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Asymptotic Solution of a Boundary Layer Suction Problem

By

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Summary.—The theory of the boundary layer on a flat plate in a uniform stream with a velocity of suction proportional to $x^{-1/2}$ (x being the distance from the leading edge of the plate), has been developed by Thwaites[1] in a report which contains numerical solutions of the problem obtained on the differential analyser. The behaviour of the solution when the rate of suction is large is investigated here, and it is found that the velocity distribution in the boundary layer approximates to the Griffith-Meredith[2] or asymptotic suction profile. The solution is developed in the form of a series of descending powers of the suction velocity and the coefficients of this series are obtained successively by the solution of linear differential equations. The first four coefficients are obtained explicitly and numerical values are given in Table 1. Series are also obtained for the displacement and momentum thicknesses and for the skin friction and form parameter H. Comparisons are made with Thwaites's solutions, and good agreement is found when the rate of suction is large.

1. *Introduction.*—One of the cases in which the boundary layer equations can be reduced to the solution of an ordinary differential equation is Blasius' problem of uniform flow over a flat plate. This can be generalised to solve the problem of flow over a porous plate at which there is a velocity normal to the surface proportional to $x^{-1/2}$, where x denotes the distance from the leading edge of the plate. This represents suction or expulsion of fluid according to the sign of this velocity. The method is based on the following considerations.

For constant stream velocity the equation of motion is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}. \qquad (1)$$

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

enables a stream function ψ to be defined such that

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y}, \\ v &= -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \qquad (3)$$

The substitution

$$\left.\begin{aligned} \eta &= \tfrac{1}{2}\left(\frac{U}{\nu x}\right)^{1/2} y \,, \\ \psi &= (\nu U x)^{1/2} f(\eta) \,, \end{aligned}\right\} \quad \ldots \quad (4)$$

reduces (1) to the ordinary differential equation

$$f''' + ff'' = 0 \,, \quad \ldots \quad (5)$$

and the velocity components are then given by

$$\left.\begin{aligned} u &= \tfrac{1}{2} U f' \,, \\ v &= \tfrac{1}{2}\left(\frac{U\nu}{x}\right)^{1/2} (\eta f' - f) \end{aligned}\right\} \quad \ldots \quad (6)$$

The boundary conditions for u then give

$$\left.\begin{aligned} f'(0) &= 0 \,, \\ f'(\infty) &= 2 \end{aligned}\right\} . \quad \ldots \quad (7)$$

If we take

$$f(0) = K \,, \quad \ldots \quad (8)$$

the normal velocity at the boundary is

$$v_0 = -\tfrac{1}{2} K \left(\frac{U\nu}{x}\right)^{1/2} , \quad \ldots \quad (9)$$

and is therefore proportional to $x^{-1/2}$.

The problem is thus equivalent to the solution of the equation (5) with boundary conditions (7) and (8). Taking $K = 0$ gives the ordinary Blasius' problem of flow over an impermeable plate.

This argument was given by Schlichting and Bussmann[3], and also by Preston[4] and Thwaites[1]. Schlichting and Bussmann considered both suction and blowing, and gave numerical solutions for $K = 5, 3, \frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -\frac{3}{4}, -1$. Thwaites's report contains solutions obtained on the differential analyser for $K = 1, 2, 5, 10$ and 20. The graphs given by Thwaites indicate that when K is large δ^* and θ tend to 0 and H tends to 2. In fact the velocity profile becomes the asymptotic suction profile, as will be seen below.

2. *Transformation of the Equation.*—When η is small, and K is large but fixed,

$$f(\eta) = K + O(\eta^2) \,, \quad \ldots \quad (10)$$

and so equation (5) is approximated by

$$f''' + Kf'' = 0 \,, \quad \ldots \quad (11)$$

whence

$$f'' = A e^{-K\eta} \,, \quad \ldots \quad (12)$$

and

$$f' = B - \frac{A}{K} e^{-K\eta} \,, \quad \ldots \quad (13)$$

where A and B are constants. Since $f'(0) = 0$, $B = A/K$ and

$$f' = \frac{A}{K}(1 - e^{-K\eta}) \,. \quad \ldots \quad (14)$$

This argument is based on the assumption that η is small, and so it cannot immediately be deduced from the boundary condition at $\eta = \infty$ that $A/K = 2$. It does, however, suggest forcibly that the important variable is not η but $K\eta$. Hence the following transformation is made.

