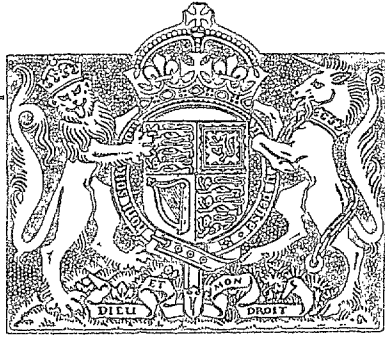


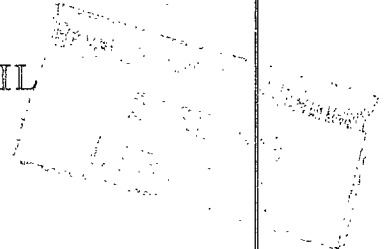
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A Calculation of the Complete
Downwash in Three Dimensions
due to a Rectangular Vortex

By

DORIS E. LEHRIAN, B.Sc.,
of the Aerodynamics Division, N.P.L.

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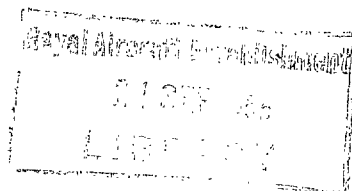
A Calculation of the Complete Downwash in Three Dimensions due to a Rectangular Vortex

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Summary.—A calculation of the complete downwash in three dimensions due to a rectangular vortex, is given for the limited range $Z = \pm 4$. The downwash is computed at selected positions, in planes normal to the plane of the vortex; these planes are spaced at even integral multiples of the semi-width of the vortex, measured from the line of symmetry. Values are tabulated for Z in the range (0,4) and a set of graphs is also included for $0 \leq Z \leq 2$; they are to be used in conjunction with the "Tables of Complete Downwash due to a Rectangular Vortex" (R. & M. 2461¹).

1. *Introduction.*—The complete downwash due to a rectangular vortex is calculated from the Glauert formula²

$$F(X, Y, Z) = \left\{ -\frac{X}{X^2 + Z^2} \left[\frac{Y + 1}{\sqrt{X^2 + (Y + 1)^2 + Z^2}} - \frac{Y - 1}{\sqrt{X^2 + (Y - 1)^2 + Z^2}} \right] \right. \\ \left. - \frac{Y - 1}{(Y - 1)^2 + Z^2} \left[1 - \frac{X}{\sqrt{X^2 + (Y - 1)^2 + Z^2}} \right] \right. \\ \left. + \frac{Y + 1}{(Y + 1)^2 + Z^2} \left[1 - \frac{X}{\sqrt{X^2 + (Y + 1)^2 + Z^2}} \right] \right\}$$

where the relation between the downwash ω/V due to a vortex of strength $K = 2Vy_v$ and the downwash factor F is defined as $F = 2\pi\omega/V$. The co-ordinates X, Y, Z are expressed in terms of y_v , the semi-width of the vortex, where the positive X -axis is upstream and the positive Z -axis downwards.

The presentation of the downwash is simplified by the following characteristics of the function F :—

$$F(X, +Y, Z) = F(X, -Y, Z)$$

$$F(X, Y, +Z) = F(X, Y, -Z).$$

The asymptotes of the function are

$$F(X = +\infty, Y, Z) = 0$$

and

$$F(X = -\infty, Y, Z) = -\frac{2(Y - 1)}{(Y - 1)^2 + Z^2} + \frac{2(Y + 1)}{(Y + 1)^2 + Z^2}.$$

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Also

$$F(+X, Y, Z) + F(-X, Y, Z) = F(X = -\infty, Y, Z).$$

Hence it is only necessary to tabulate the values of $F(X, Y, Z)$ for positive values of X, Y and Z , and the values of $F(X = -\infty, Y, Z)$. To simplify subsequent interpolation the functions $G(Z) = -F(X, Y, Z)$, $C_0 = +F(X = -\infty, Y = 0, Z)$, and $C_y = -F(X = -\infty, Y, Z)$ for $Y > 0$ are tabulated.

The function $F(X, Y = 0, Z)$ is a special case, since $F \rightarrow \infty$ at the position $X = 0$ when $Z \rightarrow 0$. This discontinuity is eliminated by considering the function

$$G'(Z) = \frac{2X}{X^2 + Z^2} \left[1 - \frac{1}{\sqrt{X^2 + Z^2 + 1}} \right] - \frac{2}{Z^2 + 1} \left[1 - \frac{X}{\sqrt{X^2 + Z^2 + 1}} \right]$$

where the downwash factor $F(+X, Y = 0, Z) = -\frac{2X}{X^2 + Z^2} + G'(Z)$.

Thus for values of Z in the range (0,2), values of $G'(Z)$ are computed for $0 \leq X \leq 3.2$ and values of $G(Z)$ for $X \geq 2.6$. For Z in the range (2,4) the function $G(Z)$ is computed for all values of X considered.

2. *Range of the Calculation.*—The intervals of Z in the range (0,2) are chosen so that linear interpolation gives an error not exceeding 0.001, but the intervals of Z in the range (2,4) are arbitrary and interpolation may give a considerable error. The values of C_0 and C_y are tabulated at intervals suitable for linear interpolation. The functions $G'(Z)$, $G(Z)$, C_0 and C_y are tabulated correct to four decimal places in Tables 1 to 9 for values of X, Y, Z as shown below.

Table	Y	X	Z	Linear Interpolation
1	0	$0 \leq X \leq 3.2$	$Z = 0(0.05)0.3(0.1)2.0$	w.r.t. Z for $G'(Z)$
2		$2.6 \leq X \leq 30$	$0(0.5)2.0$	w.r.t. Z for $G(Z)$
3		$0 \leq X \leq 30$	$2.0(0.1)4.0$	may be inaccurate
		$-\infty$	$0(0.01)2.0(0.05)4.0$	w.r.t. Z for C_0
4	2	$0 \leq X \leq 30$	$Z = 0(0.05)0.3(0.1)2.0$	w.r.t. Z for $G(Z)$
5		$-\infty$	$2.0(0.1)4.0$	may be inaccurate
			$0(0.01)2.0(0.05)4.0$	w.r.t. Z for C_2
6	4	$0 \leq X \leq 30$	$Z^2 = 0(1.0)4.0$	w.r.t. Z^2 for $G(Z)$
7		$-\infty$	$Z = 2.0(1.0)4.0$	may be inaccurate
			$0(0.05)4.0$	w.r.t. Z for C_4
8	6(2)20.	$0 \leq X \leq 30$	$Z = 0$ and 2.0	w.r.t. Z^2 for $G(Z)$
9		$-\infty$	$2.0(1.0)4.0$ 0 to 4.0	may be inaccurate w.r.t. Z for C_y

The variation between the downwash $-G(Z)$ for $Z \leq 4$ and the downwash for $Z = 0$, is less than 0.001 for all values of X when $Y > 20$, and for $X \geq 15$ when $Y \leq 20$. Hence for all $X \geq 15$ and $Y > 20$ the critical tables (R. & M. 2461¹) for $Z = 0$ are used. When $Y > 20$ the variation in C_y due to Z is also less than 0.001.

3. *Subsidiary Tables of Complete Downwash.*—The subsidiary downwash factors are presented in a similar manner to the main factors, and they are used when corrector vortices (R. & M. 2591³), which are one-quarter the width of the main vortex, have to be considered. The complete downwash at the position (X, Y, Z) due to a rectangular corrector vortex is $4F(X, Y, Z)$ where X, Y, Z are now co-ordinates in terms of one-quarter the semi-width of the main vortex, i.e., $\frac{1}{4}y_v$.

In general the known co-ordinates are $\frac{1}{4}X$, Y , $\frac{1}{4}Z$; and so values of $-4F(X, Y, Z) = G(\frac{1}{4}X, \frac{1}{4}Z)$ have been tabulated against these quantities. The function $G(\frac{1}{4}X, \frac{1}{4}Z)$ is calculated for values of $\frac{1}{4}Z$ in the range (0,4). For $0 \leq \frac{1}{4}Z \leq 2$, the intervals of $\frac{1}{4}Z$ are suitable for linear interpolation, but for $2 \leq \frac{1}{4}Z \leq 4$, interpolation may be inaccurate. The values of $C_y = -4F(X = -\infty, Y, Z)$ are also calculated at intervals of $\frac{1}{4}Z$ arranged for linear interpolation. The functions G and C_y are tabulated in Tables 10 to 15 for the values of $\frac{1}{4}X$, Y , $\frac{1}{4}Z$ as shown below.

Table	Y	$\frac{1}{4}X$	$\frac{1}{4}Z$	Linear Interpolation
10	13	$0 \leq \frac{1}{4}X \leq 15$	0(0.25)1.0(1.0)2.0	w.r.t. $\frac{1}{4}Z$ for G
11		$-\infty$	2.0(0.5)4.0 0(0.05)4.0	may be inaccurate w.r.t. $\frac{1}{4}Z$ for C_{13}
12	21	$0 \leq \frac{1}{4}X \leq 15$	0(0.5)1.0(1.0)2.0	w.r.t. $\frac{1}{4}Z$ for G
13		$-\infty$	2.0(1.0)4.0 0(0.10)4.0	may be inaccurate w.r.t. $\frac{1}{4}Z$ for C_{21}
14	29(8)53	$0 \leq \frac{1}{4}X \leq 15$	0 and 2.0	w.r.t. $\frac{1}{4}Z$ for G
15		$-\infty$	2.0 and 4.0 0 to 4.0	may be inaccurate w.r.t. $\frac{1}{4}Z$ for C_y

The variation between the downwash $-G(\frac{1}{4}X, \frac{1}{4}Z)$ for $\frac{1}{4}Z \leq 4$ and the downwash for $\frac{1}{4}Z = 0$ is less than 0.001 for all values of $\frac{1}{4}X$ when $Y > 53$, and for $\frac{1}{4}X \geq 10$ when $Y \leq 53$. The variation of C_y is also less than 0.001 for $Y > 53$.

4. *Graphs of the Downwash Factors.*—A graphical presentation of the factors avoids double interpolation, and if the intervals of Z are arranged for linear interpolation with intervals of X suitable for graphing, then for each value of Y a set of Z curves can be plotted against the X co-ordinate. Because of the great variation of the downwash with Z for the smaller values of X and Y , it is difficult to graph the values concisely and still retain sufficient accuracy.

A set of graphs has only been made for the range $0 \leq Z \leq 2$ (Figs. 1 to 7). The original graphs can be read to an accuracy of 0.001, but the originals of Figs. 1 to 4 are too large to reproduce in this report and the copies included are less than half size. The subsidiary downwash factors are also graphed in Figs. 8 to 10 for the range $0 \leq \frac{1}{4}Z \leq 2$.

5. *Conclusion.*—The downwash $F(X, Y, Z)$ at a position (X, Y, Z) for values of Z in the range (0,4) is calculated, by using Tables 1 to 9 of this report together with the critical tables (R. & M. 2461¹) for $Z = 0$, as indicated by the following diagram and formulae; the error in the value obtained, when corrected to three decimal places, will not exceed 0.001.

Y	$X < 0$	$0 \leq X \leq 15$	$X > 15$
0(2)20	Find $G(Z)$ for value $ X $, and value C_y	Find $G(Z)$	Use $Z = 0$ tables
22(2)88	Use $Z = 0$ tables	Use $Z = 0$ tables	Use $Z = 0$ tables

When $X \geq 0$

$$(i) \quad X \leq 3, Y = 0, Z \leq 2; \quad F(X, 0, \pm Z) = -G(Z) = -\frac{2X}{X^2 + Z^2} + G'(Z)$$

$$\left. \begin{array}{l} \text{(ii) } X \leq 3, Y = 0, Z > 2 \\ \text{(iii) } X \geq 3, Y = 0, Z \geq 0 \\ \text{(iv) } X \geq 0, Y > 0, Z \geq 0 \end{array} \right\} F(X, \pm Y, \pm Z) = -G(Z)$$

When $X < 0$

$$\text{(i) } Y = 0, F(X, \pm Y, \pm Z) = +C_0 + G(Z)$$

$$\text{(ii) } Y > 0, F(X, \pm Y, \pm Z) = -C_y + G(Z).$$

The subsidiary downwash $4F(X, Y, Z)$ for values of $\frac{1}{4}Z$ in the range (0,4) is calculated using Tables 10 to 15 and the critical tables (R. & M. 2461¹) for $Z = 0$ as indicated below; the error in the value obtained, when corrected to three decimal places, will not exceed 0.001.

Y	$\frac{1}{4}X < 0$	$0 \leq \frac{1}{4}X \leq 10$	$\frac{1}{4}X > 10$
13(8)53	Find $G(\frac{1}{4}X, \frac{1}{4}Z)$ for value $ \frac{1}{4}X $, and value C_y	Find $G(\frac{1}{4}X, \frac{1}{4}Z)$	Use $Z = 0$ tables
61(8)173	Use $Z = 0$ tables	Use $Z = 0$ tables	Use $Z = 0$ tables

When $\frac{1}{4}X \geq 0$

$$4F(X, Y, Z) = -G(\frac{1}{4}X, \frac{1}{4}Z).$$

When $\frac{1}{4}X < 0$

$$4F(X, Y, Z) = -C_y + G(\frac{1}{4}X, \frac{1}{4}Z).$$

The downwash for $0 \leq Z \leq 2$ can also be obtained by using Figs. 1 to 7 for the values $Y = 0(2)12$ and the range $0 \leq X \leq 10$, and the critical tables¹ for $Y > 12$ and $X > 10$, but there may then be a considerable error in the third decimal place. The subsidiary downwash for $0 \leq \frac{1}{4}Z \leq 2$ is obtained from Figs. 8 to 10, and from the critical tables (R. & M. 2461¹) for $Y > 53$ and $\frac{1}{4}X > 8$.

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3	V. M. Falkner	The Solution of Lifting Plane Problems by Vortex Lattice Theory. R. & M. 2591. September, 1947.

