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Treatment of the Stagnation Point in Arithmetical Methods

By

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Treatment of the Stagnation Point in Arithmetical Methods

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Summary.—In using any of the Relaxation Techniques near a stagnation point difficulties arise if the variable is log $1/q$. This tends to infinity and the difference equation no longer represents adequately the differential equation without special modification. Methods are given whereby larger squares can be used than had been previously practicable. As little variation as possible has been introduced into the procedure so that if an electronic calculator is used, the alterations to the circuits would be reduced to a minimum.

1. *Treatment of the Stagnation Point in Arithmetical Methods.*—The writer has long advocated the use of the conjugate functions log $1/q$ and θ in obtaining solutions to certain flow problems by 'squaring' methods. Where the physical boundaries are specified we operate on θ and then pass over to q. When the velocities are specified we operate on q and pass over to θ and so find the physical boundaries to give the specified velocities. In the case of mixed problems, *e.g.*, determination of free streamlines, part of the boundary is physical and part has a specified velocity either constant or, in hydraulic flows, a function of height. Accordingly, for these problems the θ and log $1/q$ fields are used alternatively passing from one to the other at the boundary at each alternation.

For certain problems the above methods are satisfactory, but where a stagnation point exists a difficulty arises since log $1/q$ then becomes infinite at one point. In a recent paper (Current Paper 76[1]) the difficulty is got over by comparing the field close to the stagnation point with the field $w = kz^2$ which represents the flow inside a right-angle bend. Here we have $\log dz/dw = \log 1/(4k\varrho)^{1/2} - \frac{1}{2}i\beta$ where $w = \varrho e^{i\beta}$ so that $q = 2(k\varrho)^{1/2}$ and $\theta = -\beta/2$. Putting $k = \frac{1}{4}$ we get the values of log $1/q$ shown in Fig. 6. Note that for any other value of k these figures will merely be increased or decreased by a constant.

Assume that in the arithmetic process of working over the field (Ref. 2, 5, 7) we operate on diamonds, *i.e.*, the squares with a vertical diagonal. The usual difference formula for application to Laplace fields is

$$\text{Central value} = \text{Mean of corner values} \qquad \text{(A)}$$

but near the stagnation point at the origin the neglected terms become large and a correction, Δ, has to be added to the right-hand side of (A). The actual values of Δ necessary to make (A) apply to the diamonds in Fig. 6 are shown in Fig. 7. It will be seen that as the origin is left behind Δ rapidly approaches zero. It is important to note that, in forming the values in Fig. 7, instead of using ∞ at the origin of Fig. 6 the mean of the four surrounding values has been used—in fact (A) has been used with no Δ. Also bear in mind that in using the above procedure any other value of k adopted in forming Fig. 6 will give identical values of Δ in Fig. 7.

* Oxford University Engineering Laboratory Report No. 53.

Suppose now that we are working over the field at the nose of a symmetrical aerofoil or similar blunt-nosed body. It is found that the difficulty of dealing with the infinity at the stagnation point is overcome to a great extent if we correct the value for each diamond by applying the values of Δ shown in Fig. 7. When a subdivision of the field is made the same values of Δ are again available. This is because Fig. 7 is quite independent of scale and applies for all sizes of square, provided each bears the same relation to the stagnation point as in the figure. As has already been emphasised it is also independent of velocity scale and so is universally applicable. The above process has been successfully used (Ref. 1), but it was found necessary to subdivide the grid into very small squares in the immediate neighbourhood of the nose.

The trouble arises from the fact that the comparison field is bounded by two straight lines at right-angles while the actual field is bounded on one side by the approach stagnation streamline and on the other by the curved aerofoil nose. The present paper shows how to take account of the curvature by comparison with conditions at the front of a cylinder or a parabola.