Let

$$K\eta = \zeta = \frac{-v_0 y}{\nu}, \quad \ldots \quad (15)$$

$$f(\eta) = K + \frac{1}{K}\phi(\zeta). \quad \ldots \quad (16)$$

The actual form of this transformation is chosen to make the boundary conditions for ϕ independent of K. Since

$$\left.\begin{aligned} f'(\eta) &= \phi'(\zeta), \\ f''(\eta) &= K\phi''(\zeta), \\ f'''(\eta) &= K^2\phi'''(\zeta), \end{aligned}\right\} \quad \ldots \quad (17)$$

the differential equation (5) becomes

$$K^2\phi''' + \left(K + \frac{1}{K}\phi\right)K\phi'' = 0,$$

or

$$\phi''' + \phi'' + \frac{1}{K^2}\phi\,\phi'' = 0, \quad \ldots \quad (18)$$

and the boundary conditions for ϕ are

$$\left.\begin{aligned} \phi(0) = \phi'(0) &= 0, \\ \phi'(\infty) &= 2 \end{aligned}\right\}. \quad \ldots \quad (19)$$

When K is large, the dominant terms of equation (18) are

$$\phi''' + \phi'' = 0, \quad \ldots \quad (20)$$

whence we have

$$\phi'' = Ae^{-\zeta},$$

and therefore

$$\phi' = B - Ae^{-\zeta}.$$

The boundary conditions (19) then give $B = 2$, $A = 2$ and so

$$\phi' = 2(1 - e^{-\zeta}) \quad \ldots \quad (21)$$

therefore

$$\frac{u}{U} = 1 - e^{-\zeta}, \quad \ldots \quad (22)$$

which is the equation of the asymptotic suction profile.

3. *Asymptotic Expansion for ϕ.*—The analysis is easily extended to obtain an asymptotic series for ϕ. Equation (18) involves K only as K^2 and therefore the appropriate form for the series is

$$\phi = \phi_0 + \frac{\phi_1}{K^2} + \frac{\phi_2}{K^4} + \ldots \quad \ldots \quad (23)$$

We may also regard this method as being one of successive approximation for ϕ, assuming K to be large.

Substitution in (18) gives

$$\left(\phi_0''' + \frac{\phi_1'''}{K^2} + \frac{\phi_2'''}{K^4} + \ldots\right) + \left(\phi_0'' + \frac{\phi_1''}{K^2} + \frac{\phi_2''}{K^4} + \ldots\right)$$
$$+ \frac{1}{K^2}\left(\phi_0 + \frac{\phi_1}{K^2} + \frac{\phi_2}{K^4} + \ldots\right)\left(\phi_0'' + \frac{\phi_1''}{K^2} + \frac{\phi_2''}{K^4} + \ldots\right) = 0 . \quad \ldots \quad \ldots \quad (24)$$

By considering the various powers of K in (24) we obtain a set of differential equations for the functions ϕ_r. These equations are

$$\phi_0''' + \phi_0'' = 0 , \quad \ldots \quad (25)$$
$$\phi_1''' + \phi_1'' + \phi_0\phi_0'' = 0 , \quad \ldots \quad (26)$$
$$\phi_2''' + \phi_2'' + \phi_0\phi_1'' + \phi_1\phi_0'' = 0 , \quad \ldots \quad (27)$$

and, generally,

$$\phi_r''' + \phi_r'' + \sum_{n=0}^{r-1} \phi_n\phi_{r-n-1}'' = 0 . \quad \ldots \quad (28)$$

