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On Fundamental Sets of Solutions of the Equations of Lateral Motion, and the Rapid Calculation of General Solutions

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Summary.—The lateral motion of a symmetrical aeroplane slightly disturbed from steady flight is determined, to the first order of small quantities, by the solution of a system of six simultaneous linear differential equations with constant coefficients, in which the inhomogeneous terms, representing control forces or the effects of gusts, may be arbitrary functions of time. In virtue of the general properties of such equations, as is well known, their most general solution can always be written down in a form involving definite integrals. Calculations of such theoretical expressions can be very tedious, and it is now shown that the most general solution can be much more simply obtained, by processes of addition, multiplication, and integration, from a set of three fundamental solutions. A large number of such sets of fundamental solutions has already been obtained by means of the differential analyser, and the application to these of the methods of this report will make possible a large range of more special response calculations, some of which may well develop into important matters of routine.

After an introductory statement of the equations of motion, the three fundamental solutions are defined in sect. 3.1, with four further solutions which are conveniently regarded as fundamental, though they can be derived from the original three. Relations between these seven solutions are given in sects. 3.2 to 3.6. Sect. 4 is concerned with the derivation of other solutions corresponding to constant or piecewise constant disturbances, and generalisation to disturbances given as any functions of time is made in sect. 5. A few particular examples of the technique developed are given in sect. 6, the fundamental solutions used being chosen from the differential analyser results mentioned above. A brief account of the scope of these is given in an Appendix, which includes in tabular form an index to the complete series of 1188 figures in which the results are contained.

1. *Introduction.*—During the period from Dec., 1943 to Feb., 1944, a large programme of calculations referring to the lateral response of aeroplanes was carried out on the differential analyser at Manchester University, by the author and collaborators†. One report on these results has so far been written, by Mitchell, Thorpe and Frayn[1] (1944), and the " full set of curves " referred to therein has also been reproduced. The potential usefulness of the curves obtained in the whole programme is, however, so great that it has been decided to make the complete collection of results available on loan as soon as possible, without waiting for the issue of individual reports analysing the various aspects of the work. The present report has been written with the twofold purpose of making known the existence and scope of the differential analyser results, and of indicating how more general results can be deduced, and in particular how the curves can be used to facilitate certain important types of routine calculation.

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†A. W. Thorpe and Miss E. M. Frayn, for the whole programme, with Miss M. M. Dent and F. G. H. Jones for the parts listed respectively as (i) to (iii), and (iv), in the Appendix.

2. *Equations of Motion.*—The equations of lateral motion of a symmetrical aeroplane slightly disturbed from steady motion may be written

$$\left.\begin{aligned}
&\left(\frac{d}{d\tau} - y_v\right)\hat{v} && && + \left(1 - \frac{y_r}{\mu_2}\right)\hat{r} - k\phi = \tfrac{1}{2}C_y(\tau) + y_v\hat{v}_G(\tau)\,, \\
&-\frac{\mu_2 l_v}{i_A'}\hat{v} && + \left(\frac{d}{d\tau} - \frac{l_p}{i_A'}\right)\hat{p} && + \left(-\frac{i_E}{i_A'}\cdot\frac{d}{d\tau} - \frac{l_r}{i_A'}\right)\hat{r} = \frac{\mu_2}{i_A'}C_l(\tau) + \frac{\mu_2 l_v}{i_A'}\hat{v}_G(\tau)\,, \\
&-\frac{\mu_2 n_v}{i_C'}\hat{v} && + \left(-\frac{i_E}{i_C'}\cdot\frac{d}{d\tau} - \frac{n_p}{i_C'}\right)\hat{p} && + \left(\frac{d}{d\tau} - \frac{n_r}{i_C'}\right)\hat{r} = \frac{\mu_2}{i_C'}C_n(\tau) + \frac{\mu_2 n_v}{i_C'}\hat{v}_G(\tau)\,, \\
& && -\hat{p} && - \tan\gamma_e\,.\,\hat{r} + \frac{d\phi}{d\tau} = 0\,, \\
& && && - \sec\gamma_e\,.\,\hat{r} + \frac{d\psi}{d\tau} = 0\,, \\
&-\hat{v} && && - \cos\gamma_e\,.\quad \psi + \frac{d\hat{y}}{d\tau} = 0\,.
\end{aligned}\right\} \quad (1)$$

The notation used is as follows:—

τ denotes time measured in airsecs, the dimensionless unit equal to $\hat{t}$ true seconds, where

$$\hat{t} = \frac{m}{\rho S U_e}\,, \quad \ldots \quad (2)$$

S being the wing area (sq. ft.), U_e the relative velocity in steady motion (ft./sec.), m the mass of the aeroplane (slugs), and ρ the density of the air (slugs/cu. ft.) :

γ_e is the angle of climb in steady motion :

ϕ, ψ, are the angles of bank and azimuth in the disturbed motion, in radians :

$\hat{v} + \hat{v}_G(\tau)$ is the relative velocity of sideslip in disturbed motion, taking U_e as unit, split up into a part $\hat{v}$ due to sideways velocity of the aeroplane relative to a fixed datum, and a part $\hat{v}_G(\tau)$ due to change in the velocity of the local air (*i.e.* due to a gust velocity $-\hat{v}_G(\tau)$). Since the equations are valid for small disturbances only, $\hat{v} + \hat{v}_G(\tau)$ can also be interpreted as the angle of sideslip in the disturbed motion :

$\hat{p}$, $\hat{r}$, are angular velocities in bank and yaw (rad./airsec.) :

$\hat{y}$ is the sideways displacement of the aeroplane consequent upon disturbance, in units $U_e\hat{t}$ ft. :

y_v, l_v, n_v, l_p, n_p, l_r, n_r are the usual dimensionless lateral stability derivatives ; i_A', i_C' the dimensionless moment of inertia coefficients ; μ_2 the lateral relative density, $2m/(\rho S b)$, where b is wing span (ft.) ; and $k = \frac{1}{2}C_L$; all in the notation of Bryant and Gates[2] (1937) : the additional derivative y_r (usually neglected) has been added, given in terms of its natural counterpart by

$$y_r = \frac{Y_r}{\rho S b}\,, \quad \ldots \quad (3)$$

where $Y_r r$ is the sideforce in lb. wt. due to a rate of yaw r rad./sec. We have also added the symbol i_E,

$$i_E = \frac{4E}{m b^2}\,, \quad \ldots \quad (4)$$

where E is the product of inertia (slugs ft. squared) with respect to the axes of roll and yaw :

$C_y(\tau)$, $C_l(\tau)$, $C_n(\tau)$ are the dimensionless coefficients of applied sideforce, rolling moment, and yawing moment.

Solutions of equations (1) have been calculated by means of the differential analyser, using initial conditions appropriate to three particular types of disturbance, for a large range of numerical values of the parameters. The full scheme is given in the Appendix. Our purpose in the text of this report is to show how solutions corresponding to arbitrary disturbances can be deduced. For this purpose a less cumbersome notation is to be desired, and we have used, in the remainder of the text, the modified notation of Mitchell[3] (1943), and have further taken i_E and γ_e as zero*. With these changes the equations (1) become

$$\left.\begin{aligned}
&\left(\frac{d}{d\tau}+\bar{y}_v\right)\hat{v} + \left(1-\frac{y_r}{\mu_2}\right)\hat{r} - k\phi = \mathscr{C}_y(\tau) - \bar{y}_v\hat{v}_G(\tau)\,,\\
&\mathscr{L}\,\hat{v} + \left(\frac{d}{d\tau}+l_1\right)\hat{p} - l_2\hat{r} = \mathscr{C}_l(\tau) - \mathscr{L}\hat{v}_G(\tau)\,,\\
&-\mathscr{N}\,\hat{v} + n_1\hat{p} + \left(\frac{d}{d\tau}+n_2\right)\hat{r} = \mathscr{C}_n(\tau) + \mathscr{N}\hat{v}_G(\tau)\,,\\
&-\hat{p} + \frac{d\phi}{d\tau} = 0\,,\\
&-\hat{r} + \frac{d\psi}{d\tau} = 0\,,\\
&-\hat{v} - \psi + \frac{d\hat{y}}{d\tau} = 0\,.
\end{aligned}\right\} \quad \ldots \quad (5)$$

The meanings of the new symbols can be inferred, on comparing (5) and (1) : *see* also sect. A3, and the list of symbols.

3. *Relations between Fundamental Solutions.*—3.1. *Fundamental Solutions.*—The differential analyser results are the solutions of the equations (1) or (5) for three particular combinations of initial conditions and applied moments, and for various combinations of values of the stability parameters. We shall now adopt a matrix notation, writing

$$X = \begin{Bmatrix} \hat{v} \\ \hat{p} \\ \hat{r} \\ \phi \\ \psi \\ \hat{y} \end{Bmatrix}, \quad \ldots \quad (6)$$

and express these solutions in the forms

$$X = X_v(\tau), \begin{cases} \hat{v} = 1, \hat{p} = \hat{r} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = 0, & \text{throughout} \end{cases}, \quad \ldots \quad (7)$$

$$X = X_l(\tau), \begin{cases} \hat{p} = \hat{v} = \hat{r} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = \mathscr{C}_n = 0,\ \mathscr{C}_l = 1, & \text{throughout}\dagger \end{cases}, \quad \ldots \quad (8)$$

$$X = X_n(\tau), \begin{cases} \hat{p} = \hat{v} = \hat{r} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = \mathscr{C}_l = 0,\ \mathscr{C}_n = 1, & \text{throughout}\dagger \end{cases}. \quad \ldots \quad (9)$$

*Formulæ valid in the more general case when these parameters are not zero can easily be written down.

†Some of the results (*see* Appendix) use other values than unity for $\mathscr{C}_l$ and $\mathscr{C}_n$.

These solutions form a fundamental set, enabling us to find (by elementary processes and quadratures) the solution corresponding to arbitrary initial conditions and applied forces and moments.

It is convenient to regard as fundamental four further solutions :—

$$X = X_v(\tau), \left\{ \begin{array}{ll} \hat{p} = \hat{v} = \hat{r} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = 1,\ \mathscr{C}_l = \mathscr{C}_n = 0, & \text{throughout} \end{array} \right\}, \qquad (10)$$

$$X = X_p(\tau), \left\{ \begin{array}{ll} \hat{p} = 1,\ \hat{v} = \hat{r} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_v = \mathscr{C}_l = \mathscr{C}_n = 0, & \text{throughout} \end{array} \right\}, \qquad (11)$$

$$X = X_r(\tau), \left\{ \begin{array}{ll} \hat{r} = 1,\ \hat{p} = \hat{v} = \phi = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = 0, & \text{throughout} \end{array} \right\}, \qquad (12)$$

$$X = X_\phi(\tau), \left\{ \begin{array}{ll} \phi = 1,\ \hat{p} = \hat{v} = \hat{r} = \psi = \hat{y} = 0, & \text{initially} \\ \mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = 0, & \text{throughout} \end{array} \right\}. \qquad (13)$$

The solutions corresponding to initial ψ and initial y are, of course, trivial.

These solutions will be referred to frequently as response to initial unit sideslip (X_v), unit constant rolling moment (X_l), or the like. It must be clearly understood that the unit referred to is the dimensionless, and not the natural, unit.

3.2. *Relations between the Seven Fundamental Solutions*.—Let us now integrate equations (5) formally with respect to τ, from 0 to τ, neglecting terms involving $\hat{v}_G(\tau)$. We have

$$\int_0^\tau \frac{dX}{d\tau}\, d\tau = X(\tau) - X(0) = \frac{d}{d\tau}\int_0^\tau X\, d\tau - X(0)\,, \qquad (14)$$

where $X(0)$ stands for the matrix of initial values $\hat{v}_0$, $\hat{p}_0$, $\hat{r}_0$, ϕ_0, ψ_0, $\hat{y}_0$. Hence, writing

$$\left. \begin{array}{lll} V = \int_0^\tau \hat{v}\, d\tau\,, & P = \int_0^\tau \hat{p}\, d\tau\,, & R = \int_0^\tau \hat{r}\, d\tau\,, \\ \Phi = \int_0^\tau \phi\, d\tau\,, & \Psi = \int_0^\tau \psi\, d\tau\,, & Y = \int_0^\tau \hat{y}\, d\tau\,, \end{array} \right\} \qquad (15)$$

we obtain the equations

$$\left. \begin{array}{l} \left(\frac{d}{d\tau} + \bar{y}_v\right) V + \left(1 - \frac{y_r}{\mu_2}\right) R - k\,\Phi = \hat{v}_0 + \int_0^\tau \mathscr{C}_y(\tau)\, d\tau\,, \\ \mathscr{L}\, V + \left(\frac{d}{d\tau} + l_1\right) P - l_2\, R = \hat{p}_0 + \int_0^\tau \mathscr{C}_l(\tau)\, d\tau\,, \\ -\mathscr{N}\, V + n_1\, P + \left(\frac{d}{d\tau} + n_2\right) R = \hat{r}_0 + \int_0^\tau \mathscr{C}_n(\tau)\, d\tau\,, \\ -P + \frac{d\Phi}{d\tau} = \phi_0\,, \\ -R + \frac{d\Psi}{d\tau} = \psi_0\,, \\ -V - \Psi + \frac{dY}{d\tau} = \hat{y}_0\,. \end{array} \right\} \qquad (16)$$

