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By

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Flutter calculations on a supersonic aircraft wing

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Ll. T. Niblett, B.Sc.Tech.

SUMMARY

Flutter calculations have been made on a straight-tapered unswept wing of low-aspect-ratio both with and without wing engines to discover the stiffness necessary to avoid flutter up to a Mach number of 2 and to find whether wing engines can be used to massbalance the wing effectively. Simple arbitrary modes of flexure and torsion were assumed and calculations were made using subsonic, transonic and supersonic derivatives.

It has been found that the stiffness required to prevent flutter of the bare wing is not excessive and that wing engines have a powerful massbalancing effect if placed forward on the wing. The transonic case is the most critical, but as the transonic derivatives used were two-dimensional this conclusion must be regarded as tentative.

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1 Introduction

Unswept wings have been proposed for aircraft designed to achieve a Mach number of 2. In this report the flutter stability of a typical unswept wing at subsonic, transonic and supersonic speeds is investigated theoretically to find (a) the structural stiffness necessary to prevent wing flutter, and (b) the amount of relief given by suitable location of the engines if these are carried on the wing. Wing flutter only is considered as control-surface flutter depends to a greater extent on the detailed shapes of the modes and less drastic modifications are needed to eradicate it.

The arbitrary modes chosen for the bare wing case are parabolic flexure and linear torsion about the flexural axis from root to tip. Additional modes of parabolic flexure and linear torsion about the flexural axis outboard of the engine centre-lines are included in the wing-with-engines case. In both cases the wing is taken to be encastré at the root. The aerodynamic derivatives used are taken from different sources and involve approximations, an extreme case being the use of the transonic derivatives given by linear two-dimensional theory for a relatively low-aspect-ratio wing. Owing to lack of data, no account is taken of the aerodynamic effects of the engines and their nacelles, the wing being considered as unbroken in planform.

The results for the bare wing are given in the form of the necessary wing stiffnesses to avoid flutter, whilst estimated stiffnesses were used in the wing-with-engines case and the variations of flutter speed with chordwise position of the engines are given.

2 General data for calculations

The half wing considered is shown in Fig.1. The mean chord is 10 feet, the taper ratio (r) is $\frac{1}{2}$ and the aspect ratio (A) of the whole wing is 2.8. The wing section of a supersonic aircraft is likely to be symmetric both about the chord and about a vertical line through the mid-chord point, and the inertia axis has accordingly been assumed at the mid-chord. Since the trailing-edge controls will cover more of the chord than the leading-edge controls the flexural axis has been taken to be ahead of the mid-chord line, at 45% chord. The sectional mass of the wing is taken to vary as the square of the local chord and the radius of gyration about the inertia axis is taken to be a quarter of the local chord. The wing engines are assumed to be concentrated masses, possessing moments of inertia in pitch, at mid semi-span. The inertia coefficients are obtained by spanwise integration. The structural stiffness coefficients are represented by overall stiffnesses ℓ_ϕ and m_θ based on the tip section.

3 Bare wing

For the case of no concentrated masses, such as engines, on the wing, the flutter characteristics are determined on a binary basis. The assumed modes are parabolic flexure and linear torsion of the wing about its flexural axis. The flutter equations were solved with the aid of desk calculating machines.

3.1 Subsonic flutter

The aerodynamic force and moment coefficients are estimated on the basis of strip theory and equivalent constant strip derivatives, the spanwise integration being carried out analytically. The derivatives

are evaluated according to the recommendations for low-aspect-ratio wings given by Minhinnick[1]. Estimated steady motion values are assumed for the stiffness derivatives. The damping derivatives are assumed the same as the equivalent stiffness derivatives where possible (e.g. ζ_z assumed to be ζ_α); otherwise they are obtained by a comparison of the three-dimensional steady-motion derivatives and the turning point values of the two-dimensional damping and stiffness derivatives. The inertia derivatives are given their two-dimensional values. All these derivatives are independent of frequency parameter. The steady-motion derivatives used are estimates for a Mach number of 0.9.

The flutter equations define the relationship between the flexural and torsional stiffnesses for flutter at a Mach number of 0.9. The critical stiffnesses at sea level are given in Fig.2 and the curve is of the usual subsonic type. The critical stiffnesses at any height are proportional to the dynamic head at that height and can be obtained from Fig.2 by altering the scale. It is obvious that flight at sea level will call for the largest stiffnesses.

3.2 Supersonic flutter

The possibility of flutter is investigated for three values of Mach number - 1.4, 1.8 and 2.5 - at heights of sea level, 10,000 feet and 25,000 feet. The derivatives for the two lower Mach numbers are taken from a report by Acum[2] which gives derivatives for rectangular low-aspect-ratio wings performing pitching oscillations at Mach numbers of 1.2, 1.4, 1.6, 1.8 and 2.0. The aspect ratio of the wing is too high for the derivatives for a Mach number of 1.2 to be applicable, and for a Mach number of 2.0 Acum's derivatives were not evaluated for sufficiently low frequency parameters. The derivatives for a rectangular wing having the same mean chord and aspect ratio as the wing under consideration are used as constant strip derivatives.

The derivatives for a Mach number of 2.5 are taken from tables[3] of Schwarz's two-dimensional derivatives. Approximations to the lift distributions over the tip and root of the wing where the flow will not be two-dimensional are obtained from a report by Watkins[4]. This gives the lift distribution over a rectangular wing of aspect ratio 4 performing pitching oscillations at a low frequency parameter in a supersonic stream. For the wing of Fig.1 the force distribution over the tip, up to the intersection of the Mach line from the tip leading edge with the trailing edge, is taken to be the same as that for Watkins' wing, and the derivative distribution over the inboard mixed region is taken to be the mean of the tip distribution and the constant two-dimensional value. The loss of aerodynamic force at the tip will be comparatively greater for a parabolic-flexure mode than for a linear mode and it is for this reason that the force distributions over the tip region, and not the derivative distributions are taken to be similar for the two wings. If the derivative distributions are assumed similar the forces on the tip will be overestimated. Little is known about the lift distribution in the inboard mixed region but the assumptions made should not be too seriously in error. The structural data used are the same as those used in the subsonic case.

