* Fig. 1). In order to maintain the greatest possible degree of generality the equations will be developed to as late a stage as possible on the basis of assumptions (i) and (ii) only. Thus either airscrew is subject to its own interference velocity w_1, which is normal to W (Fig. 1) and is given by the usual equation

$$w_1 = sC_L W/4\varkappa \sin\phi\ ; \qquad (1)$$

in addition it is subject to the interference velocity of the other airscrew whose axial and rotational components will be denoted by u and v.

The values of u and v according to assumptions (iii) and (iv) may be obtained as follows. The mean value $\bar{w}_1$ of w_1 taken round the circle of the blade element is given by the equation

$$\begin{aligned} \bar{w}_1 &= sC_L W/4 \sin\phi \\ &= \varkappa w_1\ , \end{aligned} \qquad (2)$$

and is in the same direction (normal to W) as w_1.* Then according to assumptions (iii) and (iv), denoting the front and back airscrews by suffices F and B (Fig. 1, b and c),

$$\begin{aligned} u_F &= \bar{w}_{1B} \cos\phi_B{}^{\dagger} \\ &= \varkappa_B w_{1B} \cos\phi_B, \end{aligned} \qquad (3)$$

$$u_B = \varkappa_F w_{1F} \cos\phi_F{}^{\dagger}, \qquad (4)$$

$$v_F = 0\ , \qquad (5)$$

$$v_B = -\ 2\varkappa_F w_{1F} \sin\phi_F. \qquad (6)$$

In what follows the general notation (u, v) will be retained as long as possible.

The general equations will first of all be obtained in a form convenient for ultimate reduction to a first order theory analogous to that of R. & M. 2035[3] using the following notation. Write for either airscrew

$$w_1 = W \tan\gamma\ , \qquad (7)$$

(Fig. 1) which by equation (1) implies also

$$sC_L = 4\varkappa \sin\phi \tan\gamma\ . \qquad (8)$$

Write also (as in R. & M. 1849[2])

$$\phi = \phi_0 + \beta^{\ddagger}, \qquad (9)$$

where

$$V = r\Omega \tan\phi_0. \qquad (10)$$

Resolving parallel and perpendicular to the direction of W (Fig. 1) for either airscrew[§],

$$W = r\Omega \sec\phi_0 \cos\beta + u \sin\phi - v\cos\phi\ , \qquad (11)$$

$$w_1 = W\tan\gamma = r\Omega\sec\phi_0 \sin\beta - u\cos\phi - v\sin\phi\ , \qquad (12)$$

where, if assumptions (iii) and (iv) are made, u and v are given by equations (3–6).

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* Strictly speaking the value of ϕ corresponding to u, v will differ from that appropriate to w_1 but the difference is of the second order in w_1/W and will be ignored.

† Varying degrees of closeness might be allowed for empirically by multiplying u_F by $(1 - \mu)$ and u_B by $(1 + \mu)$ where μ is a parameter varying from a small value for a close pair to a value near unity for a distant pair.

‡ For a single airscrew, $\beta = \gamma$, and the symbol γ is not used.

§ W is the projection of the broken line $C\ D\ E\ A$ on $A\ B$; w_1 is the projection of the reversed line $A\ E\ D\ C$ on $B\ C$.

For the thrust and torque acting on a blade element we have the usual equations

$$dT = N(dL \cos \phi - dD \sin \phi),$$

$$(1/r)dQ = N(dL \sin \phi + dD \cos \phi),$$

where

$$dL = \tfrac{1}{2}\rho c W^2 C_L dr,$$

$$dD = \tfrac{1}{2}\rho c W^2 C_D dr,$$

so that

$$(dT/dr) = \pi\rho r s W^2 \,(C_L \cos \phi - C_D \sin \phi), \quad \ldots \quad (13)$$

$$(1/r)\,(dQ/dr) = \pi\rho r s W^2 \,(C_L \sin \phi + C_D \cos \phi). \quad \ldots \quad (14)$$

For the total power loss (power input minus thrust power) we have

$$\Omega dQ - V dT = N dL\,(r\Omega \sin \phi - V \cos \phi) + N dD\,(r\Omega \cos \phi + V \sin \phi)\ .$$

By the geometry of Fig. 1 it follows that for the induced loss (defined here as the part of the power loss depending on the lift of the blade elements),

$$dP_1 \equiv N dL\,(r\Omega \sin \phi - V \cos \phi),$$

$$(dP_1/dr) = \pi\rho r s W^2 r\Omega \sec \phi_0\, C_L \sin \beta, \quad \ldots \quad (15)$$

and for the drag loss

$$dP_2 \equiv N dD\,(r\Omega \cos \phi + V \sin \phi),$$

$$(dP_2/dr) = \pi\rho r s W^2 r\Omega \sec \phi_0\, C_D \cos \beta. \quad \ldots \quad (16)$$

Equations (13–16) are all identical in form with those for a single airscrew.

Equations (10–16) with (3–6) will be developed into forms analogous to those of R. & M. 1849[2] and R. & M. 1674[1] in §7 and §8 respectively. The most practical and useful form is obtained by considering β and γ as small quantities and neglecting squares and products of β and γ for both airscrews. The resulting equations analogous to those of R. & M. 2035[3] are developed in §§3–5.

3. *First Order Theory.*—Consider β, γ as small quantities of the first order and write

$$\left.\begin{aligned} u_y &= \mu_{0_y} w_{1_z}, \\ v_y &= \nu_{0_y} w_{1_z}, \end{aligned}\right\} \quad \ldots \quad (17)$$

and

$$\left.\begin{aligned} \mu_{0_y} \sin \phi_{0_y} - \nu_{0_y} \cos \phi_{0_y} &= \xi_{0y}\ , \\ \mu_{0_y} \cos \phi_{0_y} + \nu_{0_y} \sin \phi_{0_y} &= \zeta_{0y}\ , \end{aligned}\right\} \quad \ldots \quad (18)$$

where either $y = F,\ z = B$ or $y = B,\ z = F$.

Thus equation (11) gives for either airscrew,

$$W_y = r\Omega_y \sec \phi_{0_y} + \xi_{0y} W_z \gamma_z + O(\gamma^2)\ , \quad \ldots \quad (19)$$

and substitution in equation (12) gives

$$(\beta - \gamma)_y r\Omega_y \sec \phi_{0_y} = \zeta_{0_y} r\Omega_z \sec \phi_{0_z} \gamma_z + O(\gamma^2)\ , \quad \ldots \quad (20)$$

for either $y = F,\ z = B$ or $y = B,\ z = F$. On the basis of equations (3–6) we have,

$$\left.\begin{aligned} \mu_{0F} &= \varkappa_{0B} \cos \phi_{0B} + O(\gamma)\ , \\ \nu_{0F} &= O(\gamma)\ , \\ \mu_{0B} &= \varkappa_{0F} \cos \phi_{0F} + O(\gamma)\ , \\ \nu_{0B} &= -\,2\varkappa_{0F} \sin \phi_{0F} + O(\gamma)\ . \end{aligned}\right\} \quad \ldots \quad (21)$$

Since

$$r\Omega_y \tan \phi_{0y} = V = r\Omega_z \tan \phi_{0z} ,$$

equation (20) may be written in the form

$$(\beta - \gamma)_y = \zeta_{0y}\lambda_{yz}\gamma_z + O(\gamma^2) , \quad \ldots \quad (22)$$

where $\lambda_{yz} = (\sin \phi_{0y}/\sin \phi_{0z})$.