2.1. *Approximations to the Circle.*—If we take the usual relation giving the plane flow (velocity $= U$) past a circle of radius a and transfer the origin to the stagnation point—at $x = a, y = 0$ we obtain

$$w + 2aU = U\{z + a + a^2/(z + a)\}. \qquad \text{(1)}$$

Reversing we get

$$z = \frac{w}{2U} + \left(\frac{aw}{U}\right)^{1/2} \left(1 + \frac{w}{4aU}\right)^{1/2}$$

or

$$z = \frac{w}{2U} + a\left\{\left(\frac{w}{Ua}\right)^{1/2} + \frac{1}{8}\left(\frac{w}{Ua}\right)^{3/2} \cdots\cdots\right\}. \qquad \text{(2)}$$

If we take the first two terms

$$z = \frac{w}{2U} + \left(\frac{wa}{U}\right)^{1/2} \qquad \text{(3)}$$

we obtain the flow past a parabola and the velocity is found to be

$$\frac{1}{q^2} = \frac{1}{4U^2} + \frac{a}{4\varrho U} + \frac{1}{2U}\left(\frac{a}{\varrho U}\right)^{1/2} \cos\beta/2 \qquad \text{(4)}$$

where

$$\phi = \varrho\cos\beta \text{ and } \psi = \varrho\sin\beta.$$

It will be noted that when $\varrho \to \infty$, $q \to 2U$ where U was the velocity originally assumed at infinity. The discrepancy is caused by the neglected terms in (3). These neglected parts would also produce terms in $\varrho^{1/2}$, ϱ, $\varrho^{3/2}$, etc., in (4) but none in $\varrho^{-1/2}$. So it appears that close to the origin only the second and third terms are signficant. It follows that in the region where the approximation is valid the flow past the circle has become the same as the flow past a parabola, the air in the latter case having twice the velocity at infinity. Fig. 1 shows the streamlines, etc., at the nose of the parabola plotted with non-dimensional variables ϕ/aU and ψ/aU. But for this diagram the velocity at infinity is $2U$, whereas near the stagnation point itself an identical diagram with identical velocities will be obtained for the flow past a cylinder of radius a (the nose radius of the parabola) and velocity U at infinity.

If we specify the velocity vector by q and θ, we also find by differentiating (3)

$$\tan\theta = -\sin\beta/2 \div ((\varrho/aU)^{1/2} + \cos\beta/2). \qquad \text{(5)}$$

In what follows we shall drop the minus sign and treat θ as a positive angle.

It is not difficult to show that (4) and (5) become approximately

$$\log 1/q = \log (a/4\varrho U)^{1/2} + (\varrho/aU)^{1/2} \cos \beta/2 \qquad (6)$$

and

$$\theta = \beta/2\, (\varrho/aU)^{1/2} \sin \beta/2 . \qquad (7)$$

We have thus simple expressions for log $1/q$ and θ in terms of the grid co-ordinates ϕ and ψ in the neighbourhood of the stagnation point.

2.2. *The Correction for Curvature.*—We shall again adopt the procedure outlined in section 1 at the stagnation point. That is, instead of ∞ write at this point the mean of the four values at the surrounding diamond points, making use if necessary of symmetry to obtain the fourth. The values of Δ to be used in recalculating the adjacent points during the next repetition should be taken from Figs. 2 to 5. The points on these graphs were determined as follows.

Using (6) we can write down the value of log $1/q$ at each of the points B, G, and C (Fig. 2). Call these L_B, L_G, and L_C. The artificial value of log $1/q$ at S follows as explained above (see also Fig. 2). Using (6) again we find L_E, L_F, and L_H. Δ_G is then determined by

$$L_G = \tfrac{1}{4}(L_S + L_E + L_H + L_F) + \Delta_G .$$

This process can be repeated for each point in the field. It is found that Δ's for the various points are all of the form

$$\Delta = K_1 + K_2(\varepsilon/aU)^{1/2} \qquad (8)$$

where ε is the side of the unit square in the grid, *i.e.*, the half-diagonal of the diamond.

Values of K_1 and K_2 are given in Table 1 for the points in the immediate neighbourhood. It will be seen that K_1 is identical with the values of Δ in Fig. 7 for flow in a right-angled corner. Thus the term $K_2(\varepsilon/aU)^{1/2}$ is the correction for curvature a being the radius. It will be found that this term gets smaller very rapidly as the origin is left behind.

An analogous form can be obtained for θ from (7), namely

$$\Delta = C_1 + C_2(\varepsilon/aU)^{1/2}. \qquad (9)$$

Values of C_1 and C_2 are given in Table 2.

It ought to be explained that in so far as (6) and (7) are assumed to give the condition at the stagnation point of a circular cylinder, the procedure outlined is correct, but if it is really intended to apply the corrections to a parabola, the velocity at infinity would actually be $2U$. Thus for example in Fig. 2 the line marked $\Delta_G = 0 \cdot 1733 - 0 \cdot 0822r$ calculated as above refers really to a cylinder. The same line displaced $1/\sqrt{2}$ to the left and marked P refers to the parabola.

