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A Theoretical Investigation into the Lateral Stability of an Aeroplane Controlled by an Automatic Pilot, with Particular Reference to the Effect of Flight Path Angle

By

T. W. PRESCOTT, M.ENG.

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Summary.—Several autopilots produce aileron deflection proportional to the movement between the aeroplane and the outer gimbal of a vertical gyroscope. In non-level flight this relative movement is not equal to the rotation of the aeroplane about its x-axis, and it was desirable to investigate the lateral stability for steep angles of climb and dive.

Calculations show that instability does occur, but that stability can be restored either by making the rudder deflection dependent on aileron movement in order to counteract the aileron drag coefficient, or by adding a rate of yaw term to the rudder circuit. The addition of both aileron and rate terms to the rudder circuit is greatly superior to the addition of either term alone.

The aileron drag coefficient can also have a detrimental effect at the start of an automatic turn, and response curves during entry into the turn have been calculated for various degrees of aileron drag compensation. The bank angle and sideslip response curves are unaffected by the compensation. The rate of turn response is improved during the first second but subsequently is little affected by aileron drag compensation.

1. *Introduction.*—1.1. In an automatic pilot using a vertical axis gyroscope for defining the vertical it is customary to move the ailerons proportional to the angle between the aeroplane and the outer gimbal ring. In level flight this will give the conventional system of aileron applied being proportional to the bank angle of the aeroplane.

In non-level flight, however, the deflection of the outer gimbal is $\{\phi + \tan^{-1}(\tan\gamma_e \sin\psi)\}$ where

ϕ is rotation of aeroplane about its x-axis,

ψ is rotation of aeroplane about its z-axis,

γ_e is the aeroplane's angle of climb.

Provided that ψ and $\psi\tan\gamma_e$ are small the outer gimbal deflection is approximately $(\phi + \psi\tan\gamma_e)$, and so the aileron equation becomes

$$\begin{aligned}\xi &= F(\phi + \psi\tan\gamma_e)\\ &= F_\phi\phi + F_\psi\psi. \qquad (1)\end{aligned}$$

The rudder equation considered is of the form

$$\zeta = H_\psi\psi, \qquad (2)$$

where ξ is amount of aileron applied,

ζ is amount of rudder applied,

F, H_ψ, are autopilot parameters.

* R.A.E. Technical Note I.A.P. 974—received 15th April, 1948.

It is known that a roll control of the form

$$\xi = F_{\phi} \phi + F_{\psi} \psi \quad \ldots \quad (3)$$

produces an unstable oscillation as F_{ψ} is increased positively. This instability is mainly due to the aileron drag coefficient, and it was desired to know at what angle of climb instability occurred. Making F_{ψ} increasingly negative in equation (3) results in a subsidence becoming negatively damped, and so instability was also feared in a dive ($\tan \gamma_e$ is negative).

In this report methods of stabilizing the aeroplane's motion are considered, including aileron drag compensation, and the addition of a rate of yaw term to the rudder equation.

1.2. The effect of the aileron drag coefficient at the start of an automatic turn has been investigated using the lateral control equations for an autopilot now under development. Level flight has been assumed as this autopilot also suffers from the defect mentioned in section 1.1. Exact compensation and over compensation of $\mathcal{N}_{\xi}$, the aileron drag coefficient, have been tried in an attempt to improve the entry into an automatic turn, and the results are included in this report.

2. *Lateral Stability in Climb and Dive.*—In the calculations the aerodynamic derivatives used are those of a Meteor jet fighter flying at 600 m.p.h. at sea level. Variation of the derivatives proportional to the lift coefficient C_L, (k, k_1, n_1, and l_2) with angles of climb and dive has been neglected, as stability boundary calculations showed it to have very little effect on the stability of the system. Similarly, changing the climbing speed to 300 m.p.h. at sea level also had little effect on stability.

2.1. *Conventional Displacement Control.*—The control equations considered,

$$\xi = 2(\phi + \psi \tan \gamma_e),$$

and

$$\zeta = 4\psi,$$

give a fifth order stability equation when combined with the non-dimensional form of the lateral equations of motion of the aeroplane (4).

$$\left.\begin{aligned} &\hat{v}' + \hat{v}\bar{y}_v + \psi' - k\phi - k_1\psi = 0 \\ &\phi'' + l_1\phi' - l_2\psi' + \mathcal{L}\hat{v} + \mathcal{L}_{\xi}\xi = 0 \\ &\psi'' + n_2\psi' + n_1\phi' - \mathcal{N}\hat{v} + \mathcal{N}_{\zeta}\zeta - \mathcal{N}_{\xi}\xi = 0 \end{aligned}\right\} \quad \ldots \quad (4)$$

In equation (4), dashes denote differentiation with respect to τ, the time measured in airsecs (one airsec is $\hat{t}$ true sec) ; $\hat{v} = v/U_e$; and the aerodynamic derivatives $\bar{y}_v$, k, l_1, etc. are defined by Mitchell[1]. The unit of aerodynamic time, $\hat{t}$, is given by the equation

$$\hat{t} = \frac{m}{\rho S U_e} \text{ true sec,}$$

where m is the mass of the aeroplane (slugs),
S is the wing area (sq ft),
U_e is the forward speed of the aeroplane (ft/sec),
ρ is the air density (slugs/cu ft).

