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J. R. FORSHAW, M.Eng., A.M.I.Mech.E.
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Summary.—The development of the method of the measurement of admittances and the solution of the frequency equation for a complex full-scale airframe-engine system is given, dividing the dynamical system at the attachment of the engine to the airframe, and using a force system of equal and opposite bending moments and shearing forces.

The values of the resonance frequencies obtained from the graphical solution of the frequency equation and from the resonance test are compared and found to be in good agreement. The method is applicable to the matching of an engine to an airframe by adjusting the flexibility of the mounting units.

1. *Introduction.*—The 'admittance' method of obtaining the natural frequencies of complex dynamic systems has been developed by Carter[1], Duncan[2] and Morris[3]. A similar method using the inverse of admittance, *i.e.*, 'impedance' has been developed by Biot[4] and Messrs. de Havilland Aircraft Co. Ltd.[5] The terms admittance and impedance are taken from the theory of alternating currents in electrical engineering and when used for a mechanical system refer to flexibility and stiffness measured under the conditions of a forced oscillation.

The applications of the admittance method in the past have mainly dealt with dynamical systems of two parts in which the force system at the intersection depends on a single parameter, *e.g.*, coupled engine and propeller vibration.

The method has been extended by Duncan[2], Carter[6], Fizdon, Cooper and Tate[7] and Sofrin[8] to deal with systems of two parts with more than one parameter at the intersection, which involves the use of 'cross admittances'.

The present note gives the results of an attempt to establish a technique for the experimental determination of the admittances and the solution of the frequency equation for a complex full-scale system, such as an engine-airframe combination, which can be divided into two component systems with a force system at the intersection comprising equal and opposite shearing force and bending moment. The application of the method to the problem of the variation of the natural frequencies of the complete system by the variation of the flexibility of the two systems is also considered.

* R.A.E. Tech. Note Structures 20, received 14th August, 1948.

The example chosen was the *Hurricane* IIc aircraft and the modes of vibration considered were the vertical and pitching modes for the frequency range 1000 to 6000 c.p.m. This aircraft was chosen because it was a single-engined aircraft for which the results of a number of vibration investigations were available and in particular the results of the standard ground resonance test[9] made in relation to flutter considerations.

The results show that in the experimental technique for the determination of the direct and cross admittances it is essential to use a suspension system which gives a small interference with the natural modes of vibration in the free condition. It was also necessary to develop a technique for the correction of the complex amplitude-frequency curves obtained experimentally to correspond to simple curves of undamped systems to which the simple admittance method is applicable.

When the method and technique were developed the solution of the frequency equation gave results for the natural frequencies of the airframe-engine combination which were in good agreement with the results of the standard ground resonance test.

The application of the method to the matching of an engine to the airframe by adjusting the flexibility of the engine mounting units showed the variations of natural frequencies that could be obtained by variation of mounting flexibility within a practical range.

2. *Expression for Admittance at Intersection of Dynamic Systems.*—2.1. *Force System of One Parameter.*—For a component of a dynamic system subject to vibration in which the force system at the intersection can be expressed in terms of one parameter, the admittance (or dynamic flexibility for each part) is the amplitude of vibration divided by the amplitude of the force (or couple) acting at the intersection, *e.g.*, the force system at an intersection for a dynamical system possessing torsional and axial modes of vibration can be expressed in terms of a single parameter as follows:—

For a torsional system,

if θ amplitude of vibration (radians),

M amplitude of torsional couple (lb/in.),

the admittance

$$V_r = \frac{\theta}{M} ; \qquad \ldots \qquad (1)$$

and for an axial mode of vibration,

if Z amplitude of vibration (in.),

F amplitude of force (lb),

the admittance

$$V_A = \frac{Z}{F} . \qquad \ldots \qquad (2)$$

The value of the admittance varies with the position in the system and is a function of the frequency of vibration. The admittance curve is usually plotted against frequency.

At an intersection of a dynamic system the admittance for the two parts are equal in magnitude but of opposite sign, and hence for a given application of the method the admittances for the two parts of the system are plotted against frequency, one of opposite sign, and the intercepts of the curves give the natural frequencies of the combined system.

In any particular case the choice of the point at which the system is divided is governed by the dynamic characteristics. The method may be conveniently applied to the determination of the torsional-flexural vibration characteristics of an engine-crankshaft-propeller system with the intersection at the propeller hub. Fig. 4 gives typical admittance curves for such an application of the method to a three-bladed 14-ft diameter De Havilland propeller fitted to a *Sabre II* engine crankshaft, with the propeller hub included in the propeller system.

The intercepts of the curves on the zero admittance line give the frequencies for the modes of vibration which have zero amplitude at the propeller hub, *i.e.*, a fixed root mode of vibration for the propeller, and a mode of vibration with a node at the propeller hub for the crankshaft system. The asymptotes indicate the frequencies for which the couple (or force) at the propeller hub is zero and hence correspond to the free-free modes of vibration for the propeller and crankshaft systems respectively when completely separate.

For systems of dissimilar flexibilities the intersections of the two sets of admittance curves lie mainly on the asymptotes, indicating that the natural frequencies of the combined system are little changed from the free-free frequencies of the more flexible component system.

The position of the point of intersection of admittance lines indicates the mode and frequency of a resonant condition of vibration, but the value of the admittance of itself gives no indication of the severity of the vibration; for this a further study of the mode and the exciting force is required.

Many problems of the torsional vibration of a crankshaft coupled with the flexural vibration of propeller blades have been solved by admittance methods[3] and good agreement has been obtained between predicted and observed frequencies up to frequencies of 10,000 c.p.m. The natural frequencies of a system consisting of two parts can be found by the admittance method when the force system at the point of connection between the parts can be expressed by a single parameter, but for a force system at the connection of more than one parameter as in the case of flexural vibration where bending moments and shearing forces and corresponding angular and linear deflections are involved, the admittance is a function of the frequency and the ratio of bending couple to shear force.

A systematic solution to this problem by the introduction of cross admittances, is as follows:

Let prefix 1 and 2 denote the component systems (in the later application 1 denotes airframe and 2 denotes engine)

Let
- M couple acting at intersection
- F shear force acting at intersection,
- γ_{yF} linear admittance for unit force $= y_F/F$,
- $\gamma_{\theta M}$ angular admittance for unit couple $= \theta_m/M$,
- $\gamma_{\theta F}$ angular cross admittance for unit force $= \theta_F/F$,
- y_1 total linear amplitude for system 1,
- θ_1 total angular amplitude for system 1.

Then
$$ {}_1\gamma_{yM} = {}_1\gamma_{\theta F} \qquad (3)$$

and then
$$ y_1 = F_1\gamma_{yF} + M_1\gamma_{yM} \qquad (4)$$

$$ \theta_1 = F_1\gamma_{\theta F} + M_1\gamma_{\theta M} \qquad (5)$$

Similarly for y_2 and θ_2 with signs of F and M reversed.

Therefore, the conditions for equality of the displacements at the intersection are

$$ F({}_1\gamma_{yF} + {}_2\gamma_{yF}) + M({}_1\gamma_{yM} + {}_2\gamma_{yM}) = 0 \qquad (6)$$

$$ F({}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}) + M({}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}) = 0 \qquad (7)$$

and the frequency equation is

$$\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}} = \frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}}. \qquad (8)$$

To solve equation (8), eight admittance curves are required, and to obtain this information by experiment two sets of readings are required, for each component system.

(i) A linear oscillating force is applied by means of an exciter, and linear and angular displacements are measured from which the direct admittances γ_{yF} and cross admittances $\gamma_{\theta F}$ can be derived at the point of application of the force.

(ii) An oscillating couple is applied by means of a couple exciter and linear and angular displacements are measured from which the admittances $\gamma_{\theta M}$ and the cross admittances γ_{yM} are derived. An interim check of the results is obtained by equating the cross admittances for each component.

3. *Description of Experiments.*—3.1. *Choice of Component Systems.*—The object of this work is to explore the possibility of determining the resonances of an engine-airframe combination, and provide a means of demonstrating the influence of the flexibility of a part of the system on the resonant frequencies of the whole. It was therefore important to choose a point of connection between the parts at which changes in flexibility could conveniently be made. For this reason the joint of the engine feet and the mounting structure was chosen for the purpose.

With complex systems, the method of application of force or couple may have some influence on the modes of vibration. A stiff support for the exciter in the plane of applied force or couple is preferred, to minimise amplification due to resonance effects. The support, however, must be constructed with the minimum additional mass and must impose the minimum of constraint on other degrees of freedom. The choice is usually a compromise, and the most suitable method of application must be chosen for each experiment.

3.2. *Experiments on Airframe.*—For the experiments on the airframe a light, stiff platform was constructed to replace the engine on the airframe and this acted as a support for the exciters and vibration pickups. It was manufactured from 10 gauge Duralumin sheet, and was of box construction, with a number of reinforcing members. It can be seen attached to the airframe in Fig. 1.

3.2.1. *Support of airframe.*—In the early stages of the experiments the airframe was supported on its main undercarriage and tail wheel as for airframe resonance testing procedure. The tyres were deflated to half pressure, but at the lower frequencies difficulty was experienced in obtaining equality of the cross admittance terms probably due to coupling between modes of vibration. A suspension with a sufficiently low natural frequency was obtained by supporting the airframe under the wing roots by air bags constructed from 3-ply rubberised fabric in canvas supporting bags, 2-ft diameter and 8-ft long excluding ends. A cross-connection was fitted at the ends of the air bags and they were inflated from a pressure bottle. A pressure of approximately 2 lb/sq in. was sufficient to support the airframe, and the frequencies of the modes of vibration of the airframe due to the elasticity of its supports were all less than 1 c.p.s. which is satisfactory for all the modes investigated. Fig. 2 shows the airframe in position for an experiment.

3.2.2. *Exciting Equipment.*—To cover the whole range of frequency investigated, it was expedient to use different items of equipment for particular frequency ranges.

A mechanical exciter of 2·5 in.lb out-of-balance was used throughout the experiments to excite the linear vibrations. It was necessary to reduce the out-of-balance of the exciter at the higher frequencies in order to limit the load on its bearings to a total oscillating load of 500 lb. A unit of similar detail design but with three wheels geared together instead of two was used for the couple excitation at the higher frequencies. The distance apart of the centres of the outer wheels was $7\frac{1}{2}$ in., and the values of the out-of-balance masses were equal. Since the exciting force varies as the square of the frequency, it was necessary to increase the out-of-balance for the lower frequencies by using two linear exciters of $1 \cdot 5$ in.lb out-of-balance and of opposite phase placed with their centres 22 in. apart. This method of excitation is shown in Fig. 2.

3.2.3. *Recording equipment.*—The amplitude of vibration was obtained throughout from electro-magnetic linear seismic vibration pick-ups connected through integrating circuits to an amplifier and cathode ray tube, and the spot deflection on the screen was measured directly. This reading was converted to amplitude by using the results of prior calibration of the pick-ups on a vibrating table.

Two types of pick-up were used, for the higher frequency range a Mullard type with a natural frequency of its seismic system of 15 c.p.s. was employed, and for the lower frequency range a Sperry type with a natural frequency of 4 c.p.s.

For the measurement of angular vibration two pick-ups were placed on the platform at points equidistant from the centre of gravity of the engine. The angular deflection was derived from the difference between the two readings, and this was done by subtracting the electrical out-puts of the two pick-ups by electronic means. For this purpose the two pick-ups were first mounted on a calibrating table and the out-puts from their respective integrating circuits were balanced for particular frequency ranges, the extent of which depended on the similarity of the calibration curves of the two pick-ups.

3.3. *Experiments on Engine.*—3.3.1. *Support of engine.*—The engine was slung in a similar manner to that shown in Fig. 3 at the three usual slinging points (*i.e.*, from both rear feet and the eye bolt in the reduction gear casing) with three loops of $\frac{5}{8}$-in. diameter rubber shock-absorber cord at each point.