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 Values of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 0, 0 \leq Z \leq 2, 0 \leq X \leq 3.2$$

$$F(+X, 0, \pm Z) = -G(Z) = -\frac{2X}{X^2 + Z^2} + G'(Z)$$

5

X	Z	G'	Z	G'	Z	G'	Z	G'	Z	G'	Z	G'
0	0	2.0000	0.10	1.9802	0.20	1.9231	0.3	1.8349	0.5	1.6000	0.7	1.3423
0.1		1.9004		1.8826		1.8318		1.7530		1.5414		1.3061
0.2		1.8020		1.7865		1.7417		1.6720		1.4832		1.2698
0.3		1.7064		1.6930		1.6540		1.5929		1.4256		1.2334
0.4		1.6148		1.6032		1.5694		1.5162		1.3690		1.1969
0.5		1.5278		1.5179		1.4887		1.4427		1.3138		1.1604
0.6		1.4460		1.4375		1.4125		1.3727		1.2602		1.1240
0.7		1.3696		1.3622		1.3408		1.3065		1.2086		1.0879
0.8		1.2984		1.2922		1.2737		1.2442		1.1590		1.0523
0.9		1.2325		1.2271		1.2113		1.1859		1.1117		1.0174
1.0		1.1716		1.1669		1.1533		1.1313		1.0667		0.9833
1.2		1.0633		1.0598		1.0496		1.0330		0.9836		0.9183
1.4		0.9708		0.9681		0.9604		0.9477		0.9096		0.8581
1.6		0.8915		0.8895		0.8835		0.8737		0.8438		0.8029
1.8		0.8232		0.8216		0.8169		0.8092		0.7856		0.7527
2.0		0.7639		0.7627		0.7589		0.7528		0.7338		0.7072
2.6		0.6264		0.6257		0.6237		0.6203		0.6098		0.5948
3.2		0.5296		0.5292		0.5280		0.5260		0.5196		0.5105
0	0.05	1.9950	0.15	1.9560	0.25	1.8824	0.4	1.7241	0.6	1.4706	0.8	1.2195
0.1		1.8958		1.8611		1.7955		1.6536		1.4237		1.1927
0.2		1.7981		1.7676		1.7096		1.5836		1.3769		1.1657
0.3		1.7031		1.6765		1.6259		1.5149		1.3303		1.1384
0.4		1.6119		1.5889		1.5450		1.4479		1.2841		1.1106
0.5		1.5253		1.5056		1.4676		1.3831		1.2385		1.0823
0.6		1.4439		1.4270		1.3943		1.3210		1.1937		1.0536
0.7		1.3677		1.3532		1.3251		1.2616		1.1499		1.0246
0.8		1.2969		1.2844		1.2603		1.2053		1.1073		0.9956
0.9		1.2312		1.2205		1.1997		1.1522		1.0662		0.9666
1.0		1.1704		1.1612		1.1433		1.1020		1.0266		0.9379
1.2		1.0624		1.0555		1.0421		1.0108		0.9525		0.8820
1.4		0.9701		0.9649		0.9546		0.9306		0.8852		0.8290
1.6		0.8910		0.8870		0.8790		0.8604		0.8246		0.7795
1.8		0.8228		0.8196		0.8134		0.7987		0.7702		0.7337
2.0		0.7636		0.7611		0.7562		0.7444		0.7214		0.6916
2.6		0.6262		0.6249		0.6222		0.6157		0.6028		0.5858
3.2		0.5295		0.5287		0.5271		0.5232		0.5154		0.5049

TABLE 1—continued

Y = 0

X	Z	G'	Z	G'	Z	G'	Z	G'	Z	G'	Z	G'
0	0.9	1.1050	1.1	0.9050	1.3	0.7435	1.5	0.6154	1.7	0.5141	1.9	0.4338
0.1		1.0862		0.8982		0.7443		0.6208		0.5222		0.4432
0.2		1.0671		0.8910		0.7448		0.6259		0.5299		0.4524
0.3		1.0475		0.8832		0.7446		0.6305		0.5372		0.4612
0.4		1.0272		0.8745		0.7436		0.6343		0.5439		0.4694
0.5		1.0061		0.8647		0.7415		0.6371		0.5498		0.4770
0.6		0.9843		0.8538		0.7382		0.6388		0.5546		0.4838
0.7		0.9618		0.8418		0.7337		0.6394		0.5585		0.4896
0.8		0.9387		0.8287		0.7279		0.6387		0.5612		0.4945
0.9		0.9152		0.8146		0.7210		0.6368		0.5628		0.4984
1.0		0.8916		0.7997		0.7129		0.6338		0.5633		0.5013
1.2		0.8444		0.7682		0.6939		0.5245		0.5611		0.5042
1.4		0.7985		0.7353		0.6722		0.6116		0.5552		0.5034
1.6		0.7546		0.7022		0.6486		0.5962		0.5462		0.4996
1.8		0.7132		0.6697		0.6243		0.5789		0.5349		0.4931
2.0		0.6747		0.6383		0.5998		0.5606		0.5219		0.4846
2.6		0.5759		0.5540		0.5299		0.5044		0.4783		0.4520
3.2		0.4987		0.4848		0.4691		0.4521		0.4342		0.4158
0	1.0	1.0000	1.2	0.8197	1.4	0.6757	1.6	0.5618	1.8	0.4717	2.0	0.4000
0.1		0.9878		0.8171		0.6791		0.5687		0.4805		0.4097
0.2		0.9753		0.8142		0.6822		0.5753		0.4891		0.4193
0.3		0.9622		0.8107		0.6847		0.5815		0.4973		0.4285
0.4		0.9482		0.8062		0.6864		0.5869		0.5049		0.4372
0.5		0.9333		0.8007		0.6871		0.5914		0.5117		0.4453
0.6		0.9174		0.7940		0.6866		0.5949		0.5176		0.4527
0.7		0.9005		0.7861		0.6848		0.5973		0.5226		0.4592
0.8		0.8828		0.7770		0.6818		0.5985		0.5266		0.4649
0.9		0.8643		0.7667		0.6776		0.5985		0.5295		0.4696
1.0		0.8453		0.7555		0.6723		0.5974		0.5313		0.4734
1.2		0.8063		0.7306		0.6585		0.5920		0.5318		0.4782
1.4		0.7671		0.7035		0.6415		0.5828		0.5287		0.4794
1.6		0.7287		0.6754		0.6222		0.5708		0.5224		0.4776
1.8		0.6918		0.6471		0.6015		0.5567		0.5137		0.4732
2.0		0.6569		0.6192		0.5802		0.5411		0.5031		0.4667
2.6		0.5652		0.5421		0.5173		0.4914		0.4651		0.4391
3.2		0.4920		0.4771		0.4608		0.4433		0.4250		0.4064

TABLE 2

*Values of Complete Downwash in Three
Dimensions due to a Rectangular Vortex*

$$Y = 0, \quad 0 \leq Z \leq 2, \quad 2.6 \leq X \leq 30$$

$$2 \leq Z \leq 4, \quad 0 \leq X \leq 30$$

$$F(+X, 0, \pm Z) = -G(Z)$$

<i>X</i>	<i>Z</i>	<i>G</i>	<i>Z</i>	<i>G</i>	<i>Z</i>	<i>G</i>
2.6	0	0.1428	1.0	0.1049	2.0	0.0442
3.2		0.0954		0.0774		0.0430
3.8		0.0681		0.0586		0.0382
4.4		0.0510		0.0456		0.0329
5.0		0.0396		0.0363		0.0281
6.0		0.0276		0.0259		0.0217
8.0		0.0156		0.0150		0.0136
10.0		0.0100		0.0098		0.0091
15.0		0.0044		0.0044		0.0043
20.0		0.0025		0.0025		0.0024
25.0		0.0016		0.0016		0.0016
30.0		0.0011		0.0011		0.0011
2.6		0.5		0.1320		1.5
3.2	0.0904		0.0603			
3.8	0.0656		0.0489			
4.4	0.0496		0.0397			
5.0	0.0387		0.0326			
6.0	0.0272		0.0240			
8.0	0.0154		0.0144			
10.0	0.0099		0.0095			
15.0	0.0044		0.0044			
20.0	0.0025		0.0025			
25.0	0.0016		0.0016			
30.0	0.0011		0.0011			

L

TABLE 2—continued

$Y = 0$

X	Z	G	Z	G	Z	G	Z	G	Z	G
0	2.0	- 0.4000	2.1	- 0.3697	2.2	- 0.3425	2.3	- 0.3180	2.4	- 0.2959
0.1		- 0.3598		- 0.3344		- 0.3113		- 0.2903		- 0.2712
0.2		- 0.3203		- 0.2995		- 0.2804		- 0.2629		- 0.2467
0.3		- 0.2818		- 0.2656		- 0.2503		- 0.2360		- 0.2228
0.4		- 0.2449		- 0.2329		- 0.2212		- 0.2101		- 0.1995
0.5		- 0.2100		- 0.2018		- 0.1935		- 0.1852		- 0.1772
0.6		- 0.1775		- 0.1726		- 0.1673		- 0.1616		- 0.1558
0.7		- 0.1474		- 0.1455		- 0.1428		- 0.1395		- 0.1358
0.8		- 0.1201		- 0.1206		- 0.1201		- 0.1188		- 0.1170
0.9		- 0.0954		- 0.0980		- 0.0994		- 0.0998		- 0.0995
1.0		- 0.0734		- 0.0776		- 0.0806		- 0.0824		- 0.0834
1.2		- 0.0370		- 0.0434		- 0.0485		- 0.0525		- 0.0555
1.4		- 0.0096		- 0.0171		- 0.0234		- 0.0285		- 0.0328
1.6		+ 0.0102		+ 0.0025		- 0.0042		- 0.0099		- 0.0148
1.8		0.0240		0.0166		+ 0.0100		+ 0.0041		- 0.0010
2.0		0.0333		0.0264		0.0201		0.0145		+ 0.0095
2.6		0.0442		0.0393		0.0347		0.0303		0.0263
3.2		0.0430		0.0398		0.0367		0.0336		0.0308
3.8		0.0382		0.0361		0.0340		0.0320		0.0300
4.4		0.0329		0.0315		0.0301		0.0288		0.0274
5.0		0.0281		0.0272		0.0262		0.0253		0.0243
6.0		0.0217		0.0212		0.0206		0.0201		0.0196
8.0		0.0136		0.0134		0.0132		0.0130		0.0128
10.0		0.0091		0.0090		0.0090		0.0089		0.0088
15.0		0.0043		0.0042		0.0042		0.0042		0.0042
20.0		0.0024		0.0024		0.0024		0.0024		0.0024
25.0		0.0016		0.0016		0.0016		0.0016		0.0016
30.0		0.0011		0.0011		0.0011		0.0011		0.0011

TABLE 2—continued

$$Y = 0$$

X	Z	G	Z	G	Z	G	Z	G	Z	G
0	2.5	- 0.2759	2.6	- 0.2577	2.7	- 0.2412	2.8	- 0.2262	2.9	- 0.2125
0.1		- 0.2538		- 0.2379		- 0.2234		- 0.2101		- 0.1979
0.2		- 0.2319		- 0.2182		- 0.2056		- 0.1940		- 0.1833
0.3		- 0.2104		- 0.1989		- 0.1882		- 0.1782		- 0.1690
0.4		- 0.1895		- 0.1800		- 0.1711		- 0.1628		- 0.1549
0.5		- 0.1693		- 0.1618		- 0.1546		- 0.1477		- 0.1412
0.6		- 0.1500		- 0.1443		- 0.1387		- 0.1332		- 0.1280
0.7		- 0.1318		- 0.1277		- 0.1235		- 0.1194		- 0.1153
0.8		- 0.1146		- 0.1120		- 0.1091		- 0.1062		- 0.1031
0.9		- 0.0986		- 0.0973		- 0.0956		- 0.0937		- 0.0916
1.0		- 0.0838		- 0.0836		- 0.0830		- 0.0820		- 0.0808
1.2		- 0.0577		- 0.0593		- 0.0603		- 0.0609		- 0.0611
1.4		- 0.0362		- 0.0390		- 0.0412		- 0.0429		- 0.0442
1.6		- 0.0190		- 0.0225		- 0.0254		- 0.0279		- 0.0299
1.8		- 0.0054		- 0.0093		- 0.0127		- 0.0156		- 0.0181
2.0		+ 0.0050		+ 0.0010		- 0.0026		- 0.0057		- 0.0084
2.6		0.0226		0.0191		+ 0.0158		+ 0.0129		+ 0.0101
3.2		0.0280		0.0254		0.0229		0.0205		0.0183
3.8		0.0281		0.0262		0.0244		0.0226		0.0209
4.4		0.0260		0.0247		0.0234		0.0221		0.0209
5.0		0.0234		0.0224		0.0215		0.0206		0.0196
6.0		0.0190		0.0184		0.0179		0.0174		0.0168
8.0		0.0126		0.0124		0.0122		0.0119		0.0117
10.0		0.0087		0.0086		0.0085		0.0084		0.0083
15.0		0.0042		0.0042		0.0041		0.0041		0.0041
20.0		0.0024		0.0024		0.0024		0.0024		0.0024
25.0		0.0016		0.0016		0.0016		0.0016		0.0016
30.0		0.0011		0.0011		0.0011		0.0011		0.0011

TABLE 2—continued

$$Y = 0$$

X	Z	G	Z	G	Z	G	Z	G	Z	G
0	3.0	- 0.2000	3.1	- 0.1885	3.2	- 0.1779	3.3	- 0.1682	3.4	- 0.1592
0.1		- 0.1867		- 0.1763		- 0.1668		- 0.1580		- 0.1499
0.2		- 0.1734		- 0.1642		- 0.1558		- 0.1479		- 0.1406
0.3		- 0.1603		- 0.1523		- 0.1448		- 0.1378		- 0.1313
0.4		- 0.1475		- 0.1406		- 0.1341		- 0.1280		- 0.1222
0.5		- 0.1350		- 0.1291		- 0.1236		- 0.1183		- 0.1133
0.6		- 0.1229		- 0.1180		- 0.1134		- 0.1089		- 0.1046
0.7		- 0.1112		- 0.1073		- 0.1035		- 0.0998		- 0.0962
0.8		- 0.1001		- 0.0970		- 0.0940		- 0.0910		- 0.0881
0.9		- 0.0894		- 0.0872		- 0.0849		- 0.0826		- 0.0802
1.0		- 0.0794		- 0.0779		- 0.0762		- 0.0745		- 0.0728
1.2		- 0.0611		- 0.0608		- 0.0603		- 0.0596		- 0.0588
1.4		- 0.0452		- 0.0458		- 0.0462		- 0.0464		- 0.0464
1.6		- 0.0316		- 0.0329		- 0.0340		- 0.0348		- 0.0354
1.8		- 0.0202		- 0.0220		- 0.0236		- 0.0249		- 0.0259
2.0		- 0.0109		- 0.0130		- 0.0148		- 0.0164		- 0.0178
2.6		+ 0.0076		+ 0.0053		+ 0.0032		+ 0.0013		- 0.0005
3.2		0.0162		0.0142		0.0124		0.0106		+ 0.0090
3.8		0.0193		0.0178		0.0163		0.0148		0.0135
4.4		0.0197		0.0185		0.0174		0.0162		0.0152
5.0		0.0188		0.0179		0.0170		0.0161		0.0153
6.0		0.0162		0.0157		0.0152		0.0146		0.0141
8.0		0.0115		0.0112		0.0110		0.0108		0.0106
10.0		0.0082		0.0181		0.0080		0.0079		0.0078
15.0		0.0041		0.0040		0.0040		0.0040		0.0040
20.0		0.0024		0.0024		0.0024		0.0024		0.0023
25.0		0.0016		0.0015		0.0015		0.0015		0.0015
30.0		0.0011		0.0011		0.0011		0.0011		0.0011

TABLE 2—continued

Y = 0

X	Z	G	Z	G	Z	G	Z	G	Z	G	Z	G
0	3.5	- 0.1509	3.6	- 0.1433	3.7	- 0.1362	3.8	- 0.1295	3.9	- 0.1234	4.0	- 0.1176
0.1		- 0.1423		- 0.1353		- 0.1288		- 0.1227		- 0.1171		- 0.1118
0.2		- 0.1337		- 0.1274		- 0.1215		- 0.1159		- 0.1108		- 0.1059
0.3		- 0.1252		- 0.1195		- 0.1142		- 0.1092		- 0.1045		- 0.1001
0.4		- 0.1168		- 0.1118		- 0.1070		- 0.1025		- 0.0983		- 0.0943
0.5		- 0.1086		- 0.1042		- 0.1000		- 0.0960		- 0.0922		- 0.0887
0.6		- 0.1006		- 0.0967		- 0.0931		- 0.0896		- 0.0863		- 0.0831
0.7		- 0.0928		- 0.0895		- 0.0864		- 0.0833		- 0.0804		- 0.0777
0.8		- 0.0852		- 0.0825		- 0.0798		- 0.0772		- 0.0747		- 0.0724
0.9		- 0.0780		- 0.0757		- 0.0735		- 0.0713		- 0.0692		- 0.0672
1.0		- 0.0710		- 0.0692		- 0.0674		- 0.0656		- 0.0639		- 0.0622
1.2		- 0.0579		- 0.0570		- 0.0560		- 0.0549		- 0.0538		- 0.0527
1.4		- 0.0462		- 0.0460		- 0.0456		- 0.0451		- 0.0446		- 0.0440
1.6		- 0.0359		- 0.0361		- 0.0363		- 0.0363		- 0.0362		- 0.0361
1.8		- 0.0268		- 0.0275		- 0.0280		- 0.0285		- 0.0288		- 0.0290
2.0		- 0.0190		- 0.0200		- 0.0209		- 0.0216		- 0.0222		- 0.0227
2.6		- 0.0021		- 0.0035		- 0.0048		- 0.0060		- 0.0071		- 0.0080
3.2		+ 0.0074		+ 0.0060		+ 0.0047		+ 0.0034		+ 0.0023		+ 0.0012
3.8		0.0122		0.0109		0.0098		0.0087		0.0076		0.0066
4.4		0.0141		0.0131		0.0121		0.0112		0.0103		0.0095
5.0		0.0145		0.0137		0.0129		0.0122		0.0115		0.0108
6.0		0.0136		0.0130		0.0125		0.0120		0.0115		0.0110
8.0		0.0103		0.0101		0.0099		0.0096		0.0094		0.0092
10.0		0.0076		0.0075		0.0074		0.0073		0.0072		0.0071
15.0		0.0039		0.0039		0.0039		0.0038		0.0038		0.0038
20.0		0.0023		0.0023		0.0023		0.0023		0.0023		0.0023
25.0		0.0015		0.0015		0.0015		0.0015		0.0015		0.0015
30.0		0.0011		0.0011		0.0011		0.0011		0.0011		0.0011

