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THE FLYING QUALITIES OF AIRCRAFT WITH ACTIVELY AUGMENTED PITCH AND YAW STABILITY

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SUMMARY

Ignoring the dynamics of the automatic control loop hardware, the effects have been studied on rigid body mode stability, gust response and pilot's control of aircraft in which relaxed longitudinal or directional stability is augmented by various alternative feedbacks to the elevator and rudder respectively. It is shown that 'indirect' augmentation by such methods as feedback of pitch rate, integral pitch rate or normal acceleration in the longitudinal case or lateral acceleration in the directional case can give rise to significant adverse characteristics, which could limit the amount of instability which can be so corrected or which may demand additional corrective feedback loops. 'Direct' feedback of incidence or sideslip on the other hand will restore in most respects behaviour virtually indistinguishable from that of naturally stable configurations, provided these quantities can be sensed accurately and reliably. Particular attention is drawn to difficulties with indirect feedbacks in maintaining stability near and beyond the stall and to gust response characteristics.

* Replaces RAE Technical Report 79029 - ARC 38234

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1 INTRODUCTION

If the aircraft designer is permitted to ignore the customary requirement for natural weathercock-stability in pitch and in yaw, he will be able to produce configurations with substantially enhanced performance. Such designs depend on active control to achieve stable and controllable flight. Unless only very modest relaxation in airframe stability is to be corrected, the control system providing the necessary augmentation must satisfy extreme reliability requirements, since failure may render the aircraft instantly unflyable. These integrity requirements dominate all aspects of system design, such as sensor choice, systems architecture, hard and software implementation etc. We shall not be addressing these here except for the very vital choice of the primary motion sensors. It is generally understood that stability can be restored to an aerodynamically unstable airframe by a number of fundamentally differing feedback concepts, using alternatively or perhaps in combination airflow direction sensors, accelerometers or gyroscopes as the basic aircraft state detectors. Since these sensors offer distinctly different reliability as well as performance prospects, it is clearly of interest to establish in depth the capabilities and limitations of these various possible loop-closure techniques.

The basic deficiency to be corrected is either lack of pitch stability m_w in the longitudinal case or of directional stability n_v in the lateral case. The most obvious and direct technique for synthesising a stabilising contribution to these derivatives is to feed back incidence to the pitch control (elevator) or sideslip to the yaw control (rudder) respectively. If the elements in the servo loop making up such a control system can be assumed to have adequate bandwidth and negligible time delays then such a loop closure should at least in theory restore completely normal aircraft behaviour. Unfortunately, the only devices known today for measuring airflow direction have to be directly exposed to the airstream and cannot therefore be located in a secure environment within the airframe. Such devices are vulnerable to damage by, *eg* hail and birds on the one hand and by mishandling on the ground on the other. Another problem is that any airflow sensor mounted in a practical location will see a flow strongly distorted by the proximity of the airframe itself and not what is understood as aircraft 'incidence' or 'sideslip'. Moreover a multiplicity of such sensors, as would be needed to satisfy failure-survival requirements, would not be able to register - for the same reason - identical returns and this poses severe problems for interlane comparison on which automatic failure isolation may depend.

For all these reasons there is considerable reluctance to commit an actively controlled static stability augmentor to such a feedback philosophy, however attractive it might be in all other respects.

One's attention is therefore drawn to alternatives which utilise sensors not suffering the uncertainties of the flow-direction detector. Indeed the first two serious attempts to invoke active control for augmenting vanishing directional stability, one on the Avro (Canada) Arrow and the other for the TSR2 project, used lateral accelerometers as indirect sensors of sideslip. Both projects were cancelled before flight development reached the high speeds for which this facility was designed and the practicality of the techniques was thus never demonstrated. Equal interest has been shown in the use of pitch-rate gyros for longitudinal stability augmentation. Both these techniques use inertial sensors which can be afforded a sheltered position within the airframe and which also have a well established record of accuracy and reliability.

These sensors do not of course directly measure the quantity of immediate interest, *ie* incidence or sideslip, but the response of the aircraft to changes in these quantities. Their acceptability as a proper substitute for direct flow direction measurements rest on the assumption that these response relationships remain positive and unambiguous for all conceivable flight conditions and manoeuvres. The assumption needs careful examination before such indirect techniques can safely be recommended as a viable alternative to direct flow direction feedback.

