R. & M. No. 2244
(9364)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HIS MAJESTY'S STATIONERY OFFICE
1948
Price 5s. 6d. net

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1. *Summary.*—A brief review of existing work is given and the possibility of certain simple solutions for velocity distributions of the type $U = kx^m$ with their appropriate suction distributions is indicated. An improved approximate calculation of the " entry flow " along a flat plate, through which constant suction is applied, is given in some detail. Also Prandtl's original calculation (based on the momentum equation) for boundary-layer flow with constant suction and a constant adverse velocity gradient is repeated, using Howarth's accurate solution for flow without suction. It is also demonstrated (subject to the accuracy of the approximations) that distributed suction should be much more economical in quantity than suction flow through the minimum number of isolated slots required to prevent separation in the flow under a constant adverse velocity gradient.

Practical applications of porous suction are then considered and illustrated by simple examples. These fall under two headings :—(*a*) the stabilisation of laminar flow against disturbances, (*b*) the prevention of separation. If the stability calculations made by Pretsch are correct, then a suction velocity v_1, given by $v_1/U \geqslant 1 \cdot 82 \times 10^{-5}$, where U is the free-stream velocity, should make the boundary-layer flow past a flat plate stable against all small disturbances. Thus by use of a very small suction flow it may be possible to stabilise the flow over a laminar flow type wing against the adverse effects of waviness. The prevention of laminar separation, coupled with the increase of stability, makes possible a wing with 100 per cent. laminar flow. Bluff shapes as extreme as a circular cylinder require only a comparatively small suction flow to overcome laminar separation. The application of porous suction to the attainment of a high $C_{L\,\max.}$ is also considered, and it is demonstrated that, even for a thin wing, a very high $C_{L\,\max.}$ should be made possible by a surprisingly small suction flow applied over less than 10 per cent. of the chord.

It is also suggested that porous suction could be used as a valuable research tool to thin the boundary layer and thus simulate high Reynolds number conditions at small test Reynolds numbers for both incompressible and compressible flow.

Some consideration is given to the practical realisation of a porous surface which approximates to the mathematical concept. It is concluded that porous bronze, made by sintering metallic powder, is the most suitable existing material for laboratory experiments. There seems to be no reason why a similar " surface " should not be made in light alloy for the flight applications. It is considered that the simulation of a porous surface by the use of isolated slots is not suitable unless their spacing and width are small compared with the boundary-layer thickness.

It is concluded therefore that porous suction may have important practical applications to flight at both small and large C_L's. Experiments are needed to confirm the ideas put forward in this report. Also accurate solutions of the boundary-layer equations for the flow under an adverse pressure gradient with porous suction are required to check the approximate treatment used herein.

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2. *Introduction.*—Suction as a means of boundary-layer control dates back to the inception of the boundary-layer theory itself. Prandtl[1] (1904) appears to have been the first to consider it. He also devotes some space to it in his article *The Mechanics of Viscous Fluids* in *Aerodynamic Theory*, Vol. III (1935)[2]. Both distributed and concentrated suction are considered. It is the latter which has received most attention both in this country and abroad, the latest stage of development here being the thick Griffith aerofoil and the Lighthill nose suction aerofoil.

Distributed suction has received less attention experimentally, presumably because of the difficulty of simulating the permeable surface envisaged in the theory. Prandtl[2] shows how laminar separation can be prevented in the flow under a constant adverse velocity gradient. Griffith and Meredith in an unpublished note (1936) obtained the simple solution of the boundary-layer equations for the flow past an infinite plate with uniform suction. They also showed that considerable improvement in performance could be obtained if, under suction, the flow proved to be more stable (as seemed likely) than without suction, so that much more extensive laminar flows could be maintained. No satisfactory experiments were made to check this. Schlichting[3] (1942) made approximate calculations of the boundary-layer flow along a semi-infinite flat plate under conditions of constant suction and air injection both for laminar and turbulent flows. For laminar flow with suction at large distances from the nose of the plate he obtains the solution already derived by Griffith and Meredith. The merit of Schlichting's work is that it gives an approximate picture of the " entry flow " near the nose of the plate and how this joins on to the exact " asymptotic " solution corresponding to large distances from the nose.

The stability of the laminar boundary layer of a flat plate with constant suction has recently been examined by Pretsch[4] (1942). His conclusions are startling and of the utmost practical importance. For a flat plate without suction, oscillations in the laminar boundary layer are amplified if $U\delta^*/\nu > 680$. With suction $U\delta^*/\nu > 5\cdot52 \times 10^4$ before amplification occurs and the maximum amplification is $\frac{1}{7}$th that occurring without suction. If $v_1/U > 1\cdot82 \times 10^{-5}$, the flow is always stable. This is of course an exceedingly small suction velocity. Thus the possibility of totally laminar flow wings (as was visualised by Griffith and Meredith) is presented. The only experimental evidence which supports this conclusion is contained in a brief paper by Ackeret and Pfenninger[5] (1941). The authors simulated a porous surface by a large number of fine slots disposed in the region of adverse pressure gradient along a flat wall. Without suction, the flow downstream of the region with adverse gradient was turbulent ; with suction the flow remained laminar. The Reynolds number of the tests appears to have been about 5×10^5 and no details of the slot width nor of the suction flow are given.

The purpose of this report is (*a*) to point out the possibility of certain exact solutions of the boundary-layer equations with suction, (*b*) to give an improved treatment of the flow along a flat plate with constant suction along the lines given by Schlichting[3], (*c*) to extend this solution so that together with a solution similar to one given by Prandtl[2] practical calculations for wings, etc. can be made, (*d*) to suggest possible applications and experiments for testing the theory.

3. *Exact Solutions of the Boundary-layer Equations with Suction.*—(*a*) *Solution of Griffith and Meredith—Infinite Plate with Constant Suction.*—The equation of motion (with zero pressure gradient) is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,. \qquad (2)$$

We assume that $\dfrac{\partial u}{\partial x} = 0$,

then from (2) $\quad v = \text{const.} = -v_1 \qquad (3)$

and from (1)

$$v_1\frac{\partial u}{\partial y} + \nu\frac{\partial^2 u}{\partial y^2} = 0\,,$$

whence

$$\frac{u}{U_0} = 1 - e^{-v_1 y/\nu} \qquad (4)$$

gives a solution satisfying the boundary conditions and which represents conditions far from the leading edge of a semi-infinite flat plate with constant suction.

(*b*) *Semi-infinite Plate. Suction* $\propto 1/x^{1/2}$.—If we put in equation (1) (as in *Modern Developments in Fluid Dynamics* (Vol. 1, p. 135)[6],

$$\eta = \tfrac{1}{2}(U/\nu x)^{1/2}y\,, \qquad \psi = (\nu U x)^{1/2} f\,, \qquad (5)$$

where
$$u = \frac{\partial \psi}{\partial y} = \frac{U}{2} f' \qquad v = -\frac{\partial \psi}{\partial x} = \tfrac{1}{2}(U\nu/x)^{1/2}\,(\eta\,.\,f' - f)\,, \qquad (6)$$

we obtain
$$f''' + f\,.\,f'' = 0\,, \qquad (7)$$

(where the primes denote differentiation with respect to η).

The boundary conditions are $u = U$ at $y = \infty$ and $x = 0$ and $u = 0$, $v = -v_1$ at $y = 0$ which in terms of the new variables give

$$f' = 2 \text{ at } \eta = \infty\,, \qquad (8)$$

$$\left.\begin{aligned} &f' = 0 \text{ at } \eta = 0\,, \\ &f = 2v_1\Big/\left(\frac{U\nu}{x}\right)^{1/2} \text{ at } \eta = 0\,, \end{aligned}\right\} \qquad (9)$$

so that if $v_1 \propto 1/x^{1/2}$

$$f = \text{const. at } \eta = 0\,. \qquad (10)$$

The solution of (7) with $f = 0$ at $\eta = 0$ is the well-known Blasius solution. The solution of (7) with $f =$ const. and the other boundary-layer conditions the same, will therefore correspond to $v_1 \propto 1/x^{1/2}$ for each value of f_0; the boundary-layer thickness will develop at a slower rate than without suction, but its variation with x will still be parabolic. The velocity profile will still be the same at all sections, but will differ from the Blasius profile by being more convex. In fact it follows from (7) that $f_0''' \equiv u_0'' \neq 0$, when f_0 has a value different from zero. The actual solution of (7) can be carried through by a numerical step-by-step process, but it is most conveniently done on a differential analyser for a whole series of values of f_0. B. Thwaites is co-operating with the Mathematics Division, National Physical Laboratory, in connection with this and other examples. Its interest is largely academic, but experimentally the condition $v_1 \propto 1/x^{1/2}$ could be simulated by applying a constant suction head across a wall, the thickness of which $\propto x^{1/2}$, if the flow through the surface is of the viscous type.

(*c*) *Flow near a Stagnation Point.* $U = \beta_1 x$. *Constant Suction Velocity through Surface.*—The equations of motion are (Ref. 6, p. 139)

$$\left.\begin{aligned} &u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} = \beta_1^2 x + \nu \cdot \frac{\partial^2 u}{\partial y^2}\,, \\ &u = \frac{\partial \psi}{\partial y}\,, \qquad v = -\frac{\partial \psi}{\partial x}\,, \end{aligned}\right\} \qquad (11)$$

with $u = 0$, $v = -v_1$ at $y = 0$; $u = 0$, $x = 0$, $u = \beta_1 x$ at $y = \infty$, where $\beta_1 =$ const.

By taking
$$\left.\begin{aligned} &\psi = (\nu\beta_1)^{1/2}\,.\,x\,.\,f(\eta)\,, \\ &\eta = (\beta_1/\nu)^{1/2} y\,, \end{aligned}\right\} \qquad (12)$$

$$u = \beta_1 x f'(\eta)\,, \qquad v = -(\nu\beta_1)^{1/2} f(\eta)\,; \qquad (13)$$

the equation for f is (Ref. 6, p. 139)

$$f'^2 - ff'' = 1 + f'''\,. \qquad (14)$$

The boundary conditions give

$$f' = 1 \text{ at } y = \infty\,, \qquad (15)$$

$$\left.\begin{aligned} f' &= 0 \text{ at } y = 0\,, \\ f &= v_1/(\nu\beta_1)^{1/2} \text{ at } y = 0\,. \end{aligned}\right\} \qquad (16)$$

Hence if

$$v_1 = \text{const.},\ f_0 = \text{const.} \qquad (17)$$

Equation (12) can therefore be solved numerically or preferably by machine for a series of values of f_0. The solution is of some importance if, for any reason, constant suction is applied from the front stagnation point of a wing over a part or whole of the chord.

The solutions indicated in (*b*) and (*c*) are particular cases which are included in the more general flow given by $U = kx^m$ and a particular distribution of suction can be found for any value of m, which will enable the partial differential equation to be reduced to an ordinary differential equation of the same form as for zero suction. (*See* Ref. 6, p. 140–141.)

4. *Approximate Solution for the Laminar Boundary-layer Flow along a Permeable Flat Plate with Suction.*—(*a*) *Note on Schlichting's Solution.*—Schlichting[3] (1942) has given an approximate solution of this problem, which can be improved on in detail, though the method of attack is the same. This is worked out in some detail in this section, as it has applications to wings with permeable surfaces.

As mentioned in the introduction (Section 2) Schlichting[3] has derived Griffith and Meredith's solution for the flow at large distances from the leading edge. This has been given in Section 3a. Schlichting also noted that the flow at the leading edge is given by the Blasius solution for zero suction. That this is correct can be seen from equation (9), where at

$$y = 0\ ,$$

$$f = 2v_1/(U\nu/x)^{1/2}$$

gives the value of f at the surface.

If v_1 = const., then as $x \to 0$ — the leading edge, $f \to 0$ and conditions then are identical for those with zero suction.

Schlichting takes

$$u/U = F_1(\eta) + KF_2(\eta)\,,$$

where $\eta = y/\delta_1$, δ_1 being a measure of the boundary-layer thickness and K is a form parameter. He then takes $F_1(\eta) = 1 - e^{-\eta}$ corresponding to the asymptotic solution given in § 3a and

$$F_2(\eta) = e^{-\eta}\,.$$

When $K = -\frac{1}{2}$, corresponding to the leading edge of the plate,

$$\frac{u}{U} = 1 - e^{-\eta} - \frac{\eta}{2}\,e^{-\eta}$$

is taken to correspond to the Blasius profile. This is a rather poor representation, since, when inserted in the momentum equation for zero suction, it gives for the momentum thickness θ

$\theta = 0\cdot83\sqrt{(\nu x/U)}$, instead of this exact value

$\theta = 0\cdot664\sqrt{(\nu x/U)}$;

and for the displacement thickness δ^*

$\delta^* = 1\cdot81\sqrt{(\nu x/U)}$, instead of the exact value

$\delta^* = 1\cdot721\sqrt{(\nu x/U)}$;

whilst

$H \equiv \delta^*/\theta$ is given by

$H = 2\cdot182$, instead of by its exact value

$H = 2\cdot591$.

The actual profile is shown in Fig. 5a for comparison with the Blasius profile and the "asymptotic" profile of Griffith and Meredith. It is nearer to the latter than to the former, which it is attempting to represent. Hence some doubt as to the accuracy of Schlichting's subsequent calculation must exist.

(*b*) *Improved Solution.*—It is proposed to take a one-parameter family of velocity profiles having as their limiting forms the *exact Blasius profile* and the *asymptotic profile* given in section (3a). These profiles are distributed along the plate so as to satisfy the momentum equation and the differential equation of motion 3.1 near the surface; the boundary conditions being automatically satisfied.