TABLE 3

Table of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 0, \quad X = -\infty, \quad C_0 = + F(-\infty, 0, \pm Z)$$

Z	C ₀	Δ	Z	C ₀	Δ
0	4.0000	4	0.50	3.2000	256
0.01	3.9996	12	0.51	3.1744	258
0.02	3.9984	20	0.52	3.1486	258
0.03	3.9964	28	0.53	3.1228	259
0.04	3.9936	36	0.54	3.0969	259
0.05	3.9900	44	0.55	3.0710	259
0.06	3.9856	51	0.56	3.0451	260
0.07	3.9805	59	0.57	3.0191	260
0.08	3.9746	67	0.58	2.9931	260
0.09	3.9679	75	0.59	2.9671	259
0.10	3.9604	82	0.60	2.9412	260
0.11	3.9522	90	0.61	2.9152	259
0.12	3.9432	97	0.62	2.8893	258
0.13	3.9335	104	0.63	2.8635	258
0.14	3.9231	111	0.64	2.8377	257
0.15	3.9120	118	0.65	2.8120	257
0.16	3.9002	126	0.66	2.7863	256
0.17	3.8876	131	0.67	2.7607	255
0.18	3.8745	139	0.68	2.7352	254
0.19	3.8606	144	0.69	2.7098	252
0.20	3.8462	152	0.70	2.6846	252
0.21	3.8310	157	0.71	2.6594	250
0.22	3.8153	163	0.72	2.6344	250
0.23	3.7990	168	0.73	2.6094	248
0.24	3.7822	175	0.74	2.5846	246
0.25	3.7647	180	0.75	2.5600	245
0.26	3.7467	185	0.76	2.5355	244
0.27	3.7282	190	0.77	2.5111	242
0.28	3.7092	195	0.78	2.4869	240
0.29	3.6897	200	0.79	2.4629	239
0.30	3.6697	204	0.80	2.4390	237
0.31	3.6493	209	0.81	2.4153	235
0.32	3.6284	212	0.82	2.3918	234
0.33	3.6072	217	0.83	2.3684	232
0.34	3.5855	220	0.84	2.3452	230
0.35	3.5635	224	0.85	2.3222	228
0.36	3.5411	228	0.86	2.2994	227
0.37	3.5183	230	0.87	2.2767	224
0.38	3.4953	234	0.88	2.2543	223
0.39	3.4719	236	0.89	2.2320	221
0.40	3.4483	239	0.90	2.2099	218
0.41	3.4244	242	0.91	2.1881	217
0.42	3.4002	244	0.92	2.1664	215
0.43	3.3758	246	0.93	2.1449	213
0.44	3.3512	248	0.94	2.1236	211
0.45	3.3264	250	0.95	2.1025	209
0.46	3.3014	251	0.96	2.0816	207
0.47	3.2763	253	0.97	2.0609	205
0.48	3.2510	254	0.98	2.0404	203
0.49	3.2256	256	0.99	2.0201	201
0.50	3.2000		1.00	2.0000	

TABLE 3—continued

$$Y = 0$$

Z	C ₀	Δ	Z	C ₀	Δ	Z	C ₀	Δ
1.00	2.0000		1.50	1.2308		2.00	0.8000	
1.01	1.9801	199	1.51	1.2195	113	2.05	0.7689	311
1.02	1.9604	197	1.52	1.2083	112	2.10	0.7394	295
1.03	1.9409	195	1.53	1.1973	110	2.15	0.7114	280
1.04	1.9216	193	1.54	1.1864	109	2.20	0.6849	265
1.05	1.9025	191	1.55	1.1756	108	2.25	0.6598	251
1.06	1.8836	189	1.56	1.1650	106	2.30	0.6359	239
1.07	1.8649	187	1.57	1.1544	106	2.35	0.6133	226
1.08	1.8464	185	1.58	1.1440	104	2.40	0.5917	216
1.09	1.8281	183	1.59	1.1338	102	2.45	0.5712	205
1.10	1.8100	181	1.60	1.1236	102	2.50	0.5517	195
1.11	1.7920	180	1.61	1.1136	100	2.55	0.5332	185
1.12	1.7743	177	1.62	1.1036	100	2.60	0.5155	177
1.13	1.7568	175	1.63	1.0938	98	2.65	0.4986	169
1.14	1.7394	174	1.64	1.0841	97	2.70	0.4825	161
1.15	1.7223	171	1.65	1.0746	95	2.75	0.4672	153
1.16	1.7053	170	1.66	1.0651	95	2.80	0.4525	147
1.17	1.6886	167	1.67	1.0557	94	2.85	0.4385	140
1.18	1.6720	166	1.68	1.0465	92	2.90	0.4251	134
1.19	1.6556	164	1.69	1.0373	92	2.95	0.4123	128
1.20	1.6393	163	1.70	1.0283	90	3.00	0.4000	123
1.21	1.6233	160	1.71	1.0193	90	3.05	0.3883	117
1.22	1.6075	158	1.72	1.0105	88	3.10	0.3770	113
1.23	1.5918	157	1.73	1.0018	87	3.15	0.3662	108
1.24	1.5763	155	1.74	0.9932	86	3.20	0.3559	103
1.25	1.5610	153	1.75	0.9846	86	3.25	0.3460	99
1.26	1.5458	152	1.76	0.9762	84	3.30	0.3364	96
1.27	1.5309	149	1.77	0.9678	84	3.35	0.3273	91
1.28	1.5161	148	1.78	0.9596	82	3.40	0.3185	88
1.29	1.5014	147	1.79	0.9514	82	3.45	0.3100	85
1.30	1.4870	144	1.80	0.9434	80	3.50	0.3019	81
1.31	1.4727	143	1.81	0.9354	80	3.55	0.2941	78
1.32	1.4586	141	1.82	0.9276	78	3.60	0.2865	76
1.33	1.4446	140	1.83	0.9198	78	3.65	0.2793	72
1.34	1.4308	138	1.84	0.9121	77	3.70	0.2723	70
1.35	1.4172	136	1.85	0.9045	76	3.75	0.2656	67
1.36	1.4037	135	1.86	0.8969	76	3.80	0.2591	65
1.37	1.3904	133	1.87	0.8895	74	3.85	0.2528	63
1.38	1.3772	132	1.88	0.8822	73	3.90	0.2468	60
1.39	1.3642	130	1.89	0.8749	73	3.95	0.2409	59
1.40	1.3514	128	1.90	0.8677	72	4.00	0.2353	56
1.41	1.3386	128	1.91	0.8606	71			
1.42	1.3261	125	1.92	0.8535	71			
1.43	1.3137	124	1.93	0.8466	69			
1.44	1.3014	123	1.94	0.8397	69			
1.45	1.2893	121	1.95	0.8329	68			
1.46	1.2773	120	1.96	0.8262	67			
1.47	1.2655	118	1.97	0.8195	67			
1.48	1.2538	117	1.98	0.8129	66			
1.49	1.2422	116	1.99	0.8064	65			
1.50	1.2308	114	2.00	0.8000	64			

TABLE 4
Values of Complete Downwash in Three Dimensions due to a Rectangular Vortex
 $Y = 2, \quad 0 \leq Z \leq 4, \quad 0 \leq X \leq 30$ $F(+X, \pm 2, \pm Z) = -G(Z)$

X	Z	G	Z	G	Z	G	Z	G	Z	G
0	0	0.6667	0.10	0.6571	0.20	0.6297	0.3	0.5874	0.5	0.4757
0.4		0.4962		0.4908		0.4747		0.4497		0.3810
0.8		0.3596		0.3567		0.3483		0.3348		0.2963
1.2		0.2625		0.2610		0.2565		0.2493		0.2280
1.6		0.1957		0.1949		0.1925		0.1884		0.1763
2.0		0.1496		0.1491		0.1476		0.1453		0.1380
2.6		0.1042		0.1040		0.1033		0.1021		0.0985
3.2		0.0759		0.0758		0.0754		0.0748		0.0728
3.8		0.0573		0.0572		0.0570		0.0567		0.0556
4.4		0.0446		0.0446		0.0444		0.0442		0.0435
5.0		0.0356		0.0356		0.0355		0.0354		0.0349
6.0		0.0256		0.0256		0.0255		0.0254		0.0252
8.0		0.0149		0.0149		0.0149		0.0148		0.0148
10.0		0.0097		0.0097		0.0097		0.0097		0.0096
15.0		0.0044		0.0044		0.0044		0.0044		0.0044
20.0		0.0025		0.0025		0.0025		0.0025		0.0025
25.0		0.0016		0.0016		0.0016		0.0016		0.0016
30.0	0.0011	0.0011	0.0011	0.0011	0.0011					
0	0.05	0.6643	0.15	0.6455	0.25	0.6102	0.4	0.5346	0.6	0.4148
0.4		0.4949		0.4840		0.4632		0.4177		0.3418
0.8		0.3588		0.3532		0.3421		0.3172		0.2732
1.2		0.2621		0.2591		0.2532		0.2397		0.2147
1.6		0.1955		0.1939		0.1906		0.1830		0.1685
2.0		0.1494		0.1485		0.1466		0.1420		0.1333
2.6		0.1042		0.1037		0.1028		0.1005		0.0962
3.2		0.0759		0.0756		0.0751		0.0739		0.0715
3.8		0.0573		0.0571		0.0568		0.0562		0.0548
4.4		0.0446		0.0445		0.0443		0.0439		0.0431
5.0		0.0356		0.0355		0.0354		0.0352		0.0346
6.0		0.0256		0.0255		0.0254		0.0253		0.0250
8.0		0.0149		0.0149		0.0148		0.0148		0.0147
10.0		0.0097		0.0097		0.0097		0.0096		0.0096
15.0		0.0044		0.0044		0.0044		0.0044		0.0044
20.0		0.0025		0.0025		0.0025		0.0025		0.0025
25.0		0.0016		0.0016		0.0016		0.0016		0.0016
30.0	0.0011	0.0011	0.0011	0.0011	0.0011					

TABLE 4—continued

Y = 2

X	Z	G	Z	G	Z	G	Z	G	Z	G
0	0.7	0.3550	0.9	0.2467	1.1	0.1587	1.3	0.0911	1.5	0.0410
0.4		0.3020		0.2260		0.1599		0.1060		0.0635
0.8		0.2488		0.1995		0.1533		0.1127		0.0786
1.2		0.2002		0.1695		0.1387		0.1099		0.0841
1.6		0.1598		0.1408		0.1207		0.1008		0.0821
2.0		0.1280		0.1160		0.1028		0.0893		0.0760
2.6		0.0934		0.0871		0.0800		0.0722		0.0643
3.2		0.0700		0.0665		0.0624		0.0579		0.0531
3.8		0.0539		0.0518		0.0494		0.0466		0.0436
4.4		0.0425		0.0412		0.0397		0.0379		0.0360
5.0		0.0342		0.0334		0.0324		0.0312		0.0299
6.0		0.0248		0.0244		0.0239		0.0232		0.0225
8.0		0.0146		0.0145		0.0143		0.0141		0.0138
10.0		0.0096		0.0095		0.0094		0.0093		0.0092
15.0		0.0044		0.0044		0.0043		0.0043		0.0043
20.0	0.0025	0.0025	0.0024	0.0025	0.0024					
25.0	0.0016	0.0016	0.0016	0.0016	0.0016					
30.0	0.0011	0.0011	0.0011	0.0011	0.0011					
0	0.8	0.2986	1.0	0.2000	1.2	0.1225	1.4	0.0641	1.6	0.0214
0.4		0.2630		0.1915		0.1314		0.0834		0.0461
0.8		0.2240		0.1758		0.1322		0.0948		0.0639
1.2		0.1850		0.1540		0.1240		0.0966		0.0725
1.6		0.1505		0.1308		0.1107		0.0913		0.0734
2.0		0.1222		0.1095		0.0961		0.0826		0.0696
2.6		0.0904		0.0836		0.0761		0.0683		0.0604
3.2		0.0684		0.0645		0.0602		0.0555		0.0506
3.8		0.0529		0.0506		0.0480		0.0451		0.0420
4.4		0.0419		0.0405		0.0388		0.0370		0.0350
5.0		0.0338		0.0329		0.0318		0.0306		0.0292
6.0		0.0246		0.0242		0.0236		0.0229		0.0222
8.0		0.0146		0.0144		0.0142		0.0140		0.0137
10.0		0.0096		0.0095		0.0094		0.0093		0.0092
15.0		0.0043		0.0043		0.0043		0.0043		0.0043
20.0	0.0025	0.0025	0.0025	0.0025	0.0024					
25.0	0.0016	0.0016	0.0016	0.0016	0.0016					
30.0	0.0011	0.0011	0.0011	0.0011	0.0011					

TABLE 4—continued

Y = 2

X	Z	G	Z	G	Z	G	Z	G
0	1.7	0.0048	1.9	— 0.0210	2.1	— 0.0389	2.3	— 0.0510
0.4		0.0309		+ 0.0064		— 0.0117		— 0.0249
0.8		0.0507		0.0284		+ 0.0111		— 0.0023
1.2		0.0618		0.0429		0.0274		+ 0.0147
1.6		0.0651		0.0500		0.0370		0.0258
2.0		0.0634		0.0518		0.0413		0.0320
2.6		0.0565		0.0488		0.0416		0.0350
3.2		0.0481		0.0432		0.0384		0.0337
3.8		0.0404		0.0372		0.0339		0.0307
4.4		0.0339		0.0317		0.0295		0.0272
5.0		0.0285		0.0270		0.0255		0.0239
6.0		0.0217		0.0209		0.0200		0.0191
8.0		0.0135		0.0132		0.0129		0.0125
10.0	0.0091	0.0090	0.0088	0.0087				
15.0	0.0042	0.0042	0.0042	0.0042				
20.0	0.0024	0.0024	0.0024	0.0024				
25.0	0.0016	0.0016	0.0016	0.0016				
30.0	0.0011	0.0011	0.0011	0.0011				
0	1.8	— 0.0092	2.0	— 0.0308	2.2	— 0.0455	2.4	— 0.0553
0.4		+ 0.0178		— 0.0034		— 0.0188		— 0.0300
0.8		0.0390		+ 0.0193		+ 0.0040		— 0.0078
1.2		0.0519		0.0348		0.0207		+ 0.0093
1.6		0.0573		0.0432		0.0312		0.0209
2.0		0.0575		0.0464		0.0365		0.0278
2.6		0.0526		0.0452		0.0382		0.0318
3.2		0.0456		0.0407		0.0360		0.0314
3.8		0.0388		0.0355		0.0323		0.0291
4.4		0.0328		0.0306		0.0284		0.0261
5.0		0.0278		0.0262		0.0247		0.0231
6.0		0.0213		0.0205		0.0196		0.0186
8.0		0.0134		0.0130		0.0127		0.0123
10.0	0.0090	0.0089	0.0087	0.0086				
15.0	0.0042	0.0042	0.0042	0.0041				
20.0	0.0024	0.0024	0.0024	0.0024				
25.0	0.0016	0.0016	0.0016	0.0016				
30.0	0.0011	0.0011	0.0011	0.0011				