The question now arises as the validity of these values of Δ when the diamond becomes large compared with the region in which the aerofoil approximates to a circular form. To investigate this a large number of actual values were determined in the fields round a parabola, a circle and an aerofoil. The full expressions (4) and (5) were used for the parabola. The points so obtained, although shown under 'Parabola,' have not been displaced as explained above. They thus run into the straight line (8). A number of points were calculated from the full expression (1) for flow past a cylinder. These are shown under 'Circle' and apparently lie on or close to the straight line from (8). They ought to do so near the origin but it has not been established analytically that (8) holds far out in the field.

Finally a number of points were calculated round the nose of the aerofoil of Ref. 4—a laborious process in which help was voluntarily given by Mr. Hume-Rothery. An aerofoil with a nose radius a cannot be expected to give values identical with a circle of the same radius, but its values may well be between the circle and the parabola. That is between the line for the circle and that marked P, and it appears that the actual aerofoil points are running into this region in Fig. 2.

Fig. 1 gives some idea of the region in which we are working. Thus the 'square' marked SEHF centred at G has, on Fig. 1 an ε/aU of $\frac{1}{2}$ and so according to Fig. 2 is still of such a size that the uncertainty in the log values would be less than $\pm 0 \cdot 01$, *i.e.*, less than 1 per cent on the value of q at the point G. This is not large enough to have an appreciable effect on the velocities further along the surface.

2.3. *Values of θ.*—A similar comparison has been made for the θ values (Fig. 3). It should be noted that the value of θ at the stagnation point is assumed to be $\pi/4$ since this is the limiting value as $\psi \to 0$. Again it is not possible to distinguish the values for the circle from the straight line approximation.

2.4. *Values of log $1/q$ when the Nose Radius is not known.*—If we are constructing a shape to give a specified velocity distribution on the surface and the squaring has to be carried over a stagnation point, in general the leading-edge radius will not be known until the solution is complete.

A way out of the difficulty is to find some parameter dependent only on the velocity field from which the Δ's can be determined. It is necessary to discover a parameter which will as far as possible suit all cases. The writer has no idea how this could be done rigorously, but after many attempts an approximate solution has been found.

Referring to Fig. 2 define L_Q by

$$3L_Q = (L_B + L_G + L_C) - (L_A + L_H + L_D) \quad \ldots \quad (10)$$

Using the approximate relation (6) it is possible to express the Δ at any point in terms of L_Q. Thus consider the diamond SEHF centred on G. We easily find after some work

$$\Delta_G = 0 \cdot 05239 + 0 \cdot 34885 L_Q .$$

For the diamond centred on C we find

$$\Delta_C = 0 \cdot 09420 + 0 \cdot 22818 L_Q .$$

These lines have been plotted in Fig. 5 and compared with values for the parabola, circle and aerofoil. As the diamond shrinks, *i.e.*, as L_Q increases, all values presumably converge on the straight lines, and this would apply with any other similar parameter, but other parameters, so far tried show a much wider divergence for the larger diamonds.

Comparing Fig. 5 with Fig. 2 it seems that L_Q is a better parameter than $(\varepsilon/aU)^{1/2}$. The reason probably is that L_Q is built up from values well out in the field which are affected not only by the nose but by other parts of the surface. Only by trying other shapes can it be established that this method is really reliable for large diamonds. It must work for the finer mesh.

Values of Δ in terms of L_Q have also been determined for two other diamonds of half-diagonal ε.

Point	Co-ordinates of centre	Δ
E	$-\varepsilon, +\varepsilon$	$\Delta = 0 \cdot 02183 + 0 \cdot 01747 L_Q$
F	$+\varepsilon, +\varepsilon$	$\Delta = 0 \cdot 01330 + 0 \cdot 04212 L_Q$

For other points it is probably sufficient to neglect curvature and use the constant part from Fig. 7 without further correction.

3. *Obtaining the Conjugate Function.*—In the above sections the method is outlined for settling the L and θ fields, but it is also necessary to pass from one to the other on the boundaries. As has been explained in several previous papers (R. & M. 2440[2], R. &. M. 1604[3]) the straightforward formula for this purpose is of the form

$$L_a - L_b = \int_b^a \frac{\partial \theta}{\partial \psi} \, d\phi \quad \ldots \quad (11)$$

where L and θ or ϕ and ψ may be interchanged to give three other similar expressions, any one of which may be required. Any other pair of conjugate functions such as x and y can be treated in the same way.