In the present study only the most fundamental aspects of this problem are assessed; in particular we shall assume that the servo system providing loop closure has sufficient bandwidth and is adequately lag-free to allow its dynamic properties to be ignored in the treatment of the stability and response of the aircraft rigid body modes. Also coupling with structural modes is ignored in the analytic treatment. This Report can therefore only be offered as a first step in the assessment of this branch of active control technology. We shall, however, pay particular attention to those aspects of aircraft control and behaviour which may not become apparent in the conventional approach to servo control design with its emphasis on system stability and transient control response. In particular we shall consider conditions outside the principal operating regime where safety rather than performance is the dominant concern.

2 THE AIRCRAFT WITH RELAXED INCIDENCE STABILITY

Weathercock-stability in pitch as expressed by the aerodynamic derivative $m_w = \partial C_m/\partial\alpha$ is generally considered one of the fundamental design criteria and considerable effort is expended in the design and flight development of each project to ensure it to be within acceptable limits over the whole flight and manoeuvre envelope. m_w controls three distinct aspects of aircraft stability and control:

(a) It provides stability of the short period longitudinal oscillation, the aircraft mode dominating the short term response to pilot's control and external disturbances.

(b) It provides static stability and through it long term flight stability especially of airspeed.

(c) It ensures a positive relationship between stick displacement or stick force on the one hand and medium term normal acceleration response on the other.

Positive incidence stability is ensured by locating the centre of gravity of the aircraft forward of the so called aerodynamic centre of the airframe. Variations in aircraft loading, fuel consumption etc result in variations in the centre of gravity positions whereas changes in configuration, airspeed and Mach number affect the aerodynamic centre. The critical design case for stability is normally that where the margin between the centre of gravity and the aerodynamic centre is a minimum, that for control requirements and therefore tailplane power is that when it is a maximum.

These are the constraints dictating conventional configuration design, further complicated by inevitable nonlinearities in the aerodynamic characteristics. The result is an airframe in which much potential performance is sacrificed for the provision of safe handling. These penalties can be minimised or even turned to profit (for instance by arranging elevator trim lift to be proverse) if flight stability can be provided by active control rather than configuration layout. The chosen control system must of course satisfy all the requirements for stability and controllability listed above and in addition must not introduce new handling problems.

An aircraft to which the present investigation applies clearly represents very advanced control technology. In such a design active control is likely to be used not only for augmenting basic longitudinal stability but for more subtle shaping and control of response. In such a case the time honoured stability parameters of the 'natural' aircraft, such as the manoeuvre margin and the static margin may no longer play the unique and dominating role we customarily associated them with. Similarly we may no longer recognise in such a case the traditional aircraft modes, such as the short period and the phugoid as defining short and long term response respectively. It also implies that the considerable store of experience embodied in current handling criteria may no longer be relevant and one finds oneself in uncharted territory.

We felt it necessary here to stay clear of this area and base our work on the assumption that at worst such distortions to aircraft response will not be so drastic as to completely invalidate conventional aircraft stability and response criteria.

We shall proceed therefore on the assumption that the primary function of the stability augmentation loop is to restore to the aircraft effective incidence stability and with this more or less conventional behaviour over the whole flight and manoeuvre envelope.

According to classical stability theory incidence stability is basically required to achieve adequate positive values of the two fundamental stability parameters defined as the manoeuvre margin:

$$H_m = -\frac{\partial C_m}{\partial C_L} - \frac{m_q}{\mu} = -\frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu} \qquad (1)$$

and the static margin:

$$H_n = -\frac{\partial C_m}{\partial C_L} + \frac{m_u}{2C_{L_0}} = -\frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} + \frac{m_u}{2C_{L_0}} \ . \qquad (2)$$

These quantities are stability margins which express as a fraction of the reference length used to nondimensionalise the pitching moment derivatives the tolerance in stability and controllability to unforeseen changes in the mass distribution or aerodynamic properties of the airframe.

