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Summary.—The present paper advances a formula which can be used as a criterion for the degree of mass-balance necessary for the avoidance of spring-tab flutter. The formula shows that if the tab is of sufficiently light construction, mass-balance may not be required at all; on the other hand, the usual static balance may be inadequate for a tab of high inertia. The criterion comprehends within itself the requirement (given elsewhere) limiting the length of a mass-balance arm. While the formula is based on theoretical considerations (which are set out in the Appendix) the numerical values for the quantities to be used have been deduced from flight experience, which shows excellent correlation with the theory. Two forms for the criterion are given: a simple form suitable for general application, and a slightly elaborated form intended for application to unusually large tabs.

The Appendix, besides containing the main analysis, also gives consideration to certain factors which for simplicity are omitted in the main text. In particular it is shown that the 'limiting length' for a balance arm may be generalised to a 'limiting circle' for the position of the balance mass: the circle can often be found from simple geometrical considerations.

1. *Introduction.*—The problem of flutter of spring-tab systems has received considerable attention at various times[1 to 6], but no adequate general rule has emerged for the degree of mass-balance necessary for flutter prevention. An early research[1], which examined an actual case of rudder-tab flutter, showed that, with a balance mass arbitrarily (and, as it happened, correctly) disposed, flutter would not have occurred. There followed a general experimental research[2], but this led to the conclusion that no general rule could be given for mass-balancing, since in some cases flutter was promoted by the addition of a balance mass. The considerable research of Frazer and Jones[3], and some consequent parallel investigations[4,5], showed that if mass-balance is to be attempted, the balance mass must be disposed within a certain limiting distance from the tab hinge. This result was shown[6] to be derivable from simple considerations relating to elastic and inertia couplings only; it was also shown that all the previous investigations, when viewed in the light of this limiting distance, became parts of the same consistent story.

As a result, recommendations specifying the limiting distance for tab mass-balance were officially issued[7]; the degree of mass-balance called for was based on the special cases previously dealt with, in which static balance—applied empirically, as was habitual—had proved adequate. In the present paper a formula will be developed which provides a simple criterion for the degree of mass-balance required in the general case. The criterion comprehends within itself the 'limiting length' requirement for a balance arm, and therefore supersedes the earlier recommendation[7]: it shows moreover that static balance may in some cases be an unnecessarily severe requirement and in other cases may not be sufficient to prevent flutter.

The account given in the following paragraphs has been kept as far as possible free from mathematical analysis; the derivation of the formulae is given in the Appendix. The main text is in two Parts. In Part I an especially simple form of the criterion is derived; it is based on an examination of actual systems which have flown. In Part II the criterion is elaborated somewhat, to take some account of the geometry of the tab system: the elaboration is based on a theoretical investigation, but comparisons with actual systems are again made.

* R.A.E. Reports S.M.E. 3346 and 3378.

PART I

The Simple Criterion

2. *Choice of Co-ordinates to Eliminate Elastic Coupling.*—It has been shown[1, 6] that, if control surface angle and tab angle as usually defined are chosen as co-ordinates specifying the displacement of the system, there is a considerable elastic coupling; this coupling can promote flutter when the tab is statically balanced or even over-balanced. Analysis employing the usual co-ordinates does not suggest any solution of this difficulty; it is, therefore, necessary to choose new co-ordinates for which the elastic coupling is absent, and to vary the inertia coupling for these new co-ordinates.

Fig. 1(a) shows a simplified system which covers in principle all spring tab systems as used at present. For simplicity, the masses constituting the control surface and tab are regarded as lying in a single plane in the neutral position, and only small displacements from the plane are examined (*see* Fig. 1(b)). AB represents part of the main surface; BC is the control surface and CD the tab. At the control surface and tab hinges B and C are two swinging levers BE and CF; a rod EF completes a four-bar mechanism. CF has length L and BE has length NL, where N is the 'follow-up ratio' of the system.*

The lever CF is connected to the tab through a spring G (normally a torque bar or tube).† There is also a spring H constraining the four-bar mechanism BCFE: it is shown in the angle CBE, but may equally well be in the angle BCF (H is also usually a torque tube). Finally, the point E is connected to the control circuit, which is also regarded as a spring, J.

The displacements of the system may be defined by the control surface angle ξ, the tab angle β, and the rotation θ of CF from the normal to BC. In this general displacement it is clear that the relative angular movement of CF and CD is $(\beta - \theta)$; that the relative angular movement between BE and BC is θ/N; and finally that the linear displacement of E is $NL(\xi - \theta/N)$. Accordingly the potential energy V of the system may be written as

$$2V = C_1(N\xi - \theta)^2 + C_2\theta^2 + C_3(\beta - \theta)^2\,, \qquad \ldots \quad (1)$$

where C_1, C_2 and C_3 are appropriate stiffnesses, corresponding to the springs J, H, and G respectively.

There are three co-ordinates in equation (1); however, we can eliminate one of these. For the inertia of the lever CF is in practice negligibly small; it is, therefore, subjected neither to inertia nor aerodynamic forces, but is in equilibrium under the elastic forces only. We may, therefore, use the condition

$$\frac{\partial V}{\partial \theta} = 0\,;$$

and if for simplicity we write

$$C_2 = rC_1,\ \ C_3 = sC_1\,,$$

we have in place of equation (1)

$$2V/C_1 = (N\xi - \theta)^2 + r\theta^2 + s(\beta - \theta)^2\,, \qquad \ldots \quad (2)$$

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* To find the follow-up ratio, imagine the point E to be fixed in space, and let BC be displaced through ξ. Then the tab will move, in the anti-balance sense, through $N\xi$.

† G is sometimes called the 'blow-off' or 'blow-back' spring. It is often absent, and CF is directly connected to CD: this case is covered if G is regarded as infinitely stiff.

and the equilibrium condition therefore gives

$$0 = \frac{1}{C_1}\frac{\partial V}{\partial \theta} = -(N\xi - \theta) + r\theta - s(\beta - \theta). \qquad (3)$$

Substitution for θ from equation (3) in equation (2) yields, after a little reduction

$$\frac{2(1 + r + s)}{s}\frac{V}{C_1} = (\beta - N\xi)^2 + r\beta^2 + \frac{r}{s}N^2\xi^2. \qquad (4)$$

The equation (4) contains a term in the product $\beta\xi$, and accordingly there is an elastic coupling present for the co-ordinates β, ξ. This is, of course, physically obvious: if ξ is constrained to be zero and a tab angle β is imposed, the control circuit is extended and in consequence a moment about the main hinge develops.

We must therefore choose new co-ordinates in which there is no such product term[6]. The choice for general values of the spring stiffnesses is dealt with in the Appendix; for the moment, however, we shall simplify equation (4) very considerably by regarding r as negligibly small. In practice, the spring stiffness C_2 is always small compared with C_1; and Naylor and Pellew[4] have shown that the flutter characteristics of spring-tab systems are in practical cases almost independent of the spring stiffness C_2: a conclusion which has been reinforced by another investigation[8]. It may be noted that if the spring H giving rise to the term in C_2 is actually absent, the system becomes a pure aerodynamic servo; and such systems are, therefore, included in the present investigation.