The fifth order stability discriminant yields a poorly damped subsidence, a well damped 'roll' oscillation, and a poorly damped 'yaw' oscillation. Thus for the level flight case the stability factors are

$$(\lambda + 0{\cdot}1639)\,(\lambda^2 + 6{\cdot}4200\lambda + 112{\cdot}3189)\,(\lambda^2 + 0{\cdot}3991\lambda + 51{\cdot}2717)$$
$$= (\lambda + 0{\cdot}1639)\,(\lambda + 3{\cdot}2100 \pm 10{\cdot}1002i)\,(\lambda + 0{\cdot}1996 \pm 7{\cdot}1576i)$$

The damping factors are plotted in Fig. 1 for $\gamma_e = 70$, 0, and -70 deg, and it can be seen that the yaw oscillation becomes unstable at an angle of climb of 27 deg. This instability at such a small angle of climb is mainly due to the aileron drag coefficient, $\mathcal{N}_{\xi}$.

2.2. *Addition of a Rate of Yaw Term to the Rudder Equation.*—The rudder equation was modified to

$$\zeta = 4\psi + 0\cdot 98\,\psi', \qquad (5)$$

and the damping factors of the resulting motion plotted in Fig. 2 for $\gamma_e = 70$, 0 and -70 deg. The addition of the rate term $0\cdot 98\psi'$ increases the total damping in the system by an amount $0\cdot 98\mathscr{N}_\zeta = 10\cdot 78$ ($\mathscr{N}_\zeta = 11$ for a Meteor flying at sea level), and it was expected that all this extra damping would appear on the yaw oscillation. In fact this happens when $\gamma_e = 0$, the stability factors being

$$(\lambda + 0\cdot 1639)\,(\lambda^2 + 6\cdot 4200\lambda + 112\cdot 3189)\,(\lambda^2 + 0\cdot 3991\lambda + 51\cdot 2717)$$

and

$$(\lambda + 0\cdot 1629)\,(\lambda^2 + 6\cdot 5193\lambda + 112\cdot 6298)\,(\lambda^2 + 11\cdot 0808\lambda + 51\cdot 4371),$$

without and with the rate term respectively. In a climb, however, the yaw oscillation also acquires some damping from the roll oscillation, whilst in a dive it loses some damping to the roll oscillation. The poorly damped subsidence is unaltered by either the change in γ_e or the addition of the rate term.

It may happen that the amount of rate term that can be injected into the rudder circuit is limited by physical considerations, and it is uncertain how much of it may be necessary to overcome the inherent phase lags in the system. Hence there may not be enough rate term available in the rudder equation to restore stability in a steep climb.

2.3. *Compensation for the Aileron Drag Coefficient.*—By moving the rudder an amount proportional to the aileron deflection it is possible to compensate for the aileron drag coefficient, $\mathscr{N}_\xi$, of the aeroplane. The rudder equation becomes

$$\zeta = 4\psi + H_\xi \xi, \qquad (6)$$

where H_ξ is an autopilot parameter, exact compensation for $\mathscr{N}_\xi$ being obtained when $H_\xi \mathscr{N}_\zeta = \mathscr{N}_\xi$. For the Meteor flying at sea level, $\mathscr{N}_\xi = 3$, and therefore for exact compensation, $H_\xi = 3/11 = 0\cdot 2727$. The damping factors of the motion when $H_\xi = 0\cdot 2727$ are plotted in Fig. 3 for γ_e from -70 deg to $+70$ deg. In level flight the damping factors are very nearly equal to those of the uncompensated case, but are practically unaltered by change in γ_e. However, the damping of the yaw oscillation decreases slightly as γ_e increases positively and the oscillation becomes unstable at a climb angle of 87 deg, a great improvement on the 27 deg climb angle of the uncompensated case. The instability is now mainly caused by the aerodynamic derivative n_p (the yawing moment due to rate of roll).

In Fig. 4, $H_\xi = 0\cdot 5454$ (100 per cent over compensation for $\mathscr{N}_\xi$), and the damping factors are plotted for $\gamma_e = 70$, 0, and -70 deg. In the over-compensated case the yaw oscillation becomes unstable when the aeroplane is in a dive; but as γ_e increases positively, the yaw oscillation becomes better-damped at the expense of the roll oscillation. The subsidence is unaffected by change in either H_ξ or γ_e.

2.4. *Addition of Both Rate and Aileron Terms to the Rudder Equation.*—The rudder equation used was

$$\zeta = 4\psi + 0\cdot 98\psi' + H_\xi \xi. \qquad (7)$$

The damping factors are plotted in Figs. 5 and 6 for $H_\xi = 0\cdot 2727$ and $0\cdot 5454$ respectively, with γ_e varied from -70 deg to $+70$ deg. Again, with exact compensation of the aileron drag coefficient, variation in γ_e has little effect on the stability of the motion. The extra damping added to the system by the rate term has all appeared on the yaw oscillation without affecting either the roll oscillation or the subsidence. Hence by combining exact aileron drag compensation with the addition of a rate of yaw term to the rudder equation it is possible to obtain the benefits of both sections 2.2 and 2.3. Over compensation of $\mathscr{N}_\xi (H_\xi = 0\cdot 5454)$ has the same destabilizing effect in dives as under compensation has in climbs.

3. *Automatic Turns Entry.*—The proposed lateral control equations of an autopilot are

$$\xi = 0 \cdot 7905\phi' + 3\phi - 2\phi_D + 2 \cdot 53 \int (\phi - \phi_D) d\tau, \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

and

$$\zeta = 0 \cdot 5\psi' - 1 \cdot 0 \int \hat{v} d\tau + H_\xi \xi, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

where ϕ_D is a constant under the control of the human pilot. ϕ_D does not affect the stability of the system, and is zero in straight flight. If the pilot wishes to execute an automatic turn he changes ϕ_D by moving a controller. This causes aileron to be applied in response to which the aeroplane will roll and finally execute a co-ordinated turn at a bank angle equal to ϕ_D. Aileron and rudder to trim in the turn are supplied by the appropriate integral terms in equations (8) and (9).