Thus we take

$$\frac{u}{U} = f(\bar{y}) = F_1(\bar{y}) + K\{F_2(\bar{y}) - F_1(\bar{y})\} \quad , \qquad (4.1)$$

where

$$\bar{y} = y/\delta^*, \qquad \delta^* = \int_0^\infty (1 - u/U)dy \quad , \qquad (4.2)$$

$$\left.\begin{aligned} F_1(\bar{y}) &\equiv \text{Blasius Profile (See Fig. 1a} \\ F_2(\bar{y}) &\equiv \text{Asymptotic Profile} = 1 - e^{-\bar{y}} \end{aligned}\right\} , \qquad (4.3)$$

whilst K is a form parameter and is a function of x the distance along the plate.

The equation of motion 3.1, when $y = 0$, gives :—

$$-v_1\left(\frac{\partial u}{\partial y}\right)_{y=0} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} \qquad (4.4)$$

which, with $u = U.f(\bar{y})$, $y = \bar{y} . \delta^*$, gives :

$$f_0'' = -\frac{v_1\delta^*}{\nu} \cdot f_0' \qquad (4.5)$$

where primes denote differentiation with respect to $\bar{y}$.

Now from 4.1

$$\begin{aligned} f_0'' &= (1 - K)\,(F_1'')_0 + K(F_2'')_0 \\ &= 0 - K \quad , \end{aligned} \qquad (4.6)$$

and $\quad f_0' = (1 - K)\,(F_1')_0 + K(F_2')_0 \quad ;$

where $\quad (F_2')_0 = 1 \quad ,$

and

$$(F_1')_0 = \frac{U.\delta_1^*}{\nu} \cdot \left(\frac{\tau_0}{\rho U^2}\right)_1 \quad , \qquad (4.7)$$

where subscript $_1$ denotes the Blasius profile.

Now in *Modern Developments in Fluid Dynamics*, Vol. 1, p. 136[6]

$$\delta_1^* = 1{\cdot}7208x\left(\frac{\nu}{Ux}\right)^{1/2} ,$$

and

$$\left(\frac{\tau_0}{\rho U^2}\right)_1 = 0{\cdot}33206\left(\frac{\nu}{Ux}\right)^{1/2} ,$$

whence $\quad (F_1')_0 = 0{\cdot}57141 \equiv a \quad , \qquad (4.8)$

so that $\quad f_0' = a + (1 - a)K . \qquad (4.9)$

Substituting (4.6) and (4.9) in (4.5) we obtain

$$\frac{v_1\delta^*}{\nu} = \frac{K}{a + (1 - a)K} \quad \text{(4.10)}$$

Now $\quad u/U = F_1(\bar{y}) + K\{F_2(\bar{y}) - F_1(\bar{y})\}$,

and $\quad \delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \delta^* \int_0^\infty \left(1 - \frac{u}{U}\right) d\bar{y}$,

so that $\quad 1 = \int_0^\infty \{(1 - F_1) + K(F_1 - F_2)\} \, d\bar{y}$,

$$= \int_0^\infty (1 - F_1) \, d\bar{y} + K \left\{ \int_0^\infty (1 - F_2) \, d\bar{y} - \int_0^\infty (1 - F_1) \, d\bar{y} \right\} ,$$

but $\int_0^\infty (1 - F_1) \, d\bar{y} = \int_0^\infty (1 + F_2) \, d\bar{y} = 1$ .. (4.11)

by definition, so that a velocity profile compounded from two other velocity profiles having the same value of δ^* also has the same value of δ^*.

Now $\quad \theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \delta^* \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$,

or

$$\frac{\theta}{\delta^*} = \int_0^\infty F_1(1 - F_1) d\bar{y} - K.2 \int_0^\infty F_1(F_2 - F_1) d\bar{y} + K^2 \left\{ \int_0^\infty F_2(1 - F_1) d\bar{y} - \int_0^\infty F_1(1 - F_1) d\bar{y} + 2 . \int_0^\infty F_1(F_2 - F_1) d\bar{y} \right\} \quad \text{(4.12)}$$

Now for the Blasius profile

$$\int_0^\infty F_1(1 - F_1) d\bar{y} = \frac{0 \cdot 66412}{1 \cdot 7208} = 0 \cdot 38594 \quad , \quad \text{(4.13)}$$

and for the asymptotic profile from (4.3)

$$\int_0^\infty F_2(1 - F_2) d\bar{y} = \tfrac{1}{2} \quad , \quad \text{(4.14)}$$

and by numerical integration we found

$$2 \int_0^\infty F_1(F_2 - F_1) d\bar{y} = -0 \cdot 12800 \quad , \quad \text{(4.15)}$$

so that

$$\left. \begin{aligned} \frac{\theta}{\delta^*} &= b + cK + dK^2 \\ \text{where} \quad b &= 0 \cdot 38594 \\ c &= 0 \cdot 12800 \\ d &= -0 \cdot 01394 \end{aligned} \right\} \quad \text{(4.16)}$$

We shall require $\dfrac{d\theta}{dx}$,

$$\frac{d\theta}{dx} = \frac{d\delta^*}{dx}(b + cK + dK^2) + \delta^* \cdot \frac{dK}{dx}(c + 2d)$$

On integration this gives :

$$\xi = \frac{1}{(1-a)^4} \cdot \frac{1}{(E+1)^2(E^2-1)} \Big[A(E+1)^2(E^2-1)K - (A+B+C+D)\log_e(1-K)$$
$$+(AE^6+BE^4+CE^2+D)\log_e(E^2-K) - \frac{(AE^3-BE^2+CE-D)(E^2-1)}{(E+K)}\Big] + C \quad , (4.23)$$

which on inserting numbers gives

$$\xi = \Big[-0{\cdot}35419K - 2{\cdot}7014\log_e(1-K) + 3{\cdot}5798\log_e(1.7775-K) + \frac{0{\cdot}59291}{(1.3332+K)}\Big] + C . \quad (4.24)$$

If suction is applied over the whole plate starting at the leading edge, then $K = 0$ *for* $\xi = 0$ and

$$\xi = \left[-0{\cdot}35419K - 2{\cdot}7014\log_e(1-K) + 3{\cdot}5798\log_e\left(1-\frac{K}{1{\cdot}7775}\right) - 0{\cdot}59291 \frac{\dfrac{K}{1{\cdot}7775}}{1+\dfrac{K}{1{\cdot}3332}} \right] \quad (4.25)$$

which we note makes $\xi \to \infty$ $K \to 1$—the asymptotic profile.

If suction starts at some point $\xi = \xi_0$, *with a suction profile corresponding to* $K = K_0$, then

$$\xi - \xi_0 = \left[-0{\cdot}35419(K-K_0) - 2{\cdot}7014\log_e\left(\frac{1-K}{1-K_0}\right) + 3{\cdot}5798\log_e \frac{\left(1-\dfrac{K}{1{\cdot}7775}\right)}{\left(1-\dfrac{K_0}{1{\cdot}7775}\right)} \right.$$
$$\left. - 0{\cdot}59291 \left\{ \frac{\dfrac{K}{1{\cdot}7775}}{1+\dfrac{K}{1{\cdot}3332}} - \frac{\dfrac{K_0}{1{\cdot}7775}}{1+\dfrac{K_0}{1{\cdot}3332}} \right\} \right] \quad \ldots \quad (4.26)$$

ξ_0 is found as follows. The momentum thickness θ must be continuous at the point $\xi = \xi_0$. Now, from equations (4.16), (4.10) writing $K = K_0$,

$$\left(\frac{v_1\theta}{\nu}\right)_0 = \left(\frac{v_1\delta^*}{\nu}\right)_0 (0{\cdot}38594 + 0{\cdot}1280K_0 - 0{\cdot}01394K_0^2) \quad \ldots \quad (4.27)$$

$$\left(\frac{v_1\delta^*}{\nu}\right)_0 = \frac{K_0}{0{\cdot}57141 + 0{\cdot}42859\,K_0} . \quad \ldots \quad (4.28)$$

Now for zero suction the Blasius flow gives

$$\frac{v_1\theta}{\nu} = 0{\cdot}66412 \left\{ \left(\frac{v_1}{U}\right)_1^2 \cdot \frac{Ux}{\nu} \right\}^{1/2} = 0{\cdot}66412\,\xi^{1/2} .$$

Hence

$$\xi_0 = \left(\frac{v_1\theta}{\nu}\right)_0^2 \times \frac{1}{(0{\cdot}66412)^2} , \quad \ldots \quad (4.29)$$

so that when K_0 is given, ξ_0 follows from (4.29), (4.28) and (4.27). Note that there will be a discontinuity at $\xi = \xi_0$ in $\frac{v_1\delta^*}{\nu}$, and in $\frac{U}{v_1} \cdot \frac{\tau_0}{\rho U^2}$, which is given by

$$\frac{U}{v_1} \cdot \frac{\tau_0}{\rho U^2} = \frac{\{0{\cdot}57141 + 0{\cdot}42859\,K\}^2}{K} . \quad \ldots \quad (4.30)$$

The Drag.—The drag is computed as follows. In the first place it can be split up into two parts—the "traverse" or "wake" drag and the "pump" drag. Consider one surface.

Then the "wake" drag coefficient is given by

$$(C_D)_w = 2\frac{\theta}{x} \qquad (4.31)$$

where θ is obtained from (4.16) and (4.10) or from Fig. 6. If p_0 is the pressure external to the surface corresponding to a free stream velocity of U and if p_1 is the suction pressure on the other side, then we can write:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + H = p_0 + \tfrac{1}{2}\rho U^2 \qquad (4.32)$$

where $H \equiv$ loss of head and since v_1 is small compared with U

$$\frac{H}{\frac{1}{2}\rho U^2} = \frac{p_0 - p_1}{\frac{1}{2}\rho U^2} + 1 . \qquad (4.33)$$

If no further losses of head occur, *i.e.* we neglect duct losses, then the pump power required to eject a quantity Q at the free stream pressure and velocity is (for a pump efficiency of η_2)

$$P = \frac{1}{\eta_2} \cdot Q \cdot H \qquad (4.34)$$

and the "pump" drag coefficient is

$$(C_D)_p = \frac{P}{\frac{1}{2}\rho U^3 . x} = \frac{\eta_1}{\eta_2} \cdot \frac{Q}{Ux} \cdot \frac{H}{\frac{1}{2}\rho U^2} , \qquad (4.35)$$

where η_1 is the efficiency of the propulsive system.

Now $$Q = v_1(x - x_0) , \qquad (4.36)$$

if suction commences at x_0 .

Hence

$$(C_D)_p = \frac{\eta_1}{\eta_2} \frac{v_1}{U}\left(1 - \frac{x_0}{x}\right)\left(\frac{p_0 - p_1}{\frac{1}{2}\rho U^2} + 1\right).$$

The total effective drag coefficient is

$$\underline{C_D} = (C_D)_w + (C_D)_p = \underline{2\frac{\theta}{x} + \frac{\eta_1}{\eta_2} \frac{v_1}{U}\left(1 - \frac{x_0}{x}\right)\left(\frac{p_0 - p_1}{\frac{1}{2}\rho U^2} + 1\right)} \qquad (4.37)$$

Let us suppose (*a*) that suction commences at the leading edge, (*b*) that the resistance of the porous surface is low so that $\frac{p_0 - p_1}{\frac{1}{2}\rho U^2}$ is small compared with unity, (*c*) that the pump efficiency equals the propulsive efficiency and (*d*) that asymptotic conditions are attained, *i.e.* ξ is large.

Then $$\theta = \theta_\infty = \tfrac{1}{2}\nu/v_1 \qquad (4.38)$$

and for one surface we have

$$\underline{C_D} = \frac{v_1}{U} + \frac{U}{v_1} \cdot \frac{\nu}{Ux} = \underline{\frac{v_1}{U}\left(1 + \frac{1}{\left(\frac{v_1}{U}\right)^2 \frac{Ux}{\nu}}\right)} = \frac{v_1}{U}\left(1 + \frac{1}{\xi}\right) \qquad (4.39)$$

For example $Ux/\nu = 10^7$, $v_1/U = 10^{-3}$, then $\xi = 10$ and $C_D = 0{\cdot}0011$. for one surface. If Pretsch's[4] conclusions are true, then laminar flow would be maintained and very large improvements in performance are seen to be possible, as was noted by Griffith and Meredith.

Results.—Figs. 5a and 5b show the profiles corresponding to $K = 0$, $K = 1$, *e.g.* the Blasius and asymptotic profiles, where they are compared on the axis of equal " displacement thickness " and " momentum thickness." Schlichting's profile for zero suction is also shown in Fig. 5a. Table 1 gives ξ, $\frac{v_1\delta^*}{\nu}$, $\frac{v_1\theta}{\nu}$, $H = \frac{\delta^*}{\theta}$, and $\frac{U}{v_1} \cdot \frac{\tau_0}{\rho U^2}$ for various values of K ranging from 0 to 1·6. The values of ξ correspond to suction over the whole plate. If suction does not start at the leading edge, this table enables ξ to be found, via equations (4.26) and (4.29), for chosen values of K_0.