The suspension frame permitted adjustment of the slinging point so that it could be placed above the c.g. of the engine. The natural frequencies for the vertical and pitching modes were less than 2 c.p.s.

The linear exciting force was applied and the linear movement in the vertical plane passing through the centre of gravity of the engine and propeller was derived from the pick-up measurements. This choice was arbitrary because the engine-propeller system does not behave as a concentrated mass at the centre of gravity, as is clearly shown by the results.

3.3.2. *Exciting equipment.*—The construction of a rigid platform similar to that used for the exciter for the airframe would have resulted in a complicated and heavy structure in order to clear the bottom of the crankcase. To simplify the support two linear and two couple exciters connected in phase were used in each case. The exciters were supported by a steel I beam having a maximum depth of 8 in. which was attached to two feet on each side of the engine. The shaft interconnecting the two exciters cleared the crankcase by approximately half an inch. The I beams were lightened as much as possible.

For the linear excitation phased exciters set to $1 \cdot 5$ in.lb out-of-balance were used on each I beam. The out-of-balance was reduced at the higher frequencies. For the angular excitation exciters with 3 wheels geared together similar to those described for the airframe excitation were used. Of the two sizes used, the smaller had $0 \cdot 75$ in.lb maximum unbalance per shaft and $7\frac{7}{8}$ in. between the centres of the outer wheels, and the larger exciter had $1 \cdot 25$ in.lb unbalance per shaft and was set to give the same value of the couple.

3.3.3. *Recording equipment.*—Vibration pick-ups were placed on the centre-line of each exciter, and two were placed at points equidistant from the centre of gravity for recording the angular motion. The pick-ups were of the Sperry electro-magnetic seismic type and were connected to a Miller 6-channel integrating and amplifying circuit. The outputs were fed to a 6-channel Miller galvanometer camera and a record was taken of the vibration on 70-mm recording paper. A mean of opposite pick-ups was taken and the linear and angular movement in vertical and pitching modes of the centre-line of the engine were obtained.

3.4. *Measurement of Phase Angle.*—Throughout the experiments the phase angle between the exciting force and the various pick-ups was noted, but the damping present was large, and the change in phase angle at the resonances could not be used to indicate the various resonant frequencies.

4. *Results.*—4.1. *Plotting of Results.*—Figs. 5, 6 and 7 show the results obtained from the experiments on the airframe. Figs. 5 and 6 give the admittances ${}_1\gamma_{yF}$ (linear deflection per unit load) and ${}_1\gamma_{\theta M}$ (angular deflection per unit couple). Fig. 7 shows the cross admittances ${}_1\gamma_{yM}$ (linear deflection per unit couple) and ${}_1\gamma_{\theta F}$ (angular deflection per unit load) with the mean value plotted. Care was taken that the frequencies given by different admittance and cross-admittance curves for the same resonance coincided. If the frequencies did not coincide, the observations were repeated, paying particular attention to readings at and near resonances. It was then found that the frequencies for the same resonance did coincide.

4.2. *Effect of Suspension.*—As already stated in section 3.2.1, the airframe was originally supported on half-deflated tyres, and later on air bags. Figs. 7 and 8 show the two cross admittances measured in each case. The agreement between the two cross admittances for the former case is shown in Fig. 7 and the results were unsatisfactory for frequencies below 2000 c.p.m.; therefore all the airframe admittances were measured with the airframe supported on air bags.

4.3. *Modification of Admittances, With and Without Damping.*—The theory of the admittance method given in section 2 has been developed for simple systems without damping. The measured admittances were therefore modified to correspond to undamped systems. Figs. 9 and 10 show the admittances with damping removed for the airframe, and Figs. 12 and 13 the corresponding curves for the engine system.

Figs. 11 and 14 show cross-admittance curves for airframe and engine, using the measured values throughout, *i.e.*, retaining damping but with an abrupt 180 deg phase shift at resonance.

4.4. *Plotting of Frequency Equation.*—Repeating equation (8)

$$\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}} = \frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}} .$$

This expression is plotted in above form rather than with cross-multiplied terms, to retain similarity with single parameter cases and to reduce the number of multiplication or division steps.

In plotting this expression for the undamped case, a mathematical difficulty is introduced which gives rise to indefinite intersections for the ranges where the linear and cross admittance in numerator and denominator on one side of the frequency equation approach infinity along asymptotes of equal values of the frequency.

For the damped case, a number of intersections are lost, particularly if any modification is made to the system, *e.g.*, inclusion of flexibility. To overcome both difficulties, and to facilitate the computing, the linear expressions were plotted undamped, and for the cross admittances the damped values were taken. This assumption is justified by the agreement between predicted values and values obtained from a resonance test. Fig. 15 gives the sum of the direct linear admittances of the two systems, Fig. 16 the sum of the direct angular admittances and Fig. 17 the sum of the cross admittances. In Fig. 18 the two sides of the frequency equation are plotted, the curves in full lines being a function of the angular expressions and the chain dotted curves a function of the linear expressions.

The frequencies for the case in which damping is retained are found in the same way: Fig. 20 shows the curves for the solution of the frequency equation. There is no difficulty in plotting the frequency equation in this case, however, as the admittances have no infinite values.

5. *Discussion of Results.*—5.1. *Effect of Damping.*—The calculated frequencies are shown in Table 2 for the two cases of damping removed at resonance for linear and angular admittances and damping retained. It can be seen that the effect of damping is slight, the most important effect being that the resonance of 2180 c.p.m. is omitted when damping is included.

5.2. *Comparison of Predicted and Observed Resonance Frequencies.*—The results of the standard resonance tests are given in Ref. 9 and the curves appropriate to the vertical and pitching modes of vibration are reproduced in Fig. 19 and recorded in Table 2, together with the calculated frequencies. It was necessary to make a detailed check on the curves of Fig. 19 to record minor resonances in addition to the obvious major resonances.

The comparison of the results from the two methods as given in Table 2 shows very good agreement. It will be noted that no results are given by the admittance method for frequencies below 1000 c.p.m. Above 1000 c.p.m. one frequency predicted by the admittance method could not be found in the resonance test results and one minor resonance obtained in the resonance test was not predicted by the admittance method. All the remaining results predicted by the admittance method were within 15 per cent of the observed resonance test values, and the majority within 5 per cent. There were slight changes in the conditions of the aircraft between the resonance and admittance tests, in the fitting of cowlings, amount of fuel, etc.

6. *Application of Admittance Technique to the Matching of Engine and Airframe.*—As an application of the method, a solution was obtained for the natural frequencies of a system with the same engine and airframe as before, but with a flexible mounting unit between the engine and airframe. The values taken for the flexibilities were the design values appropriate to a test 'decoupled' mounting which was to be fitted to the *Hurricane* airframe.

The vertical stiffnesses are 8,200 lb/in. and 3670 lb/in. for the front and rear feet respectively; the direct linear and angular admittances were increased by $84 \cdot 3 \times 10^{-6}$ in./lb and $2 \cdot 209 \times 10^{-6}$ radn/lb/in. respectively, but there was no increase in the cross-admittance terms. The admittance curves for the engine and the curves of the sums of the admittances for aircraft and engine were modified by transferring the origin, and the frequency equation was solved graphically with damping included and without damping in linear and angular admittance terms as shown in Figs. 20 and 21. The curves of Fig. 18 are repeated in Fig. 21 to show the effect of the flexible mounting.

From Fig. 21 it can be seen that the relatively large change in the direct linear admittance has produced the greatest changes in the frequencies at 1745, 4600 and 5770 c.p.m. which have been altered to 1430, 4200 and 5190 c.p.m. respectively. If resonances at 1745, 4600 or 5770 c.p.m. should be undesirable, therefore, they could readily be changed by a flexible mounting unit similar to the one investigated. Should any other resonance be dangerous, then the admittance having the greatest influence on the frequency can be ascertained, and the most suitable changes in the flexibility can be made.

In Table 3 the frequencies for the flexibly mounted engine are given, with and without damping. The changes in frequency are again slight, but nine fewer intersections are obtained for the solution with damping.

The comparison of these results indicates that it is safer to use the solution with damping removed in linear and angular admittances in the regions of resonance, as certain resonances may be overlooked if damping is included.

7. *Conclusions.*—The application of admittance methods to the determination of the natural frequencies of a full-scale complex system with equal and opposite shearing forces and bending moments at the intersection, in particular the join between engine and airframe of an aircraft, gave good agreement between predicted and observed values of the natural frequency. Care had to be taken during the experiments that the natural frequency for all degrees of freedom of the engine and airframe on their suspensions were much lower than the exciting frequency.

The technique is applicable to the matching of an engine to an airframe by adjusting the flexibility of the mounting units. For a particular investigation the admittance having most influence on the appropriate intersection can be ascertained and changes made to adjust the natural frequencies as desired. The maximum change that can be achieved in the frequency will decide whether a change in flexibility is practicable, or whether a major structural modification is required.

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TABLE 1

Details of Aircraft-Engine-Propeller System

Aircraft	Hurricane IIC	Z.3564		
		Weight	7000 lb (approx.)	
		Wing Span	40 ft 0 in.	
		Length	32 ft 3 in.	
		Construction	Fuselage	Tubular with fabric covering
			Wings	Stressed skin
Engine				
	Merlin XX			
		Weight	1430 lb (dry)	
		Rating	1240 b.h.p.	
Propeller				
	Rotol Mk. R.S.S./3			
		Diam.	10 ft 10 in.	
		Weight	283 lb	
		Number of blades	3	

TABLE 2

Natural Frequencies of Airframe-Engine-Propeller System with Rigidly Mounted Engine

Comparison of Admittance and Resonance Test Results

Natural Frequencies from Solution of Equation in Admittance Test		Natural Frequencies from Resonance Test	
Damping removed at resonance	Damping included	Major resonance	Minor resonance
c.p.m.	c.p.m.	c.p.m.	c.p.m.
		595	
		760	
			900
			990
1050—			
1150	1070		
1130	1130		
1745	1730		
1990	1990	1940	
2025—			
2130	2145	2130	
2180	—	2260	
2410	2400		
			2500
			2700
2900	2900	2940	
3180	3180		3160
3320	3270		3360
3550	3550		3500
3700	3700		3840
4140	4100		4200
4600	4600		4800
5770	5730		5760

TABLE 3

Natural Frequencies of Airframe-Engine-Propeller System with Flexibly Mounted Engine

Natural Frequencies from Solution of Equation in Admittance Test	
Damping removed at resonance	Damping included
c.p.m.	c.p.m.
1110	
1430	1420
2160	
2520	
2780	2770
3320	
3450	
3530	3510
3710	
4030	
4200	4200
5190	
5880	

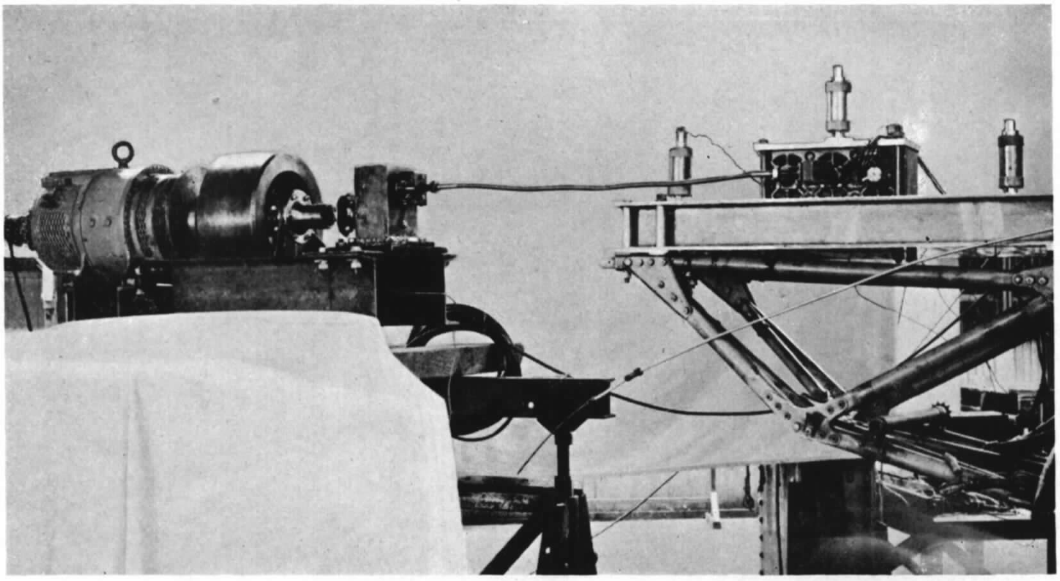

FIG. 1. Support for exciter and vibration pick-ups in admittance experiments.

