R. & M. No. 2075
(6052)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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Communicated by the Director-General of Scientific Research (A) Ministry of Supply.

Reports and Memoranda No. 2075

July, 1942*

Summary.—Reasons for Enquiry.—As part of a general investigation of stability problems a review of the theoretical aspects of dynamic longitudinal stability was required.

Range of Investigation.—A summary is given of the theory of dynamic stability in gliding flight, including an approximate method of calculating the period and damping of the phugoid. The effects of weights and springs in the elevator circuit are examined and compared with qualitative evidence from flight tests. Stability at altitudes is also considered.

Results and Conclusions.—It is shown that, with positive static stability, the low degree of phugoid damping on some modern aircraft cannot be attributed to low drag or to inadequate tail area for damping out the pitching motion, unless there is a large loss of tail-plane effectiveness on freeing the stick. It is more probably due to too small a static margin combined with friction in the elevator circuit. A weight moment about the elevator hinge improves static stability, but with the assumptions† made here, it does not appear to be as efficient dynamically as an equivalent change in static margin by an increase in tail effectiveness or a movement of the centre of gravity. A spring or inertialess weight moment improves static stability, but may have a very unfavourable effect on dynamic stability, particularly at high altitudes.

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*R.A.E. Report No. Aero. 1755, received 27th August, 1942.

† Subsequent calculations have shown that, for an incompletely mass-balanced elevator or with most practicable arrangements of inertia weights, there is no unfavourable effect on phugoid damping due to the weight moment at the same static margin.

1. *Introduction.*—Considerable attention is being given to the problem of providing sufficient longitudinal stability to give satisfactory flying qualities, particularly for long-range cruising, night fighting and at high altitudes. With a large number of present-day aircraft the trouble appears to be due to insufficient static stability under some conditions of flight, but it is not clear how much static margin is needed and whether this alone will ensure satisfactory flying qualities for long periods.

Preliminary investigations suggest that, although slipstream may have an important effect on the position of the neutral point, it produces no serious change in the relationship between dynamic and static stability, and, in general, it increases the phugoid damping for the same static margin. The simpler case of motion in the glide is therefore being analysed in detail, in an attempt to explain the observed behaviour in flight and to form a basis for estimates of dynamic stability in future designs. The present paper gives a review of the work which has already been done and includes some comparisons with full-scale results. The theory is summarised in mathematical appendices.

It is apparent that there is in some cases a marked difference between calculated and measured stick free phugoids for small degrees of static stability, and it is thought that this may be due to the effect of friction in the control circuit.

2. *Theory of Longitudinal Stability in Gliding Flight.*—2.1. *Definition of Static and Dynamic Stability.*—Static stability is positive, stick fixed, when the downward elevator angle to trim in steady flight with a fixed trimmer setting increases with speed. It is positive, stick free, when the pull on the stick required with fixed trimmer setting (or the downward tab angle to trim with zero stick force) decreases as speed increases. For the cases considered here the magnitude of the static stability margin can be defined[1,2] as the distance of the centre of gravity in front of the neutral point (centre of gravity position for neutral stability).

Positive dynamic stability means that after a disturbance the aircraft tends to return to the steady trimmed speed and attitude, either with fixed elevator or with the stick left free, if there is no load on the stick in the trimmed condition. It may either approach the trimmed position directly from the disturbed state (subsidence) or oscillate about it with decreasing amplitude. Departure from the trimmed position without oscillation (divergence) generally implies static instability, but it is shown in §4.2 that in special circumstances a dynamic divergence can occur with positive static stability. The conditions for static and dynamic stability are expressed mathematically in Appendix I, equations (14)–(18).

2.2. *Exact Method of Solution of the Dynamic Stability Equations.*—As there are differences in notation in various published papers[3-6] on dynamic stability, it has been thought advisable to summarise the classical theory in Appendix I and to develop the formulae for the derivatives in terms of the aerodynamic characteristics which can be calculated or measured in the wind tunnel. The non-dimensional notation adopted is based on that of R. & M. 1801[5] and is summarised in the list of symbols.

On a stable aircraft with fixed elevator the solution of the stability equations gives two possible types of motion, a heavily damped quick oscillation with a period of from 2 to 10 seconds and a slow " phugoid " motion with a period of from 30 seconds to 2 minutes and a low order of damping. Typical calculated values of the period and of the time to damp to half amplitude for a stable aircraft, given in Tables 6A and 6B, are based on model tests on the Halifax with fixed or free elevator.

With the elevator free and mass-balanced it is usual to neglect the moment of inertia and damping of the elevator itself and to include only the change in effectiveness of the tail plane due to the floating angle of the elevator. The neutral point h_n stick fixed is replaced by h_n' stick free, and a_1, the slope of the lift incidence curve for the tail plane, by

$$a_1' = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) .$$

There seems to be little doubt that this method gives the phugoid damping with sufficient accuracy, provided that there is no friction in the elevator circuit and that the whole circuit is mass-balanced, since the motion is too slow for any appreciable lag in the elevator floating angle. The effect of a constant out-of-balance weight moment is considered later. For the short period oscillation it is necessary to include the elevator motion in more detail. The short period oscillation is important in connection with manœuvrability, since it dominates the motion of a stable aircraft immediately after a disturbance. It can be analysed approximately by neglecting the variation of forward speed in the stability equations. The present paper is concerned only with the phugoid oscillation, in which speed changes are most important.

When the static stability is nearly neutral, the solution of the equations gives four real roots or subsidences, one of which becomes a divergence as the centre of gravity passes through the neutral point, and one is a very rapid subsidence (*see* Table 6C). As the static margin becomes more negative, the two intermediate roots combine to form a damped oscillation of slow period, similar to a phugoid, while the subsidence and divergence represent the motion associated with negligible changes in speed, which is of importance in the study of manœuvrability.

Typical values of the roots for a statically unstable aircraft are given for the Spitfire I in Table 7A, (i) and (ii), where the unstable modes are underlined. The normal centre of gravity is at $0 \cdot 314\bar{c}$ (6 in. aft of datum), while $0 \cdot 352\bar{c}$ (9 in. aft of datum) gives the furthest aft position tested. With the centre of gravity further forward at 0·25, outside the normal flying range, the behaviour is similar to that of the Halifax. Neutral points stick fixed and stick free with a weight moment were determined from flight tests and the effect of freeing the stick with the elevator mass-balanced has been estimated from these values and calculated elevator characteristics.

2.3. *Approximate Method of Solution*.—An approximate analysis of the phugoid motion for a fixed or a free mass-balanced elevator was developed in R. & M. 1118[4]. A more convenient expression for the damping has since been developed by S. B. Gates but has not previously been published. It is described in full in Appendix II, where it is shown that the damping factor r can be expressed in the form

$$r = f\, C_{D0} + F\, C_L^2 \,, \qquad (1)$$

where f and F depend on the geometrical properties of the aircraft, mainly the aspect ratio of the wings, and on the two fundamental stability and damping parameters,

$$\left.\begin{aligned} \omega &= -\mu_1 \frac{m_w}{i_B} = \frac{Wc}{Sk_B^2}\frac{1}{\rho g}\cdot\frac{a}{2}(h_n - h)^* \\ \nu &= -\frac{m_q}{i_B} \simeq \frac{a_1}{2}\frac{S'}{S}\frac{l^2}{k_B^2} \end{aligned}\right\} \qquad (2)$$

The effects of wing loading and altitude appear only in ω, while the tail plane size affects ν directly and ω through h_n, if h is fixed. Typical curves for f and F are given in Fig. 1.

The time T in seconds to halve the amplitude is given by

$$T = \frac{0 \cdot 312}{r}\sqrt{\left(\frac{W}{S}\frac{C_L}{\sigma}\right)} \,. \qquad (3)$$

For an unstable phugoid (r negative) the time to double the amplitude is $-T$.

* This expression for ω holds only when $m_u = 0$, *see* footnote on page 15.

Unless $h_n - h$ is very small, the period P in seconds is approximately

$$P = 4 \cdot 0 \sqrt{\left\{\frac{W}{S}\frac{1}{\sigma C_L}\left(1 + \frac{av}{2\omega}\right)\right\}}$$

$$= 0 \cdot 138\, V \sqrt{\left\{1 + \frac{a_1 \bar{V}}{2\mu_1 (h_n - h)}\right\}} \quad \ldots \quad (4)$$

and is proportional to speed, if the static stability margin and the tail plane effectiveness (a_1) do not vary with incidence. The number of cycles to halve the amplitude is

$$\frac{T}{P} = \frac{0 \cdot 078}{r} C_L \Big/ \sqrt{\left(1 + \frac{av}{2\omega}\right)}. \quad \ldots \quad (5)$$

The factor r represents the damping of a phugoid only when ω is greater than a small positive value. It has been shown already (Table 6C) that when $\omega \to 0$ the phugoid increases in period until it splits up into two subsidences. For very small positive values of ω, r represents the mean rate of decay of these two subsidences. This is illustrated in Fig. 3, which also gives a comparison between the exact and approximate methods of calculation. The validity of the approximate method is discussed in Appendix II.

3. *Effect of Profile Drag and Tail Size.*—At low values of C_L the damping of the phugoid of a statically stable aircraft is approximately $r = f\, C_{D0}$, where f lies between $\frac{1}{2}$ and $\frac{3}{4}$, and an average value of r at high speeds is $\frac{2}{3}\, C_{D0}$ (*see* Fig. 1). Thus, the cleaner the aircraft, the less the damping. Typical calculated values of the stick free period and the damping are given in the following Table 1 for an assumed C_{D0} of $0 \cdot 02$, which represents an average value for modern fighters, but is definitely low for bombers. For the Halifax $C_{D0} = 0 \cdot 027$ is a more reasonable value.

TABLE 1

Aircraft	C_{D0}	C_L	Height ft.	$h_n' - h$	W/S lb./sq. ft.	T min.	P min.	T/P
Beaufighter	0·02	0·2	10,000	0·050	31	1·1	1·2	0·9
$W = 15{,}500$ lb.	0·02	0·6	10,000	0·050	31	1·3	0·7	1·85
	0·02	0·6	40,000	0·050	31	1·9	1·05	1·85
Halifax	0·02	0·2	10,000	0·085	48	1·3	1·4	0·9
$W = 60{,}000$ lb.	0·02	0·6	10,000	0·070	48	1·6	0·85	1·95
	0·02	0·6	40,000	0·070	48	2·4	1·3	1·85

There is as yet no evidence that this order of damping disturbs the pilot, since periods of the order of $\frac{1}{2}$–2 minutes and damping times of from 1 to 2 cycles to halve the amplitude are common among types, such as the Havoc, which are considered pleasant to fly. Any apparent instability of modern aircraft cannot therefore be attributed solely to their clean design.

Fig. 4 shows the effect on phugoid damping of a reduction in the static margin of a stable aircraft due (1) to a change in centre of gravity position (dotted curves) and (2) to a reduction in tail area or effectiveness (the figures on the full-line curves are proportional to $S'a_1'/a_1$). As would be expected, the damping is greater with a large tail, but over a wide range of static margin the variation in damping is small. This is consistent with the results of flight records and pilots' reports, which show that pilots cannot distinguish between different degrees of static stability provided they are all markedly positive. The behaviour for small degrees of static stability is discussed in §6.

Negative damping may occur at high values of C_L if the tail plane is exceptionally small or if there is a large loss of effectiveness. It is clear from equation (1) that r can never be less than its value at $C_L = 0$, provided F is positive. It is shown in Appendix III that F can be expressed in terms of ν, ω and the aspect ratio A of the wings. For any given aircraft A and ν are fixed, while ω may vary over a wide range with varying centre of gravity position, wing loading and altitude. Typical curves for F plotted against ω for various values of ν (Fig. 1) show that there is a value of ω for which F is a minimum and that this minimum value of F decreases with ν. Thus there is theoretically a minimum value of ν or of effective tail size for a given design, which will ensure that F cannot be negative whatever the value of ω (for $\omega > 0$), and that r cannot fall below fC_{D_0}. It is shown in Appendix III that this limiting value of ν is given approximately by

$$\nu_{min} = \frac{\pi A}{1 \cdot 2\,[1 + \sqrt{(10A + 36)}]} \quad \ldots \quad (6)$$

The curve for ν_{min} against A is plotted as a full line in Fig. 2 and the corresponding values of ω are marked on the curve. At all other values of ω the damping is greater for the same value of ν. Some typical full scale values of ν are given below:—

TABLE 2

Aircraft	A	S'/S	Minimum l^2/k_B^2	Stick fixed a_1	Stick fixed ν	Stick free a_1'	Stick free ν
Spitfire	5·67	0·136	11	3·2	2·4	2·8	2·1
Halifax	7·85	0·179	8	3·4	2·4	2·9	2·1
Beaufighter	6·7	0·174	12	2·8	2·9	2·6	2·7
Havoc	8·1	0·209	12	3·0	3·8	2·4	3·0

This suggests that, unless there is a large loss of stability on freeing the stick, it is unlikely that any modern aircraft with mass-balanced elevators has a value of ν less than the theoretical minimum required to give no reduction in damping with increasing C_L.

A further illustration of the effect of tail size on the damping and period for a given aircraft is given in Figs. 6 and 10 for $C_L = 0 \cdot 6$ and $1 \cdot 2$. For the smallest tail the minimum damping decreases with increasing C_L and is negative at $C_L = 1 \cdot 2$. It increases very slowly with C_L for the normal or medium tail and more rapidly for the large one.

4. *Weights* and Springs in the Elevator Circuit.*—So far it has been assumed that the elevator and control system are completely mass-balanced about the elevator hinge and that the stick is also balanced independently, but this is not true in general. Also inertia weights are sometimes inserted deliberately in the elevator circuit either to increase the static stability or to prevent the aircraft from being pulled out of a dive too quickly. Another method of increasing static stability is to insert in the elevator circuit an initially tensioned spring which applies an approximately constant force tending to pull the elevator downwards. These two methods of increasing static stability have very different effects on dynamic stability.

* In the development of the formulae and curves of this report it is assumed that the hinge moment due to the gravity force on the weight remains constant during the motion of the aircraft. It can be shown, however, that, for a weight attached to an arm of fixed length, the effect of the change in gravity moment due to the change in attitude of the aircraft, combined with the inertia moment due to the acceleration along the x-axis, has an important effect on the hinge moment due to the weight, which should not be neglected. If the angle between the weight arm and the horizontal is initially small and the weight is situated near the aircraft C.G., the effect of the weight on the phugoid damping is the same as that of an equivalent C.G. movement to give the same change in static margin. The damping curves for a weight moment given in this report are therefore pessimistic for most practicable arrangements.

With both spring and weight the tendency of the elevator to float downwards is reduced as the speed increases, because the weight or spring moment is independent of forward speed. This has a favourable effect on static stability and appears as a term in m_u in the equations for dynamic stability. The essential difference between the weight and the tensioned spring is that the weight has inertia and responds to normal and centrifugal accelerations. It therefore gives an important contribution to the derivatives m_q and $m_{\dot{w}}$ and adds to the damping effect of the tail. The spring has no such effect. If the weight is situated far from the centre of gravity, say on the elevator itself, it also contributes a term $m_{\dot{q}}$, which has the same effect as a reduction in the apparent moment of inertia of the whole aircraft and has a favourable effect on the dynamic stability.

The effects of different sizes of inertia weight on the phugoid damping are shown in non-dimensional form for $C_L = 0 \cdot 6$ and $1 \cdot 2$ in Figs. 13 and 15, where the damping factor r for three values of ν is plotted against ω, the static stability factor stick free without a weight. These diagrams can be applied well enough to any conventional aircraft with an aspect ratio of about 6, and can be used for predicting the effect of a constant weight moment, provided the elevator characteristics are known. Similar curves for a spring are given in Figs. 14 and 16. The same data have been plotted in dimensional form for an assumed aircraft in Figs. 6–12. The scale in Fig. 9 has been reduced to show the large negative damping with a spring for negative values of the initial static margin.

4.1. *Weight Moment.*—In Appendix IV the approximate method of calculating the damping and period of the phugoid is extended to include the effect of a constant weight moment in the elevator circuit. The curves of Figs. 7 and 11 compared with those of Figs. 6 and 10 illustrate this effect for a weight inserted near the centre of gravity of the aircraft and giving a rearward movement of the neutral point, $\Delta h_n' = 0 \cdot 05$.

If the aircraft is statically stable, stick free, with complete mass-balance $(h_n' - h > 0)$, the addition of a weight moment reduces the period by the same amount as if the change in static margin were due to a forward movement of the centre of gravity (equation (83)). The effect on damping depends on the value of $h_n' - h$ and the tail area, but in general the weight moment reduces the damping. This reduction in period and damping is confirmed by flight experiments in Germany[7], in which both the stick and the elevator were mass-balanced and a weight was added in the circuit near the centre of gravity of the aircraft. It is also consistent with the results of tests on the Mosquito in this country.

If the addition of the weight converts a negative static margin into a positive one, it improves stability by eliminating divergence, but it gives less damping* than the equivalent change in the centre of gravity position (equations (82) and (53)). With a small tail or at high altitudes there may be an unstable oscillation (Fig. 7 and 11). This is because the minimum value of ν required to prevent a loss of damping with increasing C_L is larger with than without a constant weight moment (Fig. 2). If the moment is due to the unbalanced mass of the elevator itself, the effective moment of inertia or k_B^2 of the whole aircraft is reduced (by about 20 per cent. for $\Delta h_n' = 0 \cdot 05$, (*see* equation (73)) and this increases the value of ν for the same tail size, but not by the required amount.

Since it is generally agreed that it is desirable to have positive static stability with stick both fixed and free, the only legitimate use for a weight, except as an emergency measure, is to counteract a loss of tail plane effectiveness on freeing the stick with a convergent elevator ($b_1 < 0$), say one with a set-back hinge and servo tab. The present tendency towards complete mass-balancing of elevators as a measure of flutter prevention has in fact made it more difficult to obtain close aerodynamic balance without loss of static stability.