The boundary conditions are

$$\left.\begin{aligned} \phi_0(0) = \phi_0'(0) &= 0 , \\ \phi_0'(\infty) &= 2 , \end{aligned}\right\} \quad \ldots \quad (29)$$

and

$$\phi_r(0) = \phi_r'(0) = \phi_r'(\infty) = 0 \ (r > 0) . \quad \ldots \quad (30)$$

As in section 2 we find from (25)

$$\left.\begin{aligned} \phi_0'' &= 2e^{-\zeta}, \\ \phi_0' &= 2(1 - e^{-\zeta}) \end{aligned}\right\}, \quad \ldots \quad (31)$$

and, since $\phi_0(0) = 0$,

$$\phi_0 = 2(\zeta - 1 + e^{-\zeta}) . \quad \ldots \quad (32)$$

Now equation (26) becomes, on substituting for ϕ_0 and ϕ_0'' the values already obtained,

$$\phi_1''' + \phi_1'' + 2(\zeta - 1 + e^{-\zeta}) \,.\, 2e^{-\zeta} = 0 .$$

This equation integrates on multiplying by e^{ζ} to give

$$e^{\zeta}\phi_1'' + 4(\tfrac{1}{2}\zeta^2 - \zeta - e^{-\zeta}) = A_1 ,$$

so that

$$\phi_1'' = -(2\zeta^2 - 4\zeta - A_1)e^{-\zeta} + 4e^{-2\zeta}$$

and

$$\phi_1' = B_1 + (2\zeta^2 - A_1)e^{-\zeta} - 2e^{-2\zeta} .$$

The boundary conditions give $B_1 = 0$, $A_1 = -2$,

therefore

$$\phi' = 2(\zeta^2 + 1)e^{-\zeta} - 2e^{-2\zeta} , \quad \ldots \quad (33)$$
$$\phi_1 = C_1 - 2(\zeta^2 + 2\zeta + 3)e^{-\zeta} + e^{-2\zeta} ,$$

and, since $\phi_1(0) = 0$, $C_1 = 5$ therefore

$$\phi_1 = 5 - 2(\zeta^2 + 2\zeta + 3)e^{-\zeta} + e^{-2\zeta} . \quad \ldots \quad (34)$$

Using the known values of ϕ_0 and ϕ_1 equation (27) is solved similarly to (26), and it is found that

$$\left.\begin{aligned} \phi_2'' &= (\zeta^4 - 4\zeta^3 + 6\zeta^2 - 14\zeta + 20\tfrac{1}{3})\,e^{-\zeta} - 8\,(\zeta^2 + \zeta + 4)\,e^{-2\zeta} + 5e^{-3\zeta}\,, \\ \phi_2' &= -\,(\zeta^4 + 6\zeta^2 - 2\zeta + 18\tfrac{1}{3})\,e^{-\zeta} + 4\,(\zeta^2 + 2\zeta + 5)\,e^{-2\zeta} - \tfrac{5}{3}e^{-3\zeta}\,, \\ \phi_2 &= -\,39\tfrac{8}{9} + (\zeta^4 + 4\zeta^3 + 18\zeta^2 + 34\zeta + 52\tfrac{1}{3})\,e^{-\zeta} - (2\zeta^2 + 6\zeta + 13)\,e^{-2\zeta} + \tfrac{5}{9}e^{-3\zeta}\,. \end{aligned}\right\} \quad (35)$$