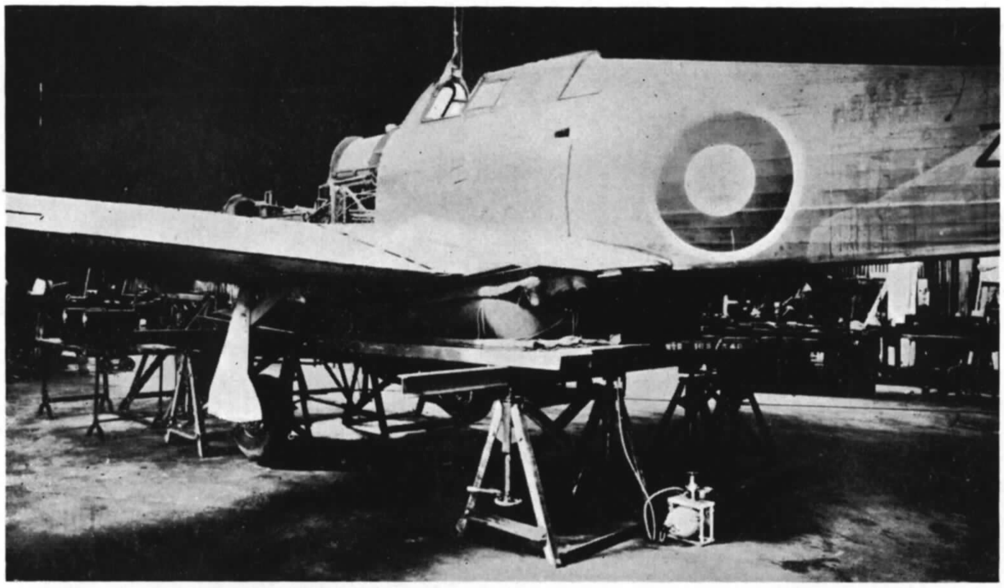

FIG. 2. Airframe suspended on air bags in admittance experiments.

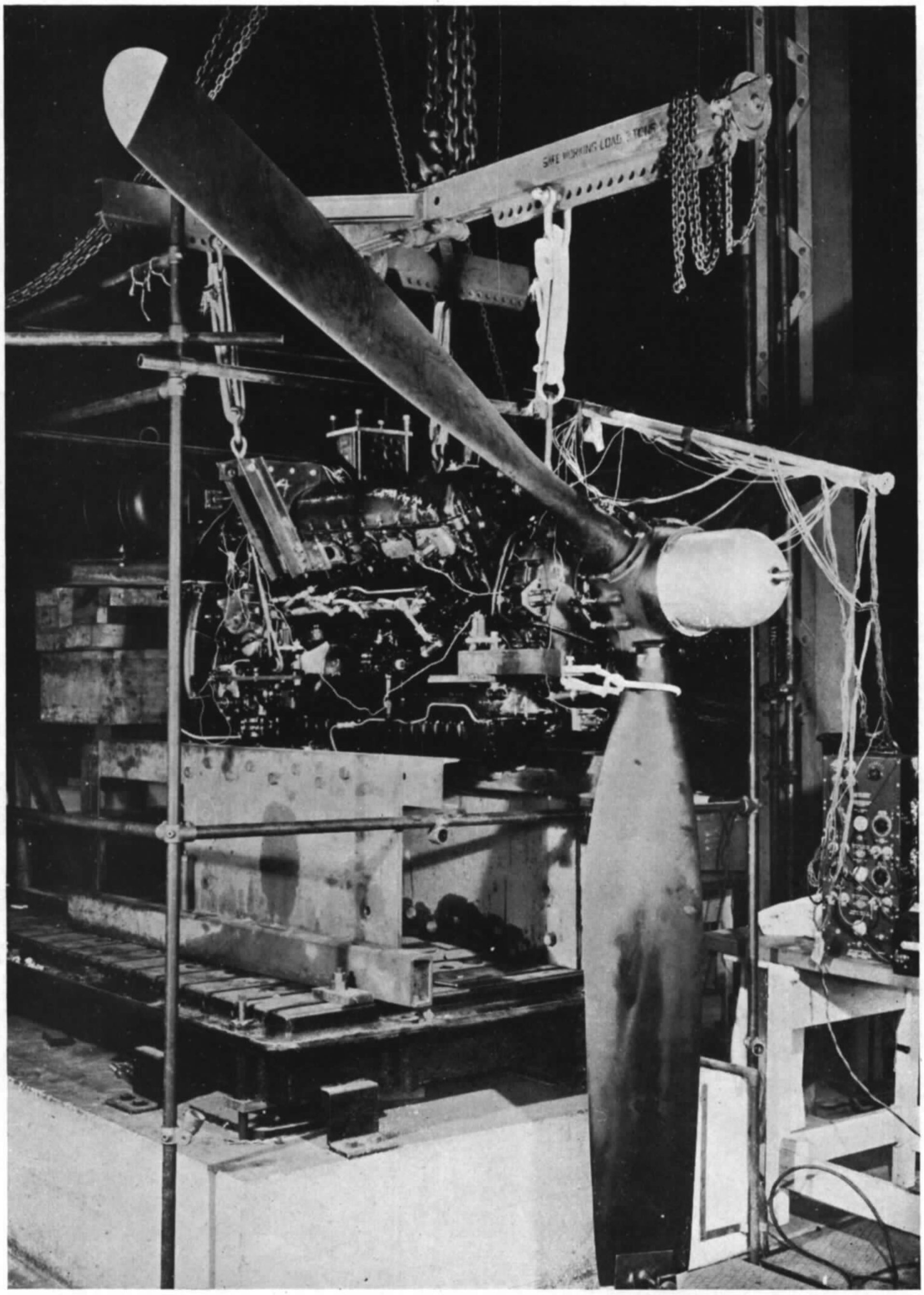

FIG. 3. Method of suspension of engine in admittance experiments.

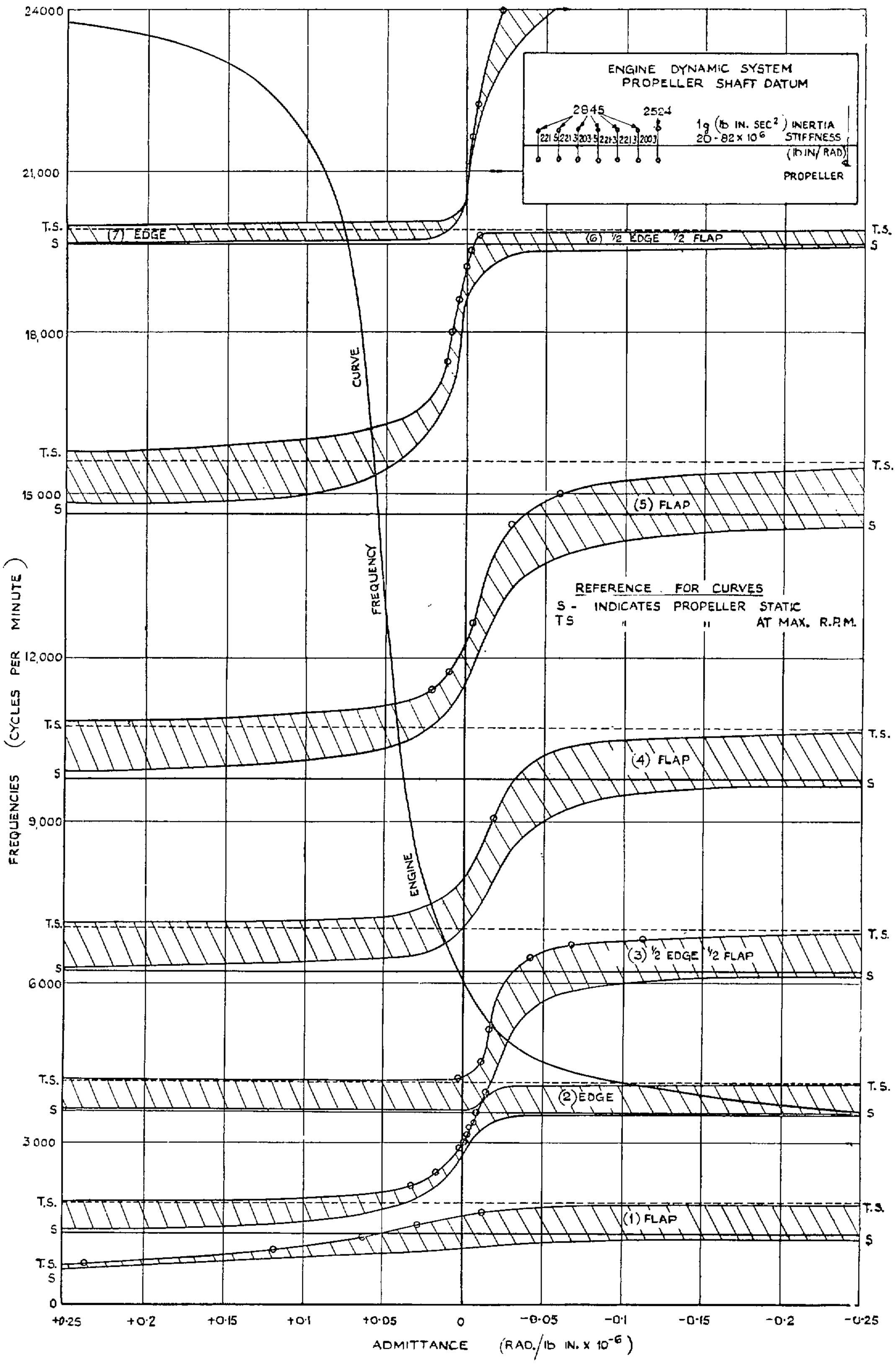

FIG. 4. Admittance curves for force system of single parameter at intersection. *Sabre* engine with 14-ft diameter 3-blade hydromatic propeller.