With $r = 0$, equation (4) reduces to

$$\frac{2(1 + s)}{s}\frac{V}{C_1} = (\beta - N\xi)^2, \qquad (5)$$

so that in this case the only effect of the blow-off spring G giving rise to the term in C_3 is to modify the spring stiffness C_1: we can regard G as infinitely stiff, so that the lever FC is rigidly attached to the tab CD, the spring stiffness C_1 being reduced in the ratio $s/(1 + s)$. If we write this reduced stiffness as C we have finally

$$2V = C(\beta - N\xi)^2. \qquad (6)$$

We now choose as new co-ordinates the angles $\bar{\xi}$, $\bar{\beta}$ such that

$$\left.\begin{aligned} \bar{\xi} &= \xi, \\ \bar{\beta} &= \beta - N\xi \end{aligned}\right\}. \qquad (7)$$

Physically, this means that $\bar{\xi}$ is as before the control surface angle, and $\bar{\beta}$ (with FC rigidly attached to CD) is proportional to the angular rotation of the lever BE. In terms of the new co-ordinates, equation (6) becomes

$$2V = C\bar{\beta}^2. \qquad (8)$$

Since this contains no product terms, there is no elastic coupling.

3. *The New Inertias.*—To find the inertias appropriate to any co-ordinate system we write the kinetic energy in terms of the velocities in those co-ordinates. From Fig. 1, if the control surface has angular velocity $\dot{\xi}$ and the tab has angular velocity $\dot{\beta}$, we see that any element of mass dm has the linear velocity $\dot{\xi}x_c$, where x_c is its distance aft of the control surface hinge. If dm lies on the tab it has the additional velocity $\dot{\beta}x_t$, where x_t is the distance aft of the tab hinge. The kinetic energy T is thus given by

$$2T = \int_c (\dot{\xi} x_c)^2 dm + \int_t (\dot{\xi} x_c + \dot{\beta} x_t)^2 dm$$
$$= \dot{\xi}^2 \int_{c+t} x_c^2 dm + 2\dot{\xi}\dot{\beta} \int_t x_c x_t dm + \dot{\beta}^2 \int_t x_t^2 dm . \qquad \ldots \qquad \ldots \qquad (9)$$

From this expression, the control surface moment of inertia I_c (including tab), the product of inertia P, and the tab moment of inertia I_t are given by

$$\left.\begin{aligned} I_c &= \int_{c+t} x_c^2 dm , \\ P &= \int_t x_c x_t dm = \int_t (d_0 + x_t) x_t dm = d_0 \int_t x_t dm + I_t , \\ I &= \int_t x_t^2 dm , \end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad (10)$$

where in the expression for P we have written the distance between the hinges (assumed parallel) as d_0. Equations (10) give the inertias appropriate to the co-ordinates ξ, β.

However, we require to know the new inertias appropriate to the co-ordinates $\bar{\xi}$, $\bar{\beta}$. Substitution from equation (7) for ξ, β in equation (9) yields

$$2T = \dot{\bar{\xi}}^2 I_c + 2\dot{\bar{\xi}}(\dot{\bar{\beta}} + N\dot{\bar{\xi}})P + (\dot{\bar{\beta}} + N\dot{\bar{\xi}})^2 I_t$$
$$= \dot{\bar{\xi}}^2 (I_c + 2NP + N^2 I_t) + 2\dot{\bar{\xi}}\dot{\bar{\beta}}(P + NI_t) + \dot{\bar{\beta}}^2 I_t , \qquad \ldots \qquad \ldots \qquad (11)$$

and accordingly the new inertias are

$$\left.\begin{aligned} \bar{I}_c &= I_c + 2NP + N^2 I_t , \\ \bar{P} &= P + NI_t = d_0 \int_t x_t dm + (N + 1) I_t , \\ \bar{I}_t &= I_t . \end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (12)$$

It is these inertias which are appropriate to the co-ordinates for which there is no elastic coupling.

4. *The Stability Boundary.*—It is shown in the Appendix that the condition for an infinitesimally small flutter speed range defines a relation between the inertias $\bar{P}$ and $\bar{I}_c$ only. If $\bar{P}$ and $\bar{I}_c$ are regarded as Cartesian co-ordinates in a plane, the relation is a hyperbola. This hyperbola, for a different set of co-ordinates, has been discussed by Fraser[9], who has shown that only one branch is significant as a stability boundary. Fig. 2 shows a typical hyperbola, which has been drawn for the rudder-tab system examined in R. & M. 1527[1]; curve A is the boundary between stability and instability. If the actual inertias of the system give a point ($\bar{I}_c$, $\bar{P}$) lying above curve A, the system will flutter; if the point plots below curve A, the system is stable. In the former case, the problem of flutter prevention can be regarded as that of reducing $\bar{P}$ until the point ($\bar{I}_c$, $\bar{P}$) lies in the safe region; while the general design problem is reduced to consideration of $\bar{P}$ in relation to $\bar{I}_c$ and the relevant hyperbola. It may be noted that alteration of $\bar{P}$ will usually involve alteration of $\bar{I}_c$ also; however, in practice the change in $\bar{I}_c$ is negligibly small.

5. *The Limiting Length.*—We may at once deduce the formula for the limiting length of balance arm from the above considerations. Suppose we require to reduce $\bar{P}$, and we effect the reduction by adding a balance mass M to the tab, in the plane of the tab and on an arm of length $x_t = - l$,

where l is positive. The contribution of M to $\bar{P}$ is to be negative, so that from equations (12)

$$d_0(-l)M + (N+1)l^2M < 0 \, ,$$

whence

$$l < d_0/(N+1) \, , \qquad \ldots \qquad (13)$$

which is the usual result. If the balance mass does not lie in the plane of the tab, the appropriate formula can be found in the same way.

6. *A Balance Mass or a Tab of Light Construction.*—While it is true that the addition of an appropriate balance mass, on an arm of length complying with the inequality (13), will reduce $\bar{P}$ as required, it is also evident, from equations (12), that $\bar{P}$ can be reduced alternatively by reduction of the intrinsic masses dm in the formula for $\bar{P}$. Thus, since Fig. 2 shows that it is not necessary to reduce $\bar{P}$ to zero to avoid flutter, replacement of a heavy tab by one of lighter construction is an alternative—and obviously a desirable alternative—to the addition of a balance mass. It is only necessary to ensure that the point $(\bar{I}_c, \bar{P})$ plots in the safe region.

7. *A Simple Criterion for the Avoidance of Flutter.*—The general expression for the hyperbolic boundary, which involves many aerodynamic terms, is too complicated for our present purpose. However, it is found that in practical cases the relevant branch of the hyperbola lies above and close to its asymptote in the region of the inertia points, so that it is safe to use the upper asymptote instead of the hyperbola itself (Fig. 2, line B). It is still safer to use a straight line through the origin and parallel to the asymptote (Fig. 2, line C). We may, therefore, tentatively write as a boundary giving a certain margin of safety

$$\bar{P} = k\bar{I}_c \, , \qquad \ldots \qquad (14)$$

where k, the slope of the relevant asymptote, depends only on the aerodynamical derivatives of the system concerned. Various possible forms for k are discussed in the Appendix.