Figs. 6 and 7 show $\frac{v_1\delta^*}{\nu}$ and $\frac{v_1\theta}{\nu}$ as functions of ξ, where they are compared with Schlichting's calculations. The differences are appreciable for the smaller values of ξ, the present calculations giving a slower approach to the asymptotic values. Fig. 8a shows H as a function of ξ as computed by Schlichting and the present method. The greatest discrepancy is at $\xi \to 0$. Fig. 8b shows the form parameter K as a function of ξ. The most rapid changes in velocity profile will therefore occur near the leading edge. Figs. 9, 10, 11 and 12 show the effect of suction commencing downstream of the leading edge, K, $\frac{v_1\theta}{\nu}$, $\frac{v_1\delta^*}{\nu}$ and $\frac{U}{v_1} \cdot \frac{\tau_0}{\rho U^2}$ are shown as functions of ξ. θ is made continuous at the commencement of suction so that discontinuities occur in δ^* and τ_0.

The approximate solution given in this section might be expected to be a very good one, as it is exact at the leading edge and far down the plate. Moreover the boundary conditions are satisfied and the differential equation is satisfied at the surface and at the " edge " of the boundary layer, whilst the integral of the differential equation, *e.g.* the momentum equation is satisfied exactly. The range of velocity profiles lies between the Blasius profile and the asymptotic profile of Griffith and Meredith and these cannot be said to be greatly different, so that the possibility of appreciable errors existing is small. Nevertheless it would be very interesting to compare the present approximate solution with an accurate solution if this could be obtained.

5. *Approximate Calculations of Laminar Boundary Layer Flow for Aerofoils with Permeable Surfaces across which Suction is Applied.*—(*a*) *Extension of the Flat Plate Theory of Section* 4.—The momentum equation for boundary layer flow with suction and with pressure gradient is

$$\frac{\tau_0}{\rho U^2} = \frac{v_1}{U} + \frac{1}{U} \cdot \frac{dU}{dx} \cdot (H + 2)\theta + \frac{d\theta}{dx} \quad \ldots \quad (5.1)$$

and the differential equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \quad \ldots \quad (5.2)$$

yields at the surface, where $y = 0, u = 0, v = v_1$

$$-v_1\left(\frac{\partial u}{\partial y}\right)_0 = U\frac{dU}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_0 . \quad \ldots \quad (5.3)$$

As in section 4 with $\frac{u}{U} = f(y/\delta^*)$

these become :

$$f'_0 = \gamma + \lambda \cdot \frac{H+2}{H} + \frac{U\delta^*}{\nu} \cdot \frac{d\theta}{dx} \quad \ldots \quad (5.4)$$

and

$$f''_0 = -\gamma \cdot f'_0 - \lambda \quad \ldots \quad (5.5)$$

where

$$\left.\begin{aligned} \gamma &= \frac{v_1 \cdot \delta^*}{\nu} \\ \lambda &= \frac{(\delta^*)^2}{\nu}\frac{dU}{dx} \end{aligned}\right\} \quad \ldots \quad (5.6)$$

Now if the suction flow is sufficiently large, the displacement thickness δ^* will be very small and hence λ will be small compared with γ, and so can be neglected in (5.4) and (5.5). These, with U variable, are now identical with (4.19) and (4.5) and so the solution will be the same as that of section 4 if ξ is given by (from the integration of (4.20) with U variable)

$$\xi = \left(\frac{v_1}{U_0}\right)^2 \cdot \frac{U_0 c}{\nu} \int \frac{U_0}{U}\, d(x/c) \qquad \ldots \qquad (5.7)$$

when $c =$ chord, $U_0 =$ free stream velocity.

This assumes that the starting profile is the Blasius one. Near a front stagnation point this is not true, as $U = \beta_1 x$ and the solution mentioned in § 3c is required. However it is assumed that this will not influence the calculations about to be made as asymptotic conditions are assumed—that is we assume that we are far enough from the starting point of the suction for the boundary layer profile to have become constant. Then from Appendix I for $K = 1\cdot 0$

$$\left.\begin{aligned} \gamma_\infty &= \frac{v_1 \delta^*}{\nu} = 1\cdot 0 \\ f_0' &= 1 \\ f_0'' &= -1 \\ H &= 2 \end{aligned}\right\} \qquad \ldots \qquad (5.8)$$

Hence λ must be small compared with 1.0.
From (5.8)

$$v_1 = \frac{\nu}{\delta^*}$$

and from (5.6)

$$\frac{1}{\delta^*} = \left(\frac{1}{\lambda\,.\,\nu} \cdot \frac{dU}{dx}\right)^{1/2},$$

whence v_1

$$v_1 = \left(\frac{\nu\,.\,dU/dx}{\lambda}\right)^{1/2},$$

or

$$\frac{v_1}{U_0} = \left\{\frac{\nu}{U_0 C} \cdot \frac{1}{\lambda} \cdot \frac{d(U/U_0)}{d(x/c)}\right\}^{1/2} . \qquad \ldots \qquad (5.9)$$

As we are concerned with adverse velocity gradients and the prevention of separation, then λ and $\frac{d(U/U_0)}{d(x/c)}$ are negative.

If $\lambda = -0\cdot 1$

then

$$\frac{v_1}{U_0} = 3\cdot 16 \left(\frac{1}{R} \times -\frac{d(U/U^0)}{d(x/c)}\right)^{1/2} . \qquad \ldots \qquad (5.10)$$

If $\lambda = -0\cdot 01$;

then

$$\frac{v_1}{U_0} = 10 \left(\frac{1}{R} \times -\frac{d(U/U_0)}{d(x/c)}\right)^{1/2} . \qquad \ldots \qquad (5.11)$$

Hence for a given aerofoil, where $\frac{d(U/U_0)}{d(x/c)}$ is known, we can compute the suction flow at any K so that conditions are remote from separation, (*b*) *Alternative Calculation to that of Prandtl*[2] *for the Suction necessary to just prevent Laminar Separation for a Constant Adverse Velocity Gradient.*—Prandtl[2], in "*Mechanics of Viscous Fluids*"—*Aerodynamic Theory*, Vol. III, p. 118, uses the Polhausen method to demonstrate that suction can be used to prevent separation. The result of his calculation can be written in the form

$$\left.\begin{aligned} \frac{v_1}{U_0} &= 2\cdot 18\left(\frac{1}{R} \times -\frac{d(U/U_0)}{d(x/c)}\right)^{1/2} \\ R &= \frac{U_0 c}{\nu} \end{aligned}\right\} \quad \ldots \quad (5.12)$$

which is similar in form to (5.9) and the constant can be compared with those in (5.10) and (5.11).

However, what should be a more accurate result can easily be obtained by using Howarth's accurate calculations of laminar boundary flow under a constant adverse velocity gradient. Following Prandtl, we assume that the flow against the adverse gradient is just on the verge of separation, so that $\tau_0 = 0$. We also assume that δ^*, θ, H and $\frac{dU}{dx}$ are constant and that the separation profile is that computed for zero suction by Howarth[7] (1938). The momentum equation (5.1) becomes

$$v_1 = -\frac{dU}{dx}\cdot(H+2)\theta = -\frac{dU}{dx}\left(\frac{H+2}{H}\right)\delta^* \quad \ldots \quad (5.13)$$

and equation (5.3) is satisfied by Howarth's separation profile since $\left(\frac{\partial u}{\partial y}\right)_0 = 0$ and so suction does not affect this equation.

Now at separation Howarth finds:

$$-\lambda = -\frac{dU}{dx}\cdot\frac{\delta^{*2}}{\nu} = 1\cdot 110 \quad \ldots \quad (5.14)$$

$$H = 1\cdot 110 \div 0\cdot 290 = 3\cdot 83 \quad \ldots \quad (5.15)$$

so $$\frac{H+2}{H} = 1\cdot 525\,.$$

Thus $$\delta^* = \left(\frac{1\cdot 110\nu}{-\frac{dU}{dx}}\right)^{1/2}$$

and substituting in (5.13), we obtain

$$v_1 = 1\cdot 607\,(-\nu\,.\,dU/dx)^{1/2},$$

or $$\left.\begin{aligned} \frac{v_1}{U_0} &= 1\cdot 607\left(-\frac{d(U/U_0)}{d(x/c)} \times \frac{1}{R}\right)^{1/2} \\ \text{where}\quad R &= U_0 c/\nu \end{aligned}\right\} \quad \ldots \quad (5.16)$$

Thus (5.16) gives the minimum suction flow to just avoid laminar separation. This is less than that found by Prandtl (equation (5.12)) using the Polhausen separation profile. Equation (5.16) should be considerable service in applications to thin wings for which a high $C_{L\,\max.}$ is desired. For by sucking over a small part of the upper surface at the nose (say $< 0\cdot 1c$) laminar separation and the subsequent stall caused by the large adverse gradients in the neighbourhood of the nose can be prevented. For wings in which laminar flow to the trailing edge is envisaged, prevention of separation is not enough—a stable *i.e.* a convex velocity profile is necessary. Thus $-\lambda$ must be small and so the suction given by (5.10) or (5.11) will be needed. In this connection an accurate solution of the boundary layer equations for the flow past a flat plate under a constant adverse gradient under all suction conditions or even an approximate solution of the momentum equation (5.4) and equation (5.5) would be very valuable, as it would remove the arbitrary value attached to $-\lambda$ in deriving (5.10) and (5.11).

(c) *Examples of the Application of Suction through a Permeable Surface as a Means of Boundary-layer Control.*—The best way of demonstrating the advantages of this type of boundary-layer control is to compute the suction flow in typical applications.

Example 1. *Circular Cylinder.*—This example is chosen because of its simplicity and because it might be made the subject of an experiment to illustrate the effects of suction, since without suction a strongly separated flow exists.

The potential flow velocity at the surface is :

$$\frac{U}{U_0} = 2 \sin \theta \, .$$

The distance around the surface from the stagnation point is

$$x = \frac{D}{2} \theta \, ,$$

where D is the diameter.

Whence $$\frac{d(U/U_0)}{d(x/D)} = 4 \cos \theta$$

and so the adverse gradient has a maximum value of

$$-\frac{d(U/U_0)}{d(x/D)} = 4 \quad \text{at } \theta = \pi \, .$$

Suction will be confined to the rear half of the cylinder as there is no danger of separation over the front half. The suction flow will be computed on the basis of a constant velocity v_1 through the surface of the rear half based on a value of $-\dfrac{d(U/U_0)}{d(x/D)} = 4$, which should over-estimate the suction required as the gradient ranges from 0 to -1, as θ ranges from $\pi/2$ to π. Equation (5.16) with D written for c is :

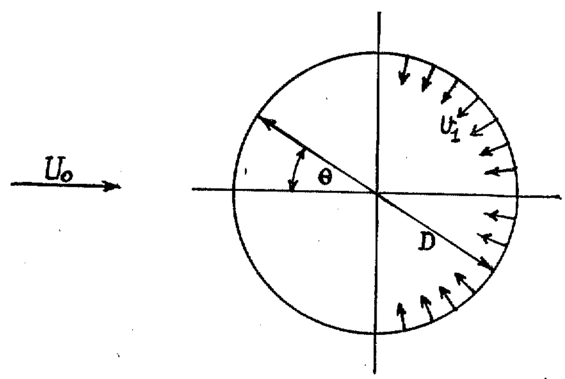

FIG. 1.

$$\frac{v_1}{U_0} = 1{\cdot}607 \left(-\frac{dU/U_0)}{d(x/D)} \cdot \frac{\nu}{U_0 D} \right)^{1/2} ,$$

whence

$$\frac{v_1}{U_0} = 2 \times 1{\cdot}607 \cdot \left(\frac{\nu}{U_0 D}\right)^{1/2} = 3{\cdot}214 \left(\frac{\nu}{U_0 D}\right)^{1/2} .$$

The quantity sucked per ft. run is given by :

$$Q = \frac{\pi}{2} \cdot v_1 D = 3{\cdot}214 \cdot \frac{\pi}{2} \cdot U_0 D \left(\frac{\nu}{U_0 D}\right)^{1/2} ,$$

or

$$C_Q = \frac{Q}{U_0 D} = 3{\cdot}214 \cdot \frac{\pi}{2} \cdot \left(\frac{\nu}{U_0 D}\right)^{1/2} .$$

If we take $D = 1$ in. dia., $U_0 = 100$ ft./sec., then $\frac{U_0 D}{\nu} = 5 \cdot 25 \times 10^4$

$v_1 = 1 \cdot 40$ ft./sec.

$Q = 11 \cdot 0$ cu. ft./min. per ft. run.

$C_Q = 0 \cdot 022.$

Note that $v_1 \propto \frac{U_0^{1/2}}{D^{1/2}}$, $Q \propto U_0^{1/2} . D^{1/2}$.

Example 2. *Wing of Low-drag Type for High Speed.*

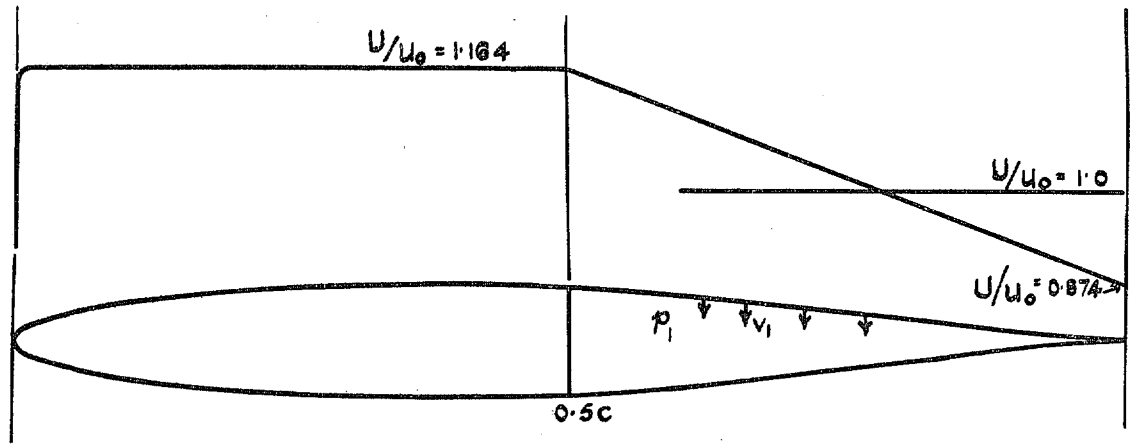

FIG. 2. Aerofoil *MR* 513–010 (Ref. 9) Velocity Distribution on Upper Surface $C_L = 0 \cdot 13$.