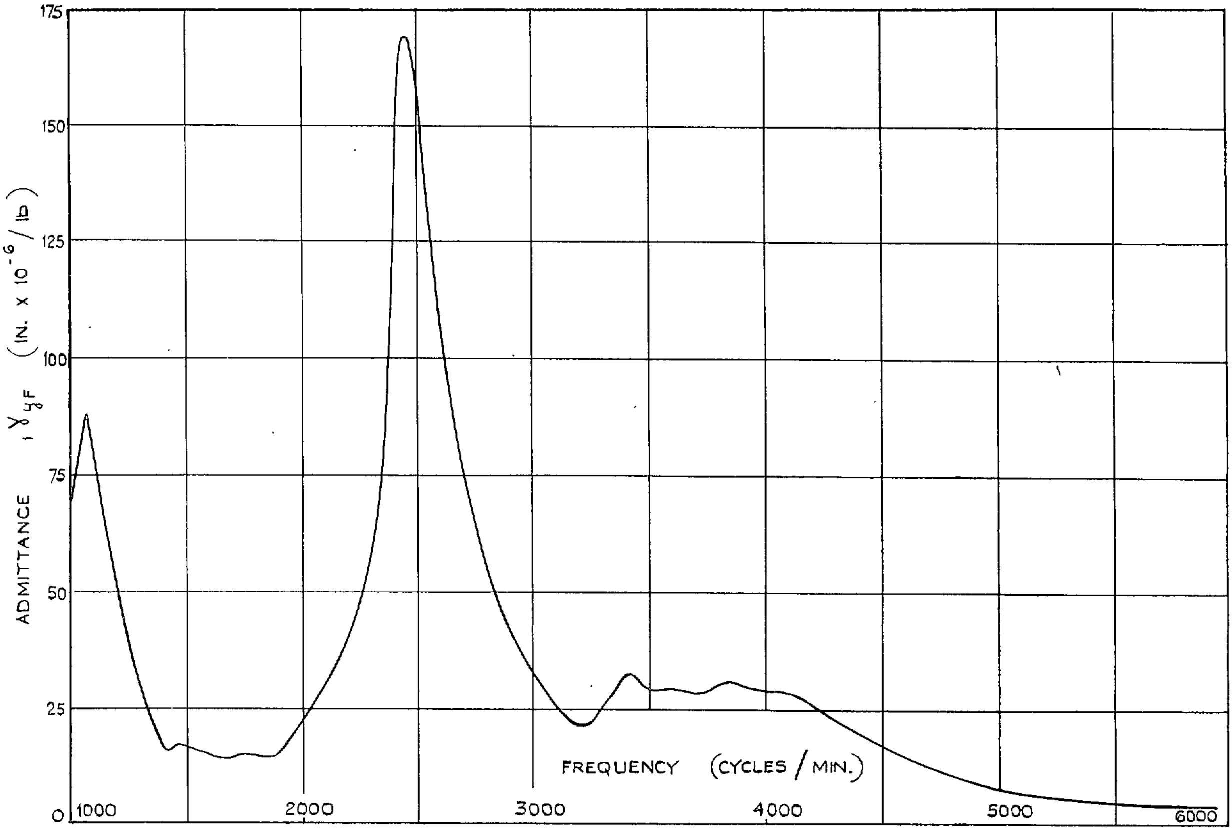

FIG. 5. Admittance measured on airframe. Linear deflection per unit load.

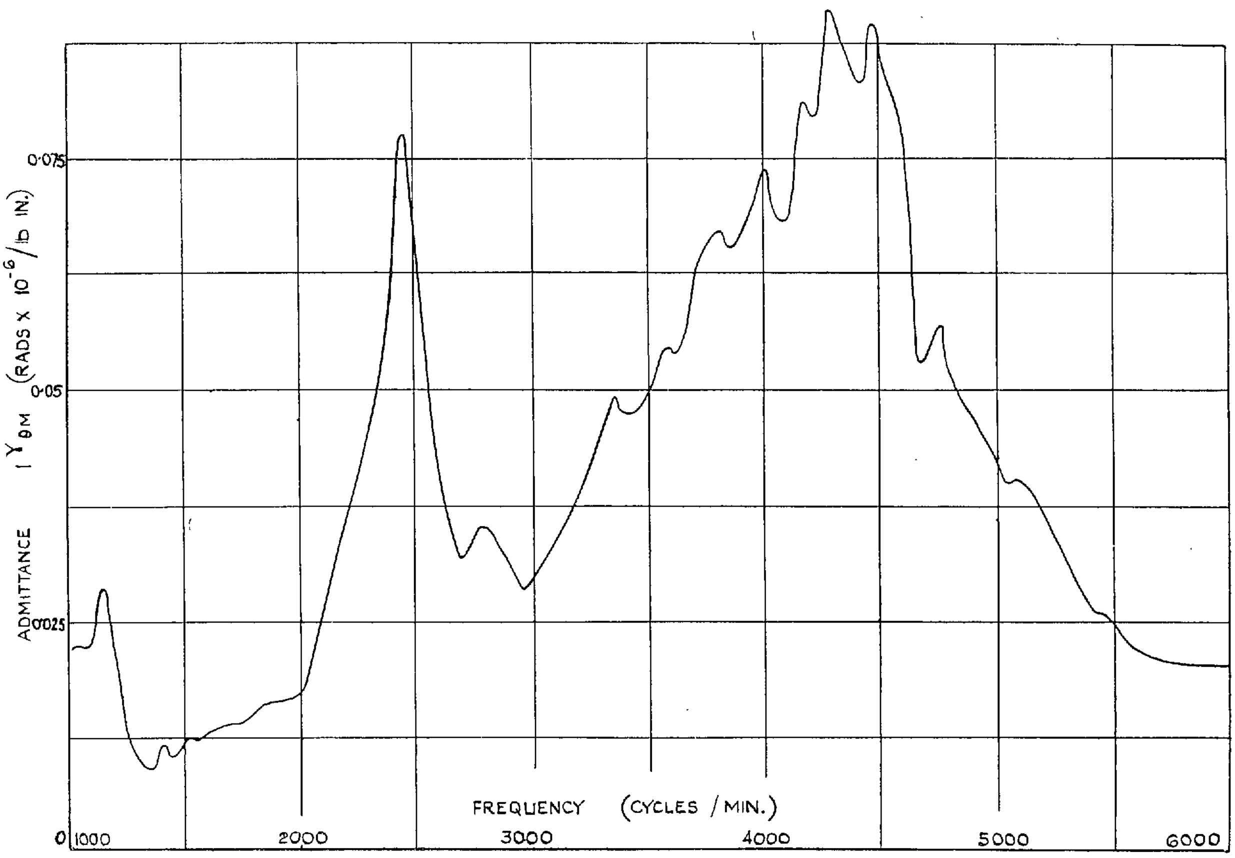

FIG. 6. Admittance measured on airframe. Angular deflection per unit moment.

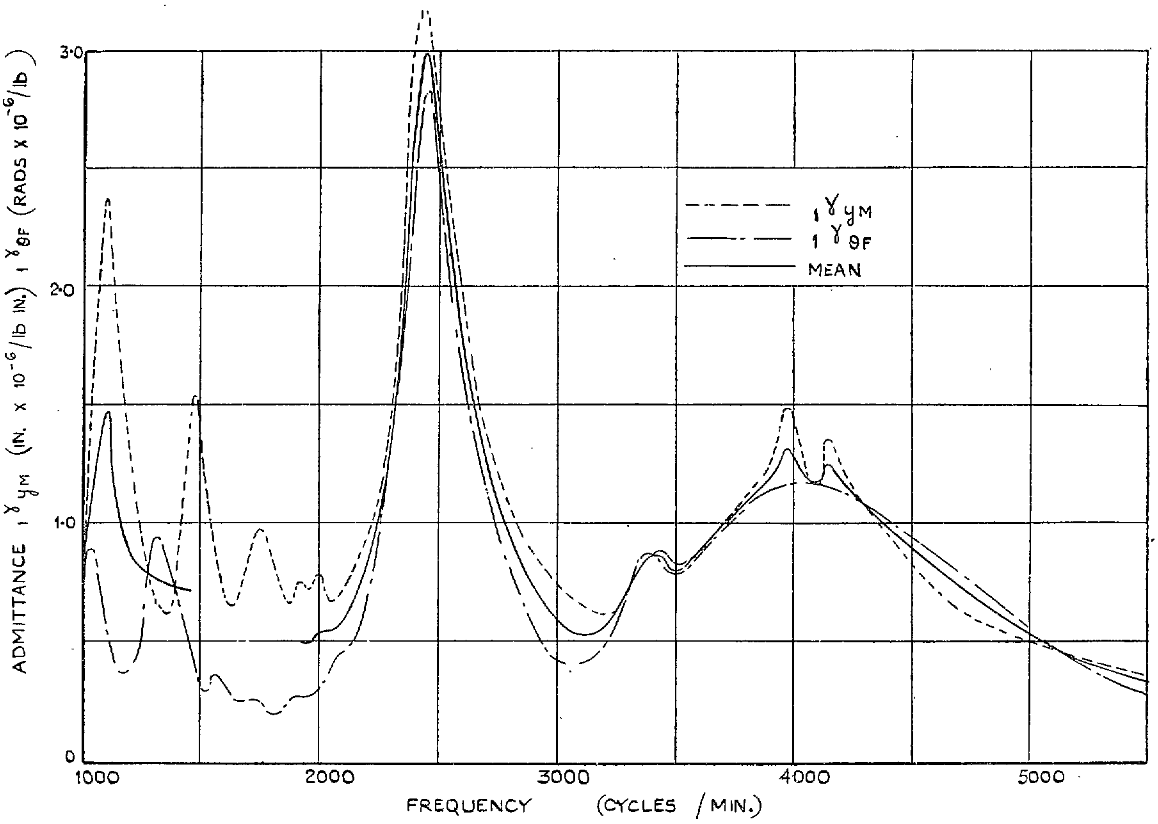

FIG. 7. Cross admittance measured on airframe supported on part-deflated tyres. Linear deflection per unit moment. Angular deflection per unit load.

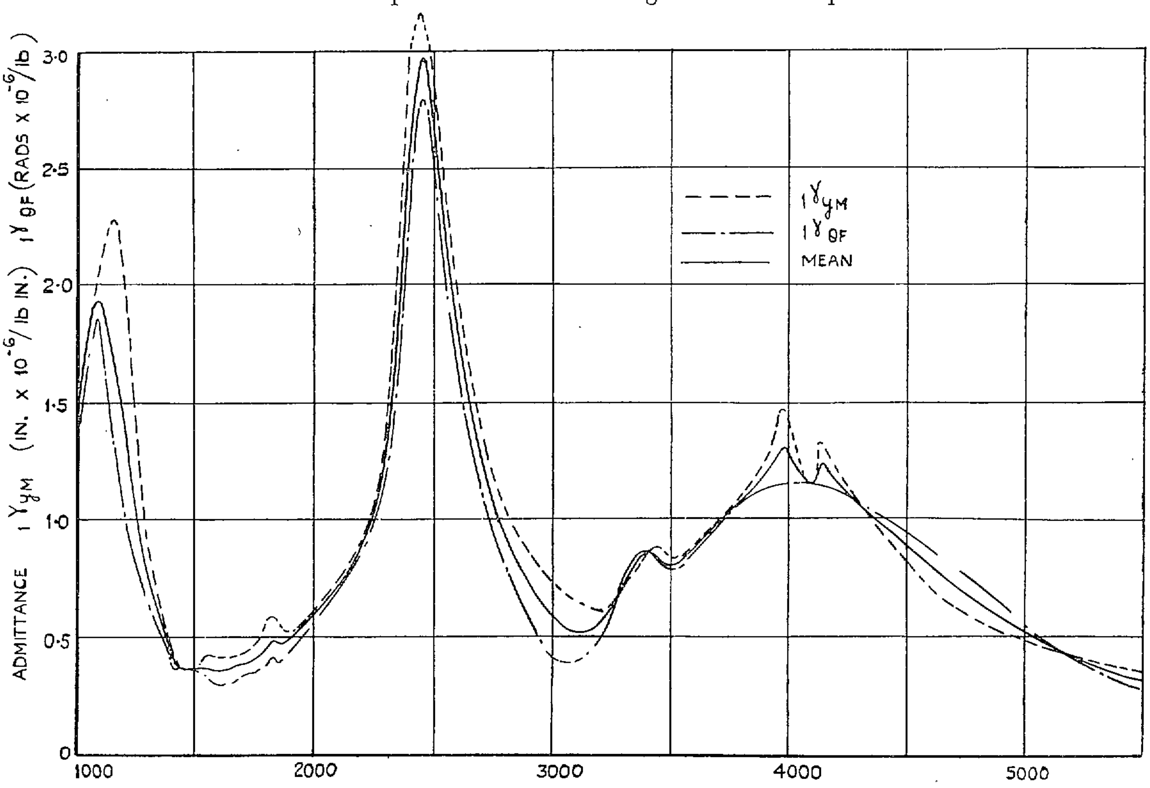

FIG. 8. Cross admittance measured on airframe supported on air bags. Linear deflection per unit moment. Angular deflection per unit load.