The flutter equations are solved by assuming a frequency parameter and determining the flexural and torsional stiffnesses for the critical flutter condition. Sets of derivatives for two frequency parameters are used, the assumed frequency parameters being 0.4 and 0.6 for the Acum derivatives and 0.336 and 0.672 for the Schwarz derivatives. The results are given in Figs.3 and 4. It was found that the critical stiffnesses were very insensitive to frequency parameter, and the curves

of Figs.3 and 4 make no differentiation in this respect. It will be seen that the curves for the different Mach numbers and heights are very similar in shape and show that for practical stiffnesses the torsional stiffness necessary to avoid flutter is greater the greater the flexural stiffness. This is a well known feature, and it arises from the fact that increasing the flexural stiffness brings the natural frequencies closer together and increases the effect of any couplings that are present. The uncoupled flexural and torsional frequencies are coincident when the flexural stiffness (ℓ_ϕ) is twenty times the torsional stiffness (m_θ), and it will be seen that wings with this stiffness ratio lie practically in the middle of the unstable region in every case. As expected, the flutter problem becomes easier at high altitudes due to the reduction in the dynamic head of the airflow at a given Mach number. In most cases slightly less stiffness is needed as the Mach number increases and the flutter requirements are severest at the lower Mach numbers. This agrees with the conclusion of Green and Peattie[5] who have investigated theoretically the roll-torsion flutter of low-aspect-ratio rectangular missile wings.

4 Wings with engines

Wing engines are likely to give lower modal frequencies, especially for the overtone modes, and possible flutter modes cannot in general be represented closely enough by two degrees of freedom. For the wing with engines, therefore, modes of parabolic flexure and linear twist about the flexural axis of the wing outboard of the engine centre-lines are included in addition to the modes used in the bare wing case. Estimates of the associated stiffnesses of a wing with a thickness/chord ratio of 3% were made and these stiffnesses are used throughout the calculations.

The quaternary flutter equations were solved on the R.A.E. flutter simulator[6] for critical flutter speed and frequency with different positions of the engine centres-of-gravity.

The estimated flexural and torsional stiffnesses of the whole wing measured at the wing tip in the semi-rigid modes of the binary calculation are 2.74 and 1.21 lb ft 10^6 respectively, in the semi-rigid modes of the quaternary calculations are 2.74 and 0.91 lb ft 10^6 and for concentrated loads at the tip are 2.74 and 0.76 lb ft 10^6. The flexural and torsional stiffnesses of the outer wing alone are 2.66 and 1.21 lb ft 10^6 in the semi-rigid modes and 2.66 and 0.99 lb ft 10^6 for concentrated loads at the tip.

4.1 Subsonic flutter

The aerodynamic derivatives used are those previously used in the bare-wing case and no account is taken of the flow through and over the engine nacelles, which are ignored in the evaluation of the aerodynamic coefficients. The curve of sea-level flutter speed against chordwise position of the engine is given in Fig.5.

Flutter is shown over an extensive speed range but the derivatives are for a Mach number of 0.9 and, strictly, the curve is accurate only in the region of 600 knots. The results at other speeds have some practical significance, however. The elements of the flutter determinant can be expressed in the form $-\omega^2 A + i\omega VB + V^2C + E$ where ω and V are the frequency and airspeed, A, B, C and E are coefficients and in particular E is the structural stiffness coefficient. If ω and V are such that the determinant is equal to zero (corresponding to a critical flutter condition) the determinant will also be zero for $k\omega$ and kV when the structural stiffness coefficient is k^2E. The frequency parameter $\left(\nu = \frac{\omega c}{V}\right)$, on which the coefficients B and C depend, will remain the same. Thus 1000 ft/sec for

one wing is equivalent to 500 ft/sec for the same wing with all the structural stiffnesses quartered. With the stiffnesses assumed it is necessary to have the engine centre-of-gravity forward of 40% chord to avoid flutter. If this is achieved it would seem from the steeply rising curve that the wing stiffness could be reduced quite considerably without any ill effects on the flutter stability.

The results show some agreement with low-speed wind-tunnel tests by Gaukroger[7] in respect of the massbalancing effect of the localised mass over the c.g. range of Fig.5. He also obtained an overtone flutter, for forward centres of gravity of the mass, which was essentially flexure-torsion flutter of the wing outboard of the mass. If the effect of the lower aspect ratio is ignored, Fig.2 shows that, for the estimated stiffnesses (Section 4), the outboard wing is well clear of binary flutter. The lower aspect ratio of the outboard part of the wing would probably make for even greater stability.

The rapid disappearance of flutter is due to the massbalancing of the fundamental wing modes by the engines when they are forward on the wing. The effect is probably accentuated by there being a near coincidence of the fundamental frequencies when the engines are in this area.

4.2 Transonic flutter

Unfortunately no derivatives were available for low-aspect-ratio wings in transonic flow and the derivatives used are those given by Jordan[8], which are based on linear two-dimensional theory. Figure 6 shows the results that were obtained for a height of 25,000 feet. The flutter curve has two branches and the range of speeds covered is great. In fact the vertical extent of the curves is such that the only way of achieving stability is by careful location of the engines. For the stiffnesses assumed the engine centres-of-gravity have to be between 20% and 45% of the chord. The extent of the stable region remains nearly constant with variation of the stiffness but the range itself moves rearward as the stiffnesses are increased. The left-hand branch has not been investigated fully but it is probably due to the forward position of the engine inducing a normal mode which has negative aerodynamic damping. A reasonable amount of structural damping was included when the equations were solved on the simulator.