Comparing (16) and (5) we deduce the following conclusions :—

(i) The solution of (16) with the initial conditions $\hat{p}_0 = 1$, $\hat{v}_0 = \hat{r}_0 = \phi_0 = \psi_0 = \hat{y}_0 = 0$, and with $\mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = 0$ throughout is identical with the solution of (5) for unit applied rolling moment. Formally this is expressed by the equation

$$\int_0^\tau X_p(\tau)\, d\tau = X_l(\tau) \,. \qquad (17)$$

(ii) Similarly

$$\int_0^\tau X_r(\tau)\, d\tau = X_n(\tau) \qquad (18)$$

and

$$\int_0^\tau X_v(\tau)\, d\tau = X_y(\tau) \,. \qquad (19)$$

Again, writing $\phi = 1 + \phi'$ in (5), and taking $\mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = \hat{v}_G = 0$, we obtain the system of equations

$$\left.\begin{aligned}
\left(\frac{d}{d\tau} + \bar{y}_v\right)\hat{v} \qquad\qquad + \left(1 - \frac{y_r}{\mu_2}\right)\hat{r} - k\phi' &= k \,, \\
\mathscr{L}\,\hat{v} + \left(\frac{d}{d\tau} + l_1\right)\hat{p} \qquad - l_2\,\hat{r} &= 0 \,, \\
-\mathscr{N}\,\hat{v} \qquad + n_1\hat{p} + \left(\frac{d}{d\tau} + n_2\right)\hat{r} &= 0 \,, \\
-\hat{p} \qquad\qquad + \frac{d\phi'}{d\tau} &= 0 \,, \\
-\hat{r} + \frac{d\psi}{d\tau} &= 0 \,, \\
-\hat{v} \qquad\qquad - \psi + \frac{d\hat{y}}{d\tau} &= 0 \,,
\end{aligned}\right\} \qquad (20)$$

from which we deduce that

$$X'_\phi(\tau) = k\,X_y(\tau) \,, \qquad (21)$$

where

$$X' = \begin{Bmatrix} \hat{v} \\ \hat{p} \\ \hat{r} \\ \phi - 1 \\ \psi \\ \hat{y} \end{Bmatrix} \,. \qquad (22)$$

We can now proceed to investigate methods of calculating the four added fundamental solutions (10) to (13).

3.3. *Calculation* of *Response to Initial Rate of Roll.*—The solution $X_l(\tau)$ being known, we have from (17)

$$\left.\begin{aligned} \phi_p(\tau) &= \hat{p}_l(\tau) , \\ \psi_p(\tau) &= \hat{r}_l(\tau) \end{aligned}\right\} \qquad (23)$$

immediately, and

$$\hat{y}_p(\tau) = d\hat{y}_l/d\tau = \hat{v}_l(\tau) + \psi_l(\tau) , \qquad (24)$$

from the last equation of (5).

Also, on using the equations of motion,

$$\left.\begin{aligned} \hat{p}_p(\tau) &= d\hat{p}_l(\tau)/d\tau = 1 + l_2\hat{r}_l(\tau) - l_1\hat{p}_l(\tau) - \mathscr{L}\,\hat{v}_l(\tau) , \\ \hat{r}_p(\tau) &= d\hat{r}_l(\tau)/d\tau = \mathscr{N}\,\hat{v}_l(\tau) - n_1 p_l(\tau) - n_2\hat{r}_l(\tau) , \\ \hat{v}_p(\tau) &= d\hat{v}_l(\tau)/d\tau = k\phi_l(\tau) - \left(1 - \frac{y_r}{\mu_2}\right)\hat{r}_l(\tau) - \bar{y}_v\hat{v}_l(\tau) . \end{aligned}\right\} \qquad (25)$$

The appropriate curves can therefore be obtained by multiplication and addition from the curves for constant applied rolling moment.

3.4. *Calculation of Response to Initial Rate of Yaw.*—Similarly, from (18), we have

$$\left.\begin{aligned} \phi_r(\tau) &= \hat{p}_n(\tau) , \\ \psi_r(\tau) &= \hat{r}_n(\tau) , \end{aligned}\right\} \qquad (26)$$

$$\hat{y}_r(\tau) = \hat{v}_n(\tau) + \psi_n(\tau) , \qquad (27)$$

$$\left.\begin{aligned} \hat{p}_r(\tau) &= l_2\hat{r}_n(\tau) - l_1\hat{p}_n(\tau) - \mathscr{L}\,\hat{v}_n(\tau) , \\ \hat{r}_r(\tau) &= 1 + \mathscr{N}\,\hat{v}_n(\tau) - n_1\hat{p}_n(\tau) - n_2\hat{r}_n(\tau) , \\ \hat{v}_r(\tau) &= k\phi_n(\tau) - \left(1 - \frac{y_r}{\mu_2}\right)\hat{r}_n(\tau) - \bar{y}_v\hat{v}_n(\tau) . \end{aligned}\right\} \qquad (28)$$

3.5. *Calculation of Response to Sideforce.*—In this case, by (19), we have immediately

$$\left.\begin{aligned} \hat{p}_v(\tau) &= \phi_v(\tau) , \\ \hat{r}_y(\tau) &= \psi_v(\tau) . \end{aligned}\right\} \qquad (29)$$

Also

$$\left.\begin{aligned} \phi_y(\tau) &= \int_0^\tau \phi_v d\tau , \\ \psi_y(\tau) &= \int_0^\tau \psi_v d\tau , \\ \hat{v}_y(\tau) &= \hat{y}_v(\tau) - \psi_y(\tau), \\ \hat{y}_y(\tau) &= \int_0^\tau \hat{y}_v d\tau . \end{aligned}\right\} \qquad (30)$$

These can be evaluated with the aid of a continuous integraph.

3.6. *Calculation of Response to Initial Angle of Bank.*—When response to sideforce has been obtained, this follows from the relations

$$\left.\begin{aligned} \hat{p}_\phi &= k\hat{p}_y , \\ \hat{r}_\phi &= k\hat{r}_y , \\ \hat{v}_\phi &= k\hat{v}_y , \\ \phi_\phi &= 1 + k\phi_y , \\ \psi_\phi &= k\psi_y , \\ \hat{y}_\phi &= k\hat{y}_y . \end{aligned}\right\} \qquad (31)$$

4. *Derived Solutions for Constant or Piecewise Constant Disturbances.*—4.1. *Response to Sharp-edged Sidegusts.*—If an aeroplane moving steadily with controls central encounters a sidegust of velocity $\hat{v}_G(\tau)$, the motion is determined by solving (5), with $\mathscr{C}_y = \mathscr{C}_l = \mathscr{C}_n = 0$, and with the initial conditions $\hat{v} = \hat{p} = \hat{r} = \phi = \psi = \hat{y} = 0$. For a constant $\hat{v}_G = 1$, we may remove the terms in $\hat{v}_G$, by replacing $\hat{v}$ and $\hat{y}$ in (5) by $\hat{v}'$ and $\hat{y}'$, where

$$\left.\begin{aligned} \hat{v}' &= \hat{v} + 1 , \\ \hat{y}' &= \hat{y} + \tau ; \end{aligned}\right\} \qquad (32)$$

we thus see that the motion required, in $\hat{v}'$, $\hat{p}$, $\hat{v}$. ϕ, ψ, $\hat{y}'$, is the same as that already determined, in $\hat{v}$, $\hat{p}$, $\hat{v}$, ϕ, ψ, $\hat{y}$, for no disturbance force or moment, and for the initial conditions $\hat{v} = 1$, $\hat{p} = \hat{r} = \phi = \psi = \hat{y} = 0$. The motion is therefore given, in $\hat{v}'$, $\hat{p}$, $\hat{r}$, ϕ, ψ, $\hat{y}'$, by the response to initial sideslip. In fact, if the suffix g identifies the solution for constant unit gust velocity,

$$\left.\begin{aligned} \hat{p}_g(\tau) &= \hat{p}_v(\tau) , & \phi_g(\tau) &= \phi_v(\tau) , \\ \hat{r}_g(\tau) &= \hat{r}_v(\tau) , & \psi_g(\tau) &= \psi_v(\tau) , \end{aligned}\right\} \qquad (33)$$

with

$$\left.\begin{aligned} \hat{v}_g(\tau) &= -1 + \hat{v}_v(\tau) , \\ \hat{y}_g(\tau) &= \hat{y}_v(\tau) - \tau . \end{aligned}\right\} \qquad (34)$$

4.2. *Any Combination of Conditions involving Constant or Piecewise Constant Applied Forces, Moments, or Gusts.*—The fundamental solutions can be made to yield an extraordinary variety of results under this heading. Most generally, we may consider the disturbance motion following arbitrary initial values $\hat{p}_0$, $\hat{v}_0$, $\hat{r}_0$, ϕ_0 at $\tau = 0$, with a sharp-edged gust velocity $\hat{v}_G$, and with constant control forces $\mathscr{C}_y$, $\mathscr{C}_l$, $\mathscr{C}_n$. The motion, as long as gust velocity and control forces remain unchanged, is given by

$$\begin{aligned} X(\tau) = {} & \hat{p}_0 X_p(\tau) + \hat{v}_0 X_v(\tau) + \hat{r}_0 X_r(\tau) + \phi_0 X_\phi(\tau) \\ & + \mathscr{C}_y X_y(\tau) + \mathscr{C}_l X_l(\tau) + \mathscr{C}_n X_n(\tau) + \hat{v}_G X_g(\tau) . \end{aligned} \qquad (35)$$

If at time τ_0 the gust velocity changes to $\hat{v}_G + \hat{v}_G'$, and the control forces to $\mathscr{C}_y + \mathscr{C}_y'$, $\mathscr{C}_l + \mathscr{C}_l'$, $\mathscr{C}_n + \mathscr{C}_n'$, the solution will be given by (35) up to time τ_0, and subsequently by

$$\begin{aligned} X(\tau) = {} & \hat{p}_0 X_p(\tau) + \hat{v}_0 X_y(\tau) + \hat{r}_0 X_r(\tau) + \phi_0 X_\phi(\tau) \\ & + \mathscr{C}_y X_y(\tau) + \mathscr{C}_l X_l(\tau) + \mathscr{C}_n X_n(\tau) + \hat{v}_G X_g(\tau) \\ & + \mathscr{C}_y' X_y(\tau - \tau_0) + \mathscr{C}_l' X_l(\tau - \tau_0) + \mathscr{C}_n' X_n(\tau - \tau_0) \\ & + \hat{v}_G' X_g(\tau - \tau_0) \ldots \end{aligned} \qquad (36)$$

Further changes can follow by addition in the same way.

Particular examples of this technique follow.

4.31. *Sharp-edged sidegust, on and off, duration τ_0, motion initially steady*

$$\left.\begin{aligned} X(\tau) &= \hat{v}_G X_g(\tau)\,, & 0 \leqslant \tau \leqslant \tau_0\,, \\ &= \hat{v}_G \{X_g(\tau) - X_g(\tau - \tau_0)\}, & \tau_0 \leqslant \tau\,. \end{aligned}\right\} \quad \ldots \quad (37)$$

4.32. *Picking up a dropped wing by constant rolling moment.*—We represent the dropped wing by initial ϕ_0, the motion is then given by

$$X(\tau) = \phi_0 X_\phi(\tau) + \mathscr{C}_l X_l(\tau)\,, \quad \ldots \quad (38)$$

or by

$$X(\tau) = \phi_0 X_\phi(\tau) + \mathscr{C}_l X_l(\tau) + \mathscr{C}_n X_n(\tau)\,, \quad \ldots \quad (39)$$

if the yawing moment produced by aileron application is taken into account.

If the ailerons are centralised after time τ_0, the motion is given by the above expressions for $0 \leqslant \tau \leqslant \tau_0$, and thereafter by

$$X(\tau) = \phi_0 X_\phi(\tau) + \mathscr{C}_l \{X_l(\tau) - X_l(\tau - \tau_0)\} + \mathscr{C}_n \{X_n(\tau) - X_n(\tau - \tau_0)\}\,. \quad \ldots \quad (40)$$

4.33. *Engine cut.*—If $\mathscr{C}_{le}$, $\mathscr{C}_{ne}$ are rolling and yawing moment due to engine failure,

$$X(\tau) = \mathscr{C}_{le} X_l(\tau) + \mathscr{C}_{ne} X_n(\tau)\,. \quad \ldots \quad (41)$$

If at time τ_0 the controls are moved so as instantaneously to balance the engine cut moments, we have subsequently

$$X(\tau) = \mathscr{C}_{le} \{X_l(\tau) - X_l(\tau - \tau_0)\} + \mathscr{C}_{ne} \{X_n(\tau) - X_n(\tau - \tau_0)\}\,. \quad \ldots \quad (42)$$

Alternatively, if the applied rolling and yawing moments do not balance the engine cut moment,

$$X(\tau) = \mathscr{C}_{le} X_l(\tau) + \mathscr{C}_l X_l(\tau - \tau_0) + \mathscr{C}_{ne} X_n(\tau) + \mathscr{C}_n X_n(\tau - \tau_0)\,. \quad \ldots \quad (43)$$

4.34. *Fin stall following engine failure or rudder application.*—This is a much more complicated example of the possible utility of the curves and it should be remarked that it is not likely to be one to which linear equations can be applied. It is not recommended that calculations of this type should be carried out. The case considered, however, is an example of the potential range of calculations which can be made, when sufficient thought is given to the possibilities.

We shall suppose that stalling the fin causes a decrease in both $\mathscr{N}$ and n_2: we then have two systems of equations to deal with, system 1 applying until the fin stalls, and system 2 thereafter. We shall denote the solutions corresponding to the two systems by upper suffices enclosed in brackets, *e.g.* $X_l^{(1)}(\tau)$, *etc.* We shall suppose also that the fin stalls when the fin incidence reaches a certain value, say

$$\hat{v} - \frac{l\hat{r}}{\mu_2} = \alpha\,, \quad \ldots \quad (44)$$

where l is the dimensionless fin and rudder arm.