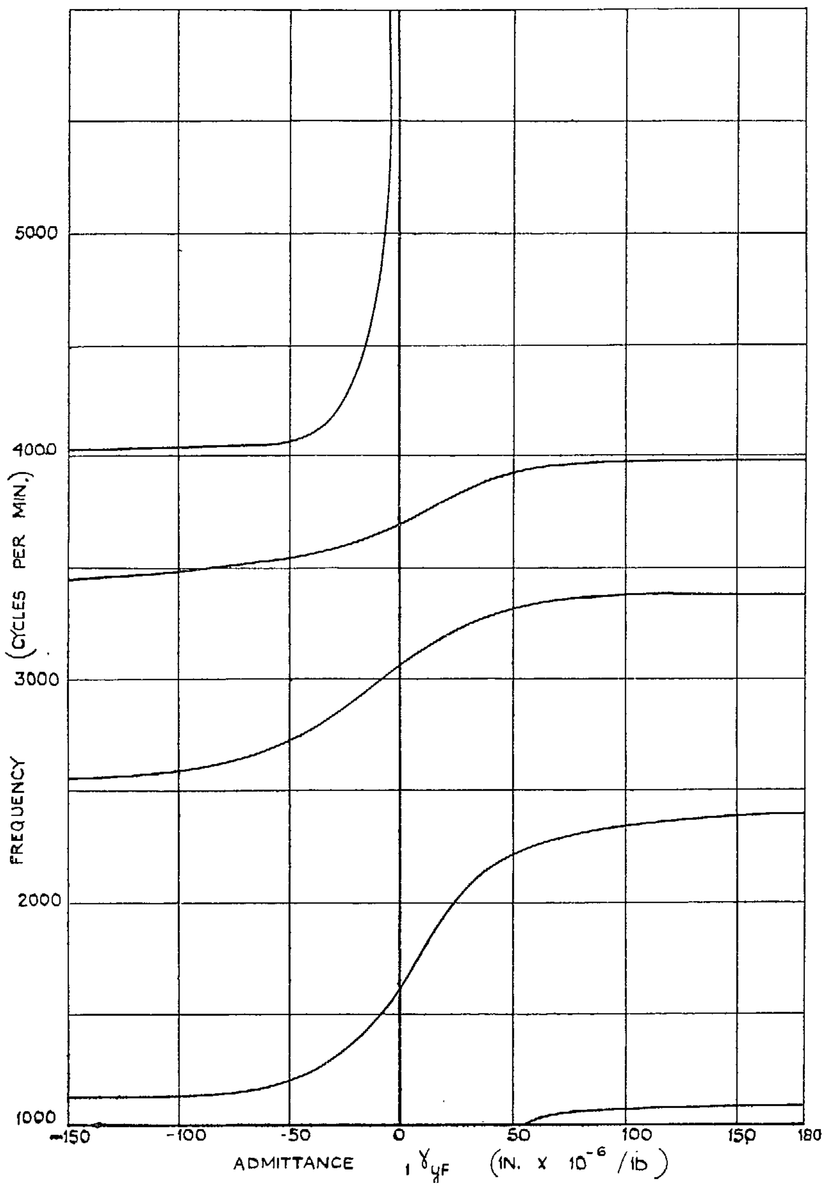

FIG. 9. Admittance for airframe with damping removed. Linear deflection per unit load.

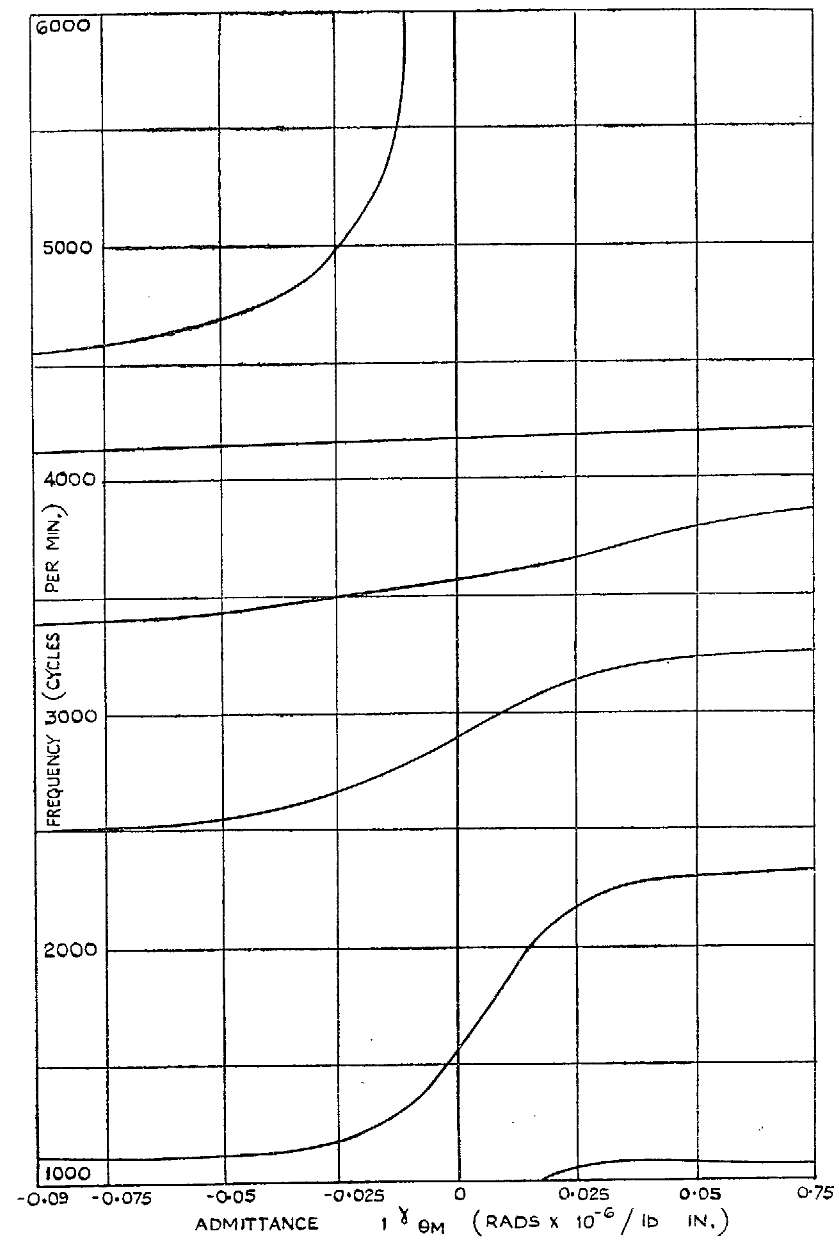

FIG. 10. Admittance for airframe with damping removed. Angular deflection per unit moment.

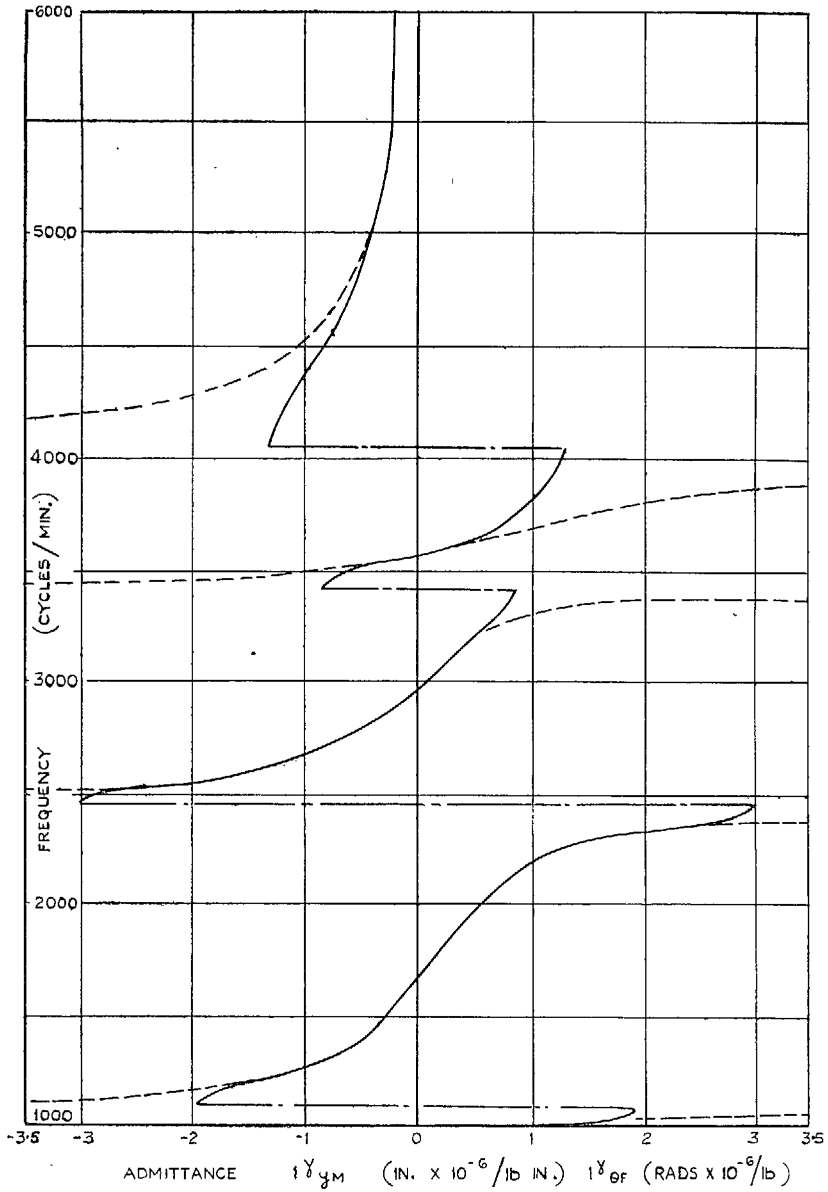

FIG. 11. Cross admittance for airframe. Linear deflection per unit moment. Angular deflection per unit load.