TABLE 4—continued

Y = 2

X	Z	G	Z	G	Z	G	Z	G
0	2.5	- 0.0588	2.7	- 0.0635	2.9	- 0.0660	3.1	- 0.0670
0.4		- 0.0342		- 0.0407		- 0.0450		- 0.0477
0.8		- 0.0126		- 0.0202		- 0.0259		- 0.0299
1.2		+ 0.0045		- 0.0036		- 0.0099		- 0.0149
1.6		0.0164		+ 0.0087		+ 0.0023		- 0.0030
2.0		0.0239		0.0169		0.0109		+ 0.0058
2.6		0.0288		0.0233		0.0184		0.0140
3.2		0.0292		0.0251		0.0213		0.0178
3.8		0.0275		0.0245		0.0216		0.0189
4.4		0.0250		0.0228		0.0207		0.0186
3.0		0.0223		0.0207		0.0191		0.0175
6.0		0.0182		0.0172		0.0162		0.0152
8.0		0.0121		0.0118		0.0114		0.0109
10.0		0.0085		0.0083		0.0081		0.0079
15.0		0.0041		0.0041		0.0040		0.0040
20.0		0.0024		0.0024		0.0024		0.0023
25.0		0.0016		0.0016		0.0015		0.0015
30.0	0.0011	0.0011	0.0011	0.0011				
0	2.6	- 0.0615	2.8	- 0.0650	3.0	- 0.0667	3.2	- 0.0670
0.4		- 0.0378		- 0.0431		- 0.0465		- 0.0485
0.8		- 0.0167		- 0.0233		- 0.0281		- 0.0315
1.2		+ 0.0002		- 0.0070		- 0.0126		- 0.0169
1.6		0.0124		+ 0.0053		- 0.0005		- 0.0052
2.0		0.0202		0.0137		+ 0.0082		+ 0.0036
2.6		0.0260		0.0207		0.0161		0.0120
3.2		0.0271		0.0232		0.0195		0.0161
3.8		0.0260		0.0230		0.0203		0.0176
4.4		0.0239		0.0217		0.0196		0.0176
5.0		0.0215		0.0199		0.0183		0.0168
6.0		0.0177		0.0167		0.0157		0.0148
8.0		0.0119		0.0116		0.0112		0.0107
10.0		0.0084		0.0082		0.0080		0.0078
15.0		0.0041		0.0040		0.0040		0.0040
20.0		0.0024		0.0024		0.0024		0.0023
25.0		0.0016		0.0015		0.0015		0.0015
30.0	0.0011	0.0011	0.0011	0.0011				

TABLE 4—continued

$$Y = 2$$

X	Z	G	Z	G	Z	G	Z	G
0	3.3	- 0.0667	3.5	- 0.0657	3.7	- 0.0642	3.9	- 0.0622
0.4		- 0.0491		- 0.0497		- 0.0495		- 0.0489
0.8		- 0.0328		- 0.0346		- 0.0358		- 0.0363
1.2		- 0.0186		- 0.0214		- 0.0235		- 0.0250
1.6		- 0.0072		- 0.0105		- 0.0132		- 0.0153
2.0		+ 0.0015		- 0.0020		- 0.0050		- 0.0074
2.6		0.0101		+ 0.0067		+ 0.0038		+ 0.0013
3.2		0.0146		0.0117		0.0091		0.0068
3.8		0.0164		0.0140		0.0118		0.0099
4.4		0.0166		0.0148		0.0130		0.0114
5.0		0.0160		0.0146		0.0132		0.0119
6.0		0.0143		0.0133		0.0124		0.0115
8.0		0.0105		0.0101		0.0096		0.0092
10.0		0.0077		0.0075		0.0073		0.0070
15.0		0.0039		0.0039		0.0038		0.0038
20.0	0.0023	0.0023	0.0023	0.0023				
25.0	0.0015	0.0015	0.0015	0.0015				
30.0	0.0011	0.0011	0.0010	0.0010				
0	3.4	- 0.0663	3.6	- 0.0650	3.8	- 0.0632	4.0	- 0.0612
0.4		- 0.0495		- 0.0496		- 0.0493		- 0.0485
0.8		- 0.0338		- 0.0353		- 0.0361		- 0.0364
1.2		- 0.0201		- 0.0226		- 0.0243		- 0.0255
1.6		- 0.0089		- 0.0119		- 0.0143		- 0.0161
2.0		- 0.0003		- 0.0036		- 0.0062		- 0.0084
2.6		+ 0.0084		+ 0.0052		+ 0.0025		+ 0.0002
3.2		0.0131		0.0104		0.0079		0.0057
3.8		0.0152		0.0129		0.0108		0.0089
4.4		0.0157		0.0139		0.0122		0.0106
5.0		0.0153		0.0139		0.0125		0.0112
6.0		0.0138		0.0128		0.0119		0.0110
8.0		0.0103		0.0099		0.0094		0.0090
10.0		0.0076		0.0074		0.0072		0.0069
15.0		0.0039		0.0039		0.0038		0.0038
20.0	0.0023	0.0023	0.0023	0.0023				
25.0	0.0015	0.0015	0.0015	0.0015				
30.0	0.0011	0.0011	0.0010	0.0010				

TABLE 5

Tables of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 2, \quad X = -\infty, \quad C_2 = -F(-\infty, \pm 2, \pm Z)$$

Z	C ₂	Δ	Z	C ₂	Δ
0	1.3333	2	0.50	0.9514	122
0.01	1.3331	5	0.51	0.9392	121
0.02	1.3326	10	0.52	0.9271	122
0.03	1.3316	13	0.53	0.9149	122
0.04	1.3303	18	0.54	0.9027	122
0.05	1.3285	21	0.55	0.8905	122
0.06	1.3264	25	0.56	0.8783	122
0.07	1.3239	28	0.57	0.8661	122
0.08	1.3211	32	0.58	0.8539	122
0.09	1.3179	36	0.59	0.8417	121
0.10	1.3143	40	0.60	0.8296	122
0.11	1.3103	43	0.61	0.8174	121
0.12	1.3060	47	0.62	0.8053	121
0.13	1.3013	50	0.63	0.7932	120
0.14	1.2963	53	0.64	0.7812	120
0.15	1.2910	57	0.65	0.7692	119
0.16	1.2853	60	0.66	0.7573	119
0.17	1.2793	63	0.67	0.7454	119
0.18	1.2730	67	0.68	0.7335	118
0.19	1.2663	69	0.69	0.7217	117
0.20	1.2594	73	0.70	0.7100	116
0.21	1.2521	75	0.71	0.6984	116
0.22	1.2446	79	0.72	0.6868	115
0.23	1.2367	80	0.73	0.6753	114
0.24	1.2287	84	0.74	0.6639	114
0.25	1.2203	86	0.75	0.6525	112
0.26	1.2117	89	0.76	0.6413	112
0.27	1.2028	91	0.77	0.6301	111
0.28	1.1937	94	0.78	0.6190	110
0.29	1.1843	95	0.79	0.6080	109
0.30	1.1748	98	0.80	0.5971	108
0.31	1.1650	99	0.81	0.5863	107
0.32	1.1551	102	0.82	0.5756	107
0.33	1.1449	104	0.83	0.5649	105
0.34	1.1345	105	0.84	0.5544	104
0.35	1.1240	107	0.85	0.5440	104
0.36	1.1133	108	0.86	0.5336	102
0.37	1.1025	110	0.87	0.5234	101
0.38	1.0915	111	0.88	0.5133	100
0.39	1.0804	113	0.89	0.5033	99
0.40	1.0691	114	0.90	0.4934	99
0.41	1.0577	115	0.91	0.4835	97
0.42	1.0462	115	0.92	0.4738	96
0.43	1.0347	117	0.93	0.4642	95
0.44	1.0230	118	0.94	0.4547	94
0.45	1.0112	119	0.95	0.4453	92
0.46	0.9993	119	0.96	0.4361	92
0.47	0.9874	119	0.97	0.4269	91
0.48	0.9755	121	0.98	0.4178	90
0.49	0.9634	121	0.99	0.4088	88
0.50	0.9514	120	1.00	0.4000	

TABLE 5—continued

$$Y = 2$$

Z	C ₂	A	Z	C ₂	A	Z	C ₂	A
1.00	0.4000	88	1.50	0.0820	42	2.00	- 0.0615	85
1.01	0.3912	86	1.51	0.0778	41	2.05	- 0.0700	77
1.02	0.3826	85	1.52	0.0737	41	2.10	- 0.0777	70
1.03	0.3741	84	1.53	0.0696	40	2.15	- 0.0847	64
1.04	0.3657	84	1.54	0.0656	40	2.20	- 0.0911	57
1.05	0.3573	82	1.55	0.0616	39	2.25	- 0.0968	51
1.06	0.3491	81	1.56	0.0577	38	2.30	- 0.1019	46
1.07	0.3410	80	1.57	0.0539	38	2.35	- 0.1065	41
1.08	0.3330	79	1.58	0.0501	37	2.40	- 0.1106	37
1.09	0.3251	78	1.59	0.0464	36	2.45	- 0.1143	33
1.10	0.3173	77	1.60	0.0428	36	2.50	- 0.1176	29
1.11	0.3096	76	1.61	0.0392	35	2.55	- 0.1205	25
1.12	0.3020	75	1.62	0.0357	35	2.60	- 0.1230	22
1.13	0.2945	73	1.63	0.0322	34	2.65	- 0.1252	19
1.14	0.2872	73	1.64	0.0288	34	2.70	- 0.1271	16
1.15	0.2799	72	1.65	0.0254	33	2.75	- 0.1287	14
1.16	0.2727	71	1.66	0.0221	32	2.80	- 0.1301	11
1.17	0.2656	70	1.67	0.0189	32	2.85	- 0.1312	9
1.18	0.2586	69	1.68	0.0157	31	2.90	- 0.1321	7
1.19	0.2517	67	1.69	0.0126	31	2.95	- 0.1328	5
1.20	0.2450	67	1.70	0.0095	30	3.00	- 0.1333	4
1.21	0.2383	66	1.71	0.0065	30	3.05	- 0.1337	2
1.22	0.2317	65	1.72	0.0035	29	3.10	- 0.1339	1
1.23	0.2252	64	1.73	0.0006	29	3.15	- 0.1340	- 1
1.24	0.2188	64	1.74	- 0.0023	28	3.20	- 0.1339	- 2
1.25	0.2124	62	1.75	- 0.0051	28	3.25	- 0.1337	- 2
1.26	0.2062	61	1.76	- 0.0079	27	3.30	- 0.1335	- 4
1.27	0.2001	61	1.77	- 0.0106	27	3.35	- 0.1331	- 5
1.28	0.1940	59	1.78	- 0.0133	26	3.40	- 0.1326	- 6
1.29	0.1881	59	1.79	- 0.0159	26	3.45	- 0.1320	- 6
1.30	0.1822	58	1.80	- 0.0185	25	3.50	- 0.1314	- 7
1.31	0.1764	57	1.81	- 0.0210	25	3.55	- 0.1307	- 7
1.32	0.1707	56	1.82	- 0.0235	25	3.60	- 0.1300	- 8
1.33	0.1651	55	1.83	- 0.0260	24	3.65	- 0.1292	- 9
1.34	0.1596	54	1.84	- 0.0284	24	3.70	- 0.1283	- 9
1.35	0.1542	54	1.85	- 0.0308	23	3.75	- 0.1274	- 9
1.36	0.1488	52	1.86	- 0.0331	23	3.80	- 0.1265	- 10
1.37	0.1436	52	1.87	- 0.0354	22	3.85	- 0.1255	- 10
1.38	0.1384	51	1.88	- 0.0376	22	3.90	- 0.1245	- 10
1.39	0.1333	51	1.89	- 0.0398	22	3.95	- 0.1235	- 10
1.40	0.1282	49	1.90	- 0.0420	21	4.00	- 0.1224	- 11
1.41	0.1233	49	1.91	- 0.0441	21			
1.42	0.1184	48	1.92	- 0.0462	20			
1.43	0.1136	47	1.93	- 0.0482	20			
1.44	0.1089	47	1.94	- 0.0502	20			
1.45	0.1042	45	1.95	- 0.0522	20			
1.46	0.0997	46	1.96	- 0.0542	18			
1.47	0.0951	44	1.97	- 0.0560	19			
1.48	0.0907	44	1.98	- 0.0579	18			
1.49	0.0863	43	1.99	- 0.0597	18			
1.50	0.0820		2.00	- 0.0615	18			

TABLE 6

Values of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 4, \quad Z^2 = 0(1)4, \quad 0 \leq X \leq 30$$

$$Z = 2(1)4, \quad 0 \leq X \leq 30$$

$$F(+X, \pm 4, \pm Z) = -G(Z)$$

X	Z^2	G	Z^2	G	Z^2	G
0	0	0.1333	2.0	0.0875	4.0	0.0584
0.4		0.1192		0.0805		0.0552
0.8		0.1055		0.0735		0.0520
1.2		0.0928		0.0668		0.0487
1.6		0.0812		0.0604		0.0454
2.0		0.0709		0.0544		0.0421
2.6		0.0579		0.0463		0.0373
3.2		0.0474		0.0393		0.0328
3.8		0.0392		0.0334		0.0287
4.4		0.0326		0.0286		0.0250
5.0		0.0274		0.0245		0.0219
6.0		0.0210		0.0192		0.0176
8.0		0.0132		0.0125		0.0118
10.0		0.0089		0.0086		0.0083
15.0		0.0042		0.0042		0.0041
20.0	0.0025	0.0024	0.0024			
25.0	0.0016	0.0016	0.0016			
30.0	0.0011	0.0011	0.0011			
0	1.0	0.1077	3.0	0.0714		
0.4		0.0976		0.0666		
0.8		0.0878		0.0618		
1.2		0.0785		0.0570		
1.6		0.0699		0.0523		
2.0		0.0620		0.0478		
2.6		0.0517		0.0415		
3.2		0.0431		0.0359		
3.8		0.0362		0.0309		
4.4		0.0305		0.0267		
5.0		0.0259		0.0231		
6.0		0.0201		0.0184		
8.0		0.0128		0.0121		
10.0		0.0087		0.0084		
15.0		0.0042		0.0041		
20.0	0.0024	0.0024				
25.0	0.0016	0.0015				
30.0	0.0011	0.0011				

TABLE 6—*continued*

$$Y = 4$$

X	Z	G	Z	G	Z	G
0	2.0	0.0584	3.0	0.0196	4.0	— 0.0020
0.4		0.0552		0.0206		+ 0.0006
0.8		0.0520		0.0216		0.0030
1.2		0.0487		0.0222		0.0052
1.6		0.0454		0.0226		0.0071
2.0		0.0421		0.0226		0.0087
2.6		0.0373		0.0221		0.0104
3.2		0.0328		0.0211		0.0114
3.8		0.0287		0.0198		0.0119
4.4		0.0250		0.0182		0.0119
5.0		0.0219		0.0167		0.0116
6.0		0.0176		0.0142		0.0107
8.0		0.0118		0.0102		0.0085
10.0		0.0083		0.0075		0.0066
15.0		0.0041		0.0039		0.0036
20.0	0.0024	0.0023	0.0022			
25.0	0.0016	0.0015	0.0015			
30.0	0.0011	0.0010	0.0010			

TABLE 7

Table of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 4, \quad X = -\infty, \quad C_4 = -F(-\infty, \pm 4, \pm Z)$$

Z	C_4	A	Z	C_4	A
0	0.2667		2.00	0.1167	47
0.05	0.2665	2	2.05	0.1120	46
0.10	0.2661	4	2.10	0.1074	45
0.15	0.2654	7	2.15	0.1029	45
0.20	0.2644	10	2.20	0.0984	44
0.25	0.2631	13	2.25	0.0940	43
0.30	0.2615	16	2.30	0.0897	42
0.35	0.2597	18	2.35	0.0855	41
0.40	0.2576	21	2.40	0.0814	40
0.45	0.2552	24	2.45	0.0774	40
0.50	0.2526	26	2.50	0.0734	38
0.55	0.2498	28	2.55	0.0696	37
0.60	0.2467	31	2.60	0.0659	37
0.65	0.2434	33	2.65	0.0622	36
0.70	0.2399	35	2.70	0.0586	34
0.75	0.2363	36	2.75	0.0552	34
0.80	0.2324	39	2.80	0.0518	33
0.85	0.2284	40	2.85	0.0485	32
0.90	0.2242	42	2.90	0.0453	31
0.95	0.2198	44	2.95	0.0422	30
1.00	0.2154	44	3.00	0.0392	29
1.05	0.2108	46	3.05	0.0363	28
1.10	0.2061	47	3.10	0.0335	28
1.15	0.2014	47	3.15	0.0307	26
1.20	0.1965	49	3.20	0.0281	26
1.25	0.1916	49	3.25	0.0255	25
1.30	0.1866	50	3.30	0.0230	24
1.35	0.1816	50	3.35	0.0206	23
1.40	0.1765	51	3.40	0.0183	22
1.45	0.1715	51	3.45	0.0161	22
1.50	0.1664	51	3.50	0.0139	21
1.55	0.1613	51	3.55	0.0118	20
1.60	0.1562	51	3.60	0.0098	20
1.65	0.1511	51	3.65	0.0078	18
1.70	0.1461	50	3.70	0.0060	18
1.75	0.1411	50	3.75	0.0042	18
1.80	0.1361	50	3.80	0.0024	17
1.85	0.1312	49	3.85	0.0007	16
1.90	0.1263	49	3.90	- 0.0009	15
1.95	0.1215	48	3.95	- 0.0024	15
2.00	0.1167	48	4.00	- 0.0039	