The section shown in Fig. 2 was designed by Thwaites[9]; it is approximately 10 per cent. thick. It is designed to achieve laminar flow up to $0 \cdot 5c$ and has a good C_L range—$0 \cdot 13 < C_L < 0 \cdot 13$ and for its thickness a low U/U_0 max. $= 1 \cdot 164$ on the upper surface at $C_L = 0 \cdot 13$. This is flat to $0 \cdot 5c$ when it falls in a linear manner to $U/U_0 = 0 \cdot 874$ at the trailing edge. Hence

$$-\frac{d(U/U_0)}{d(x/c)} = \frac{0 \cdot 290}{0 \cdot 5} = 0 \cdot 58.$$

Let us suppose that by suction completely laminar flow up to the trailing edge can be obtained. Thus it will be necessary to suck sufficiently hard for a convex, *i.e.* stable velocity profile to be obtained over the rear half of the aerofoil. Very slight suction would also be applied over the front half to ensure that the flow under zero pressure gradient was sufficiently stabilised against the effect of waves. Over the rear half the suction must be greater than that required to prevent separation (equation (5.16)). It will probably lie between the values given by formulae (5.10) and (5.11). The calculations which follow will be based on equation (5.11) and will neglect compressibility.

Take the mean chord $c = 7 \cdot 5$ ft.

and $U_0 = 600$ m.p.h. $= 880$ ft./sec.,

then $\frac{U_0 c}{\nu} \simeq 36 \times 10^6$.

Hence from (5.11) with $-\frac{dU/U_0}{dx/c} = 0 \cdot 58$

we obtain for the upper surface $\frac{v_1}{U_0} = \frac{10 \times (0 \cdot 58)^{1/2}}{6 \times 10^3} = 0 \cdot 00127$.

Thus $v_1 = 1 \cdot 12$ ft./sec.

and $C_Q = \frac{Q}{U_0 c} = \frac{v_1}{U_0} \times \frac{1}{2} = 0 \cdot 00064.$

These values are quite small and for a wing 300 sq. ft. in area suction involves a quantity flow of 336 cu. ft./sec. through the upper surface at 600 m.p.h. The momentum thickness at the trailing edge, assuming asymptotic conditions to be obtained in the boundary layer, is given from (4.38) by

$$\frac{\theta}{c} = \tfrac{1}{2}\frac{\nu}{v_1 c} = \tfrac{1}{2}\frac{U_0}{v_1}\frac{\nu}{U_0 c} = \tfrac{1}{2}\frac{10^3}{1\cdot 27} \times \frac{1}{6 \times 10^6} = 0\cdot 0000656$$

so the " wake " drag will prove to be negligible against the pump drag.

The " pump " drag is computed as follows. Let p_1 be the pressure inside the rear half of the wing and p the pressure outside corresponding to the velocity U. Then neglecting compressibility :

$$p_1 + \tfrac{1}{2}\rho v_1^2 + H = p + \tfrac{1}{2}\rho U^2 = p_0 + \tfrac{1}{2}\rho U_0^2$$

where H = loss of head through crossing the surface. Since v_1 is very small

$$\frac{H}{\frac{1}{2}\rho U_0^2} = \frac{p_0 - p_1}{\frac{1}{2}\rho U_0^2} + 1 .$$

Neglecting duct losses and assuming the pump ejects the air at the free stream velocity and pressure, *i.e.* completely restores its loss of head, the power required by a pump of efficiency η_2 is:

$$P = \frac{Q \, . \, H}{\eta_2}$$

If the efficiency η_2 is equal to that of the propulsive system of the aircraft η_1, we can define a drag coefficient $(C_D)_P$ by

$$(C_D)_P = \frac{\eta_1}{\eta_2} \cdot \frac{P}{\frac{1}{2}\rho U_0^3 \, . \, S} = C^Q \cdot \frac{H}{\frac{1}{2}\rho U_0^2} = C_Q\left(\frac{p_0 - p_1}{\frac{1}{2}\rho U_0^2} + 1\right).$$

Now $\frac{p_0 - p_1}{\frac{1}{2}\rho U_0^2} > \frac{p_0 - p_m}{\frac{1}{2}\rho U_0^3}$ where p_m corresponds to U_m the max. velocity on the surface $= 1\cdot 164 U_0$ in this case, otherwise flow into the wing in this region would not take place. Ideally we should like p_1 to be low enough to swamp external variations of pressure and so obtain a constant suction velocity v_1 through a surface of constant thickness, but the low suction head would give rise to large loads on the wing surface and at the same time increase the power required. Hence we assume that the thickness of the porous surface or the porosity can be adjusted to suit the pressure difference.

Now for the present example

$$\frac{p_0 - p_m}{\frac{1}{2}\rho U_0^2} = (1\cdot 164)^2 - 1 = 0\cdot 36$$

and if p_t is the pressure at the trailing edge

$$\frac{p_0 - p_t}{\frac{1}{2}\rho U_0^2} = (0\cdot 874)^2 - 1 = -0\cdot 235 .$$

Thus $$\frac{p_t - p_m}{\frac{1}{2}\rho U_0^2} = 0\cdot 595 .$$

If we take $$\frac{p_0 - p_1}{\frac{1}{2}\rho U_0^2} = 0\cdot 5 ,$$

then at the half-chord position

$$\frac{p_m - p_1}{\frac{1}{2}\rho U_0^2} = 0\cdot 5 - 0\cdot 36 = 0\cdot 14$$

and at the trailing edge,

$$\frac{p_t - p_1}{\frac{1}{2}\rho U_0^2} = 0\cdot14 + 0\cdot595 = 0\cdot735$$

which means a variation in porosity or of thickness of 5 : 1. With the above value of p_1,

$$(C_D)_P = 0\cdot0064 \times 1\cdot5 = 0\cdot00096 \text{ for the upper surface.}$$

The lower surface should require slightly less, for the adverse gradient will be rather less.

Hence the total drag coefficient may be expected to be

$$C_D \doteqdot 0\cdot002$$

on the present basis of estimating suction by (5.11). This C_D is roughly half that to be expected for this wing with zero suction, assuming laminar flow is maintained at $0\cdot5c$.

Example 3. Thin High-Speed Wing with Suction over the Nose to give a High $C_{L\ max.}$

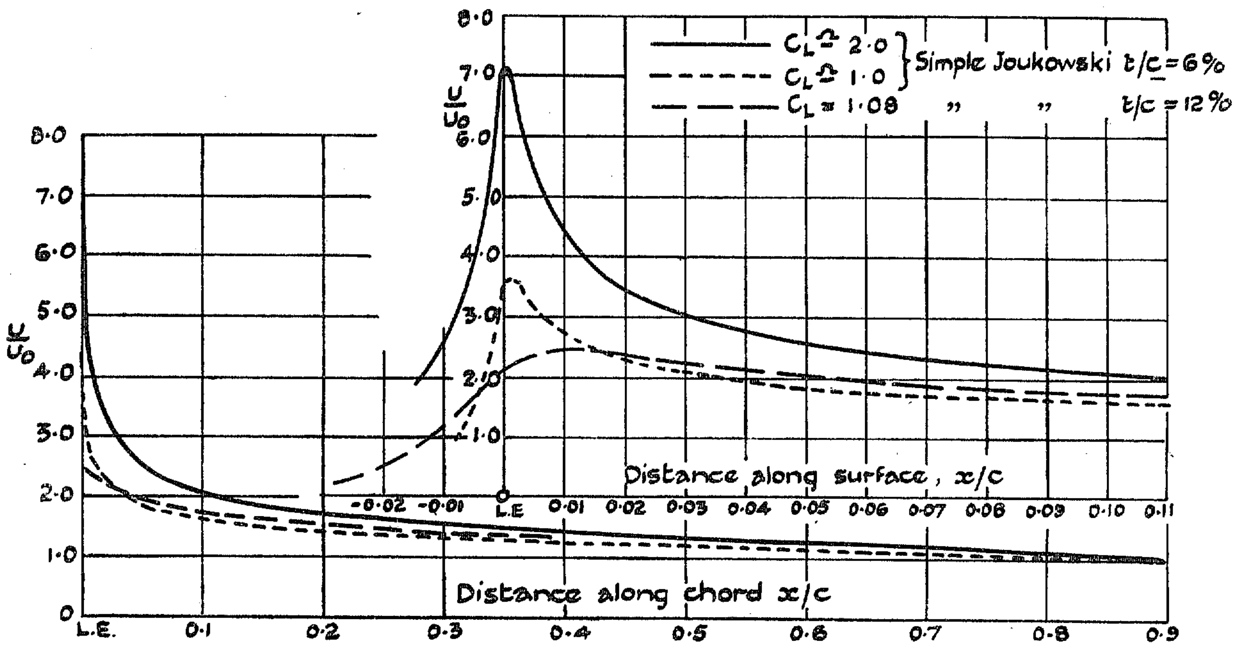

FIG. 3. Velocity Distributions over Nose of Joukowski Aerofoils.

Thin aerofoils are known to have a smaller $C_{L\ max.}$ than aerofoils of medium thickness. For instance from NACA TR 460, NACA 0006 aerofoil has a $C_{L\ max.}$ of about $0\cdot9$ at $R = 3 \times 10^6$, but the lift and moment curves are no longer linear above $C_L = 0\cdot6$. On the other hand NACA 0012 has a $C_{L\ max.} = 1\cdot5$ and the lift and moment curves are closely linear up to this C_L. The different behaviour is due to the smaller leading edge radius of curvation of NACA 0006, it being exactly $\frac{1}{4}$ that of NACA 0012. This results in high velocity peaks at the nose at high lifts for the thinner section and in larger adverse gradients which causes early separation and an early stall. Fig. 3 illustrates this for two symmetrical Joukowski aerofoils 6 per cent. and 12 per cent. thick. From experiments on a 12 per cent. simple Joukowski aerofoil the velocity distribution for $C_L = 1\cdot0$ is known to be safe, *i.e.* no turbulent separation occurs before the trailing edge. Hence if suction is applied over such an extent of the thin aerofoil nose so that the velocity gradients outside the suction region are not appreciably different for those on the 12 per cent. aerofoil without suction, then turbulent separation and the stall will be prevented.

To illustrate an important application of this type of boundary layer control and to obtain an idea of the suction flow required, we consider a fighter (using a symmetrical 6 per cent. thick Joukowski section) at landing or take-off. Let us take

$$\left.\begin{aligned} \text{mean } c &= 7 \text{ ft.} \\ U_0 &= 180 \text{ ft./sec.} \end{aligned}\right\} \quad \frac{U_0 c}{\nu} = 7 \cdot 94 \times 10^6 ,$$

$$\text{span } b = 30 \text{ ft.}$$

Suction is assumed to be uniform and sufficient to prevent laminar separation (equation 5.16) for the largest velocity gradient occurring at a specified C_L.

For $C_L = 1 \cdot 0$.—From Fig. 3 suction must take place from the leading edge to $x/c = 0 \cdot 015$ along the surface. The average value of $-\dfrac{d(U/U_0)}{d(x/c)} \simeq 100$ whence from (5.16).

$$\frac{v_1}{U_0} = \frac{1 \cdot 607 \times 10}{2 \cdot 82 \times 10^3} = 0 \cdot 0057$$

The quantity is given by :

$$Q = b \,.\, v_1 \,.\, x = \frac{v_1}{U_0} \cdot \frac{x}{c} \cdot V_0 \,.\, b \,.\, c = 3 \cdot 23 \text{ cu ft./sec.} = 194 \text{ cu. ft./min.}$$

For $C_L = 2 \cdot 0$.—From Fig. 3 it would seem that suction should be applied between $0 < x/c < 0 \cdot 10$. The largest value of the velocity gradient is

$$-\frac{d(U/U_0)}{dx/c} \simeq 333$$

whence

$$\frac{v_1}{U_0} = \frac{1 \cdot 607 \times 18 \cdot 25}{2 \cdot 82 \times 10^3} = 0 \cdot 0104 .$$

If the suction flow is maintained from $0 < x/c < 0 \cdot 10$

$$Q = 39 \cdot 3 \text{ cu. ft./sec.} = 2{,}360 \text{ cu. ft./min.}$$

This calculation neglects compressibility effects.

If equation (5.16) is correct, then this is an over estimate, since the gradient falls very rapidly as the distance from the leading edge increases. However it is plain from both cases that the suction flow required is very small and that the scheme offers great possibilities as a high-lift device.