Although effective in preventing divergence, the constant weight moment combined with a convergent elevator is much less efficient dynamically than a neutral elevator ($b_1 = 0$). As shown in Fig. 2, a larger effective tail plane area $(S'a_1'/a_1)$ is needed with than without a weight

* With the assumptions made here (*see* footnote on page 5).

to ensure adequate damping at high values of C_L, but the loss in effectiveness with the convergent elevator is actually reducing a_1' when the stick is free. It is therefore preferable to design the elevator to give as little loss of stability, stick free, as is possible ($a_1' = a_1$) without a weight, and to use the weight only as a last resource if the stability is unsatisfactory in flight.

4.2. *Spring Moment.*—Approximate expressions for period and damping have been developed in Appendix IV for the case of a spring in the elevator circuit, but they are not applicable when $h_n' - h$ (or ω) is negative. As a spring is more likely to be used when the static margin is negative without it, a more accurate method of solution is needed for the most interesting part of the curves for damping and period in Figs. 8, 9, 12. This is discussed in Appendix IV (A.4.2).

If there is a small positive static margin initially, the addition of the spring reduces both period and damping appreciably and the damping may become negative (*cf.* Figs. 6 and 8, 10 and 12).

For an initial negative static margin the damping is compared in Fig. 9 ($C_L = 0{\cdot}6$) with the negative damping (or rate of growth) of the divergence, which occurs without the spring. It is seen that at best the divergence is replaced by an unstable oscillation, and there may even be a dynamic divergence, although the static stability is positive. This means that, although the pilot must push on the stick to hold it at the diving speed when it is trimmed for level flight, he will have to pull to keep the aircraft from going into too steep a dive until the speed has increased to the diving speed. The spring gives static stability because the floating angle of the elevator changes with speed, but at constant speed the aircraft is still unstable stick-free. In a manoeuvre the speed changes relatively slowly and, if at the level flight speed the trimmer is set correctly for the dive and the stick left free, the aircraft will diverge before the stabilising effect of the speed change has had time to have any effect. This may not be important in practice because the pilot will take control during the manoeuvre. A device like the weight moment, which adds to the damping in pitch as well as to the static stability, is much more effective dynamically, as is shown in Fig. 7, although still not as good as a larger tail plane.

Even a small spring moment has a large effect on the minimum value of ν or effective tail plane size required to prevent a loss of damping with increasing C_L (Fig. 2). The method used for calculating these curves breaks down for larger values of ω_s, but it is clear from Figs. 8, 9, 12 that no practicable size of tail plane can prevent instability with a larger spring ($\omega_s = 10$ at 10,000 ft. and 30 at 40,000 ft.).

The loss of phugoid damping with a spring has been observed in tests on the Mosquito and the Beaufighter, and is confirmed by the more systematic German tests referred to above[8]. Calculated values for the Spitfire with a spring moment of $5{\cdot}5$ lb. ft., giving $\Delta h_n' = 0{\cdot}05$, are given in Table 7A (iii) at the end of the report, and Fig. 5 shows an appreciable loss of damping for the Halifax due to a spring giving only $0{\cdot}007$ for $\Delta h_n'$, which is equivalent to the unbalanced weight moment of 17 lb. ft. on the production aircraft.

The calculated effect of an allowance for the extensibility of a typical spring, as used on the Hampden, has been found to be small but unfavourable on the Spitfire. In Germany[7], apparently, elevator oscillations have been set up by a flexible spring, but no such case is known to have occurred in England.

5. *Effect of Altitude.*—Theoretically the static margin in the glide is unaffected by altitude and this is consistent with results of full-scale tests on the Spitfire.

When the control system is mass-balanced and frictionless, the effects of altitude on dynamic stability are not very marked. If the static margin is negative, the rate of divergence becomes more rapid as the altitude increases (Fig. 9). The minimum damping of the phugoid for a given C_L or indicated air speed, if expressed in non-dimentional units or in terms of the number of cycles to damp to half amplitude, is independent of height, but it occurs at a further aft centre of gravity as the height increases. If expressed as the reciprocal of the time in seconds to halve the amplitude, the minimum damping is reduced with increase in height, except at high C_L and with a very small tailplane (Figs. 6 and 10). At a given static margin the damping ($1/T$) is reduced by an increase in height, except with a small tailplane over a limited range of C.G. position. The period corresponding with the minimum damping increases with height at the same indicated air speed (Figs. 6, 10), but it is unaffected at the same true speed (equation (4)) unless the tail-plane effectiveness varies with C_L. The period of the short period oscillation (Table 6) decreases with increasing altitude.

A weight or spring moment gives the same increment $\Delta h_n'$ in the static margin at all altitudes, but the effect on dynamic stability becomes more severe as the height increases. With a constant weight moment in the circuit, a larger tail is required at the higher altitude to prevent a loss of damping with the increasing C_L, since ω_w in Fig. 2 is proportional to $1/\rho$. Even with the large tail of Figs. 7, 11, the damping with the weight moment falls to a very low value.

With a spring the effect is even more marked. For a large initial static margin the effect of altitude is favourable, (Figs. 8, 12), but for small positive values a rapidly growing oscillation may occur at 40,000 ft. If the aircraft is statically unstable without the spring, the rate of growth of the oscillation or divergence increases with altitude and at 40,000 ft. is almost as large with as without the spring, (Fig. 9), in spite of the increase of 0·05 in the static margin. The estimated increase in instability with altitude for the Spitfire is shown in Section (iii), Tables 7A, 7B. Calculations for the Halifax (Fig. 5) show that a relatively small spring may cause a large loss of damping at 40,000 ft., while at 5,000 ft. the effect is comparatively small.

6. *Comparison with Flight Tests.*—Attempts have been made to compare estimates of dynamic stability with phugoids measured with free elevators in flight. The results are far from conclusive owing to the inadequacy of model and flight data, particularly on the positions of the neutral points with free elevator. In general, however, if there is an ample margin of static stability, the period and damping of the phugoid are in quite good agreement with estimated values. The main discrepancy lies in the region of small margins of static stability. Here theory (Fig. 3) would indicate increased damping as the centre of gravity moves aft, followed by a subsidence which changes over to a divergence as the centre of gravity passes through the neutral point. Such behaviour is observed in some cases and a subsidence or very mild divergence is usually considered satisfactory by the test pilot. In many cases, however, unstable phugoids appear as the centre of gravity moves aft, followed by very erratic behaviour, such as a brief oscillation followed by a divergence or an oscillation of such large amplitude that the pilot takes charge almost immediately. These may be genuine divergences for small disturbances within the order considered by the theory, and they are certainly undesirable. Apparent divergences, however, may occur with nearly neutral static stability if the aircraft is not exactly in trim at the equilibrium speed. With the centre of gravity still further aft, no oscillation is possible and there is a definite divergence indicating serious static instability.

The position of the neutral point stick free is rarely measured in flight, and the accuracy of the tests when available is not high, so it cannot be stated definitely on which side of the neutral point the unstable phugoids occur, but they appear to be on the stable side, and in some cases there are signs of a dynamic divergence with positive $h_n' - h$[10]. On theoretical grounds such unstable phugoids would be expected only with a very small tail or for a normal tail with a large loss of effectiveness on freeing the stick (large b_1/b_2) or with a spring in the elevator circuit, while divergences can occur with a positive static margin only with a spring.

Unpublished tests on the Halifax prototype (L.7244) may be quoted as giving an example of a disagreement between estimated and measured phugoids, while a production aircraft (L.9505)[8] gives good agreement. The estimated and measured values are compared in the following Table 3 and the estimated values are shown as points * in Fig. 4.

TABLE 3

Aircraft No.	Period		Cycles to halve amplitude		W	$h_n' - h$	C_L
	Measured	Calculated	Measured	Calculated			
L.7244	$\simeq$33	32	Unstable	2·0	43,300	0·065	1·12
L.9505	$\simeq$40	39	$\simeq$2·0	1·9	53,000	0·053	0·9

The elevator nose of the L.7244 was subsequently modified to a shape adopted for the production type, but the firm's model tests indicate little change in tail-plane effectiveness with free elevator due to the modification, and the calculated values are based on the National Physical Laboratory tests with the original elevator nose shape. Also Fig. 4 shows that the estimated damping varies little with tail plane effectiveness; the figures marked on the full line curves give the ratio of a_1' (stick free) to a_1 (stick fixed). Another possible explanation was a change in the degree of mass-balance of the elevator, but Fig. 5 shows that this effect cannot be large. The weight moment on the production elevator is about 17 lb. ft., but this is relatively small for such a large aircraft and gives an estimated $\Delta h_n'$ of only 0·007. Measurements of neutral points in gliding flight should throw some light on the problem, but this is no longer possible on the L.7244 with the original elevators.

Tests on the Spitfire at various altitudes[9] show a change from a neutrally stable oscillation at 6,000 ft. to an unstable one at 30,000 ft. As the elevator is not completely mass-balanced, such a change can be explained qualitatively by the theory (*see* Fig. 7). From quantitative estimates (Table 7), based admittedly on inadequate flight and model data on elevator characteristics, a stable phugoid would be expected at either height. In this case Dr. Neumark has pointed out that the calculated movements, which change neutral stability or a divergence with stick fixed into a stable phugoid with stick free and a weight moment on the elevator, are so extremely small that they would be prevented or considerably modified by friction in the elevator circuit. The relative amplitudes for the phugoid motion, based on an assumed value for u, are given in Table 4 below. The equivalent value of the hinge moment is shown to be small compared with the moment required to overcome the static friction on the ground.

TABLE 4

V (T.A.S.) m.p.h. at 40,000 ft.	Assumed u m.p.h.	θ	η	$H_\eta = b_2 \eta \frac{1}{2} \rho V^2 S_\eta c_\eta$ lb. ft.	Friction moment on ground lb. ft.
240	12	3·6°	0·05°	0·09	1·6

It is generally assumed that friction in the elevator circuit gives a phugoid intermediate between the estimated phugoids for stick fixed and stick free, but it may be more unstable than either. If the effect were similar to that of an initially tensioned spring, which applies a nearly constant moment about the elevator hinge, it could easily account for the existence of unstable phugoids or even divergence with positive static stability and for a deterioration in dynamic stability with altitude. In this connection the spring is typical of any device whose effect on static stability depends only on the variation of forward speed (through m_u) and which provides no damping of the pitching motion.

7. *Summary of Conclusions.*—(1) In modern aircraft the period of the phugoid is of the order of $\frac{1}{2}$ to 2 minutes. Its damping is essentially small. The phugoid problem is to avoid conditions in which this small damping falls to zero.

(2) A good approximation to the damping factor in aerodynamic time is $fC_{D0} + FC_L^2$, where f is roughly constant at about $\frac{2}{3}$ and F has large variations with ω (static stability) and ν (tail damping), (*see* Fig. 1 and Appendix II).

(3) The order of the damping is governed primarily by C_{D0}, which fixes its value at high speed at about $\frac{2}{3}$ C_{D0}, and excludes an increasing oscillation in that region.

(4) F increases with ν at all values of ω and may be negative if ν is small enough: ν, or $l^2S'/k_B{}^2S$, is therefore the chief secondary factor in determining the damping; the larger the tail, the better.

There is a value ν_0 for which the minimum value of F is zero. If ν is greater than ν_0 the damping is never less than fC_{D0}. If ν is less than ν_0 the damping falls below fC_{D0} for some values of ω and may become negative at high C_L (Fig. 15, full lines). An average value of ν_0 is 1·5 (Fig. 2, full line), corresponding to a tail volume of about 0·3, which is well below the values now in common use. Accordingly, an increasing oscillation in the glide should be rare, unless it is caused by an unexpectedly large decrease in tail effectiveness on freeing the stick.

(5) At low values of C_L the damping is nearly independent of static margin, provided it is positive, while at high values an envelope curve for the flying range of wing loading and altitude would show little change in damping with static margin (*see* Fig. 10). Thus choice of static margin cannot be dictated by a logical argument from damping. The lower limits now being recommended (0·02 to 0·06 according to type) are governed by wider considerations, and may fall on either side of the minimum damping, according to size of tail. They avoid the critical region from 0 to 0·02, where factors (*e.g.* friction) not included in the theory may have a powerful adverse effect on the damping.

(6) Increase in altitude has no effect on the static margin, but if this is negative it increases the rate of divergence. If the static margin is positive, it moves the point of minimum damping nearer the origin (Fig. 10). Unless the tailplane is exceptionally small the time to damp to half amplitude is increased by an increase in altitude over the whole range of static margin, but the change is likely to be most pronounced with a small static margin.

(7) A deficiency in stick free static margin can often most easily be made up by putting a weight or a long spring in the elevator circuit. These affect the damping in different ways, since the effect of the weight, unlike that of the spring, depends on changes in acceleration as well as changes in forward speed.

(*a*) A weight is in general an effective remedy. If applied at a negative static margin, it converts a divergence into a damped oscillation, although this may become unstable at high altitudes (Fig. 11), but, if applied at a small positive margin, there may be an appreciable decrease in damping* (Fig. 13).

(*b*) A spring is in general an ineffective remedy. At a negative margin it makes the margin less negative without producing stability; at a moderate positive margin it produces an increasing oscillation (Fig. 14). Its adverse effects increase with altitude.

(8) Application of the phugoid analysis to flight records shows that the agreement is reasonably good when the static margin of the experiment is definitely positive, but when it is small and uncertain in sign the theory is a less reliable guide. In particular it fails to account for the frequent occurrence of increasing oscillations, mainly at small static margins. It is clear from the analysis itself that the region between small negative and positive margins is a critical one in which factors such as friction, which are not included in the theory, may have large effects. Further experimental and theoretical work is in progress to clear up these discrepancies.

(9) As this analysis ignores thrust and slipstream effects, no comprehensive rules can be given for design against negative damping. The following rough guides can however be given :—

(*a*) The static margin, both stick fixed and stick free, should never be less than 0·02.

(*b*) In working to a given margin, it is better to use a large than a small tail; in general the tail volume should not be less than 0·5.

(*c*) If an easy remedy is needed for a negative margin, stick free, a weight should be used, not a spring.

* *See* footnote on page 5.

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APPENDIX I

Summary of Theory of Longitudinal Stability in Gliding Flight

A.I.1. *Equations of Motion and Solution.*—The system of axes and the notation are based on R. & M. 1801[5] with some modifications suggested by Dr. Neumark. The origin is at the centre of gravity of the aircraft. The x-axis is forward along the wind direction in the equilibrium condition and is fixed in the aircraft during the disturbed motion. The z-axis is downwards in the plane of symmetry of the aircraft and perpendicular to the x-axis, while the y-axis is to starboard. Only motion in the xz-plane is considered here.

In the equilibrium condition the aircraft is moving with velocity V_e along the x-axis, which is inclined at an angle γ_e to the horizontal; (γ_e is positive in the climb and is denoted by θ_0 in R. & M. 1801[5]). In the disturbed motion the component velocities are $V + u$ along the x-axis, w along the z-axis and an angular velocity q about the y-axis, where u, w, θ, q are so small that second order terms can be neglected. The aerodynamic force has components X, Z along the x- and z-axes, and produces a moment M about the y-axis.

During a disturbance the x-axis is inclined at an angle $(\gamma_e + \theta)$ to the horizontal, where $d\theta/dt = q$, the angle of climb is $(\gamma_e + \theta - w/V_e)$ and the angle of incidence $(\alpha_e + w/V_e)$, where α_e is the incidence in the equilibrium condition.

The gravity forces are $+ mg \cos (\gamma_e + \theta) = + mg \cos \gamma_e - mg\theta \sin \gamma_e$ along the z-axis and

$$- mg \sin (\gamma_e + \theta) = - mg \sin \gamma_e - mg\theta \cos \gamma_e$$

along the x-axis.

The accelerations in the disturbed motion are $\dot{u}$ along the x-axis, $\dot{w} - V_e q$ along the z-axis and $\dot{q}$ about the y-axis, the corresponding forces and moment being $- m\dot{u}$, $- m(\dot{w} - V_e q)$, and $- B\dot{q}$, where B is the moment of inertia of the whole aircraft about the y-axis.

The aerodynamic forces may be expressed in the form

$$\left.\begin{aligned} X &= X_e + X_u u + X_w w + X_q q \,, \\ Z &= Z_e + Z_u u + Z_w w + Z_q q \,, \\ M &= M_e + M_u u + M_w w + M_q q + M_{\dot{w}} \dot{w} + M_\eta \eta \,, \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

where X_e, Z_e, M_e are the values of X, Z, M in the equilibrium condition, and X_u *etc.* represent $\partial X/\partial u$ *etc.* Strictly, X, Z, M should include acceleration terms due to $\dot{u}$, $\dot{w}$, $\dot{q}$, but $M_{\dot{w}}$ is the only one which is sufficiently important to be included. $M_\eta \eta$ represents the moment applied by the elevator in controlled motion, where η is the elevator angle measured from its value in the equilibrium condition. The corresponding forces $X_\eta \eta$ or $Z_\eta \eta$ are negligible.

By equating the sum of the inertia, gravity and aerodynamic forces or moments to zero and subtracting the terms for the equilibrium conditions which are themselves zero, we get the equations

$$\left.\begin{aligned} - m(\dot{u} + g\theta \cos \gamma_e) + X_u u + X_w w + X_q q &= 0 \\ - m(\dot{w} - V_e q + g\theta \sin \gamma_e) + Z_u u + Z_w w + Z_q q &= 0 \\ - B\dot{q} + M_u u + M_w w + M_q q + M_{\dot{w}} \dot{w} + M_\eta \eta &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

These can be expressed in non-dimensional units in the form*

$$\hat{u} = \frac{u}{V_e} \,, \quad \hat{w} = \frac{w}{V_e} \,, \quad \hat{q} = \hat{t} q, \quad \tau = \frac{t}{\hat{t}} \,, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where

$$\hat{t} = \frac{m}{\rho S V_e} = \frac{V_e C_L}{2g} = \frac{1}{g} \sqrt{\left(\frac{W}{S} \cdot \frac{C_L}{2\rho}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

and $\hat{t}$ seconds is the unit of aerodynamic time τ.