Suppose the initial motion is due to rudder application. We have then

$$X(\tau) = \mathscr{C}_n X_n^{(1)}(\tau)\,, \quad \ldots \quad (45)$$

which holds until a time τ_0 such that

$$\mathscr{C}_n \left| v_n^{(1)}(\tau_0) - \frac{l}{\mu_2} r_n^{(1)}(\tau_0) \right| = \alpha\,. \quad \ldots \quad (46)$$

At this instant the values of the disturbances are

$$\left.\begin{matrix} \mathscr{C}_n p_n^{(1)}(\tau_0)\,, & \mathscr{C}_n v_n^{(1)}(\tau_0)\,, & \mathscr{C}_n r_n^{(1)}(\tau_0)\,, \\ \mathscr{C}_n \phi_n^{(1)}(\tau_0)\,, & \mathscr{C}_n \psi_n^{(1)}(\tau_0)\,, & \mathscr{C}_n y_n^{(1)}(\tau_0)\,. \end{matrix}\right\} \quad \ldots \quad (47)$$

Subsequently system 2 moves from these initial conditions under the same applied moment and with an extra yawing moment $\mathscr{C}_n'$ needed to give the correct fin lift at the stall. The motion is then given by

$$\begin{aligned} X(\tau) = {} & \mathscr{C}_n p_n^{(1)}(\tau_0)\, X_p^{(2)}(\tau - \tau_0) + \mathscr{C}_n v_n^{(1)}(\tau_0)\, X_v^{(2)}(\tau - \tau_0) \\ & + \mathscr{C}_n r_n^{(1)}(\tau_0)\, X_r^{(2)}(\tau - \tau_0) + \mathscr{C}_n \phi_n^{(1)}(\tau_0)\, X_\phi^{(2)}(\tau - \tau_0) \\ & + (\mathscr{C}_n + \mathscr{C}_n')\, X_n^{(2)}(\tau - \tau_0) , \end{aligned} \quad \ldots \quad (48)$$

apart from correction terms in ψ and $\hat{y}$. We may proceed to move the controls at time τ_1 in an attempt to unstall the fin. If $\mathscr{C}_l''$, $\mathscr{C}_n''$ are the extra moments introduced, we have subsequently

$$\begin{aligned} X(\tau) = {} & \mathscr{C}_n p_n^{(1)}(\tau_0)\, X_p^{(2)}(\tau - \tau_0) + \mathscr{C}_n v_n^{(1)}(\tau_0)\, X_v^{(2)}(\tau - \tau_0) \\ & + \mathscr{C}_n r_n^{(1)}(\tau_0)\, X_r^{(2)}(\tau - \tau_0) + \mathscr{C}_n \phi_n^{(1)}(\tau_0)\, X_\phi^{(2)}(\tau - \tau_0) \\ & + (\mathscr{C}_n + \mathscr{C}_n')\, X_n^{(2)}(\tau - \tau_0) + \mathscr{C}_l'' X_l^{(2)}(\tau - \tau_1) + \mathscr{C}_n'' X_n^{(2)}(\tau - \tau_2) \end{aligned} \quad \ldots \quad (49)$$

Equations (48) and (49) will remain valid as long as the fin incidence exceeds α or some lower critical angle at which the fin unstalls. If it drops to this value, we return to system 1 with the original applied forces but with the initial conditions determined at the instant when the fin ceases to be stalled.

5. *Derived Solutions for Variable Applied Forces, Moments, or Gust Velocities.*—5.1. *Disturbances Varying Linearly with Time.*—The general curves can also be used, though not so simply, for calculations on variable applied forces and moments, or on variable gust velocities. The simplest obvious cases are those in which the disturbances vary linearly with the time. The results here, in the case of applied forces and moments, can be obtained by comparing with the integrated equations (16). We then see that integration of the response to unit sideforce, rolling moment, or yawing moment yields the response to linearly applied sideforce, rolling moment, or yawing moment respectively, the rate of growth of the applied force or moment being unity. If we denote these solutions by $X_{dy}(\tau)$, $X_{dl}(\tau)$, $X_{dn}(\tau)$, we thus have

$$\left.\begin{aligned} X_{dy}(\tau) &= \int_0^\tau X_y(\tau)\, d\tau , \\ X_{dl}(\tau) &= \int_0^\tau X_l(\tau)\, d\tau , \\ X_{dn}(\tau) &= \int_0^\tau X_n(\tau)\, d\tau . \end{aligned}\right\} \quad \ldots \quad (50)$$

In full, for $X_{dl}(\tau)$, we have

$$\left.\begin{aligned} \hat{p}_{dl}(\tau) &= \phi_l(\tau) , \\ \hat{r}_{dl}(\tau) &= \psi_l(\tau) \end{aligned}\right\} \quad \ldots \quad (51)$$

immediately, and

$$\left.\begin{aligned} \phi_{dl}(\tau) &= \int_0^\tau \phi_l(\tau)\, d\tau , \\ \psi_{dl}(\tau) &= \int_0^\tau \psi_l(\tau)\, d\tau , \\ \hat{v}_{dl}(\tau) &= \hat{y}_l(\tau) - \psi_{dl}(\tau) , \\ \hat{y}_{dl}(\tau) &= \int_0^\tau \hat{y}_l(\tau)\, d\tau . \end{aligned}\right\} \quad \ldots \quad (52)$$

Exactly similar equations can be written down for response to linearly applied sideforce or yawing moment. The use of a continuous integraph is indicated.

We may also define a further solution for a linearly increasing sidegust, with $d\hat{v}_G/d\tau = 1$. Calling this $X_{dg}(\tau)$, we have

$$X_{dg}(\tau) = \int_0^\tau X_g(\tau)\, d\tau\ , \qquad \ldots \qquad (53)$$

leading to equations like (51) and (52) as before.

5.2. *Mixed Disturbances.*—With the four extra solutions thus obtained much more general calculations may be made. We may take as an example a sudden engine failure, at time 0, which the pilot attempts to correct, after an interval τ_0, by moving the controls with constant velocity until the asymmetric moments due to engine failure, $\mathscr{C}_{le}$, $\mathscr{C}_{ne}$, say, are balanced at time τ_1, after which the controls are held fixed. The solution is then given by

$$\left.\begin{aligned}
X(\tau) &= \mathscr{C}_{le}X_l(\tau) + \mathscr{C}_{ne}X_n(\tau)\ , && 0 \leqslant \tau \leqslant \tau_0\ ,\\
X(\tau) &= \mathscr{C}_{le}X_l(\tau) + \mathscr{C}_{ne}X_n(\tau)\\
&\quad - \frac{\mathscr{C}_{le}}{\tau_1 - \tau_0} X_{dl}(\tau - \tau_0) - \frac{\mathscr{C}_{ne}}{\tau_1 - \tau_0} X_{dn}(\tau - \tau_0)\ , && \tau_0 \leqslant \tau \leqslant \tau_1\ ,\\
X(\tau) &= \mathscr{C}_{le}X_l(\tau) + \mathscr{C}_{ne}X_n(\tau) - \frac{\mathscr{C}_{le}}{\tau_1 - \tau_0}\{X_{dl}(\tau - \tau_0) - X_{dl}(\tau - \tau_1)\}\\
&\quad - \frac{\mathscr{C}_{ne}}{\tau_1 - \tau_0}\{X_{dn}(\tau - \tau_0) - X_{dn}(\tau - \tau_1)\}\ , && \tau_1 \leqslant \tau\ .
\end{aligned}\right\} \qquad (54)$$

As another example, if the motion is due to a gust which grows linearly to velocity $\hat{v}_G$ in time τ_0, and then falls linearly to zero in the same time, thereafter remaining zero, the motion is given by

$$\left.\begin{aligned}
X(\tau) &= \frac{\hat{v}_G}{\tau_0} X_{dg}(\tau)\ , && 0 \leqslant \tau \leqslant \tau_0\ ,\\
&= \frac{\hat{v}_G}{\tau_0}\{X_{dg}(\tau) - 2X_{dg}(\tau - \tau_0)\}\ , && \tau_0 \leqslant \tau \leqslant 2\tau_0\ ,\\
&= \frac{\hat{v}_G}{\tau_0}\{X_{dg}(\tau) - 2X_{dg}(\tau - \tau_0) + X_{dg}(\tau - 2\tau_0)\}\ , && 2\tau_0 \leqslant \tau\ .
\end{aligned}\right\} \quad \ldots \quad (55)$$

5.3. *Use of the Curves as Influence Functions.*—Variable applied forces, moments, or gust velocities can also be dealt with by a much more powerful method, which may also be found preferable, in the case of linear variations, to the method given above. We shall illustrate this by considering the case of a variably applied rolling moment.

The solution $X_l(\tau)$ can be regarded as an influence function giving the magnitudes of the disturbances at time τ after the sudden application of a unit rolling moment $\mathscr{C}_l$. Hence if at time τ_1 rolling moment $d\mathscr{C}_l$ is applied, the corresponding magnitudes of the disturbances will be $X_l(\tau - \tau_1)\, d\mathscr{C}_l$. All such magnitudes are additive, and the motion due to any variations of $\mathscr{C}_l$ can therefore be expressed in the form

$$X(\tau) = \int_0^\tau X_l(\tau - \tau_1) \frac{d\mathscr{C}_l}{d\tau_1} d\tau_1\ . \qquad \ldots \qquad (56)$$

Similar equations can be written down for the motion due to arbitrary changes of gust velocity or to arbitrary control motions. The most general motion, starting with arbitrary $\hat{p}_0$, $\hat{v}_0$, $\hat{r}_0$, ϕ_0, with arbitrary disturbances, can then be written down in the form

$$\begin{aligned}
X(\tau) &= \hat{p}_0X_p(\tau) + \hat{v}_0X_v(\tau) + \hat{r}_0X_r(\tau) + \phi_0X_\phi(\tau)\\
&+ \int_0^\tau \left\{X_l(\tau - \tau_1)\frac{d\mathscr{C}_l}{d\tau_1} + X_n(\tau - \tau_1)\frac{d\mathscr{C}_n}{d\tau_1} + X_v(\tau - \tau_1)\frac{d\mathscr{C}_Y}{d\tau_1} + X_g(\tau - \tau_1)\frac{d\hat{v}_G}{d\tau_1}\right\} d\tau\ . \quad \ldots \quad (57)
\end{aligned}$$

This type of integral can be evaluated very simply by means of the Stieltjes-planimeter, Nyström[4] (1935).

6. *Examples of Solutions Derived from the Fundamental Solutions.*—A number of calculations have been made to illustrate the possible applications of the curves, discussed above. The results are given in Figs. 1–10, each of which shows all six components of the motion. The curves and the method of calculation, are as follows :—

6.1. *Fundamental Solutions* (*Figs.* 1–3).—The fundamental solutions X_l, X_n, X_v are shown in Figs. 1–3, together with the derived solution X_g. These solutions were obtained by the differential analyser, and belong (*see* Appendix), to the basic stage of the programme on the lateral response of conventional aircraft at high speeds. The numerical data used are as follows:—

$$\left.\begin{array}{lllll} k = 0\cdot 1\,, & \mu_2 = 20\,, & i_A' = 0\cdot 12\,, & i_C' = 0\cdot 18\,, & i_E' = 0\,, \\ y_v = 0\cdot 2,\ y_r = 0, & l_p = -0\cdot 42\,, & l_r = 0\cdot 06\,, & n_{v0} = -0\cdot 024\,, & n_p = -0\cdot 03\,, \\ l = 1\,, & l_v = -0\cdot 06\,, & n_{vf} = 0\cdot 072\ (n_v = 0\cdot 048\,, & n_r = -0\cdot 072)\,, & \end{array}\right\} \quad (58)$$

and the solutions X_l, X_n, X_v are as defined in sect. 3.1, corresponding to unit applied rolling and yawing moments (the modified moments $\mathscr{C}_l$ and $\mathscr{C}_n$) and unit initial sideslip ($\hat{v} = 1$); while X_g gives the response to a sidegust of unit velocity ($\hat{v}_G = 1$) given by

$$\left.\begin{array}{llll} \hat{p}_g = \hat{p}_v, & \hat{r}_g = \hat{r}_v\,, & \phi_g = \phi_v\,, & \psi_g = \psi_v\,, \\ \hat{v}_g = -1 + \hat{v}_v\,, & \hat{y}_g = -\tau + \hat{y}_v\,. & & \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (59)$$

6.2. *Solutions for Unit Constant Sideforce, and Unit Initial Angle of Bank* (*Fig.* 4).—The response to unit constant sideforce ($\mathscr{C}_y$) has been derived by the formulæ of sect. 3.5, and is shown in Fig. 4. The solution for unit initial angle of bank differs from this only in scale, and the addition of a constant to ϕ, and the alternative scales for this solution have been added to Fig. 4. The extreme slowness of the motion which develops when the wings are not level should be noted.