Lighthill[9] (1945) has designed a series of nose-suction aerofoils in which, above a certain C_L, he relies on " sink " effect to reduce the velocity peaks and adverse gradients. A typical example of an 8·6 per cent. wing is given in p. 3 of A.R.C. Report No. 8658, where at a $C_L = 1 \cdot 715$ the quantity sucked per unit span is $0 \cdot 016\, U_0 c$. Taking c, U_0 and b as above, we find :

$$Q \text{ (Lighthill)} = 0 \cdot 016 \times 180 \times 7 \times 30 \text{ cu. ft./sec.} \simeq 600 \text{ cu. ft./sec.}$$

$$\simeq 36{,}000 \text{ cu. ft./min.}$$

which is an enormous flow and impossible to accommodate on a fighter.

We might note that the high velocity peak for the 6 per cent. Joukowski aerofoil at $C_L = 2 \cdot 0$ would be appreciably modified by compressibility which has been neglected here. There is obviously considerable scope for skilful design as regards the nose shape of thin aerofoils, in order to keep down the high velocity peaks and gradients by increasing the leading edge radius of curvature.

6. *Applications of Distributed Suction.*—(*a*) Prevention of Separation—*High* $C_{L\,\text{max.}}$—Examples (1) and (3) of section 5 show that quite small amounts of suction are needed to prevent laminar separation, and the most immediate important practical application is to thin high-speed wings or even to wings of medium thickness in order to attain a high $C_{L\,\text{max.}}$. In this connection it is greatly superior to the " sink " effect suggested by Lighthill[10], because of the prohibitively large quantities of air which have to be handled in practical applications of this device. If a symmetrical or slightly cambered wing is used then a large $C_{L\,\text{max.}}$ may possibly be obtained without the use of flaps and hence with no change of trim, which makes it of great importance in tailless aircraft. Moreover, the stall when it occurs will be gentle, as it will start by turbulent separation at the trailing edge. The performance of the wing at low C_L's, at high speeds, should be better as the porous nose should in no way interfere with the attainment of laminar flow, as would be the case with slots or nose flaps. The scheme should also work for wings with sharp leading edges, provided suction commences at the front stagnation point on the lower surface, so that no boundary layer exists as the flow passes round the sharp nose.

In this paper we have been concerned solely with laminar flow, but it is evident that suction could be used to prevent turbulent separation, say in diffusers. Unfortunately, conditions in a turbulent boundary layer near separation are not very well understood and so calculations are not possible.

(*b*) *Laminar Flow Wings.*—The work of Pretsch[4] (1942) shows that, with suction, the stability of the laminar boundary layer is greatly increased for a flat plate and that the suction required is extremely small. This should be even more marked in the region of favourable pressure gradients on a laminar-flow wing. The chief obstacle to the achievement of laminar flow over say 60 per cent. of the chord is the difficulty of obtaining a stressed-skin construction free from waves. Hence by a very small amount of suction in the region of favourable pressure gradients the adverse effects of waviness might be nullified and by a stronger suction in the region of adverse gradient laminar flow may be held to the trailing edge, if the wing surface is everywhere convex. Example 2 of section 5 shows the order of suction required under these circumstances and the C_D which might be expected. Lower drags and suction quantities would be obtained by having the pressure minimum well back—say as far as 75 per cent. of the chord as $C_Q \propto \{d(U/U_0)/(x/c)\}^{\frac{1}{2}}$, thus localising the region of strong suction. The example shows that the " wake " drag is negligible compared with the " pump " drag and so great emphasis must be laid on pump and ducting efficiency. It may be noted that there is no restriction on thickness and that it should be possible to design an aerofoil, say 30 per cent. thick, which in the event of suction failure would have better qualities than the Griffith type designed for concentrated suction.

A point which may have considerable importance for high-speed wings, is that when distributed suction is in operation, the boundary layer is extremely thin and so the drag rise associated with boundary layer separation due to the formation of shock waves might not take place under suction conditions.

(*c*) *As a Research Tool.*—Scale effect on the sectional characteristics of wings and controls is known to be closely connected with the boundary layer thickness near the trailing edge, which, for given transition points, decreases as the Reynolds number increases. Thus by a judicious use of distributed suction, full scale boundary layer conditions might be simulated at moderate Reynolds numbers in a small wind tunnel and thus enable design data to be obtained without doubtful extrapolation.

In the same way by sucking away the boundary layer Mach number effects might be isolated from Reynolds number effects in tests on models in the high-speed tunnels, where in general the Reynolds numbers are small and so the boundary layers are relatively thick. Also shock wave formation could be studied in the absence of boundary layers.

7. *Experimental and Practical Aspects.*—(*a*) *The Surface.*—Theory envisages a continuous suction flow through the surface. The examples show that although the suction velocity through the surface is low, considerable pressure differences may have to be tolerated. Thus considerable

resistance to the flow is required. The ideal surface would be one composed of extremely fine honeycomb cells. Experiments by Perring and Diprose (unpublished) (1937) have been made in the past to simulate distributed suction by a finite number of slots whose spacing was large compared with the slot width. Instead of stabilising the flow as was hoped, it appears from these experiments that when the flow into the slot exceeded a certain amount, turbulent flow was set up. Slot entry shape may have had considerable influence on the stability of flow. In Ackeret's experiments the slots were stated to have been closely spaced and his experiments were successful. If slots are to be used, then their spacing should probably at least be equal to the slot width, which in turn should be less than the boundary layer thickness. In Appendix I it is shown that if the minimum number of slots is used to prevent separation, then probably 8 times as much air as is required by porous suction will have to be sucked. Obviously the building up of a porous surface of any extent in this manner is a difficult undertaking. Likewise a porous surface made by drilling a large number of small holes will be tedious if not impossible to construct as the hole spacing must be close. If this is not so, then there will be a danger of transition to turbulent flow, as some unpublished experiments show that by sucking through an isolated pressure orifice 1/100 in. dia., transition can be brought about. Ceramics such as are used for filtration purposes seemed to offer possibilities and the advice of Chemical Research Laboratory, D.S.I.R. was sought. It was Mr. Roff of C.R.L. who suggested the use of porous bronze.

This material is obtained by pressure moulding powdered metal using a particle size to give the desired porosity and size of passages. It can be obtained in sheet form up to a maximum size of 12 in. × 6 in. and the thickness ranges from $\frac{1}{16}$ in. to $\frac{1}{8}$ in.

The size of pores range from $2\frac{1}{2}$ microns to 100 microns and their uniformity appears to be good. Samples representing extremes in the size of pores have been obtained and are being tested. A report on these will be issued shortly. The use of metal offers obvious advantages. It can be brazed or soldered. It has some mechanical strength, though the surface cannot be machined as the pores would be closed up. From the point of view of model experiments it would appear to meet all our requirements. For application to flight it will probably turn out to be too heavy for surfaces of considerable extent, even if the light alloys can be used, as its strength is not great.

(*b*) *Suggested Experiments.*—(1)Hollow cylinders of porous bronze can be obtained from stock, in diameters ranging from 1 in. to 3 in. It is proposed to test a 1 in. dia. cylinder in a 1 ft. tunnel to check the calculation made in the section 5c. This would also be suitable for test in the 20 in. × 8 in. High Speed Tunnel and would enable valuable information to be obtained on the effect of the boundary layer in the formation of shock waves.

(2) The problem of $C_{L\,\max.}$ for thin high-speed wings is now a matter of urgency. It is suggested that a porous nose could be fitted to a 30 in. chord aerofoil for test in the 9 ft. × 7 ft. tunnel to check the calculation of Section 5. If this were successful, flight tests might follow on an existing machine.

(3) As model wings for the 20 in. × 8 in. High Speed Tunnel are usually of 5-in. chord, it should be possible to have these made in two halves from this material. Suction over various portions of the wing could be effected by filling up the pores of the portions for which zero suction is required; with wax this could be dissolved out again if required. This would enable the effect of suction at high Mach numbers to be studied.

(4) The Compressed Air Tunnel provides an easy method of effecting suction by allowing air from the wing to leak to atmosphere. The standard models are 4 ft. span × 8 in. chord giving a maximum Reynolds number of 8×10^6, so that landing conditions could be simulated and scale effects on suction could be explored.

(5) Experiments should be undertaken to test out Pretsch's conclusion that the stability of the laminar boundary layer is greatly increased by suction and to substantiate its proposed use for low-drag wings. These however will probably have to wait until the problem of constructing a porous skin of considerable area has been overcome.

8. *Conclusions.*—As regards the theory, certain simple exact solutions of the boundary layer equation indicated in section 3 would be worth obtaining, as also would be an accurate solution of the flow along a semi-infinite plate for which an approximate solution is given in section 4. From the point of view of application to the design of suction wings, an accurate or even approximate solution for the boundary layer flow under an adverse gradient with suction is required immediately to amplify the simple formulae obtained in section 5.

The most immediate application of distributed suction is to thin high-speed wings, in order to obtain a high $C_{L\,\max.}$ by suction over the nose. The numerical example worked in section 5 suggests that the suction flow required will be quite small, compared with that required for "nose-suction" aerofoils relying on "sink" effect. There is the possibility that two cabin superchargers, using the auxiliary drives on existing engines, could be able to cope with this flow. The practical attainment of laminar flow wings may also be brought nearer by use of suction. Use may also be made of distributed suction to simulate high Reynolds number conditions by thinning the boundary layer and for study of Mach number effects with boundary layers absent. From an experimental standpoint the new porous metal should go a long way towards meeting the mathematical requirement of a surface through which a continuous flow can take place. The simulation of a porous surface by a number of isolated slots or holes is to be discouraged, because of their distabilising influence on the laminar boundary layer. A big new field for experimental investigation is presented, which up to the present has remained almost untouched.

TABLE 1

K	$\xi = \left(\frac{v_1}{U}\right)^2 \cdot \frac{UX}{\nu}$	$\frac{v_1 \delta^\times}{\nu}$	$\frac{v_1 \theta}{\nu}$	H	$\frac{U\tau_0}{v_1\ \rho U^2}$
0	0	0		2·591	∞
0·1	0·01088	0·1628	0·0648	2·509	3·773
0·2	0·04663	0·3044	0·1251	2·433	2·159
0·3	0·1138	0·4286	0·1813	2·364	1·633
0·4	0·2230	0·5385	0·2342	2·299	1·380
0·5	0·3916	0·6364	0·2841	2·240	1·235
0·6	0·6505	0·7241	0·3315	2·185	1·144
0·7	1·059	0·8033	0·3765	2·134	1·085
0·8	1·757	0·8750	0·4195	2·086	1·045
0·9	3·195	0·9403	0·4606	2·041	1·018
0·91					
0·92	3·706	0·9527	0·4686	2·033	1·014
0·93					
0·94	4·389	0·9648	0·4766	2·025	1·010
0·95					
0·96	5·389	0·9767	0·4844	2·016	1·006
0·97					
0·98	7·161	0·9885	0·4923	2·008	1·003
0·99	8·986	0·9943	0·4961	2·004	1·001
0·995	10·83	0·9971	0·4981	2·002	1·001
1·000	∞	1·0000	0·5000	2·000	1·000
1·1	+2·1765				
1·2	−0·3126	1·105	0·5741	1·925	0·982
1·3	−2·133				
1·4	−3·7950	1·195	0·6428	1·859	0·9802
1·5	−5·5425				
1·6	−7·677	1·273	0·7064	1·802	0·9878

APPENDIX I

Suction Quantity to Prevent Separation using the Minimum Number of Isolated Slots in the Flow against a Constant Adverse Velocity Gradient

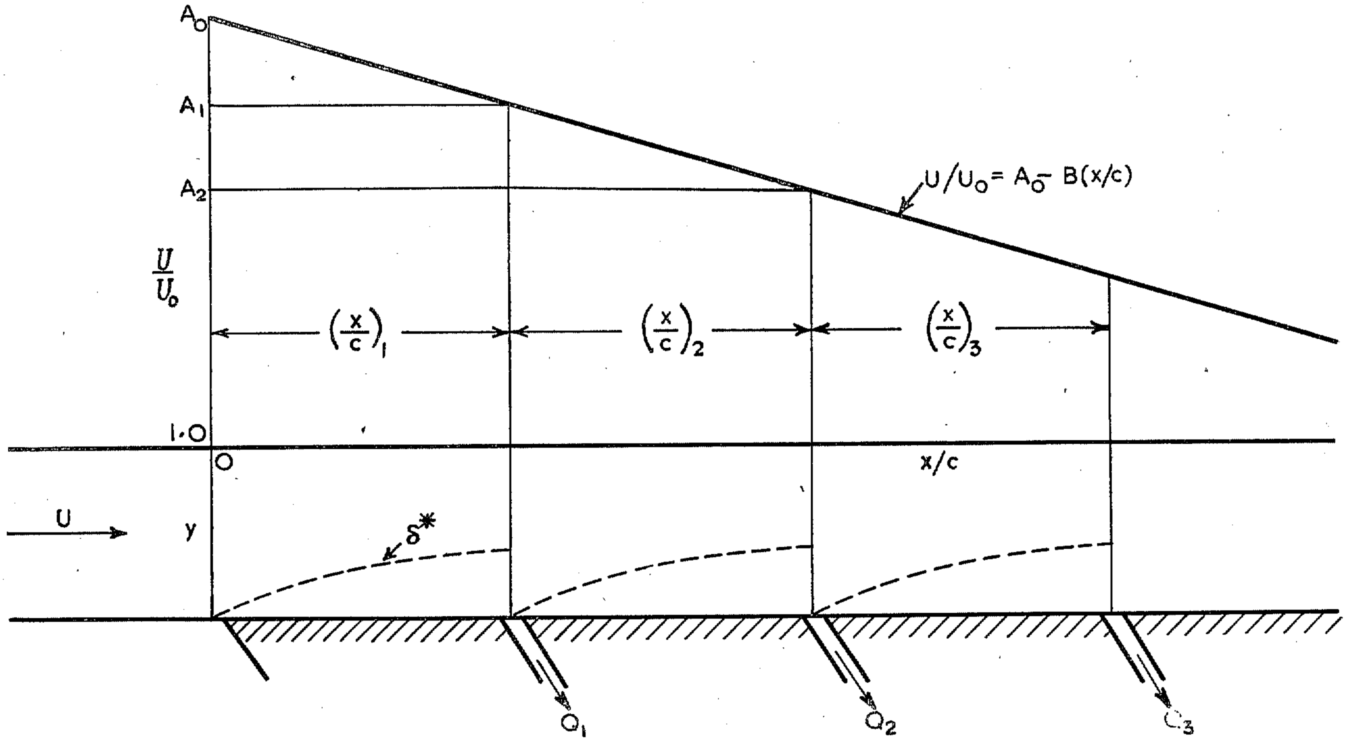

FIG. 4

The details of the boundary-layer flow against a constant adverse velocity gradient have been given by Howarth[7] (1938).