If $H_\xi = 0$ no rudder is applied at $\tau = 0$, the commencement of the turn. But since aileron is applied at $\tau = 0$ the adverse yawing moment due to the aileron drag makes the aeroplane turn the wrong way. This causes the aeroplane to sideslip and rudder will be applied to make the aeroplane turn in the right direction. Hence with no compensation for $\mathcal{N}_\xi$ the turns entry is not particularly smooth (*see* Fig. 7).

Response curves for $\hat{v}$, ϕ, and ψ', using the above two control equations are plotted in Figs. 7, 8, and 9, respectively for $H_\xi = 0$, $0 \cdot 2727$, and $0 \cdot 5454$. These values of H_ξ are equivalent to no compensation, exact compensation, and 100 per cent over compensation for $\mathcal{N}_\xi$. The aeroplane derivatives used in these calculations are for a Meteor flying at 110 m.p.h. at sea level with flaps and undercarriage down as it was considered that these derivatives would produce the worst turns entries.

On examination of Figs. 7, 8, and 9, it is seen that change in H_ξ has very little effect on the ϕ and $\hat{v}$ curves. At the larger values of H_ξ the initial ψ' motion is improved but after one second change in H_ξ has little effect even on the ψ' motion.

4. *Conclusions.*—(a) With no compensation for aileron drag the motion becomes unstable when the aeroplane is at a climb angle of greater than 27 deg. An unexpected result was the continued stability in a dive. This is probably due to the fact that an uncontrolled aeroplane is more stable in a dive than in level flight (Frayn and Parnell[2]).

(b) Whilst the addition of a rate of yaw term to the rudder circuit can restore stability, physical reasons may limit the amount which can be applied and insufficient may be available when the aeroplane is in a steep climb.

(c) With exact compensation for aileron drag the stability of the motion is uninfluenced by variation in climb and dive angle, but over compensation can lead to instability in a dive.

(d) Exact compensation for aileron drag and the addition of all the available rate of yaw term to the rudder circuit is the best combination to counteract instability in steep climbs and dives.

(e) Exact aileron drag compensation reduces the swing in the wrong direction but otherwise has little effect on the entry into an automatic turn ; over compensation prevents the initial swing in the wrong direction experienced in the uncompensated case.

LIST OF SYMBOLS

Symbol	*Section defined*	*Meaning*
b	Appendix	Aeroplane wing span
C_L	2	Lift coefficient, $mg.\cos \gamma_e / \frac{1}{2}\rho S U_e^2$
γ_e	1.1	Angle of climb in undisturbed flight

Symbol	*Section defined*	*Meaning*
F, F_ϕ, F_ψ	1.1	Autopilot parameters
ϕ	1.1	Angular displacement of aeroplane about x-axis
ϕ_D	3	Bank datum, desired bank angle
g	Appendix	Acceleration due to gravity
H_ψ, H_ξ	1.1, 2.3	Autopilot parameters
i_A	Appendix	Aeroplane inertia coefficient about x-axis
i_C	Appendix	Aeroplane inertia coefficient about z-axis
k	2.1	$\frac{1}{2}C_L$
k_1	2.1	$k \tan \gamma_e$
l_1	2.1	Rotary damping coefficient in roll $- l_p/i_A$
l_2	2.1	l_r/i_A
l_p, l_r, l_v, l_ξ	Appendix	Rolling moments due to rate of roll, rate of yaw, sideslip, and aileron angle
$\mathscr{L}$	2.1	$- \mu_2 l_v/i_A$
$\mathscr{L}_\xi$	2.1	Aileron rolling effect coefficient, $- \mu_2 l_\xi/i_A$
m	2.1	Mass of aeroplane
μ_2	Appendix	Relative density of aeroplane. $2m/\rho Sb$
n_1	2.1	$- n_p/i_C$
n_2	2.1	Rotary damping coefficient in yaw, $- n_r/i_C$
n_p, n_r, n_v, n_ξ, n_ζ	Appendix	Yawing moments due to rate of roll, rate of yaw, sideslip, aileron angle, and rudder angle
$\mathscr{N}$	2.1	$\mu_2 n_v/i_C$
$\mathscr{N}_\xi$	1.2	Aileron yawing effect coefficient, $\mu_2 n_\xi/i_C$
$\mathscr{N}_\zeta$	2.1	Rudder yawing effect coefficient, $- \mu_2 n_\zeta/i_C$
ψ	1.1	Angular displacement of aeroplane about z-axis
ρ	2.1	Air density
S	2.1	Aeroplane wing area
$\hat{t}$	2.1	Unit of time in non-dimensional system, $m/\rho SU_e$
τ	2.1	Time in airsecs
U_e	2.1	Forward speed in undisturbed flight
v	2.1	Component of speed along y-axis (sideslip)
$\hat{v}$	2.1	Non-dimensional form of v, v/U_e (angle of sideslip)
ξ	1.1	Aileron movement from equilibrium position
y_v	Appendix	Non-dimensional form of force component along y-axis due to sideslip
$\bar{y}_v$	2.1	$- y_v$
ζ	1.1	Rudder movement from equilibrium position.