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* In R. & M. 1801[5] the non-dimensional forms are $\mu_1 (u/V_e)$, $\mu_1 (w/V_e)$, $\hat{t}q$ (τq in the notation of R. & M. 1801[5]).

The first two equations are then multiplied by $1/\rho SV_e^2$ and the third by $\mu_1/\rho SlV_e^2 i_B$, where $i_B = B/ml^2 = (k_B/l)^2$, k_B is the radius of gyration of the aircraft and $\mu_1 = m/\rho Sl$ and represents the relative density of the aircraft. Also $\hat{u}$, $\hat{w}$, $\hat{q}$, θ are expressed in the form

$$\hat{u} = Ue^{\lambda\tau},\ \hat{w} = We^{\lambda\tau},\ \hat{q} = Qe^{\lambda\tau},\ \theta = \Theta e^{\lambda\tau} = \frac{\hat{q}}{\lambda}\ ,$$

where λ is given by the roots of the determinantal equation

$\frac{u}{V_e}$	$\frac{w}{V_e}$	θ	η
$\lambda - x_u$	$- x_w$	$-\lambda \frac{x_q}{\mu_1} + \frac{C_L}{2}$	0
$- z_u$	$\lambda - z_w$	$\lambda\left(\frac{- z_q}{\mu_1} - 1\right) + \frac{C_L \tan \gamma_e}{2}$	0
$- \mu_1 m_u'$	$- \mu_1 m_{\dot{w}}' \lambda - \mu_1 m_w'$	$- m_q' \lambda + \lambda^2$	$- \mu_1 m_\eta'$

$= 0$ .. (11)

where

$$x_u = X_u/\rho SV_e\ ,\qquad z_u = Z_u/\rho SV_e\ ,$$
$$x_w = X_w/\rho SV_e\ ,\qquad z_w = Z_w/\rho SV_e\ ,\qquad \text{.. .. .. (12)}$$
$$x_q = X_q/\rho SV_e l\ ,\qquad z_q = Z_q/\rho SV_e l\ ,$$

$$m_u = i_B m_u' = M_u/\rho SlV_e\ ,\qquad \mu_1 m_{\dot{w}}{}^* = i_B \mu_1 m_{\dot{w}}' = M_{\dot{w}}/\rho Sl^2\ ,$$
$$m_w = i_B m_w' = M_w/\rho SlV_e\ ,\qquad m_q = i_B m_q' = M_q/\rho Sl^2 V_e\ ,\qquad \text{.. .. (13)}$$
$$m_\eta = i_B m_\eta' = M_\eta/\rho SlV_e^2\ .$$

The equation for λ is

$$\lambda^4 + B_1\lambda^3 + C_1\lambda^2 + D_1\lambda + E_1 = 0\ ,\qquad \text{.. .. .. (14)}$$

where, with fixed elevator ($\eta = 0$) and $x_q = 0$,

$$B_1 = -(x_u + z_w) - m_q' - \left(1 + \frac{z_q}{\mu_1}\right)\mu_1 m_{\dot{w}}'\ ,$$

$$C_1 = (x_u z_w - x_w z_u) - \left(1 + \frac{z_q}{\mu_1}\right)\mu_1 m_w' + (x_u + z_w)m_q' + \mu_1 m_{\dot{w}}'\left[x_u\left(1 + \frac{z_q}{\mu_1}\right) + \frac{C_L}{2}\tan\gamma_e\right]\ ,$$

$$D_1 = \mu_1 m_w'\left[x_u\left(1 + \frac{z_q}{\mu_1}\right) + \frac{C_L}{2}\tan\gamma_e\right] + \mu_1 m_u'\left[- x_w\left(1 + \frac{z_q}{\mu_1}\right) + \frac{C_L}{2}\right] - m_q'(x_u z_w - x_w z_u) + \mu_1 m_{\dot{w}}'\frac{C_L}{2}(z_u - x_u\tan\gamma_e)\ ,\qquad \text{.. .. (15)}$$

$$E_1 = \frac{C_L}{2}\left[\mu_1 m_w'(z_u - x_u\tan\gamma_e) - \mu_1 m_u'(z_w - x_w\tan\gamma_e)\right]\ .$$

It is shown later that these equations apply also to the case of a free elevator if appropriate values are used for the derivatives.

* It should be noted that in this notation $\mu_1 m_{\dot{w}}$ is independent of μ_1 (*see* equation 36).

For complete dynamic stability all the coefficients $B_1 - E_1$ must be positive and also, if there is to be no unstable oscillation, the Routhian discriminant

$$\bar{R} = B_1C_1D_1 - D_1^2 - B_1^2E_1 \qquad (16)$$

must be positive. Static stability is positive if $E_1 > 0$. In general there can be no divergence (positive real root of λ), unless $E_1 < 0$, as E_1 is usually the first of the coefficients to change sign. It is possible, however, in special cases (*see* Appendix IV) for C_1 to become negative before E_1. There may then be two positive real roots of λ and a dynamic divergence can occur with positive static stability. Before C_1 changes sign, in this case, $\bar{R} < 0$, since C_1 is small, and there is an unstable oscillation (*see* Figs. 8 and 9).

A more usual form for the static stability condition is

$$\frac{dC_m}{dC_L} = \frac{\partial C_m}{\partial \alpha}\frac{\partial \alpha}{\partial C_L} + \frac{\partial C_m}{\partial V}\frac{\partial V}{\partial C_L} > 0 , \qquad (17)$$

where $\partial V/\partial C_L$ is given by the relationship

$$C_L \frac{\rho}{2} V^2S = W.$$

This is equivalent to

$$m_wz_u - m_uz_w > 0 , \qquad (18)$$

which is approximately the same as $E_1 > 0$.

The roots of the stability quartic can be determined by a method of successive approximation described in R. & M. 1118[4]. In general there are two pairs of complex roots, of which one pair, given by $\lambda = -R \pm iJ$, represents a strongly damped oscillation of relatively high frequency, and the other, $\lambda = -r \pm ij$, represents the phugoid motion of low frequency. The phugoid period in seconds is

$$P = 2\pi \frac{\hat{t}}{j} \text{ seconds} \qquad (19)$$

and the time to damp to half amplitude is

$$T = \frac{\hat{t} \log_e 2}{r} \text{ seconds} . \qquad (20)$$

When r is negative this means that the time to double the amplitude is $-T$.

For a subsidence or divergence the time to damp to half amplitude is

$$T = \frac{\hat{t} \log_e 2}{-\lambda} . \qquad (20a)$$

With decreasing static stability ($-m_w$) the short period motion splits up into two subsidences, one of which remains heavily damped, while the damping of the other one decreases with m_w. With a further decrease in stability the phugoid is also replaced by two subsidences, one with increasing and one with decreasing damping, which becomes negative as $-m_w$ passes through zero. As the static margin becomes more negative, the two intermediate roots combine to form a damped oscillation of slow period, similar to a phugoid, and the heavily damped subsidence and the divergence persist. The behaviour is illustrated in Table 6C.

A.I.2. *Expressions for the Derivatives.*—The derivatives in equation (11) can be expressed in terms of aerodynamic characteristics which can be calculated or derived from wind-tunnel tests. The equilibrium conditions are

$$\left.\begin{aligned} L &= W \cos \gamma_e \ , \\ D &= - W \sin \gamma_e \ , \\ \tan \gamma_e &= - \frac{C_D}{C_L} \ . \end{aligned}\right| \quad \ldots \quad (21)$$

In disturbed motion due to u, w, θ, η

$$X = \tfrac{1}{2}\rho S \, (V_e + u)^2 \left[C_L \frac{w}{V_e} - \frac{\partial C_D}{\partial \alpha} \left(\alpha_e + \frac{w}{V_e} \right) \right] - W \sin (\gamma_e + \theta) \quad \ldots \quad (22)$$

and therefore

$$X_u = \rho S V_e \, (- C_D) \quad \text{and} \quad x_u = - C_D \ . \quad \ldots \quad (23)$$

$$X_w = \tfrac{1}{2}\rho S V_e \left(C_L - \frac{\partial C_D}{\partial \alpha} \right) \ , \ x_w = \tfrac{1}{2} \left(C_L - \frac{\partial C_D}{\partial \alpha} \right) \ . \quad \ldots \quad (24)$$

Similarly

$$Z = \tfrac{1}{2}\rho S \, (V_e + u)^2 \left[- \frac{\partial C_L}{\partial \alpha} \left(\alpha_e + \frac{w}{V_e} \right) - C_D \frac{w}{V_e} \right] + W \cos (\gamma_e + \theta) \quad \ldots \quad (25)$$

and therefore

$$Z_u = \rho S V_e \, (- C_L) \ , \qquad z_u = - C_L \ , \quad \ldots \quad (26)$$

$$Z_w = \tfrac{1}{2}\rho S V_e \left(- \frac{\partial C_L}{\partial \alpha} - C_D \right) \ , \qquad z_w = - \tfrac{1}{2} \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) \ . \quad \ldots \quad (27)$$

Also

$$M = \tfrac{1}{2}\rho S c \, (V_e + u)^2 \left[\frac{\partial C_{Mn}}{\partial \alpha} \left(\alpha_e + \frac{w}{V_e} \right) - \bar{V} \, (a_1 \alpha_T + a_2 \, (\eta_e + \eta) + a_3 \beta) \right] , \quad \ldots \quad (28)$$

where $\alpha_T = \alpha_e + \dfrac{w}{V_e} + \eta_T - \dfrac{d\varepsilon}{d\alpha} \left(\alpha_e + \dfrac{w}{V_e} \right)$,

$$C_{Mn} = C_{M \text{ no tail}} = C_{M0} + C_L \, (h - h_0) + k \left(C_{D0} - \frac{C_L^2}{6} \right) \quad \ldots \quad (29)$$

and k is the distance of the centre of gravity below the wing chord. In the equilibrium condition

$$\frac{\partial C_{Mn}}{\partial \alpha} \alpha_e - \bar{V} \, [\, a_1 \, (\alpha_e + \eta_T - \varepsilon) + a_2 \eta_e + a_3 \beta \,] = 0 \quad \ldots \quad (30)$$

and therefore $M_u = 0$, $m_u = 0$.

With the stick fixed, $\eta = 0$,

$$\begin{aligned} M_w &= \tfrac{1}{2}\rho S c V_e \left[\frac{\partial C_{mn}}{\partial \alpha} - \bar{V}_{a1} \left(1 - \frac{d\varepsilon}{d\alpha} \right) \right] \\ &= \tfrac{1}{2}\rho S c V_e \frac{\partial C_m}{\partial \alpha} = \tfrac{1}{2}\rho S c V_e \frac{\partial C_L}{\partial \alpha} \, (h - h_n) \ ; \quad \ldots \quad (31) \end{aligned}$$

and $$m_w = \tfrac{1}{2} \frac{c}{l} \frac{\partial C_m}{\partial \alpha} = \tfrac{1}{2} \, a \, \frac{c}{l} \, (h - h_n)^* \ , \quad \ldots \quad (32)$$

where h_n is the neutral point and is given by

$$\frac{dC_m}{dC_L} = h - h_n \ .$$

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* It should be noted that m_w can be expressed in terms of $(h - h_n)$ only when, as in the case considered here, C_m is independent of speed ($m_u = 0$). Slipstream effects, for instance, introduce a term in $\delta Cm/\delta V$ in the expression for dC_m/dC_L in equation (17), in which case $\delta Cm/\delta\alpha$ is no longer proportional to static margin.

The derivatives with respect to the angular velocity q can be measured on a whirling arm. Of these x_q' is negligible. Theoretical values of z_q and m_q due to the wings have been calculated by H. Glauert and are given in R. & M. 1216[12]. The most important part of m_q is due to the change of incidence $\Delta\alpha_T = lq/V_e$ on the tailplane. Thus

$$\left.\begin{aligned} M_q\ (tail) &= \tfrac{1}{2}\rho ScV_e^2\left(-\bar{V}a_1\frac{l}{V_e}\right), \\ m_q\ (tail) &= -\tfrac{1}{2}\frac{S'}{S}a_1 . \end{aligned}\right\} \qquad (33)$$

Similarly

$$\left.\begin{aligned} Z_q\ (tail) &= M_q/l , \\ z_q\ (tail) &= -\tfrac{1}{2}\frac{S'}{S}a_1 . \end{aligned}\right\} \qquad (34)$$

The term z_q is small and can be neglected in general, but $m_{q\ (tail)}$ represents the main contribution to the damping of the pitching motion.

The importance of the term $M_{\dot{w}}$ arises from the fact that the tail plane at time t is influenced by the vortices which were cast off by the wing at a time $(t - l/V_e)$. Thus, when $\alpha = \alpha_e + \dot{w}t/V$, $\varepsilon = \frac{d\varepsilon}{d\alpha}\left\{\alpha_e + \frac{\dot{w}}{V}\left(t - \frac{l}{V_e}\right)\right\}$. Therefore, at time $t = 0$,

$$M_{\dot{w}} = \tfrac{1}{2}\rho ScV_e^2\left[-\bar{V}a_1\frac{d\varepsilon}{d\alpha}\frac{l}{V_e^2}\right] \qquad (35)$$

and
$$\mu_1 m_{\dot{w}} = -\frac{a_1}{2}\frac{S'}{S}\frac{d\varepsilon}{d\alpha} = m_{q\ (tail)}\frac{d\varepsilon}{d\alpha} . \qquad (36)$$

The term is not sufficiently important to warrant a closer approximation.

The above formulae apply strictly to the stick fixed condition, but they are applicable to the stick free case when elevator, stick and control circuit are completely mass-balanced, provided that

$$a_1' = a_1 - \frac{a_2 b_1}{b_2}$$

is substituted for a_1. This is shown by equating the hinge moment coefficient to zero,

$$C_H = b_1\alpha' + b_2(\eta_e + \eta) + b_3\beta = 0 \qquad (37)$$

and substituting for $\eta_e + \eta$ in equation (28), with

$$\alpha_T = \alpha_e + \frac{w}{V_e} + \eta_T + \frac{ql}{V_e} - \frac{d\varepsilon}{d\alpha}\left(\alpha_e + \frac{w}{V_e} - \frac{\dot{w}l}{V_e^2}\right) . \qquad (38)$$

The formulae are summarised below :—

TABLE 5

	Stick Fixed	Stick Free
x_u	$-C_D$	$-C_D$
x_w	$\frac{1}{2}\left(C_L - \frac{\partial C_D}{\partial\alpha}\right)$	$\frac{1}{2}\left(C_L - \frac{\partial C_D}{\partial\alpha}\right)$
z_u	$-C_L$	$-C_L$
z_w	$-\frac{1}{2}(a + C_D)$	$-\frac{1}{2}(a + C_D)$
$z_{q\ (tail)}$	$-\frac{1}{2}\frac{S'}{S}a_1$	$-\frac{1}{2}\frac{S'}{S}a_1'$
m_u	0	0
m_w	$\frac{1}{2}\frac{c}{l}\frac{\partial C_m}{\partial\alpha} =$ $\frac{1}{2}\frac{c}{l}a(h - h_n)$*	$\frac{1}{2}\frac{c}{l}\frac{\partial C_m}{\partial\alpha} =$ $\frac{1}{2}\frac{c}{l}a(h - h_n')$*
$m_{q\ (tail)}$	$-\frac{1}{2}\frac{S'}{S}a_1$	$-\frac{1}{2}\frac{S'}{S}a_1'$
$+ \mu_1 m_{\dot{w}}$	$-\frac{1}{2}\frac{S'}{S}a_1\frac{d\varepsilon}{d\alpha}$	$-\frac{1}{2}\frac{S'}{S}a_1\frac{d\varepsilon}{d\alpha}$

A.I.3. *Modified Formulae for Analysis of Stability Quartic.*—A more convenient form for computational work, suggested by Dr. Neumark, is obtained by the following substitutions, with their appropriate values for the gliding case :—

$$\omega = -\mu_1 m_w' = \frac{mc}{\rho S k_B^2}\frac{a}{2}(h_n - h)^*,$$

$$\varkappa = -\mu_1 m_u' = 0\ ,$$

$$\nu = -m_q' \simeq \frac{a_1}{2}\frac{S'}{S}\frac{l^2}{k_B^2}\ , \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

if the part of m_q' due to wing and body is neglected,

$$\chi = -\mu_1 m_{\dot{w}}' = \frac{a_1}{2}\frac{S'}{S}\frac{l^2}{k_B^2}\frac{d\varepsilon}{d\alpha}\ ,$$

* *See* footnote on page 15.

$$
\left.
\begin{aligned}
N_1 &= -(x_u + z_w) = \frac{a}{2} + \frac{3}{2} C_D \ , \\
P_1 &= x_u z_w - x_w z_u = \frac{C_D}{2}(a + C_D) + \frac{C_L}{2}\left(C_L - a\frac{dC_D}{dC_L}\right) , \\
Q_1 &= -x_u\left(1 + \frac{z_q}{\mu_1}\right) - \frac{C_L}{2}\tan\gamma_e = C_D\left(\frac{3}{2} + \frac{z_q}{\mu_1}\right) \simeq \frac{3}{2} C_D \ , \\
R_1 &= -\frac{C_L}{2}(z_u - x_u \tan\gamma_e) = \tfrac{1}{2}(C_L^2 + C_D^2) \ , \\
S_1 &= \frac{C_L}{2} - x_w\left(1 + \frac{z_q}{\mu_1}\right) = \tfrac{1}{2}\left[C_L - \left(C_L - a\frac{dC_D}{dC_L}\right)\left(1 + \frac{z_q}{\mu_1}\right)\right] \\
&\simeq \frac{a}{2}\frac{dC_D}{dC_L} \ , \\
T_1 &= -\frac{C_L}{2}(z_w - x_w \tan\gamma_e) = a\,\frac{C_L}{4}\left[1 + \frac{C_D}{C_L}\frac{dC_D}{dC_L}\right] .
\end{aligned}
\right\} \quad \ldots \quad \ldots \quad (40)
$$

The constants N_1 and T_1 depend mainly on the condition of flight and to a lesser extent on the geometry of the aircraft. The fundamental qualities of the aircraft which determine its stability characteristics are ω, depending on the static stability and relative density μ_1, ν which defines the damping in pitch, and χ which depends on the damping and the downwash.