Let the velocity at the edge of the boundary layer be given by

$$\frac{U}{U_0} = A_0 - B\frac{x}{c}\,, \qquad \text{(A.I.1)}$$

or

$$U = A_0 U_0 - \frac{BU_0}{c}\,x \qquad \text{(A.I.2)}$$

$$= \beta_0 - \beta_1 x\,, \qquad \text{(A.I.3)}$$

in Howarth's notation.

Howarth introduces the variable ξ for the distance along the surface and η for the distance normal to the surface, defined by

$$\xi = \frac{\beta_1}{\beta_0}\,x \equiv \frac{Bx}{Ac}\,, \qquad \text{(A.I.4)}$$

$$\eta = \frac{1}{2}\left(\frac{\beta_0}{\nu x}\right)^{1/2} y. \qquad \text{(A.I.5)}$$

Let the boundary layer commence at $x = 0$ and continue until laminar separation is imminent. The *whole* of the boundary layer is now sucked away via a slot at this position and a new boundary layer starts and continues until separation is again imminent, when it is sucked away by a second slot, and so on. In this way we shall use the *minimum* number of slots. We ignore the " sink " effect of the slots on the flow.

Howarth finds that separation occurs at

$$\left.\begin{aligned} \xi &= 0 \cdot 12\,, \\ \text{i.e.} \qquad \frac{x}{c} &= \frac{A}{B} \times 0 \cdot 12\,. \end{aligned}\right\} \quad \ldots \quad (\text{A.I.6})$$

For the separation profile, the " displacement thickness " δ^* is given by

$$\delta^* = 1 \cdot 11 \left(\frac{\nu}{\beta_1}\right)^{1/2} \quad \ldots \quad (\text{A.I.7})$$

and also the value of y, where $u/U = 1 \cdot 000$, *i.e.* the boundary-layer thickness δ corresponds to

$$\eta_\delta = 4 \cdot 2\,; \quad \ldots \quad (\text{A.I.8})$$

whence from (A.I.5)

$$\delta = 2\eta_\delta \left(\frac{\nu x}{\beta_0}\right)^{1/2} = 8 \cdot 4 \left(\frac{\nu x}{\beta_0}\right)^{1/2}. \quad \ldots \quad (\text{A.I.9})$$

(If $u/U = 0 \cdot 990$ defines the edge of the boundary-layer, then $\eta_\delta = 3 \cdot 2$ and there is an arbitrary element arising from the definition of the edge of the boundary layer.)

At separation $\xi \equiv (\beta_1/\beta_0)x = 0 \cdot 12$, whence

$$\delta = 8 \cdot 4\sqrt{0 \cdot 12} \left(\frac{\nu}{\beta_1}\right)^{1/2} = 2 \cdot 91 \left(\frac{\nu}{\beta_1}\right)^{1/2} \quad \ldots \quad (\text{A.I.10})$$

and

$$\delta - \delta^* = (2 \cdot 91 - 1 \cdot 11) \left(\frac{\nu}{\beta_1}\right)^{1/2} = 1 \cdot 80 \left(\frac{\nu}{\beta_1}\right)^{1/2},$$

which, since $\beta_1 = BU_0/c$, gives

$$\delta - \delta^* = 1 \cdot 80 \left(\frac{\nu c}{BU_0}\right)^{1/2}. \quad \ldots \quad (\text{A.I.11})$$

The quantity sucked at the first slot is

$$Q_1 = U_1(\delta - \delta^*) \quad \ldots \quad (\text{A.I.12})$$

and

$$(C_Q)_1 = \frac{Q_1}{U_0 c} = \frac{1 \cdot 80}{R^{1/2}} \left(\frac{U}{U_0}\right)_1 \frac{1}{B^{1/2}}, \quad \ldots \quad (\text{A.I.13})$$

where

$$R = \frac{U_0 c}{\nu} \quad \ldots \quad (\text{A.I.14})$$

and $(U/U_0)_1$ is the value of (U/U_0) at the first slot ($\xi = 0 \cdot 12$).

Now from (A.I.1)

$$\left(\frac{U}{U_0}\right)_1 = A_0 - B\left(\frac{x}{c}\right)_1,$$

but from (A.I.6)

$$\left(\frac{x}{c}\right)_1 = \frac{A_0}{B} \times 0 \cdot 12\,, \quad \ldots \quad (\text{A.I.15})$$

whence

$$\left(\frac{U}{U_0}\right)_1 = A_0(1 - 0 \cdot 12) = 0 \cdot 88 A_0\,. \quad \ldots \quad (\text{A.I.16})$$

Hence from (A.I.13) and (A.I.16)

$$(C_Q)_1 = 0\cdot 88 \frac{A_0}{B} \times 1\cdot 80 \frac{B^{1/2}}{R^{1/2}},$$

or using (A.I.6), giving $A_0/B = (x/c)_1\,(1/\xi_1) = (x/c)\,(1/0\cdot 12)$,

$$(C_Q)_1 = \frac{0\cdot 88 \times 1\cdot 80}{0\cdot 12} \frac{B^{1/2}}{R^{1/2}}\left(\frac{x}{c}\right)_1 = 13\cdot 2 \frac{B^{1/2}}{R^{1/2}}\left(\frac{x}{c}\right)_1 . \quad \text{(A.I.17)}$$

The position of and the flow into the second slot can be computed in the same way. The starting value of U/U_0 is now

$$\left(\frac{U}{U_0}\right)_1 = 0\cdot 88A_0 = A_1, \text{ say},$$

and the distance of the second slot from the first is

$$\left(\frac{x}{c}\right)_2 = \frac{0\cdot 12A_1}{B} = \frac{0\cdot 12 \times 0\cdot 88A_0}{B}, \quad \text{(A.I.18)}$$

and exactly as before

$$(C_Q)_2 = 13\cdot 2 \frac{B^{1/2}}{R^{1/2}}\left(\frac{x}{c}\right)_2 . \quad \text{(A.I.19)}$$

Hence for n slots, the total C_Q is

$$(C_Q)_T = \sum_{r=1}^{n} (C_Q)_r = \frac{13\cdot 2B^{1/2}}{R^{1/2}} \sum_{r=1}^{n} (x/c)_r , \quad \text{(A.I.20)}$$

where

$$(x/c)_n = (0\cdot 12A_0/B)\,(0\cdot 88)^{n-1}, \quad \text{(A.I.21)}$$

or

$$(C_Q)_T = 13\cdot 2 \frac{B^{1/2}}{R^{1/2}} \cdot \frac{x}{c}, \quad \text{(A.I.22)}$$

where x/c is the distance of the nth slot from the origin or leading edge and is the distance over which suction is taking place through isolated slots.

Now, for distributed suction, Prandtl's method (Section 5*b*) using the Polhausen separation profile gives

$$C_Q = 2\cdot 18 \frac{B^{1/2}}{R^{1/2}} \cdot \frac{x}{c}, \quad \text{(A.I.23)}$$

and using the Howarth separation profile, it gives

$$C_Q = 1\cdot 607 \frac{B^{1/2}}{R^{1/2}} \cdot \frac{x}{c}, \quad \text{(A.I.24)}$$

both of which are very considerably less than the suction necessary with the *maximum* spacing of isolated slots.

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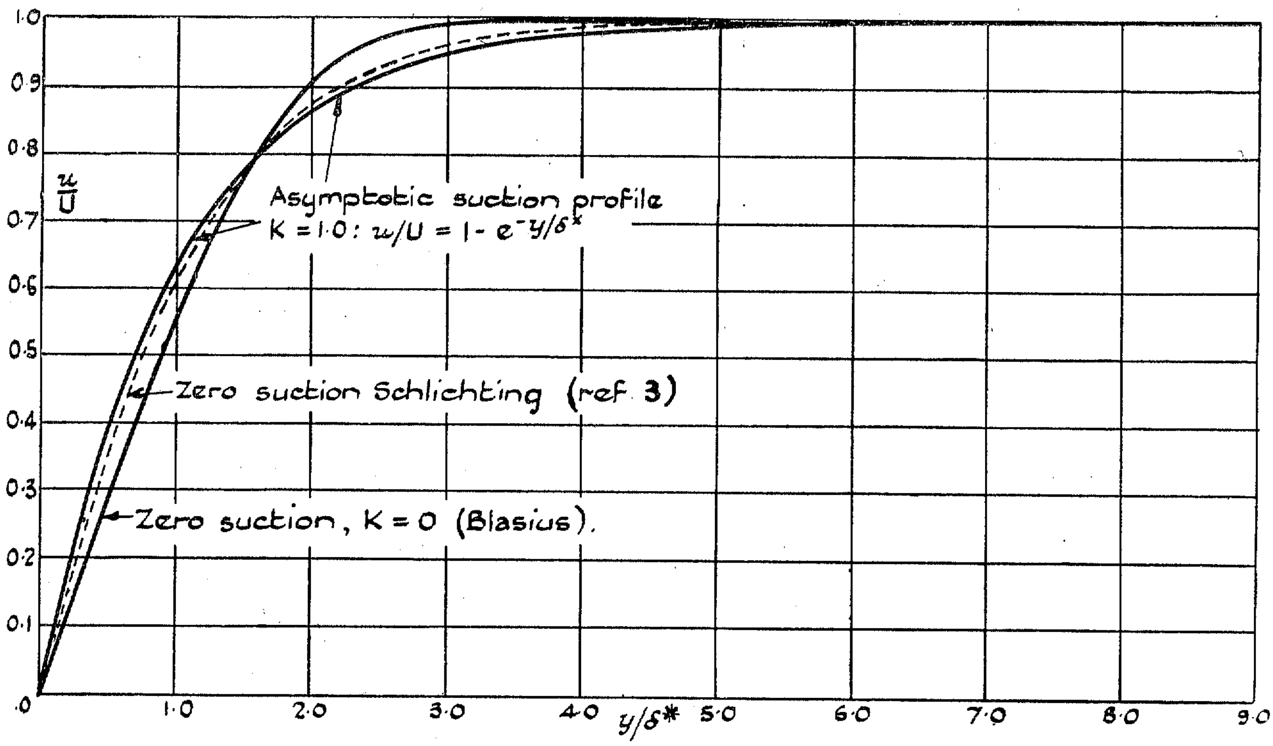
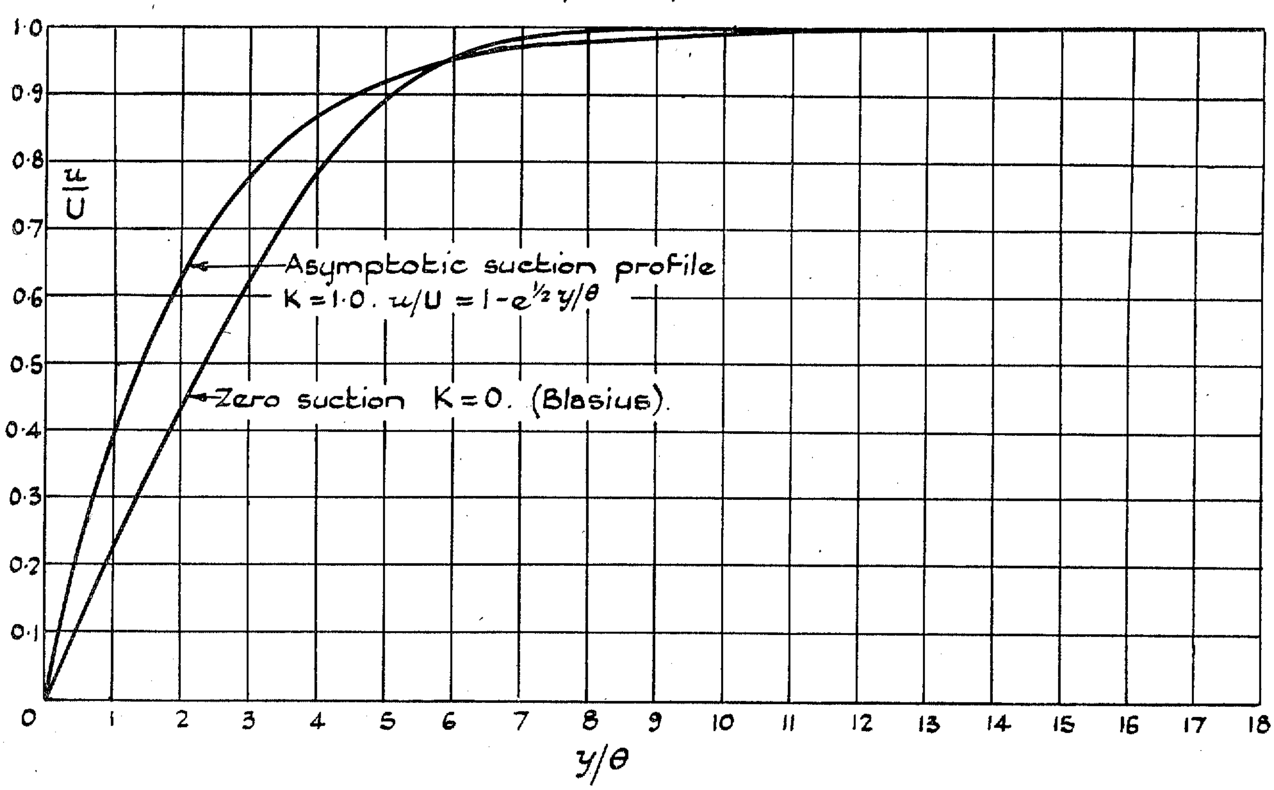


(a) Profiles of equal displacement thickness = δ^*



(b) Profiles of equal momentum thickness = θ

FIG. 5. Velocity Profiles.