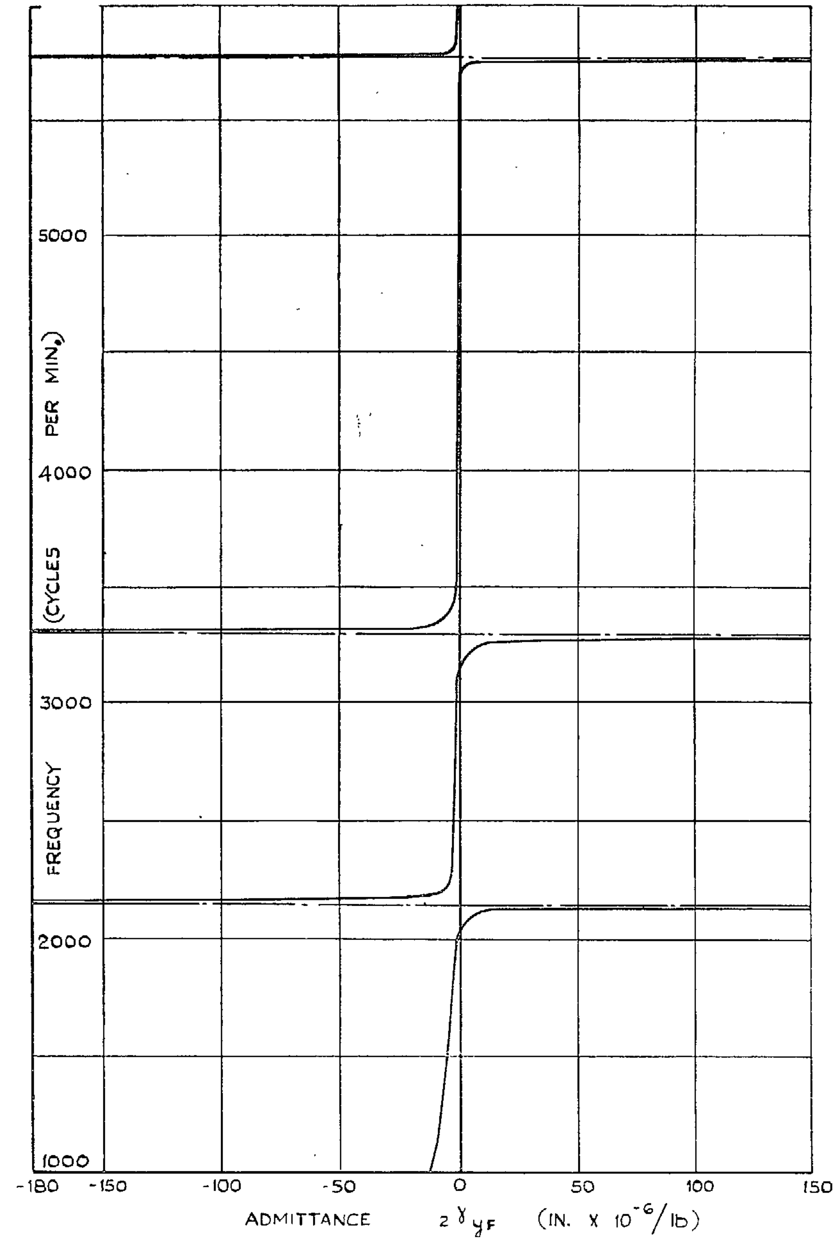

FIG. 12. Admittance for engine with damping removed. Linear deflection per unit load.

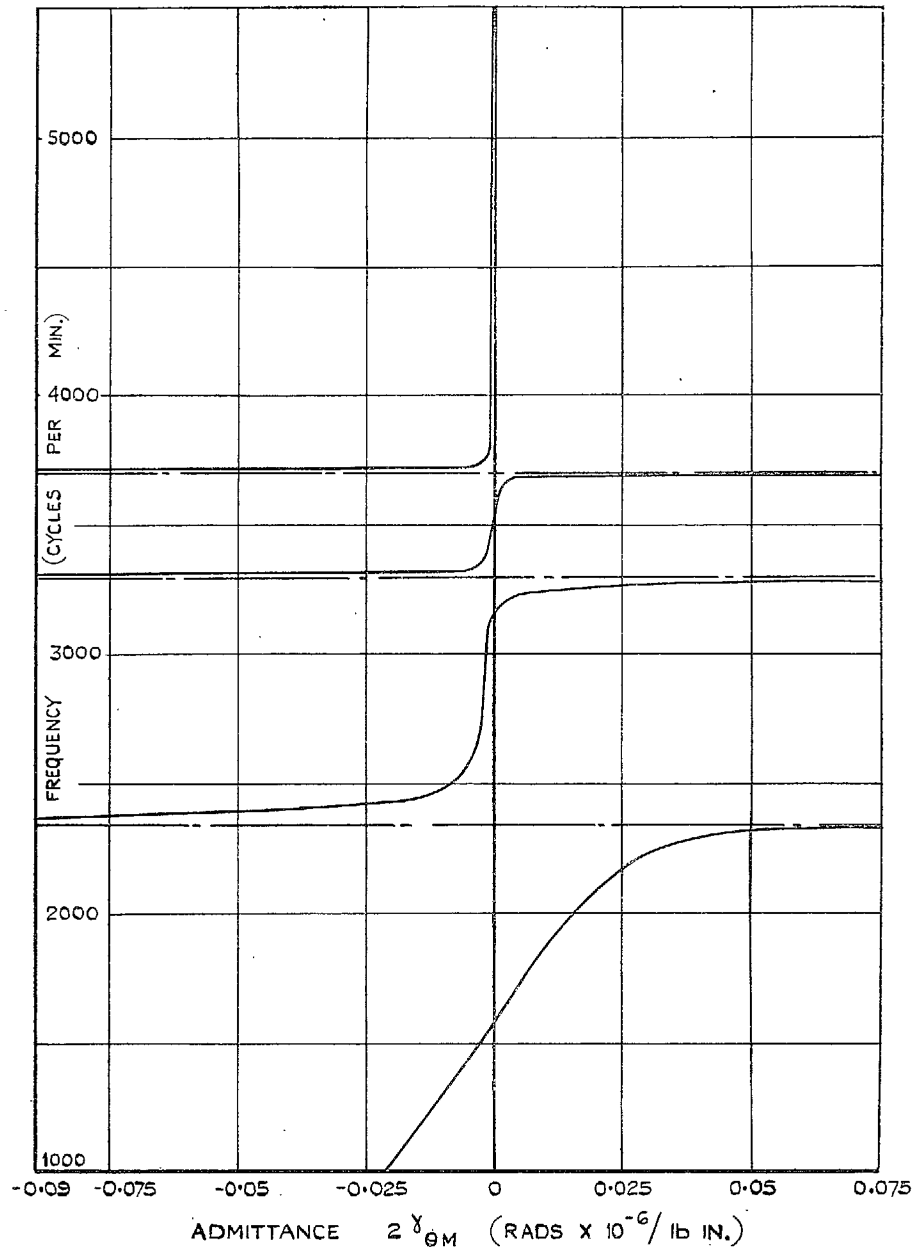

FIG. 13. Admittance for engine with damping removed. Angular deflection per unit couple.

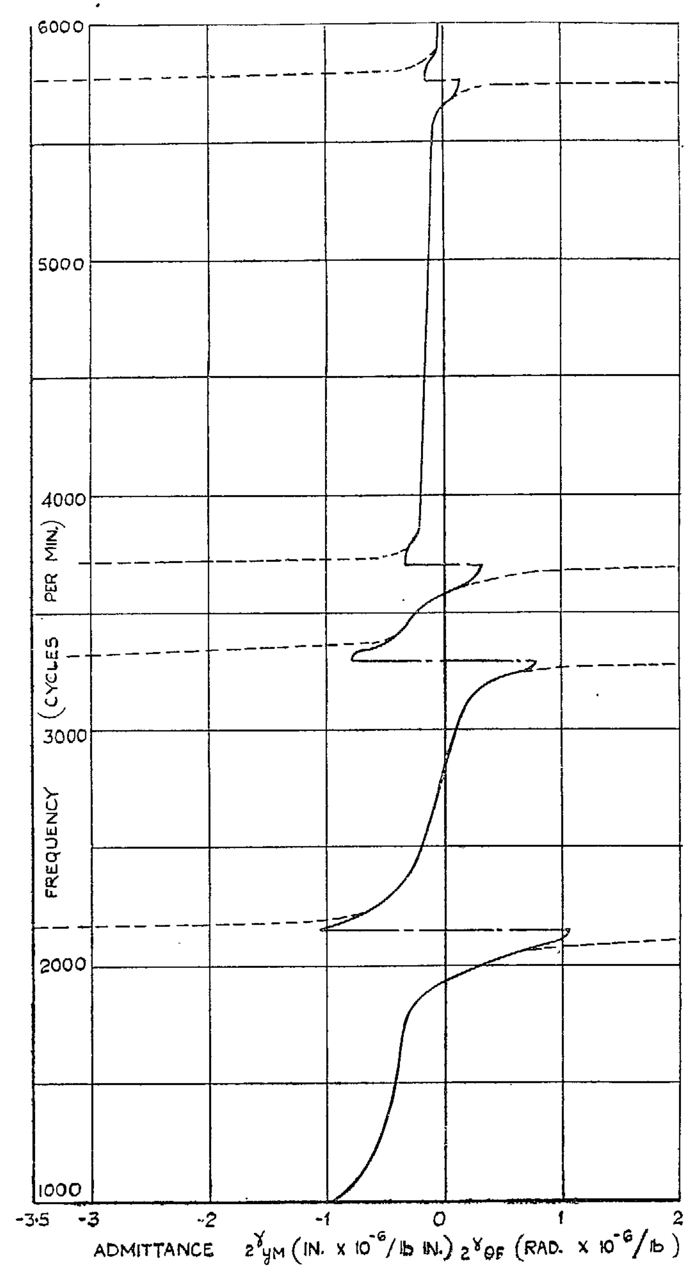

FIG. 14. Cross admittance for engine with 180 deg phase shift at resonance. Linear deflection per unit couple. Angular deflection per unit load.

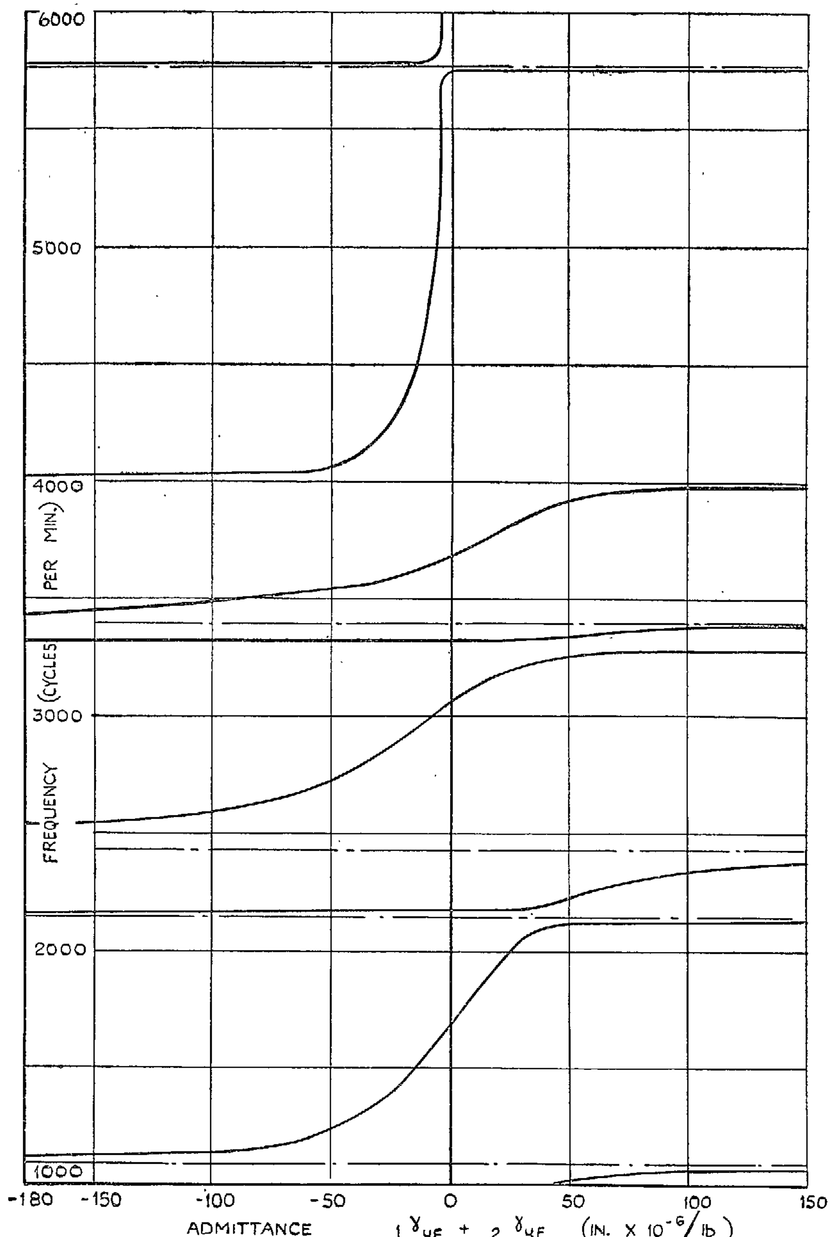

FIG. 15. Sum of admittances for airframe and engine with damping removed. Linear deflection per unit load.