The coefficients in the stability quartic become (from (15))

$$
\left.
\begin{aligned}
B_1 &= N_1 + \nu + \left(1 + \frac{z_q}{\mu_1}\right)\chi \ , \\
C_1 &= P_1 + \omega\left(1 + \frac{z_q}{\mu_1}\right) + N_1\nu + Q_1\chi \ , \\
D_1 &= Q_1\omega + P_1\nu + R_1\chi - S_1\varkappa \ , \\
E_1 &= R_1\omega - T_1\varkappa \ .
\end{aligned}
\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (41)
$$

Here B_1 represents the total damping and is independent of the static margin, while E_1 is proportional to the static margin and is unaffected directly by the damping.

APPENDIX II

Approximate Theory of Phugoid Damping

If, in the stability quartic

$$\lambda^4 + B_1\lambda^3 + C_1\lambda^2 + D_1\lambda + E_1 = 0 \;, \qquad (42)$$

C_1 is large compared with D_1 and E_1, the phugoid roots are given approximately by

$$\lambda^2 + \lambda\left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^{\,2}}\right) + \frac{E_1}{C_1} = 0 \,, \qquad (43)$$

or

$$\lambda = -r \pm ij \;, \qquad (44)$$

where r, the damping of the phugoid motion, is given by

$$r = \tfrac{1}{2}\left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^{\,2}}\right) \qquad (45)$$

and j, the frequency, is given by

$$j = \sqrt{\left(\frac{E_1}{C_1} - r^2\right)} \,. \qquad (46)$$

If r^2 is small compared with E_1/C_1, the non-dimensional period is approximately

$$\hat{P} = 2\pi\sqrt{\frac{C_1}{E_1}} \,. \qquad (47)$$

A closer approximation,

$$\hat{P} = \frac{2\pi\sqrt{\dfrac{C_1}{E_1}}}{\sqrt{\left(1 - \dfrac{r^2C_1}{E_1}\right)}} \;, \qquad (48)$$

is needed when ω is small.

For further simplification only the important terms in B_1, C_1, E_1, D_1 are retained. These are

$$\left.\begin{aligned} B_1 &= N_1 + \nu + \chi \;, \\ C_1 &= N_1\nu + \omega \;, \\ D_1 &= Q_1\omega + P_1\nu + R_1\chi \;, \\ E_1 &= R_1\omega \;, \end{aligned}\right\} \qquad (49)$$

where, if

$$C_D = C_{D0} + sC_L^{\,2} \;, \qquad (50)$$

$$\left.\begin{aligned} N_1 &= \frac{a}{2} \;, \\ P_1 &= \frac{a}{2}\,(C_{D0} + sC_L^{\,2}) + \frac{C_L^{\,2}}{2}\,(1 - 2as) \;, \\ Q_1 &= \frac{3}{2}\,(C_{D0} + sC_L^{\,2}) \;, \\ R_1 &= \frac{C_L^{\,2}}{2} \,. \end{aligned}\right\} \qquad (51)$$

The damping r then becomes

$$r = fC_{D0} + FC_L^2 \; , \qquad \ldots \qquad (52)$$

where

$$2f = 1 + \frac{\omega}{2\omega + a\nu} \; ,$$

$$2F = \frac{(3\omega - a\nu)s}{(2\omega + a\nu)} - \frac{a\{\omega - \nu(\nu + \chi)\}}{(2\omega + a\nu)^2} \; . \qquad \ldots \qquad (53)$$

The non-dimensional period $\hat{P}$ is approximately

$$\hat{P} = \frac{2\pi\sqrt{2}}{C_L}\sqrt{\left(1 + \frac{a\nu}{2\omega}\right)} \; , \qquad \ldots \qquad (54)$$

or more accurately for small values of ω

$$\hat{P} = 2\pi \Bigg/ \sqrt{\left(\frac{\omega C_L^2}{2\omega + a\nu} - r^2\right)} \; . \qquad \ldots \qquad (55)$$

In dimensional units the time T in seconds to halve the amplitude is

$$T = \frac{0\cdot 69315\hat{t}}{r} = \frac{0\cdot 312}{r}\sqrt{\left(\frac{W}{S}\frac{C_L}{\sigma}\right)} \; , \qquad \ldots \qquad (56)$$

and the period P in seconds is

$$\left.\begin{aligned} P = \hat{t}\,\hat{P} &= 4\cdot 0\sqrt{\left[\frac{W}{S}\frac{1}{\sigma C_L}\left(1 + \frac{a\nu}{2\omega}\right)\right]} \\ &= 0\cdot 138\, V_e\sqrt{\left(1 + \frac{a\nu}{2\omega}\right)} \; . \end{aligned}\right\} \qquad \ldots \qquad (57)$$

When $r^2 = \dfrac{\omega C_L^2}{2\omega + a\nu}$, the period $\hat{P}$ becomes infinite (from equation (55)), and, as ω decreases, the phugoid splits up into two subsidences given by

$$\lambda = -r \pm \sqrt{\left(r^2 - \frac{\omega C_L^2}{2\omega + a\nu}\right)} \; , \qquad \ldots \qquad (58)$$

and r represents the mean damping of the two subsidences. It is seen from Fig. 3 and Table 6C that this happens over a very small range of ω or $h_n' - h$, and it cannot be shown on the scale of the diagrams of Figs. 6 and 10. When ω passes through zero one subsidence changes sign and becomes a divergence, with static instability.

In the approximate values for the coefficients B_1, C_1, D_1, E_1, (equations (49), (51)), it is assumed that

$$\left.\begin{aligned} C_D^2 &\ll C_L^2 \ll a\nu \; , \\ C_D &\ll \nu \text{ or } a \; , \\ z_q &\ll \mu \; . \end{aligned}\right\} \qquad \ldots \qquad (59)$$

The order of error introduced by these assumptions and by the approximate method of solution of the quartic is shown in Table 8 for positive values of ω or $(h_n - h)$. The error at high values of C_L is due to the first of the assumptions in equation (59), but the order of accuracy is quite good enough for practical purposes. The approximate values of the coefficients of the quartic given in equations (49) and (51) may therefore be assumed to hold up to $C_{L\,max}$ with flaps closed.

The approximate method of solution of the quartic given by equation (43), or more completely by

$$(\lambda^2 + B\lambda + C)\left[\lambda^2 + \lambda\left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^2}\right) + \frac{E_1}{C_1}\right] = 0 \quad \ldots \quad (60)$$

is not valid for negative values of ω, but it does hold remarkably well down to quite small positive values, as is shown in Fig. 3. When ω is negative, C_1 becomes small and an exact solution of equation (42) is required to give the four roots. A curve for the real divergent root can be obtained quite quickly, as in Figs. 6, 9, 10, by assigning values to λ and calculating ω from the equation

$$\lambda^4 + B_1\lambda^3 + (C_{11} + C_{12}\omega)\lambda^2 + (D_{11} + D_{12}\omega) + (E_{11} + E_{12}\omega) = 0 \quad \ldots \quad (61)$$

The behaviour when C_1 is small or even negative, while E_1 is positive, is discussed in A.I.1 and more fully in Appendix IV.

APPENDIX III

Minimum Tail Size required for Phugoid Damping

It is clear that ν can never fall below its value at $C_L = 0$ so long as $F \nless 0$, where

$$2F = \frac{s(3\omega - a\nu)}{(2\omega + a\nu)} - \frac{a(\omega - \nu(\nu + \chi))}{(2\omega + a\nu)^2} \quad \ldots \quad (62)$$

F is positive when $\omega = 0$ and when ω is ∞, and can be negative for some value of ω only if $F(\omega, \nu) = 0$ has two real roots in ω. But F is always positive if

$$6s\omega^2 - \omega\,(a - sa\nu) + \{a\nu(\nu + \chi) - sa^2\nu^2\} > 0 \quad \ldots \quad (63)$$

for all values of ω. This is true if

$$(a - sa\nu)^2 < 24s^2a^2\nu^2\left(\frac{\nu + \chi}{sa\nu} - 1\right), \quad \ldots \quad (64)$$

where $\dfrac{\nu + \chi}{\nu} = 1 + \dfrac{d\varepsilon}{d\alpha}$.

Thus $$\nu_{min} = \frac{1}{\xi + s}, \quad \ldots \quad (65)$$

where $$\frac{\xi^2}{s^2} = 24\left\{\frac{\left(1 + \frac{d\varepsilon}{d\alpha}\right)}{as} - 1\right\}. \quad \ldots \quad (66)$$

The quantities a, s and $\dfrac{d\varepsilon}{d\alpha}$ are functions of aspect ratio A, and the generalised curve for ν_{min} against A in Fig. 2 is based on the following assumptions :—

$$a = \frac{2\pi}{1 + 2/A}, \quad s = \frac{1\cdot2}{\pi A}, \quad \frac{d\varepsilon}{dC_L} = \frac{2}{\pi A}$$

and therefore

$$\frac{d\varepsilon}{d\alpha} = \frac{4}{A + 2}, \quad \frac{\xi^2}{s^2} = 10A + 36. \quad \ldots \quad (67)$$

The minimum value of ν is therefore

$$\nu_{min} = \frac{\pi A}{1\cdot2\,[1 + \sqrt{(10A + 36)}]} \quad \ldots \quad (68)$$

APPENDIX IV

The Effect of a Weight or Spring in the Elevator Circuit on the Phugoid

A.IV.1. *Stability derivatives.*—The increment η_K in the floating angle of the elevator due to a constant* weight moment K about the elevator hinge is given by

$$0 = H_K = \tfrac{1}{2}\rho S_\eta c_\eta (V_e + u)^2 b_2 \eta_K + K\left(1 - \frac{\dot{w} - V_e q + l_K \dot{q}}{g}\right) , \qquad (69)$$

where l_K is the distance of the weight from the centre of gravity of the aircraft. The increment M_K in the pitching moment is therefore

$$\left.\begin{aligned} M_K &= \tfrac{1}{2}\rho S c V_e^2 (-\bar{V} a_2 \eta_K) \\ &= \tfrac{1}{2}\rho S c V_e^2 \left[\frac{a_2}{b_2} \bar{V} n C_L \left\{1 - 2\frac{u}{V_e} - \frac{\dot{w} - V_e q + l_K \dot{q}}{g}\right\}\right] , \end{aligned}\right\} \qquad (70)$$

where $$n = \frac{KS}{WS_\eta c_\eta} . \qquad (71)$$

The rearward movement $\Delta h_n'$ of the neutral point is

$$\Delta h_n' = -\frac{dC_m}{dC_L} = -\frac{a_2}{b_2}\bar{V} n , \qquad (72)$$

and the derivatives are

$$\left.\begin{aligned} M_u &= \frac{2}{V_e} W c \,\Delta h_n' , & m_u &= \frac{c}{l} C_L \,\Delta h_n' , \\ \Delta M_w &= 0 , & \Delta m_w &= 0 , \\ \Delta M_{\dot{w}} &= \frac{Wc}{g} \Delta h_n' , & \Delta m_{\dot{w}} &= \mu_1 \frac{c}{l} \Delta h_n' , \\ \Delta M_q &= -\frac{WcV_e}{g} \Delta h_n' & \Delta m_q &= -\mu_1 \frac{c}{l} \Delta h_n' , \\ \Delta B &= -\frac{W}{g} c l_K \,\Delta h_n' , & \Delta i_B &= -\frac{c l_K}{l^2} \Delta h_n' . \end{aligned}\right\} \qquad (73)$$

If $i_B = 0{\cdot}08$, $l_K = l$, $l/c = 3$, $\Delta h_n' = 0{\cdot}05$, the effective moment of inertia is reduced by 21 per cent.

Since a spring has no inertia, all the increments in the derivatives vanish with the exception of m_u.

A.IV.2. *Approximate Formulae for Phugoid Damping and Period with a Spring.*—The approximate method of Appendix II can be applied to a spring, if the terms in the coefficients D_1 and E_1 due to m_u are included. Let $\omega_s = (a/2C_L)\,\mu_1\, m_u'$

$$= \frac{mc}{\rho S k_B^2} \frac{a}{2} \Delta h_n' = \omega \frac{\Delta h_n'}{h_n' - h} . \qquad (74)$$

Then
$$\left.\begin{aligned} \Delta D_1 &= \frac{2C_L}{a} S_1 \omega_s = 2sC_L^2 \omega_s , \\ E_1 &= -\frac{C_L}{2} z_w \frac{2C_L}{a} \omega_s = \frac{C_L^2}{2} \omega_s , \end{aligned}\right\} \qquad (75)$$

$$\left.\begin{aligned} \Delta f &= 0 , \\ 2\Delta F &= \frac{4s\omega_s}{2\omega + a\nu} = \frac{\omega_s(a + 2\nu + 2\chi)}{(2\omega + a\nu)^2} , \end{aligned}\right\} \qquad (76)$$

---

* *See* footnote on page 5.

and

$$\left.\begin{aligned} 2f &= 1 + \frac{\omega}{2\omega + a\nu} , \\ 2F &= \frac{(3\omega - a\nu + 4\omega)s}{2\omega + a\nu} \\ &\quad - \frac{a\{\omega - \nu(\nu + \chi)\} + \omega_s(a + 2\nu + 2\chi)}{(2\omega + a\nu)^2} , \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (77)$$

where ω and ν have the same values as without the spring. The non-dimensional period is approximately

$$\hat{P} = 2\pi\sqrt{\frac{C_1}{E_1}} = \frac{2\pi\sqrt{2}}{C_L}\sqrt{\left(1 + \frac{a\nu - 2\omega_s}{2(\omega + \omega_s)}\right)}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (78)$$

as compared with

$$\hat{P} = \frac{2\pi\sqrt{2}}{C_L}\sqrt{\left(1 + \frac{a\nu}{2(\omega + \omega_s)}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (79)$$

if the increase in static stability is produced by moving the centre of gravity instead of by adding the spring.

The above formulae hold only when $\omega > 0$, *i.e.* when there is a positive static margin without the spring. It will be noticed that C_1 is unaffected by the addition of the spring and is still approximately (from equations (49) and (51))

$$C_1 = \frac{a\nu}{2} + \omega .$$

As ω increases negatively, C_1 becomes small and eventually negative, while E_1 is still positive so long as $\omega_s > -\omega$. Even for positive values of ω, C_1 may not be large enough compared with E_1 and D_1 to keep $\bar{R} = B_1C_1D_1 - D_1^2 - B_1^2E_1$ positive. This shows that the spring is likely to give unstable phugoids for small positive and negative values of ω and may even give divergences for larger negative values, while the static stability margin is still positive because of the stabilising effect of ω_s. This is illustrated in Figs. 8, 9 and 12 and discussed in §4.2. For very small positive values and for all negative values of ω it is necessary to solve the quartic equation exactly and not by the approximate method of equation (62).

A.IV.3. *Approximate Formulae for Phugoid Damping and Period with a Constant Weight Moment.**—If the weight is near the centre of gravity of the aircraft, i_B is unaltered and the effect of the term m_u on D_1 and E_1 is the same as for the spring. In addition, if

$$\omega_w = \omega\Delta h_n'/(h_n' - h) ,$$

$$\left.\begin{aligned} \Delta\nu &= \Delta(-m_q') = \frac{2}{a}\,\omega_w , \\ \Delta\chi &= -\mu\Delta m_{\dot{w}}' = -\frac{2}{a}\,\omega_w , \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (80)$$

* *See* footnote on page 5.

and the values of $B_1 - E_1$ become

$$\left.\begin{aligned} B_1 &= \frac{a}{2} + \nu + \chi \ , \\ C_1 &= \frac{a\nu + 2(\omega + \omega_w)}{2} \ , \\ D_1 &= (C_{D0} + s\,C_L^2)\,\frac{(a\nu + 3\omega + 2\omega_w)}{2} + \frac{C_L^2}{2}\,(\nu + \chi - 2as\nu) \, , \\ E_1 &= \frac{C_L^2}{2}\,(\omega + \omega_w) \ , \end{aligned}\right\} \qquad \ldots \quad \ldots \quad (81)$$

where ω and ν have the same values as without the weight, and the effect of the weight is represented solely by the terms in ω_w.

The damping r is given by

$$r = fC_{D0} + F\,C_L^2 \ ,$$

where

$$2f = 1 + \frac{\omega}{a\nu + 2(\omega + \omega_w)} \ ,$$

$$2F = \frac{\{3\omega + 2\omega_w - a\nu\}\,s}{2(\omega + \omega_w) + a\nu} - \frac{a\{\omega + \omega_w - \nu\,(\nu + \chi)\}}{[2(\omega + \omega_w) + a\nu]^2} \quad . \quad \ldots \quad \ldots \quad \ldots \quad (82)$$

If the weight is some distance l_K away from the centre of gravity, say on the elevator itself, these formulae still hold if i_B is replaced by

$$i_B - \frac{l_K c}{l^2}\,\Delta h_n' \ .$$

The non-dimensional period with a weight moment is approximately

$$\hat{P} = 2\pi\sqrt{\frac{C_1}{E_1}} = \frac{2\pi\sqrt{2}}{C_L}\sqrt{\left(1 + \frac{a\nu}{2(\omega + \omega_w)}\right)} \ , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (83)$$

and has the same value as if the static stability were increased by moving the centre of gravity aft.

In this case C_1 depends on the static margin with and not without the weight and the approximate method is still valid for negative values of ω, provided $(\omega + \omega_w)$ is not very small or negative. The effect of the weight is shown in Figs. 7, 11, 13, 15.