Equation (14) has the great advantage that it defines a maximum value of $\bar{P}$ which is directly proportional to $\bar{I}_c$; in other words, flutter will be avoided provided that

$$\bar{P}/I_c < k \, ,$$

or, since $\bar{I}_c$ can in practice be replaced without serious error by its major component I_c,

$$\frac{P + NI_t}{I_c} < k \, . \qquad \ldots \qquad (15)$$

It remains to consider the value to be assigned to k; as remarked above, this is a function of the aerodynamical derivatives in any particular case. This raises a considerable difficulty; for very little is known of the aerodynamics of an oscillating spring-tab system even in particular cases, so that to assign a value to k in the general case is virtually impossible. However, since the geometry of spring-tab systems does not vary widely (except perhaps in the choice of N) it appears possible that a uniform value of k may be assumed without serious error. Moreover, we can obtain some insight into the value of k by an examination of the values of $\bar{P}/\bar{I}_c$ appropriate to spring-tab systems which have flown and the histories of which are known. Table 1 lists the values of $(P + NI_t)/I_c$ for twenty-six such systems: it has been compiled from information supplied by aircraft firms.

The entries in the Table have been arranged in decreasing order of the value of $(P + NI_t)/I_c$; the range in this quantity is very considerable, as indeed are the ranges of the inertias (the ratio

TABLE 1

Spring tab system No.	Control (A ≡ aileron etc.)	I_c slug ft²	P slug ft²	I_t slug ft²	N	$\frac{P + NI_t}{I_c}$	Trouble	Mass-balance
1	A	0·168	0·00405	0·00405	2·75	0·0905	Flutter	Yes
2	R	6·00	0·220	0·037	2·73	0·0535	Flutter	No
3	A	1·22	0·00888	0·013	3·00	0·0393	Flutter	Yes
4	A	0·904	0·0114	0·0230	1·00	0·0381	Vibration	Yes
5	E	22·0	0·143	0·143	3·39	0·0286	Flutter	Yes
6	A	0·0856	0·000807	0·000245	4·00	0·0208	Flutter	Yes
7	A	1·49	0·00704	0·0124	1·83	0·0199	Flutter	Yes
8	A	1·05	0·00289	0·00487	3·50	0·0189	Flutter	Yes
9	R	1·27	0·0109	0·00842	1·52	0·0187	Vibration	Yes
10	A	1·40	0·00458	0·00710	3·00	0·0185	Flutter	Yes
11	A	0·231	0·00280	0·00054	2·51	0·0180	No	No
12	A	0·152	—0·00030	0·00149	1·85	0·0162	No	Yes
13	A	0·390	0·00039	0·0012	4·54	0·0149	No	Yes
14	R	0·991	0·00914	0·00141	2·66	0·0130	No	No
15	R	2·36	0·00017	0·00965	2·90	0·0119	No	Yes
16	A	2·72	0·00866	0·00866	2·38	0·0108	No	Yes
17	E	1·96	0·00224	0·00398	3·55	0·0083	No	Yes
18	R	0·991	0·00060	0·00225	2·66	0·0066	No	Yes
19	A	1·23	0·00175	0·00175	3·50	0·0064	No	Yes
20	A	2·49	0·00318	0·00367	3·33	0·0062	No	Yes
21	E	7·15	0·00633	0·00633	3·00	0·0035	No	Yes
22	R	4·06	0·00370	0·00370	2·17	0·0029	No	Yes
23	E	5·25	0·000612	0·00475	2·00	0·0019	No	Yes
24	R	3·07	0·000512	0·00345	1·55	0·0019	No	Yes
25	E	5·10	0·00311	0·00311	1·80	0·0017	No	Yes
26	R	1·46	—0·00636	0·00636	1·26	0·0011	No	Yes

of the values of I_c for systems 5 and 6 is over 250). What is quite remarkable, however, is that the first ten systems have all suffered from some trouble—eight cases of flutter and two of vibration—while all the remainder have been trouble-free. Clearly this precise separation is fortuitous to a certain extent; but it does strongly suggest that we may adopt a uniform value for k in applying the inequality (15).

Some information concerning certain of the systems in the table may be useful here. No. 1 was an experimental spring-tab balance for an aileron, in which the tab was statically balanced by means of weights on two arms inclined at about 50 deg to the plane of the tab; these arms were much too long, so that the effect of the balance masses was nearly to double the value of $P + NI_t$ appropriate to the bare tab. System No. 2 was the very early aerodynamic servo discussed in R. & M. 1652[2] (1933); no mass-balance was fitted to the tab. System No. 3, like No. 1, was mass-balanced incorrectly; the balance mass was outside the limiting length and so increased the value of $P + NI_t$ instead of decreasing it. System No. 4, though not actually fluttering, had some vibration trouble; it had an unusually large tab chord. On No. 5 the tab balance masses were on projecting arms above the plane of the tab, and in consequence the state of balance was a function of tab angle. Flutter developed during a pull-out, when the tab was almost certainly down, with its centre of gravity unusually far aft. Systems 6, 7, 8 and 10 were all mass-balanced within the limiting length, but owing to the relative inertias and follow-up ratios the values of $(P + NI_t)/I_c$ are fairly high. In all these cases the flutter that occurred was mild in nature, indicating that they were borderline cases: in fact, on System No. 8 the slight variations due to production gave mild flutter on one aeroplane while another was immune. The trouble on System No. 9 was cured by repositioning the rudder balance mass; it seems probable, however, that the tab was playing a part in the vibration reported.

The dividing line between those systems which have experienced trouble and those which have not occurs in Table 1 for a value of $(P + NI_t)/I_c$ between 0·018 and 0·0185. Theory, as applied to System No. 2 in R. & M. 1652[2], indicates a rather higher value; so also does the theory of Part II of the present paper. However, since the aerodynamical derivatives assumed in both cases are very uncertain, it is clearly better to rely on experience, and to use the information contained in Table 1. With a suitable safety margin, therefore, we may suggest, in the light of Table 1, the formula

$$\frac{P + NI_t}{I_c} < 0 \cdot 015 \quad \ldots \quad (16)$$

as a criterion for the avoidance of spring-tab flutter.

It may be remarked that System No. 14 shows that the criterion (16) can be satisfied in the absence of any mass-balance, and that such absence does not necessarily result in flutter; System No. 11, though it does not satisfy the criterion (16), also shows that absence of tab mass-balance is not necessarily dangerous. Indeed, it seems probable that on many of the Systems lower in the table than No. 14 the balance masses could be discarded without violating the criterion (16) and, therefore, without serious risk of promoting flutter.

8. *Remarks and a Caution.*—The criterion derived in the foregoing paragraphs depends on the assumption that the principal dangerous flutter mode is that involving only tab and control surface movement in the manner dealt with above. While this assumption is probably justified, it must be remembered that other modes of motion are possible. The control surface itself, for example, must be mass-balanced in the usual way if flutter involving main surface movement is to be avoided. One particular caution is necessary: *if appreciable backlash is present in the system, the considerations relating to elastic coupling will not apply within the range of the backlash, and static balance of the tab may be necessary even though the criterion* (16) *may be satisfied in the absence of static balance.* Great care should, therefore, be taken in regard to the possible development of backlash; and it is clearly desirable in any case that when a tab is being designed its centre of gravity should be as near its hinge as possible. This would help, for example, at large tab angles, when the conclusions of the present report (which are based on a linear theory) would not fully apply.