It has also been shown that if, for example, θ is constant along a boundary ψ = constant, then (11) can be replaced with good accuracy by a simple form involving only multiples of four neighbouring values of L (*see* Ref. 4 and 7).

But the above breaks down at a stagnation point and it is now necessary to obtain approximate relations which will operate.

Again after experiment a simple method was found. In Fig. 4, A is at $(-\varepsilon, 0)$ and B is at $(-2\varepsilon, +2\varepsilon)$. We find for the parabola

$$q_A/U = 2^{1/4}(\tfrac{3}{8}\pi - \theta_B) \div \sin \tfrac{3}{8}\pi$$
$$= 1\cdot518 - 1\cdot290\ \theta_B .$$

This value is compared in Fig. 4 with values for circle and aerofoil.

It is possible that a better method may be developed, but in the meantime this method will prove satisfactory for diamonds of a reasonable size.

3.1. *The Trailing Edge.*—Let τ = half the trailing-edge angle. Then the flow near the trailing edge approximates closely to

$$w = (Kz)^n$$

where

$$n = \pi/(\pi - \tau) .$$

With the notation of section 2.1 the velocity is

$$q = nK\varrho^m \text{ where } m = (n - 1)/n = \tau/\pi .$$

If, as before, we agree to put a nominal value of log $1/q$ at the trailing-edge equal to the arithmetical mean of the surrounding points, then it is easy to show from (11) that Δ at any point is equal to the Δ for a right-angle multiplied by $2m$. It is not known what the exact effect of curvature will be but in many cases the curvature is very small at the trailing-edge so that the first term derived above is probably sufficient.

4. *Determination of x on Surface.*—When, on any particular approximation, we have determined θ and q, and it is necessary to determine x along the surface, we apply the expression

$$x = \int \frac{1}{q} \cos \theta \, d\phi .$$

This, however, cannot be used too close to the stagnation points. For many cases x is proportional to ϕ with considerable accuracy, *e.g.*, for a circle

$$x = \phi/2U.$$

For the front portion of the aerofoil previously considered, we find

ϕ/ϕ_c	uX/ϕ
0·05	0·8124
0·1	0·8147
0·15	0·8171
0·2	0·8196

where
$\phi_c = \phi$ interval corresponding to the whole chord.

It is tentatively suggested that for other aerofoils, a relation such as

$$Ux(1 + \tfrac{3}{2}t/c)/\phi = 1$$

will give results of reasonable accuracy in the region of the leading edge.

At the trailing edge, on the other hand, using the notation of section 3.1, we find for flow in a corner

$$xq/\phi n \cos \pi/n = 1.$$

This expression applies satisfactorily for an aerofoil, as is shown by the following values calculated for the PPP aerofoil[4] :—

$\dfrac{\phi}{\phi_c}$	$\dfrac{xq}{\phi n \cos \pi/n}$
0·025	1·005
0·05	1·010
0·1	1·014
0·2	1·020
0·3	1·030

ϕ measured from trailing edge

5. *Squaring on x and y.*—Since $\nabla_w^2 = \nabla_w^2 y = 0$, we can operate on either co-ordinate. At a stagnation point, the work is easier for x than for $\log 1/q$. The corrections are smaller ; *e.g.*, for the diamond centred on G (Fig. 2), we find that Δ is the same for both x/a and y/a, being given by

$$\Delta = 0 + 0 \cdot 0686\, (\varepsilon/aU)^{1/2}.$$

Thus only the small curvature term remains.

6. *Conclusion.*—An attempt has been made to produce constants which will enable the usual squaring method to be carried over the stagnation point. The values in Fig. 5 have been used by Dr. A. S. Thom in one or two actual solutions[8]. In these solutions it was found advantageous to use values of Δ rather larger than those shown, but this was probably due to the peculiar nature of the problem which was asymmetric.

If the method is to be used on some type of electronic calculator, it seems desirable to upset the ordinary procedure as little as possible, and there would almost certainly be, in any case, some means of applying Δ's to the means. Certainly this would be so if the machine were intended to solve Poisson's Equation.