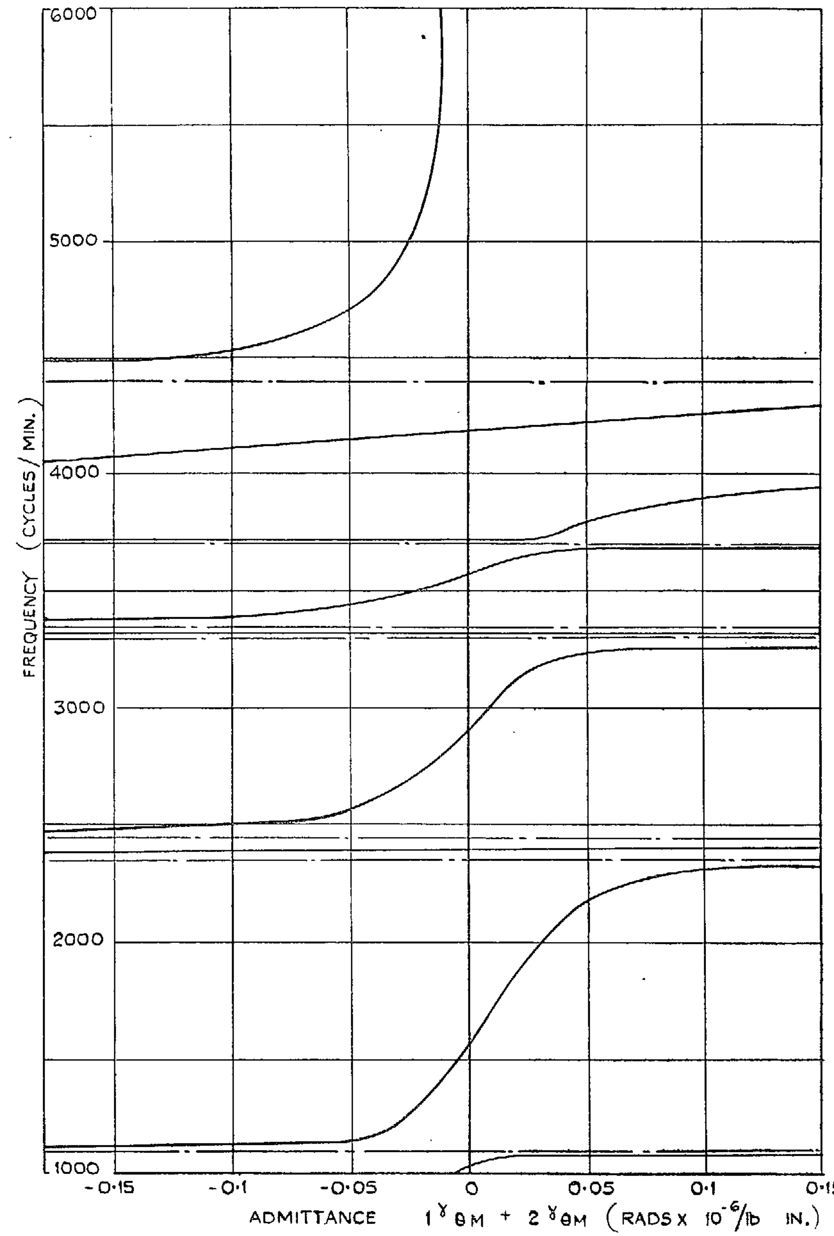

FIG. 16. Sum of admittances for airframe and engine with damping removed. Angular deflection per unit moment.

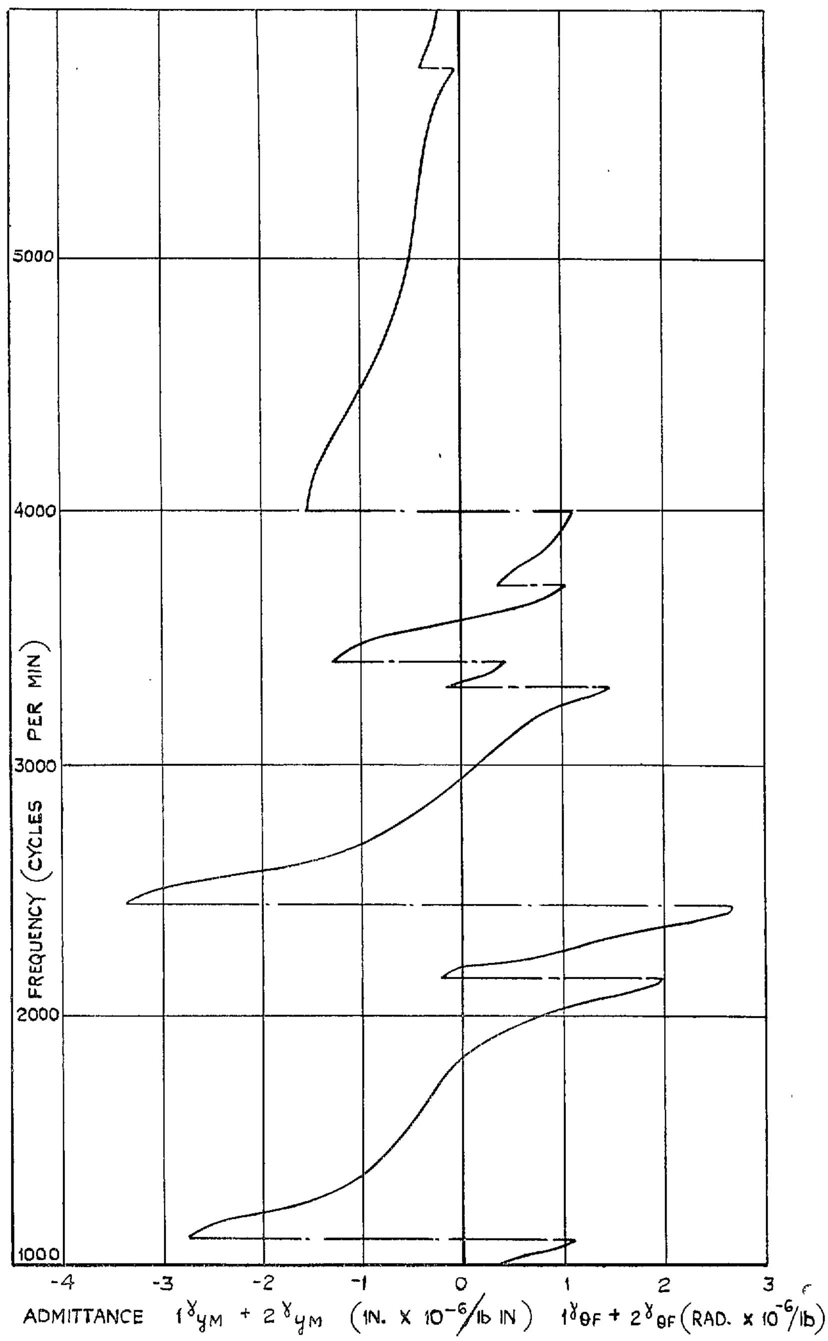

FIG. 17. Sum of cross admittances for airframe and engine with 180-deg phase shift at resonance. Linear deflection per unit moment. Angular deflection per unit load.

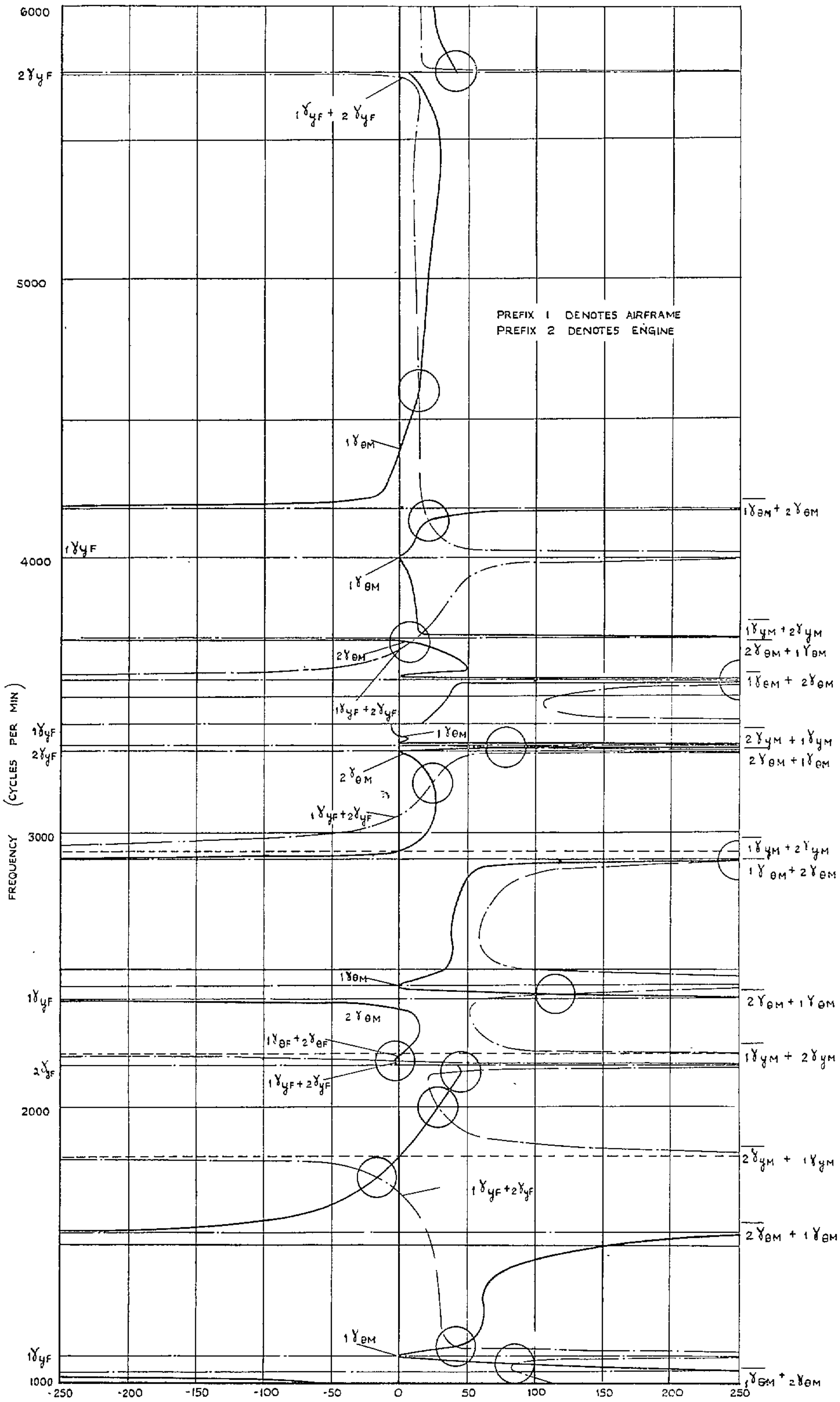

FIG. 18. Solution of frequency equation with damping removed for linear and angular admittances.

$$\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}} \begin{pmatrix}\text{curve shown}\\ \text{chain dotted}\end{pmatrix} = \frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}} \begin{pmatrix}\text{curve shown}\\ \text{in full}\end{pmatrix}$$

