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Note on the Influence of Spanwise Flow on Lift Distribution

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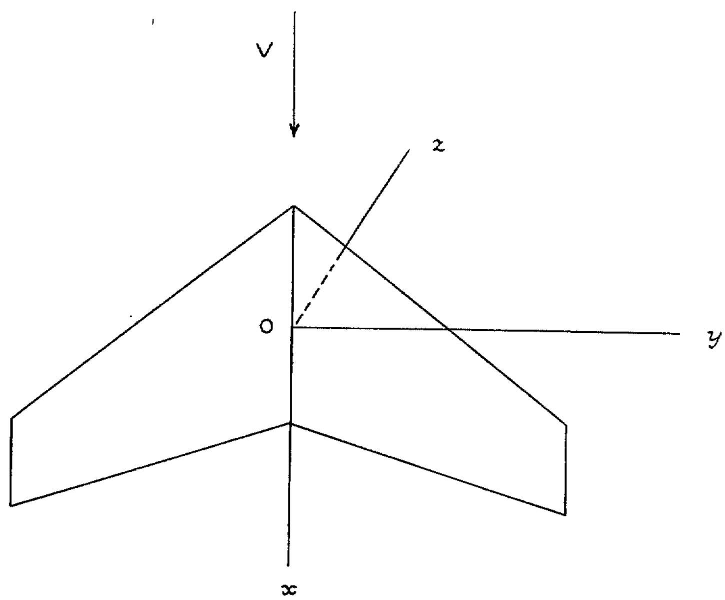
1. *Summary*.—The influence of spanwise flow on the lift distribution for a thin flexible wing of any plan form is considered. By the use of Euler's equations for incompressible, inviscid flow, it is shown that the lift distribution is not appreciably affected provided the displacements of the wing are small.

2. *Theory*.—The wing is replaced by an infinitely thin surface, and it is assumed, for simplicity, that this representative surface is flat when the wing is in its mean position, and that it lies in the plane $z = 0$.



Let V be the velocity of the airstream and let u, v, w be the velocity components of the disturbed flow corresponding to small oscillations or displacements of the flexible surface. Euler's equations for incompressible inviscid flow then gives

$$\left.\begin{aligned} -\frac{1}{\rho}\frac{\partial p}{\partial x} &= \frac{Du}{Dt} = \frac{\partial u}{\partial t} + (V+u)\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} &= \frac{Dv}{Dt}, \quad -\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{Dw}{Dt}, \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where p and ρ represent the pressure and the air density respectively. On integration (1) yields

$$\phi = \frac{\partial \Phi}{\partial t} + \tfrac{1}{2}V^2 + Vu + \tfrac{1}{2}(u^2 + v^2 + w^2) + F(t) \,, \qquad (2)$$

where $\phi \; (\equiv - p/\rho)$ is the usual acceleration potential and Φ denotes the velocity potential. If u, v, w are small compared to V, equation (2) can be replaced by the approximate relation

$$\phi = \frac{\partial \Phi}{\partial t} + \tfrac{1}{2}V^2 + Vu + F(t) \,. \qquad (3)$$

Let ϕ_a, ϕ_b and Φ_a, Φ_b represent respectively the values of ϕ and Φ immediately above and immediately below a point on the surface. Then, (3) leads to

$$\frac{p_b - p_a}{\rho} = \phi_a - \phi_b = \left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)(\Phi_a - \Phi_b) \,, \qquad (4)$$

where $p_b - p_a$ represents the lift or the discontinuity in the pressure field. It is known that this discontinuity in the acceleration potential field can be reproduced by a distribution of doublets of strength $\phi_a - \phi_b$ over the wing area, with axes normal to the surface. Since the displacements of the wing from its mean position are small, it is sufficiently accurate to assume that the doublets are distributed over the projected wing area in the plane $z = 0$ for all values of t. It then follows from (4) that the discontinuity in the velocity potential field can be reproduced by a distribution of doublets of strength $K(\equiv \Phi_a - \Phi_b)$ over the projected wing area and the wake in the plane $z = 0$. The velocity potential at any point x_1, y_1, z_1 at time t is given by

$$\Phi(x_1, y_1, z_1, t) = \frac{1}{4\pi}\iint K \frac{\partial}{\partial z_1}\left(\frac{1}{r}\right) dx\, dy \,, \qquad (5)$$

where $r^2 = (x - x_1)^2 + (y - y_1)^2 + z_1^2$, and where K is independent of z_1. By definition,

