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Aerodynamic Coefficients for an Oscillating Delta Wing

By

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Summary.—A delta wing of aspect ratio 3 is assumed to be describing pitching and vertical translational oscillations of small amplitude in incompressible inviscid flow. The corresponding aerodynamic coefficients are calculated by the method of Ref. 1 for values 0·26 and 0·8 of the mean frequency parameter $\omega_m (\equiv pc_m/V)$, and certain of the coefficients are also obtained for $\omega_m \to 0$. The results are given in Table 3 and are plotted in Figs. 2a to 2h.

There is little experimental evidence available for comparison, apart from some measurements of the pitching-moment damping coefficient at low values of ω_m which are included in Fig. 2h.

From comparisons of calculated values with experimental results for the delta wing, and other plan-forms[2], it appears that the method[1] provides a reasonable basis for calculating flutter derivatives, but as indicated in section 5 of this note it is not satisfactory for estimating all the stability derivatives. For the calculation of the latter, a modified version of the method has been developed[3], which is relatively easy to use and which should give reasonable accuracy.

1. *Introduction.*—In this note, the routine method of calculating flutter derivatives outlined in Ref. 1 is developed in more detail, and applied to a delta wing of aspect ratio 3 and 90 deg apex angle describing pitching and translational oscillations. Measurements of the pitching-moment damping derivative for low values of ω_m have been made for this wing at the National Physical Laboratory and the Royal Aircraft Establishment at both low and high speeds, and the results for low speeds are given for comparison.

By using tables* of the downwash W_e due to a semi-infinite doublet strip of strength $s_1 e^{-ipX/V}$, and the chordwise factors $L_0'(k)$, $k = 1, 2 \ldots 6$ included in this note, flutter derivatives can be computed without much difficulty. For oscillations of lower frequency such as occur in stability problems, an alternative method suggested by W. P. Jones has been used by the writer to calculate stability derivatives[3]. This method avoids the use of the complex factors $L_0'(k)$ and the two-dimensional oscillatory lift function $C(\omega')$ involved in the present method. As $\omega \to 0$, the function $C \to 1 + \frac{1}{2}i\omega(\gamma + \log_e \frac{1}{4}\omega)$, and it is thought that the $\log_e \omega$ term may introduce errors into the calculation when ω_m is very small, or when the spanwise variation of ω due to a high taper ratio is taken into account. Although the derivatives $l_{\dot{\alpha}}$ and $-m_{\dot{\alpha}}$ become infinite according to two-dimensional theory when $\omega \to 0$, it is probable that the corresponding coefficients for a wing of finite span will tend to finite values as $\omega_m \to 0$. The experimental value of $-m_{\dot{\alpha}}$ quoted here appears to support this view. Hence the present method should not be used to calculate stability derivatives, but it appears to be satisfactory for estimating flutter derivatives for wings of normal aspect ratio, since the variation in the $\log_e \omega$ term is not so rapid for the higher values of ω.

Compressibility effects are not considered in this note, but the calculation of flutter derivatives for a wing at high Mach number can be related to a similar calculation for a wing of reduced

* *See* section 3.

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aspect ratio in incompressible flow[4]. For example, the delta wing of aspect ratio 3 at $M = 0{\cdot}8$ reduces to a wing of aspect ratio $3\sqrt{(1 - M^2)} = 1{\cdot}8$ at $M = 0$. This approach therefore requires a method which can be applied to wings of very low aspect ratio. Derivative measurements on such wings are proposed to test the present theory and to aid further development of methods of derivative calculation.

2. *Theory.*—The flow induced by the oscillating wing is reproduced by a distribution of doublets of strength $K\,e^{ipt}$ over the wing and wake, where K represents the complex amplitude of the discontinuity in the velocity potential. When the wing oscillations are of small amplitude the doublets can be assumed to lie in the plane $z = 0$. As in Ref. 1, the distribution K is expressed in the form

$$K = cV\Big[(S_0' + S_0'')\sum_{m=1} C_{0m}A_m + \sum_{n=1} S_n \sum_{m=1} C_{nm}A_m\Big], \quad \ldots \quad (1)$$

where C_{nm} are arbitrary coefficients, and with $\xi \equiv -\cos\theta$ on the wing the chordwise distributions are defined by

$$\left.\begin{aligned} S_0' &= e^{-i\omega'\xi}\int_{-1}^{\xi}\Big\{\operatorname{cosec}\theta - [1 - C(\omega')]\cot\frac{\theta}{2}\Big\}e^{i\omega'\xi}\,d\xi && \ldots -1 \leqslant \xi \leqslant 1 \\ &= \pi\,e^{-i\omega'\xi}X_0(\omega') && \ldots\ \xi \geqslant 1 \\ S_0'' &= \sin\theta && \\ S_1 &= \tfrac{1}{2}(\sin\theta + \tfrac{1}{2}\sin 2\theta) && \\ S_n &= \tfrac{1}{2}\Big(\frac{\sin(n+1)\theta}{n+1} - \frac{\sin(n-1)\theta}{n-1}\Big) && \ldots\ n \geqslant 2 \end{aligned}\right\} \quad \ldots \quad (2)$$