REFERENCES

No.	Author	Title, etc.
1	K. Mitchell	A Supplementary Notation for Theoretical Lateral Stability Investigations. R.A.E. Tech. Note No. Aero. 1183 (Misc.). May, 1943. A.R.C. 6797. (Unpublished).
2	E. M. Frayn and M. V. Parnell	The Theoretical Effect of Flight Path Angle on the Lateral Stability and Response of an Aeroplane. R. & M. 2529. November, 1945.

APPENDIX

Data for Meteor type jet fighter flying at sea level

$\frac{mg}{S} = 31$ lb/sq ft $\qquad \frac{b}{2} = 20$ ft

$\rho = 0\cdot002378$ slugs/cu ft

$i_A = 0\cdot064 \qquad \mu_2 = 21$

$i_C = 0\cdot140$

$y_v = -0\cdot19$

$l_v = -0\cdot03 \qquad l_p = -0\cdot415 \qquad l_r = +0\cdot24C_L$

$n_v = +0\cdot053 \qquad n_p = -0\cdot05C_L \qquad n_r = -0\cdot044$

$l_\xi = -0\cdot18$

$n_\xi = +0\cdot02 \qquad n_\zeta = -0\cdot074$

The value of n_r is derived from flight tests, the wind tunnel value is much higher ($n_r = -0\cdot13$). The low value of n_r was chosen as it gives the worst stability conditions.

	Meteor flying at 600 m.p.h. at sea level	*Meteor flying at 110 m.p.h. at sea level, with flaps and undercarriage down*
U_e ft/sec	880	160
C_L	0.034	1.0
$\hat{t}$	0.46	2.53
k	0.017	0.50
$\bar{y}_v$	0.19	0.19
l_1	6.48	6.48
l_2	0.13	3.75
n_1	0.012	0.357
n_2	0.313	0.313
$\mathscr{L}$	9.84	9.84
$\mathscr{N}$	7.95	7.95
$\mathscr{L}_\xi$	56.0	56.0
$\mathscr{N}_\xi$	3.0	3.0
$\mathscr{N}_\zeta$	11.0	11.0

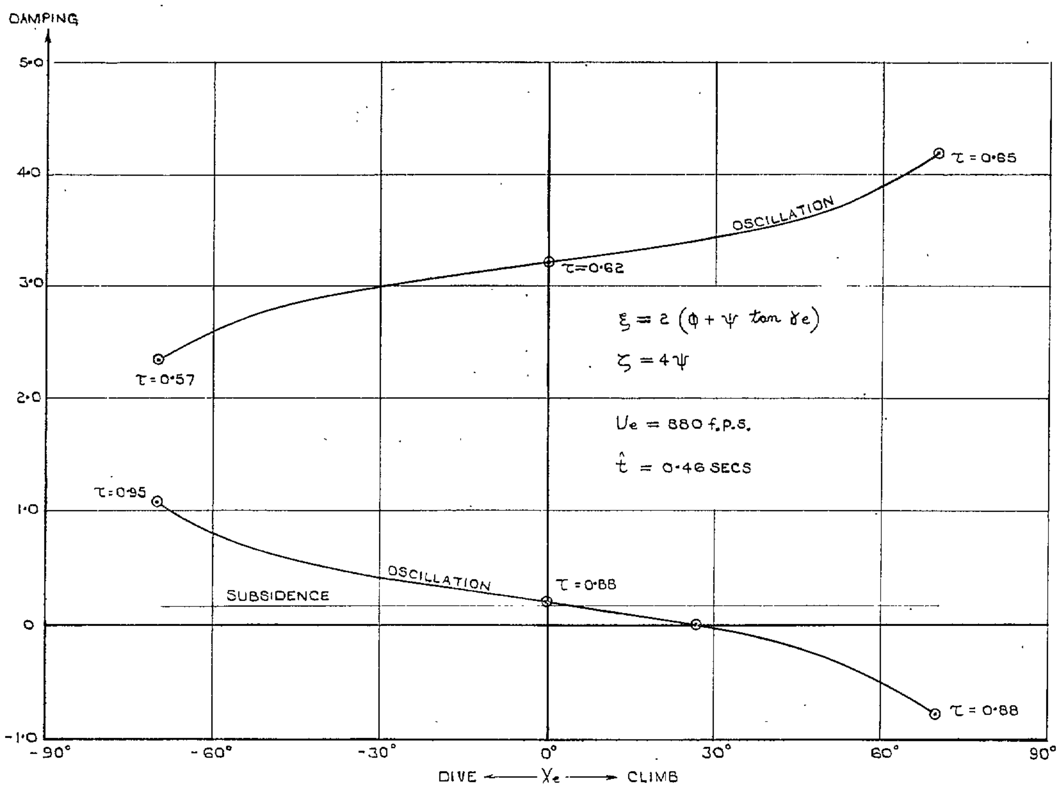

FIG. 1. Lateral stability. Aileron drag uncompensated.