On the subject of static balance of the tab, we may remark that

$$\bar{P} = P + NI_t = d_0 \int_t x_t dm + (N + 1)I_t \, .$$

Now $\int_t x_t dm$ is the static unbalance of the tab; and it is evidently not significant of itself in the avoidance of flutter. However, if the tab is statically balanced, the criterion (16) reduces to the very simple form

$$I_t/I_c < 0 \cdot 015/(N + 1) \, . \quad \ldots \quad (17)$$

One further point of importance may be noted: it is that for any given control surface and tab the value of $\bar{P}$ may be reduced (very often considerably) by appropriate reduction in the value of the follow-up ratio N; from the flutter viewpoint, the smaller N, the better. This consideration may very well conflict with aerodynamic requirements, of course; but in the design of a spring-tab system it is a point that should be borne in mind.

PART II

Consideration of the Effects of Tab Dimensions

9. *Reasons for an Elaboration of the Simple Criterion.*—In the derivation of the simple criterion in section 7 it was remarked that the quantity k appearing on the right-hand side of the inequality (15) was strictly a function of the aerodynamical derivatives appropriate to the particular system concerned. Since the geometry of spring-tab systems does not in practice vary widely, a uniform value was suggested; and Table 1 gave support to this. Nevertheless, it may be that tabs of unusual proportions will be required in the future, and some consideration of this possibility is, therefore, clearly desirable.

10. *General Effects of Tab Dimensions.*—Let p be the ratio of tab chord to control surface chord (both measured from hinge line to trailing edge) and q the ratio of tab span to control surface span. Since p is usually small, we may make some rough estimates of the way in which certain of the aerodynamical derivatives depend on p and q. For example, the control surface hinge moment due to unit tab angle we should expect to be proportional (roughly) to pq; the tab hinge moment itself to p^2q. But although this kind of estimated dependence may be included in the theoretical estimate of k, it is not possible easily to assess the relative importance of the various terms.

We shall, therefore, assume that the functional dependence of k on p, q, may be expressed in the form

$$k = Kp^m q^n$$

where K is a universal constant. To evaluate m and n we shall make a theoretical investigation in which p and q are varied and k is calculated; we then write

$$\log k = \log K + m \log p + n \log q$$

and evaluate m and n as the average gradients of the curves of $\log k$ against $\log p$ and $\log q$.

11. *The Theoretical Investigation.*—The system studied theoretically consists of a main surface of constant chord 8 ft (including control): this surface is fixed. It carries an aileron of chord $1\cdot2$ ft and span $9\cdot4$ ft; the aileron has a tab of chord $1\cdot2\,p$ ft and span $9\cdot4\,q$ ft. A value of 3 is assigned to the follow-up ratio of the mechanism; this is considered to be the best representative figure than can be chosen. Three values of p (2/15, 4/15, and 6/15), have been used in the calculations, and three values of q (1/4, 2/4, and 3/4), giving nine cases in all. To simplify the calculations, and in view of previous work[4, 8], the spring stiffness C_2 has been taken to be zero, so that (*see* section 2) only one spring, of stiffness C, appears in the analysis.

Two-dimensional vortex sheet theory has been used to give the aerodynamical derivatives, a fixed value of the frequency parameter ($\omega c/V$, based on main surface chord) of unity being assumed; no empirical modification to the theoretical values of the air forces (such as is often applied) has been made here. It is not considered that this will greatly affect the estimates of the stability boundaries; and in any case we are mainly concerned with comparative values of k as deduced from the boundaries for different p, q.

Some further details of the calculations are given in the Appendix; the results, however, may be summarised in the form given in Table 2. If we write $\bar{I}_c = x$, $\bar{P} = y$, then the stability boundary is a hyperbola in the (x, y)-plane. Table 2 gives, for each value of p, q, the co-ordinates (x_0, y_0) of the centre of the hyperbola,* and the slope k of the asymptote corresponding to the significant branch of the hyperbola.

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* The coordinates (x_0, y_0) of the centre are given as an indication of its position in relation to a typical inertia point, as defined by the actual values of $\bar{I}_c$, $\bar{P}$ for a practical system. It is clearly necessary for (x_0, y_0) to be close to the origin O compared with $(\bar{I}_c, \bar{P})$ if a line through O parallel to the asymptote is to be a reasonable substitute for the true boundary. Fig. 2 shows that, for the system considered, the distance from O to (x_0, y_0) is only about one-tenth of the distance from O to $(\bar{I}_c, \bar{P})$.

TABLE 2

p	q	x_0	y_0	k	$K_1 = k/p^{7/4}q^{1/4}$	$K_2 = k/p^{3/2}$
		$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$		
$\frac{2}{15}$	$\frac{1}{4}$	4·74	0·222	6·64	0·319	0·136
	$\frac{2}{4}$	4·06	0·218	9·41	0·381	0·193
	$\frac{3}{4}$	3·82	0·214	10·98	0·402	0·225
$\frac{4}{15}$	$\frac{1}{4}$	25·4	2·23	26·1	0·373	0·190
	$\frac{2}{4}$	20·3	2·15	32·8	0·395	0·239
	$\frac{3}{4}$	18·2	2·07	33·3	0·361	0·242
$\frac{6}{15}$	$\frac{1}{4}$	64·3	7·39	54·0	0·380	0·214
	$\frac{2}{4}$	47·5	6·53	65·5	0·387	0·259
	$\frac{3}{4}$	38·9	5·66	64·9	0·347	0·256

It will be seen from Table 2 that the range of values of k is from 0·0066 for the smallest to 0·065 for the largest tab; and that the ratio of the extreme values of k is about the same as the ratio of the areas of the corresponding tabs. However, when we plot log k against log p for q constant and against log q for p constant, we deduce as mean values for the indices m and n (*see* section 10) the values

$$m = 1 \cdot 76$$
$$n = 0 \cdot 29 .$$

If we round off these figures we find approximately

$$k = K_1 p^{7/4} q^{1/4} ,$$

and accordingly we add in Table 2 the values of the quantity

$$K_1 = k/p^{7/4} q^{1/4} . \qquad \text{(18)}$$

The tabulated values of K_1 are fairly closely constant, the mean value being

$$K_1 = 0 \cdot 372$$

with variations of about $\pm$ 10 per cent.

Since the dependence of k on q is so slight, a simpler form than equation (18) may be found if we assume that in practice the area of a tab is likely to be roughly a constant proportion of the control surface area. With pq constant equation (18) can be written alternatively

$$K_2 = k/p^{3/2} . \qquad \text{(19)}$$

Although in Table 2 the range of variation of pq is 9 : 1, it is of interest to see that the variation in the values of K_2 is not excessive: the mean value is

$$K_2 = 0 \cdot 217$$

with variations of about $\pm$ 25 per cent.

12. *Interpretation in the Light of Practical Experience.*—In view of the above considerations, it appears that, as an alternative to the criterion (16), we may write

$$\left(\frac{P + NI_t}{I_c}\right) p^{-3/2} < K \qquad \text{(20)}$$

where K, according to the foregoing theory, would have a value of about 0·2. However, just as in Part I the value of k ultimately proposed was derived not from theory but from a study of the history of actual systems which have flown, so the value to be assigned to K can be similarly found. Table 3 lists the same twenty-six systems as are given in Table 1, with the appropriate values of p, q; using the value of p and the results of Table 1, a value of the left-hand side of the inequality (20) has been found, and the systems have been arranged in descending order of magnitude of this quantity.