4.3 Supersonic flutter

The aerodynamic coefficients used are those used in the bare-wing case and no account is taken of the airflow through and over the engine nacelles, which are ignored in the evaluation of the aerodynamic coefficients. The curves of flutter speed at 25,000 feet against chordwise position of the engine are given in Fig.6 for Mach numbers of 1.4 and 1.8.

The flutter curve has one branch at rearward positions of the engine centres-of-gravity. It has much the same shape as the equivalent curves in the transonic and subsonic cases and shows a large increase in flutter speed when the engine centres-of-gravity are near to the flexural axis. Again the curves are only correct near one speed in each case with the assumed stiffnesses, but the variation of the conditions for flutter stability with aircraft stiffness can be estimated as before. The curves near the points where the speeds are identical with the assumed Mach numbers are nearly vertical and agree well with the transonic result. If all the stiffnesses of the aircraft are increased in the same proportion

the furthest rearward position of the engine for flutter stability moves aft and eventually a condition can be reached in which the stiffness is sufficient to prevent flutter with the engine in any position. Such a stiffness however is not likely to be countenanced on weight grounds and correct location of the engines must be relied on for the prevention of flutter. For the assumed stiffnesses stability is achieved if the engine centres-of-gravity are forward of 45% chord. The wing stiffnesses can then be decreased without incurring much further limitation of the chordwise position of the engines.

It might be noticed that the assumed frequency parameter for the M = 1.4 case was 0.6 whilst that for the M = 1.8 case was 0.2. A value of 0.2 is more appropriate for the M = 1.4 case but the pertinent aerodyanmic coefficients led to oscillations of an ill-defined character on the simulator. The M = 1.8 case was done for both frequency parameters and the results differed little from each other. It is reasonable to assume that the same insensitivity to assumed frequency parameter will hold at a Mach number of 1.4.

5 Discussion

5.1 Assumptions

The value of the results is limited by the number of assumptions that have been made, especially in evaluating the aerodynamic forces on the wing. Such assumptions are inevitable until more data are obtained on wings in three-dimensional flow, but until they are obtained as much information as possible must be gleaned from calculations of this kind.

The structure of a wing with a thickness/chord ratio of 3% is likely to be comparatively simple, and the assumption of simple modes of vibration should be better than it is for conventional wings. The flexibility of the wing-nacelle joints will affect the modes but the amount present will depend on their design and is difficult to estimate. Chordwise distortion of the wing will probably be present but as yet there is no evidence of its effect either from calculations or experience with aircraft already flying. It will probably be most severe on the inboard wing which does not play a very great part in flutter. Ignoring the nacelles aerodynamically should not introduce a large error; steady-motion results suggest that whilst the nacelle centres of pressure will probably be ahead of those for the wing the forces on the nacelles will be smaller than those on the wing, and these two effects will tend to cancel each other.

5.2 Results

The results of the calculations show that flutter stability of a supersonic ($M \approx 2$) aircraft wing carrying no concentrated masses should be achieved with reasonable stiffnesses. Considering the supersonic results the stiffnesses show a tendency to increase as the Mach number is reduced, bringing the transonic range into a position of importance, but unfortunately no adequate aerodynamic derivatives exists with which to investigate this range effectively. The evidence available from rocket tests suggests there is no sudden change in flutter stability at sonic speeds. In the wing-with-engines case the transonic derivatives used give results, for flutter with rearward engine positions, that compare well with those from supersonic derivatives.

For the wing-with-engines case the flutter stability depends on the chordwise position of the engine. With favourable location of the engine flutter will be avoided with quite low stiffnesses, and divergence and

aileron reversal will set the lower limit to the wing stiffness required. Limitations on engine position due to overall c.g. requirements do not appear likely to conflict seriously with these flutter requirements. The forward limit set to the engine position in the transonic case might prove troublesome; it is considered that this feature requires further consideration, particularly in view of the aerodynamic assumptions used in the present investigation.

6 Conclusions

A limited theoretical investigation of the flutter stability of an unswept aircraft wing of low aspect ratio both with and without wing engines has been made up to Mach numbers in the region of 2. It has been found that the stiffness required to prevent flutter of the bare wing is not excessively large and that wing engines can have a powerful massbalancing effect. The transonic case appears to be the most critical, but as the transonic derivatives used were two-dimensional this conclusion must be regarded as tentative.