Also from (8) for either airscrew,

$$\left.\begin{aligned} \gamma &= bsC_L + O(\gamma^2) , \\ \text{where} \quad 1/b &= 4\varkappa_0 \sin \phi_0{}^*, \end{aligned}\right\} \quad \ldots \quad (23)$$

so that sC_L is of the same order as γ.

If C_L is given for both airscrews, equations (23) determine γ and equations (20, 18 and 21) determine β for both airscrews. Then equations (15) and (16) in the form

$$(dP_1/dr) = \pi\rho rs . C_L r^3\Omega^3 \sec^3 \phi_0 . \beta + O(\gamma^3) , \quad \ldots \quad (24)$$

$$(dP_2/dr) = \pi\rho rs . C_D r^3\Omega^3 \sec^3 \phi_0 + O(\gamma^3) \quad \ldots \quad (25)$$

give the power losses of either airscrew. In general it is convenient to consider sC_D as a small quantity of order γ^2 so that both dP_1/dr and dP_2/dr are of order γ^2. To the first order the power input to either airscrew is

$$\Omega(dQ/dr) = \pi\rho rsr^3\Omega^3 \sec^2 \phi_0 \, (C_L \sin \phi_0 + C_D \cos \phi_0) + O(\gamma^2) . \quad \ldots \quad (26)$$

The further development analogous to that of R. & M. 2035[3] required to determine C_L for either airscrew for given blade angle setting is given in §5, but it is convenient first to consider the application of equations (24–26) to determine explicitly the power input and power wastage to the first order, for given C_L, for the particular case of equal rotational speed and power input for the two airscrews.

4. *Special Case. Equal Rotational Speed and Power Input.—Equal Rotational Speed.*—It follows from equations (10) and (23) that equal rotational speed of the two airscrews implies equal values of ϕ_0, $\varkappa_0$ and b so that λ_{yz} is unity. Equation (22) then gives

$$\beta_y = \gamma_y + \zeta_{0y}\gamma_z + O(\gamma^2) , \quad \ldots \quad (27)$$

so that from (24)

$$(dP_1/dr)_y = \pi\rho r^4\Omega^3 \sec^3 \phi_0 \, (sC_L)_y \, (\gamma_y + \zeta_{0y} \gamma_z) + O(\gamma^3), \quad \ldots \quad (28)$$

and using equations (21)

$$\zeta_{0F} = \varkappa_0 \cos^2 \phi_0 + O(\gamma), \quad \zeta_{0B} = \varkappa_0 (\cos^2 \phi_0 - 2 \sin^2 \phi_0) + O(\gamma) .$$

and

$$(dP_1/dr)_F = \pi\rho r \, (r\Omega \sec \phi_0)^3 \, (sC_L)_F\{\gamma_F + \varkappa_0\gamma_B \cos^2 \phi_0\} + O(\gamma^3), \quad \ldots \quad (29)$$

$$(dP_1/dr)_B = \pi\rho r \, (r\Omega \sec \phi_0)^3 \, (sC_L)_B\{\gamma_B + \varkappa_0\gamma_F \, (\cos^2 \phi_0 - 2 \sin^2 \phi_0)\} + O(\gamma^3) . \quad \ldots \quad (30)$$

Equal Rotational Speed and Power Input.—Equation (26) shows that equal power input to the blade element at radius r combined with equal rotational speed implies that

$$\left.\begin{aligned} (sC_L)_F - (sC_L)_B &= O(\gamma^2) \\ \text{and} \quad \gamma_F - \gamma_B &= O(\gamma^2) ; \end{aligned}\right\} \quad \ldots \quad (31)$$

equations (29) and (30) then become

$$(dP_1/dr)_F = \pi\rho r \, (r\Omega \sec \phi_0)^3 \, sC_L\gamma \, (1 + \varkappa_0 \cos^2 \phi_0) + O(\gamma^3),$$

$$(dP_1/dr)_B = \pi\rho r \, (r\Omega \sec \phi_0)^3 \, sC_L\gamma \, (1 + \varkappa_0 \cos^2 \phi_0 - 2\varkappa_0 \sin^2 \phi_0) + O(\gamma^3) \quad \ldots \quad (32)$$

* R. & M. 2035[3], equation (10).

For the combination of two airscrews

$$(dP_1/dr)_C = \pi \rho r \, (r\Omega \sec \phi_0)^3 \, 2sC_L\gamma \, (1 + \varkappa_0 \cos 2\phi_0) + O(\gamma^3) \, . \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

Equations (31–33) and (25) transformed into equations for the coefficient p_{C1}, p_{C2} of induced drag power loss, analogous to equations (31) and (33) of R. & M. 2035[3], may be used to calculate the power loss grading for all radii for a given distribution of sC_L (equal for the two airscrews); the corresponding blade angle distribution may be obtained from §5. The power input grading (torque grading) may be obtained from equation (26) or more accurately (as in R. & M. 2035[3]) from equation (14) using the more accurate value of W obtained below in §6. In the latter case the power input will not be exactly equal for the two airscrews if the values of sC_L are equal. The second order difference in sC_L required to make the power inputs equal to the second order is determined in §6. Or, the performance for a given blade angle distribution may be deduced from the equations of §5; the blade angles at standard radius (0·7) might be adjusted to give equal power input at that radius.