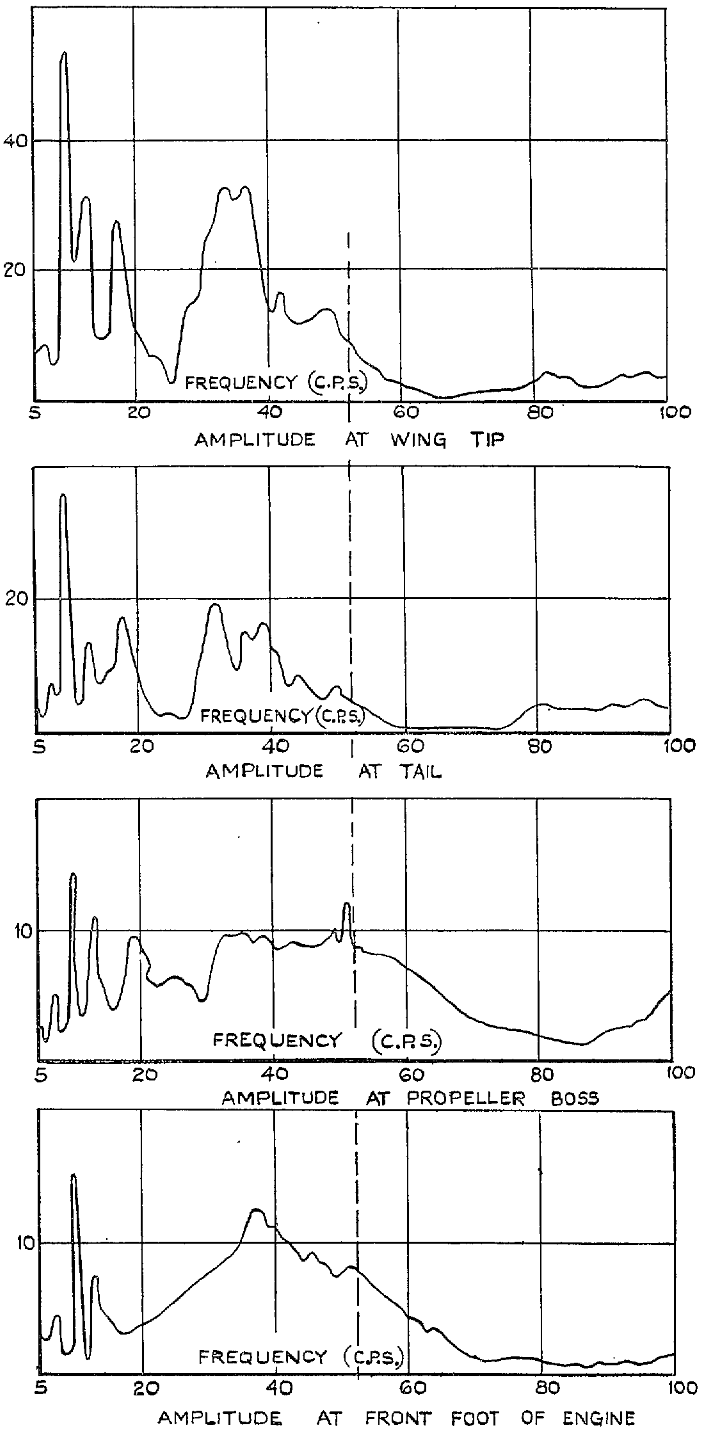

FIG. 19. Amplitude-frequency curves for airframe-engine system from resonance test. (Vertical excitation at propeller hub.)

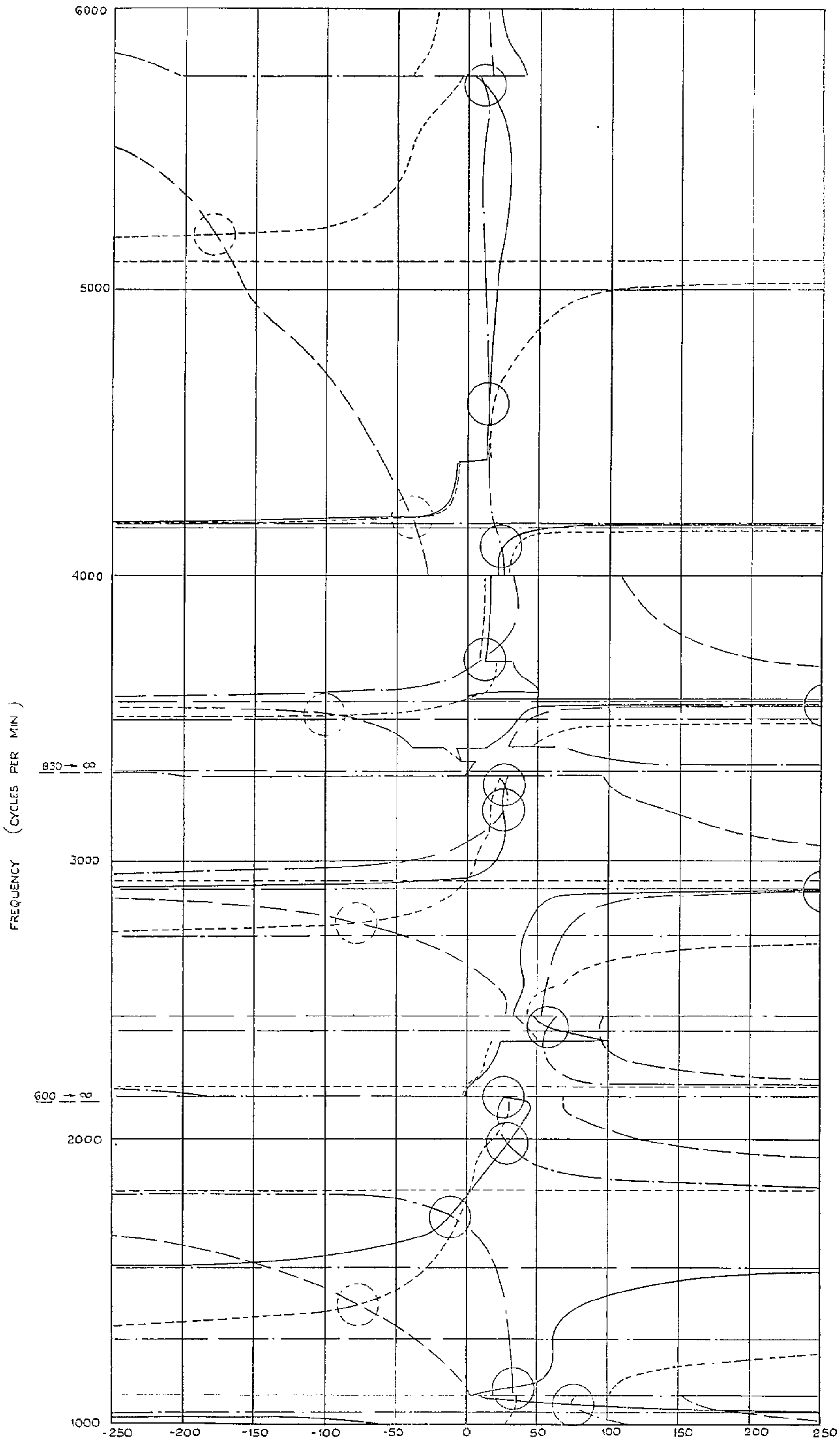

FIG. 20.

Solution of frequency equation $\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}}$ (curve shown chain dotted) $=$ $\frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}}$ (curve shown in full) with damping included for rigid engine mounting.

Solution of frequency equation $\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}}$ (curve shown in dashes) $=$ $\frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}}$ (curve shown close dotted) with damping included for rigid engine mounting.

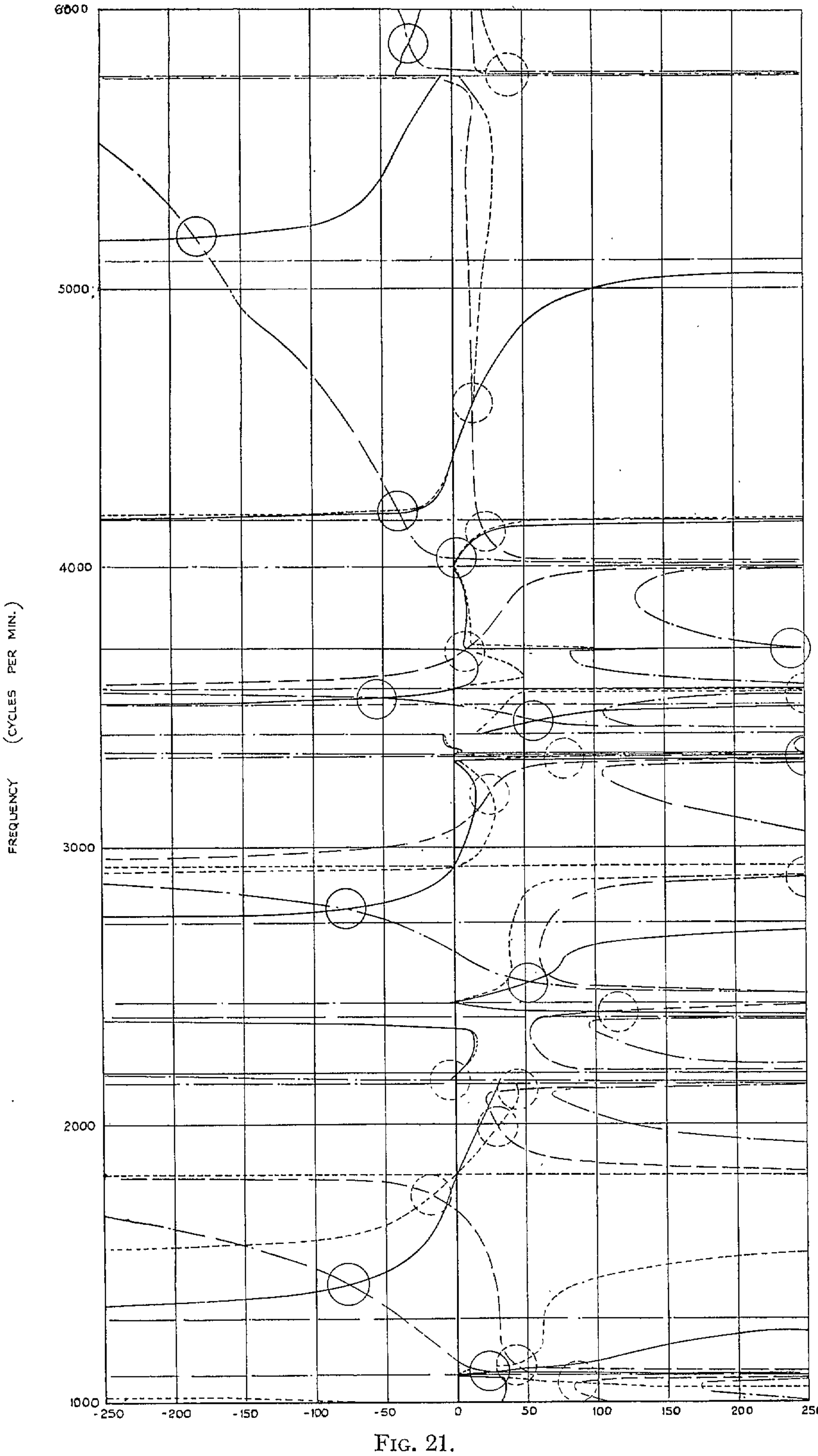

FIG. 21.

Solution of frequency equation $\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}}$ (curve shown chain dotted) $=$ $\frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}}$ (curve shown in full) with damping removed for linear and angular admittances for flexibly mounted engine.

Repeat of solution given in Fig. 18 $\frac{{}_1\gamma_{yF} + {}_2\gamma_{yF}}{{}_1\gamma_{yM} + {}_2\gamma_{yM}}$ (curve shown in dashes) $=$ $\frac{{}_1\gamma_{\theta F} + {}_2\gamma_{\theta F}}{{}_1\gamma_{\theta M} + {}_2\gamma_{\theta M}}$ (curve shown close dotted) with damping removed for linear and angular admittances for rigidly mounted engine.

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