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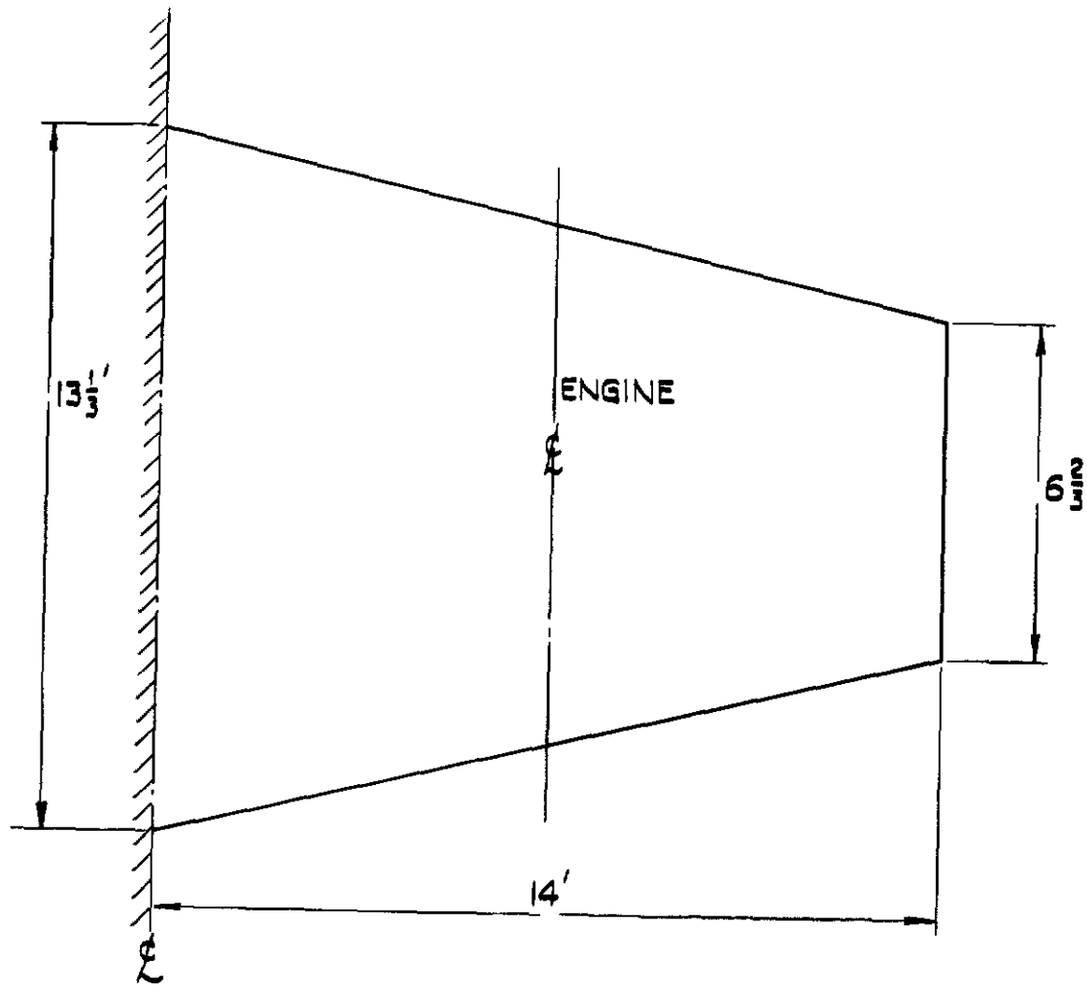


FIG. 1. PLAN VIEW OF HALF WING.