We can now calculate ϕ_3 and find

$$\left.\begin{aligned} \phi_3'' = &-[\tfrac{1}{3}\zeta^6 - 2\zeta^5 + 5\zeta^4 - 16\tfrac{2}{3}\zeta^3 + 44\tfrac{1}{3}\zeta^2 - 130\tfrac{4}{9}\zeta + 275\tfrac{7}{27}]\,e^{-\zeta} \\ &+ [8\zeta^4 + 16\zeta^3 + 96\zeta^2 + 168\zeta + 409\tfrac{1}{3}]\,e^{-2\zeta} - [15\zeta^2 + 30\zeta + 78]\,e^{-3\zeta} \\ &+ 5\tfrac{1}{27}e^{-4\zeta}\,, \\ \phi_3' = &\,[\tfrac{1}{3}\zeta^6 + 5\zeta^4 + 3\tfrac{1}{3}\zeta^3 + 54\tfrac{1}{3}\zeta^2 - 21\tfrac{7}{9}\zeta + 253\tfrac{13}{27}]\,e^{-\zeta} \\ &- [4\zeta^4 + 16\zeta^3 + 72\zeta^2 + 156\zeta + 282\tfrac{2}{3}]\,e^{-2\zeta} \\ &+ [5\zeta^2 + 13\tfrac{1}{3}\zeta + 30\tfrac{4}{9}]\,e^{-3\zeta} - 1\tfrac{7}{27}e^{-4\zeta}\,, \\ \phi_3 = &\,524\tfrac{13}{18} - [\tfrac{1}{3}\zeta^6 + 2\zeta^5 + 15\zeta^4 + 63\tfrac{1}{3}\zeta^3 + 244\tfrac{1}{3}\zeta^2 + 466\tfrac{8}{9}\zeta + 720\tfrac{10}{27}]\,e^{-\zeta} \\ &+ [2\zeta^4 + 12\zeta^3 + 54\zeta^2 + 132\zeta + 207\tfrac{1}{3}]\,e^{-2\zeta} \\ &- [\tfrac{5}{3}\zeta^2 + 5\tfrac{5}{9}\zeta + 12]\,e^{-3\zeta} + \tfrac{17}{54}e^{-4\zeta}\,. \end{aligned}\right\} \quad (36)$$

The functions ϕ_0, ϕ_1, ϕ_2, ϕ_3 and their first derivatives are tabulated in Table 1 for various values of ζ (chosen to facilitate comparison with Thwaites's results) between $\zeta = 0$ and $\zeta = 4$. Clearly ϕ_r and its derivatives are polynomials in ζ and $e^{-\zeta}$, of degree $2r$ in ζ and $(r + 1)$ in $e^{-\zeta}$.

4. *The Characteristics of the Boundary Layer.*—From the expansion of $\phi(\zeta)$ now obtained it is a simple matter to deduce the corresponding expressions for δ^*, θ, τ_0 and H,

The skin friction is given by

$$\frac{\tau_0}{\rho U^2}\left(\frac{Ux}{\nu}\right)^{1/2} = \tfrac{1}{4}f''(0) \quad \ldots \quad (37)$$

$$= \tfrac{1}{4}K\phi''(0)$$

$$= \tfrac{1}{4}K\left[2 + \frac{2}{K^2} - \frac{6\frac{2}{3}}{K^4} + \frac{61\frac{1}{9}}{K^6} + O\,(K^{-8})\right]$$

$$= \tfrac{1}{2}K + \frac{1}{2K} - \frac{5}{3K^3} + \frac{275}{18K^5} + O\,(K^{-7})\,. \quad \ldots \quad (38)$$

The displacement thickness is

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy\,, \quad \ldots \quad (39)$$

$$= \left(\frac{\nu x}{U}\right)^{1/2} \int_0^\infty [2 - f'(\eta)]\,d\eta\,. \quad \ldots \quad (40)$$

Thus we have

$$\left(\frac{U}{\nu x}\right)^{1/2} \delta^* = \frac{1}{K}\int_0^\infty [2 - \phi'(\zeta)]\, d\zeta\,,$$

$$= \frac{1}{K}\lim_{\zeta \to \infty} [2\zeta - \phi(\zeta)]\,.$$

$$= \frac{2}{K} - \frac{\phi_1(\infty)}{K^3} - \frac{\phi_2(\infty)}{K^5} - \frac{\phi_3(\infty)}{K^7} - \ldots \qquad \ldots \quad \ldots \quad \ldots \quad (41)$$