where $\omega' = \frac{1}{2}\omega = pc/2V$. The spanwise distribution $cA_m \equiv s\eta^{m-1}\sqrt{(1-\eta^2)} \equiv sT_m$, and for symmetrical motion only odd values of m are required in (1). It is known that the amplitude W of the downwash induced at the point $(x_1, y_1, 0)$ by the doublet distribution $K\,e^{ipt}$ is given by

$$W = \frac{1}{4\pi}\iint K\,\frac{\partial^2}{\partial z_1^2}\Big(\frac{1}{r}\Big)\,dx\,dy\,,$$

where $r^2 = (x - x_1)^2 + (y - y_1)^2 + z_1^2$ and $z_1 \to 0$,

and this can be expressed in the form

$$W = V\Big[\sum_{m=1} C_{0m}(W_{0m}' + W_{0m}'') + \sum_{n=1}\sum_{m=1} C_{nm}W_{nm}\Big], \quad \ldots \quad (3)$$

where W_{nm} is the downwash induced by the doublet distribution $K_{nm} \equiv sS_nT_m$.

The tangential flow condition for oscillatory motion is

$$\frac{W}{V} = \frac{ipz'}{V} + \frac{\partial z'}{\partial x}, \quad \ldots \quad (4)$$

where $z'e^{ipt}$ is the normal downward displacement of any point (x, y) on the wing. When the downwash terms W_{nm} in (3) are known at a sufficient number of collocation points (x_1, y_1) on the wing, the arbitrary coefficients C_{nm} can be chosen to satisfy (4). Then K is given by (1), and as shown in Ref. 1, the corresponding lift distribution

$$\rho V\Gamma = \rho V^2\Big[(\Gamma_0' + \Gamma_0'')\sum_{m=1} C_{0m}A_m + \sum_{n=1}\Gamma_n\sum_{m=1} C_{nm}A_m\Big] \quad \ldots \quad (5)$$

where

$$\begin{aligned} \Gamma_0' &= 2\{\operatorname{cosec}\theta - [1 - C(\omega')]\cot\tfrac{1}{2}\theta\} \\ \Gamma_0'' &= 2\{\cot\tfrac{1}{2}\theta - \operatorname{cosec}\theta + i\omega'\sin\theta\} \\ \Gamma_1 &= -2\sin\theta + \cot\tfrac{1}{2}\theta + i\omega'(\sin\theta + \tfrac{1}{2}\sin 2\theta) \\ n \geqslant 2 \ \ldots\ \Gamma_n &= -2\sin n\theta + i\omega'\Big[\frac{\sin(n+1)\theta}{n+1} - \frac{\sin(n-1)\theta}{n-1}\Big]. \end{aligned}$$

For the present calculation the amplitude z' of the displacement at (x, y) due to translational and pitching motion of the wing is represented by

$$z' = c_m z + x\alpha \quad \ldots \quad (6)$$

where $c_m z$ and α are the amplitudes of the translational and angular displacements. It is also assumed that the K distribution can be represented to sufficient accuracy by

$$K = cV\left[(S_0' + S_0'')\sum_{m=1} C_{0m}A_m + S_1 \sum_{m=1} C_{1m}A_m\right] \quad (7)$$

with this expression limited to terms $m = 1, 3, 5$. The corresponding downwash distribution W is then given by (3), and this must satisfy condition (4), which, for the motion considered yields the relation

$$\frac{W}{V} = \frac{ip}{V}(c_m z + x\alpha) + \alpha . \quad \ldots \quad (8)$$

In order to determine the six unknown constants in (7) and the corresponding downwash expression, equation (8) is satisfied at the six collocation points indicated in Fig. 1.

3. *Calculation of the Downwash.*—If the doublet distribution K_{nm} is independent of the frequency, the downwash W_{nm} can be calculated approximately[1,2] by replacing the continuous distribution $K_{nm} = s.S_n(\theta).T_m(\eta)$, by a '126 Falkner lattice'[5] of rectangular vortices of constant strength $sL_n(k).T_m(\eta_1), k = 1, 2 \ldots 6$. The factors $L_n(k)$ are chosen on a two-dimensional theory basis as indicated in Ref. 1, and are tabulated in Table 1 for the distributions S_0'' and S_1. The downwash at a particular point due to a rectangular vortex of unit strength is equal to $F/4\pi s_1$, where values of F are tabulated in Ref. 6*, and the total downwash W_{nm} due to the lattice $sL_n(k).T_m(\eta_1)$ is obtained by summation.

The doublet distribution $K_{0m}' = s.S_0'(\theta).T_m(\eta)$ is dependent on the frequency and does not vanish in the wake. The downwash W_{0m}' is calculated approximately[1] by replacing K_0' by narrow doublet strips of width $2s_1 = 0 \cdot 1$ semispan and constant strength spanwise, which are in turn replaced by superimposed doublet strips of strength $sL_0'(k).T_m(\eta_1)e^{-ip(x-x_k)/V}$, $k=1, 2 \ldots 6$ extending downstream from $x = x_k$ to $x = \infty$. The factors $L_0'(k)$ which satisfy the two-dimensional theory conditions indicated in Ref. 1, are calculated for values of the frequency parameter ranging from $0 \cdot 12$ to $2 \cdot 40$. Approximate formulae for these factors are given in Table 2 so that values can easily be obtained for any ω in the range.