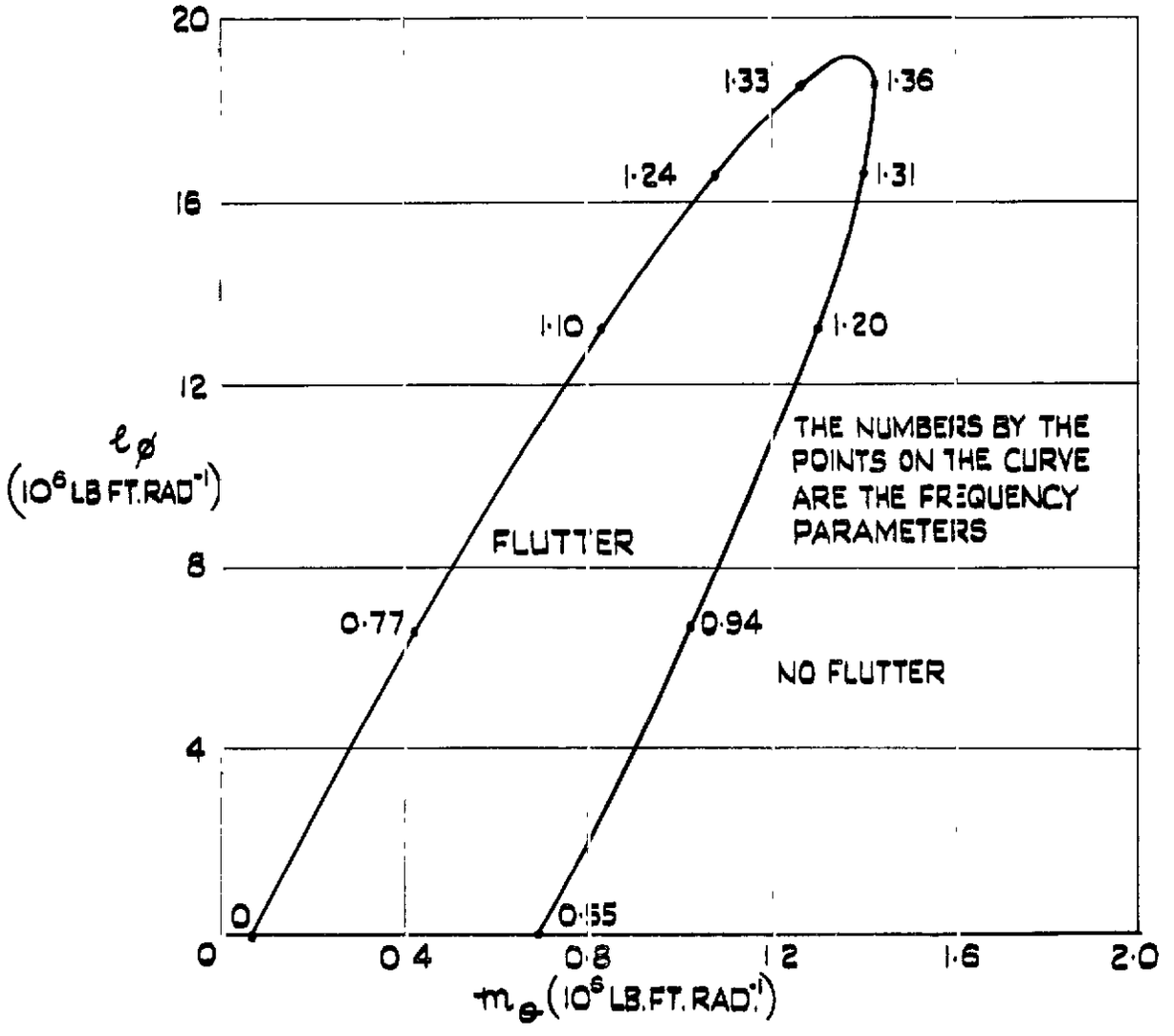
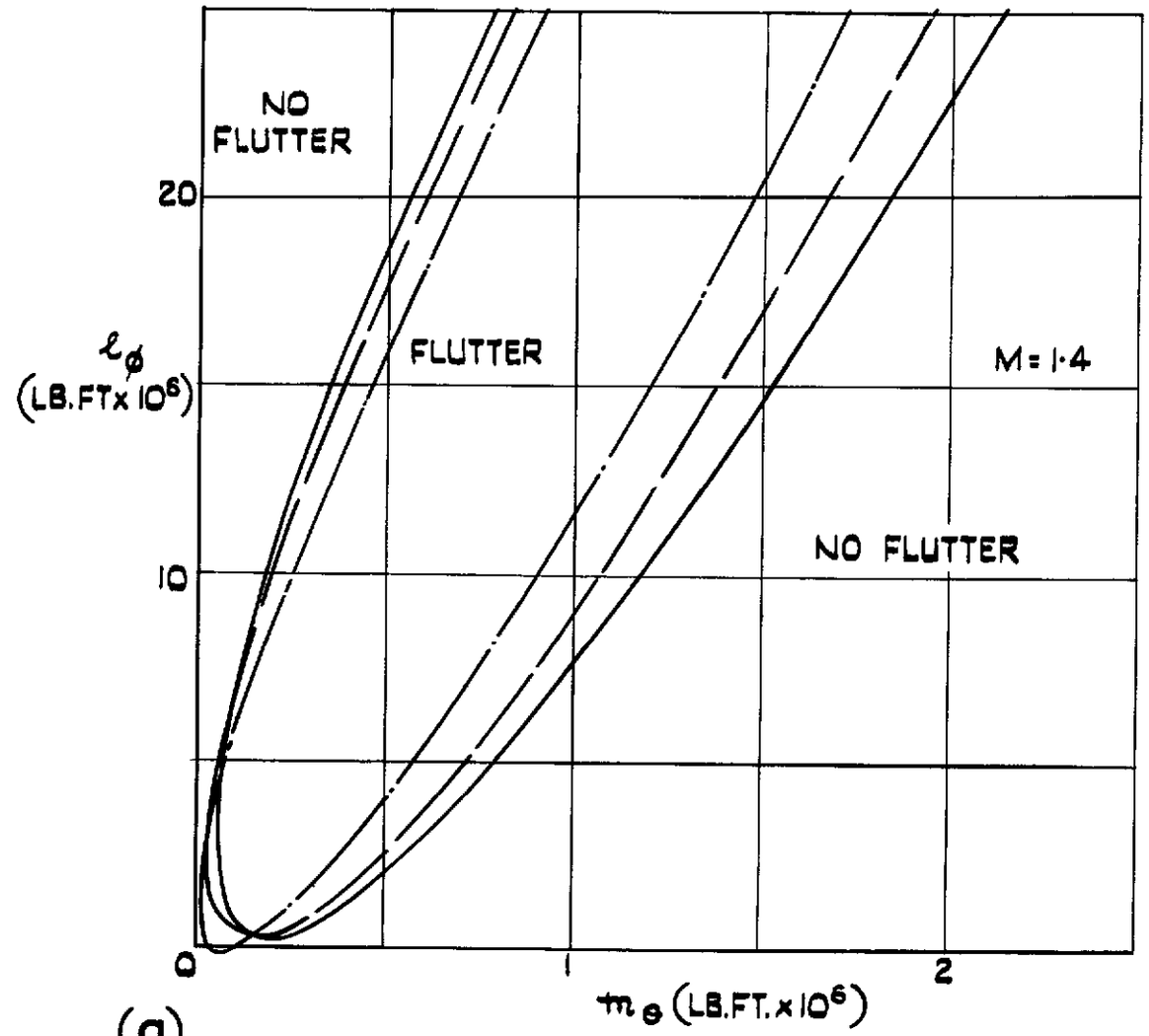
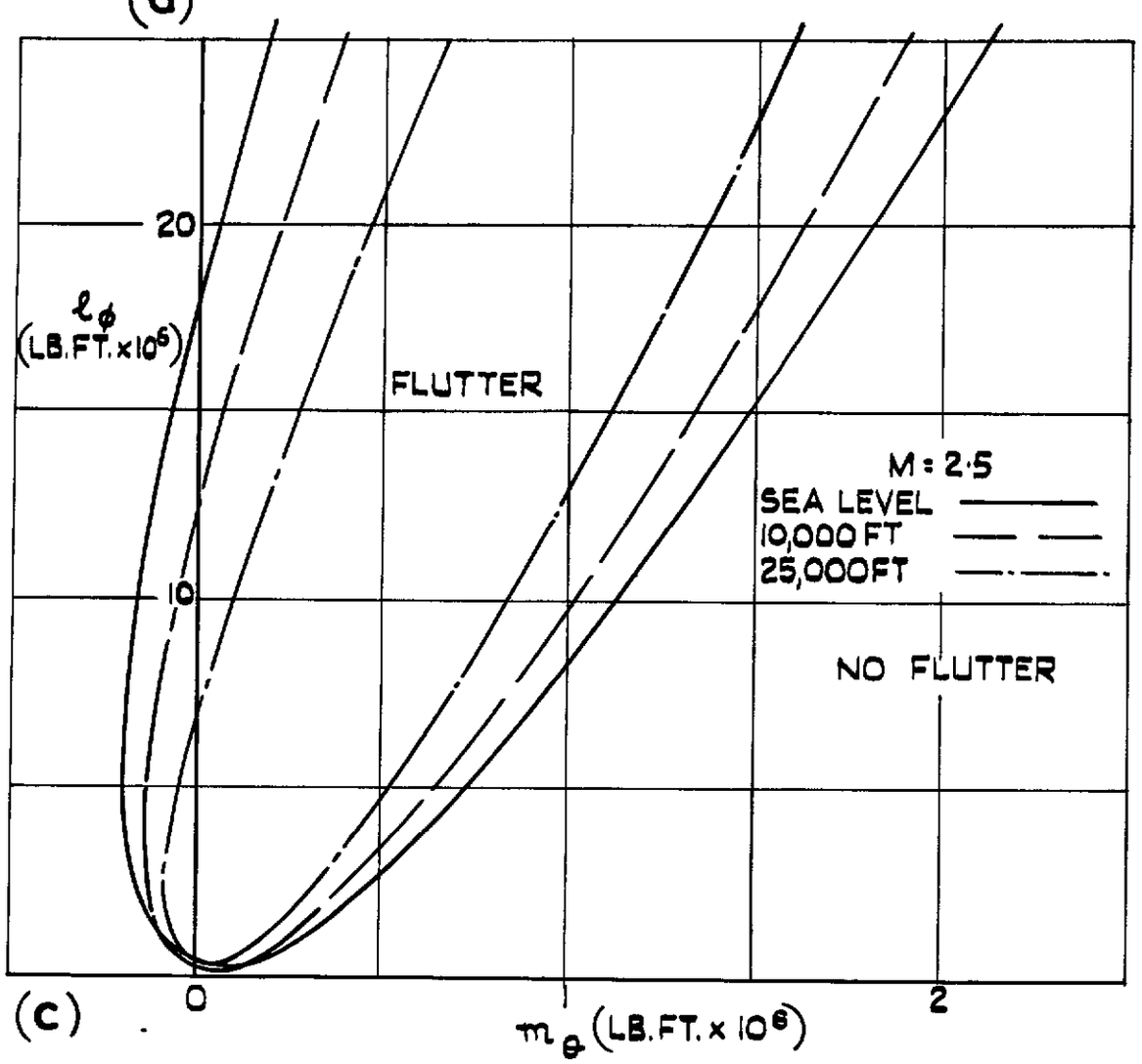