A.IV.4. *The Effect of a Spring or a Weight on the Minimum Tail Size required for Stability of the Phugoid.*—The minimum tail size required to ensure that r does not fall below its value at $C_L = 0$ can be found by the same method as in Appendix III for any value of ω_s or ω_w over the range of values of ω for which the approximate method of solution of the quartic is valid.

For the spring the method of Appendix III applied to equation (77) for F gives

$$A\nu^2 - 2B\nu + C > 0 \qquad (84)$$

$$\nu > \frac{B}{A} \pm \sqrt{\left(\frac{B^2}{A^2} - \frac{C}{A}\right)}, \qquad (85)$$

where

$$\left.\begin{aligned} A &= a^2 (\xi^2 - s^2) , \\ B &= -as (a + 16s\omega_s) + a\xi^2\omega_s , \\ C &= -(64s^2\omega_s^2 + 8as\omega_s + a^2) . \end{aligned}\right\} \qquad (86)$$

If the vlaue of ω corresponding with the minimum ν is negative, this formula no longer holds, because the expression for ν on which it is based is not sufficiently accurate. The curve for $\omega_s = 5$ in Fig. 2, however, does give an indication of the destabilising effect of the spring and of the increase in tail area which may be necessary to avoid oscillatory instability at high C_L. It is shown in Figs. 8, 9, 12 that no practicable size of tail is likely to be adequate with larger effective spring moments, (in these figures $\omega_s = 10$ at 10,000 ft. and 30 at 40,000 ft.).

For the constant weight moment, equation (82) for F gives

$$\nu > \frac{B}{A} \pm \sqrt{\left(\frac{B^2}{A^2} - \frac{C}{A}\right)},$$

where

$$\left.\begin{aligned} A &= a^2 (\xi^2 - s^2) , \\ B &= -as (a - 10s\omega_w) , \\ C &= -(a + 2s\omega_w)^2 . \end{aligned}\right\} \qquad (87)$$

In this case the approximate method of solution holds well enough down to $\omega = -\omega_w$, and the curves for ν_{min} in Fig. 2 give a reliable indication of the loss of damping due to the weight. This loss is considerably less than with the spring, but is still appreciable and it reduces the value of the weight as a stabilising device.*

* *See* footnote on page 5.

TABLE 6

Estimated Damping and Periods for the Halifax

6A. 10,000 ft.

(i) Stick fixed ($h_n = 0\cdot433$) (ii) Stick free (mass-balanced elevator)

W	h	C_L	(i) Short period oscillation: Time to ½ ampl.	(i) Short period oscillation: Period	(i) Phugoid: Time to ½ ampl.	(i) Phugoid: Period	(ii) Short period oscillation: Time to ½ ampl.	(ii) Short period oscillation: Period	(ii) Phugoid: Time to ½ ampl.	(ii) Phugoid: Period	h_n'
37,000	0·227	0·2	0·26	1·77	60·32	63·85	0·27	1·83	60·56	64·06	0·418
		0·6	0·45	3·07	75·21	36·85	0·49	3·30	78·01	37·10	0·403
		1·0	0·59	3·96	62·47	28·56	0·64	4·39	68·34	28·85	0·393
	0·350	0·2	0·32	3·44	65·74	74·33	0·33	3·87	66·55	76·94	
		0·6	0·55	5·97	86·44	42·94	0·59	7·83	82·98	46·72	
		1·0	0·71	7·71	74·88	33·42	0·78	11·52	60·61	38·21	
60,000	0·248	0·2	0·25	1·52	74·00	78·65	0·26	1·58	74·24	78·91	
		0·6	0·43	2·63	83·94	45·37	0·47	2·86	85·89	45·69	
		1·0	0·57	3·40	64·21	35·15	0·62	3·83	67·59	35·50	
	0·333	0·2	0·30	2·41	77·91	84·31	0·31	2·62	78·73	85·70	
		0·6	0·52	4·17	94·16	48·64	0·56	5·07	98·17	50·60	
		1·0	0·67	5·39	75·97	37·73	0·74	7·14	81·29	40·11	
					6B. 40,000 ft.						
37,000	0·227	0·2	0·45	1·66	97·59	102·70	0·47	1·71	97·77	102·84	0·418
		0·6	0·79	2·87	107·04	59·24	0·84	3·11	108·98	59·38	0·403
		1·0	1·01	3·70	79·81	45·84	1·11	4·11	83·10	46·00	0·393
	0·350	0·2	0·56	3·06	104·46	109·45	0·57	3·41	106·04	111·23	
		0·6	0·96	5·30	133·54	63·14	1·02	6·75	150·28	65·74	
		1·0	1·23	6·87	113·95	48·89	1·34	9·77	153·27	52·31	
60,000	0·248	0·20	0·43	1·42	121·97	129·14	0·45	1·48	122·14	129·32	
		0·6	0·75	2·45	127·50	74·52	0·81	2·69	128·92	74·71	
		1·0	0·97	3·17	91·82	57·68	1·08	3·59	94·05	57·88	
	0·333	0·2	0·52	2·20	125·51	132·63	0·54	2·39	126·35	133·53	
		0·6	0·90	3·80	138·46	76·52	0·97	4·57	145·68	77·75	
		1·0	1·16	4·91	103·79	59·23	1·29	6·42	116·60	60·73	

TABLE 6C

Stability Roots for Small Static Stability

Halifax at 5,000 ft.; $W = 53{,}000$; $C_L = 1 \cdot 23$ (*see* Fig. 3) stick free, mass-balanced elevator

ω	$h_n' - h$	λ_1	λ_2	λ_3	λ_4
−1·50	−0·010	−4·8107	−0·7400±0·5778i		+0·2702
−0·30	−0·002	−4·4654	−0·9549	−0·6793	+0·0792
0	0	−4·3660	−1·2198	−0·4347	0
+0·15	+0·001	−4·3135	−1·3146	−0·3314	−0·0610
+0·30	+0·002	−4·2590	−1·4023	−0·1796±0·0782i	
+0·75	+0·005	−4·0798	−1·6520	−0·1444±0·2533i	
+1·50	+0·010	−3·6907	−2·1118	−0·1090±0·3676i	

ω	$h_n' - h$	Subsidence Time to ½ ampl.	Stable Oscillation Time to ½ ampl.	Stable Oscillation Period	Divergence Time to 2 ampl.
−1·50	−0·010	0·50	3·28	38·11	8·98
			Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	
−0·30	−0·002	0·54	2·54	3·57	30·64
0	0	0·56	1·99	5·58	∞
					Subsidence Time to ½ ampl.
+0·15	+0·001	0·56	1·85	7·32	39·78
				Stable Oscillation Time to ½ ampl.	Stable Oscillation Period
+0·30	+0·002	0·57	1·73	13·51	281·59
+0·75	+0·005	0·59	1·47	16·81	86·93
+1·50	+0·010	0·66	1·15	22·26	59·90

TABLE 7A

Estimated Damping and Periods for the Spitfire

10,000 ft.

(i) Stick fixed ($h_n = 0 \cdot 310$)

W	h	C_L	Short period oscillation		Stable phugoid	
			Time to ½ ampl.	Period	Time to ½ ampl.	Period
5,270	0·25	0·2	0·24	2·33	53·87	59·39
5,960	0·25	1·0	0·58	5·20	59·37	27·74
			Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Divergence Time to 2 ampl.
5,270	0·314	0·2	0·15	0·72	10·04	18·12
		0·6	0·25	1·40	4·58	11·89
				Stable oscillation Time to ½ ampl.	Stable oscillation Period	
		1·0	0·33	2·58	47·36	11·08
5,960	0·352	0·2	0·12	14·55	33·88	1·36
		0·6	0·21	8·13	23·19	1·75
		1·0	0·28	6·34	20·06	1·85

(ii) Stick free ($h_n' = 0 \cdot 295$) (mass-balanced elevator)

W	h	C_L	Short period oscillation		Stable phugoid	
			Time to ½ ampl.	Period	Time to ½ ampl.	Period
5,270	0·25	0·2	0·25	2·75	54·99	61·53
5,960	0·25	1·0	0·62	6·10	64·60	28·71
			Subsidence Time to ½ ampl.	Stable oscillation Time to ½ ampl.	Stable oscillation Period	Divergence Time to 2 ampl.
5,270	0·314	0·2	0·14	2·93	43·43	4·80
		0·6	0·24	3·53	25·13	3·66
		1·0	0·31	3·64	21·49	3·42
5,960	0·352	0·2	0·12	26·17	39·69	0·73
		0·6	0·21	13·35	24·40	1·13
		1·0	0·27	8·88	20·22	1·31

TABLE 7A—*contd.*

(iii) Spring ($\Delta h_n' = 0{\cdot}05$)

W	h	C_L	Short period oscillation		Stable phugoid	
			Time to ½ ampl.	Period	Time to ½ ampl.	Period
5,270	0·25	0·2	0·25	2·75	59·22	42·36
5,960	0·25	1·0	0·62	6·20	448·45	19·67
			Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Unstable Time to 2 ampl.	phugoid Period
5,270	0·314	0·2	0·14	1·69	17·7	30·55
		0·6	0·24	1·95	8·98	21·88
		1·0	0·31	2·00	7·91	19·02
				Stable oscillation		Divergence
				Time to ½ ampl.	Period	Time to 2 ampl.
5,960	0·352	0·2	0·12	50·22	109·59	0·75
				Unstable oscillation		
				Time to 2 ampl.	Period	
		0·6	0·21	228·68	62·20	1·38
		1·0	0·27	34·79	45·84	2·05

(iv) Weight moment ($\Delta h_n' = 0{\cdot}05$)

W	h	C_L	Short period oscillation		Stable phugoid	
			Time to ½ ampl.	Period	Time to ½ ampl.	Period
5,270	0·25	0·2	0·25	1·72	61·40	55·06
5,960	0·25	1·0	0·60	3·90	76·15	25·73
5,270	0·314	0·2	0·25	3·79	86·61	67·47
		0·6	0·43	6·52	114·31	38·85
		1·0	0·55	8·33	98·44	30·27
			Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Divergence Time to 2 ampl.
5,960	0·352	0·2	0·16	0·91	8·64	0·62
		0·6	0·28	1·99	3·36	7·13
				Stable oscillation		
				Time to ½ ampl.	Period	
		1·0	0·37	2·87	39·66	6·54

TABLE 7B

Estimated Damping and Periods for the Spitfire

40,000 ft.

(i) Stick fixed ($h_n = 0 \cdot 310$)

W	h	C_L	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Divergence Time to 2 ampl.
5,270	0·314	0·2	0·23	2·02	8·82	13·79
				Stable oscillation		
				Time to ½ ampl.	Period	
		0·6	0·41	4·70	61·98	9·52
		1·0	0·52	5·26	44·05	8·69
5,960	0·352	0·2	0·16	67·59	79·77	0·54
		0·6	0·28	44·71	46·55	0·92
		1·0	0·36	27·04	36·69	1·15

(ii) Stick free ($h_n{}' = 0 \cdot 295$) (mass-balanced elevator)

W	h	C_L	Subsidence Time to ½ ampl.	Stable oscillation		Divergence Time to 2 ampl.
				Time to ½ ampl.	Period	
5,270	0·314	0·2	0·19	37·89	62·47	1·39
		0·6	0·33	19·11	39·20	2·08
		1·0	0·43	13·19	32·85	2·35
5,960	0·352	0·2	0·15	74·77	83·14	0·39
		0·6	0·26	53·77	48·24	0·67
		1·0	0·34	32·97	37·71	0·85

TABLE 7B—*contd.*

(iii) Spring ($\Delta h_n' = 0 \cdot 05$)

W	h	C_L	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Divergence Time to 2 ampl.	Divergence Time to 2 ampl.
5,270	0·314	0·2	0·19	5·58	4·39	1·61
					Unstable phugoid	
					Time to 2 ampl.	Period
		0·6	0·33	4·04	3·19	32·12
		1·0	0·43	3·57	3·54	25·77
				Stable oscillation		Divergence Time to 2 ampl.
				Time to ½ ampl.	Period	
5,960	0·352	0·2	0·15	94·31	235·84	0·39
		0·6	0·26	360·08	136·26	0·68
				Unstable oscillation		
				Time to 2 ampl.	Period	
		1·0	0·34	318·54	105·57	0·90

(iv) Weight moment ($\Delta h_n' = 0 \cdot 05$)

W	h	C_L	Short period oscillation		Stable phugoid	
			Time to ½ ampl.	Period	Time to ½ ampl.	Period
5,270	0·314	0·2	0·42	2·99	170·80	96·76
		0·6	0·73	5·16	296·28	55·66
		1·0	0·93	6·67	388·45	43·17
			Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Subsidence Time to ½ ampl.	Divergence Time to 2 ampl.
5,960	0·352	0·2	0·24	4·45	5·91	5·81
		0·6	0·42	6·07	53·42	5·35
		1·0	0·55	6·05	45·43	5·06

TABLE 8

Comparison of Exact and Approximate Methods of Calculating Phugoid Damping

Spitfire ($h = 0 \cdot 25$) at 10,000 ft.

Elevator	W lb.	C_L	r	
			Exact	Approximate
Fixed ($h_n = 0 \cdot 310$)	5,270	0·2	0·0141	0·0141
Fixed ($h_n = 0 \cdot 310$)	5,960	1·0	0·0304	0·0351
Free (mass-balanced) ($h_n' = 0 \cdot 295$)	5,270	0·2	0·0138	0·0138
Free (mass-balanced) ($h_n' = 0 \cdot 295$)	5,960	1·0	0·0279	0·0317
Free with spring ($\Delta h_n' = 0 \cdot 05$)	5,270	0·2	0·0128	0·0128
	5,960	1·0	0·0040	0·0076
Free with weight ($\Delta h_n' = 0 \cdot 05$)	5,270	0·2	0·0123	0·0124
	5,960	1·0	0·0237	0·0261

Halifax $W = 60{,}000$ lb., elevator fixed

Height	h_n	h	C_L	r	
				Exact	Approximate
40,000 ft.	0·433	0·333	0·2	0·0156	0·0155
			0·6	0·0244	0·0245
			1·0	0·0421	0·0423
10,000 ft.	0·433	0·333	0·2	0·0144	0·0144
			0·6	0·0207	0·0212
			1·0	0·0331	0·0346

LIST OF SYMBOLS

A = aspect ratio (except in equations 86, 87).
a = $\partial C_L/\partial \alpha$ for the complete aircraft.
a_1 = $\partial C_L'/\partial \alpha_T$ for the tail plane with fixed elevator.
$a_1' = a_1 - \frac{a_2 b_1}{b_2}$, effective value of $\partial C_L'/\partial \alpha_T$ for the tail plane with free elevator.
a_2 = $\partial C_L'/\partial \eta$.
B = moment of inertia of aircraft about the lateral axis.
B_1 (*See* Equations (14), (15)).
b_1 = $\partial C_H/\partial \alpha_T$.
b_2 = $\partial C_H/\partial \eta$.
C_1 (*See* equations (14), (15)).
C_D = $D/\frac{1}{2}\rho V^2 S$.
C_{D_0} = value of C_D at $C_L = 0$.
C_H = $H/\frac{1}{2}\rho V^2 S_\eta c_\eta$ due to aerodynamic forces.
C_L = $L/\frac{1}{2}\rho V^2 S$.
C_L' = $L'/\frac{1}{2}\rho V^2 S'$.
C_m = $M/\frac{1}{2}\rho V^2 Sc$.
C_{m_n} = value of C_m without tail.
C_{m_0} = value of C_{m_n} at $C_L = 0$.
c = mean wing chord.
c_η = mean elevator chord.
D = drag of complete aircraft.
D_1 (*See* equations (14), (15)).
E_1 (*See* equations (14), (15)).
F (*See* equations (52), (53)).
f (*See* equations (52), (53)).
g = acceleration due to gravity.
H = moment about elevator hinge.
H_K = H due to moment K.
hc = distance of the centre of gravity aft of leading edge of mean wing chord.
h_n = value of h for neutral static stability, stick fixed.
h_n' value of h for neutral static stability, stick free, without weight or spring.
$\Delta h_n'$ increment in h_n' due to weight or spring.
i_B $B/ml^2 = k_B^2/l^2$.
J large roots of quartic (equation 14) are $-R \pm iJ$.
j small roots (phugoid) of quartic are $-r \pm ij$.
K static moment about elevator hinge due to a weight or a spring. Assumed independent of altitude of aircraft.
kc distance of centre of gravity below mean wing chord.
k_B radius of gyration of aircraft about the lateral axis.
L lift on complete aircraft.
L' lift on tail plane.
l distance from aircraft centre of gravity to mean $\frac{1}{4}$-chord point of tail plane.
l_K distance from aircraft centre of gravity to inertia weight or elevator centre of gravity.
M pitching moment on complete aircraft.
M_K pitching moment due to K (equation (70)).
m = W/g = mass of the aircraft.
m_u, m_w *etc.* (*See* equation (13) and Table 5).
m_u', m_w', *etc.* (*See* equation (13)).
N_1 (*See* equation (40)).
n = $KS/WS_\eta c_\eta$.
P period of phugoid oscillation in seconds.
$\hat{P}$ period of phugoid oscillation in non-dimensional units ($P = \hat{t}\hat{P}$).