Example.—For the purpose of illustration equations (33), (25) and (26) have been used to calculate the partial efficiency for a section at standard radius (0·7) for equal rotational speed and power input. The formulae (deducible from equations (31–33), (25) and (26)) are

$$1 - \eta_F = \frac{\gamma C_L \, (1 + \varkappa_0 \cos^2 \phi_0) + C_D}{\cos \phi_0 \, (C_L \sin \phi_0 + C_D \cos \phi_0)} \, , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

$$1 - \eta_B = \frac{\gamma C_L \, (1 + \varkappa_0 \cos^2 \phi_0 - 2\varkappa_0 \sin^2 \phi_0) + C_D}{\cos \phi_0 \, (C_L \sin \phi_0 + C_D \cos \phi_0)} \, , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$1 - \eta_C = \frac{\gamma C_L \, (1 + \varkappa_0 \cos 2\phi_0) + C_D}{\cos \phi_0 \, (C_L \sin \phi_0 + C_D \cos \phi_0)} \, , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

with

$$\gamma = sC_L/(4\varkappa_0 \sin \phi_0) = bsC_L \, . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

In Fig. 2 values of $(1 - \eta_C)$ are plotted for a range of values of J for (1) a pair of contra-rotating two bladers and (2) a pair of contra-rotating three-bladers and for the following values of s, C_L and C_D:—

$$s = 0{\cdot}090, \qquad C_L = 0{\cdot}56, \qquad C_D = 0{\cdot}017.$$

The values of s and C_L are those at radius 0·7 for airscrew B in R. & M. 2021[4], while the value of C_D is adjusted to give a partial efficiency for this radius equal to the calculated efficiency (0·878) for the whole airscrew. The calculations correspond therefore, to a power input to each airscrew of 2,000 h.p. at 450 m.p.h. equal to that assumed in R. & M. 2021[4] (a total of 4,000 h.p. for the two airscrews) for the same diameter, rotational speed and height. They were made for a range of values of J from 1·27 to 4·54.* They are compared with the corresponding efficiency figures for a single airscrew of double (the same total) number of blades and solidity and also with airscrews having the same number of blades as one of the contra-rotating pairs and the same total solidity. The equation corresponding to (36) for a single airscrew is

$$1 - \eta_S = \frac{\gamma C_L + C_D}{\cos \phi_0 \, (C_L \sin \phi_0 + C_D \cos \phi_0)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

with (37) in which it must be remembered that values of $\varkappa_0$ and s must be used, appropriate to the *total* number of blades and solidity. Thus the value of s for the single propeller has twice the value for the corresponding contra-rotating pair, and so the value of γ in (38) would be double that in (36) apart from the change in $\varkappa_0$ due to doubling the number of blades.

The results of Fig. 2 show that for the present case the increase of efficiency as between the 2-bladers (contra-rotating) and the 4-bladers (single-rotating) varies from 1·0 per cent. to 4·6 per cent., and the increase as between the 3-bladers (contra-rotating) and the 6-bladers (single-rotating) varies from 1·7 per cent. to 4·8 per cent. for the particular values of s, C_L and C_D chosen.

* The actual efficiency figures for the highest values of J would in practice be reduced by the increase of C_D due to increased compressibility effect.

5. *The Relation between* sC_L and *Blade Angle* θ to the *First Order, for the General Case.*—This may be obtained by a similar method to that of R. & M. 2035[3], §3, as follows :—

Write

$$\theta - \phi_0 + \varepsilon = \Theta \; , \qquad (39)$$

and

$$asC_L = \alpha + \varepsilon$$

$$= \Theta - \beta \; , \qquad (40)$$

where a and ε define the (straight line) lift curve as in R. & M. 2035[3], equation (11), and are, in general, functions of the Mach number.

Comparison of (40) and (23) gives

$$b\Theta = b\beta + a\gamma + O(\gamma^2) \; , \qquad (41)$$

which with (22) determines Θ_F, Θ_B as functions of γ_F, γ_B and so of $(sC_L)_F$ and $(sC_L)_B$ in the form

$$\Theta_y = \left(\frac{a+b}{b}\right)_y \gamma_y + \zeta_{0y}\lambda_{yz}\gamma_z + O(\gamma^2) \; , \qquad (42)$$

with $y = F$, $z = B$ or $y = B$, $z = F$. Using the relation $\lambda_{yz}\,\lambda_{zy} = 1$, the pair of equations represented by (42) may then be solved for γ_y, γ_z in the form

$$(sC_L)_y = (\gamma/b)_y = \{(a+b)_z\,\Theta_y - b_z\zeta_{0y}\lambda_{yz}\,\Theta_z\}/\{(a+b)_F\,(a+b)_B - b_F b_B \zeta_{0F}\zeta_{0B}\} + O(\gamma^2) \; , \qquad (43)$$

which reduces to equation (13) of R. & M. 2035[3] on putting $\zeta_{0F} = \zeta_{0B} = 0$. In this pair of equations, using (18) and (21) we have

$$\zeta_{0F} = \varkappa_{0B} \cos\phi_{0B} \cos\phi_{0F} + O(\gamma) \; , \qquad (44)$$

$$\zeta_{0B} = \varkappa_{0F} (\cos\phi_{0B}\cos\phi_{0F} - 2\sin\phi_{0B}\sin\phi_{0F}) + O(\gamma) \; , \qquad (45)$$

and

$$\lambda_{FB} = 1/\lambda_{BF} = \sin\phi_{0F}/\sin\phi_{0B} \; . \qquad (46)$$

Special Case. For Equal Rotational Speed, using the results of §4 equation (43) becomes

$$(sC_L)_F = \gamma_F/b$$

$$= \{(a+b)\,\Theta_F - \varkappa_0 b\cos^2\phi_0\Theta_B\}/\{(a+b)^2 - \varkappa_0{}^2b^2\cos^2\phi_0\,(\cos^2\phi_0 - 2\sin^2\phi_0)\} + O(\gamma^2) \; ,$$

$$(sC_L)_B = \gamma_B/b$$

$$= \{(a+b)\,\Theta_B - \varkappa_0 b\,(\cos^2\phi_0 - 2\sin^2\phi_0)\,\Theta_F\}/\{(a+b)^2 - \varkappa_0{}^2b^2\cos^2\phi_0\,(\cos^2\phi_0 - 2\sin^2\phi_0)\} + O(\gamma^2) \; . \qquad (47)$$

For equal rotational speed and equal power input to the blade element at radius r equation (42) becomes (using 31)

$$\Theta_y = \left\{\left(\frac{a+b}{b}\right) + \zeta_{0y}\right\}\gamma + O(\gamma^2) \; ,$$

and so

$$\Theta_F - \Theta_B = \gamma\,(\zeta_{0F} - \zeta_{0B}) + O(\gamma^2)$$

$$= 2\gamma\varkappa_0\sin^2\phi_0 + O(\gamma^2)$$

$$= \tfrac{1}{2}sC_L\sin\phi_0 + O(\gamma^2) \; . \qquad (48)$$

This value is plotted against J in Fig. 3 for the values of sC_L used in §4 and varies from 0·7 deg. to 1·3 deg. over the range of J considered.