FIG. 2. CRITICAL WING STIFFNESSES AT TIP SECTION - BARE WING, M = 0·9, SEA LEVEL.



(a)



(c)

FIG. 3 (a & c) CRITICAL WING STIFFNESSES AT TIP SECTION - BARE WING - EFFECT OF HEIGHT.

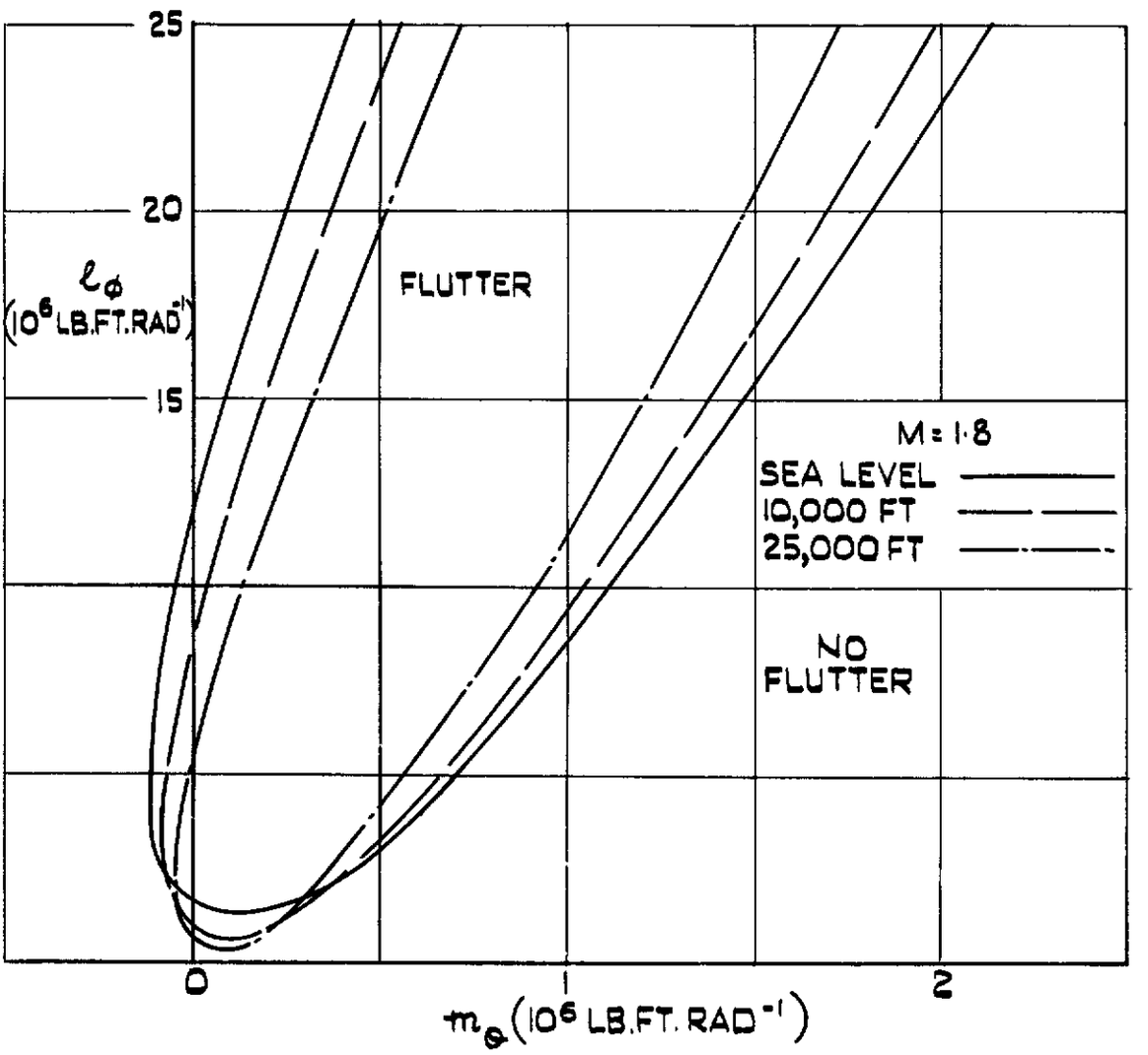


FIG. 3(b) CRITICAL WING STIFFNESSES AT TIP SECTION - BARE WING - EFFECT OF HEIGHT.