The downwash W_e, induced at points (X_1, Y_1) in the plane of a semi-infinite doublet strip of width $2s_1$ and strength $s_1 e^{-ipX/V}$, has already been tabulated (*see* Refs. 1 and 2). Retabulation of these values of W_e has been undertaken by the Mathematics Division, N.P.L., and tables are now available which are straightforward to use. Each table is for a particular value of a parameter $\bar{\omega} = ps_1/V$ in the range $0 \cdot 01$ to $0 \cdot 24$. For each value of the spanwise parameter $n_1 = |Y_1/s_1|$, corresponding to the spanwise lattice spacing i.e., $n_1 = 0, 2, 4 \ldots$ etc., $4\pi W_e$ tabulated against the chordwise parameter $t_1 = X_1/s_1$ at intervals of t_1 suitable for linear interpolation. Unfortunately the tables are extensive and cannot conveniently be reproduced with this note. Copies of the tables are available, (Ref. 7), for $\bar{\omega}$ equal to $0 \cdot 01$ $(0 \cdot 01)$ $0 \cdot 04$ $(0 \cdot 02)$ $0 \cdot 08$, $0 \cdot 09$, $0 \cdot 12$, $0 \cdot 16$, $0 \cdot 18$, $0 \cdot 24$. These values were chosen to cover the practical range of values of ω_m for wings of various aspect ratio.

By using the $\bar{\omega}$ table appropriate to the aspect ratio and mean frequency parameter value considered, the downwash W_{0m}' at any collocation point corresponding to the lattice of doublet strips $sL_0'(k).T_m(\eta_1).e^{-ip(x-x_k)/V}$, is obtained by summation.

4. *Aerodynamic Derivative Coefficients.*—When the lift distribution $\rho V\Gamma$ is known, the amplitudes of the lift L and the pitching moment M for the half wing are given by

* It should be noted that the positive $X \equiv x/s_1$ axis of these tables extends upstream, whereas the x-axis of present theory extends downstream.

$$L = \int_0^s \int_{x_l}^{x_t} \rho V \Gamma \, dx \, dy$$

$$M = - \int_0^s \int_{x_l}^{x_t} \rho V \Gamma (x - x_f) \, dx \, dy$$

where $x = x_f$ is the reference axis.

The aerodynamic derivative coefficients are then defined by the relations

$$\frac{L}{\rho s c_m V^2} = (l_z + i\omega_m l_{\dot{z}})z + (l_\alpha + i\omega_m l_{\dot{\alpha}})\alpha$$

$$\frac{M}{\rho s c_m{}^2 V^2} = (m_z + i\omega_m m_{\dot{z}})z + (m_\alpha + i\omega_m m_{\dot{\alpha}})\alpha$$

where the interia terms are included in the stiffness coefficients l_z, m_z, etc.

In the present calculation, the reference axis is taken as $x_f = 0$, and the derivatives $l_z{}'$, $m_z{}'$, etc. for any axis position hc_m behind $x_f = 0$, are then given by the usual transformation formulae

$$l_z{}' = l_z$$

$$l_\alpha{}' = l_\alpha - hl_z$$

$$-m_z{}' = -m_z - hl_z$$

$$-m_\alpha{}' = -m_\alpha - h[l_\alpha - m_z] + h^2 l_z$$

and so on.

5. *Results and Remarks on the Calculation.*—All the derivatives are given for $\omega_m = 0 \cdot 26$ and $0 \cdot 8$, for a reference axis at $0 \cdot 973 c_m$ ($= 0 \cdot 556\, c_0$) behind the apex of the wing; this corresponds to one axis of oscillation used in the N.P.L. experiments. The derivative values are tabulated in Table 3 and graphed in Figs. 2a to 2h. As indicated, two variations of the scheme of calculation have been used.

It should be noted that the factors $L_0{}'(k)$ in section 3 are dependent on the local frequency parameter ω, and this leads to difficulty in the case of a highly tapered wing, since ω becomes small towards the tip. It is not known whether allowance should be made for the large spanwise variation in ω by taking a set of factors $L_0{}'(k)$ appropriate to each spanwise position η_1 of the lattice, or whether it is sufficient to use the set $L_0{}'(k)_m$ appropriate to the mean frequency parameter ω_m. Furthermore, the lift distribution $\rho V \Gamma$, given by equation (5), includes the two-dimensional function $C(\omega')$ in the $\Gamma_0{}'$ term. To be consistent, $C(\omega')$ is considered variable* with ω in the solution in which the factors $L_0{}'(k)$ are taken variable, but $C(\omega_m{}')$ is used with the constant set $L_0{}'(k)_m$. The terms $\Gamma_0{}''$ and Γ_1 also involve ω but here ω is regarded as the local parameter and a function of the spanwise parameter η; for the present delta wing,

$$\omega = \omega_m \{ \tfrac{1}{4}(7 - 6|\eta|) \} .$$

Both methods were used for $\omega_m = 0 \cdot 8$, and allowing for the spanwise variation in ω seems to have little effect on the final results. For $\omega_m = 0 \cdot 26$ only the method using factors appropriate to ω_m was used.