LIST OF SYMBOLS

z	=	$(x + iy)$; physical plane
q, θ		Modulus and amplitude of velocity vector
w	=	$\phi + i\psi$
ϱ	=	$(\phi^2 + \psi^2)^{1/2}$
β	=	$\tan^{-1} \psi/\phi$
ε		Side of square (or half-diamond diagonal) in w-plane
L	=	$\log_e 1/q$
Δ		Central value in a diamond less mean of corner values
a		Radius of cylinder, or nose radius
2τ		Trailing-edge angle
n	=	$\pi/(\pi - \tau)$
m	=	$(n - 1)/n = \tau/\pi$
ϕ_c	=	(ϕ at leading edge) — (ϕ at trailing edge)
U	=	Undisturbed velocity for cylinder and aerofoil. (*See* section 2.1 for parabola)
r	=	$(\varepsilon/aU)^{1/2}$
t/c		Thickness/chord ratio

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TABLE 1

$\psi = 3\varepsilon$	$-0\cdot0004$	$-0\cdot0015$	$-0\cdot0004$	$+0\cdot0005$
$\psi = 2\varepsilon$	$+0\cdot0015$	$-0\cdot0081$	$+0\cdot0015$	$+0\cdot0019$
$\psi = \varepsilon$	$0\cdot0279 - 0\cdot0041r$	$+0\cdot1733 - 0\cdot0822r$	$+0\cdot0279 - 0\cdot0099r$	$+0\cdot0015$
$\psi = 0$	—	0	$+0\cdot1733 - 0\cdot0538r$	$-0\cdot0081$
	$\phi = -\varepsilon$	$\phi = 0$	$\phi = +\varepsilon$	$\phi = +2\varepsilon$

Values of Δ to be used squaring log $1/q$ near a stagnation point. The value in each rectangle is the value of Δ to be used for the diamond with corners in the four neighbouring points. Log $1/q$ at origin is to be taken as mean of four neighbouring points. $r = (\varepsilon/aU)^{1/2}$.

TABLE 2

$\psi = 2\varepsilon$	$-0\cdot0048 + 0\cdot0018r$	$0 - 0\cdot0024r$	$+0\cdot0048 - 0\cdot0015r$	$0 + 0\cdot0004r$	$-0\cdot0005 + 0\cdot0004r$
$\psi = \varepsilon$	$0 + 0\cdot0100r$	$0 - 0\cdot0687r$	$0 + 0\cdot0041r$	$-0\cdot0048 + 0\cdot0023r$	$-0\cdot0011 + 0\cdot0008r$
$\psi = 0$	—	—	—	—	—
	$\phi = -\varepsilon$	$\phi = 0$	$\phi = +\varepsilon$	$\phi = +2\varepsilon$	$\phi = +3\varepsilon$

Values of Δ to be used squaring θ near a stagnation point. $r = (\varepsilon/aU)^{1/2}$. Take θ at the stagnation point as $\pi/4$.

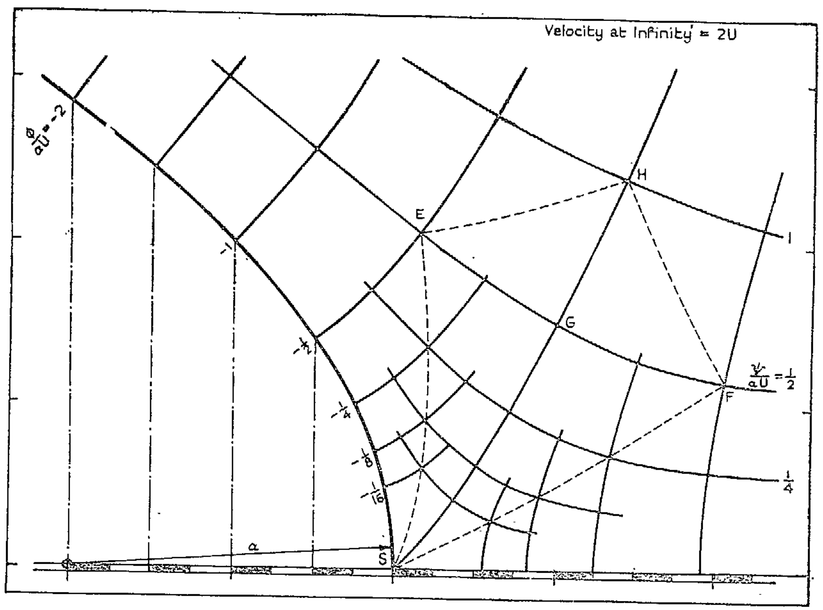

FIG. 1. Grid near vertex of a parabola.