TABLE 8

Values of Complete Downwash in Three Dimensions due to a Rectangular Vortex

$$Y = 6(2)20, \quad Z = 0 \text{ and } 2(1)4, \quad 0 \leq X \leq 30$$

$$F(+X, \pm Y, \pm Z) = -G(Z)$$

X	Z	G				
		Y = 6	Y = 8	Y = 10	Y = 12	
0	0	0.0571	0.0318	0.0202	0.0140	
0.4		0.0532	0.0302	0.0194	0.0135	
0.8		0.0494	0.0285	0.0186	0.0131	
1.2		0.0456	0.0270	0.0178	0.0126	
1.6		0.0421	0.0254	0.0170	0.0121	
2.0		0.0386	0.0240	0.0162	0.0117	
2.6		0.0339	0.0218	0.0150	0.0110	
3.2		0.0297	0.0198	0.0140	0.0104	
3.8		0.0260	0.0180	0.0129	0.0098	
4.4		0.0228	0.0163	0.0120	0.0092	
5.0		0.0201	0.0148	0.0111	0.0086	
6.0		0.0163	0.0126	0.0097	0.0077	
8.0		0.0111	0.0092	0.0075	0.0062	
10.0		0.0079	0.0069	0.0058	0.0051	
15.0		0.0040	0.0037	0.0033	0.0031	
20.0		0.0024	0.0023	0.0020	0.0021	
25.0		0.0015	0.0015	0.0014	0.0014	
30.0		0.0011	0.0011	0.0010	0.0010	
0		2.0	0.0403	0.0262	0.0179	0.0129
0.4			0.0381	0.0250	0.0172	0.0124
0.8	0.0359		0.0238	0.0165	0.0120	
1.2	0.0337		0.0226	0.0159	0.0116	
1.6	0.0316		0.0215	0.0152	0.0112	
2.0	0.0295		0.0203	0.0146	0.0108	
2.6	0.0266		0.0187	0.0136	0.0102	
3.2	0.0238		0.0172	0.0127	0.0097	
3.8	0.0214		0.0157	0.0118	0.0091	
4.4	0.0192		0.0144	0.0110	0.0086	
5.0	0.0171		0.0132	0.0102	0.0081	
6.0	0.0143		0.0113	0.0091	0.0073	
8.0	0.0102		0.0085	0.0071	0.0059	
10.0	0.0074		0.0064	0.0056	0.0048	
15.0	0.0039		0.0035	0.0033	0.0030	
20.0	0.0023		0.0022	0.0021	0.0020	
25.0	0.0015		0.0014	0.0014	0.0014	
30.0	0.0011		0.0010	0.0010	0.0010	

TABLE 8—continued

X	Z	G			
		Y = 6	Y = 8	Y = 10	Y = 12
0	3.0	0.0264	0.0207	0.0154	0.0116
0.4		0.0254	0.0199	0.0148	0.0112
0.8		0.0243	0.0191	0.0143	0.0109
1.2		0.0233	0.0183	0.0138	0.0105
1.6		0.0223	0.0175	0.0133	0.0102
2.0		0.0213	0.0167	0.0128	0.0099
2.6		0.0198	0.0156	0.0120	0.0094
3.2		0.0183	0.0145	0.0113	0.0089
3.8		0.0168	0.0134	0.0106	0.0084
4.4		0.0154	0.0124	0.0099	0.0079
5.0		0.0142	0.0115	0.0092	0.0075
6.0		0.0122	0.0101	0.0083	0.0068
8.0		0.0090	0.0078	0.0066	0.0056
10.0		0.0068	0.0060	0.0053	0.0046
15.0		0.0037	0.0034	0.0031	0.0029
20.0	0.0022	0.0021	0.0020	0.0019	
25.0	0.0015	0.0014	0.0014	0.0013	
30.0	0.0010	0.0010	0.0010	0.0010	
0	4.0	0.0143	0.0149	0.0125	0.0100
0.4		0.0142	0.0145	0.0121	0.0097
0.8		0.0141	0.0140	0.0118	0.0095
1.2		0.0139	0.0136	0.0114	0.0092
1.6		0.0138	0.0132	0.0110	0.0089
2.0		0.0136	0.0127	0.0106	0.0086
2.6		0.0132	0.0121	0.0101	0.0082
3.2		0.0127	0.0114	0.0096	0.0079
3.8		0.0121	0.0108	0.0090	0.0075
4.4		0.0115	0.0102	0.0085	0.0071
5.0		0.0109	0.0096	0.0081	0.0068
6.0		0.0098	0.0086	0.0073	0.0062
8.0		0.0077	0.0069	0.0060	0.0052
10.0		0.0060	0.0055	0.0049	0.0043
15.0		0.0034	0.0033	0.0030	0.0028
20.0	0.0021	0.0021	0.0020	0.0019	
25.0	0.0014	0.0014	0.0014	0.0013	
30.0	0.0010	0.0010	0.0010	0.0010	

TABLE 8—continued

X	Z	G				
		Y = 14	Y = 16	Y = 18	Y = 20	
0	0	0.0103	0.0078	0.0062	0.0050	
0.4		0.0100	0.0076	0.0060	0.0049	
0.8		0.0097	0.0075	0.0059	0.0048	
1.2		0.0094	0.0072	0.0058	0.0047	
1.6		0.0091	0.0071	0.0056	0.0046	
2.0		0.0088	0.0069	0.0055	0.0045	
2.6		0.0084	0.0066	0.0053	0.0044	
3.2		0.0079	0.0063	0.0051	0.0042	
3.8		0.0075	0.0060	0.0049	0.0041	
4.4		0.0071	0.0058	0.0047	0.0040	
5.0		0.0068	0.0055	0.0045	0.0038	
6.0		0.0062	0.0051	0.0042	0.0036	
8.0		0.0051	0.0044	0.0037	0.0032	
10.0		0.0042	0.0037	0.0032	0.0028	
15.0		0.0027	0.0025	0.0022	0.0020	
20.0		0.0018	0.0018	0.0016	0.0015	
25.0		0.0012	0.0013	0.0012	0.0011	
30.0		0.0009	0.0010	0.0009	0.0009	
0		2.0	0.0096	0.0075	0.0060	0.0049
0.4			0.0094	0.0073	0.0058	0.0048
0.8	0.0091		0.0071	0.0057	0.0047	
1.2	0.0088		0.0069	0.0056	0.0046	
1.6	0.0086		0.0067	0.0054	0.0045	
2.0	0.0083		0.0066	0.0053	0.0044	
2.6	0.0079		0.0063	0.0051	0.0042	
3.2	0.0076		0.0060	0.0050	0.0041	
3.8	0.0072		0.0058	0.0048	0.0040	
4.4	0.0068		0.0055	0.0046	0.0038	
5.0	0.0065		0.0053	0.0044	0.0037	
6.0	0.0060		0.0049	0.0041	0.0035	
8.0	0.0050		0.0042	0.0036	0.0031	
10.0	0.0042		0.0035	0.0031	0.0027	
15.0	0.0027		0.0024	0.0022	0.0020	
20.0	0.0019		0.0017	0.0016	0.0015	
25.0	0.0013		0.0012	0.0012	0.0011	
30.0	0.0010		0.0009	0.0009	0.0009	

TABLE 8—continued

X	Z	G				
		Y = 14	Y = 16	Y = 18	Y = 20	
0	3.0	0.0089	0.0071	0.0057	0.0047	
0.4		0.0087	0.0069	0.0056	0.0046	
0.8		0.0084	0.0067	0.0054	0.0045	
1.2		0.0082	0.0066	0.0053	0.0044	
1.6		0.0080	0.0064	0.0052	0.0043	
2.0		0.0077	0.0062	0.0051	0.0042	
2.6		0.0074	0.0060	0.0049	0.0041	
3.2		0.0070	0.0058	0.0047	0.0040	
3.8		0.0067	0.0055	0.0045	0.0038	
4.4		0.0064	0.0053	0.0044	0.0037	
5.0		0.0061	0.0051	0.0042	0.0036	
6.0		0.0056	0.0047	0.0039	0.0034	
8.0		0.0047	0.0041	0.0034	0.0030	
10.0		0.0039	0.0035	0.0030	0.0026	
15.0		0.0026	0.0024	0.0021	0.0019	
20.0		0.0018	0.0017	0.0015	0.0014	
25.0		0.0012	0.0012	0.0011	0.0011	
30.0		0.0009	0.0009	0.0008	0.0008	
0		4.0	0.0080	0.0065	0.0053	0.0044
0.4			0.0078	0.0064	0.0052	0.0044
0.8	0.0076		0.0062	0.0051	0.0043	
1.2	0.0074		0.0061	0.0050	0.0042	
1.6	0.0072		0.0059	0.0049	0.0041	
2.0	0.0070		0.0058	0.0048	0.0040	
2.6	0.0067		0.0056	0.0046	0.0039	
3.2	0.0064		0.0053	0.0045	0.0038	
3.8	0.0062		0.0051	0.0043	0.0037	
4.4	0.0059		0.0049	0.0041	0.0036	
5.0	0.0056		0.0047	0.0040	0.0034	
6.0	0.0052		0.0044	0.0037	0.0033	
8.0	0.0044		0.0038	0.0033	0.0029	
10.0	0.0037		0.0033	0.0029	0.0026	
15.0	0.0025		0.0023	0.0020	0.0019	
20.0	0.0017		0.0016	0.0014	0.0014	
25.0	0.0012		0.0012	0.0011	0.0011	
30.0	0.0009		0.0009	0.0008	0.0008	

TABLE 9

*Table of Complete Downwash in Three
Dimensions due to a Rectangular Vortex*

$$Y = 6(2)22, \quad X = -\infty, \quad C_y = -F(-\infty, \pm Y, \pm Z)$$

Z	C _z	Δ	Z	C _z	Δ
0	0.1143	0	2.00	0.0807	
0.05	0.1143	1	2.05	0.0793	14
0.10	0.1142	1	2.10	0.0779	14
0.15	0.1141	2	2.15	0.0765	14
0.20	0.1139	3	2.20	0.0751	14
0.25	0.1136	2	2.25	0.0737	14
0.30	0.1134	4	2.30	0.0723	14
0.35	0.1130	3	2.35	0.0708	15
0.40	0.1127	5	2.40	0.0694	14
0.45	0.1122	4	2.45	0.0680	14
0.50	0.1118	5	2.50	0.0666	14
0.55	0.1113	6	2.55	0.0652	14
0.60	0.1107	6	2.60	0.0638	14
0.65	0.1101	7	2.65	0.0624	14
0.70	0.1094	7	2.70	0.0610	14
0.75	0.1087	7	2.75	0.0596	14
0.80	0.1080	8	2.80	0.0582	14
0.85	0.1072	8	2.85	0.0568	14
0.90	0.1064	9	2.90	0.0555	13
0.95	0.1055	9	2.95	0.0541	14
1.00	0.1046	9	3.00	0.0527	14
1.05	0.1037	10	3.05	0.0514	13
1.10	0.1027	10	3.10	0.0501	13
1.15	0.1017	10	3.15	0.0488	13
1.20	0.1007	11	3.20	0.0475	13
1.25	0.0996	11	3.25	0.0462	13
1.30	0.0985	11	3.30	0.0449	13
1.35	0.0974	12	3.35	0.0436	13
1.40	0.0962	12	3.40	0.0423	13
1.45	0.0950	12	3.45	0.0411	12
1.50	0.0938	12	3.50	0.0399	12
1.55	0.0926	13	3.55	0.0387	12
1.60	0.0913	13	3.60	0.0375	12
1.65	0.0900	12	3.65	0.0363	12
1.70	0.0888	14	3.70	0.0351	12
1.75	0.0874	13	3.75	0.0340	11
1.80	0.0861	13	3.80	0.0329	11
1.85	0.0848	14	3.85	0.0318	11
1.90	0.0834	13	3.90	0.0307	11
1.95	0.0821	14	3.95	0.0296	11
2.00	0.0807		4.00	0.0285	11

TABLE 9—continued

Y = 8			Y = 10		
Z	C ₈	A	Z	C ₁₀	A
0	0.0635	8	0	0.0404	3
0.5	0.0627	22	0.5	0.0401	9
1.0	0.0605	35	1.0	0.0392	15
1.5	0.0570	46	1.5	0.0377	19
2.0	0.0524	53	2.0	0.0358	24
2.5	0.0471	57	2.5	0.0334	26
3.0	0.0414	59	3.0	0.0308	29
3.5	0.0355	57	3.5	0.0279	29
4.0	0.0298		4.0	0.0250	

Y = 12			Y = 14		
Z	C ₁₂	A	Z	C ₁₄	A
0	0.0280	6	0	0.0205	3
1.0	0.0274	17	1.0	0.0202	9
2.0	0.0257	25	2.0	0.0193	14
3.0	0.0232	32	3.0	0.0179	18
4.0	0.0200		4.0	0.0161	

Y = 16			Y = 18		
Z	C ₁₆	A	Z	C ₁₈	A
0	0.0157	2	0	0.0124	1
1.0	0.0155	5	1.0	0.0123	4
2.0	0.0150	9	2.0	0.0119	5
3.0	0.0141	11	3.0	0.0114	7
4.0	0.0130		4.0	0.0107	

Y = 20			Y = 22		
Z	C ₂₀	A	Z	C ₂₂	A
0	0.0100	1	0	0.0083	1
1.0	0.0099	2	1.0	0.0082	1
2.0	0.0097	3	2.0	0.0081	3
3.0	0.0094	5	3.0	0.0078	3
4.0	0.0089		4.0	0.0075	

TABLE 10

*Subsidiary Values of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 13, \quad 0 \leq \frac{1}{4}Z \leq 4, \quad 0 \leq \frac{1}{4}X \leq 15$$

$$4F(+X, \pm 13, \pm Z) = -G(\frac{1}{4}X, \frac{1}{4}Z)$$

$\frac{1}{4}X$	$\frac{1}{4}Z$	G	$\frac{1}{4}Z$	G	$\frac{1}{4}Z$	G
0	0	0.0476	1.00	0.0358	3.5	- 0.0017
0.5		0.0403		0.0312		- 0.0003
1.0		0.0335		0.0267		+ 0.0010
1.5		0.0276		0.0226		0.0021
2.0		0.0226		0.0191		0.0029
2.5		0.0185		0.0160		0.0034
3.0		0.0152		0.0135		0.0037
4.0		0.0106		0.0097		0.0039
6.0		0.0057		0.0054		0.0032
8.0		0.0035		0.0034		0.0024
10.0		0.0023		0.0023		0.0018
15.0		0.0011		0.0011		0.0010
0	0.25	0.0468	2.0	0.0154	4.0	- 0.0039
0.5		0.0397		0.0146		- 0.0024
1.0		0.0331		0.0138		- 0.0011
1.5		0.0272		0.0128		+ 0.0001
2.0		0.0223		0.0117		0.0011
2.5		0.0183		0.0106		0.0018
3.0		0.0151		0.0095		0.0023
4.0		0.0105		0.0075		0.0028
6.0		0.0057		0.0047		0.0027
8.0		0.0035		0.0031		0.0022
10.0		0.0023		0.0021		0.0017
15.0		0.0011		0.0010		0.0009
0	0.50	0.0443	2.5	0.0075		
0.5		0.0378		0.0080		
1.0		0.0317		0.0082		
1.5		0.0262		0.0083		
2.0		0.0216		0.0082		
2.5		0.0178		0.0078		
3.0		0.0148		0.0074		
4.0		0.0104		0.0063		
6.0		0.0056		0.0042		
8.0		0.0035		0.0029		
10.0		0.0023		0.0020		
15.0		0.0111		0.0010		
0	0.75	0.0406	3.0	0.0020		
0.5		0.0349		0.0031		
1.0		0.0295		0.0040		
1.5		0.0247		0.0047		
2.0		0.0205		0.0052		
2.5		0.0171		0.0054		
3.0		0.0142		0.0054		
4.0		0.0101		0.0050		
6.0		0.0056		0.0037		
8.0		0.0034		0.0027		
10.0		0.0023		0.0019		
15.0		0.0011		0.0010		