An attempt was also made, to evaluate the derivatives as $\omega_m \to 0$. The downwash was calculated for steady flow conditions, using the factors $\lim_{\omega = 0} L_0{}'(k)$, and the tangential flow equation (8) was solved in the general matrix form $\{C_{nm}\} = [W_{nm}]^{-1} \{W/V\}$. The derivatives were then evaluated taking the lift function $C(\omega_m{}') = 1 - i\omega_m A$; but since

$$C(\omega') \to 1 + \tfrac{1}{2} i\omega(\gamma + \log_e \tfrac{1}{4}\omega) \text{ as } \omega \to 0 ,$$

no definite limiting value could be given to A. When arbitrary values were assigned to A, the derivatives $l_{\dot{\alpha}}$ and $m_{\dot{\alpha}}$ appeared to be particularly sensitive to the value chosen, and so the

* Values of $C(\omega')$ are tabulated in R. & M. 1958; using this method the spanwise integrations for lift and pitching moment are evaluated by integrating numerically at intervals $\eta = 0 \cdot 05$ across the span.

method proved unsatisfactory for these derivatives. The other derivatives are independent of the $i\omega_m A$ term, and the values are given in Table 3 and graphed in Fig. 2.

Values of $-m_{\dot{\alpha}}'$, which have been calculated using two-dimensional strip theory corrected for lift slope*, are also given in Fig. 2h, and these show the effect at low values of ω_m of the $\log_e\omega$ term introduced by the function $C(\omega')$. In the present theory this effect can be demonstrated by allowing for the variation of $C(\omega')$ in the lift distribution $\rho V\Gamma$ as ω varies across the span, but using the coefficients C_{nm} obtained by taking the constant set of factors $L_0'(k)_m$ appropriate to ω_m. As shown in Fig. 2h, the value $-m_{\dot{\alpha}}' = 0 \cdot 33$ obtained for $\omega_m = 0 \cdot 8$ remains approximately the same, but for $\omega_m = 0 \cdot 26$ the value of $-m_{\dot{\alpha}}'$ rises from $0 \cdot 34$ to $0 \cdot 41$. However, a modified version of the method, suggested by W. P. Jones for the calculation of stability derivatives, is described in detail in Ref. 3; some preliminary results obtained for this delta wing are included in Figs. 2d and 2h as a matter of interest.

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* Unpublished results supplied by W. E. A. Acum.

TABLE 1

Chordwise Factors of the Rectangular Vortices representing the Doublet Distribution S_0'' and S_1.

k	$L_0''(k)$	$L_1(k)$	Position of vortices on chord, from leading edge
1	$0 \cdot 20508.\pi$	$0 \cdot 17090.\pi$	$\frac{1}{12}$
2	$0 \cdot 06836.\pi$	$0 \cdot 01139.\pi$	$\frac{3}{12}$
3	$0 \cdot 01953.\pi$	$-0 \cdot 03581.\pi$	$\frac{5}{12}$
4	$-0 \cdot 01953.\pi$	$-0 \cdot 05534.\pi$	$\frac{7}{12}$
5	$-0 \cdot 06836.\pi$	$-0 \cdot 05697.\pi$	$\frac{9}{12}$
6	$-0 \cdot 20508.\pi$	$-0 \cdot 03418.\pi$	$\frac{11}{12}$

Note: As in '126 Falkner lattice solutions'[5] the lattice is reduced for values of of the spanwise parameter $n_1 = |Y_1/s_1| \geqslant 10$. The 6-step lattice is replaced by a 2-step lattice with the following factors

k	$L_0''(k)$	$L_1(k)$	Position of vortices on chord, from leading edge
1	$0 \cdot 25.\pi$	$0 \cdot 125.\pi$	$\frac{1}{4}$
2	$-0 \cdot 25.\pi$	$-0 \cdot 125.\pi$	$\frac{3}{4}$

TABLE 2

Chordwise Factors for the Lattice Representation of the Doublet Distribution S_0'

$\omega = pc/V$	$L_0'(k)$		
	$k = 1$	$k = 2$	$k = 3$
0	0·77313	0·42951	0·36816
0·12	0·61977 − i 0·20139	0·35945 − i 0·09180	0·32130 − i 0·06104
0·24	0·49772 − i 0·25342	0·30338 − i 0·11532	0·28366 − i 0·07640
0·36	0·40915 − i 0·26430	0·26248 − i 0·11990	0·25608 − i 0·07906
0·48	0·34425 − i 0·25935	0·23235 − i 0·11714	0·23567 − i 0·07677
0·60	0·29561 − i 0·24811	0·20962 − i 0·11144	0·22017 − i 0·07248
1·20	0·17050 − i 0·18151	0·14959 − i 0·07778	0·17801 − i 0·04746
1·80	0·12046 − i 0·13182	0·12323 − i 0·05189	0·15755 − i 0·02768
2·40	0·09379 − i 0·09690	0·10687 − i 0·03313	0·14303 − i 0·01287