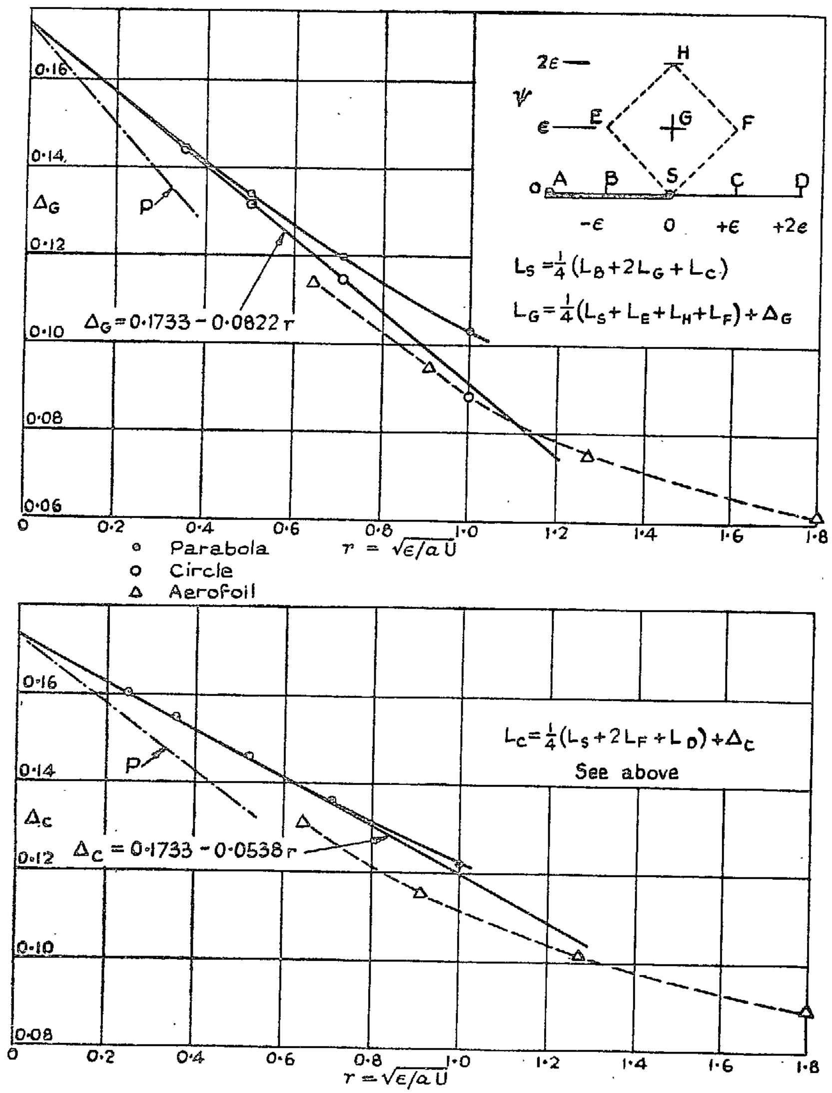

FIG. 2. Value of Δ to be used when squaring on log $1/q$ $(= L)$ in terms of $\sqrt{\varepsilon/aU}$ where $\varepsilon = \phi$ interval and $a =$ radius of nose. Stagnation point is at S.

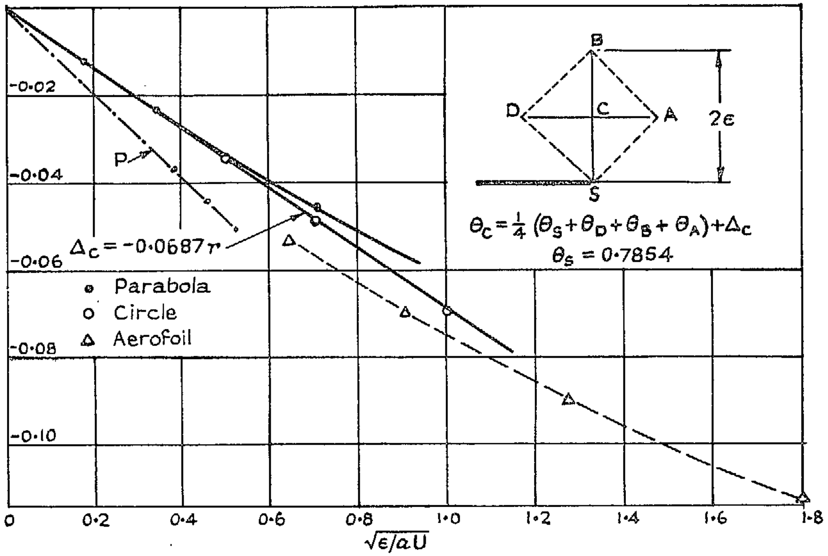

FIG. 3. Δ to be used when squaring on θ in terms of $\sqrt{\varepsilon/aU}$. Stagnation point is at S.