6.3. *Solutions for Initial Rates of Roll and Yaw* (*Figs.* 5, 6).—The response to initial unit rate of roll, and to initial unit rate of yaw, have been calculated by the formulæ of sects. 3.3 and 3.4, which in this case become

$$\left.\begin{array}{l} \phi_p = \hat{p}_l\,, \quad \psi_p = \hat{r}_l\,, \quad \hat{y}_p = \hat{v}_l + \psi_l\,, \\ \hat{p}_p = 1 + \tfrac{1}{2}\hat{r}_l - \tfrac{7}{2}\hat{p}_l - 10\hat{v}_l\,, \\ \hat{r}_p = \tfrac{16}{3}\hat{v}_l - \tfrac{1}{6}\hat{p}_l - \tfrac{2}{5}\hat{r}_l\,, \\ \hat{v}_p = \tfrac{1}{10}\phi_l - \hat{r}_l - \tfrac{1}{5}\hat{v}_l\,, \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (60)$$

$$\left.\begin{array}{l} \phi_r = \hat{p}_n\,, \quad \psi_r = \hat{r}_n\,, \quad \hat{y}_r = \hat{v}_n + \psi_n\,, \\ \hat{p}_r = \tfrac{1}{2}\hat{r}_n - \tfrac{7}{2}\hat{p}_n - 10\hat{v}_n\,, \\ \hat{r}_r = 1 + \tfrac{16}{3}\hat{v}_n - \tfrac{1}{6}\hat{p}_n - \tfrac{2}{5}\hat{r}_n\,, \\ \hat{v}_r = \tfrac{1}{10}\phi_n - \hat{r}_n - \tfrac{1}{5}\hat{v}_n\,. \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

The results are shown in Figs. 5 and 6 respectively.

6.4. *Picking up a Dropped Wing by Application of Rolling Moment* (*Fig.* 7).—The appropriate formulæ for this case are given in sect. 4.32. The case taken is where the initial angle of bank is $\frac{1}{2}$ radian, and unit rolling moment ($\mathscr{C}_l$) is applied initially, and held constant thereafter, or until the angle of bank is zero, and the controls are then centralised. The results are given in Fig. 7, the full-line curves showing the response when the rolling moment is maintained constant throughout, and the dotted curves showing the results if the controls are centralised when $\phi = 0$.

6.5. *Response to a Graded Gust* (*Figs.* 8, 9).—The response to a linearly increasing sidegust, with unit rate of growth, is shown in Fig. 8, each component being the time-integral of the corresponding component in Fig. 3. This solution has then been used with the results shown in Fig. 9, to construct the response to an on-off graded gust which grows at unit rate from 0 to 1 airsec, and immediately decreases at the same rate from 1 to 2 airsecs.

6.6. *Response to a Sharp-edged Constant Sidegust, Duration ½ Airsec* (*Fig.* 10).—This final example illustrates the technique of sect 4.2, the working formulæ being given in sect. 4.31, with $\tau_0 = \frac{1}{2}$. The curves obtained are shown in Fig. 10.

LIST OF SYMBOLS

$C_l, C_n, C_y, E, i_A', i_C', k, l_p, l_r, l_v, n_p, n_r, n_v, y_v, \mu_2$, are as defined in R. & M. 1801[2].

b span of aeroplane (ft.)

$\mathscr{C}_l = \mu_2 C_l / i_A', \quad \mathscr{C}_n = \mu_2 C_n / i_C', \quad \mathscr{C}_y = \frac{1}{2} C_y$

$\mathscr{C}_{le}, \mathscr{C}_{ne}$ Values of $\mathscr{C}_l$, $\mathscr{C}_n$ due to engine failure

$\mathscr{C}_l', \mathscr{C}_n', \mathscr{C}_y'$, *etc.* Changes of $\mathscr{C}_l$, $\mathscr{C}_n$, $\mathscr{C}_y$ during an manœuvre

$d\mathscr{C}_l$ Increment of $\mathscr{C}_l$

g As suffix, identifies the solution for sharp-edged sidegust

dg As suffix, identifies the solution for linearly increasing sidegust (unit rate)

$i_E = 4E/(Wb^2)$

l Dimensionless fin arm. As suffix, identifies the solution for unit constant rolling moment

dl As suffix, identifies the solution for linearly increasing rolling moment (unit rate)

$l_1 = -l_p/i_A', \qquad l_2 = l_r/i_A'$

$\mathscr{L} = -\mu_2 l_v / i_A'$

m Mass of aeroplane (slugs)

n As suffix, identifies the solution for unit constant yawing moment

dn As suffix, identifies the solution for linearly increasing yawing moment (unit rate)

$n_1 = -n_p/i_C', \quad n_2 = -n_r/i_C'$

n_{r0} Value of n_r for a particular fin size

n_{v0} Value of n_v for a particular fin size

n_{vf} Extra n_v due to change of fin size

$\mathscr{N} = \mu_2 n_v / i_C'$

p As suffix, identifies the solution for unit initial rate of roll

$P = \int_0^\tau p \, d\tau$

LIST OF SYMBOLS (*continued*)

$\hat{p}$	Rate of roll, rad./airsec
$\hat{p}_0$	Initial value of $\hat{p}$
r	Rate of yaw, rad./sec.
	As suffix, identifies the solution for unit initial rate of yaw
$R = \int_0^\tau \hat{r} d\tau$	
$\hat{r}$	Rate of roll, rad./airsec.
$\hat{r}_0$	Initial value of $\hat{r}$
S	Wing area of aeroplane (sq. ft.)
$\hat{t}$	Value of airsec in seconds
U_e	Steady velocity of aeroplane (ft./sec.)
v	As suffix, identifies the solution for unit initial sideslip
$V = \int_0^\tau \hat{v} d\tau$	
$\hat{v}$	Sideslip in radians
$\hat{v}_0$	Initial value of $\hat{v}$
$\hat{v}'$	$\hat{v} + 1$
$\hat{v}_G$	Gust velocity
$\hat{v}_G'$	Change of gust velocity
X	Matrix of components $\hat{v}$, $\hat{p}$, $\hat{r}$, ϕ, ψ, $\hat{y}$.
X'	Matrix of components $\hat{v}$, $\hat{p}$, $\hat{r}$, $\phi - 1$, ψ, $\hat{y}$
y	As suffix, identifies the solution for unit constant sideforce
dy	As suffix, identifies the solution for linearly increasing sideforce (unit rate)
$Y = \int_0^\tau \hat{y} d\tau$	
Y_r	Sideforce due to rate of yaw (lb. wt./rad./sec.)
$\hat{y}$	Sideways displacement, in units $U_e\hat{t}$
$\hat{y}_0$	Initial value of $\hat{y}$
$\hat{y}' = \hat{y} + \tau$	
$y_r =$	Dimensionless sideforce due to rate of yaw
y_{v0}	Value of y_v for a particular fin size
$\bar{y}_v = - y_v$	
α	Fin incidence at which the fin stalls (radians)
γ_e	Angle of climb of aeroplane (radians)
ρ	Air density (slugs./cu. ft.)
τ	Time in airsecs
τ_0, τ_1	Special values of τ

LIST OF SYMBOLS (*continued*)

ϕ Angle of bank (radians)

As suffix, identifies the solution for unit initial angle of bank

Φ $\int_0^\tau \phi d\tau$

ϕ_0 Initial value of ϕ

$\phi' = \phi - 1$

ψ = Angle of azimuth (radians)

$\Psi = \int_0^\tau \phi d\tau$

Ψ_0 Initial value of Ψ

(1), (2) As upper suffices, identify solutions for normal and fin-stalled conditions respectively

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REFERENCES

No.	*Author*	*Title, etc.*
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APPENDIX

The programme of work referred to in the text covers four aspects of the lateral response problem :—

(i) Lateral response of conventional aeroplanes at high speed ($C_L = 0{\cdot}2$).

(ii) Lateral response of tailless aeroplanes at high speed ($C_L = 0{\cdot}2$).

(iii) Lateral response of tailless aeroplanes at low speeds ($C_L = 1{\cdot}0$).

(iv) Lateral response of ultra high-lift aeroplanes ($C_L = 2{\cdot}8$).

Each stage is described, under its appropriate heading, below.

A1. *Lateral Response of Conventional Aeroplanes at High Speeds* ($C_L = 0{\cdot}2$).—The quantities

$$k = 0{\cdot}1, \quad \gamma_e = 0, \quad i_E = 0, \quad y_r = 0 \qquad \text{(A1.1)}$$

remained fixed throughout the whole of this stage of the programme. The remaining quantities were treated as follows :—

(i) Basic values were attached to all of

$$y_v,\ l_p,\ l_r,\ n_p,\ i_A{}',\ i_C{}',\ \mu_2, \qquad \text{(A1.2)}$$

and n_v and l_v were varied independently, n_r varying with n_v according to the laws

$$n_v = n_{v0} + n_{vf}, \qquad n_r = -\, l n_{vf}, \qquad \text{(A1.3)}$$

with basic values of n_{v0} and the dimensionless fin arm l. This part of the programme is referred to as " Basic ".

(ii) Similar results with independently varied n_v (n_r) and l_v were obtained with modified values of $i_A{}'$, $i_C{}'$, but with basic values for all other quantities. This part is referred to as " Variation of inertias ". The values of $\mathscr{C}_l$ and $\mathscr{C}_n$ were changed, with the inertias, so as to keep the standard dimensionless C_l and C_n constant.

(iii) Similar results with independently varied n_v (n_r) and l_v, with all quantities basic except μ_2, which was given modified values. This part is referred to as " Variation of μ_2 ".

(iv) With basic values of $i_A{}'$, $i_C{}'$, with fixed μ_2, and with independently varied n_v (n_r) and l_v, the remaining quantities were altered one by one from their basic values to new values estimated for a particular aeroplane, which will be referred to in the sequel as Aeroplane K. This part is referred to as " Transition ".

(v) Finally, further calculations were made with the values of the derivatives for Aeroplane K with two values of μ_2, and with independently varied n_v (n_r) and l_v. This part is referred to as " Aeroplane K ".

The graphs corresponding to this whole section of the programme are numbered S1 to S396, and bear, in addition to the number, a code caption indicating what they represent. The following conventions are used in making up the caption :—

(i) The type of disturbance is indicated by the letters S, A or R, indicating response to initial sideslip, unit rolling (Aileron) moment, and unit yawing (Rudder) moment, respectively.

(ii) The stage of the programme is indicated by the letters B (Basic), I (Variation of inertia), μ (Variation of μ_2) or T (Transition). In the case of Aeroplane K the results for $\mu_2 = 5$ and $\mu_2 = 10$ are distinguished by the headings K5 and K10 respectively.

(iii) The component of the motion shown in any particular graph is indicated by the corresponding small letter, placed last in the caption.

(iv) Where l_v is constant in all the results shown on a particular graph, the value of $100l_v$ appears before the code letters indicating the part of the programme or the type of disturbance applied.

(v) Similarly where n_{vf} is constant, the value of $1{,}000n_{vf}$ follows the code letters for part of programme and type of solution.

As an example, the caption BS120*v* indicates response to initial sideslip for basic conditions with $n_{vf} = 0 \cdot 120$, the component shown being sideslip.

An index to the figures S1 to S396 for this stage of the programme, with the numerical values used, is given in Table 1.

A2. *Lateral Response of Ultra High-lift Aeroplanes* ($C_L = 2 \cdot 8$).—This programme splits into parts in a very similar way. The quantities

$$k = 1 \cdot 4, \qquad \gamma_e = 0 \qquad \ldots \qquad (A2.1)$$

were constant throughout, and the derivatives were assumed to be connected by the relations

$$\left.\begin{aligned} y_v &= y_{v0} - \frac{1}{l} n_{vf}, \\ y_r &= n_{vf}, \\ n_v &= n_{v0} + n_{vf}, \\ n_r &= n_{r0} - l n_{vf}, \end{aligned}\right\} \qquad \ldots \qquad (A2.2)$$

Basic values were attached to each of

$$y_{v0},\ l_p,\ l_r,\ n_p,\ n_{r0},\ n_{v0},\ l,\ i_A',\ i_C',\ i_E,\ \mu_2, \qquad \ldots \qquad (A2.3)$$

and n_{vf} and l_v were independently varied, y_v, y_r, n_v and n_r varying with n_{vf}. This constitutes the " High-lift basic " part of the programme.

High-lift " Variation of μ_2 " and " Variation of inertias " sections were obtained by making calculations for basic values with μ_2 modified, and with i_A' and i_C' modified, respectively.

A " High-lift transition " section was obtained, in which the effect of changing i_E, and the effects of altering the rotary derivatives y_v, l_p, l_r, n_p, n_{r0}, were investigated. These parameters were altered separately from their basic values to new values *and back again*, and not cumulatively as in the high-speed programme.

An index to the figures S397 to S756 for this section of the programme, with the numerical values used, is given in Table 2. The captioning of these figures follows the general lines of those of the basic stage, except that $100n_{vf}$ is indicated instead of $1{,}000\ n_{vf}$.

A3. *Lateral Response of Tailless Aeroplanes at High or Low Speeds.*—These programmes differ markedly from those for conventional aeroplanes, and are considerably simpler. As in the first part of the programme, i_E, y_r, γ_e were assumed to be zero. Further, no relations between derivatives were assumed, and this makes it natural to work in terms of the modified quantities of equations (5), rather than to use the standard dimensionless derivatives of equations (1). Variations of μ_2 are then absorbed in those of $\mathscr{L}$ and $\mathscr{N}$, and those of i_A', i_C' in these and in the remaining quantities.

The programme was therefore as follows. For each speed, given values were assigned to the quantities

$$l_1,\ l_2,\ k,\ n_1\ ; \qquad \ldots \qquad (A3.1)$$

the quantities $\bar{y}_v$ and n_2 were then given all combinations of two values each, while $\mathscr{L}$ and $\mathscr{N}$, for each combination of $\bar{y}_v$ and n_2, were independently varied. All results have been plotted for fixed $\mathscr{N}$ and varying $\mathscr{L}$, with captions of the form $b_3$4p, where the letters b or B indicate small or large C_L, the suffix refers to the combination of $\bar{y}_v$ and n_2 used ; the next numeral is the value of $\mathscr{N}$, and p is the component shown.