P_1 (*See* equation (40)).
Q_1 (*See* equation (40)).
q angular velocity (radians per second) of the aircraft in pitch.
$\hat{q} = \hat{t}q$.
R large roots of quartic (equation 14) are $- R \pm iJ$.
$\bar{R}$ (*See* equation (16)).
R_1 (*See* equation (40)).
r small roots (phugoid) of quartic are $- r \pm ij$.
S gross wing area.
S' gross tail-plane area.
S_1 (*See* equation (40)).
S_η elevator area.
s (*See* equation (50)).
T seconds to halve amplitude (equations 20, 20a).
T_1 (*See* equation (40)).
t time in seconds.
$\hat{t} = m/\rho SV$ = unit of aerodynamic time.
u increment of velocity along the x-axis in disturbed flight.
$\hat{u} = u/Ve$.
V resultant velocity of aircraft in disturbed flight.
V_e velocity of aircraft in equilibrium condition.
$\bar{V} = S'l/Sc$ tail volume ratio.
W all-up weight of aircraft.
w increment of velocity along the z-axis in disturbed flight.
$\hat{w}$ w/Ve.
X aerodynamic force along x-axis.
$X_u, X_w, etc. = \partial X/\partial u, \partial X/\partial w, etc.$
x axis fixed in the aircraft in disturbed flight in direction of motion in equilibrium condition.
x_u, x_w, x_q (*See* equation (12) and Table 5).
Z aerodynamic force along z-axis.
z_u, z_w, z_q (*See* equation (12) and Table 5).
α wing incidence measured from zero lift.
α_e value of α in equilibrium condition.
α_T tail-plane incidence to relative wind.
β tab angle relative to elevator.
γ_e angle of climb in equilibrium condition.
ε mean downwash angle at the tail.
η increment in elevator angle during disturbed flight.
η_e elevator angle in equilibrium condition.
η_K increment in elevator floating angle due to K.
η_T tail setting relative to zero lift line of wing.
θ angle of rotation of x-axis from equilibrium condition.
$\varkappa$ (*See* equation (39)).
λ (*See* equation (11)).
$\mu_1 = m/\rho Sl$.
ν (*See* equation (39)).
ξ (*See* equation (66)).
ρ air density.
σ air density/standard air density at sea level.
τ aerodynamic time $= t/\hat{t}$.
χ (*See* equation (39)).
ω (*See* equation (39)).
$\omega_s = \omega \Delta h_n'/(h_n' - h)$, where $\Delta h_n'$ is due to a spring in the elevator circuit (*See* also equation (74)).
$\omega_w = \omega \Delta h_n'/(h_n' - h)$, where $\Delta h_n'$ is due to a weight moment.

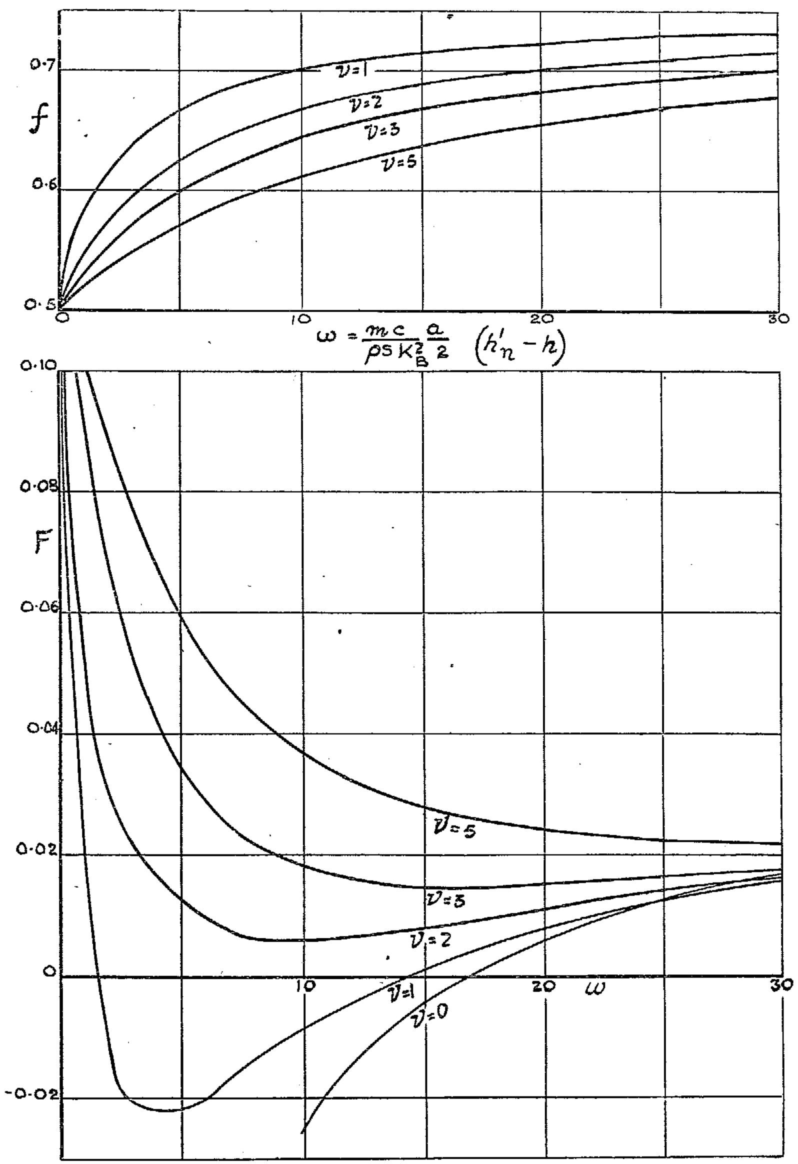

FIG. 1. Variation of f and F with ω and ν.

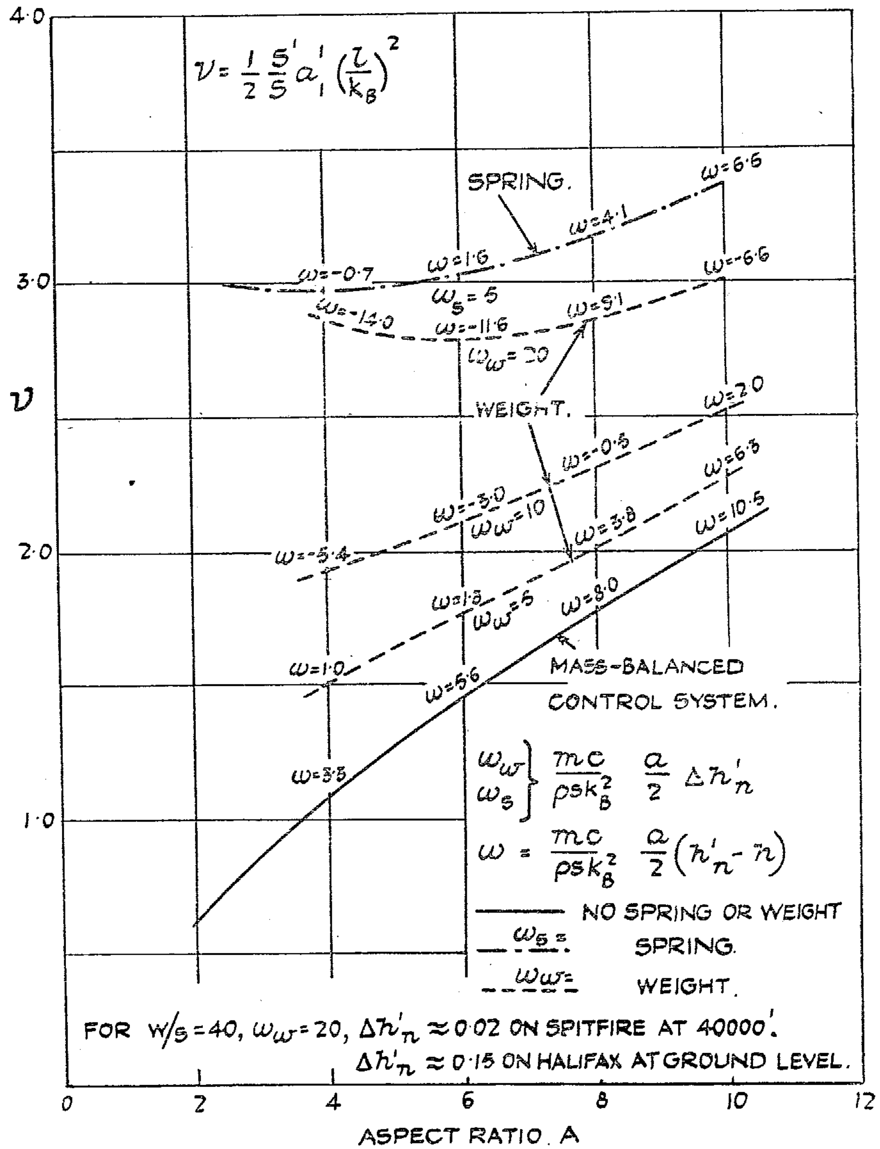

FIG. 2. Minimum Value of ν to give $F \geqslant 0$ for Various Values of ω_s and ω_w.

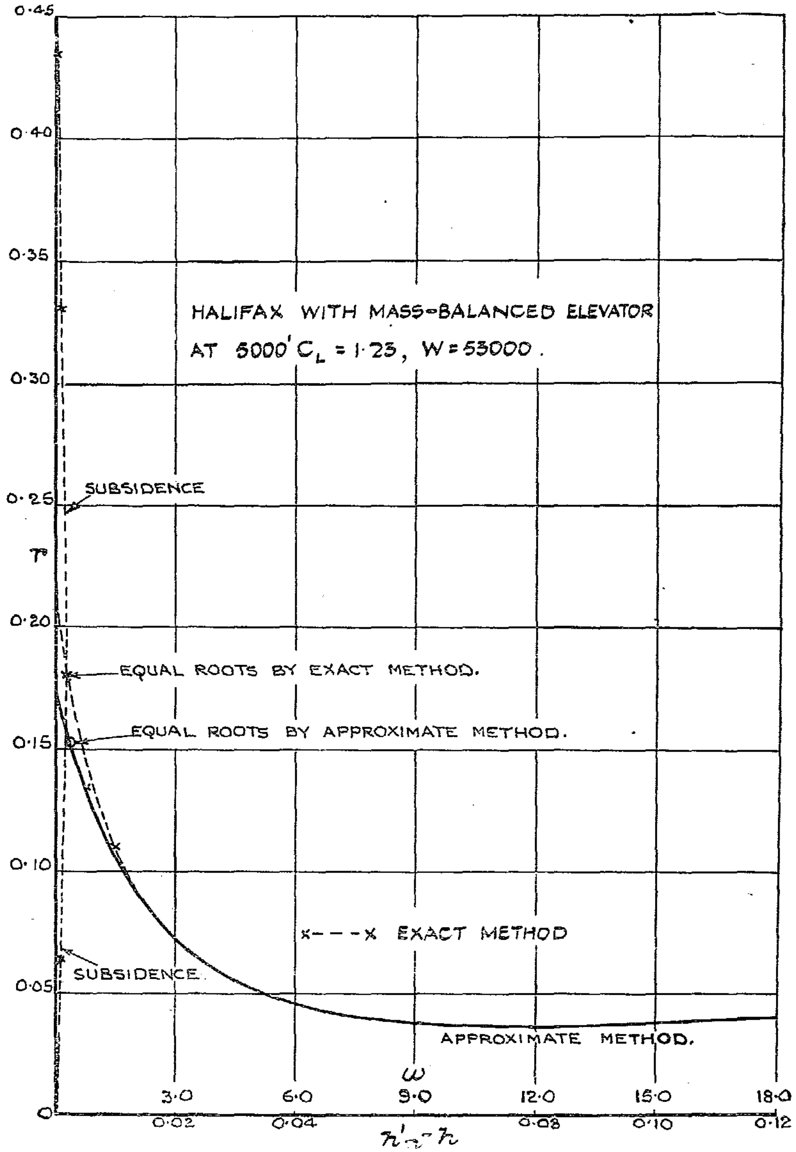

FIG. 3. Comparison of Exact and Approximate Methods of Calculating Phugoid Damping.

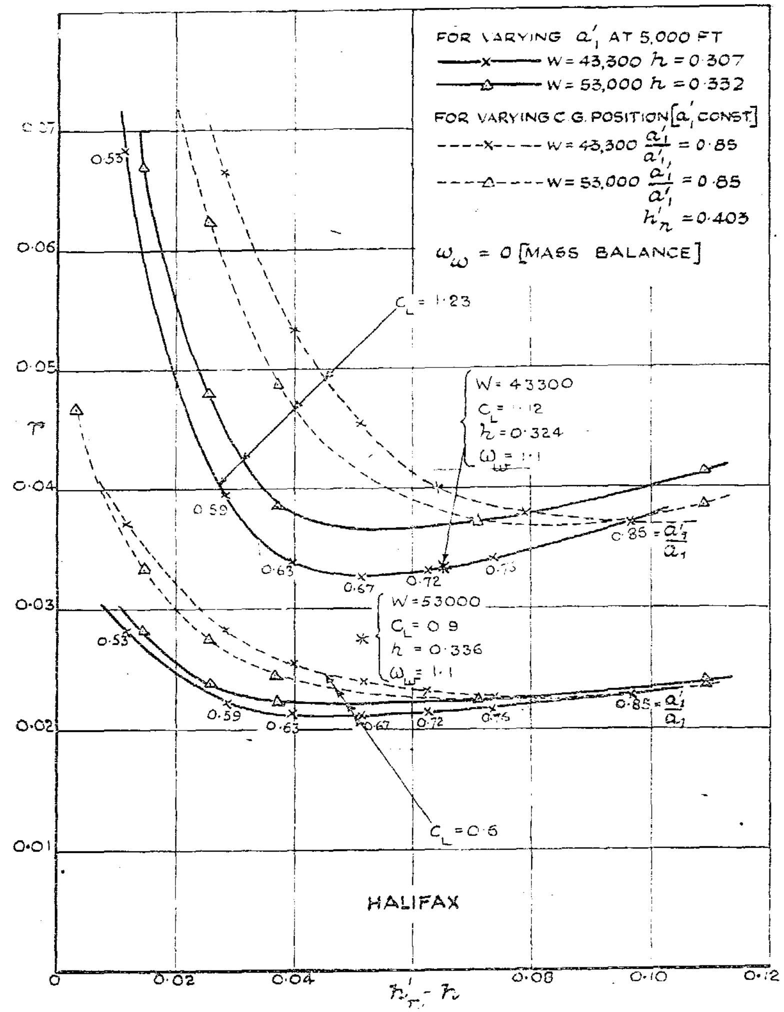

FIG. 4. Effect of Varying Tail-plane Effectiveness and C.G. Position on Damping for the Halifax.

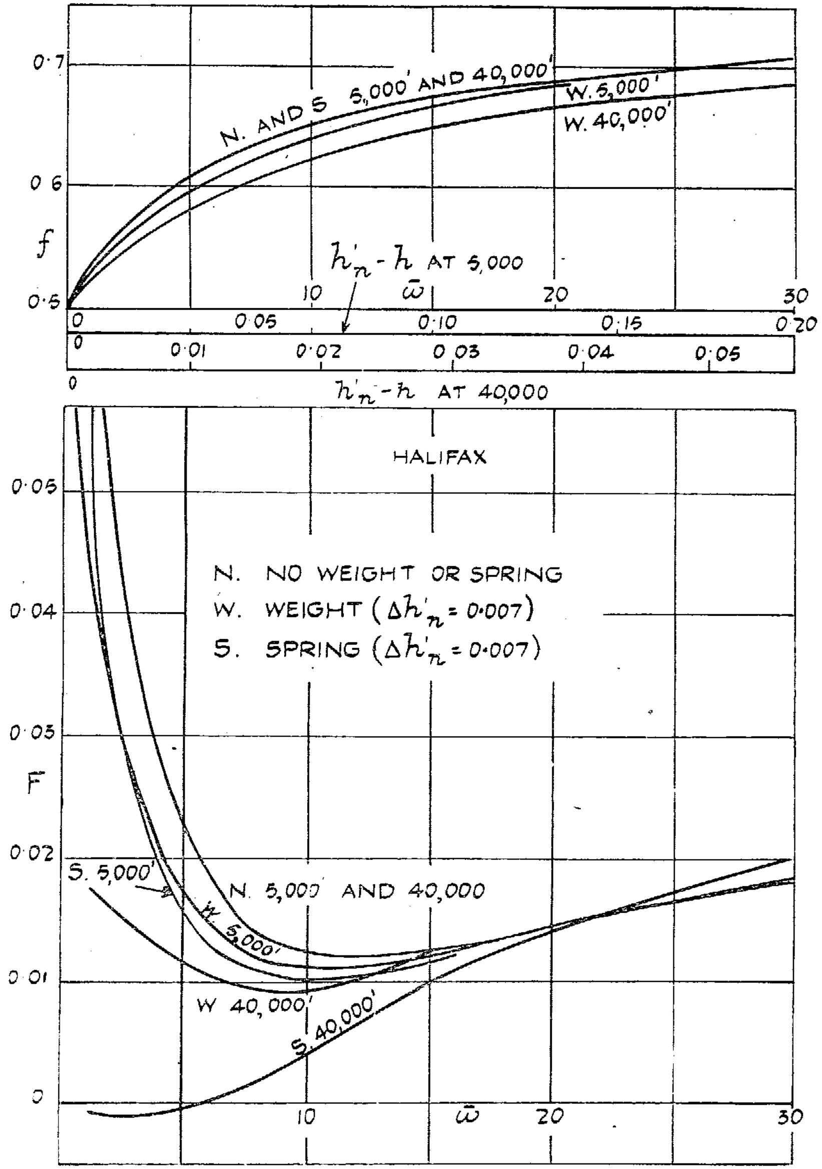

FIG. 5. Effect of Weights and Springs on Damping for the Halifax.