Substituting the numerical values from section 3,

$$\left(\frac{U}{\nu x}\right)^{1/2} \delta^* = \frac{2}{K} - \frac{5}{K^3} + \frac{39\frac{8}{9}}{K^5} - \frac{524\frac{13}{18}}{K^7} + O(K^{-9})\,. \qquad \ldots \quad \ldots \quad \ldots \quad (42)$$

Similarly we have for the momentum thickness θ,

$$\left(\frac{U}{\nu x}\right)^{1/2} \theta = \int_0^\infty f'(1 - \tfrac{1}{2}f')\, d\eta \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

$$= \frac{1}{2K}\int_0^\infty \phi'(2 - \phi')\, d\zeta\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

By substituting the series expression for ϕ', multiplying out and then integrating we get the expansion of θ in descending powers of K.

The result obtained is

$$\left(\frac{U}{\nu x}\right)^{1/2} \theta = \frac{1}{K} - \frac{3\frac{1}{3}}{K^3} + \frac{30\frac{5}{9}}{K^5} - \frac{433\frac{31}{45}}{K^7} + O(K^{-9})\,. \qquad \ldots \quad \ldots \quad \ldots \quad (45)$$

From (42) and (45) by division we have

$$H = \frac{\delta^*}{\theta} = 2 + \frac{1\frac{2}{3}}{K^2} - \frac{15\frac{2}{3}}{K^4} + \frac{239\frac{137}{270}}{K^6} + O(K^{-8})\,. \qquad \ldots \quad \ldots \quad (46)$$

The momentum equation for the flat plate is

$$\frac{d\theta}{dx} = \frac{v_0}{U} + \frac{\tau_0}{\rho U^2}\,, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (47)$$

which gives in the present circumstances

$$\theta = (-K + \tfrac{1}{2}f''(0))\left(\frac{\nu x}{U}\right)^{1/2}, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

and in this form may readily be verified.

5. *Conclusion.*—From the formulae given in section 3 for $\phi_r(\zeta)$ and $\phi_r'(\zeta)$ the tables of Table 1 were constructed. These enable the velocity distribution for large values of K to be calculated readily. Such calculations were made for $K = 5$, 10 and 20 for comparison with Thwaites's results, the values of ζ employed in Table 1 being chosen for this purpose. This comparison is shown in Table 2. For $K = 10$ and $K = 20$ the asymptotic series is certainly superior to the differential analyser result, and for $K = 5$ the accuracy is about equal. The series is probably of no use for $K < 3$, and its accuracy becomes progressively better as K increases.

The method has been extended and applied to many other boundary layer suction problems including the effect of suction in preventing separation.

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TABLE 1

The Coefficients ϕ_r and ϕ_r' of the Asymptotic Series for ϕ and ϕ'

ζ	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_0'	ϕ_1'	ϕ_2'	ϕ_3'
0	0	0	0	0	0	0	0	0
0·125	0·01499	0·01499	− 0·04998	0·458	0·23501	0·23497	−0·78342	7·181
0·250	0·05760	0·05758	− 0·19206	1·760	0·44240	0·44189	−1·47568	13·521
0·375	0·12458	0·12440	− 0·41562	3·808	0·62542	0·62315	−2·08928	19·123
0·5	0·21306	0·21237	− 0·71158	6·514	0·78694	0·78057	−2·63580	24·082
0·75	0·44473	0·44042	− 1·49098	13·614	1·05527	1·02989	−3·56744	32·391
1	0·73576	0·72078	− 2·48179	22·569	1·26424	1·20085	−4·33610	39·003
1·25	1·07301	1·03520	− 3·64914	33·010	1·42699	1·30417	−4·98540	44·347
1·5	1·44626	1·36814	− 4·96666	44·664	1·55374	1·35077	−5·54043	48·750
2	2·27067	2·04094	− 7·96634	70·851	1·72933	1·31672	−6·40494	55·654
2·5	3·16417	2·66731	−11·31714	100·054	1·83583	1·17676	−6·94206	60·951
3	4·09957	3·21014	−14·85120	131·618	1·90043	0·99078	−7·13720	65·129
4	6·03663	4·01129	−21·81012	199·598	1·96337	0·62206	−6·59744	69·951