TABLE 3

System No.	p	q	pq	$\left(\frac{P + NI_t}{I_c}\right)p^{-3/2}$	Trouble
2	0·20	1·00	0·200	0·598	Flutter
1	0·32	0·31	0·099	0·501	Flutter
3	0·23	0·44	0·101	0·356	Flutter
5	0·25	0·26	0·065	0·229	Flutter
6	0·24	0·80	0·192	0·177	Flutter
9	0·23	0·54	0·124	0·170	Vibration
11	0·23	0·22	0·051	0·163	No
16	0·17	0·59	0·100	0·154	No
8	0·27	0·37	0·100	0·135	Flutter
7	0·30	0·53	0·159	0·121	Flutter
4	0·48	0·29	0·139	0·115	Vibration
10	0·30	0·39	0·117	0·113	Flutter
14	0·24	0·31	0·074	0·111	No
12	0·31	0·28	0·087	0·094	No
19	0·17	0·48	0·082	0·091	No
17	0·21	0·51	0·107	0·086	No
13	0·32	0·25	0·080	0·082	No
15	0·30	0·38	0·114	0·072	No
20	0·20	0·55	0·110	0·069	No
18	0·24	0·31	0·074	0·056	No
22	0·17	0·65	0·110	0·041	No
21	0·25	0·25	0·062	0·028	No
25	0·19	0·74	0·140	0·021	No
24	0·27	0·41	0·111	0·014	No
23	0·29	0·40	0·116	0·012	No
26	0·38	0·32	0·122	0·005	No

Table 3 also gives values of pq; the ratio of extreme values is only about 3 : 1.

Although the new method of classification produces a fair amount of re-arrangement, it is still mainly true that those systems which have given trouble are at the head of the table. The position is not quite so clear as in Table 1, since Systems 11 and 16, which have not given trouble, now appear above four systems which have. On the other hand, System 4, which had not given serious trouble, but was nevertheless high up in Table 1, is in Table 3 brought near the borderline, since it has a quite unusually large value of p. On the whole, therefore, there may be a case for using the elaborated form of the criterion to provide some concession on the requirements of the simple criterion (16) for tabs of high chord ratio. A formula suggested by Table 3 would be

$$\left(\frac{P + NI_t}{I_c}\right)p^{-3/2} < 0\cdot10. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

It may be remarked here that in both Table 1 and Table 3 System No. 10 is a borderline case. In fact, the trouble on System No. 10 was cured by the addition of cord at the trailing edge of the tab. It is well known that the addition of cord can produce considerable changes in the hinge moment characteristics of control surfaces and tabs; and in this case, the change underlines the dependence of k on the aerodynamical derivatives. For the addition of cord clearly increases the value of $P + NI_t$, and might on this score have been expected to make the flutter worse; the fact that it provided a cure indicates that a greater change was produced, in the favourable sense, in the stability boundary defined by the aerodynamical properties.

13. *The Modified Criterion.*—Taking a broad view of the considerations of Parts I and II, it would appear that in general the criterion (16) should be used, but that for tabs with high values of the chord ratio p, the criterion (21) will give some concession; the theory suggests that such a concession can be justified. We may summarise both results by writing finally, as the suggested criterion for the avoidance of spring tab flutter in two degrees of freedom,

$$\frac{P + NI_t}{I} < 0\cdot015 \text{ or } 0\cdot10p^{3/2},$$

whichever is the greater.

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No.	*Author*	*Title, etc.*
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APPENDIX

A.1. *Derivation of the Form of the Criterion.*—In the transformed co-ordinates, the equations of motion are

$$\left.\begin{aligned} A_{11}\ddot{\beta} + B_{11}V\dot{\beta} + C_{11}V^2\beta + C\beta + A_{12}\ddot{\xi} + B_{12}V\dot{\xi} + C_{12}V^2\xi &= 0 \\ A_{21}\ddot{\beta} + B_{21}V\dot{\beta} + C_{21}V^2\beta \qquad + A_{22}\ddot{\xi} + B_{22}V\dot{\xi} + C_{22}V^2\xi &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (A.1)$$

where

$$\left.\begin{aligned} A_{11} &= \bar{I}_t \\ A_{12} = A_{21} &= \bar{P} \\ A_{22} &= \bar{I}_c \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A.2)$$

and the aerodynamic damping and stiffness terms are written B_{ij} and C_{ij} respectively; C is the single elastic stiffness defined by equation (8). If the usual solutions are taken as proportional to exp (λVt) and if we write

$$x = C/V^2$$

then the determinantal equation is

$$\begin{vmatrix} A_{11}\lambda^2 + B_{11}\lambda + C_{11} + x\,, & A_{12}\lambda^2 + B_{12}\lambda + C_{12} \\ A_{21}\lambda^2 + B_{21}\lambda + C_{21} \,, & A_{22}\lambda^2 + B_{22}\lambda + C_{22} \end{vmatrix} = 0\,.$$

On expansion, this becomes

$$q_0\lambda^4 + q_1\lambda^3 + (q_2 + xA_{22})\lambda^2 + (q_3 + xB_{22})\lambda + (q_4 + xC_{22}) = 0\,, \quad \ldots \quad \ldots \quad \ldots \quad (A.3)$$

where

$$\left.\begin{aligned} q_0 &= A_{11}A_{22} - A_{12}A_{21} \\ q_1 &= A_{11}B_{22} + B_{11}A_{22} - A_{12}B_{21} - B_{12}A_{21} \\ q_2 &= A_{11}C_{22} + B_{11}B_{22} + C_{11}A_{22} - A_{12}C_{21} - B_{12}B_{21} - C_{12}A_{21} \\ q_3 &= B_{11}C_{22} + C_{11}B_{22} - B_{12}C_{21} - C_{12}B_{21} \\ q_4 &= C_{11}C_{22} - C_{12}C_{21}\,. \end{aligned}\right\} \quad \ldots \quad \ldots \quad (A.4)$$

The condition that equation (A.3) has a purely imaginary root—that is, that the critical condition when oscillations are not damped has been reached—is

$$0 = q_1(q_2 + xA_{22})\,(q_3 + xB_{22}) - q_0(q_3 + xB_{22})^2 - q_1^2(q_4 + xC_{22})$$

which is a quadratic in x. The two roots of this quadratic define critical flutter speeds, between which the system is unstable. Clearly the flutter speed range is unaltered—that is, the difference between the two roots in x is the same—if we add the same quantity to each. Accordingly, to study the range of instability, we may make the transformation

$$q_3 + xB_{22} = yB_{22}\,;$$

the quadratic then becomes

$$0 = y^2B_{22}^2(q_0B_{22} - q_1A_{22}) - q_1yB_{22}(q_2B_{22} - q_3A_{22} - q_1C_{22}) + q_1^2(q_4B_{22} - q_3C_{22})\,.$$

The condition that this quadratic has equal roots—that is, that the difference between the roots either in y or x is zero, so that there is no range of instability in the physical problem—is, of course,

$$(q_2B_{22} - q_3A_{22} - q_1C_{22})^2 - 4(q_0B_{22} - q_1A_{22})\,(q_4B_{22} - q_3C_{22}) = 0\,. \quad \ldots \quad \ldots \quad \ldots \quad (A.5)$$