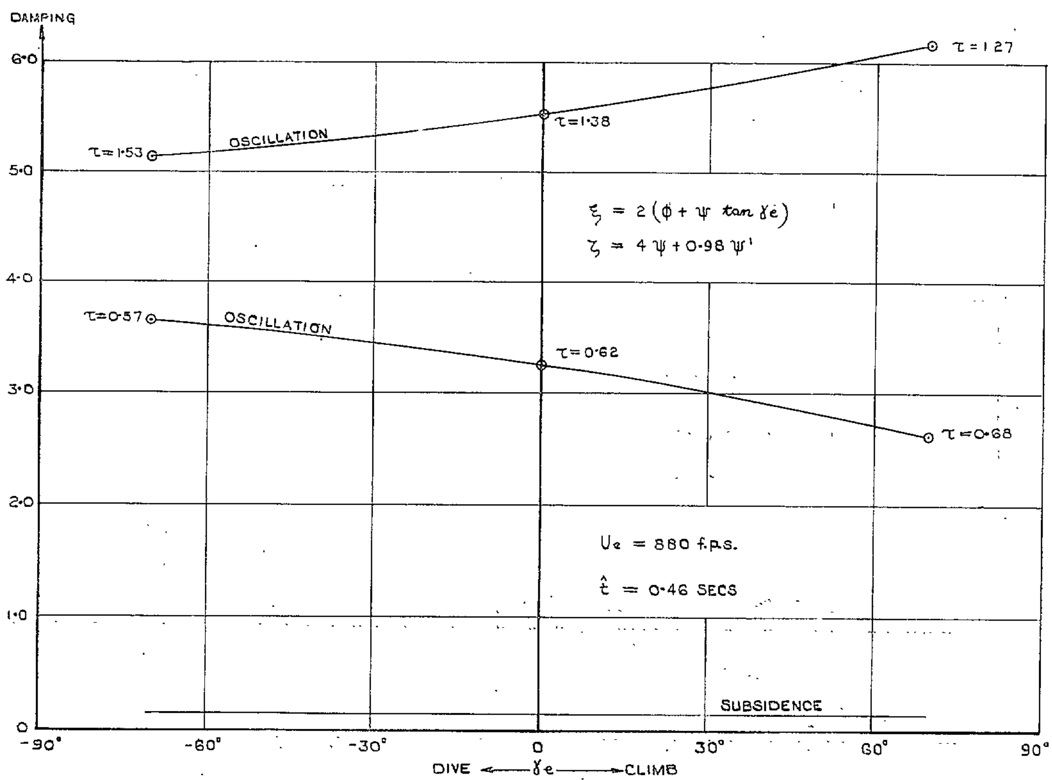

FIG. 2. Lateral stability. Aileron drag uncompensated with addition of rate term.

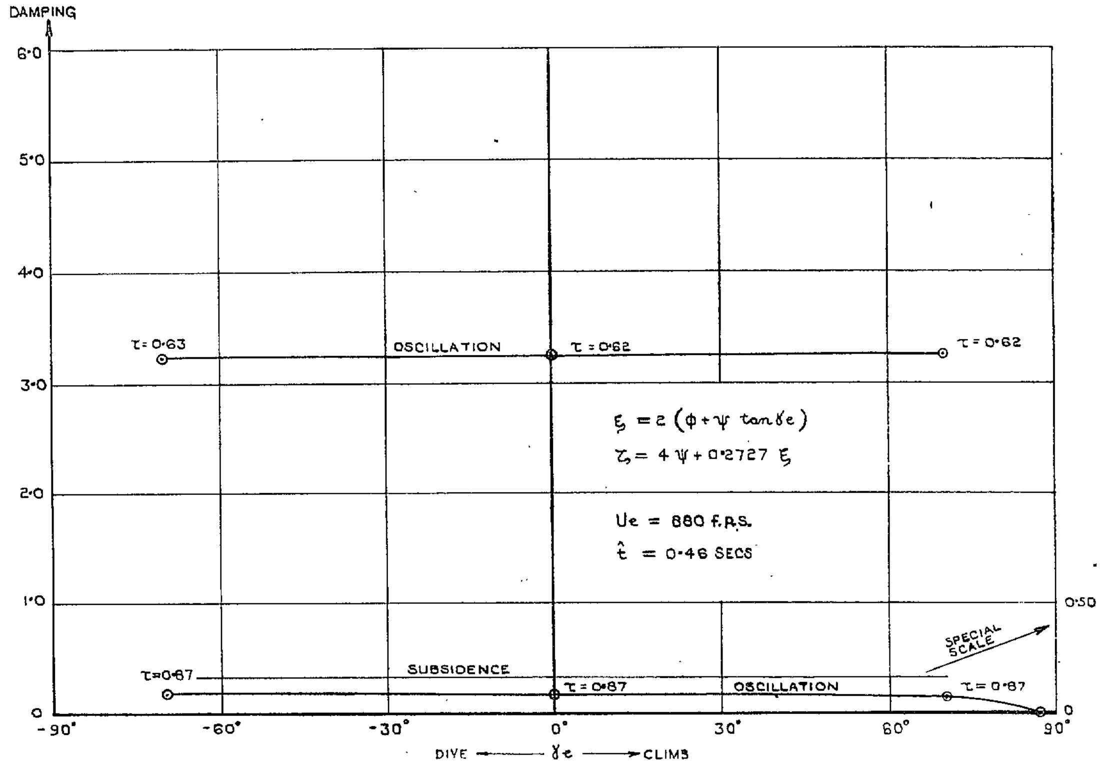

FIG. 3. Lateral stability. Aileron drag compensated.