TABLE 11

*Subsidiary Table of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 13, \quad X = -\infty$$

$$C_{13} = -4F(-\infty, \pm 13, \pm Z)$$

$\frac{1}{4}Z$	C_{13}	Δ	$\frac{1}{4}Z$	C_{13}	Δ
0	0.0952	0	2.00	0.0308	18
0.05	0.0952	2	2.05	0.0290	17
0.10	0.0950	4	2.10	0.0273	17
0.15	0.0946	5	2.15	0.0256	17
0.20	0.0941	6	2.20	0.0239	16
0.25	0.0935	7	2.25	0.0223	15
0.30	0.0928	9	2.30	0.0208	15
0.35	0.0919	9	2.35	0.0193	15
0.40	0.0910	11	2.40	0.0178	14
0.45	0.0899	13	2.45	0.0164	13
0.50	0.0886	13	2.50	0.0151	13
0.55	0.0873	14	2.55	0.0138	13
0.60	0.0859	15	2.60	0.0125	12
0.65	0.0844	16	2.65	0.0113	12
0.70	0.0828	17	2.70	0.0101	11
0.75	0.0811	18	2.75	0.0090	11
0.80	0.0793	18	2.80	0.0079	11
0.85	0.0775	19	2.85	0.0068	10
0.90	0.0756	19	2.90	0.0058	10
0.95	0.0737	20	2.95	0.0048	9
1.00	0.0717	20	3.00	0.0039	9
1.05	0.0697	21	3.05	0.0030	8
1.10	0.0676	21	3.10	0.0022	8
1.15	0.0655	21	3.15	0.0014	8
1.20	0.0634	21	3.20	+ 0.0006	7
1.25	0.0613	22	3.25	- 0.0001	7
1.30	0.0591	21	3.30	- 0.0008	7
1.35	0.0570	22	3.35	- 0.0015	7
1.40	0.0548	21	3.40	- 0.0022	6
1.45	0.0527	21	3.45	- 0.0028	6
1.50	0.0506	21	3.50	- 0.0034	5
1.55	0.0485	21	3.55	- 0.0039	5
1.60	0.0464	21	3.60	- 0.0044	5
1.65	0.0443	20	3.65	- 0.0049	5
1.70	0.0423	20	3.70	- 0.0054	5
1.75	0.0403	20	3.75	- 0.0059	4
1.80	0.0383	19	3.80	- 0.0063	4
1.85	0.0364	19	3.85	- 0.0067	4
1.90	0.0345	19	3.90	- 0.0071	3
1.95	0.0326	18	3.95	- 0.0074	4
2.00	0.0308	18	4.00	- 0.0078	4

TABLE 12

*Subsidiary Values of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 21, \quad 0 \leq \frac{1}{4}Z \leq 4, \quad 0 \leq \frac{1}{4}X \leq 15$$

$$4F(+X, \pm 21, \pm Z) = -G(\frac{1}{4}X, \frac{1}{4}Z)$$

$\frac{1}{4}X$	$\frac{1}{4}Z$	G	$\frac{1}{4}Z$	G
0	0	0.0182	2.0	0.0118
0.5		0.0165		0.0110
1.0		0.0148		0.0101
1.5		0.0132		0.0093
2.0		0.0117		0.0085
2.5		0.0104		0.0077
3.0		0.0091		0.0070
4.0		0.0071		0.0057
6.0		0.0045		0.0039
8.0		0.0030		0.0027
10.0		0.0021		0.0019
15.0		0.0010		0.0010
0		0.50		0.0177
0.5	0.0160		0.0066	
1.0	0.0144		0.0063	
1.5	0.0129		0.0060	
2.0	0.0115		0.0057	
2.5	0.0102		0.0054	
3.0	0.0090		0.0051	
4.0	0.0071		0.0044	
6.0	0.0045		0.0033	
8.0	0.0030		0.0024	
10.0	0.0021		0.0018	
15.0	0.0010		0.0009	
0	1.00		0.0163	4.0
0.5		0.0148	0.0031	
1.0		0.0134	0.0032	
1.5		0.0120	0.0033	
2.0		0.0108	0.0033	
2.5		0.0096	0.0033	
3.0		0.0085	0.0032	
4.0		0.0068	0.0031	
6.0		0.0043	0.0026	
8.0		0.0029	0.0020	
10.0		0.0020	0.0016	
15.0		0.0010	0.0009	

TABLE 13

*Subsidiary Table of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 21, \quad X = -\infty$$

$$C_{21} = -4F(-\infty, \pm 21, \pm Z)$$

$\frac{1}{2}Z$	C_{21}	Δ	$\frac{1}{2}Z$	C_{21}	Δ
0	0.0364	1	2.00	0.0237	10
0.10	0.0363	1	2.10	0.0227	11
0.20	0.0362	2	2.20	0.0216	10
0.30	0.0360	3	2.30	0.0206	10
0.40	0.0357	3	2.40	0.0196	10
0.50	0.0354	4	2.50	0.0186	10
0.60	0.0350	5	2.60	0.0176	9
0.70	0.0345	6	2.70	0.0167	10
0.80	0.0339	6	2.80	0.0157	9
0.90	0.0333	7	2.90	0.0148	9
1.00	0.0326	7	3.00	0.0139	9
1.10	0.0319	8	3.10	0.0130	9
1.20	0.0311	8	3.20	0.0121	9
1.30	0.0303	9	3.30	0.0112	9
1.40	0.0294	9	3.40	0.0104	8
1.50	0.0285	9	3.50	0.0096	8
1.60	0.0276	10	3.60	0.0089	7
1.70	0.0266	9	3.70	0.0081	8
1.80	0.0257	10	3.80	0.0074	7
1.90	0.0247	10	3.90	0.0067	7
2.00	0.0237		4.00	0.0061	6

TABLE 14

*Subsidiary Table of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 29(8)53, \quad 0 \leq \frac{1}{4}Z \leq 4, \quad 0 \leq \frac{1}{4}X \leq 15$$

$$4F(+X, \pm Y, \pm Z) = -G(\frac{1}{4}X, \frac{1}{4}Z)$$

$\frac{1}{4}X$	$\frac{1}{4}Z$	G			
		Y = 29	Y = 37	Y = 45	Y = 53
0	0	0.0095	0.0058	0.0040	0.0028
0.5		0.0089	0.0055	0.0038	0.0027
1.0		0.0082	0.0052	0.0036	0.0026
1.5		0.0076	0.0049	0.0034	0.0025
2.0		0.0070	0.0046	0.0033	0.0024
2.5		0.0064	0.0043	0.0031	0.0023
3.0		0.0059	0.0040	0.0029	0.0022
4.0		0.0049	0.0035	0.0026	0.0020
6.0		0.0034	0.0027	0.0021	0.0017
8.0		0.0025	0.0020	0.0017	0.0014
10.0		0.0018	0.0016	0.0013	0.0011
15.0		0.0009	0.0009	0.0008	0.0007
0		2.0	0.0076	0.0051	0.0036
0.5	0.0071		0.0048	0.0034	0.0026
1.0	0.0067		0.0046	0.0033	0.0025
1.5	0.0062		0.0043	0.0031	0.0024
2.0	0.0058		0.0041	0.0030	0.0023
2.5	0.0054		0.0038	0.0029	0.0022
3.0	0.0050		0.0036	0.0027	0.0021
4.0	0.0043		0.0032	0.0024	0.0019
6.0	0.0031		0.0025	0.0020	0.0016
8.0	0.0023		0.0019	0.0016	0.0013
10.0	0.0017		0.0015	0.0013	0.0011
15.0	0.0009		0.0008	0.0008	0.0007
0	4.0		0.0039	0.0034	0.0027
0.5		0.0038	0.0032	0.0026	0.0021
1.0		0.0036	0.0031	0.0025	0.0020
1.5		0.0035	0.0030	0.0024	0.0020
2.0		0.0034	0.0029	0.0023	0.0019
2.5		0.0032	0.0027	0.0022	0.0018
3.0		0.0031	0.0026	0.0021	0.0018
4.0		0.0028	0.0024	0.0020	0.0016
6.0		0.0023	0.0019	0.0016	0.0014
8.0		0.0018	0.0016	0.0014	0.0012
10.0		0.0014	0.0013	0.0011	0.0010
15.0		0.0008	0.0008	0.0007	0.0007

TABLE 15

*Subsidiary Table of Complete Downwash in
Three Dimensions due to a Rectangular Vortex*

$$Y = 29(8)61, \quad X = -\infty$$

$$C_Y = -4F(-\infty, \pm Y, \pm Z)$$

Y = 29		
$\frac{1}{4}Z$	C_{29}	Δ
0	0.0190	2
0.5	0.0188	8
1.0	0.0180	12
1.5	0.0168	16
2.0	0.0152	18
2.5	0.0134	19
3.0	0.0115	19
3.5	0.0096	18
4.0	0.0078	

Y = 37			Y = 45		
$\frac{1}{4}Z$	C_{37}	Δ	$\frac{1}{4}Z$	C_{45}	Δ
0	0.0117	4	0	0.0079	2
1.0	0.0113	11	1.0	0.0077	5
2.0	0.0102	16	2.0	0.0072	8
3.0	0.0086	19	3.0	0.0064	10
4.0	0.0067		4.0	0.0054	

Y = 53			Y = 61		
$\frac{1}{4}Z$	C_{53}	Δ	$\frac{1}{4}Z$	C_{61}	Δ
0	0.0057	4	0	0.0043	2
2.0	0.0053	4	2.0	0.0041	3
3.0	0.0049	6	3.0	0.0038	3
4.0	0.0043		4.0	0.0035	

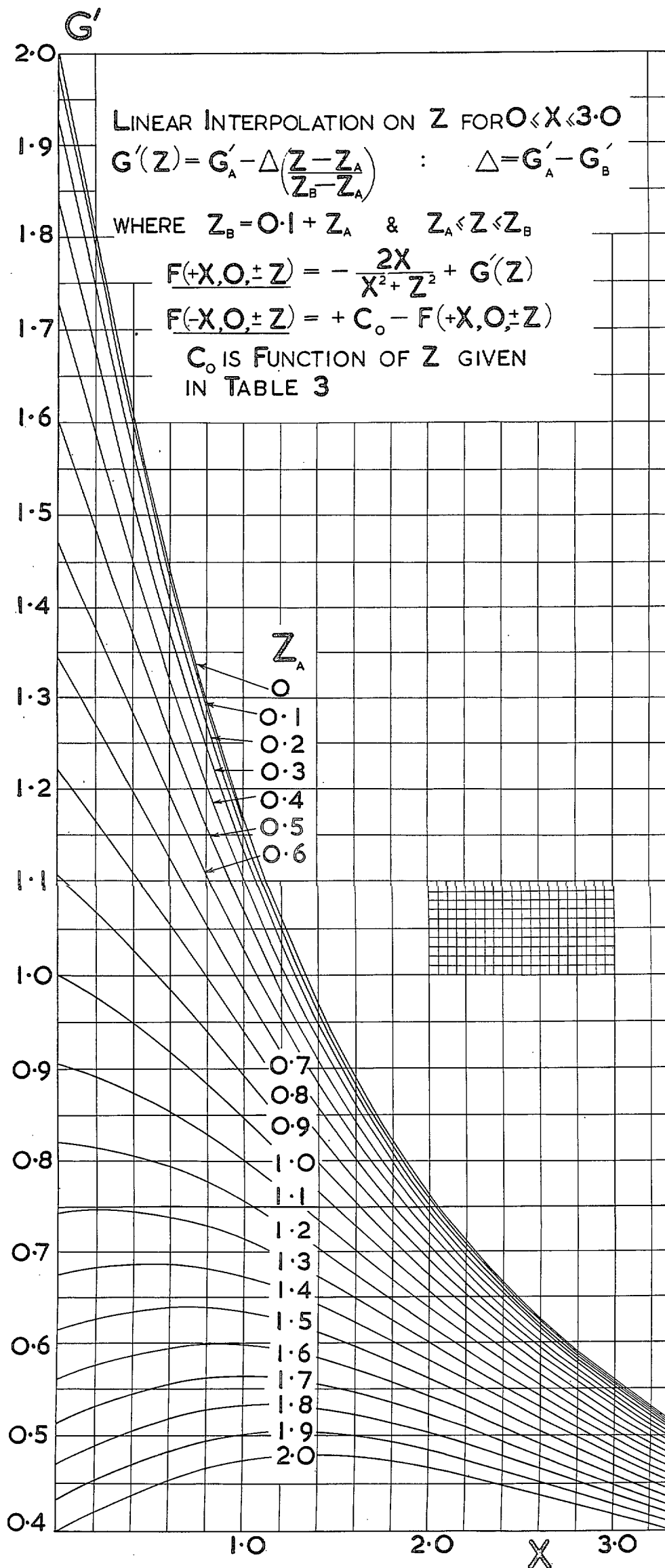


FIG. I.

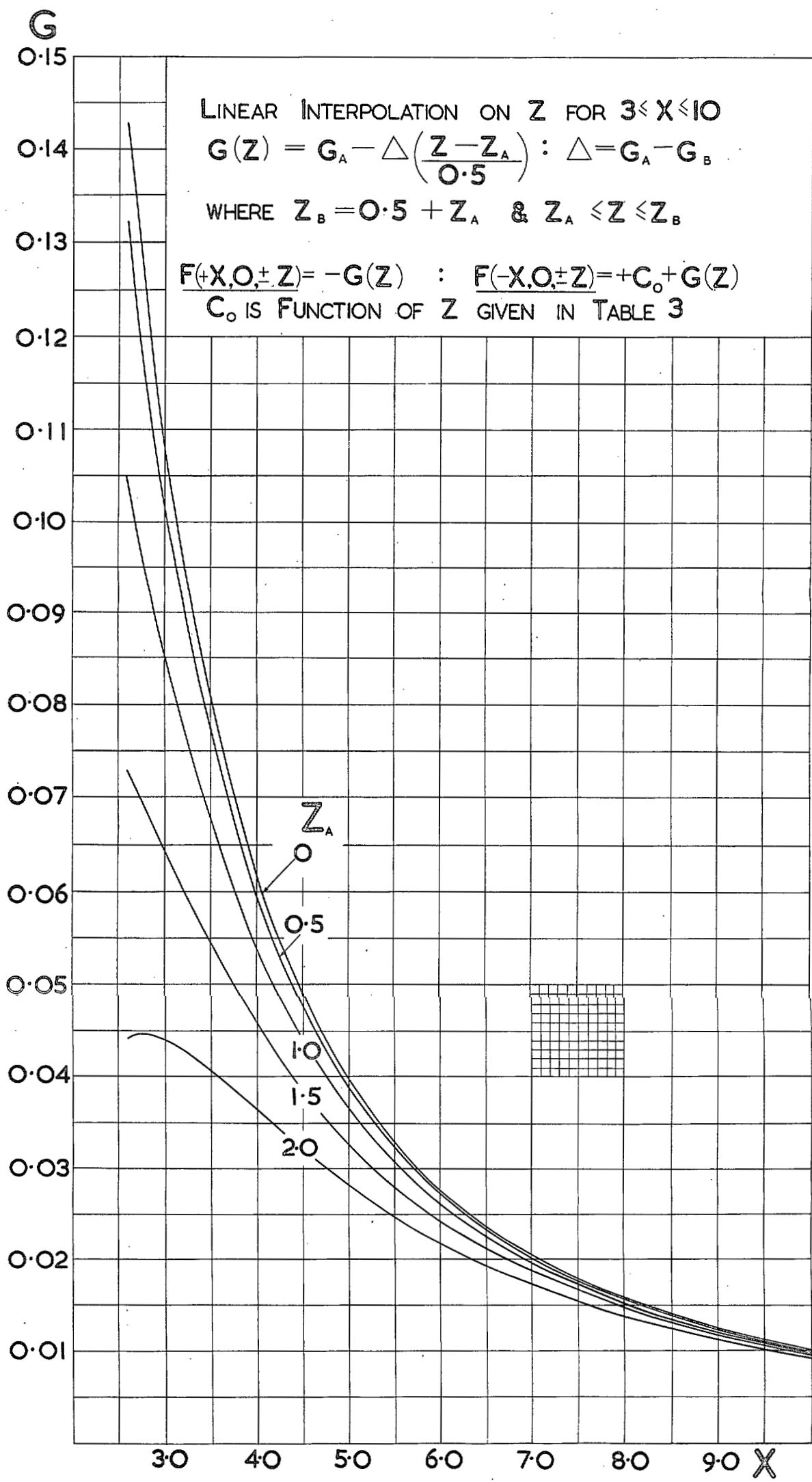


FIG. 2.