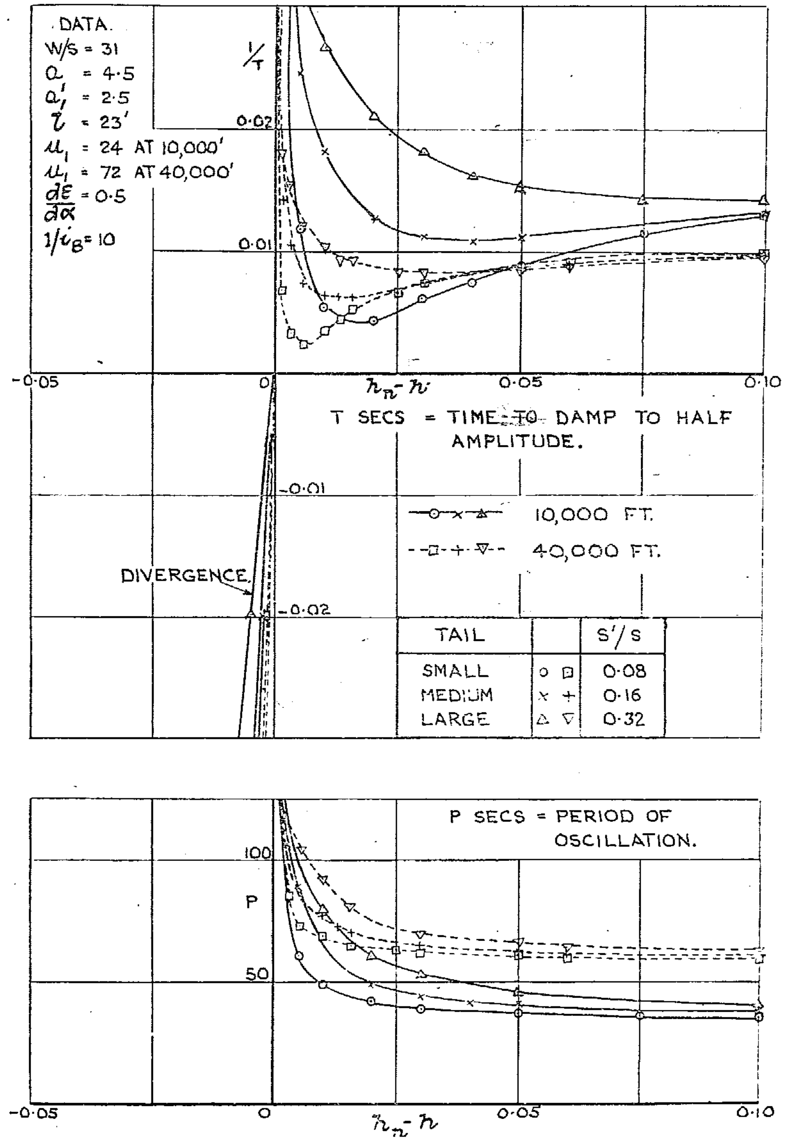

FIG. 6. Variation of Period and Damping with Static Margin. No Spring or Weight. $C_L = 0{\cdot}6$.

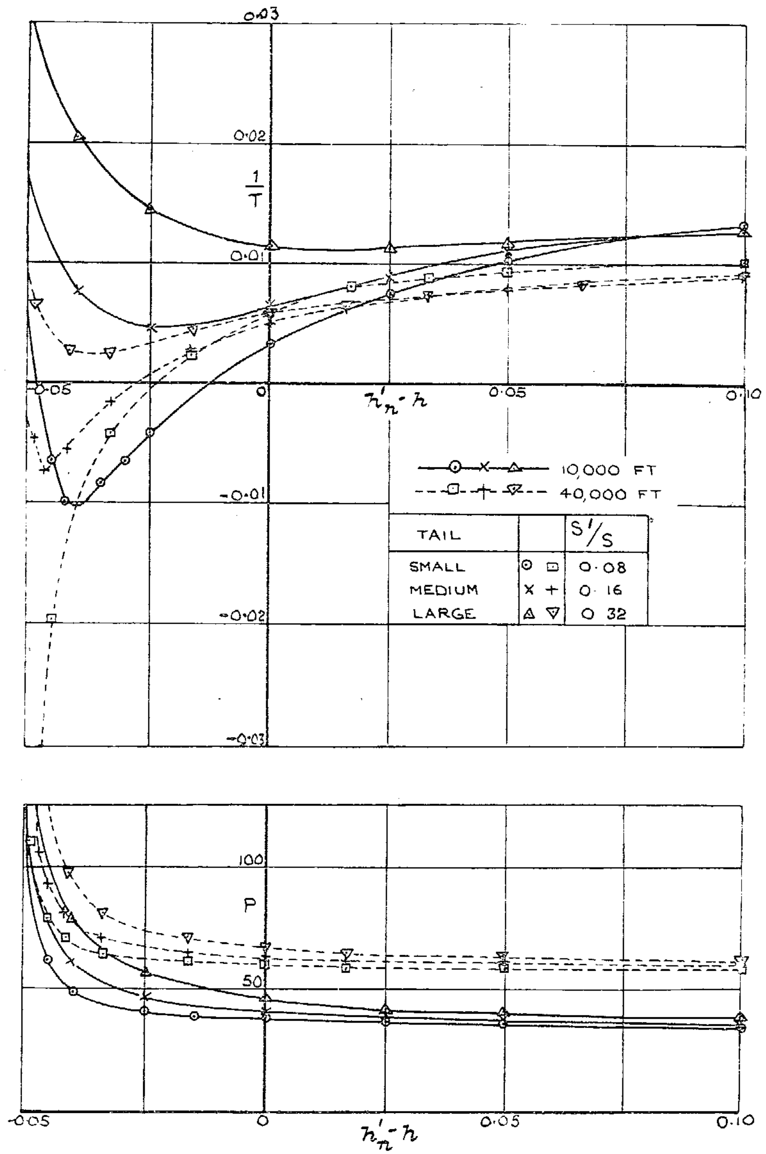

FIG. 7. Variation of Period and Damping with Static Margin.
Weight ($\Delta h_n' = 0 \cdot 05$). $C_L = 0 \cdot 6$.

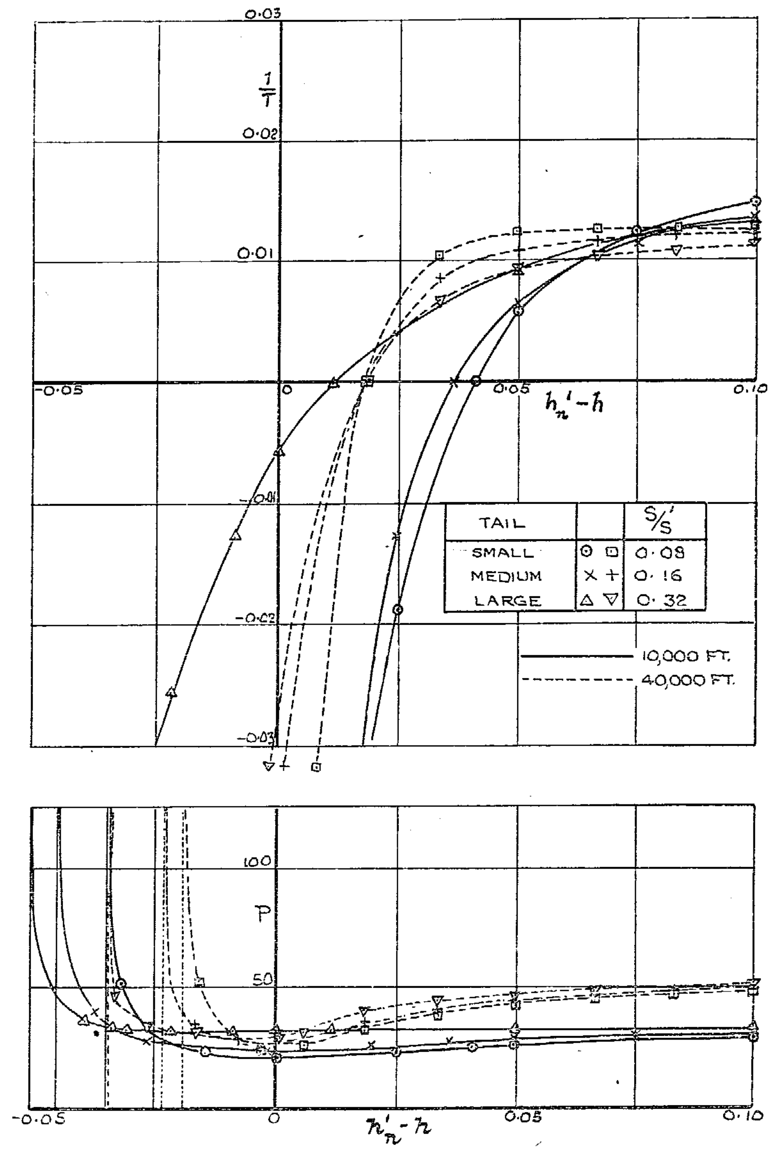

FIG. 8. Variation of Period and Damping with Static Margin.
Spring ($\Delta h_n' = 0 \cdot 05$). $C_L = 0 \cdot 6$.

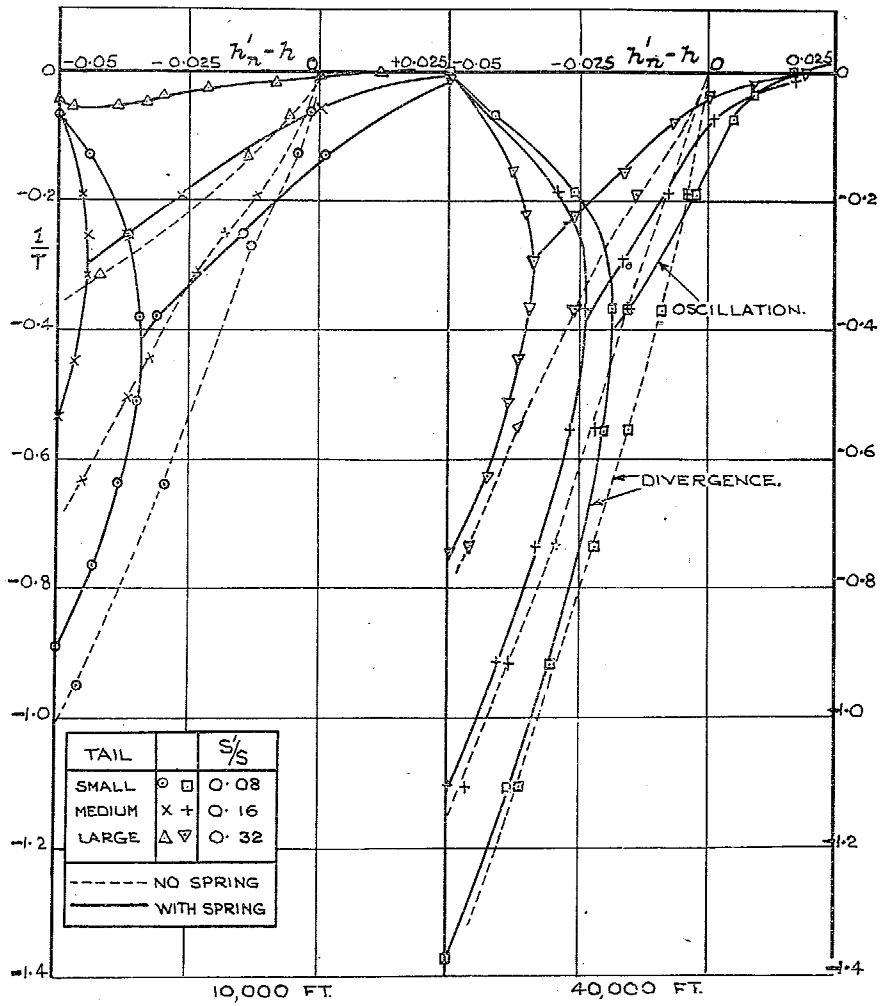

FIG. 9. Effect of Spring on Growth of Disturbance on an Initially Unstable Aircraft.

$C_L = 0 \cdot 6$.

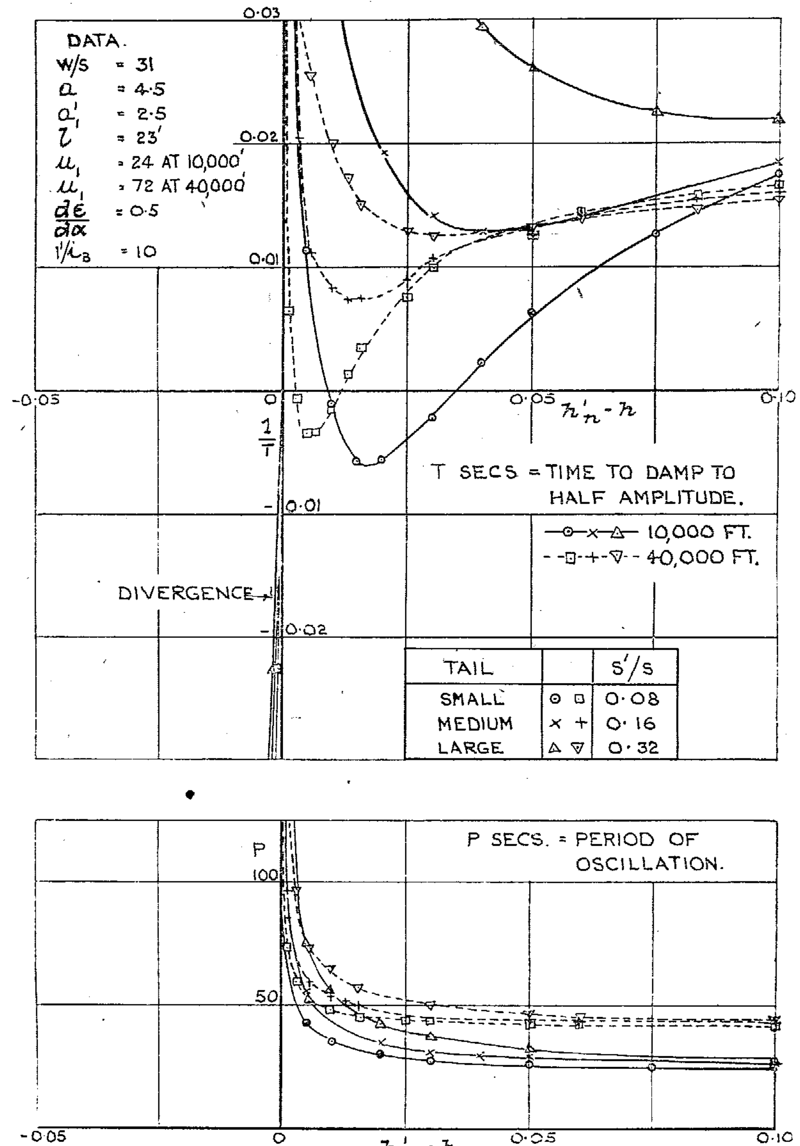

FIG. 10. Variation of Period and Damping with Static Margin.

No Spring or Weight. $C_L = 1 \cdot 2$.

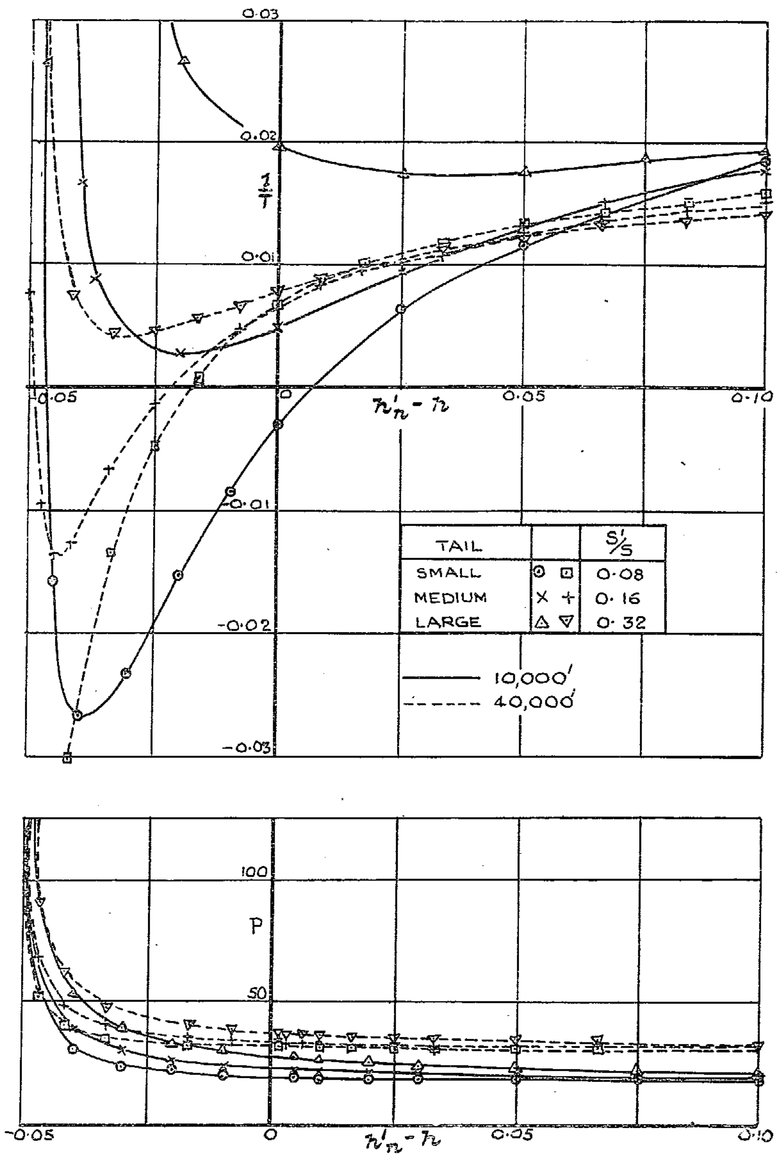

FIG. 11. Variation of Period and Damping with Static Margin. Weight ($\Delta h_n' = 0{\cdot}05$). $C_L = 1{\cdot}2$.