6. *Values of W and $\Omega(dQ/dr)$ to the Second Order in γ.*—The value of W to the second order can be obtained from equation (11) in the form

$$W_y = r\Omega_y \sec \phi_{0y} + \xi_{0y} r\Omega_z \sec \phi_{0z}\gamma_z$$

$$= r\Omega_y \sec \phi_{0y} \{1 + \xi_{0y}\lambda_{yz}\gamma_z\} + O(\gamma^2) \; . \qquad (49)$$

The expressions for dT/dr and dQ/dr involve the factors $\sin \phi$, $\cos \phi$ which may be written,

$$\sin \phi = \sin \phi_0 \, (1 + \beta \cot \phi_0) + O(\gamma^2) \; , \qquad (50)$$

$$\cos \phi = \cos \phi_0 \, (1 - \beta \tan \phi_0) + O(\gamma^2) \; , \qquad (51)$$

with

$$\beta_y = \gamma_y + \zeta_{0y}\lambda_{yz}\gamma_z + O(\gamma^2) \; , \qquad (52)$$

from (22). Equation (49) then gives

$$W_y^2 \sin \phi_y = r^2\Omega_y^2 \tan \phi_{0y} \sec \phi_{0y} \{1 + [2\xi_{0y} + \zeta_{0y} \cot \phi_{0y}] \, \lambda_{yz}\gamma_z + \gamma_y \cot \phi_{0y}\} + O(\gamma^2) \; , \qquad (53)$$

$$W_y^2 \cos \phi_y = r^2\Omega_y^2 \sec \phi_{0y} \{1 + [2\xi_{0y} - \zeta_{0y} \tan \phi_{0y}] \, \lambda_{yz}\gamma_z - \gamma_y \tan \phi_{0y}\} + O(\gamma^2) \; ; \qquad (54)$$

or

$$W_y^2 \sin \phi_y = r^2\Omega_y^2 \sec^2 \phi_{0y} \{\sin \phi_{0y} + \tfrac{1}{2}\lambda_{yz}\gamma_z \, [\mu_{0y} \, (3 - \cos 2\phi_{0y}) - \nu_{0y} \sin 2\phi_{0y}] + \gamma_y \cos \phi_{0y}\} + O(\gamma^2) \; , \qquad (55)$$

$$W_y^2 \cos \phi_y = r^2\Omega_y^2 \sec^2 \phi_{0y} \{\cos \phi_{0y} + \tfrac{1}{2}\lambda_{yz}\gamma_z \, [\mu_{0y} \sin 2\phi_{0y} - \nu_{0y} \, (3 + \cos 2\phi_{0y})] - \gamma_y \sin \phi_{0y}\} + O(\gamma^2) \; . \qquad (56)$$

In evaluating $\Omega(dQ/dr)$ and $V(dT/dr)$ it is reasonable to consider C_D/C_L, as before, as a small quantity of the same order as γ, and to write

$$\Omega(dQ/dr) = \pi\rho r^2 \Omega W^2 \sin \phi \; sC_L \{1 + (C_D/C_L) \cot \phi_0\} + O(\gamma^3) \; , \qquad (57)$$

$$V(dT/dr) = \pi\rho r^2 \Omega \tan \phi_0 \; W^2 \cos \phi \; sC_L \{1 - (C_D/C_L) \tan \phi_0\} + O(\gamma^3) \; . \qquad (58)$$

In these expressions $W^2 \sin \phi$, $W^2 \cos \phi$, are given by equations (53–56) in which γ is given by

$$\gamma = bsC_L,$$

so that the torque and thrust power loss grading may be evaluated as far as terms of order γ^2 if the value of sC_L is known to this order for each airscrew. Equations (43–46) give the values of sC_L for each airscrew in terms of the blade angle settings.

Strictly speaking, equations (23) and (40) are only correct to the first order in γ and α, but it was suggested in R. & M. 2035[3] that in practice the curves of C_L against α in the unstalled range and of sC_L against γ over a considerable range of large values of J, are straight lines to a higher order of approximation. The additional order of accuracy would then apply to equations (43) since they are deducible from (23) and (40) by linear transformations; the values of sC_L deduced from (43) for given blade angles would then be sufficiently accurate when substituted in (57) and (58) to give values of thrust and torque power correct as far as terms in γ^2. In any case the value of power loss given by taking the difference between power input deduced from (57) and useful power deduced from (58), will be consistent with (24) and (25) and correct to the same order as the latter equations.

Case of Equal Revolutions to Second Order.—Substitution of

$$\mu_{0F} = \mu_{0B} = \varkappa_0 \cos \phi_0 \ ,$$
$$\nu_{0F} = 0 \ ,$$
$$\nu_{0B} = -\ 2\varkappa_0 \sin \phi_0$$

in (53–56), gives

$$W_F^2 \sin \phi_F = r^2 \Omega^2 \tan \phi_0 \sec \phi_0 \{1 + \gamma_F \cot \phi_0 + \varkappa_0 \gamma_B (\cot \phi_0 + \sin \phi_0 \cos \phi_0)\} + O(\gamma^2) \ , \quad \text{(59)}$$

$$W_B^2 \sin \phi_B = r^2 \Omega^2 \tan \phi_0 \sec \phi_0 \{1 + \gamma_B \cot \phi_0 + \varkappa_0 \gamma_F (\cot \phi_0 + 3 \sin \phi_0 \cos \phi_0)\} + O(\gamma^2) \ , \quad \text{(60)}$$

$$W_F^2 \cos \phi_F = r^2 \Omega^2 \sec \phi_0 \{1 - \gamma_F \tan \phi_0 + \varkappa_0 \gamma_B \sin \phi_0 \cos \phi_0\} + O(\gamma^2) \ , \quad \text{(61)}$$

$$W_B^2 \cos \phi_B = r^2 \Omega^2 \sec \phi_0 \{1 - \gamma_B \tan \phi_0 + \varkappa_0 \gamma_F (2 \tan \phi_0 + 3 \sin \phi_0 \cos \phi_0)\} + O(\gamma^2) \ . \quad \text{(62)}$$

Equal Power Input to Second Order.—It is evident that the difference $C_{LF} - C_{LB}$ will be of order γ^2 and it is therefore reasonable to assume that $C_{DF} - C_{DB}$ is of order γ^3. The condition of equal power input will therefore be taken as

$$(sC_L W^2 \sin \phi)_F = (sC_L W^2 \sin \phi)_B + O(\gamma^3) \ . \quad \text{(63)}$$