We now substitute in this condition the values of q_i given by equation (A.4) and the inertias given by equation (A.2); on expansion equation (A.5) can then be written

$$0 = a\bar{I}_c^2 + 2h_c\bar{I}\bar{P} + b\bar{P}^2 + 2f\bar{I}_c + 2g\bar{P} + c\,, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A.6)$$

where

$$
\begin{aligned}
a &= (B_{12}C_{21} - B_{21}C_{12})^2 - 4|B|C_{12}C_{21} \\
h &= (B_{12}C_{21} - B_{21}C_{12})\{B_{22}(C_{12} - C_{21}) - C_{22}(B_{12} - B_{21})\} + 2|B|C_{22}(C_{12} + C_{21}) \\
b &= \{B_{22}(C_{12} - C_{21}) - C_{22}(B_{12} - B_{21})\}^2 - 4|B|C_{22}^2 \\
f &= -|B|B_{22}\{2B_{11}C_{22} - (B_{12}C_{21} + B_{21}C_{12})\} \\
g &= -|B|B_{22}\{B_{22}(C_{12} + C_{21}) - C_{22}(B_{12} + B_{21})\} \\
c &= |B|^2B_{22}^2,
\end{aligned}
$$

and

$$|B| = B_{11}B_{22} - B_{12}B_{21}.$$

Equation (A.6) defines a relation between the quantities $\bar{I}_c$ and $\bar{P}$ which gives no instability at any speed. The equation is notable in a number of particulars; first, it is independent of the inertia term $A_{11}(=\bar{I}_t)$ and of the stiffness term C_{11}. This result is otherwise obvious, since $A_{11}\lambda^2$ and C_{11} are real terms which could be added to x without alteration to the range between the roots of the quadratic. Thus the tab inertia does not affect the stability independently, but only through its appearance in the terms $\bar{P}$ and $\bar{I}_c$. Again, the relation between $\bar{P}$ and $\bar{I}_c$ defines a hyperbola; a particular case is shown in Fig. 2. It has been shown by Fraser[9] that only one branch (the upper branch in Fig. 2) is significant as a stability boundary; if for a given value of $\bar{I}_c$, $\bar{P}$ lies between zero and the corresponding value on the upper branch of the hyperbola, the system will not flutter.

Equation (A.6) might be used as it stands as a criterion for the avoidance of flutter; but it is clearly too complicated, and depends on too many aerodynamic terms, about which little is known, for general use. However, we may simplify it greatly if we admit some physical arguments. Suppose a principal damping (either B_{11} or B_{22}) to be reduced until $|B| = 0$; the system will presumably be more liable to flutter in this condition. If, therefore, we prevent flutter for $|B| = 0$ the actual system should be more stable. Now with $|B| = 0$ the hyperbola (A.6) collapses into two coincident straight lines through the origin, each given by

$$0 = \bar{I}_c(B_{12}C_{21} - B_{21}C_{12}) + \bar{P}\{B_{22}(C_{12} - C_{21}) - C_{22}(B_{12} - B_{21})\}.$$

A similar result is obtained if B_{22} is reduced to zero; two coincident straight lines through the origin result, each having the equation

$$0 = \bar{I}_c(B_{12}C_{21} + B_{21}C_{12}) - \bar{P}C_{22}(B_{12} + B_{21}).$$

While no rigorous justification for these results can be offered, they do suggest that a simple relation in the form of a straight line through the origin in the $\bar{I}_c$, $\bar{P}$ plane may well prove a useful criterion for the avoidance of flutter. We might adopt either of the lines given above, or, as suggested in the text, a line parallel to the relevant asymptote. Such a line is given by

$$\frac{\bar{P}}{\bar{I}_c} = \frac{-h \pm \sqrt{(h^2 - ab)}}{b}, \qquad \text{(A.7)}$$

the greater positive value of the right-hand side being adopted. But whichever of these alternatives is used, it is necessary to insert the appropriate aerodynamic coefficients, about which all too little is known. Accordingly it is probably more satisfactory to write

$$\bar{P} = k\bar{I}_c \qquad \text{(A.8)}$$

and to find a value for k from flight experience, as was done in the text.

A.2. *The Effect of Geometry.*—The theoretical systems discussed in Part II consisted, as has been said, of a fixed lifting surface of chord 8 ft carrying an aileron of span 9·4 ft and chord 1·2 ft, with a tab of span 9·4 q ft and chord 1·2 p ft; three values of p and three of q were used in the calculations. Aerodynamical derivatives for this system were determined from two-dimensional vortex sheet theory, a constant value of unity being assigned to the frequency parameter. In the first instance, the derivatives were obtained in terms of the co-ordinates ξ, β; they were then transformed by use of the relation

$$\begin{bmatrix} \beta \\ \xi \end{bmatrix} = \begin{bmatrix} 1, & N \\ 0, & 1 \end{bmatrix} \begin{bmatrix} \bar{\beta} \\ \bar{\xi} \end{bmatrix} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A.9)$$

in which N was given the typical value 3.

The values of the aerodynamical damping and stiffness derivatives thus obtained are as follows: they relate to standard density conditions, and are quoted for the specific values of p but with q general.

Values of Damping Derivatives (slugs ft)

$p \times 15$	$B_{11} \times 10^3$	$B_{12} \times 10^3$	$B_{21} \times 10^3$	$B_{22} \times 10^3$
2	$0 \cdot 0918q$	$0 \cdot 5800q$	$2 \cdot 655q$	$38 \cdot 60 + 8 \cdot 88q$
4	$0 \cdot 9267q$	$4 \cdot 446q$	$10 \cdot 715q$	$38 \cdot 60 + 37 \cdot 14q$
6	$2 \cdot 51005q$	$12 \cdot 014q$	$21 \cdot 804q$	$38 \cdot 60 + 78 \cdot 86q$

Values of Stiffness Derivatives (slugs)

$p \times 15$	$C_{11} \times 10^3$	$C_{12} \times 10^3$	$C_{21} \times 10^3$	$C_{22} \times 10^3$
2	$0 \cdot 2412q$	$0 \cdot 7932q$	$17 \cdot 89q$	$13 \cdot 24 + 53 \cdot 88q$
4	$0 \cdot 9609q$	$3 \cdot 2861q$	$23 \cdot 72q$	$13 \cdot 24 + 72 \cdot 38q$
6	$2 \cdot 1533q$	$7 \cdot 598q$	$28 \cdot 06q$	$13 \cdot 24 + 87 \cdot 58q$

For each of the nine cases under consideration, the appropriate values of B_{ij} and C_{ij} have been employed to determine the coefficients a, b, etc., in equation (A.6). From these coefficients the centre of the hyperbola has been determined and also the straight lines through the origin parallel to the asymptotes, given by equation (A.7). The relevant straight line (that with the greater positive slope in the $\bar{I}_c$, $\bar{P}$ plane) is given in Table 2 in Part II.

It was remarked above that the derivatives were obtained from two-dimensional theory: that is, no corrections for aspect ratio were applied. Clearly, this involves some error; and no information is available to assess the magnitude of the error. However, if (as is frequently assumed) the effect of aspect ratio is to reduce all the terms B_{ij}, C_{ij} by some constant factor R, the coefficients a, b, h in the expression for the hyperbola will all be reduced by the factor R^4, and the slope of the asymptote will accordingly be unaltered. This suggests that the criterion for stability will not depend acutely on aspect ratio effects.