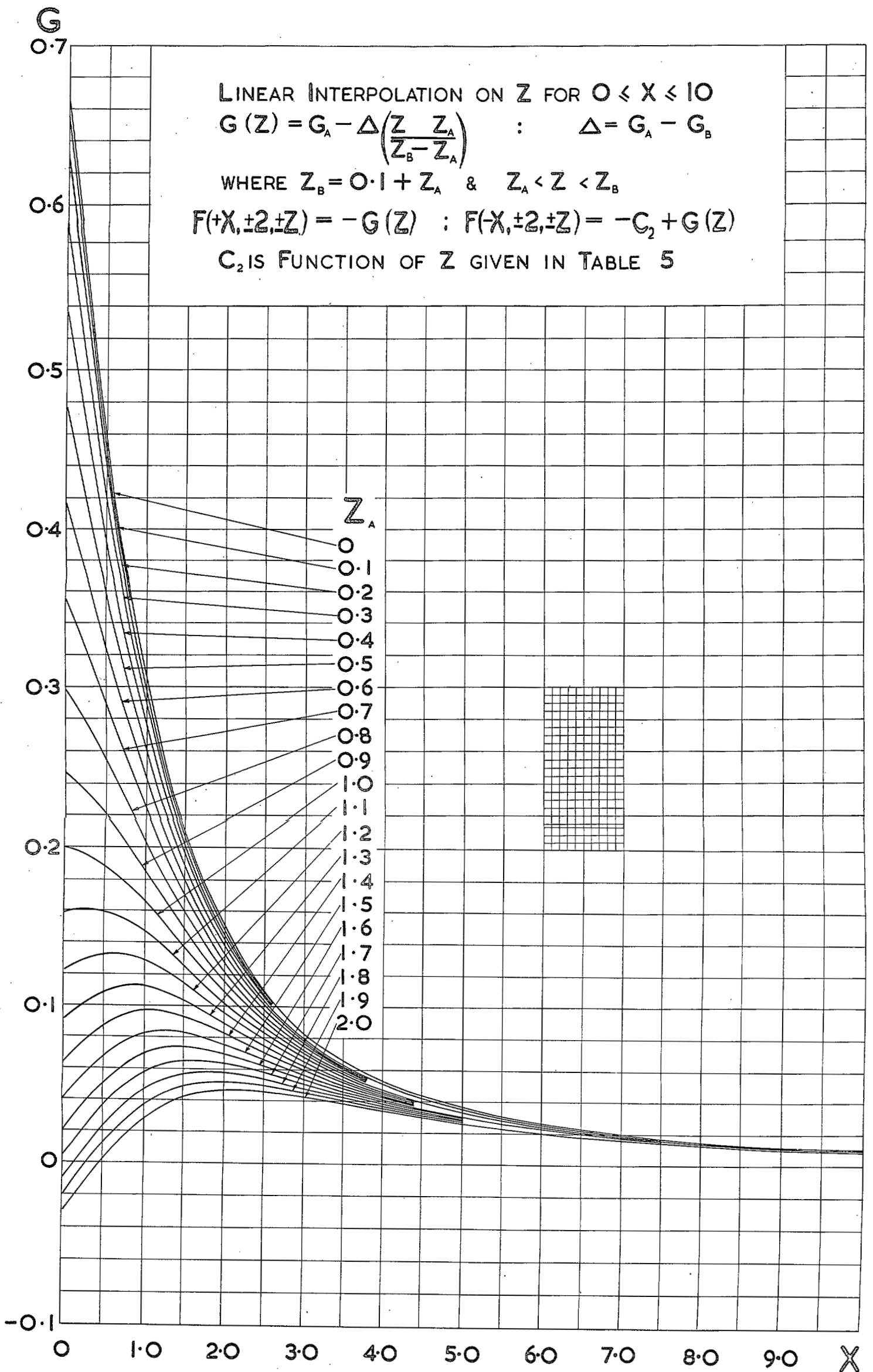


FIG. 3.

LINEAR INTERPOLATION ON Z^2 FOR $0 \leq Z \leq 2$

$$G(Z) = G_A - \Delta \left(\frac{Z^2 - Z_A^2}{1.0} \right) : \Delta = G_A - G_B$$

WHERE $Z_B^2 = 1.0 + Z_A^2$ & $Z_A \leq Z \leq Z_B$

$$\frac{F(+X, \pm 4, \pm Z)}{C_4} = -G(Z) : \frac{F(-X, \pm 4, \pm Z)}{C_4} = -C_4 + G(Z)$$

C_4 IS FUNCTION OF Z GIVEN IN TABLE 7

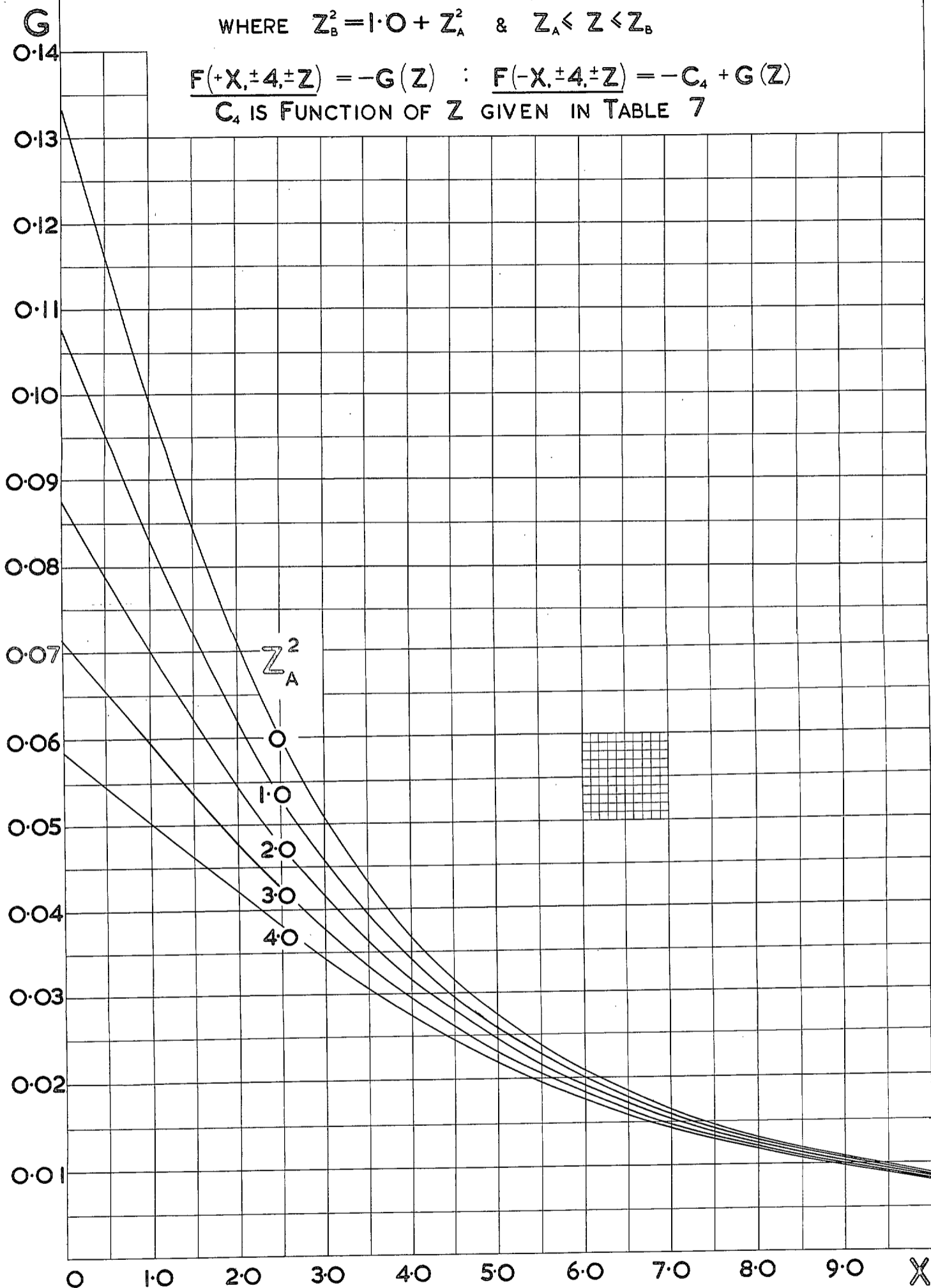


FIG. 4

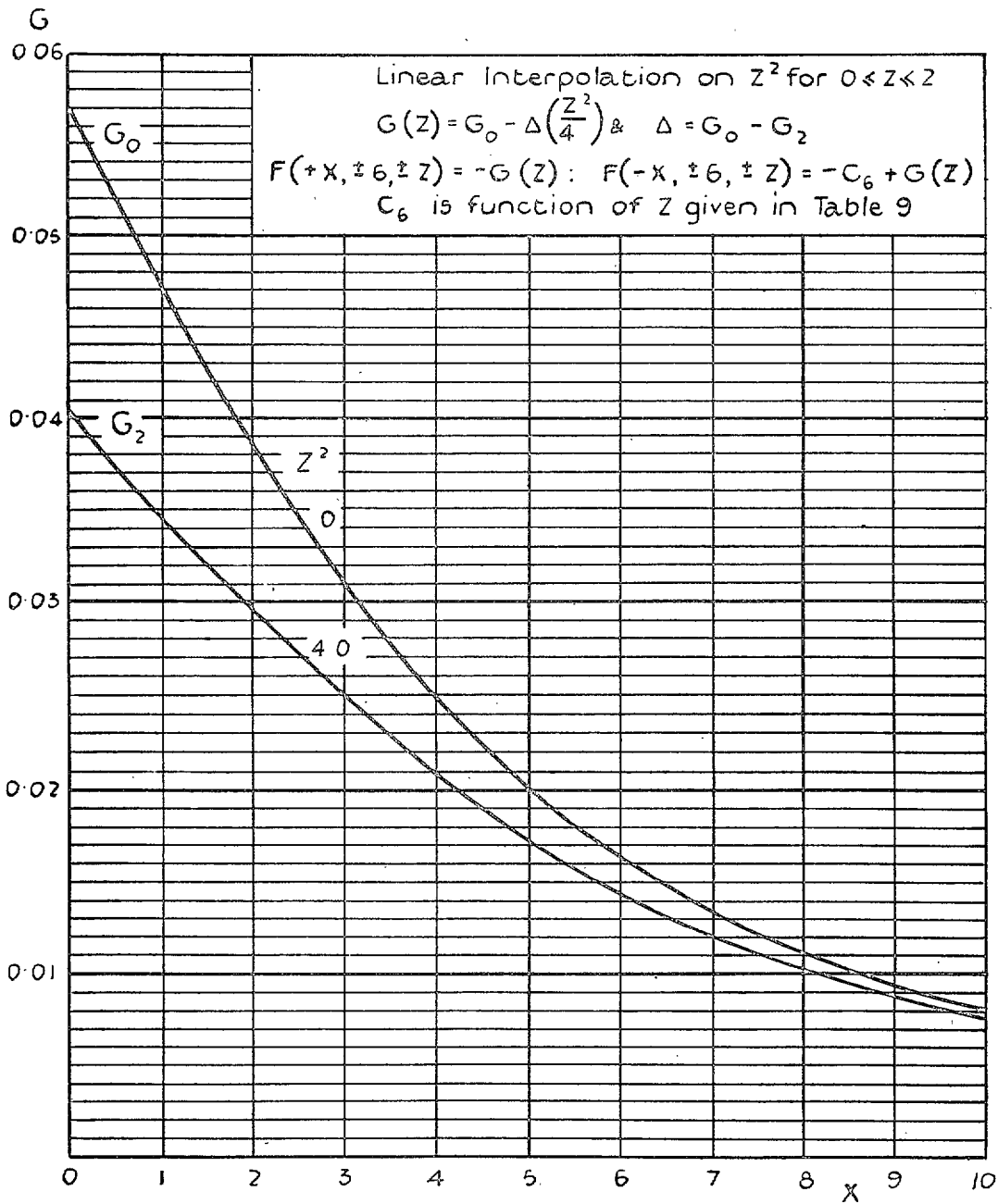


FIG. 5. Downwash factors for $Y = 6$.

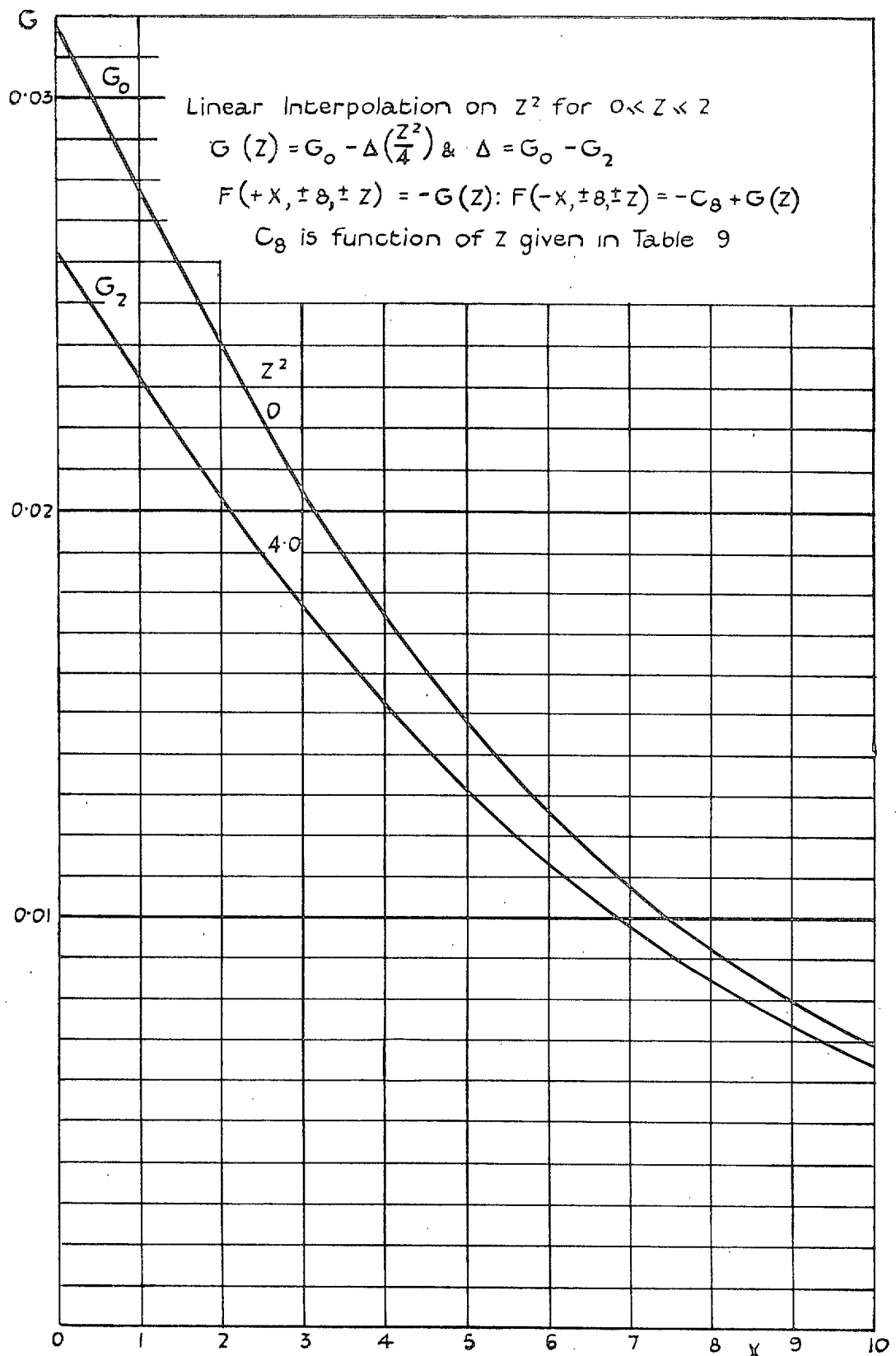


FIG. 6. Downwash factors for $Y = 8$.