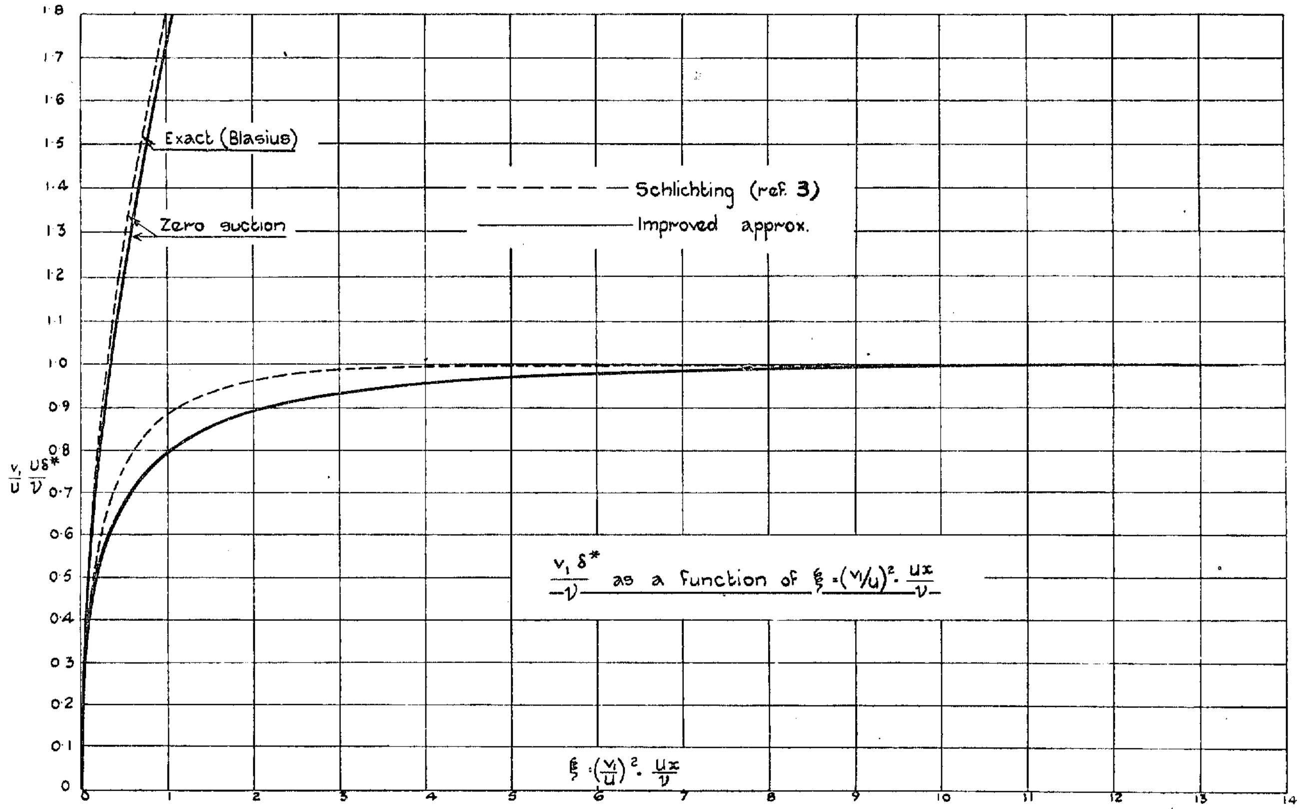

Exact (Blasius)
Zero suction
Schlichting (ref. 3)
Improved approx.
$\frac{v_1}{U}\frac{U\delta^*}{\nu}$
$\frac{v_1\delta^*}{\nu}$ as a function of $\xi = (v_1/U)^2 \cdot \frac{Ux}{\nu}$
$\xi = \left(\frac{v_1}{U}\right)^2 \cdot \frac{Ux}{\nu}$

FIG. 6.

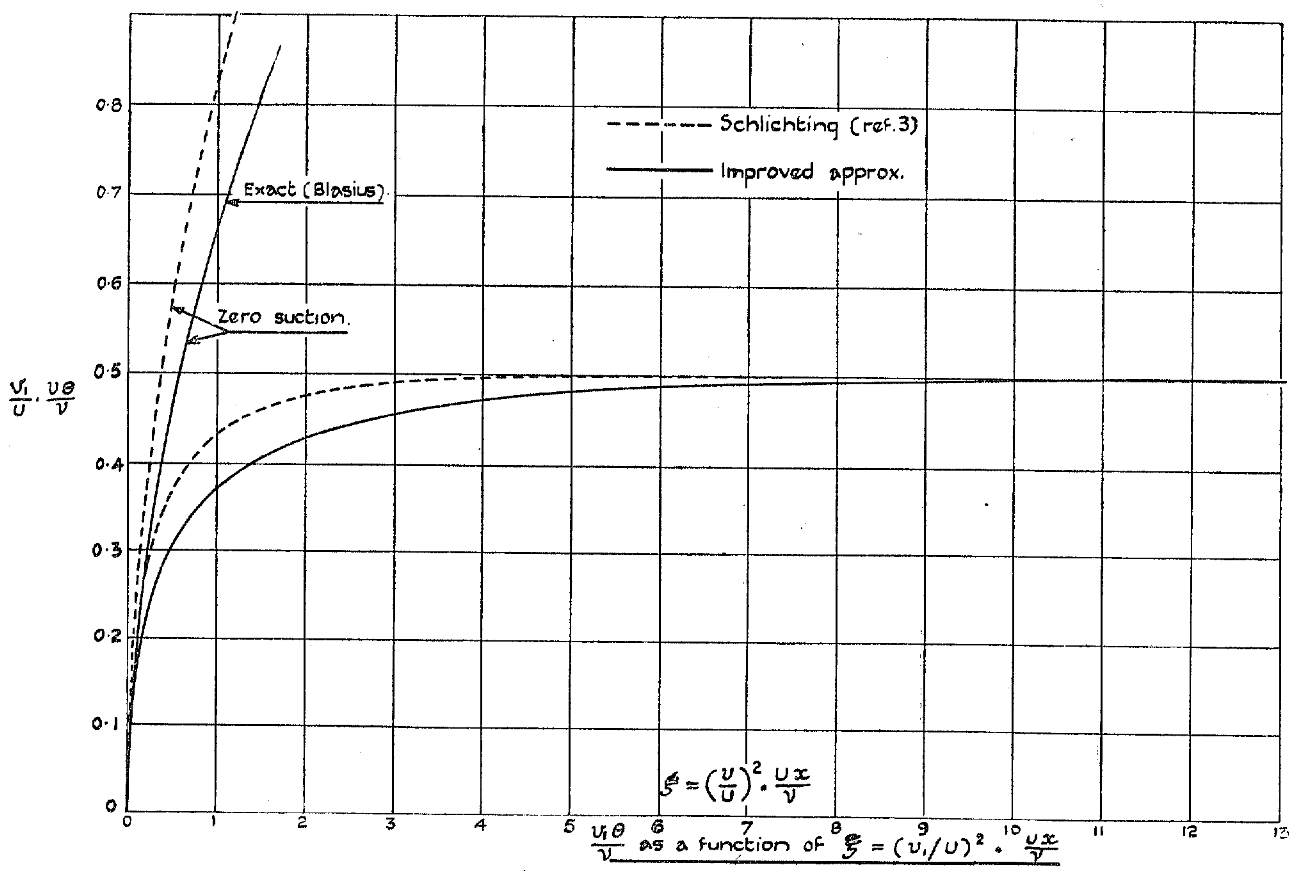

Schlichting (ref. 3)
Improved approx.
Exact (Blasius)
Zero suction.
$\frac{v_1}{U}\cdot\frac{U\theta}{\nu}$
$\xi = \left(\frac{v}{U}\right)^2 \cdot \frac{Ux}{\nu}$
$\frac{v_1\theta}{\nu}$ as a function of $\xi = (v_1/U)^2 \cdot \frac{Ux}{\nu}$

FIG 7.

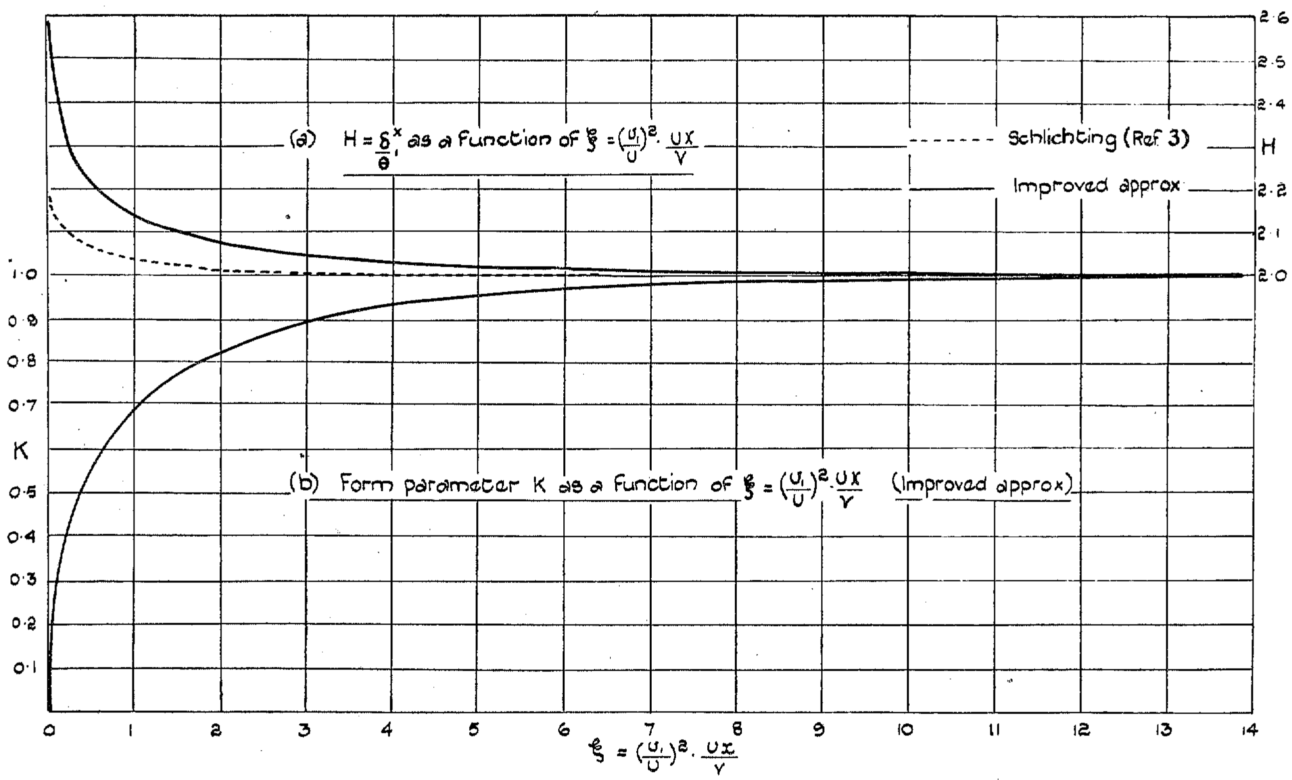

(a) H = δ*/θ as a function of ξ = (U₁/U)²· Ux/ν
Schlichting (Ref 3)
Improved approx
(b) Form parameter K as a function of ξ = (U₁/U)²· Ux/ν (Improved approx)
K
H
ξ = (U₁/U)²· Ux/ν

FIG. 8.