An index to the figures S757 to S1188 for this stage of the programme, with the numerical values used, is given in Table 3.

TABLE

Lateral Response of Conventional

List of Figures

Code Caption	Rotary Derivatives	μ_2	i_A'	i_c'	Parameters Fixed	Parameters Varied	Response to Sideslip			
							p	v	τ	φ
06B	Basic ..	20	0·12	0·18	$l_v = +0{\cdot}06$	n_{vf} (and n_r) ..	S.1	S.2	S.3	S.4
00B	Basic ..	20	0·12	0·18	$l_v = 0$	n_{vf} (and n_r) ..	S.7	S.8	S.9	S.10
−06B	Basic ..	20	0·12	0·18	$l_v = -0{\cdot}06$	n_{vf} (and n_r) ..	S.13	S.14	S.15	S.16
−12B	Basic ..	20	0·12	0·18	$l_v = -0{\cdot}12$	n_{vf} (and n_r) ..	S.19	S.20	S.21	S.22
B.000	Basic ..	20	0·12	0·18	$n_{vf} = 0$	l_v	S.25	S.26	S.27	S.28
B.024	Basic ..	20	0·12	0·18	$n_{vf} = 0{\cdot}024$	l_v	S.31	S.32	S.33	S.34
B.072	Basic ..	20	0·12	0·18	$n_{vf} = 0{\cdot}072$	l_v	S.37	S.38	S.39	S.40
B.120	Basic ..	20	0·12	0·18	$n_{vf} = 0{\cdot}120$	l_v	S.43	S.44	S.45	S.46
00I.048	Basic ..	20	Var.	Var.	$l_v = 0$ $n_{vf} = 0{\cdot}048$	i_A' and i_C' ..	S.49	S.50	S.51	S.52
−12I.048	Basic ..	20	Var.	Var.	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}048$	i_A' and i_C' ..	S.55	S.56	S.57	S.58
00I.120	Basic ..	20	Var.	Var.	$l_v = 0$ $n_{vf} = 0{\cdot}120$	i_A' and i_C' ..	S.61	S.62	S.63	S.64
−12I.120	Basic ..	20	Var.	Var.	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}120$	i_A' and i_C' ..	S.67	S.68	S.69	S.70
00μ048	Basic ..	Var.	0·12	0·18	$l_v = 0$ $n_{vf} = 0{\cdot}048$	μ_2	S.73	S.74	S.75	S.76
−12μ048	Basic ..	Var.	0·12	0·18	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}048$	μ_2	S.79	S.80	S.81	S.82
00μ120	Basic ..	Var.	0·12	0·18	$l_v = 0$ $n_{vf} = 0{\cdot}120$	μ_2	S.85	S.86	S.87	S.88
−12μ120	Basic ..	Var.	0·12	0·18	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}120$	μ_2	S.91	S.92	S.93	S.94
00T.048	Var. ..	5	0·12	0·18	$l_v = 0$ $n_{vf} = 0{\cdot}048$	Rotaries	S.97	S.98	S.99	S.100
−12T.048	Var. ..	5	0·12	0·18	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}048$	Rotaries	S.103	S.104	S.105	S.106
00T.120	Var. ..	5	0·12	0·18	$l_v = 0$ $n_{vf} = 0{\cdot}120$	Rotaries	S.109	S.110	S.111	S.112
−12T.120	Var. ..	5	0·12	0·18	$l_v = -0{\cdot}12$ $n_{vf} = 0{\cdot}120$	Rotaries	S.115	S.116	S.117	S.118
K.5	Aeroplane K	5	0·12	0·18	—	l_v n_{vf} (and n_r) ..	S.361	S.362	S.363	S.364
K.10	Aeroplane K	10	0·12	0·18	—	l_v n_{vf} (and n_r) ..	S.379	S.380	S.381	S.382

Relations between Derivatives

$$n_v = n_{vo} + n_{vf}$$

$$n_v = -\, l n_{vf}$$

$$k = 0{\cdot}1$$

Values of Fixed Derivatives

Stage		$-y_v$	$-l_p$	l_r	$-n_p$	$-n_{vo}$	l
Basic		0·2	0·42	0·06	0·03	0·024	1
Transition	(i)	0·2	0·42	0·06	0·015	0·024	1
	(ii)	0·2	0·42	0·052	0·015	0·024	1
	(iii)	0·2	0·54	0·052	0·015	0·024	1
	(iv)	0·233	0·54	0·052	0·015	0·024	1
	(v)	0·233	0·54	0·052	0·015	0·024	0·652
Aeroplane K ..		0·233	0·54	0·052	0·015	0·033	0·652

1

Aeroplanes at High Speeds ($C_L = 0\cdot2$)

nd Data

		Response to Rolling Moment						Response to Yawing Moment					
ψ	y	p	v	r	φ	ψ	y	p	v	r	φ	ψ	y
S.5	S.6	S.121	S.122	S.123	S.124	S.125	S.126	S.241	S.242	S.243	S.244	S.245	S.246
S.11	S.12	S.127	S.128	S.129	S.130	S.131	S.132	S.247	S.248	S.249	S.250	S.251	S.252
S.17	S.18	S.133	S.134	S.135	S.136	S.137	S.138	S.253	S.254	S.255	S.256	S.257	S.258
S.23	S.24	S.139	S.140	S.141	S.142	S.143	S.144	S.259	S.260	S.261	S.262	S.263	S.264
S.29	S.30	S.145	S.146	S.147	S.148	S.149	S.150	S.265	S.266	S.267	S.268	S.269	S.270
S.35	S.36	S.151	S.152	S.153	S.154	S.155	S.156	S.271	S.272	S.273	S.274	S.275	S.276
S.41	S.42	S.157	S.158	S.159	S.160	S.161	S.162	S.277	S.278	S.279	S.280	S.281	S.282
S.47	S.48	S.163	S.164	S.165	S.166	S.167	S.168	S.283	S.284	S.285	S.286	S.287	S.288
S.53	S.54	S.169	S.170	S.171	S.172	S.173	S.174	S.289	S.290	S.291	S.292	S.293	S.294
S.59	S.60	S.175	S.176	S.177	S.178	S.179	S.180	S.295	S.296	S.297	S.298	S.299	S.300
S.65	S.66	S.181	S.182	S.183	S.184	S.185	S.186	S.301	S.302	S.303	S.304	S.305	S.306
S.71	S.72	S.187	S.188	S.189	S.190	S.191	S.192	S.307	S.308	S.309	S.310	S.311	S.312
S.77	S.78	S.193	S.194	S.195	S.196	S.197	S.198	S.313	S.314	S.315	S.316	S.317	S.318
S.83	S.84	S.199	S.200	S.201	S.202	S.203	S.204	S.319	S.320	S.321	S.322	S.323	S.324
S.89	S.90	S.205	S.206	S.207	S.208	S.209	S.210	S.325	S.326	S.327	S.328	S.329	S.330
S.95	S.96	S.211	S.212	S.213	S.214	S.215	S.216	S.331	S.332	S.333	S.334	S.335	S.336
S.101	S.102	S.217	S.218	S.219	S.220	S.221	S.222	S.337	S.338	S.339	S.340	S.341	S.342
S.107	S.108	S.223	S.224	S.225	S.226	S.227	S.228	S.343	S.344	S.345	S.346	S.347	S.348
S.113	S.114	S.229	S.230	S.231	S.232	S.233	S.234	S.349	S.350	S.351	S.352	S.353	S.354
S.119	S.120	S.235	S.236	S.237	S.238	S.239	S.240	S.355	S.356	S.357	S.358	S.359	S.360
S.365	S.366	S.367	S.368	S.369	S.370	S.371	S.372	S.373	S.374	S.375	S.376	S.377	S.378
S.383	S.384	S.385	S.386	S.387	S.388	S.389	S.390	S.391	S.392	S.393	S.394	S.395	S.396

Values of n_{vf} and Associated Derivatives

	Basic to Transition (iv)		Transition (v)		Aeroplane K	
n_{vf}	n_v	$-n_r$	n_v	$-n_r$	n_v	$-n_r$
0	$-0\cdot024$	0	—	—	—	—
$0\cdot024$	0	$0\cdot024$	—	—	—	—
$0\cdot048$	$0\cdot024$	$0\cdot048$	$0\cdot024$	$0\cdot0313$	$0\cdot015$	$0\cdot0313$
$0\cdot072$	$0\cdot048$	$0\cdot072$	—	—	—	—
$0\cdot120$	$0\cdot096$	$0\cdot120$	$0\cdot096$	$0\cdot0782$	$0\cdot087$	$0\cdot0782$
$0\cdot084$	—	—	—	—	$0\cdot051$	$0\cdot0549$

Values of l_v

Basic $l_v = 0\cdot06, 0, -0\cdot06, -0\cdot12$

Aeroplane K, $l_v = 0, -0\cdot06, -0\cdot12$

All Other Cases, $l_v = 0, -0\cdot12$

TABLE

Lateral Response of Conventiona
List of Figure

Code Caption	Rotary Derivatives	μ_2	i_A'	i_C'	Parameters Fixed	Parameters Varied	Response to Sideslip p	v	r	φ
C0H	Basic High Lift	20	0·06	0·12	$l_v = 0$	$n_{vf}, n_v, y_v, y_r, n_r$	S.397	S.398	S.399	S.400
−06H	Basic High Lift	20	0·06	0·12	$l_v = -0{\cdot}06$	$n_{vf}, n_v, y_v, y_r, n_r$	S.403	S.404	S.405	S.406
−12H	Basic High Lift	20	0·06	0·12	$l_v = -0{\cdot}12$	$n_{vf}, n_v, y_v, y_r, n_r$	S.409	S.410	S.411	S.412
H. −06	Basic High Lift	20	0·06	0·12	$n_{vf} = -0{\cdot}06$	l_v	S.415	S.416	S.417	S.418
H. −01	Basic High Lift	20	0·06	0·12	$n_{vf} = -0{\cdot}01$	l_v	S.421	S.422	S.423	S.424
H. 07	Basic High Lift	20	0·06	0·12	n_{vf} 0·07	l_v	S.427	S.428	S.429	S.430
−06Hμ −06	Basic High Lift	Var.	0·06	0·12	$l_v = -0{\cdot}06$ $n_{vf} = -0{\cdot}06$	μ_2	S.433	S.434	S.435	S.436
−12Hμ −06	Basic High Lift	Var.	0·06	0·12	$l_v = -0{\cdot}12$ n_{vf} $-0{\cdot}06$	μ_2	S.439	S.440	S.441	S.442
06Hμ 01	Basic High Lift	Var.	0·06	0·12	$l_v = -0{\cdot}06$ n_{vf} $-0{\cdot}01$	μ_2	S.445	S.446	S.447	S.448
−12Hμ C1	Basic High Lift	Var.	0·06	0·12	$l_v = -0{\cdot}12$ n_{vf} $-0{\cdot}01$	μ_2	S.451	S.452	S.453	S.454
H/40	Basic High Lift	40	0·06	0·12	—	l_v, n_{vf}, etc. ..	S.457	S.458	S.459	S.460
H/10	Basic High Lift	10	0·06	0·12	—	l_v, n_{vf}, etc. ..	S.463	S.464	S.465	S.466
−06HI −06	Basic High Lift	20	Var.	Var.	$l_v = -0{\cdot}06$ $n_{vf} = -0{\cdot}06$	i_A' and i_C' ..	S.469	S.470	S.471	S.472
−12HI −06	Basic High Lift	20	Var.	Var.	$l_v = -0{\cdot}12$ $n_{vf} = -0{\cdot}06$	i_A' and i_C' ..	S.475	S.476	S.477	S.478
−06HI 01	Basic High Lift	20	Var.	Var.	$l_v = -0{\cdot}06$ $n_{vf} = -0{\cdot}01$	i_A' and i_C' ..	S.481	S.482	S.483	S.484
−12HI −01	Basic High Lift	20	Var.	Var.	$l_v = -0{\cdot}12$ $n_{vf} = -0{\cdot}01$	i_A' and i_C' ..	S.487	S.488	S.489	S.490
−06HT 06	Var.	20	0·06	0·12	$l_v = -0{\cdot}06$ $n_{vf} = -0{\cdot}06$	Rotaries	S.493	S.494	S.495	S.496
−12HT −06	Var.	20	0·06	0·12	$l_v = -0{\cdot}12$ $n_{vf} = -0{\cdot}06$	Rotaries	S.499	S.500	S.501	S.502
−06HT −01	Var.	20	0·06	0·12	$l_v = -0{\cdot}06$ $n_{vf} = -0{\cdot}01$	Rotaries	S.505	S.506	S.507	S.508
−12HT −01	Var.	20	0·06	0·12	$l_v = -0{\cdot}12$ $n_{vf} = -0{\cdot}01$	Rotaries	S.511	S.512	S.513	S.514