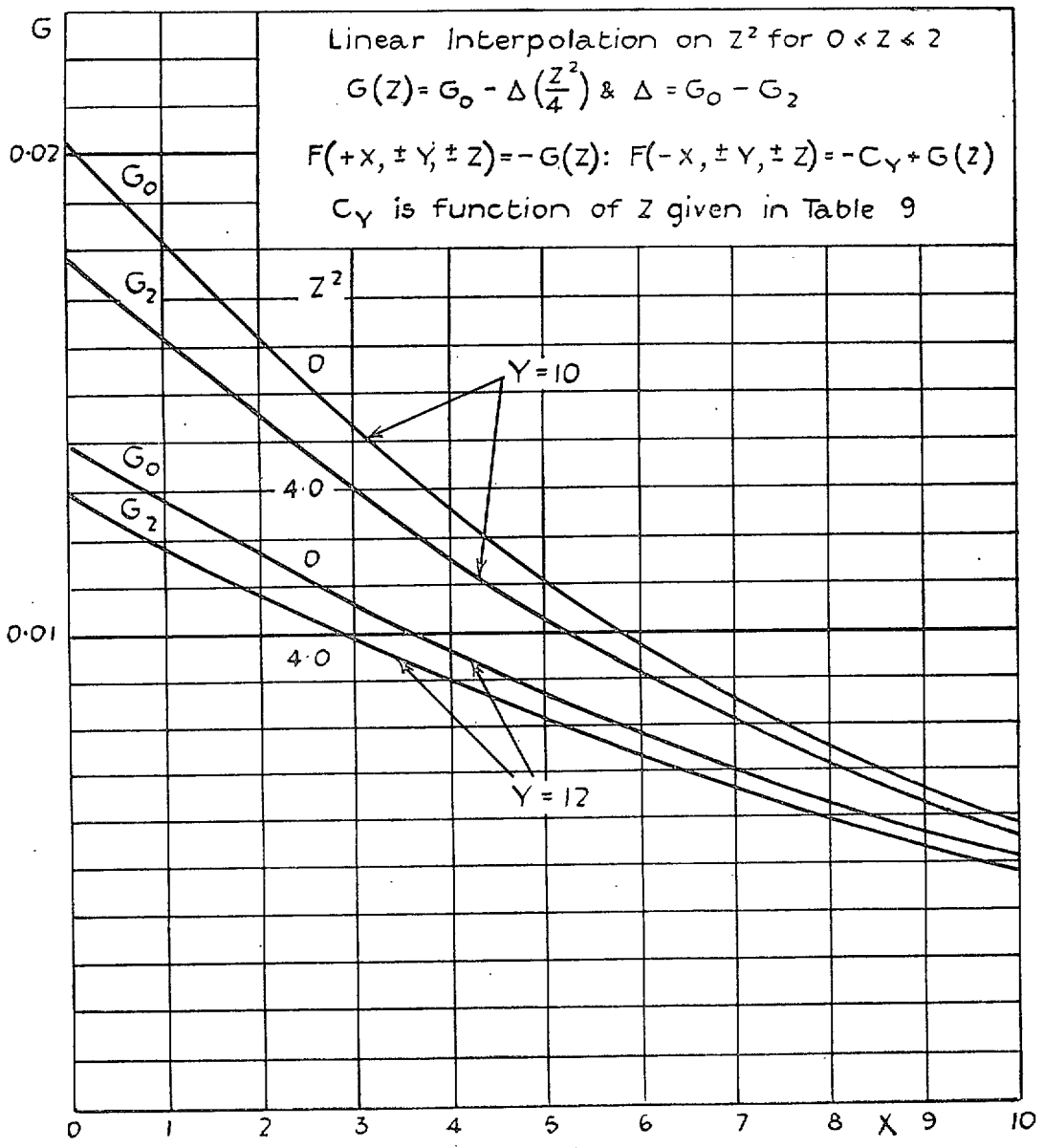


FIG. 7. Downwash factors for $Y = 10$ and 12 .

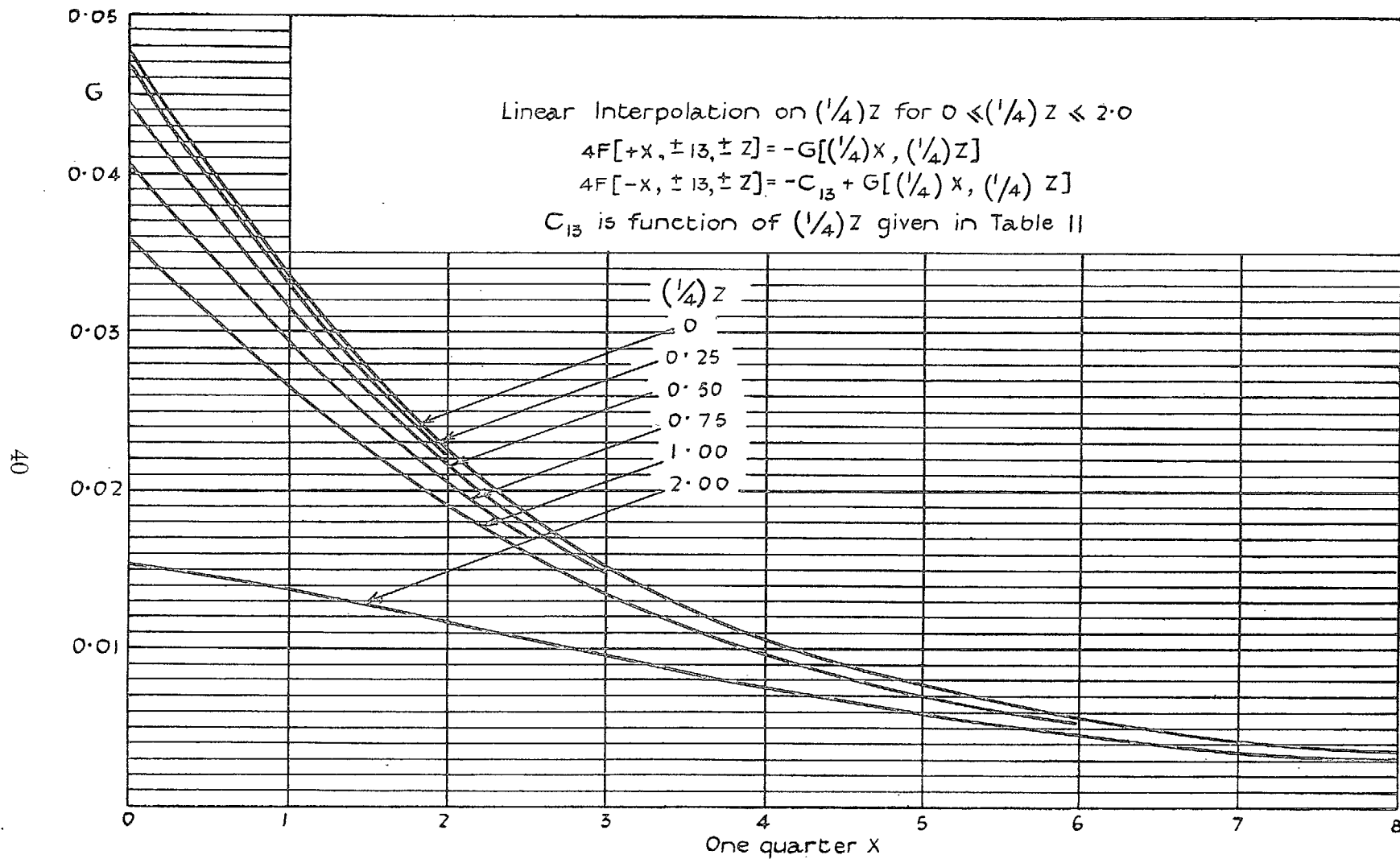


FIG. 8. Subsidiary downwash factors for $Y = 13$.

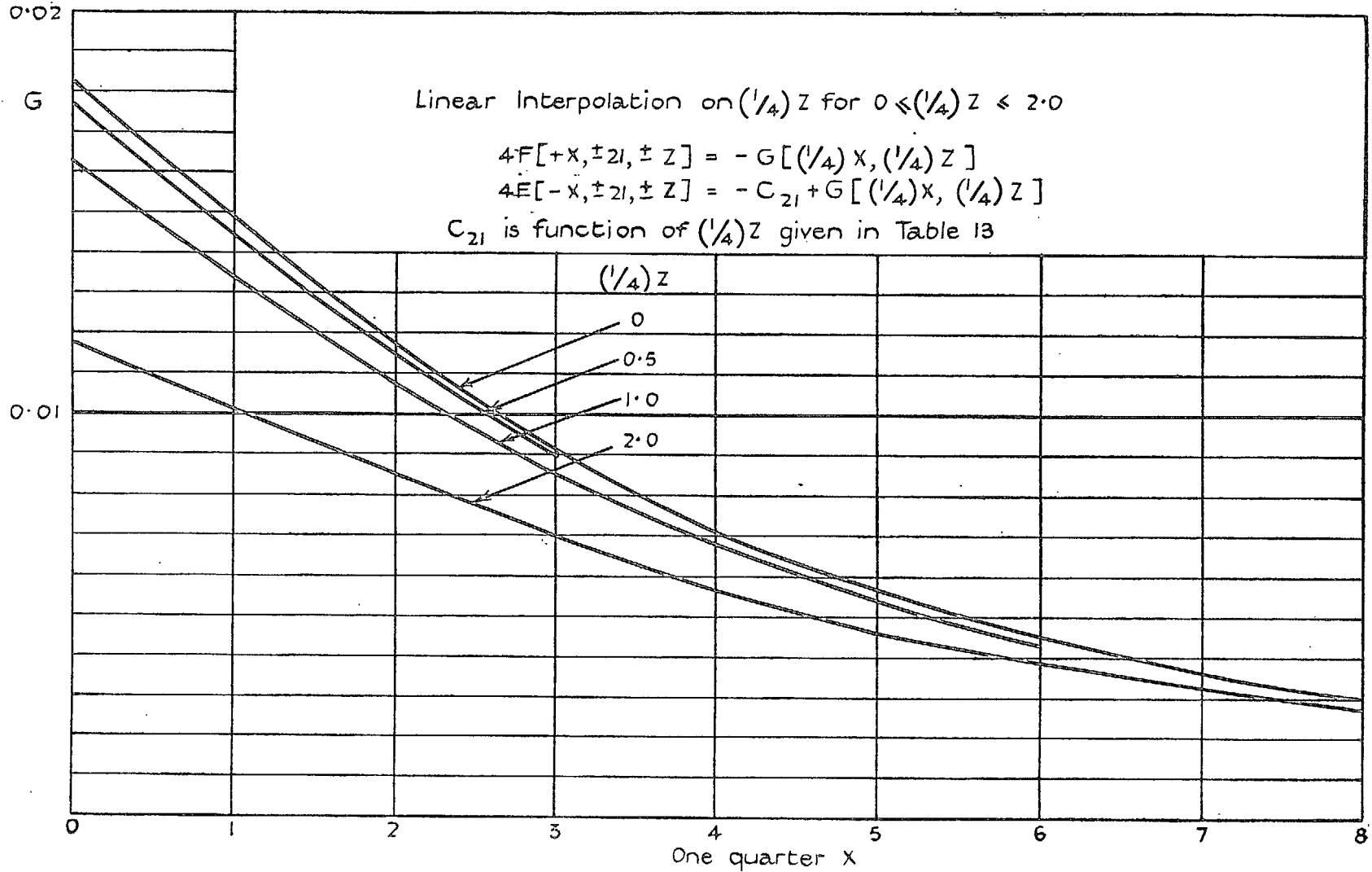


FIG. 9. Subsidiary downwash factors for $Y = 21$.

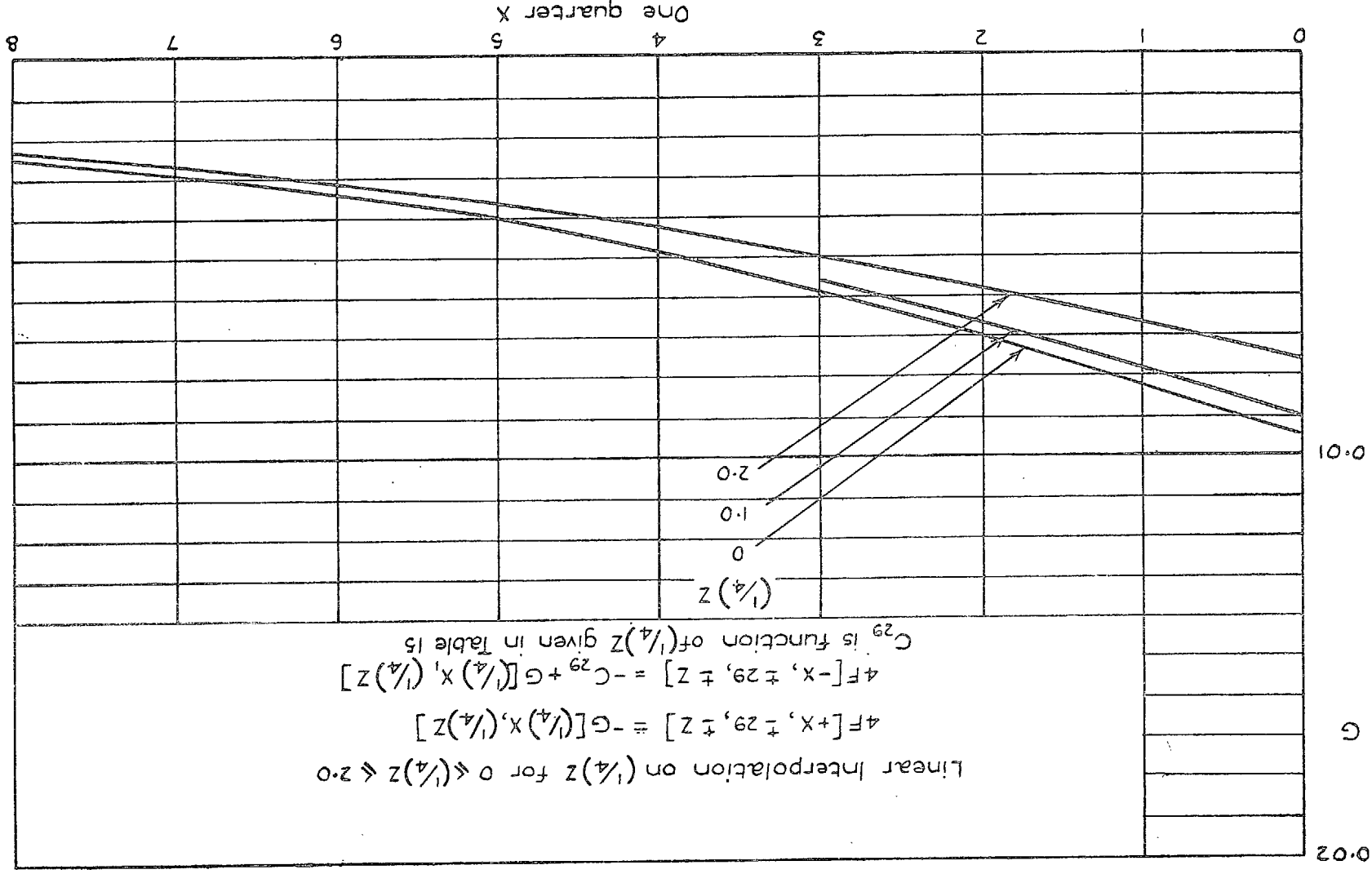


Fig. 10. Subsidiary downwash factors for $Y = 29$.

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