$$K(x, y, t) = \Phi_a - \Phi_b = \int_{-\infty}^{x} (u_a - u_b)\, dx \,,$$

and it must therefore correspond to the circulation up to the point x of the chordwise section defined by y. When $\pm z_1 \to 0$, (5) yields the following relations :—

$$\left.\begin{aligned} \Phi_a &= - \Phi_b = \frac{K}{2} \,, \\ u_a &= - u_b = \frac{1}{2}\frac{\partial K}{\partial x} \,, \\ v_a &= - v_b = \frac{1}{2}\frac{\partial K}{\partial y} \,, \\ w_a &= \quad w_b = \frac{\partial \Phi}{\partial z_1} \,, \\ u_a^2 + v_a^2 + w_a^2 &= u_b^2 + v_b^2 + w_b^2 \,. \end{aligned}\right\} \qquad (6)$$

It then follows that the strengths of the transverse vortex $u_a - u_b$ and of the trailing vortex $v_a - v_b$ are $\partial K/\partial x$ and $\partial K/\partial y$ respectively.

Next suppose that u, v, w are such that u^2, v^2, w^2 are not small enough to be neglected in comparison with the absolute value of Vu in (2). Equation (3) is then not valid, and (4) must be replaced by

$$\phi_a - \phi_b = \left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)(\Phi_a - \Phi_b) + \tfrac{1}{2}(q_a^2 - q_b^2) \,, \qquad (7)$$

where $q^2 \equiv u^2 + v^2 + w^2$. For small disturbances $q_a^{\,2} = q_b^{\,2}$, and $u_a = -u_b$, as already shown in (6), and it is reasonable to suppose that for larger disturbances $\frac{1}{2}(q_a^{\,2} - q_b^{\,2})$ can be neglected in comparison with $V(u_a - u_b)$, although $\frac{1}{2}q^2$ is not small compared to Vu. Since (7) reduces to (4), it would appear that the normal form of vortex sheet theory is applicable to problems involving relatively large disturbances. This may explain why, in the steady motion case for instance, theory gives reliable estimates for the lift for angles of incidence which are by no means small.

For any prescribed motion the doublet distribution K must be such that the normal induced velocity

$$w = \left(\frac{\partial \Phi}{\partial z_1}\right)_{z_1 \to 0} = \frac{1}{4\pi}\iint K \frac{\partial^2}{\partial z_1^{\,2}}\left(\frac{1}{r}\right) dxdy \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

satisfies the tangential flow condition. If $\zeta = f(x, y, t)$ represents the displacement of the surface, the required condition is

$$w = \frac{\partial \zeta}{\partial t} + (V + u)\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where $u \,.\, \partial\zeta/\partial x$ and $v \,.\, \partial\zeta/\partial y$ can usually be neglected. To satisfy this condition K is represented by a linear combination of simple doublet distributions $\sum_{i=1}^{n} C_i K_i$ for which the corresponding normal induced velocity is readily calculable. Condition (9) then yields

$$\sum_{i=1}^{n} C_i w_i = \frac{\partial \zeta}{\partial t} + V\frac{\partial \zeta}{\partial x}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

from which the unknown constants C_i can be determined by collocation at a sufficient number of points. The choice of simple doublet distributions is limited by the fact that $K = 0$ and $\partial K/\partial x = \infty$ along the leading edge. In the wake,

$$\frac{\partial K}{\partial t} + V\frac{\partial K}{\partial x} = 0, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

since there is no discontinuity in the pressure field.

3. *General Remarks*.—In the above analysis the mean position of the wing is assumed to be flat. If, however, the wing is bent under load so that the vertical displacement of the wing tip is not small, and if small oscillations occur about this mean position, the theory outlined would still apply, provided the doublet sheet was assumed to be curved accordingly. The axes of the doublets would be normal to the curved surface and equation (10) would still apply, provided w and ζ are interpreted as the velocity and the displacement normal to the curved surface in its mean position. It is assumed that the angles of incidence involved are small.

Equations (6) indicate a possible method of determining $\partial K/\partial x$ and $\partial K/\partial y$ by observation of the flow pattern. If, by the use of small puffs of smoke in a wind tunnel or drops of coloured liquid in a water tunnel, the speed and direction of flow at any point of a wing could be determined, the pressure distribution could then be estimated. Such experiments would provide useful checks of the relations given in (6), and should be relatively easy to carry out for the steady motion case. Furthermore, experimental flow patterns for thick wings of various plan forms might indicate some necessary modifications of the thin-wing theory.

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