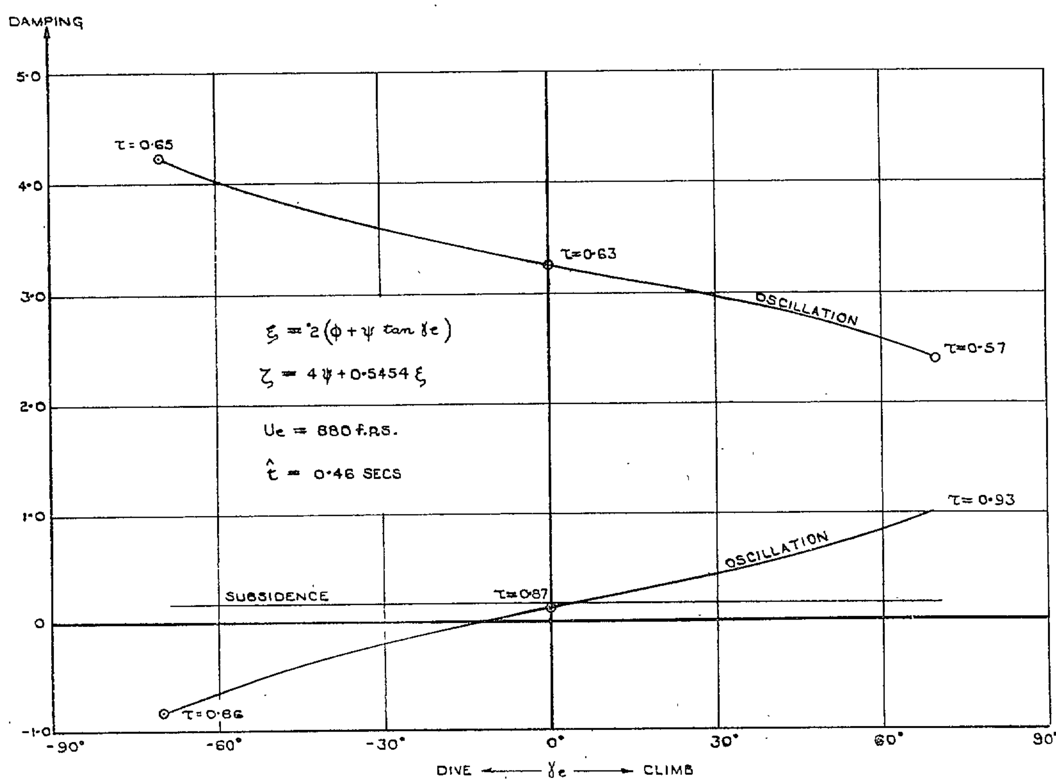

FIG. 4. Lateral stability. Aileron drag over-compensated.

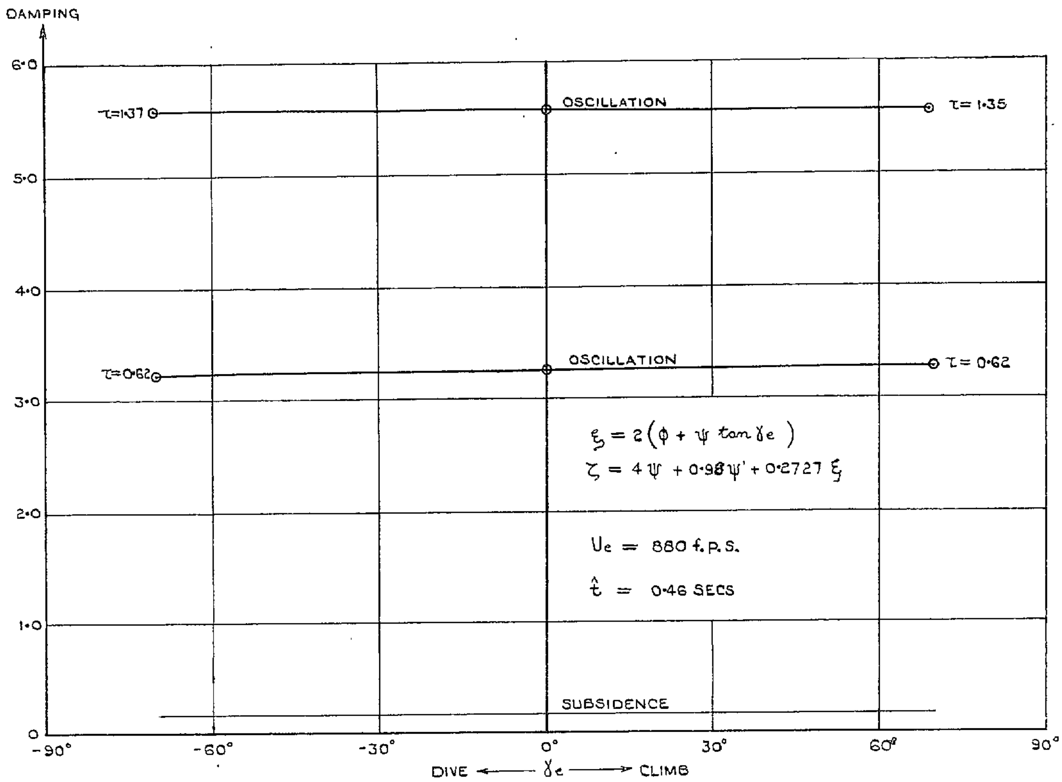

FIG. 5. Lateral stability. Aileron drag compensated with addition of rate term.