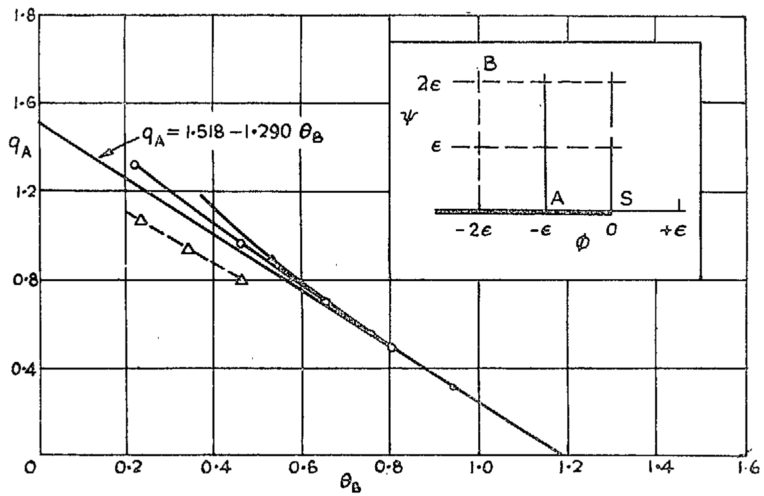

FIG. 4. Relation between q at $(-\varepsilon, 0)$ and θ at $(-2\varepsilon, +2\varepsilon)$. Stagnation point is at S.