Condition (63) may be satisfied by writing $\gamma_F = \gamma_B = \gamma$ in (59–62), since this is true to the first order, and putting

$$(sC_L)_F = sC_L (1 + \varkappa_0 \gamma \sin \phi_0 \cos \phi_0) \ , \quad \text{(64)}$$

$$(sC_L)_B = sC_L (1 - \varkappa_0 \gamma \sin \phi_0 \cos \phi_0) \quad \text{(65)}$$

in (63), where sC_L is a mean value between the two airscrews. The final expressions for the thrust and torque grading will be for either airscrew,

$$\Omega(dQ/dr) = \pi \rho r^4 \Omega^3 \tan \phi_0 \sec \phi_0 \{sC_L\{1 + \gamma [\cot \phi_0 + \varkappa_0 (\cot \phi_0 + 2 \sin \phi_0 \cos \phi_0)]\} + sC_D \cot \phi_0\} \ ; \quad \text{(66)}$$

and for the front and back airscrews separately,

$$V(dT/dr)_F = \pi \rho r^4 \Omega^3 \tan \phi_0 \sec \phi_0 \Big(sC_L\{1 + \gamma [-\tan \phi_0 + 2\varkappa_0 \sin \phi_0 \cos \phi_0]\} - sC_D \tan \phi_0\Big) \ , \quad \text{(67)}$$

$$V(dT/dr)_B = \pi \rho r^4 \Omega^3 \tan \phi_0 \sec \phi_0 \Big(sC_L\{1 + \gamma [-\tan \phi_0 + 2\varkappa_0 (\tan \phi_0 + \sin \phi_0 \cos \phi_0)]\} - sC_D \tan \phi_0\Big) \ . \quad \text{(68)}$$

The difference between these expressions for torque and thrust power agrees with the first order value of power loss given in (31–33). Expressions for the blade angle to the second order could be deduced from §7, equation (76) below, but would be rather complicated.

7. *Exact Transformation of Equations* (11) *and* (12) *into a Form Analogous to the Equations of R. & M.* 1849[2].—Write

$$\left.\begin{aligned} u_y &= \mu_y w_{1z} \ , \\ v_y &= \nu_y w_{1z} \ , \end{aligned}\right\} \quad \text{(69)}$$

where either $y = F$, $z = B$, or $y = B$, $z = F$, and μ_y, ν_y are functions of ϕ_y, ϕ_z (according to equations (3–6), of ϕ_z only).

Write

$$\left.\begin{aligned} \mu \sin \phi - \nu \cos \phi &= \xi \ , \\ \mu \cos \phi + \nu \sin \phi &= \zeta \ . \end{aligned}\right\} \quad \text{(70)}$$

These definitions are analogous to the first order definitions of (17) and (18). Write also

$$\left.\begin{aligned} r\Omega \sec\phi_0 \cos\beta &= C \ , \\ r\Omega \sec\phi_0 \sin\beta &= D \ ; \end{aligned}\right\} \qquad (71)$$

also by (10)

$$(r\Omega \tan\phi_0)_F = (r\Omega \tan\phi_0)_B \ .$$

Then equations (11) and (12) become

$$W_y = C_y + \xi_y w_{1_z} \ , \qquad (72)$$

$$w_{1y} = W_y \tan\gamma_y = D_y - \zeta_y w_{1_z} \ . \qquad (73)$$

The pair of equations,

$$\left.\begin{aligned} w_{1y} + \zeta_y w_{1_z} &= D_y \\ \zeta_z w_{1y} + w_{1_z} &= D_z \ , \end{aligned}\right\} \qquad (74)$$

may be solved giving

$$w_{1y} = (D_y - \zeta_y D_z)/(1 - \zeta_y \zeta_z) \ . \qquad (75)$$

Then (72) gives

$$W_y = \{C_y(1 - \zeta_y\zeta_z) + \xi_y D_z - \xi_y \zeta_z D_y\}/(1 - \zeta_y \zeta_z) \ , \qquad (76)$$

and substitution in (73), using (71), gives

$$\tan\gamma_y = \tan\beta_y\left(1 - \zeta_y \frac{D_z}{D_y}\right)\Big/\left\{1 - \zeta_y\zeta_z + \xi_y \frac{D_z}{C_y} - \xi_y\zeta_z \frac{D_y}{C_y}\right\}$$

$$= \tan\beta_y\left(1 - \zeta_y \frac{\sin\phi_y \sin\beta_z}{\sin\phi_z \sin\beta_y}\right)\Big/\left\{1 - \zeta_y\zeta_z + \xi_y \frac{\sin\phi_{0y}\sin\beta_z}{\sin\phi_{0z}\cos\beta_y} - \xi_y\zeta_z \tan\beta_y\right\}. \qquad (77)$$

The two equations (77) determine γ_F, γ_B and so in virtue of (8) $(sC_L)_F$ and $(sC_L)_B$ as functions of ϕ_F, ϕ_B only, ϕ_{0F}, ϕ_{0B} being known. Since equations (3)–(6) are only claimed to be correct to the first order, the advantage of the present equations over first order equations is doubtful.

It would be possible to plot $(sC_L)_F$ against ϕ_F, giving for each J a series of curves for various values of ϕ_B and similarly for $(sC_L)_B$, (Fig. 4). It would then be necessary to determine intersections with $(sC_L)_F$ against α_F and $(sC_L)_B$ against α_B curves giving consistent values of ϕ_F and ϕ_B and this could be done by a very rapid successive approximation between the two figures. This represents the analogue of the use of Chart I in R. & M. 1849[2].

8. *Equations of the type of R. & M.* 1674[1].—From Fig. 1.

$$AC = w_1 \operatorname{cosec}\gamma \ .$$

Resolving parallel to AF, we have

$$AC \cos(\phi - \gamma) = r\Omega - v \ ,$$

giving

$$w_1 = (r\Omega - v)\sin\gamma \sec(\phi - \gamma)$$

$$= (r\Omega - v)\tan\gamma \sec\phi/(1 + \tan\gamma \tan\phi) \ , \qquad (78)$$

with $\quad \tan\gamma = sC_L/4\varkappa \sin\phi \ .$

For the front airscrew $v = 0$ and the equation becomes identical with equation (8) of R. & M. 1674[1]. For the back airscrew $v/r\Omega$ is of order γ and might be calculated by writing $\alpha_F = \alpha_B$.

V is then given by (Fig. 1)

$$V = r\Omega \tan \phi - GC - CD - HK$$

$$= r\Omega \tan \phi - w_1 \sec \phi - u - v \tan \phi \quad \ldots \quad (79)$$

The most convenient form for W is (Fig. 1)

$$W = HA - GB - HG$$

$$= (r\Omega - v) \sec \phi - w_1 \tan \phi \,, \quad \ldots \quad (80)$$

which is identical with equation (2) of R. & M. 1674[1] for $v \to 0$.

The equations (78–80) may be transformed so as to involve non-dimensional coefficients only, by dividing by convenient multiples of $R\Omega_F$, $R\Omega_B$.