In a similar manner, it may be shown that the criterion is independent of altitude. The terms B_{ij} and C_{ij} are both proportional to the relative density; and this constant factor will not affect the slope of the asymptote.

A.3. *Retention of the Spring Stiffness* C_2.—In section 2 it was shown how to choose new co-ordinates for which elastic couplings would be absent; in the actual analysis, however, it was assumed that the spring stiffness C_2 ($= rC_1$) was insignificant and could be neglected. We now consider the effect of retaining this term. The expression giving the potential energy V is (*see* equation (4))

$$\frac{2(1 + r + s)}{s} \frac{V}{C_1} = (\beta - N\xi)^2 + r\beta^2 + \frac{r}{s} N^2\xi^2 . \qquad \text{(A.10)}$$

In place of equations (7) we now choose co-ordinates $\bar{\xi}$, $\bar{\beta}$, given by

$$\left.\begin{aligned} \xi &= (1 + r)\bar{\xi} \\ \beta &= \bar{\beta} + N\bar{\xi} \end{aligned}\right\} \qquad \text{(A.11)}$$

or

$$\left.\begin{aligned} \bar{\xi} &= \xi/(1 + r) \\ \bar{\beta} &= \beta - N\xi/(1 + r) , \end{aligned}\right\}$$

which reduce to equations (7) when $r = 0$. Substitution from equations (A.11) in (A.10) now removes the product terms; the equation becomes, after a little reduction

$$\frac{2}{1+r} \frac{V}{C_1} = \frac{s}{1 + r + s} \bar{\beta}^2 + rN^2\bar{\xi}^2 , \qquad \text{(A.12)}$$

which reduces to equation (8) when r vanishes. Since equation (A.12) contains no product terms there is no elastic coupling; but we now have two direct stiffnesses instead of one. However, in practice the presence of the second stiffness does not materially alter the conclusions of section A.1.

We may, however, make some useful deductions from a study of the new inertias. Substitution from equations (A.11) in (9) yields a new expression for the kinetic energy which is an extension of equation (11) and which gives in place of equation (12)

$$\left.\begin{aligned} \bar{I}_c &= (1 + r)^2 I_c + 2(1 + r) NP + N^2 I_t \\ \bar{P} &= (1 + r)P + NI \\ \bar{I}_t &= I_t \end{aligned}\right\} \qquad \text{(A.13)}$$

When consideration is given to the relative importance of the quantities determining $\bar{I}_c$, $\bar{P}$ in equations (A.13) it is seen that the inclusion of r (which is always positive) increases $\bar{I}_c$ relatively more than $\bar{P}$; we may conclude that introduction of the spring stiffness C_2 is likely to have a slightly beneficial effect on the flutter characteristics. Moreover, if it is necessary in a particular case to reduce $\bar{P}$ by the addition of a balance mass, the spring stiffness C_2 gives a slight extension of the limiting length. For if the mass is M on an arm of length $- l$ (l positive) a diminution in $\bar{P}$ implies

$$(1 + r)(Ml^2 - d_0 lM) + NMl^2 < 0$$

or

$$l < \frac{d_0}{1 + \dfrac{N}{1 + r}} ,$$

which is slightly greater than the value for $r = 0$.

A.4. *The Effect of Offset Masses : a 'Limiting Circle'*.—In the main text the analysis was simplified by the assumption that all masses lie in the plane containing the two hinge lines. In fact, the deviations from this assumption are usually quite small; it is, however, a simple matter to derive the appropriate formulae when some parts of the system or some balance masses are offset from the plane of the hinges.

In Fig. 3, B is the control surface hinge and C the tab hinge. Consider a mass dm disposed as shown, at a distance x_c aft of the hinge B, and a distance y from the plane of the hinges. When the control surface has angular velocity $\dot{\xi}$, the components of velocity of dm normal and parallel to the plane containing BC are clearly $\dot{\xi}x_c$ and $\dot{\xi}y$ respectively. If the mass dm is attached to the tab it will have additional components $\dot{\beta}x_t$ and $\dot{\beta}y$; hence the total kinetic energy T is given by

$$2T = \int_c (\dot{\xi}^2 x_c^2 + \dot{\xi}^2 y^2)dm + \int_t \left\{(\dot{\xi}x_c + \dot{\beta}x_t)^2 + (\dot{\xi} + \dot{\beta})^2 y^2\right\} dm$$

$$= \dot{\xi}^2 \int_{c+t} (x_c^2 + y^2)dm + 2\dot{\xi}\dot{\beta} \int_t (x_c x_t + y^2)dm + \dot{\beta}^2 \int_t (x_t^2 + y^2)dm \qquad \text{(A.15)}$$

which is the generalisation of equation (9). The coefficients of $\dot{\xi}^2$ and $\dot{\beta}^2$ are, however, still the moments of inertia I_c and I_t; while if we write as before

$$x_c = d_0 + x_t ,$$

then

$$P = \int_t (d_0 x_t + x_t^2 + y^2)dm = d_0 \int_t x_t dm + I_t , \qquad \text{(A.16)}$$

where I_t is the true moment of inertia of the tab. Thus all the results of the main text, deduced for masses lying in the plane of the hinge lines, are still correct if the inertias are regarded as the complete moments of inertia and if P is interpreted as the sum of the tab moment of inertia and the product of d_0 and the static unbalance of the tab.

The question of offset balance masses is covered by the foregoing analysis. Suppose the appropriate value of $P + NI_t$ is to be achieved by the addition of a balance mass M on an arm of length l (radial from the tab hinge) in a direction making an angle θ with the plane of the hinges (Fig. 4). Its contribution to $P + NI_t$ is, by equation (A.16)

$$- d_0 Ml \cos\theta + (N + 1)Ml^2 = Ml\{(N + 1)l - d_0 \cos\theta\}. \qquad \text{(A.17)}$$

If this is to be negative we must have

$$l < \frac{d_0 \cos\theta}{N + 1} , \qquad \text{(A.18)}$$

an inequality which is the generalisation of the inequality (13).

We may make some further deductions of interest.

If

$$l = \frac{d_0 \cos\theta}{(N + 1)} , \qquad \text{(A.19)}$$

the contribution of the mass M to $P + NI_t$ is, by equation (A.17), zero. Now the relation (A.19) between l and θ defines a circle on QC as diameter (*see* Fig. 4) where C is the tab hinge and Q lies on BC at a distance $d_0/(N + 1)$ forward of C. Accordingly, if a balance mass is disposed on this circle its contribution to $P + NI_t$ is nil and it is useless as a flutter preventive. If the mass is disposed anywhere within the circle it will reduce $P + NI_t$; outside the circle its effect is to increase $P + NI_t$. It is readily shown that the expression (A.17) for the contribution of the mass M to $P + NI$ can be reduced to

$$(N + 1)M\{r^2 - R^2\}$$

where r is the radial distance of M from the centre of the circle and

$$R = d_0/2(N + 1)$$

is the radius of the circle. Thus a given mass has its optimum effect at the centre of the circle, and the effect falls off parabolically with r; it is zero for $r = R$, and for $r > R$ the adverse effect rapidly increases with r. We see, therefore, that we have a limiting circle as a generalisation of the idea of a limiting length for the position of a balance mass; the limiting length is the diameter of the circle when the balance mass lies in the plane of the tab.