Relations between Derivatives

$$y_v = y_{v0} - \frac{1}{l} n_{vf} \qquad y_r = n_{vf}$$

$$n_v = n_{v0} + n_{vf} \qquad n_r = n_{r0} - l n_{vf}$$

$$k = 1{\cdot}4$$

Values of Fixed Derivatives

Stage	$-l_p$	l_r	$-y_{v0}$	n_{v0}	$-n_{r0}$	$-n_p$	l	i_E
Basic	+0·48	0·6	0·2	0·09	0·33	0·18	1	0
Transition (i)	0·48	0·6	0·4	0·09	0·33	0·18	1	0
Transition (ii)	0·384	0·6	0·2	0·09	0·33	0·18	1	0
Transition (iii)	0·48	0·8	0·2	0·09	0·33	0·18	1	0
Transition (iv)	0·48	0·6	0·2	0·09	0·33	0·27	1	0
Transition (v)	0·48	0·6	0·2	0·09	0·24	0·18	1	0
Product of Inertia	0·48	0·6	0·2	0·09	0·33	0·18	1	0·018

2

Aeroplanes at High Lift ($C_L = 2{\cdot}8$)
and Data

		Response to Rolling Moment						Response to Yawing Moment					
ψ	y	p	v	r	φ	ψ	y	p	v	r	φ	ψ	y
S.401	S.402	S.517	S.518	S.519	S.520	S.521	S.522	S.637	S.638	S.639	S.640	S.641	S.642
S.407	S.408	S.523	S.524	S.525	S.526	S.527	S.528	S.643	S.644	S.645	S.646	S.647	S.648
S.413	S.414	S.529	S.530	S.531	S.532	S.533	S.534	S.649	S.650	S.651	S.652	S.653	S.654
S.419	S.420	S.535	S.536	S.537	S.538	S.539	S.540	S.655	S.656	S.657	S.658	S.659	S.660
S.425	S.426	S.541	S.542	S.543	S.544	S.545	S.546	S.661	S.662	S.663	S.664	S.665	S.666
S.431	S.432	S.547	S.548	S.549	S.550	S.551	S.552	S.667	S.668	S.669	S.670	S.671	S.672
S.437	S.438	S.553	S.554	S.555	S.556	S.557	S.558	S.673	S.674	S.675	S.676	S.677	S.678
S.443	S.444	S.559	S.560	S.561	S.562	S.563	S.564	S.679	S.680	S.681	S.682	S.683	S.684
S.449	S.450	S.565	S.566	S.567	S.568	S.569	S.570	S.685	S.686	S.687	S.688	S.689	S.690
S.455	S.456	S.571	S.572	S.573	S.574	S.575	S.576	S.691	S.692	S.693	S.694	S.695	S.696
S.461	S.462	S.577	S.578	S.579	S.580	S.581	S.582	S.697	S.698	S.699	S.700	S.701	S.702
S.467	S.468	S.583	S.584	S.585	S.586	S.587	S.588	S.703	S.704	S.705	S.706	S.707	S.708
S.473	S.474	S.589	S.590	S.591	S.592	S.593	S.594	S.709	S.710	S.711	S.712	S.713	S.714
S.479	S.480	S.595	S.596	S.597	S.598	S.599	S.600	S.715	S.716	S.717	S.718	S.719	S.720
S.485	S.486	S.601	S.602	S.603	S.604	S.605	S.606	S.721	S.722	S.723	S.724	S.725	S.726
S.491	S.492	S 607	S.608	S.609	S.610	S.611	S.612	S.727	S.728	S.729	S.730	S.731	S.732
S.497	S.498	S.613	S.614	S.615	S.616	S.617	S.618	S.733	S.734	S.735	S.736	S.737	S.738
S.503	S.504	S.619	S.620	S.621	S.622	S.623	S.624	S.739	S.740	S.741	S.742	S.743	S.744
S.509	S.510	S.625	S.626	S.627	S.628	S.629	S.630	S.745	S.746	S.747	S.748	S.749	S.750
S.515	S.516	S.631	S.632	S.633	S.634	S.635	S.636	S.751	S.752	S.753	S.754	S.755	S.756

n_{vf} and Associated Derivatives

n_{vf}	Basic and Product of Inertia Transition (ii) (iii) (iv)				Transition (i)			
n_v	n_v	$-n_r$	$-y_v$	y_r	n_v	$-n_r$	$-y_v$	y_r
$-0{\cdot}06$	$0{\cdot}03$	$0{\cdot}27$	$0{\cdot}14$	$-0{\cdot}06$	$0{\cdot}03$	$0{\cdot}27$	$0{\cdot}34$	$-0{\cdot}06$
$-0{\cdot}01$	$0{\cdot}08$	$0{\cdot}32$	$0{\cdot}19$	$-0{\cdot}01$	$0{\cdot}08$	$0{\cdot}32$	$0{\cdot}39$	$-0{\cdot}01$
$0{\cdot}07$	$0{\cdot}16$	$0{\cdot}40$	$0{\cdot}27$	$0{\cdot}07$	$0{\cdot}16$	$0{\cdot}40$	$0{\cdot}47$	$0{\cdot}07$
	Transition (v)							
$-0{\cdot}06$	$0{\cdot}03$	$0{\cdot}18$	$0{\cdot}14$	$-0{\cdot}06$				
$-0{\cdot}01$	$0{\cdot}08$	$0{\cdot}23$	$0{\cdot}19$	$-0{\cdot}01$				
$0{\cdot}07$	$0{\cdot}16$	$0{\cdot}31$	$0{\cdot}27$	$0{\cdot}07$				

TABLE

Lateral Response of Tailless

List of Figures

Code Caption	k	l_1	l_2	n_1	$\bar{y}_r$	$12n_2$	$\mathscr{L}$	$\mathscr{N}$	Response to Sideslip			
									p	v	τ	
$b_1\frac{1}{4}$	0·1	3·5	0·5	0·25	0·1	2	Var.	$\frac{1}{4}$	S.757	S.758	S.759	S.760
$b_1 1$	0·1	3·5	0·5	0·25	0·1	2	Var.	1	S.763	S.764	S.765	S.766
$b_1 4$	0·1	3·5	0·5	0·25	0·1	2	Var.	4	S.769	S.770	S.771	S.772
$b_2\frac{1}{4}$	0·1	3·5	0·5	0·25	0·1	0	Var.	$\frac{1}{4}$	S.775	S.776	S.777	S.778
$b_2 1$	0·1	3·5	0·5	0·25	0·1	0	Var.	1	S.781	S.782	S.783	S.784
$b_2 4$	0·1	3·5	0·5	0·25	0·1	0	Var.	4	S.787	S.788	S.789	S.790
$b_3\frac{1}{4}$	0·1	3·5	0·5	0·25	0	2	Var.	$\frac{1}{4}$	S.793	S.794	S.795	S.796
$b_3 1$	0·1	3·5	0·5	0·25	0	2	Var.	1	S.799	S.800	S.801	S.802
$b_3 4$	0·1	3·5	0·5	0·25	0	2	Var.	4	S.805	S.806	S.807	S.808
$b_4\frac{1}{4}$	0·1	3·5	0·5	0·25	0	0	Var.	$\frac{1}{4}$	S.811	S.812	S.813	S.814
$b_4 1$	0·1	3·5	0·5	0·25	0	0	Var.	1	S.817	S.818	S.819	S.820
$b_4 4$	0·1	3·5	0·5	0·25	0	0	Var.	4	S.823	S.824	S.825	S.826
$B_1 1$	0·5	3·5	2	0·5	0·1	3	Var.	1	S.973	S.974	S.975	S.976
$B_1 4$	0·5	3·5	2	0·5	0·1	3	Var.	4	S.979	S.980	S.981	S.982
$B_1 16$	0·5	3·5	2	0·5	0·1	3	Var.	16	S.985	S.986	S.987	S.988
$B_2 1$	0·5	3·5	2	0·5	0·1	1	Var.	1	S.991	S.992	S.993	S.994
$B_2 4$	0·5	3·5	2	0·5	0·1	1	Var.	4	S.997	S.998	S.999	S.1000
$B_2 16$	0·5	3·5	2	0·5	0·1	1	Var.	16	S.1003	S.1004	S.1005	S.1006
$B_3 1$	0·5	3·5	2	0·5	0	3	Var.	1	S.1009	S.1010	S.1011	S.1012
$B_3 4$	0·5	3·5	2	0·5	0	3	Var.	4	S.1015	S.1016	S.1017	S.1018
$B_3 16$	0·5	3·5	2	0·5	0	3	Var.	16	S.1021	S.1022	S.1023	S.1024
$B_4 1$	0·5	3·5	2	0·5	0	1	Var.	1	S.1027	S.1028	S.1029	S.1030
$B_4 4$	0·5	3·5	2	0·5	0	1	Var.	4	S.1033	S.1034	S.1035	S.1036
$B_4 16$	0·5	3·5	2	0·5	0	1	Var.	16	S.1039	S.1040	S.1041	S.1042

Relations between Derivatives

None.

Values of $\mathscr{L}$

At $C_L = 0{\cdot}2$ $\quad \mathscr{L} = 0, \frac{1}{4}, 1, 4$

At $C_L = 1{\cdot}0$ $\quad \mathscr{L} = 1, 4, 16$

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Aeroplanes at $C_L = 0{\cdot}2$ and $C_L = 1{\cdot}0$ and Data

		Response to Rolling Moment						Response to Yawing Moment					
ψ	y	p	v	τ	φ	ψ	y	p	v	τ	φ	ψ	y
S.761	S.762	S.829	S.830	S.831	S.832	S.833	S.834	S.901	S.902	S.903	S.904	S.905	S.906
S.767	S.768	8.835	S.836	S.837	S.838	S.839	S.840	S.907	S.908	S.909	S.910	S.911	S.912
S.773	S.774	S.841	S.842	S.843	S.844	S.845	S.846	S.913	S.914	S.915	S.916	S.917	S.918
S.779	S.780	S.847	S.848	S.849	S.850	S.851	S.852	S.919	S.920	S.921	S.922	S.923	S.924
S.785	S.786	S.853	S.854	S.855	S.856	S.857	S.858	S.925	S.926	S.927	S.928	S.929	S.930
S.791	S.792	S.859	S.860	S.861	S.862	S.863	S.864	S.931	S.932	S.933	S.934	S.935	S.936
S.797	S.798	S.865	S.866	S.867	S.868	S.869	S.870	S.937	S.938	S.939	S.940	S.941	S.942
S.803	S.804	S.871	S.872	S.873	S.874	S.875	S.876	S.943	S.944	S.945	S.946	S.947	S.948
S.809	S.810	S.877	S.878	S.879	S.880	S.881	S.882	S.949	S.950	S.951	S.952	S.953	S.954
S.815	S.816	S.883	S.884	S.885	S.886	S.887	S.888	S.955	S.956	S.957	S.958	S.959	S.960
S.821	S.822	S.889	S.890	S.891	S.892	S.893	S.894	S.961	S.962	S.963	S.964	S.965	S.966
S.827	S.828	S.895	S.896	S.897	S.898	S.899	S.900	S.967	S.968	S.969	S.970	S.971	S.972
S.977	S.978	S.1045	S.1046	S.1047	S.1048	S.1049	S.1050	S.1117	S.1118	S.1119	S.1120	S.1121	S.1122
S.983	S.984	S.1051	S.1052	S.1053	S.1054	S.1055	S.1056	S.1123	S.1124	S.1125	S.1126	S.1127	S.1128
S.989	S.990	S.1057	S.1058	S.1059	S.1060	S.1061	S.1062	S.1129	S.1130	S.1131	S.1132	S.1133	S.1134
S.995	S.996	S.1063	S.1064	S.1065	S.1066	S.1067	S.1068	S.1135	S.1136	S.1137	S.1138	S.1139	S.1140
S.1001	S.1002	S.1069	S.1070	S.1071	S.1072	S.1073	S.1074	S.1141	S.1142	S.1143	S.1144	S.1145	S.1146
S.1007	S.1008	S.1075	S.1076	S.1077	S.1078	S.1079	S.1080	S.1147	S.1148	S.1149	S.1150	S.1151	S.1152
S.1013	S.1014	S.1081	S.1082	S.1083	S.1084	S.1085	S.1086	S.1153	S.1154	S.1155	S.1156	S.1157	S.1158
S.1019	S.1020	S.1087	S.1088	S.1089	S.1090	S.1091	S.1092	S.1159	S.1160	S.1161	S.1162	S.1163	S.1164
S.1025	S.1026	S.1093	S.1094	S.1095	S.1096	S.1097	S.1098	S.1165	S.1166	S.1167	S.1168	S.1169	S.1170
S.1031	S.1032	S.1099	S.1100	S.1101	S.1102	S.1103	S.1104	S.1171	S.1172	S.1173	S.1174	S.1175	S.1176
S.1037	S.1038	S.1105	S.1106	S.1107	S.1108	S.1109	S.1110	S.1177	S.1178	S.1179	S.1180	S.1181	S.1182
S.1043	S.1044	S.1111	S.1112	S.1113	S.1114	S.1115	S.1116	S.1183	S.1184	S.1185	S.1186	S.1187	S.1188

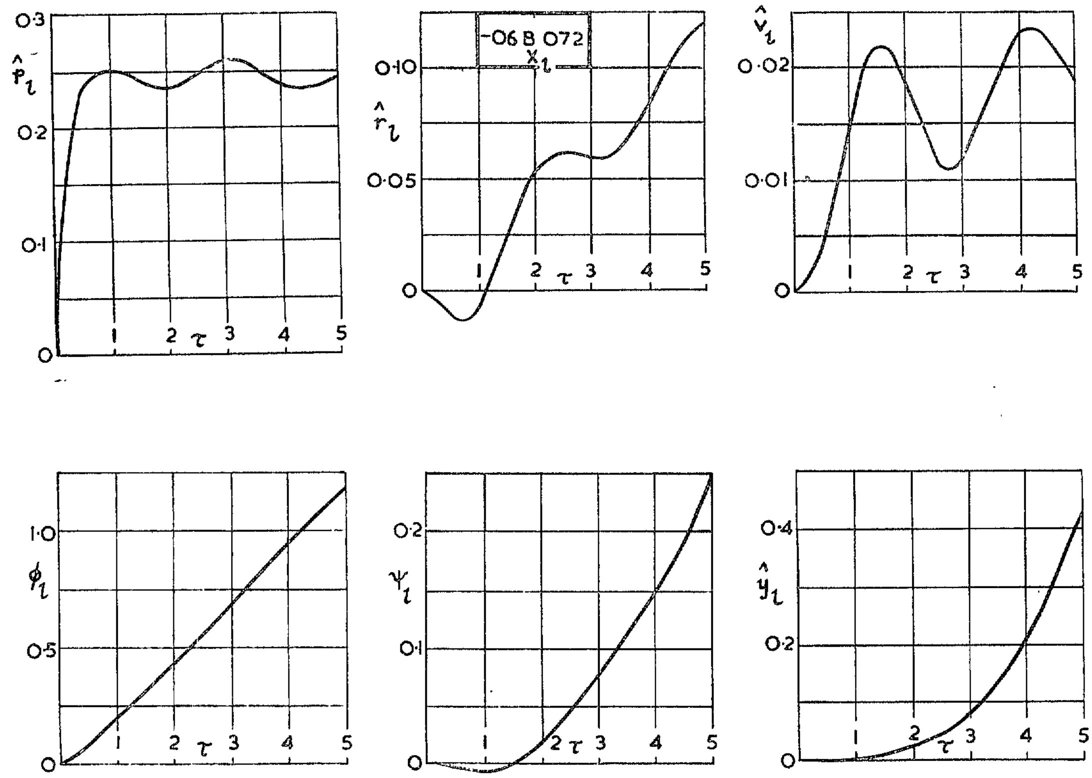


FIG. 1. Response to Unit Constant Rolling Moment (Fundamental Machine Solution).