$\omega = pc/V$	$L_0'(k)$		
	$k = 4$	$k = 5$	$k = 6$
0	0·36816	0·42951	0·77313
0·12	0·33458 − i 0·04336	0·40590 − i 0·03008	0·75874 − i 0·01855
0·24	0·30749 − i 0·05398	0·38673 − i 0·03712	0·74691 − i 0·02298
0·36	0·28755 − i 0·05546	0·37254 − i 0·03770	0·73807 − i 0·02342
0·48	0·27271 − i 0·05337	0·36191 − i 0·03573	0·73143 − i 0·02220
0·60	0·26135 − i 0·04983	0·35370 − i 0·03273	0·72629 − i 0·02032
1·20	0·22939 − i 0·02943	0·32972 − i 0·01560	0·71121 − i 0·00906
1·80	0·21219 − i 0·01275	0·31545 − i 0·00089	0·70239 + i 0·00142
2·40	0·19846 + i 0·00033	0·30289 + i 0·01148	0·69469 + i 0·01129

TABLE 2—*Continued*

For $0 \cdot 12 \leqslant \omega \leqslant 0 \cdot 60 \qquad \omega' = \omega/2$

$$L_0'(1) = (0{\cdot}78319 - 3{\cdot}11851\,\omega' + 6{\cdot}97044\,\omega'^2 - 6{\cdot}65892\,\omega'^3) - i\,(0{\cdot}09551 + 2{\cdot}32520\,\omega' - 10{\cdot}08716\,\omega'^2 + 13{\cdot}43736\,\omega'^3)$$
$$L_0'(2) = (0{\cdot}43426 - 1{\cdot}42569\,\omega' + 3{\cdot}17295\,\omega'^2 - 3{\cdot}00819\,\omega'^3) - i\,(0{\cdot}04350 + 1{\cdot}06281\,\omega' - 4{\cdot}63843\,\omega'^2 + 6{\cdot}16756\,\omega'^3)$$
$$L_0'(3) = (0{\cdot}37138 - 0{\cdot}95337\,\omega' + 2{\cdot}09388\,\omega'^2 - 1{\cdot}99762\,\omega'^3) - i\,(0{\cdot}02907 + 0{\cdot}70555\,\omega' - 3{\cdot}10710\,\omega'^2 + 4{\cdot}12463\,\omega'^3)$$
$$L_0'(4) = (0{\cdot}37052 - 0{\cdot}68348\,\omega' + 1{\cdot}49013\,\omega'^2 - 1{\cdot}42839\,\omega'^3) - i\,(0{\cdot}02079 + 0{\cdot}50040\,\omega' - 2{\cdot}23479\,\omega'^2 + 2{\cdot}96443\,\omega'^3)$$
$$L_0'(5) = (0{\cdot}43124 - 0{\cdot}48125\,\omega' + 1{\cdot}03909\,\omega'^2 - 1{\cdot}00431\,\omega'^3) - i\,(0{\cdot}01458 + 0{\cdot}34615\,\omega' - 1{\cdot}58024\,\omega'^2 + 2{\cdot}09332\,\omega'^3)$$
$$L_0'(6) = (0{\cdot}77427 - 0{\cdot}29403\,\omega' + 0{\cdot}61995\,\omega'^2 - 0{\cdot}60865\,\omega'^3) - i\,(0{\cdot}00886 + 0{\cdot}21575\,\omega' - 0{\cdot}97707\,\omega'^2 + 1{\cdot}28374\,\omega'^3)$$

For $0 \cdot 60 \leqslant \omega \leqslant 2 \cdot 40 \qquad \omega' = \omega/2$

$$L_0'(1) = (0{\cdot}54749 - 1{\cdot}10833\,\omega' + 0{\cdot}99144\,\omega'^2 - 0{\cdot}31926\,\omega'^3) - i\,(0{\cdot}32959 - 0{\cdot}29837\,\omega' + 0{\cdot}08711\,\omega'^2)$$
$$L_0'(2) = (0{\cdot}32699 - 0{\cdot}51310\,\omega' + 0{\cdot}45000\,\omega'^2 - 0{\cdot}14593\,\omega'^3) - i\,(0{\cdot}15239 - 0{\cdot}14903\,\omega' + 0{\cdot}04133\,\omega'^2)$$
$$L_0'(3) = (0{\cdot}29979 - 0{\cdot}34533\,\omega' + 0{\cdot}29567\,\omega'^2 - 0{\cdot}09741\,\omega'^3) - i\,(0{\cdot}10255 - 0{\cdot}10873\,\omega' + 0{\cdot}02833\,\omega'^2)$$
$$L_0'(4) = (0{\cdot}31936 - 0{\cdot}24933\,\omega' + 0{\cdot}20744\,\omega'^2 - 0{\cdot}06963\,\omega'^3) - i\,(0{\cdot}07328 - 0{\cdot}08463\,\omega' + 0{\cdot}01944\,\omega'^2)$$
$$L_0'(5) = (0{\cdot}39539 - 0{\cdot}17737\,\omega' + 0{\cdot}14278\,\omega'^2 - 0{\cdot}04926\,\omega'^3) - i\,(0{\cdot}05222 - 0{\cdot}06893\,\omega' + 0{\cdot}01322\,\omega'^2)$$
$$L_0'(6) = (0{\cdot}75277 - 0{\cdot}11297\,\omega' + 0{\cdot}09189\,\omega'^2 - 0{\cdot}03148\,\omega'^3) - i\,(0{\cdot}03223 - 0{\cdot}04090\,\omega' + 0{\cdot}00389\,\omega'^2)$$

TABLE 3

Derivative Coefficients for an Oscillating Delta Wing

(Reference Axis at $0.556c_0$ behind Apex)