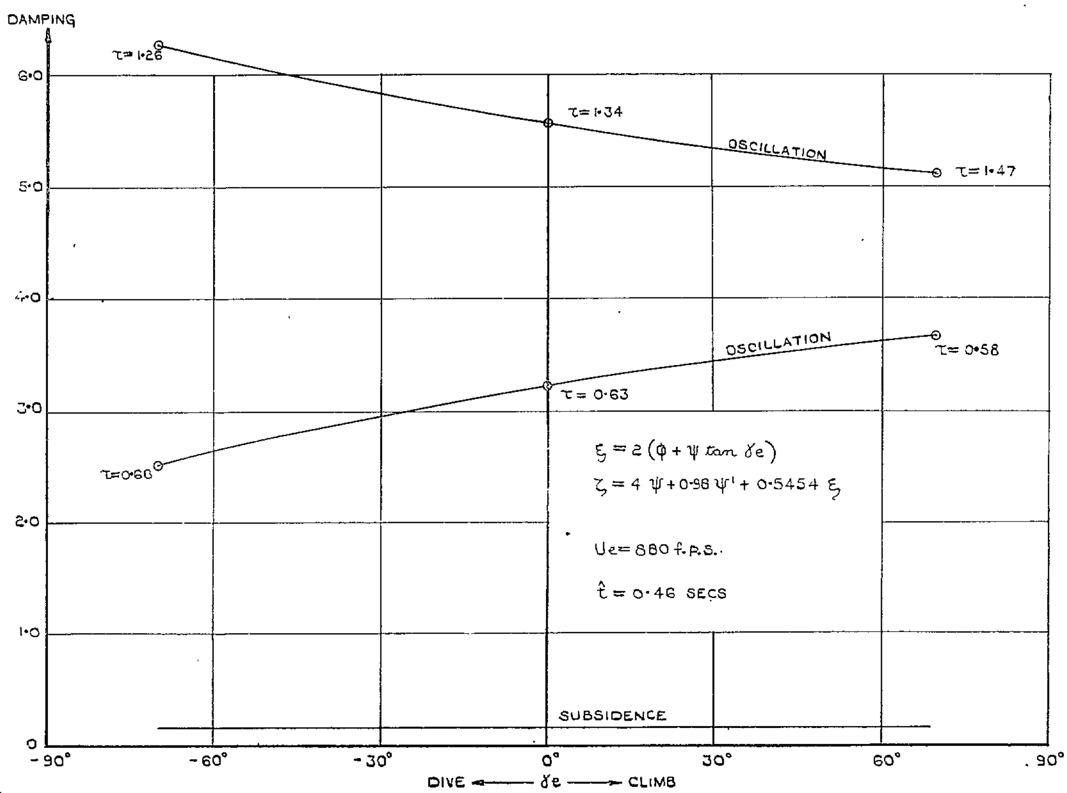

FIG. 6. Lateral stability. Aileron drag over-compensated with addition of rate term.

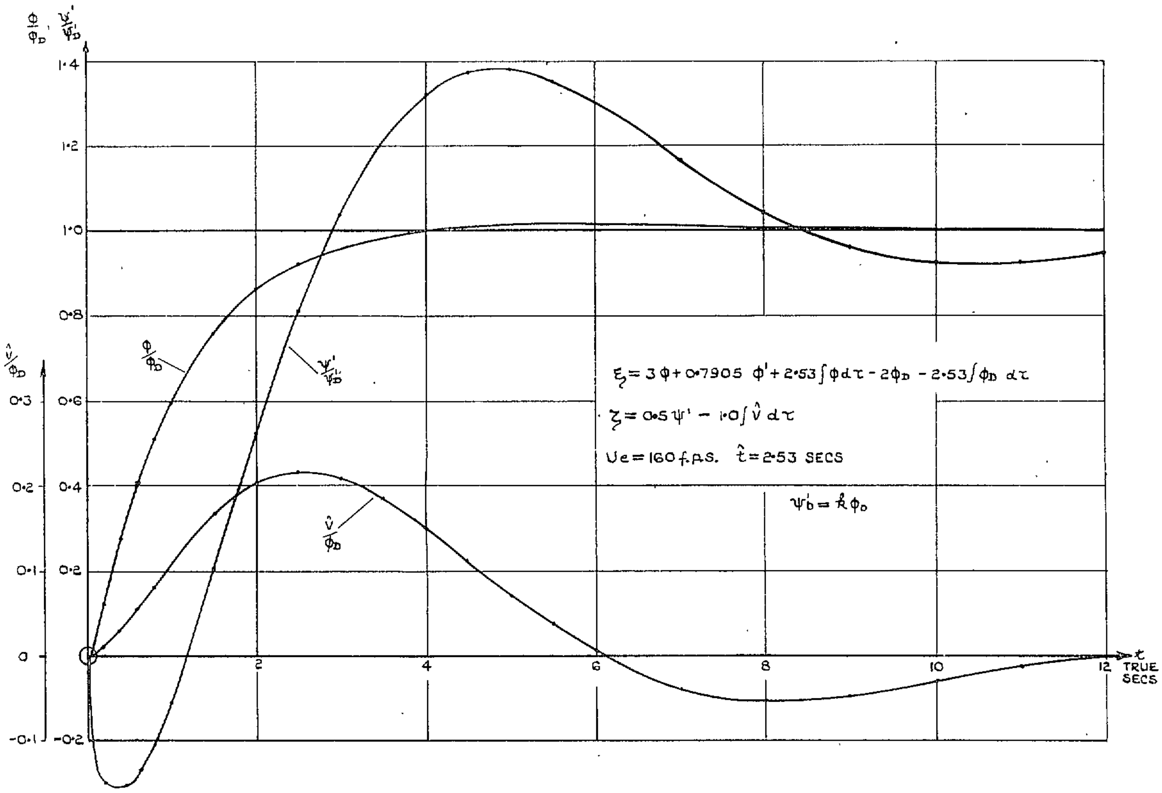

FIG. 7. Turns entry. No compensation for $\mathscr{N}_\xi$.