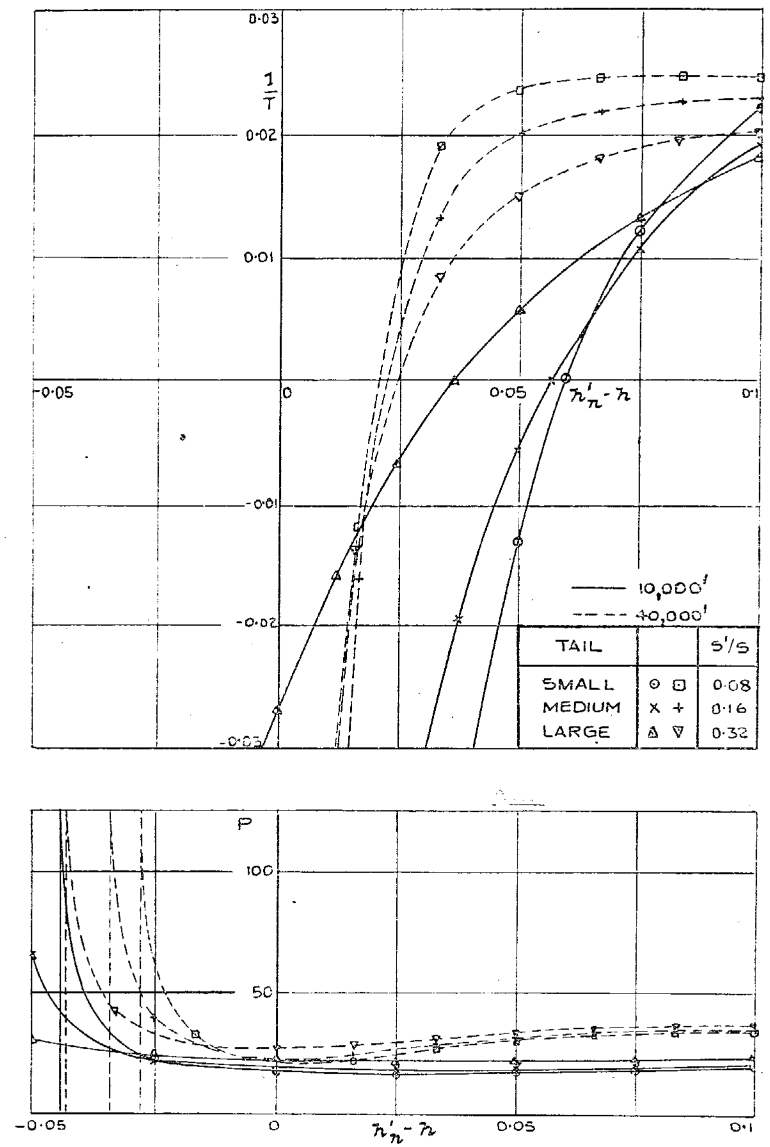

FIG. 12. Variation of Period and Damping with Static Margin. Spring ($\Delta h_n' = 0{\cdot}05$). $C_L = 1{\cdot}2$.

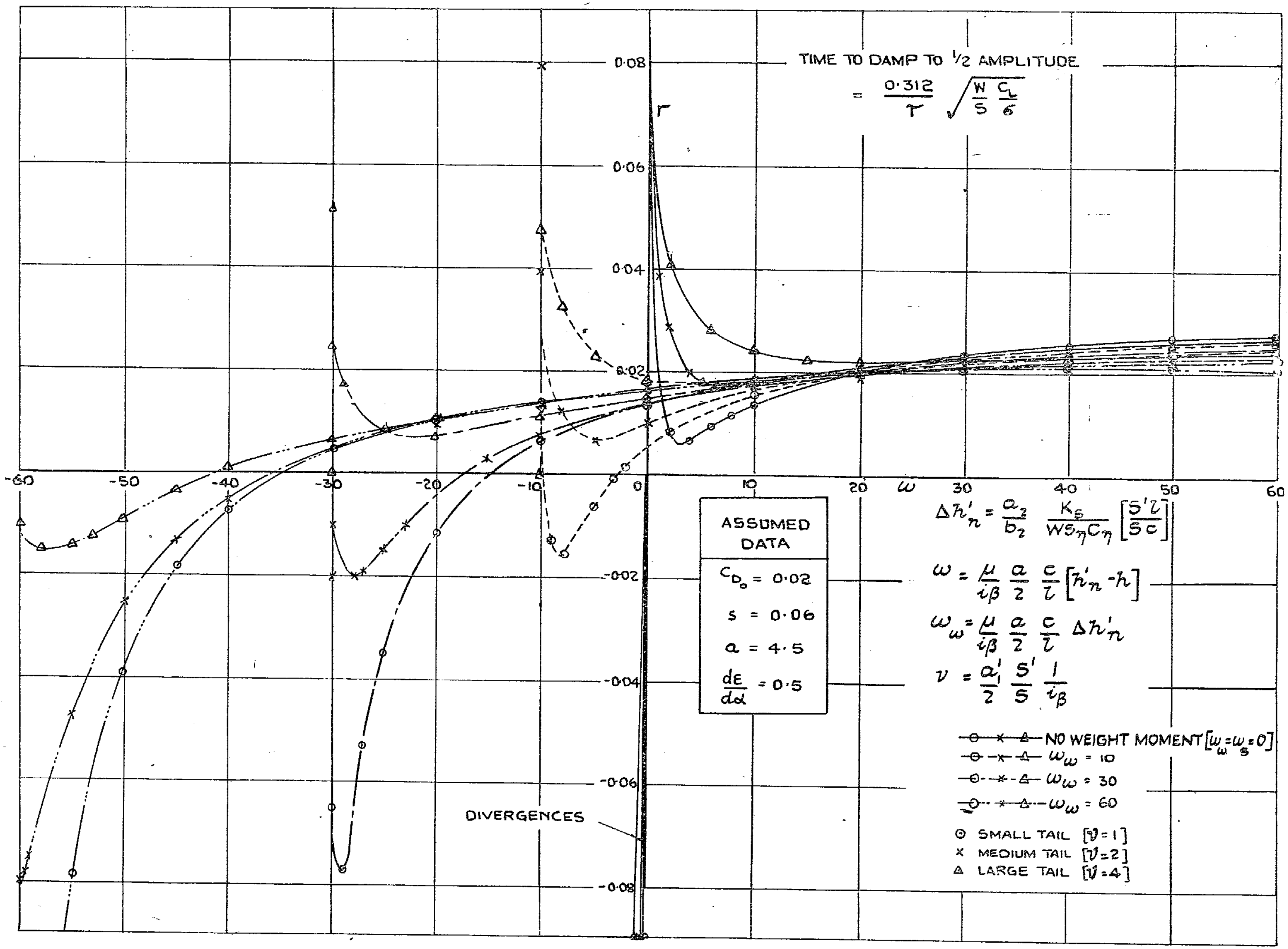

FIG. 13. Effect of a Weight Moment on Phugoid Damping. ($C_L = 0 \cdot 6$).

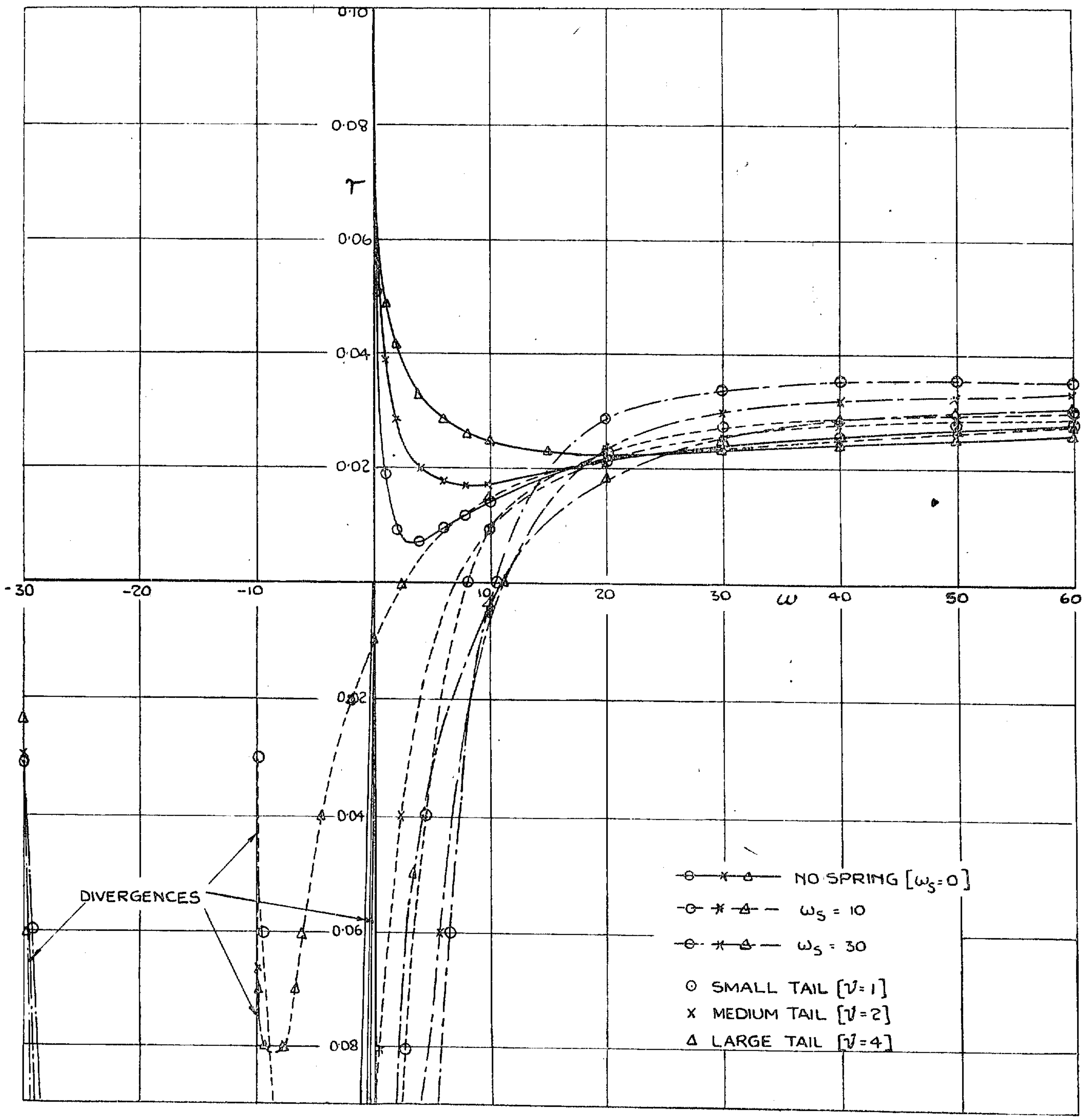

FIG. 14. Effect of a Spring on Phugoid Damping. ($C_L = 0 \cdot 6$).

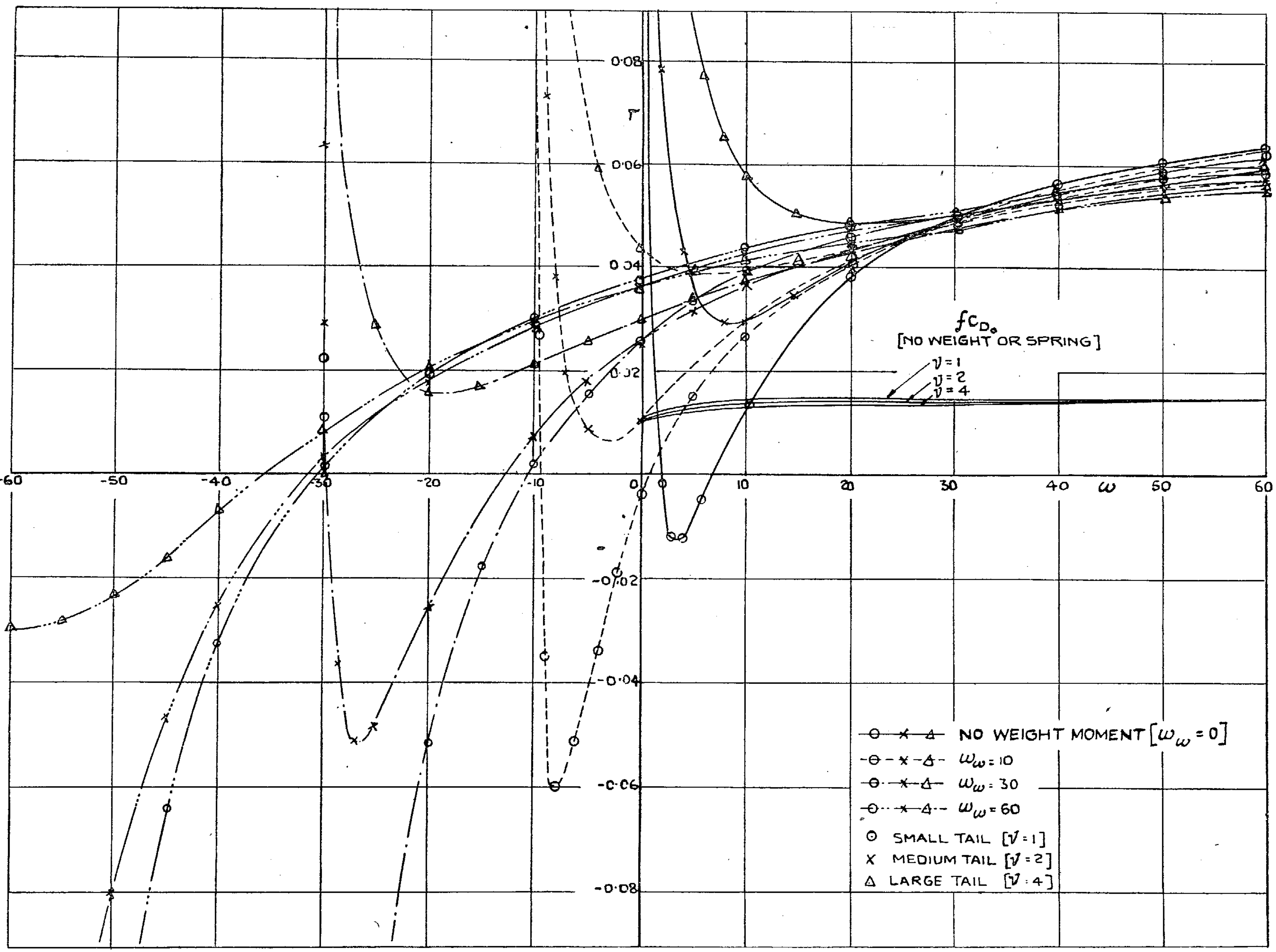

FIG. 15. Effect of a Weight Moment on Phugoid Damping. ($C_L = 1 \cdot 2$).

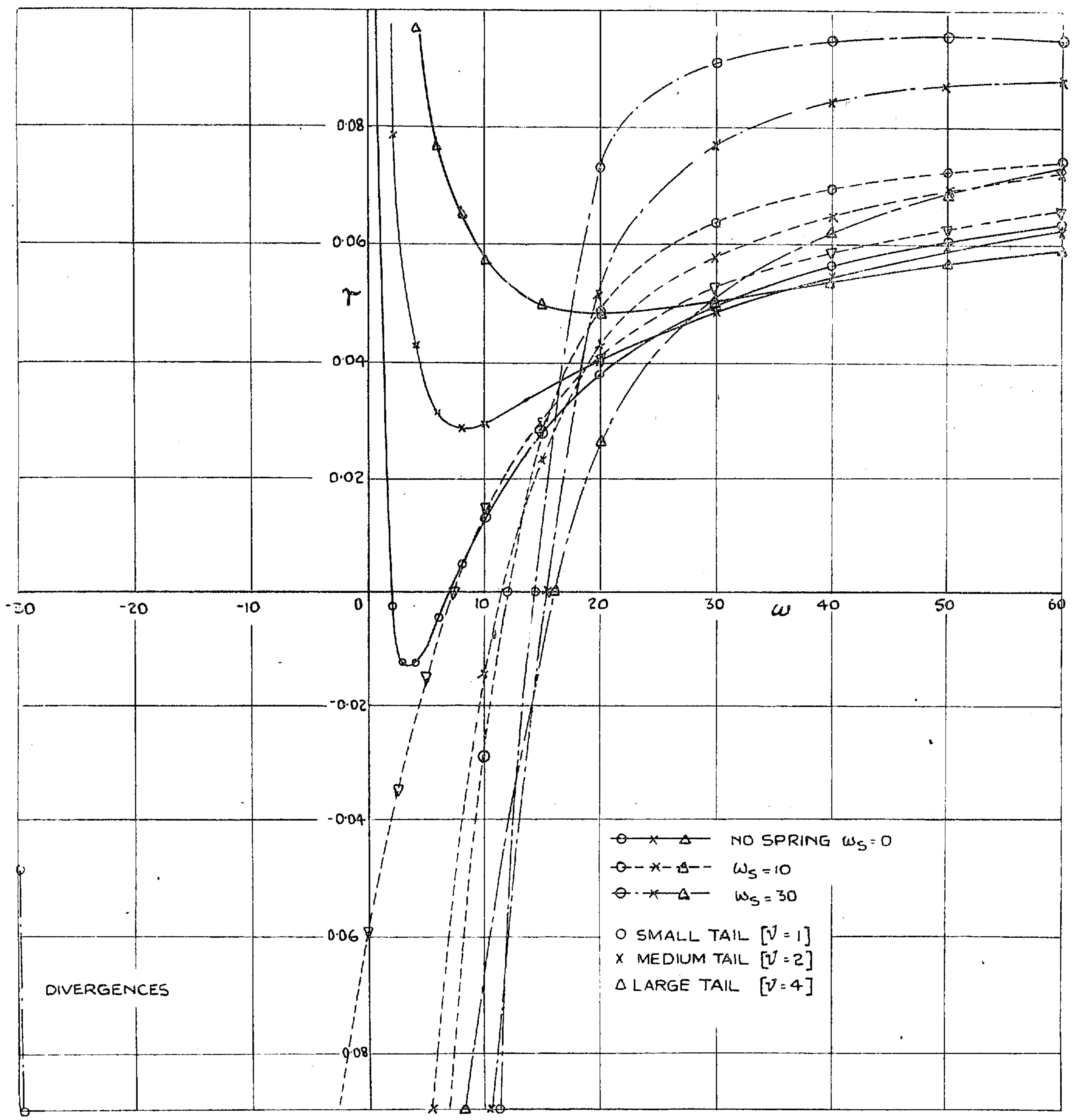

FIG. 16. Effect of a Spring on Phugoid Damping. ($C_L = 1 \cdot 2$).

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