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TABLE 2

Comparison of Results from the Asymptotic Formulae with Thwaites's Results

ζ	$K = 5\ (\sigma_1 = 2\cdot5)$					$K = 10\ (\sigma_1 = 5)$					$K = 20\ (\sigma_1 = 10)$				
	η	$f'(\eta)$ (A)	$f'(\eta)$ (T)	$f(\eta)$ (A)	$f(\eta)$ (T)	η	$f'(\eta)$ (A)	$f'(\eta)$ (T)	$f(\eta)$ (A)	$f(\eta)$ (T)	η	$f'(\eta)$ (A)	$f'(\eta)$ (T)	$f(\eta)$ (A)	$f(\eta)$ (T)
0	0	0	0	5	5	0	0	0	10	10	0	0	0	20	20
0·125	0·025	0·244	0·243	5·003	5·003	0·0125	0·23729	0·236	10·00151	10·001	0·00625	0·23559	0·234	20·00075	20·001
0·25	0·05	0·459	0·459	5·012	5·018	0·025	0·44668	0·447	10·00582	10·006	0·0125	0·44350	0·443	20·00289	20·003
0·375	0·075	0·648	0·650	5·026	5·026	0·0375	0·63146	0·631	10·01258	10·013	0·01875	0·62696	0·627	20·00624	20·006
0·5	0·1	0·815	0·813	5·044	5·044	0·05	0·79450	0·795	10·02151	10·022	0·025	0·78887	0·789	20·01068	20·011
0·75	0·15	1·093	1·09	5·092	5·092	0·075	1·06524	1·07	10·04490	10·045	0·0375	1·05782	1·06	20·02229	20·022
1	0·2	1·308	1·31	5·152	5·152	0·1	1·27585	1·28	10·07427	10·074	0·05	1·26722	1·27	20·03688	20·037
1·25	0·25	1·474	1·47	5·222	5·222	0·125	1·43957	—	10·10830	—	0·0625	1·43022	1·43	20·05378	20·054
1·5	0·3	1·602	1·60	5·299	5·299	0·15	1·56674	1·57	10·14595	10·146	0·075	1·55708	1·56	20·07248	20·072
2	0·4	1·775	1·77	5·469	5·468	0·2	1·74192	1·74	10·22903	10·229	0·1	1·73258	1·73	20·11379	20·114
2·5	0·5	1·876	1·87	5·652	5·651	0·25	1·84696	1·85	10·31898	10·320	0·125	1·83873	1·84	20·15854	20·158
3	0·6	1·933	1·93	5·843	5·842	0·3	1·90969	1·91	10·41303	10·414	0·15	1·90286	1·90	20·20537	20·205
4	0·8	1·982	1·98	6·235	6·234	0·4	1·96900	1·97	10·60748	10·608	0·2	1·96488	1·96	20·30233	20·302

	$K = 5$: Asymptotic	$K = 5$: Thwaites	$K = 5$: S–B	$K = 10$: Asymptotic	$K = 10$: Thwaites	$K = 20$: Asymptotic	$K = 20$: Thwaites
$\left(\frac{U}{\nu x}\right)^{1/2} \delta^*$	0·366	0·368	0·364	0·1954	0·193	0·09939	0·100
$\left(\frac{U}{\nu x}\right)^{1/2} \theta$	0·178	0·180	0·174	0·0969	0·095	0·04959	0·050
$\frac{\delta^*}{\theta} = H$	2·05	2·05	2·086	2·015	2·023	2·00407	2·008

Note.—(A) and (T) denote respectively values obtained from the asymptotic series and values given by Thwaites.

S–B denotes values given by Schlichting and Bussmann.

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