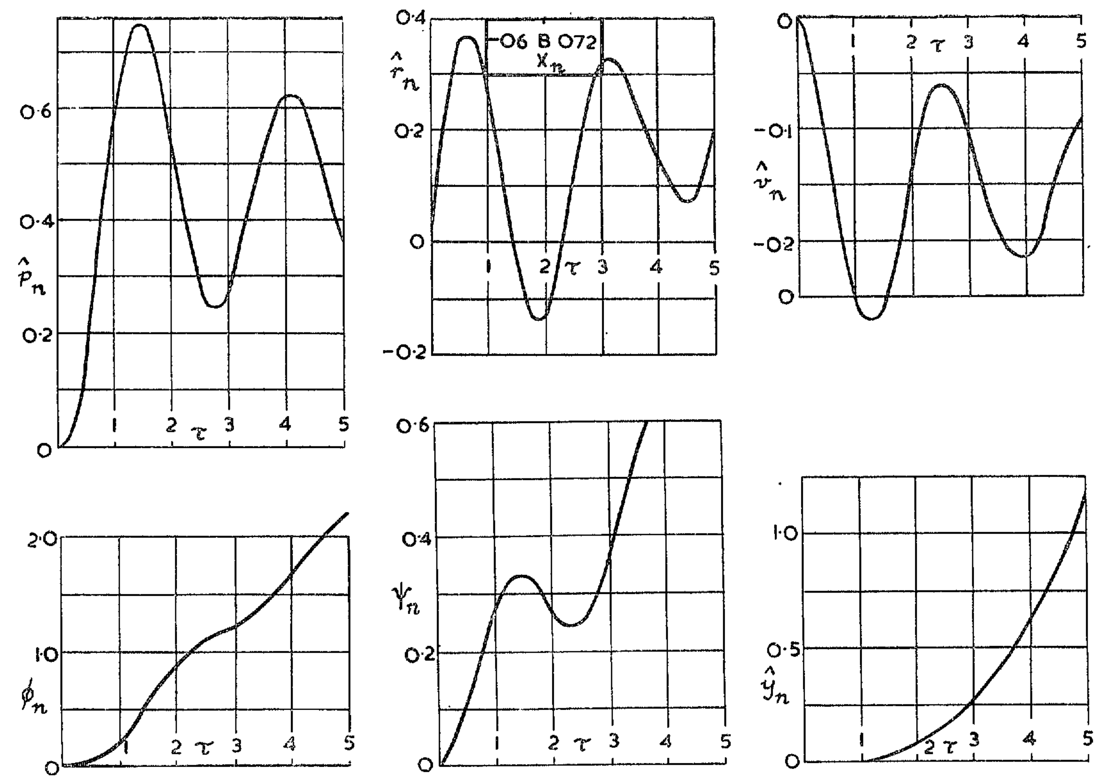


FIG. 2. Response to Unit Constant Yawing Moment (Fundamental Machine Solution).

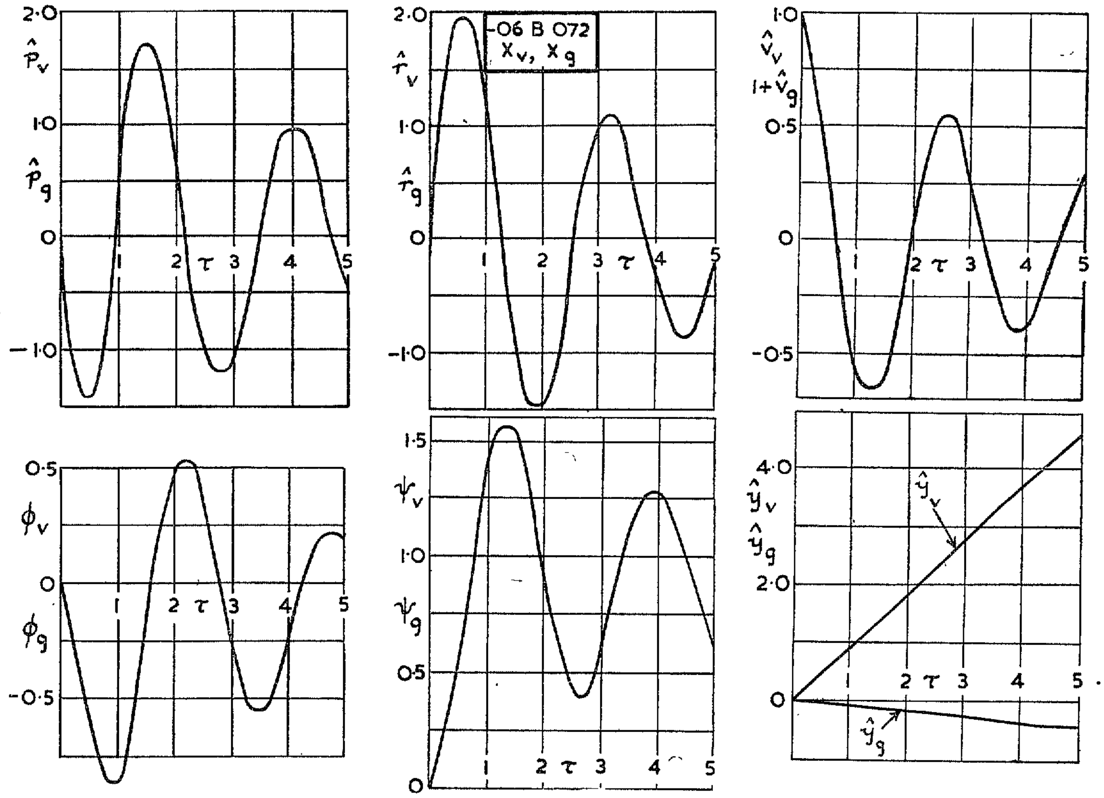


FIG. 3. Response to Unit Initial Sideslip or Sidegust (Fundamental Machine Solution).

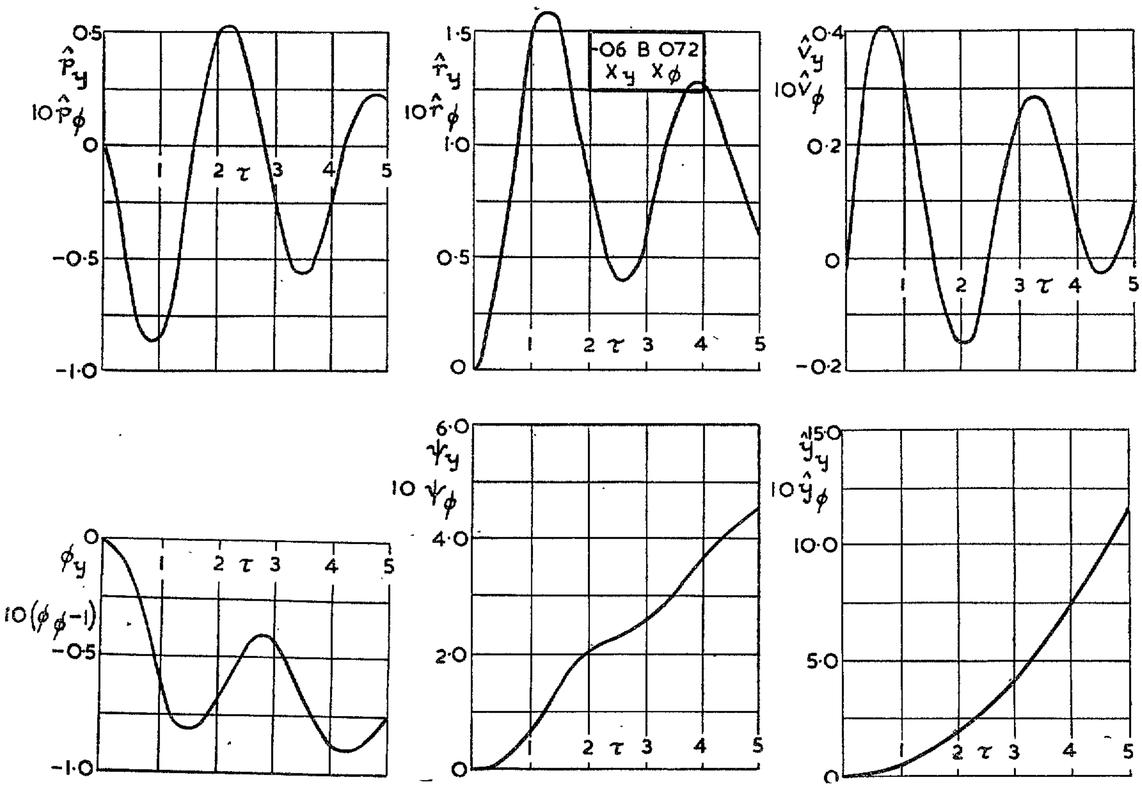


FIG. 4. Response to Unit Constant Sideforce and to Unit Initial Angle of Bank (Derived Solution).

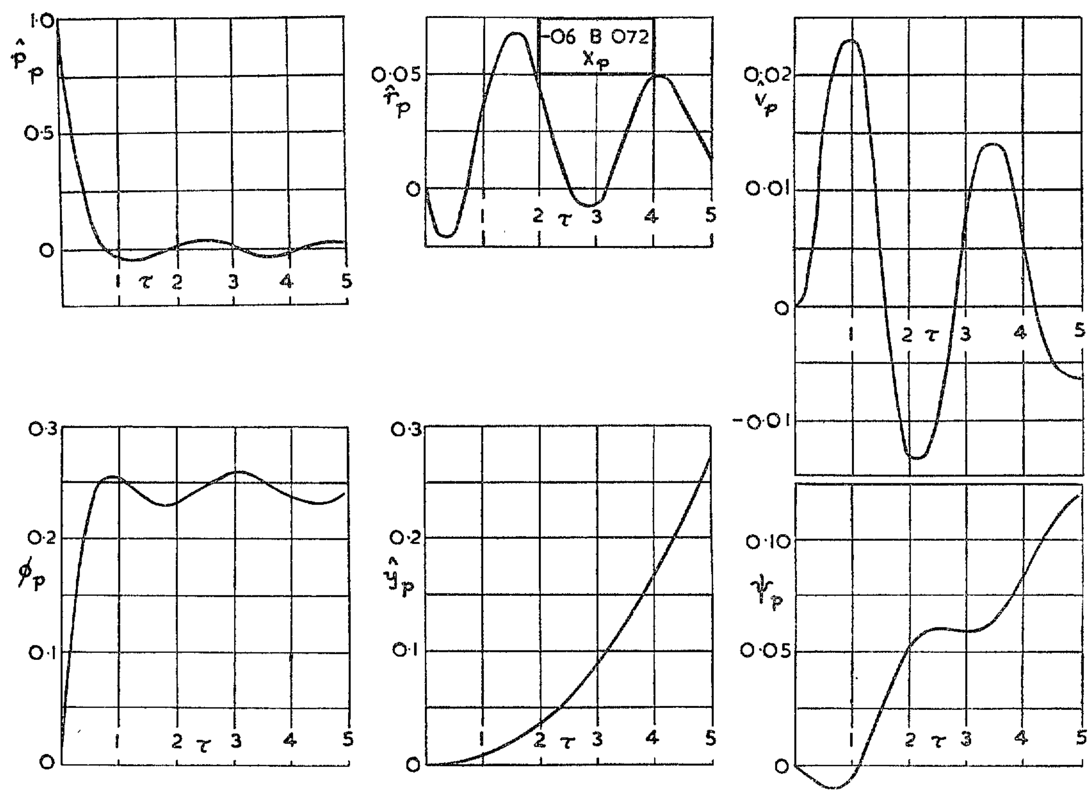


FIG. 5. Response to Unit Initial Rate of Roll (Derived Solution).