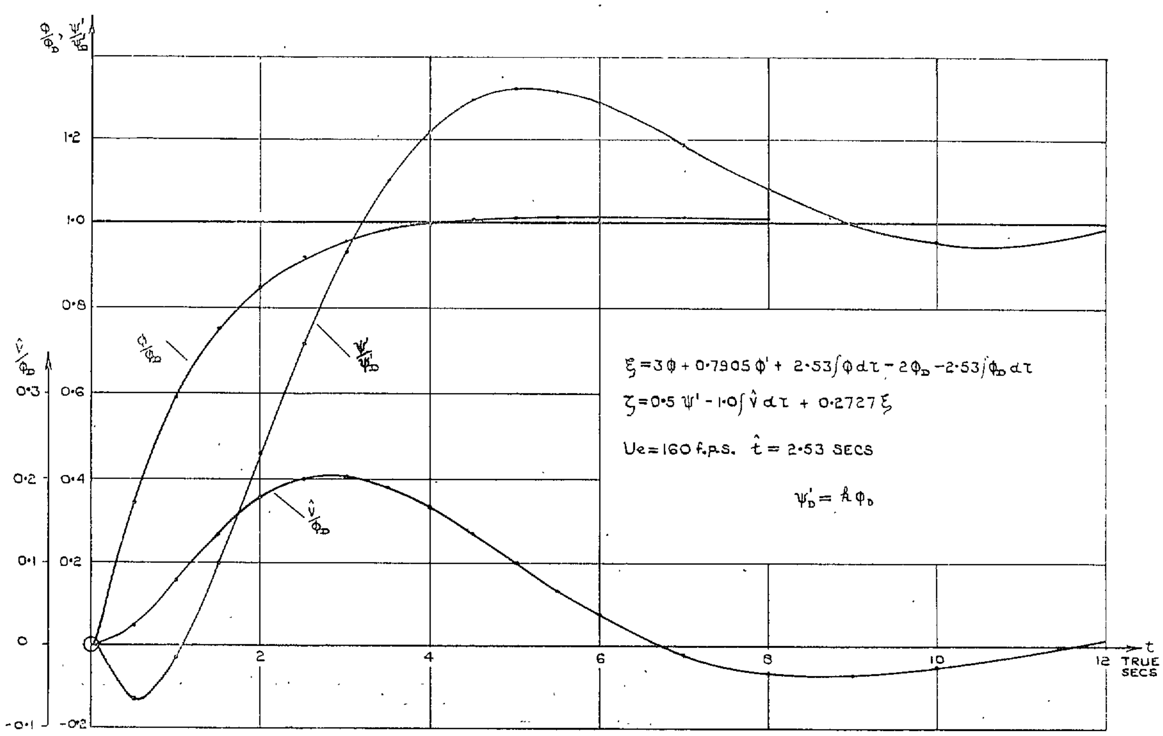

FIG. 8. Turns entry. Exact compensation for $\mathscr{N}_\xi$.

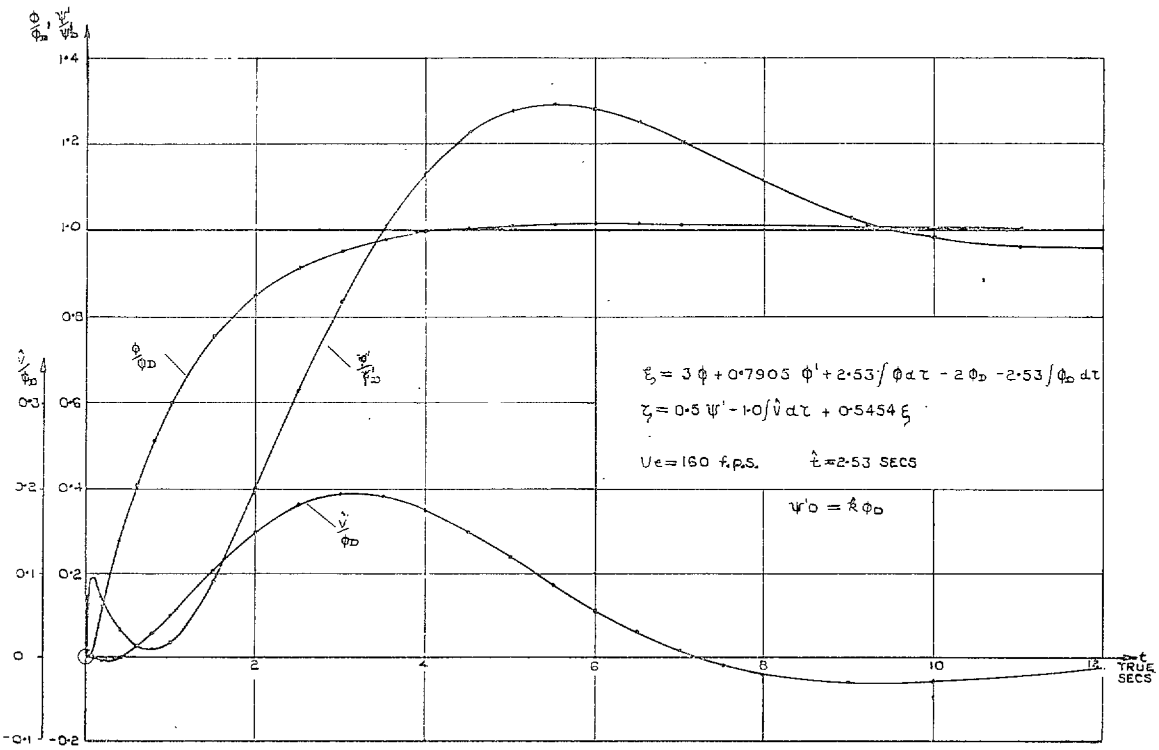

FIG. 9. Turns entry. Over compensation for $\mathcal{N}_\xi$.

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