For a simple mechanism such as that shown in Figs. 1 and 4, the intersection of the circle with BC at Q is readily found without computation. For since Q lies at a distance $d_0/(N + 1)$ from C, it divides BC in the ratio $N : 1$. But this is the ratio of the lengths of the swinging levers BE and CF; accordingly Q lies at the intersection of the straight lines BC and EF. Thus the limiting circle is at once defined by the geometry of the system.

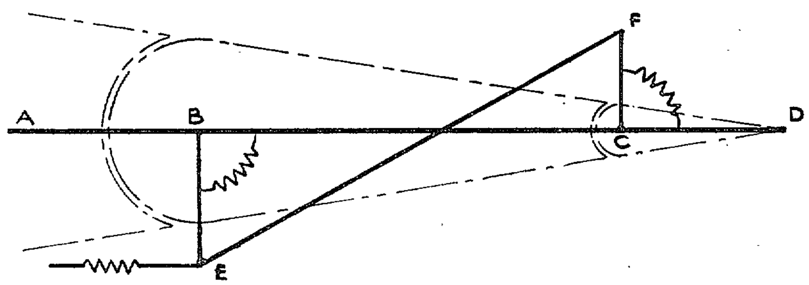

FIG. 1 (a). Typical Spring-Tab System.

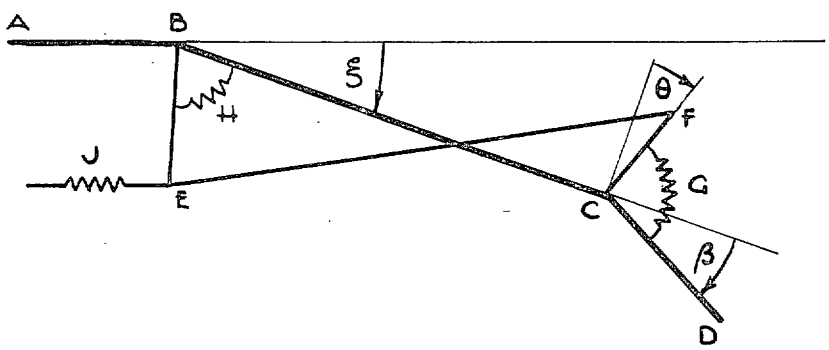

FIG. 1 (b). General Displacement.

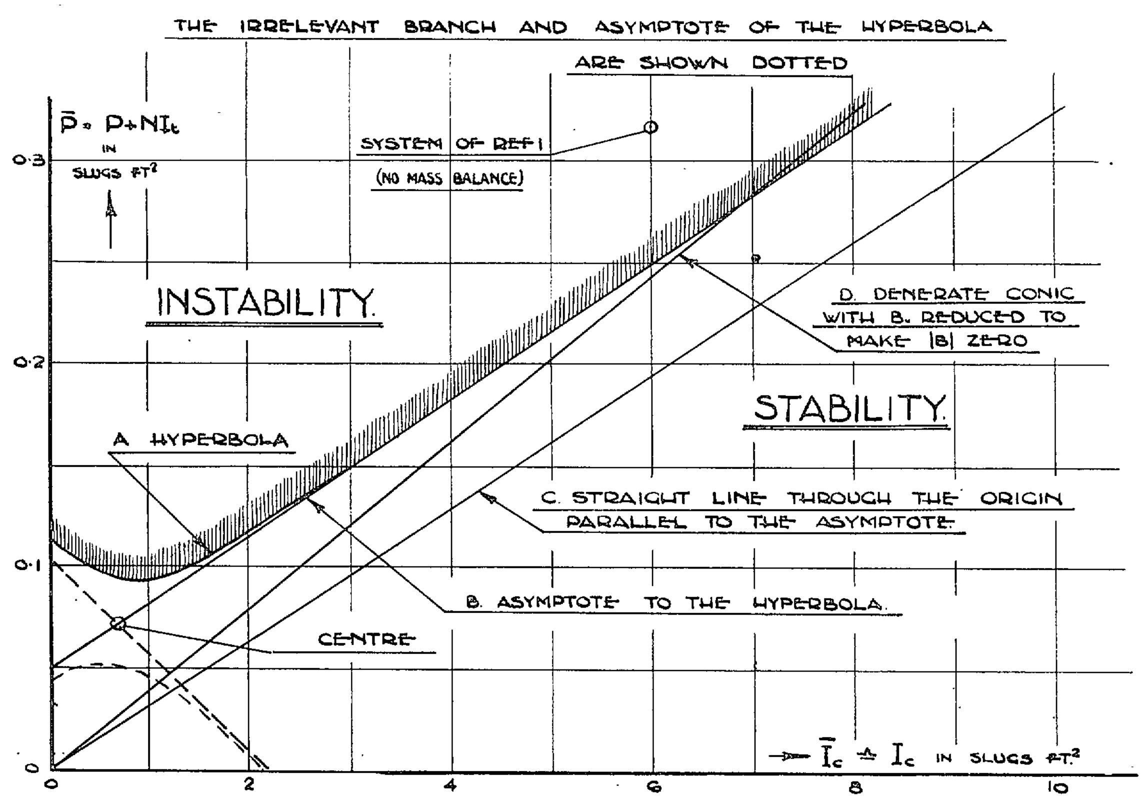

FIG. 2. Stability Boundaries for System of R. & M. 1527[2].

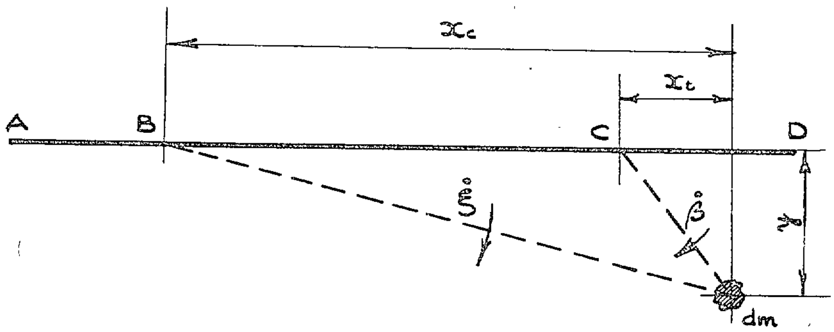

FIG. 3. Motion of an Offset Mass.

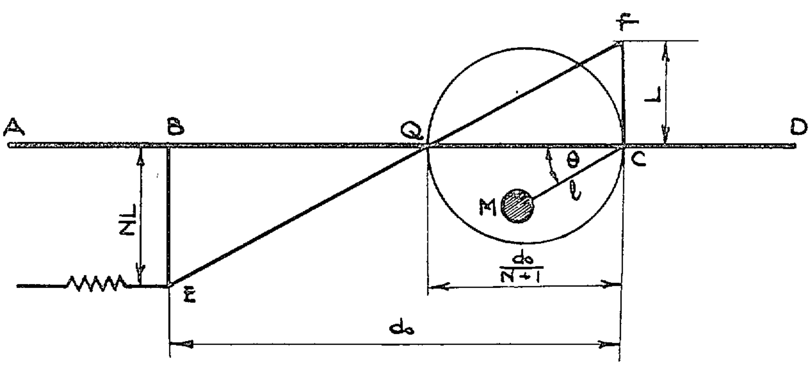

FIG. 4. The Limiting Circle.

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