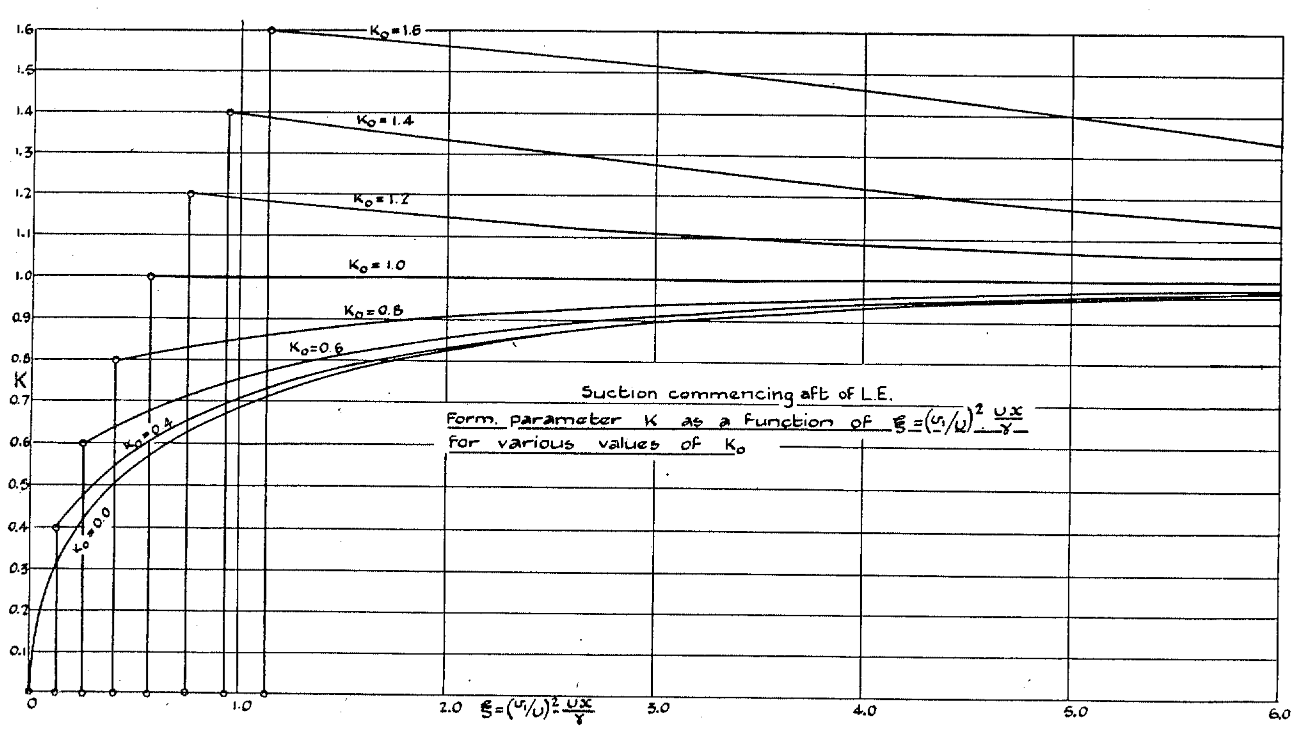
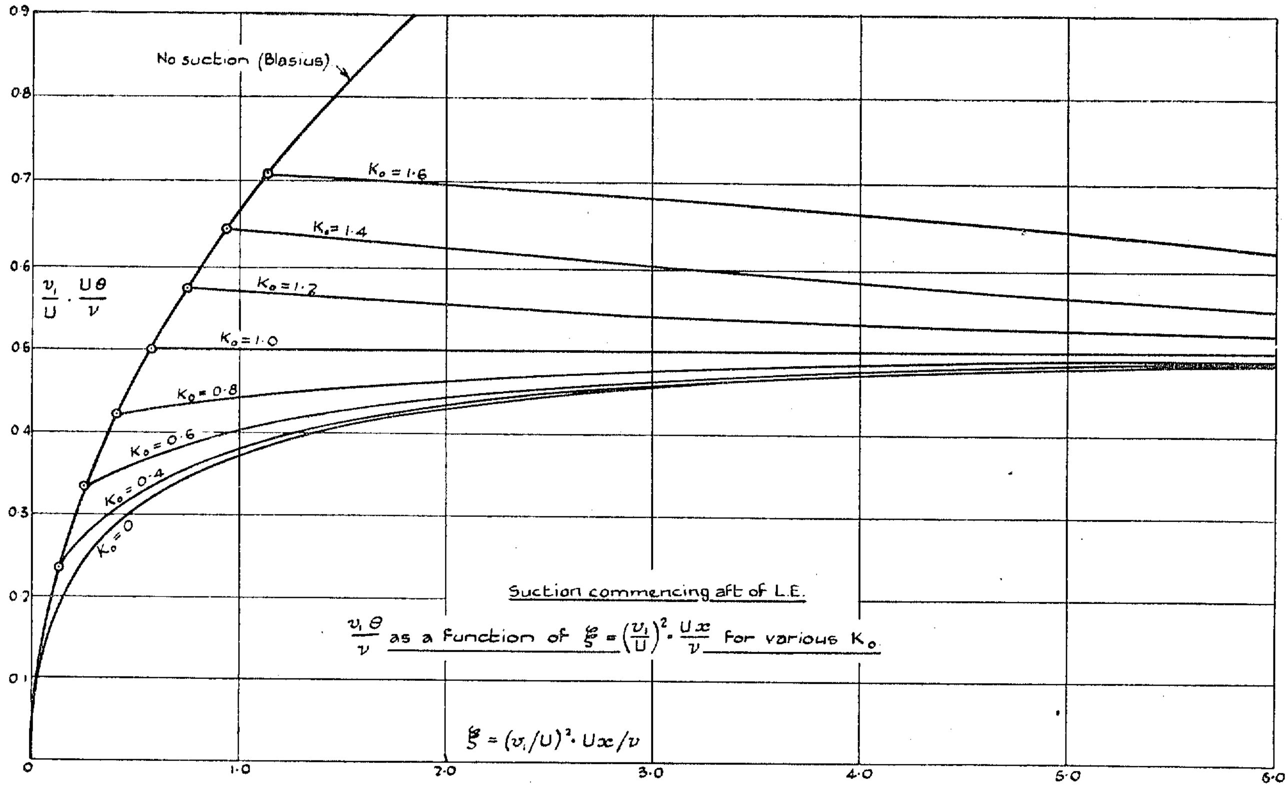
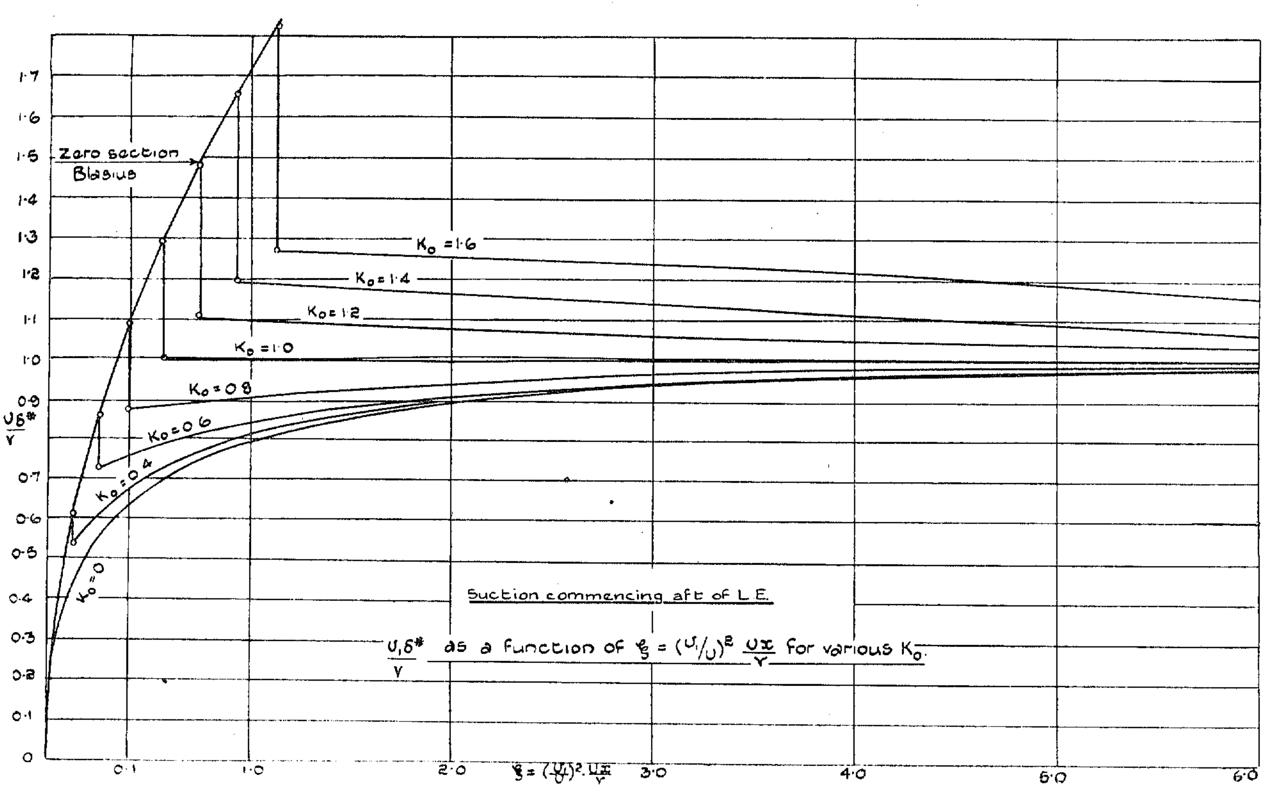

K₀ = 1.6
K₀ = 1.4
K₀ = 1.2
K₀ = 1.0
K₀ = 0.8
K₀ = 0.6
K₀ = 0.4
K₀ = 0.0
K
Suction commencing aft of L.E.
Form parameter K as a function of ξ = (U₁/U)² Ux/ν
for various values of K₀
ξ = (U₁/U)² Ux/ν

FIG. 9.



No suction (Blasius)
$K_o = 1.6$
$K_o = 1.4$
$K_o = 1.2$
$K_o = 1.0$
$K_o = 0.8$
$K_o = 0.6$
$K_o = 0.4$
$K_o = 0$
$\frac{v_1}{U} \cdot \frac{U\theta}{\nu}$
Suction commencing aft of L.E.
$\frac{v_1\theta}{\nu}$ as a function of $\xi = \left(\frac{v_1}{U}\right)^2 \cdot \frac{Ux}{\nu}$ for various K_o.
$\xi = (v_1/U)^2 \cdot Ux/\nu$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
4.0
5.0
6.0

FIG. 10.



Zero section
Blasius
$K_o = 1.6$
$K_o = 1.4$
$K_o = 1.2$
$K_o = 1.0$
$K_o = 0.8$
$K_o = 0.6$
$K_o = 0.4$
$K_o = 0$
$\frac{V_1}{U} \frac{U\delta^*}{\nu}$
Suction commencing aft of L.E.
$\frac{U_1\delta^*}{\nu}$ as a function of $\xi = (U_1/U)^2 \frac{Ux}{\nu}$ for various K_o.
$\xi = (\frac{U_1}{U})^2 \frac{Ux}{\nu}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
2.0
3.0
4.0
5.0
6.0

FIG. 11.

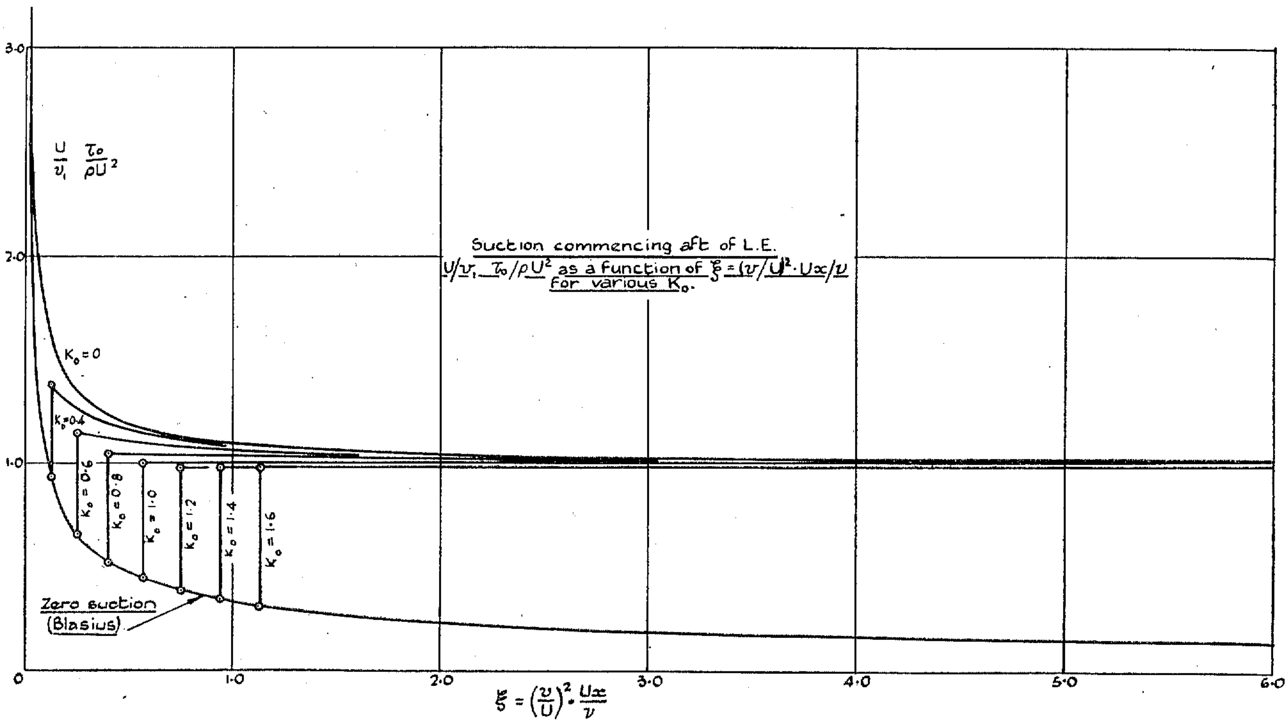


FIG. 12.

(84255) Wt. 12/818 9/48 Hw.

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