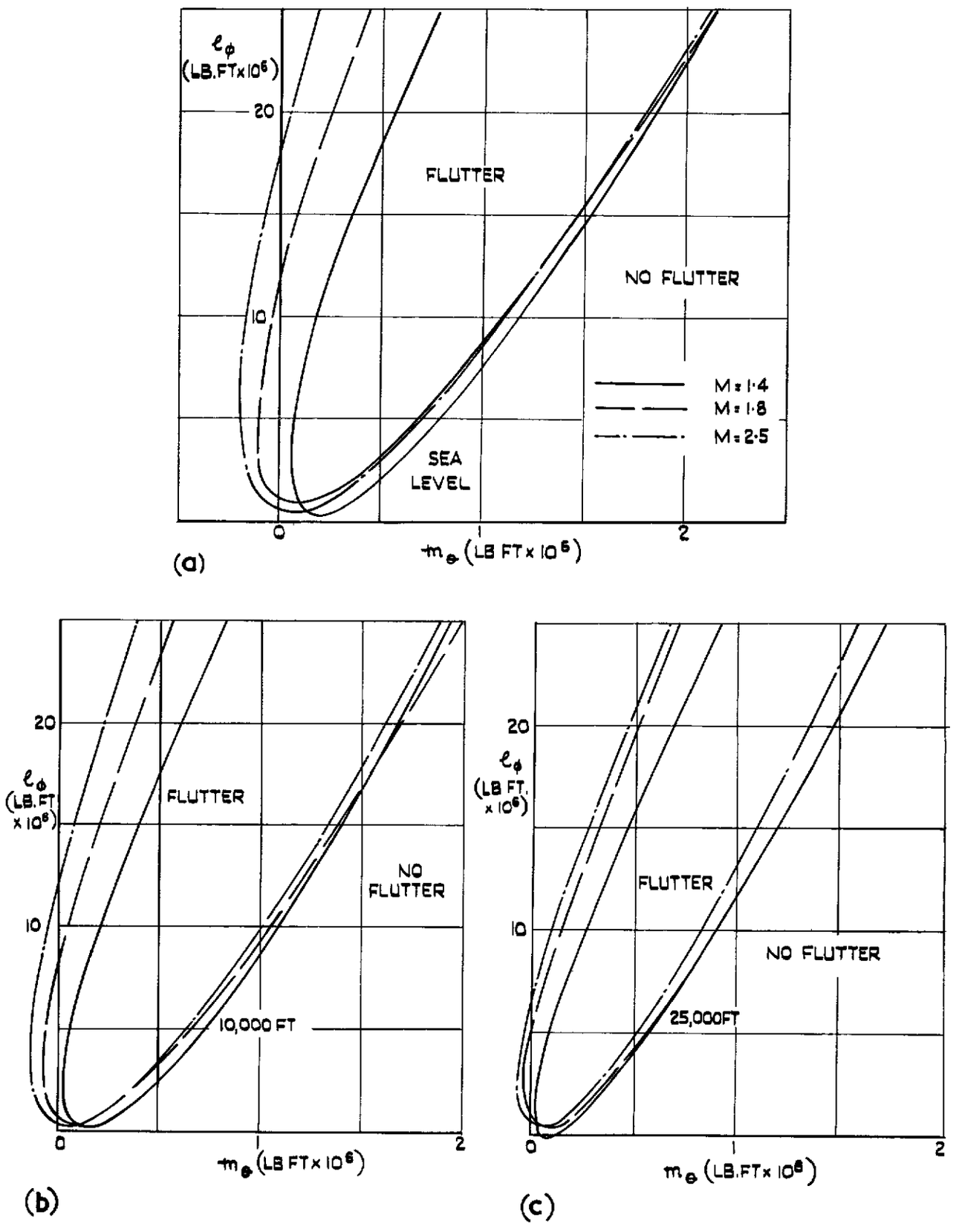


FIG. 4. (a - c) CRITICAL WING STIFFNESSES AT TIP SECTION - BARE WING - EFFECT OF MACH NUMBER.