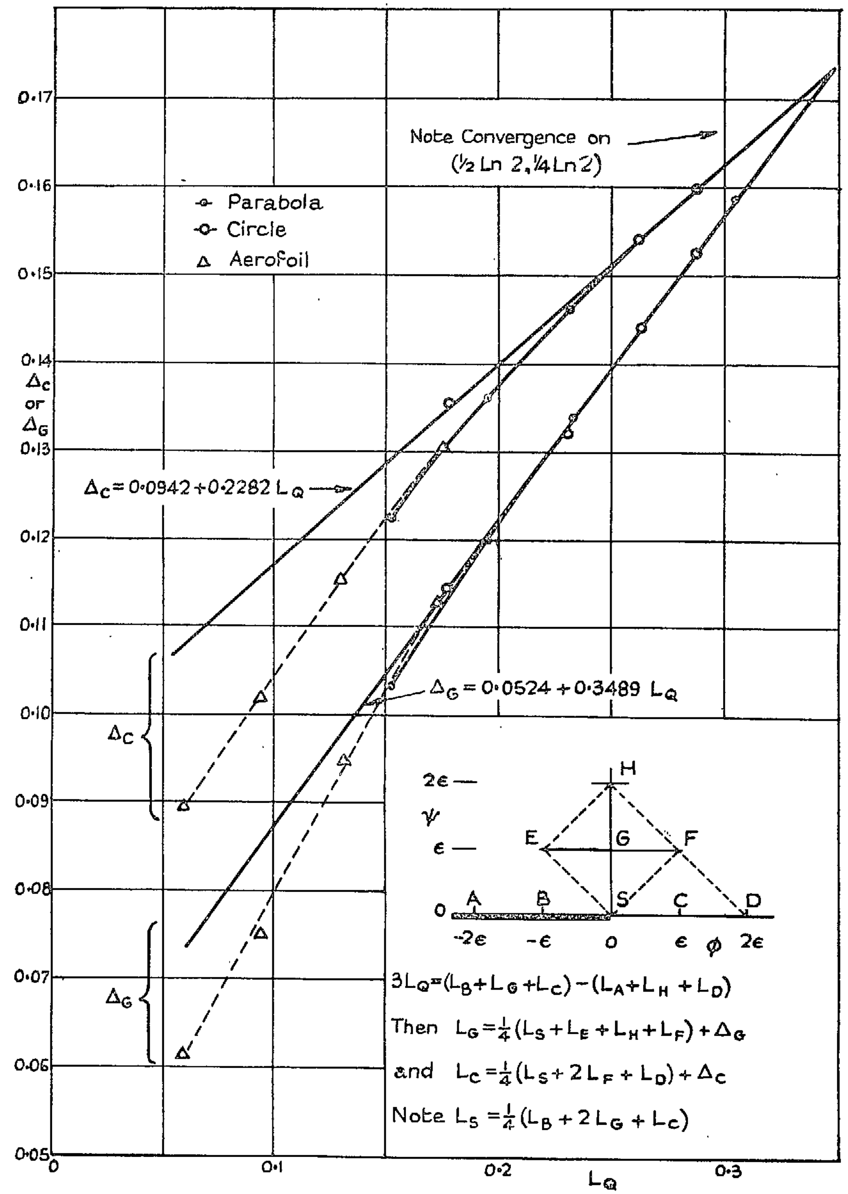

FIG. 5. Δ to be used when squaring on log $1/q$ $(= L)$, in terms of q's. Stagnation point is at S.

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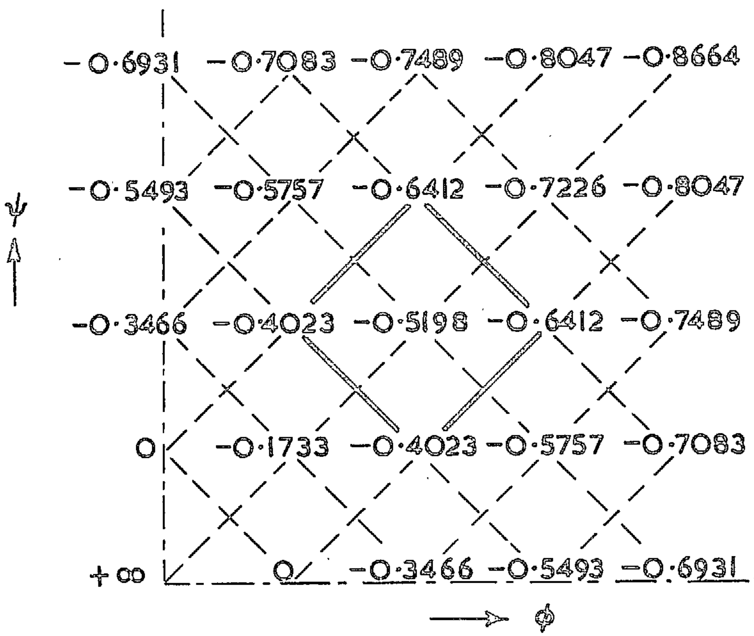
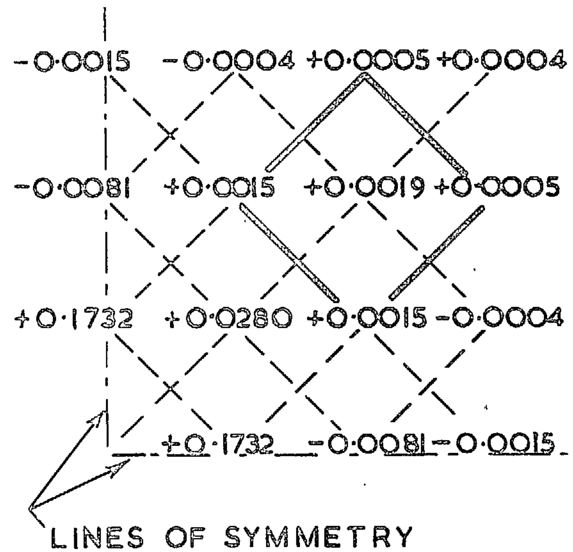


THE AXES ARE BOTH LINES OF SYMMETRY. ALL VALUES ARE NEGATIVE EXCEPT THAT AT THE ORIGIN

FIG. 6. Comparison field. Values of log $1/q$ in the field $w = kz^2$.



THESE FIGURES APPLY TO ANY SCALE PROVIDED THAT THE DIAMONDS ARE IN SIMILAR POSITIONS WITH RESPECT TO THE AXES

FIG. 7. Adjustments. Values of Δ (derived from Fig. 6) to be used when squaring the field $w = kz^2$.

A typical diamond is outlined heavily to identify it with that on Fig. 6.

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