The solution of the equations by the methods of R. & M. 1674[1] is straightforward apart from the occurrence of the term involving v in equation (78) for the back airscrew. A suitable series of values of the blade incidence α is first chosen for both airscrews for a series of standard values of the radius. Values of C_L, C_D for either screw are supposed known as function of α, and ϕ is deduced from the equation

$$\phi = \theta - \alpha \,.$$

Equations (78), (79), (80), (14), (15) and (16) then determine in succession values of w_1, W, V, $\Omega(dQ/dr)$, dP_1/dr, dP_2/dr (or of suitable coefficients of them) for both airscrews. In evaluating the term v in equation (78) it should be sufficiently accurate to write $\alpha_F = \alpha_B$. It is finally necessary to plot values of V or of its coefficients J_F and J_B and of $\Omega(dQ/dr)$, dP_1/dr, dP_2/dr or their coefficients against α, so as to deduce values of the thrust and power coefficients for the same values of V at all radii before plotting against the radius r* and integrating to obtain the power input and power loss on the whole airscrew.

9. *Recapitulation.*—§1. Of the four basic assumptions as set out in §1, the first two are considered to be of general application to an airscrew, subject to any type of external interference. The development of the equations is carried as far as possible without reference to the third and fourth assumptions and these may require further empirical modification and would in fact be modified as a result of increasing the distance between the two airscrews or varying their diameters, *etc.*

§2. Equations are given of the most general form consistent with assumptions (i) and (ii) and determine the total velocity W and the interference velocity w_1 of either screw on itself, in terms of $r\Omega$, ϕ, and (u, v) the components of the interference velocity of the second screw ; also for the thrust, torque and power loss grading in terms of W, C_L, C_D and ϕ.

§3. In this section squares and higher powers of the interference velocity ratio are neglected. This is probably not a serious limitation since it is very doubtful whether the original assumptions hold beyond the first order in the interference velocities. Explicit equations are given for (dP_1/dr), (dQ/dr), (dP_2/dr) to the first order.

§4. The equations of §3 are applied to the particular case of equal rotational speed and equal power input. Explicit equations are given for the partial efficiency at a given radius and for the induced loss for front and back airscrews separately.

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* Coefficients of the type t_c, p_{c1}, p_{c2} are plotted against $r_c^2 = (r/R)^2$.

§5. This section gives first order results for given blade angles and also the first order difference of blade angle between front and back airscrews for the case of equal angular velocity and power input. This completes the formulae necessary to obtain the numerical results given in the present note.

§6. Values of W, (dQ/dr) and (dT/dr) are given to the second order for known values of C_L. Difference of C_L between the two airscrews is determined to the second order for equal revolutions and power input. The resulting value of the difference between the thrust power and torque power checks with the first order estimation of power loss in §3.

§7. In this section equations are obtained analogous to those on which the charts of R. & M. 1849[2] are based.

§8. In this section equations analogous to those of R. & M. 1674[1] are developed which could be used in the absence of charts to calculate the exact performance of an airscrew on the basis of assumption (i)–(iv).

LIST OF SYMBOLS

a	Reciprocal of slope of lift curve. Equation (40).
b	Equation (23).
B suffix	For "back airscrew".
c	Blade chord.
C suffix	Mean value for contra-rotating pair of airscrews.
C, D	Equation (71).
C_L, C_D	Lift and drag coefficients of blade element.
dD	Drag of blade element.
F suffix	For "front airscrew".
$J = \pi V/R\Omega$.	
dL	Lift of blade element.
N	Number of blades of either component.
dP_1	Induced power loss for blade elements.
dP_2	Drag power loss for blade elements.
dQ	Torque on blade elements.
r, R	Radius of blade element, tip radius.
s	Solidity $(= Nc/2\pi r)$ of either component.
S suffix	For single airscrew.
dT	Thrust on blade elements.
u, v	Components of interference velocity of front airscrew on back airscrew or vice versa (Fig. 1).
V	Forward speed (Fig. 1).
W	Resultant velocity at blade element (Fig. 1).
W_0	Fig. 1.

List of Symbols—*continued.*

w_1	Interference velocity of either airscrew on itself (equation (1)).
$\bar{w}_1$	Equation (2).
y, z suffices	Denoting either front airscrew and back airscrew respectively, or vice versa (equation (17)).
0 suffices	Indicating limiting value for zero lift.
α	Blade incidence.
β	Fig. 1 ; equation (9).
γ	Fig. 1 ; equations (7) and (8).
ε	Zero lift angle ; equation (39).
ζ, ζ_0	Equations (70), (18).
η	Efficiency.
θ	Blade angle ; equation (39).
Θ	Equation (39).
$\varkappa$	Tip loss factor ; equation (1). $\varkappa_0$ is written for $\varkappa(\phi_0)$.
λ_{yz}	Equation (22).
$\left.\begin{matrix}\mu, \mu_0\\ \nu, \nu_0\end{matrix}\right\}$	Equations (69), (17).
ξ, ξ_0	Equations (70), (18).
ϕ, ϕ_0	Fig. 1. Equations (9), (10).
Ω	Angular velocity in radians per second.

Note. $O(\gamma^2)$. The notation used in equation (19) *etc.* The statement

$$F(\gamma) = F_1(\gamma) + O(\gamma^n)$$

implies that $F(\gamma)$ can be expanded in powers of γ in the form

$$F(\gamma) = f_0 + \gamma f_1 + \gamma^2 f_2 + \ldots$$

and that

$$F_1(\gamma) = f_0 + \gamma f_1 + \gamma^2 f_2 + \ldots + \gamma^{n-1} f_{n-1} \; .$$

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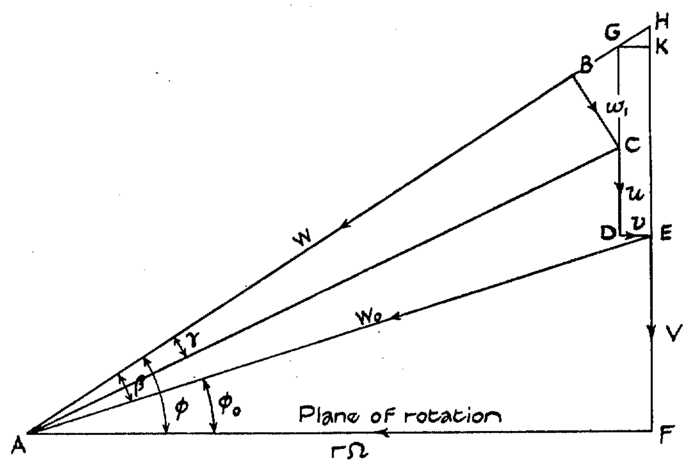

FIG. 1 (*a*). Either Airscrew.