Solution	ω_m	l_z'	$l_{\dot{z}}'$	l_α'	$l_{\dot{\alpha}}'$	$-m_z'$	$-m_{\dot{z}}'$	$-m_\alpha'$	$-m_{\dot{\alpha}}'$	Description of Solution *see* section 5
1	0	0	1·539	1·539	—	0	−0·084	−0·084	—	Limiting values as $\omega \to 0$
2	0·26	−0·019	1·480	1·489	1·055	−0·008	−0·084	−0·090	0·341	Factors $L_0'(k)$ and lift function $C(\omega/2)$ constant at values for ω_m
3	0·8	−0·306	1·315	1·324	1·183	−0·071	−0·090	−0·139	0·333	Factors $L_0'(k)$ and $C(\omega/2)$ constant at values for ω_m
4	0·8	−0·316	1·346	1·347	1·211	−0·068	−0·076	−0·125	0·333	Factors $L_0'(k)$ and $C(\omega/2)$ variable with ω across the wing span

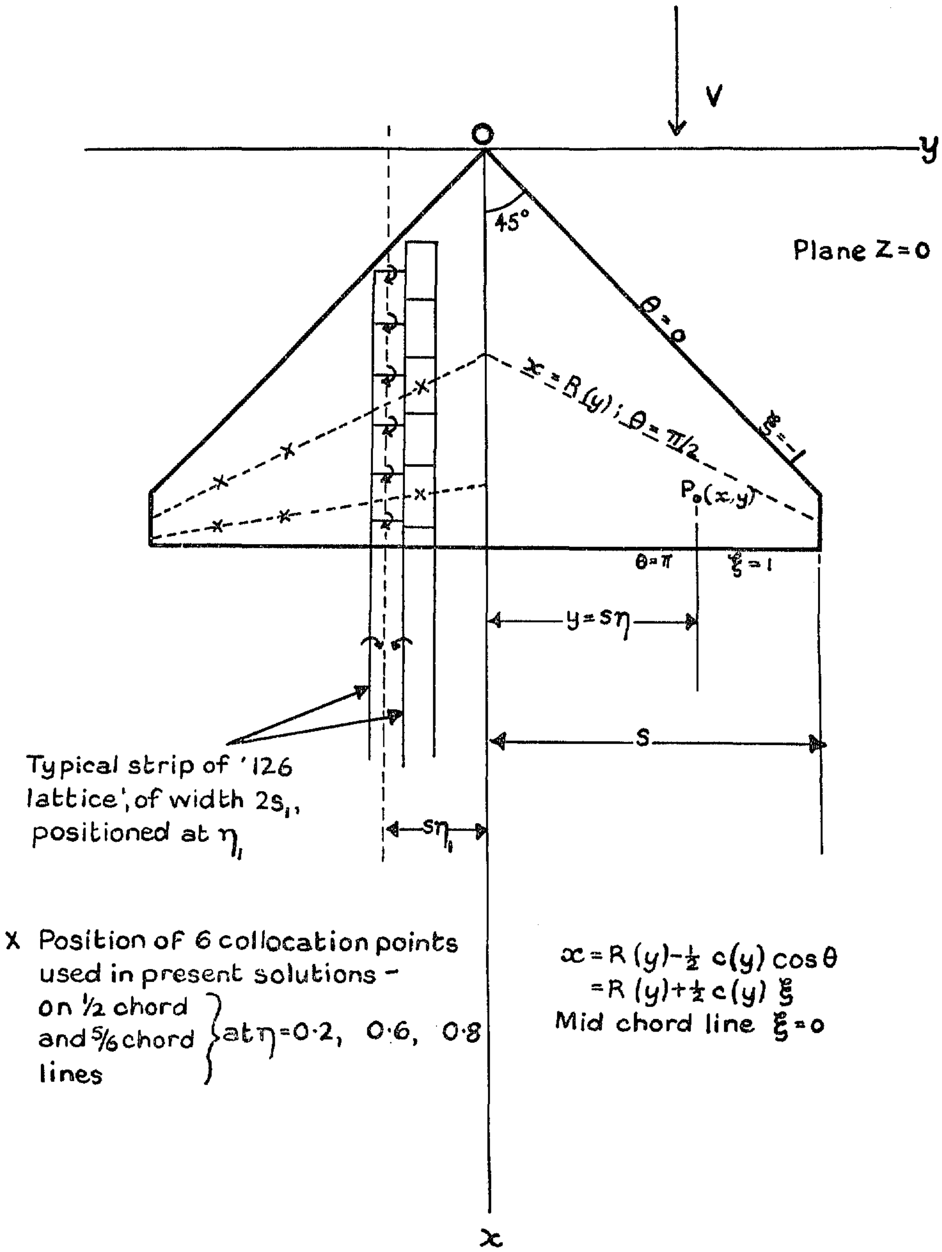
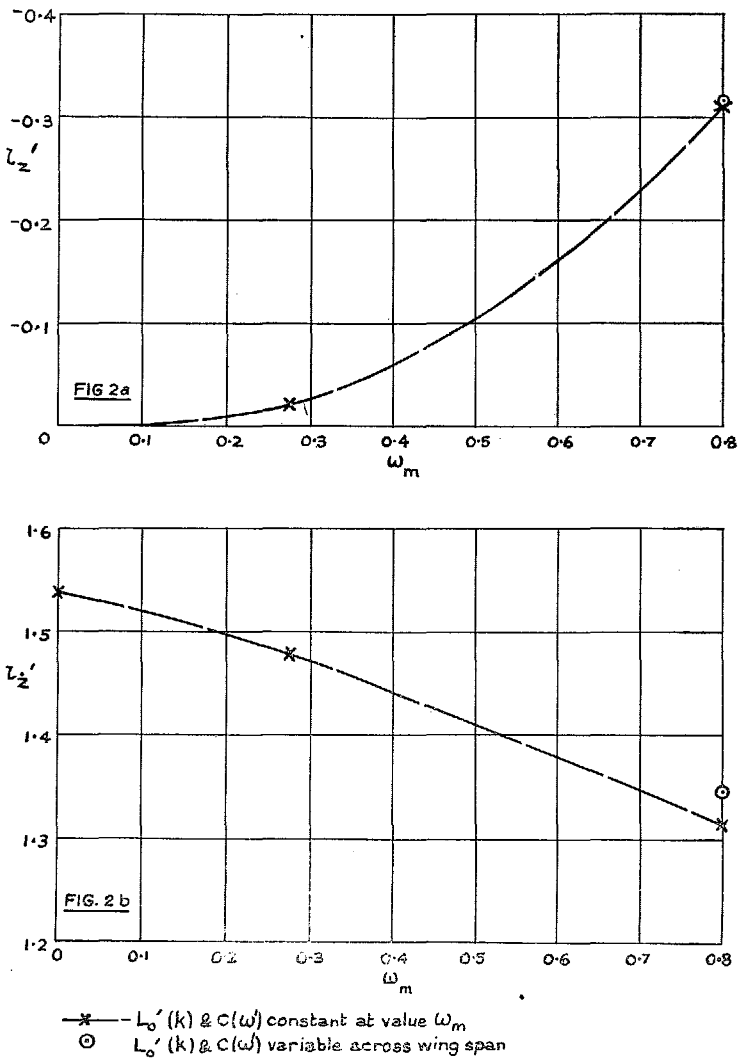
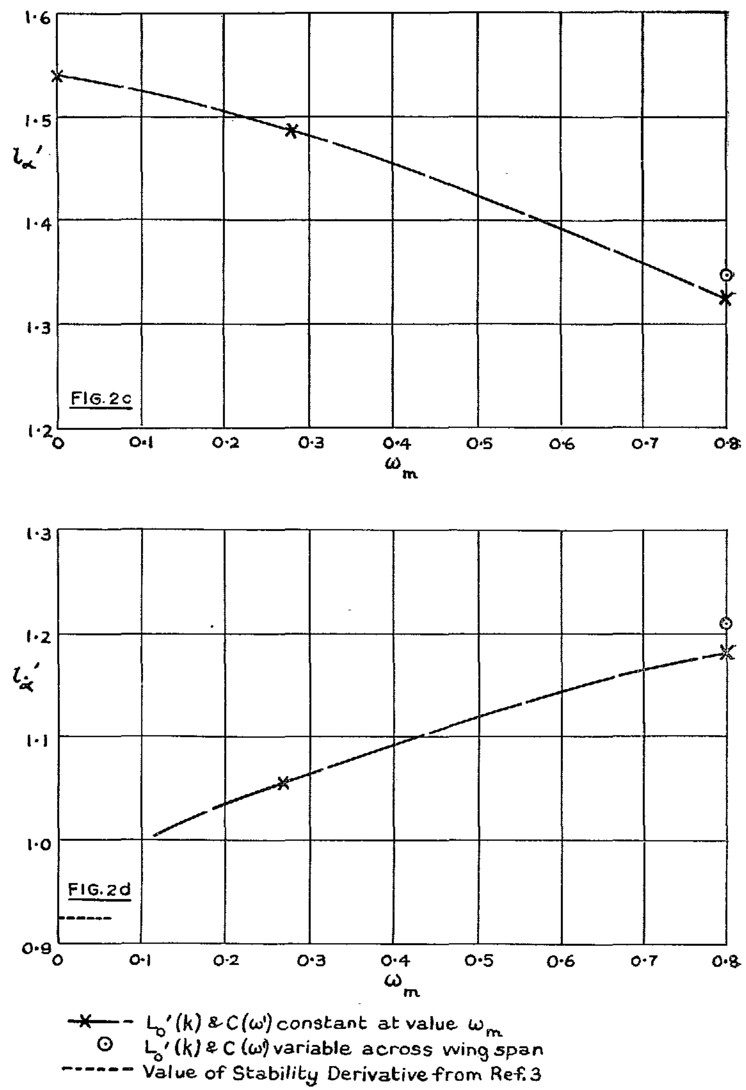


FIG. 1. Plan-form of wing.



FIGS. 2a and 2b. Derivative Coefficients for an oscillating delta wing. (Reference axis at $0 \cdot 556c_0$ behind apex.)



FIGS. 2c and 2d. Derivative coefficients for an oscillating delta wing. (Reference axis at $0 \cdot 556c_0$ behind apex.)