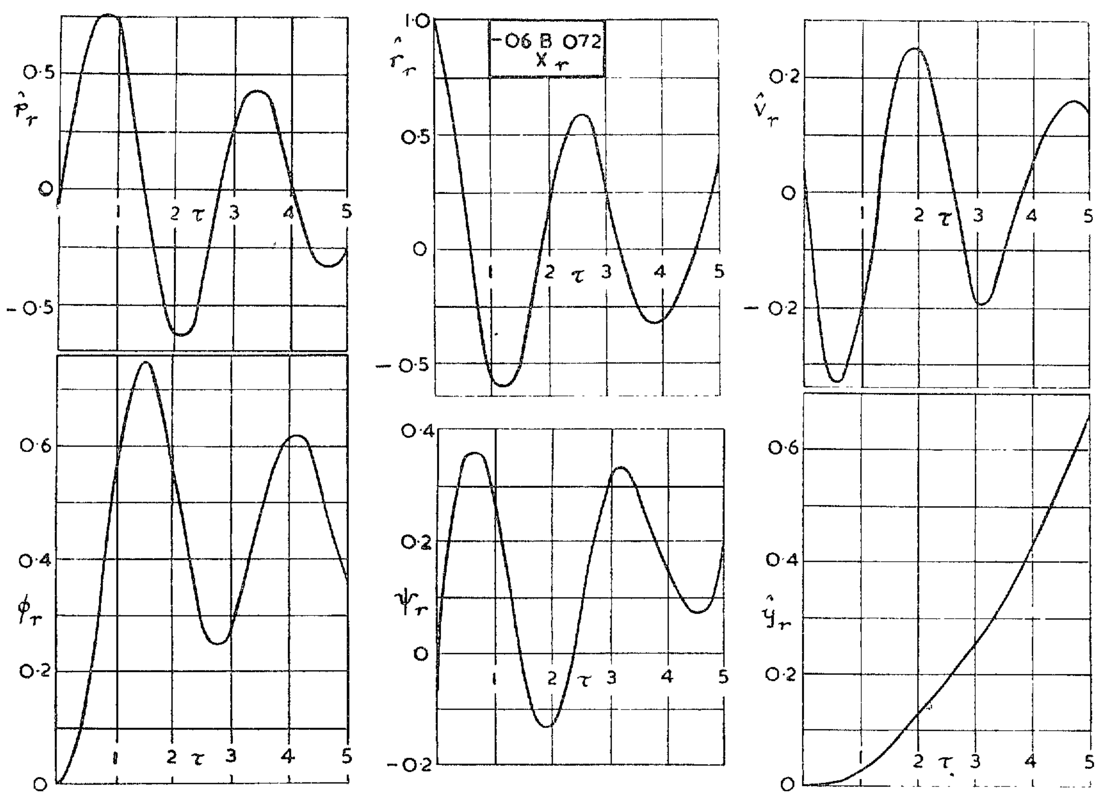
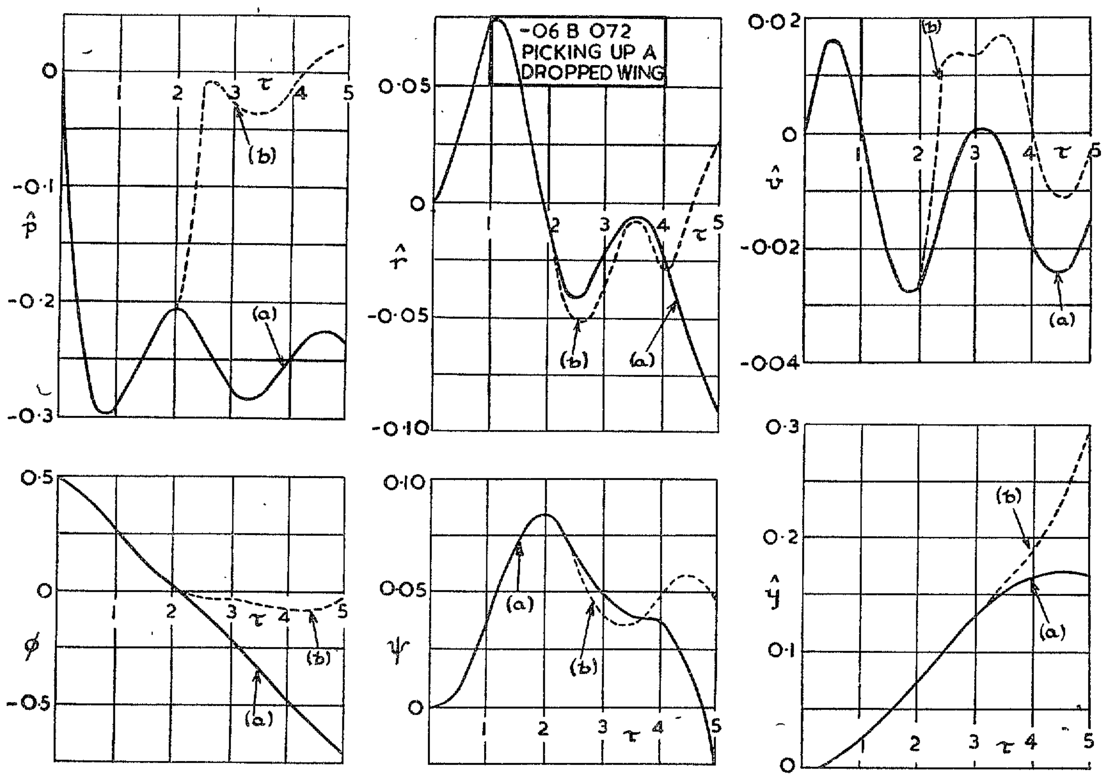


FIG. 6. Response to Unit Initial Rate of Yaw (Derived Solution).



(*a*) Full line : rolling moment constant.

(*b*) Dotted line : controls centralised when $\phi = 0$.

FIG. 7. Picking Up a Dropped Wing.

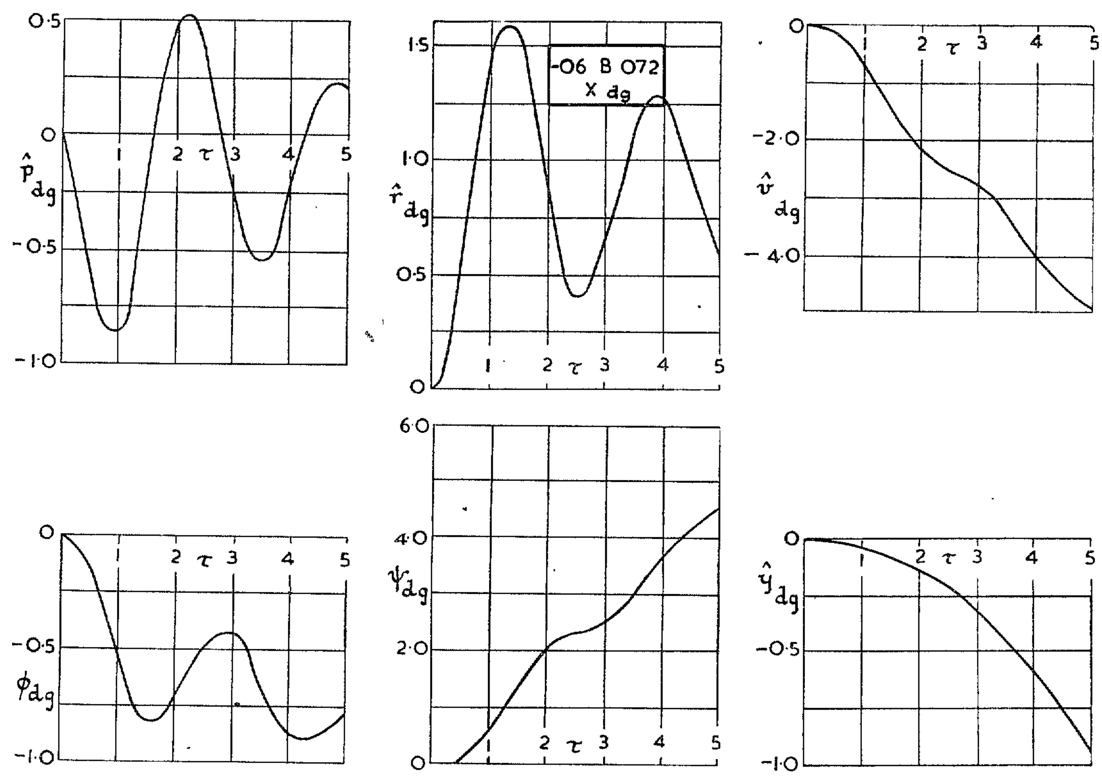


FIG. 8. Response to Linearly Increasing Sidegust (Derived Solution).

(78824) Wt. 10/7116 3/48 Hw. G.377/1

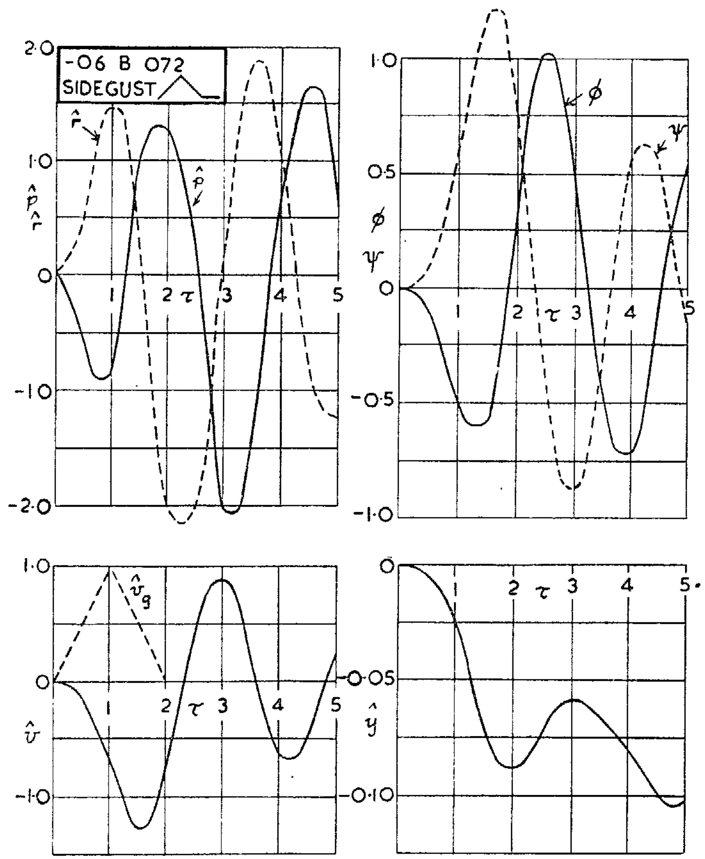


FIG. 9. Response to On–off Graded Gust, Duration 2 sec., Peak Value Unity at 1 sec., Linear Rise and Fall (Derived Solution).

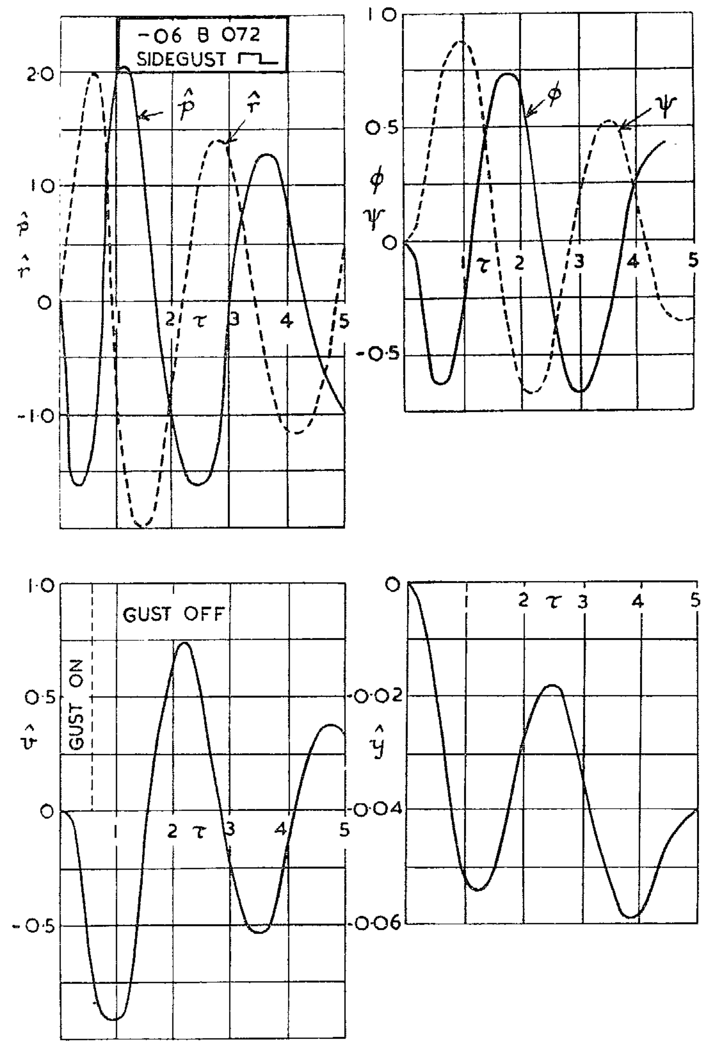


FIG. 10. Response to Sharp Edged Unit Sidegust, Duration ½ airsec. (Derived Solution).

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