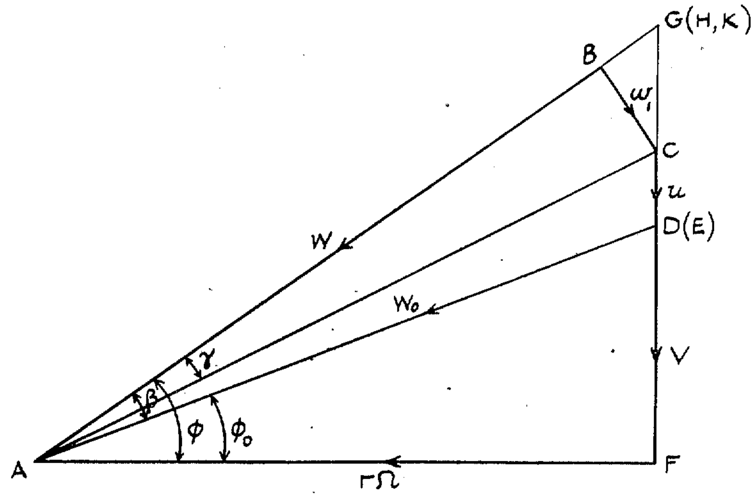

FIG. 1 (*b*). Front Airscrew.

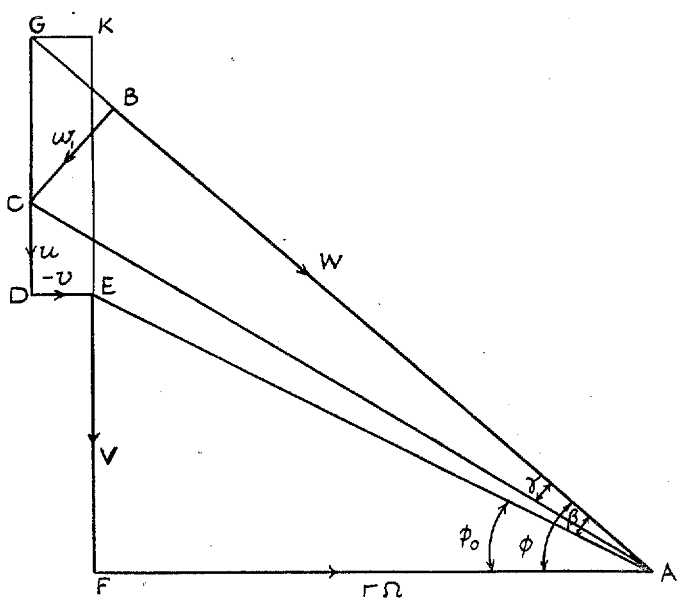

FIG. 1 (*c*). Back Airscrew.

FIG. 1. Interference Velocity Components for a Contra-rotating Pair of Airscrews.

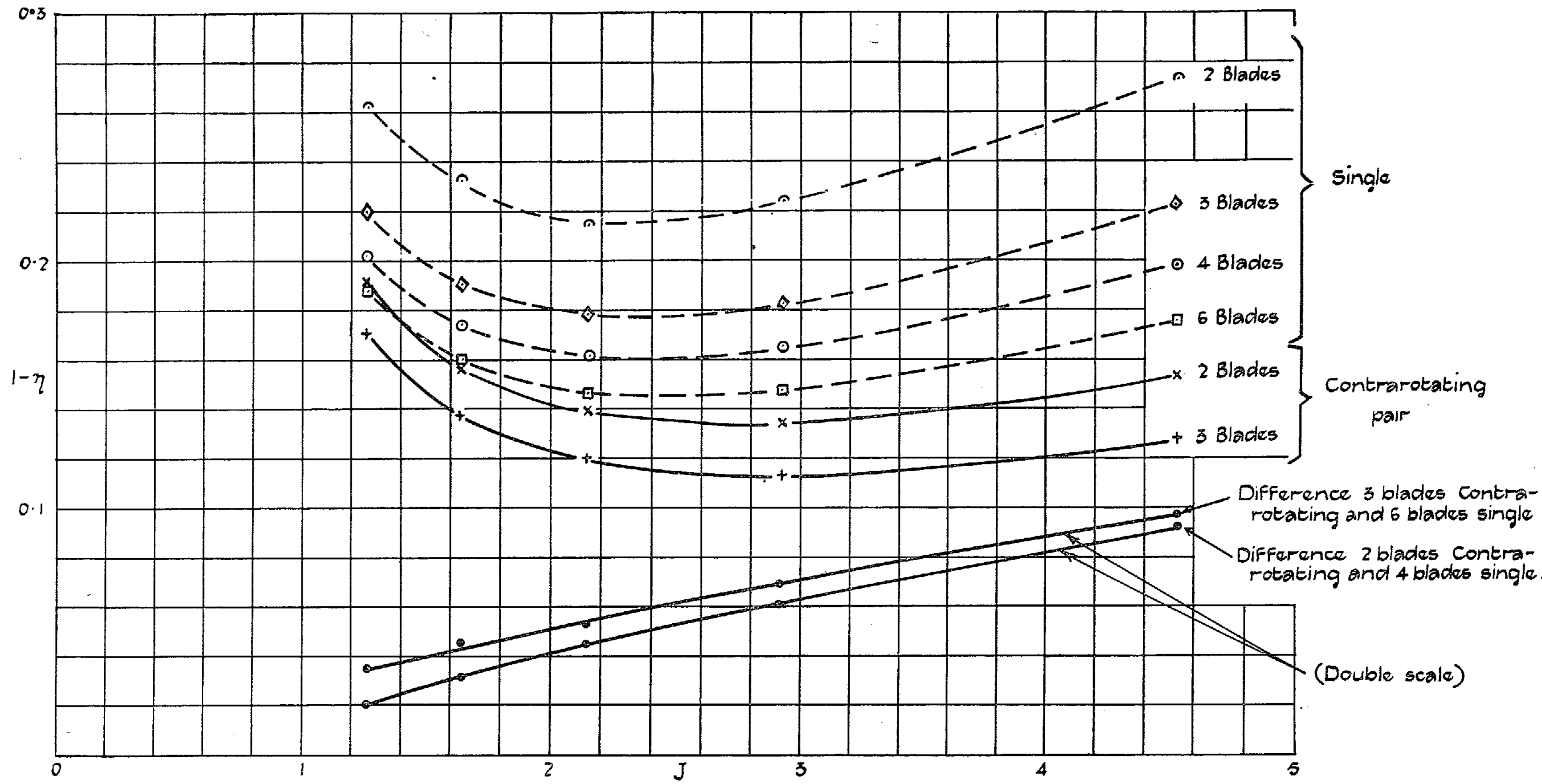

FIG. 2. Ideal Loss of Efficiency at Standard Radius 0·7, Plotted against J. All curves calculated for same values of s (0·09), C_L (0·56), C_D (0·017) (total solidity 0·18).