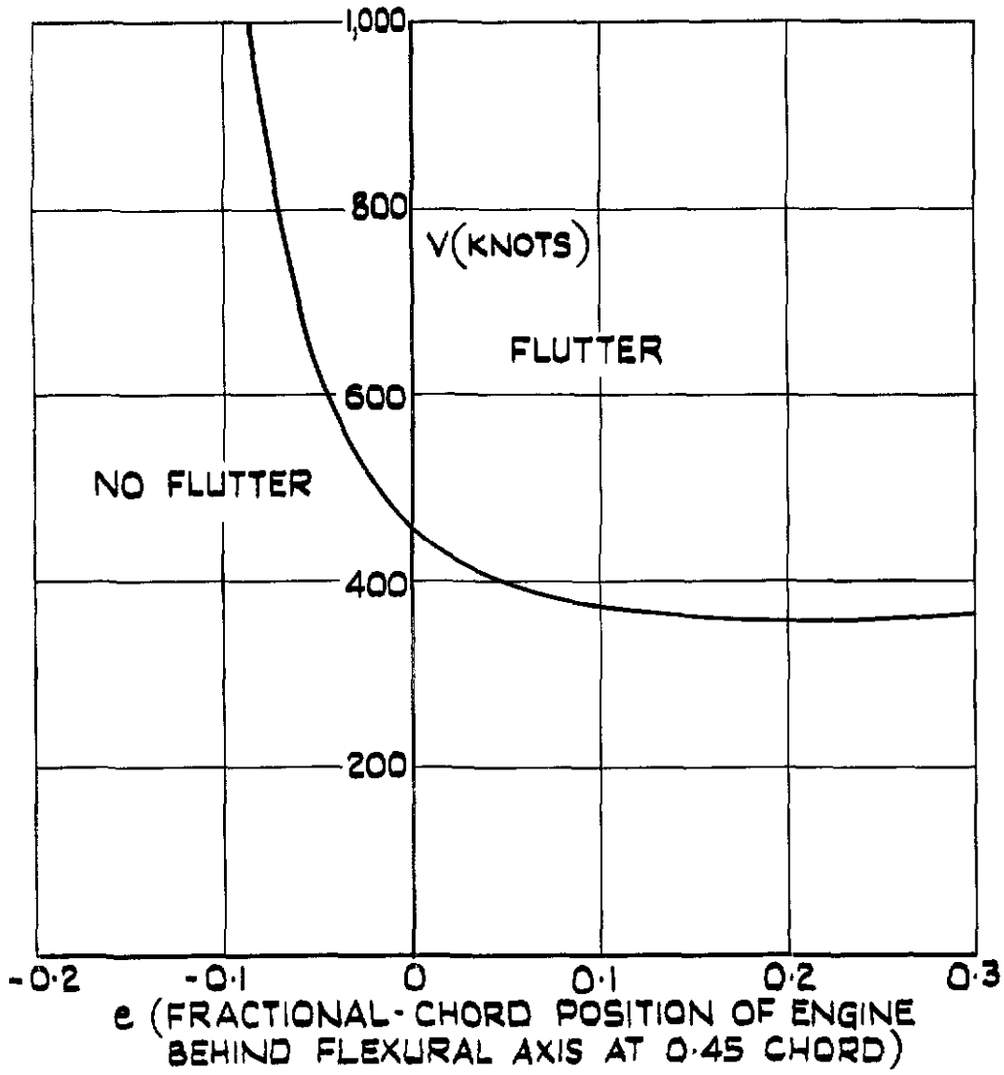


FIG. 5. CRITICAL SPEED v. WING ENGINE POSITION - M = 0·9 - SEA LEVEL.

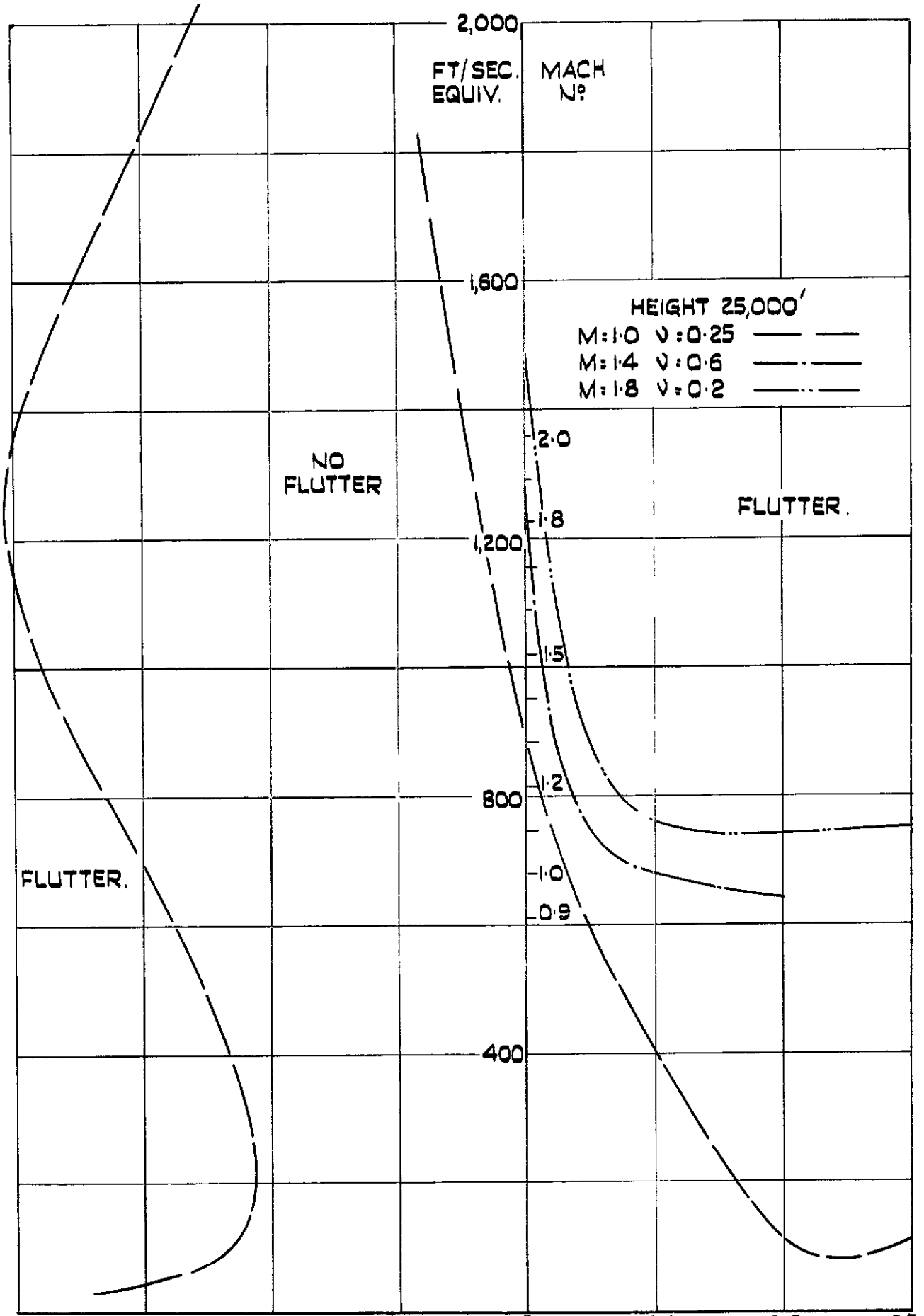


FIG. 6. CRITICAL SPEED v. WING - ENGINE POSITION - M = 1·0, 1·4, 1·8.

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