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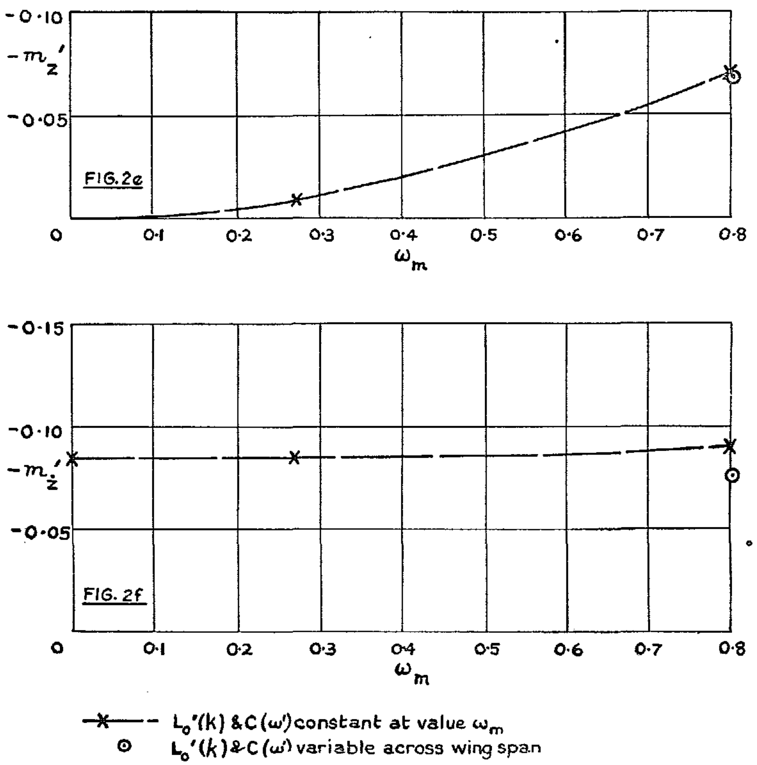
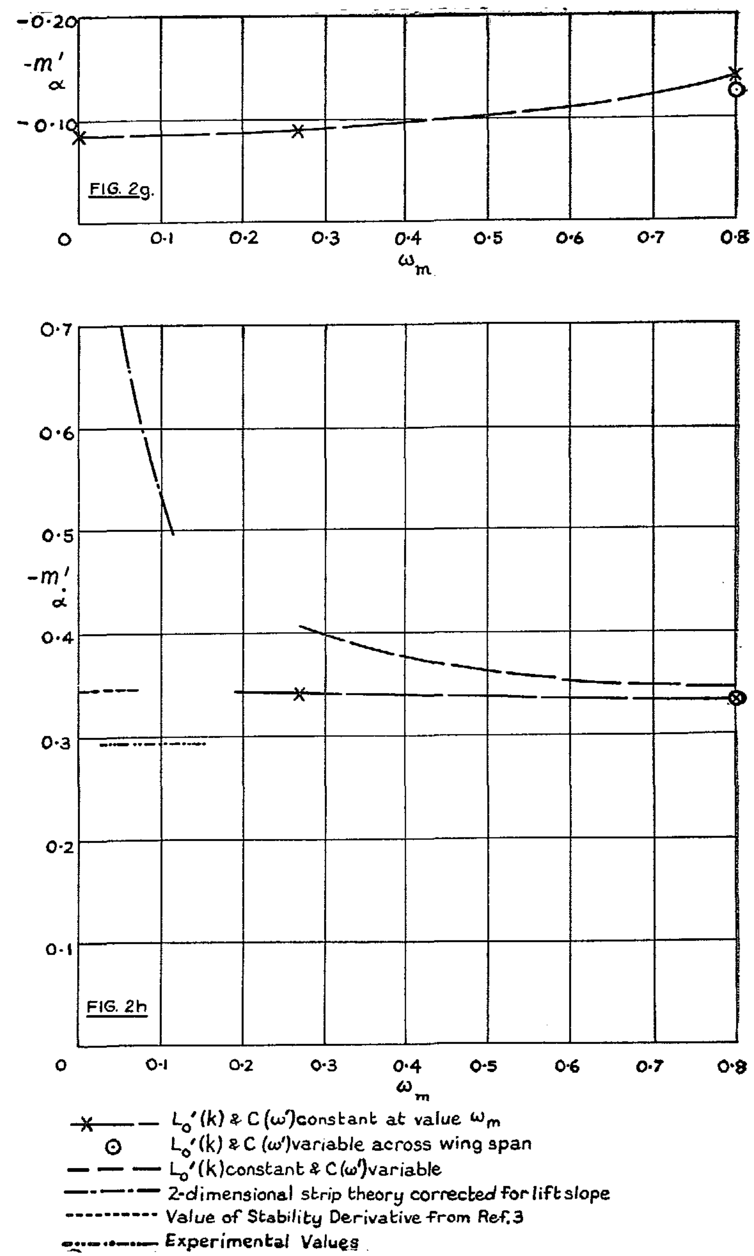


FIGS. 2e and 2f. Derivative coefficients for an oscillating delta wing. (Reference axis at $0 \cdot 556 c_0$ behind apex.)



FIGS. 2g and 2h. Derivative coefficients for an oscillating delta wing. (Reference axis at $0 \cdot 556 c_0$ behind apex.)

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