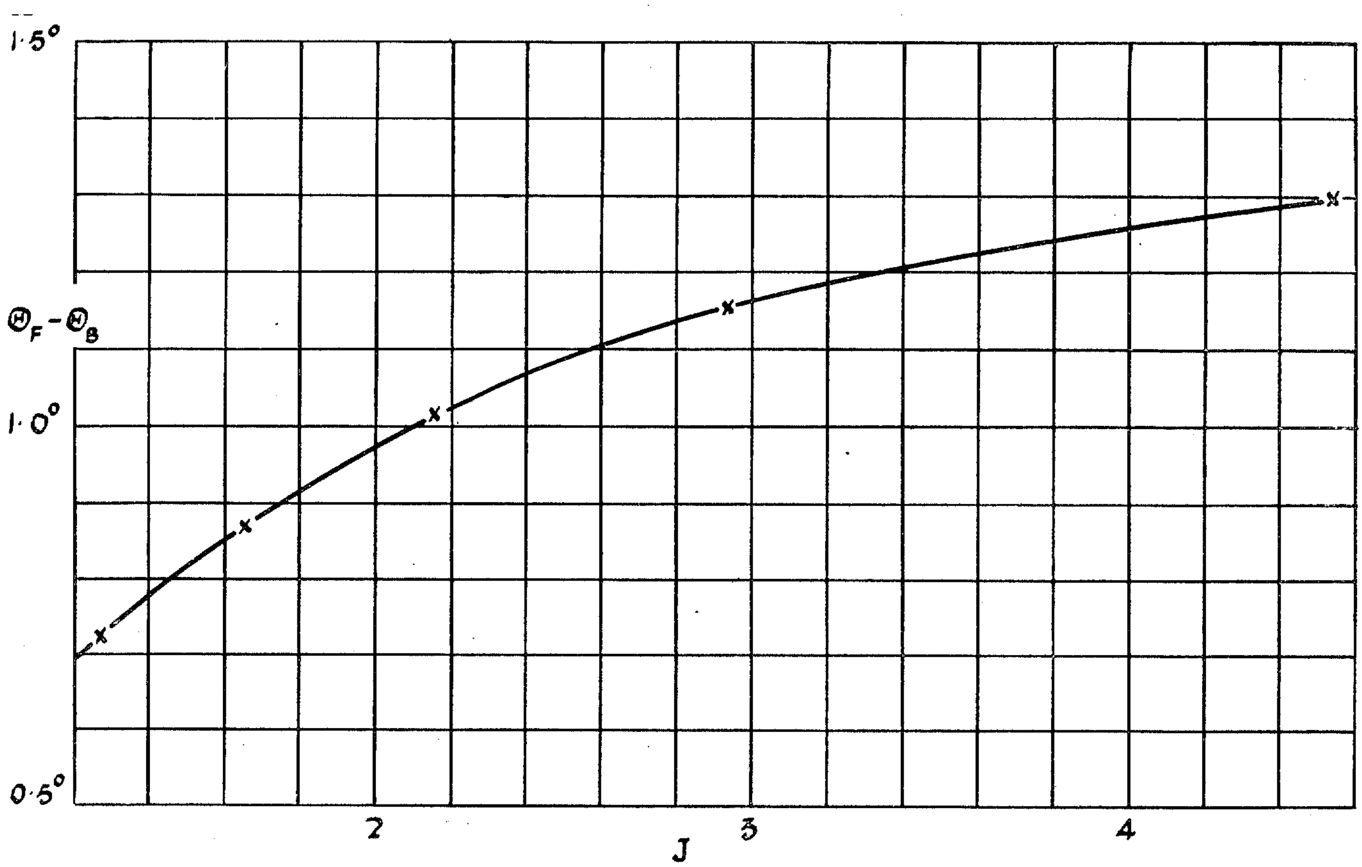

FIG. 3. Values of $(\Theta_F - \Theta_B)$ (independent of number of blades) Calculated for Same Conditions as Fig. 2 (for Equal Rotational Speed). The Curve also Represents the Difference of Blade Angles $(\theta_F - \theta_B)$ provided that the Zero Lift Angles are the Same for Both Airscrews.

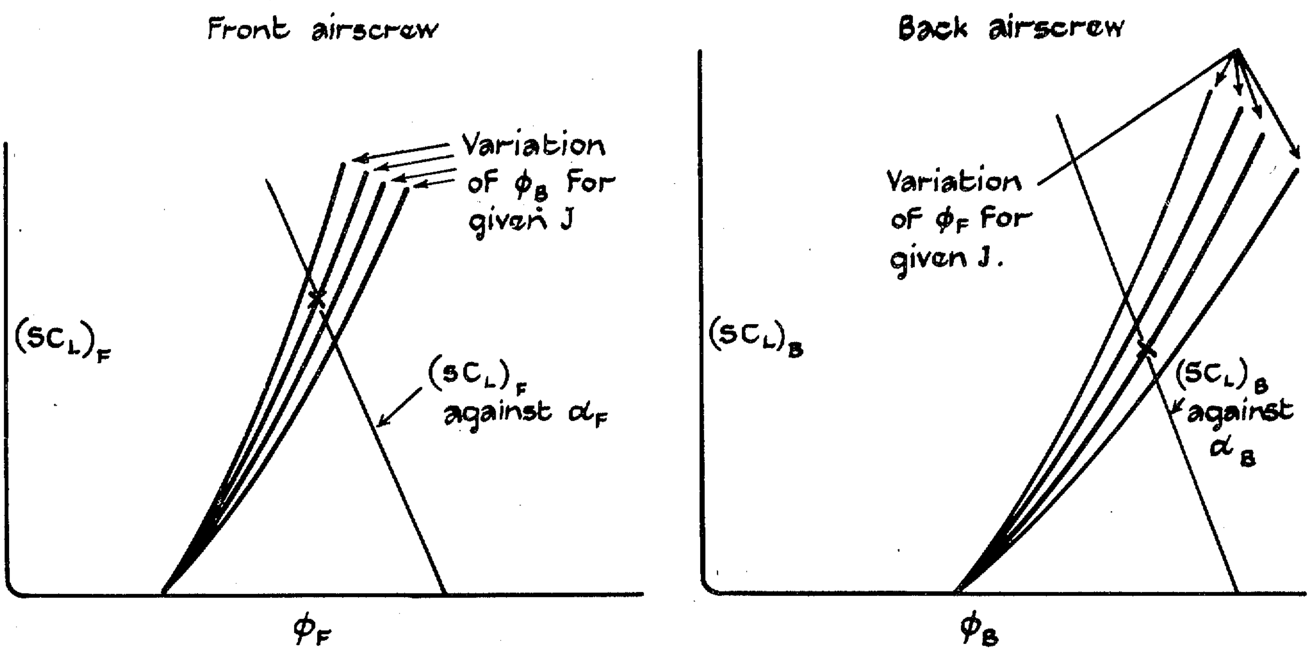

FIG. 4. Charts Analogous to Chart I of R. & M. 1849[2].

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S.O. Code No. 23-2084