When considering an aircraft stabilised by automatic control rather than orthodox aerodynamic measures it might be argued that these concepts of stability margins may no longer be relevant and be superseded by the stability criteria of automatic control theory. These latter will obviously become important and the role of the classical stability margins in this situation is far from obvious. This issue has been studied in detail in Appendix B with the important conclusion that, with the important exception of integral pitch rate system, for the class of feedback systems considered in this Report the manoeuvre margin as defined in equation (1) has the same significance as a margin against unforeseen changes in airframe properties as in the classical situation. In particular this means that the short period mode will become divergent and positive steady state control will be lost if the manoeuvre margin becomes negative.

When integral pitch rate is used as the primary stabilising term, a 'manoeuvre margin' of the form

$$H_m^* = -\frac{\partial C_m/\partial\alpha + \partial C_m/\partial\int q}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu}$$

appears in the analytical treatment of small perturbations where again it constitutes a necessary condition for stability. We note that in this expression the pitch augmentation term $\partial C_m/\partial\int q$ is directly additive to the aerodynamic incidence stability term $\partial C_m/\partial\alpha$. However, detailed analysis will show that in many cases $H_m^* > 0$ is not a sufficient condition for short period stability and this 'manoeuvre margin' does then not command the dominant importance classically associated with it.

With all the other feedbacks discussed in this Report, however, H_m retains its central significance. Moreover as shown in Appendix B this applies whatever the complexity of the transfer function describing the dynamic properties of the servo loop elements.

These observations are of particular significance here, since in the simplified analysis of this Report we shall generally ignore servo loop dynamics and the corresponding additional response modes introduced in a real system. In one important respect at least this idealised approach is therefore shown to be valid although other important and perhaps dominating aspects will be missed, such as the maximum gains that can be realised in practice with a control system having limited bandwidth and other imperfections.

Apart from considering fundamental stability margins we shall discuss, wherever significant departures from conventional behaviour are expected, quasi-steady manoeuvre control, response to turbulence, stability in inertia-coupled rapid rolling and especially behaviour near the stall when drastic changes can occur in aerodynamic factors normally considered sensibly invariant, such as the lift slope.

The two stability factors defined in equations (1) and (2) each contain in addition to the pitch stability derivative $\partial C_m/\partial\alpha$ another term which could be augmented to achieve a desired value of H_m or H_n. These alternatives are pitch damping (pitch rate into elevator) for the manoeuvre margin and m_u

corresponding to speed into elevator for static stability. Moreover since the incidence stability term actually enters the analysis in the form of a pitching moment derivative with respect to C_L, *ie* with respect to a quantity proportional to lift and therefore normal acceleration, another alternative would appear to be to feed normal acceleration to the elevator to augment both H_m and H_n.

The various feedback concepts can be visualised as either alternatives or complementary; they are schematically illustrated in Fig 1. Another promising approach is to use integral pitch rate as the primary augmentation feedback. For reasons elucidated later this feedback concept can only be implemented for a control system employing pitch rate demand as indicated in the block diagram shown in Fig 11. These alternatives, combined perhaps with airspeed feedback to control the static margin, are evidently capable of restoring both the manoeuvre margin and the static margin to any desired positive value and therefore satisfy the fundamental longitudinal stability requirements, but it is still necessary to ensure that these indirect methods lead to fully acceptable flight and control characteristics in all areas and it is to these problems that the following discussion is predominantly addressed.

Before discussing these control strategies in detail it should be noted that there is yet another conceivable method of obtaining a measurement of incidence from inertial sensors. It is commonly known that in flight through a uniform atmosphere and when the aircraft is restricted to move only in the vertical plane, of the three state variables α, q and n, one is redundant and that in theory at least it is possible to deduce α from measurements of q and n by integrating:

$$\dot{\alpha}' = q - \frac{n}{V} g \tag{3}$$

In addition to pitch rate q and normal acceleration n, however, true speed V must also be available. In general unrestricted flight the kinematic relationship expressed in equation (3) for pure longitudinal motion becomes substantially more complex and requires IN standards of measurements, but the principle remains valid. The quantity we defined as $\dot{\alpha}'$ here, however, only represents true aerodynamic incidence in flight through still air and would not register the effects of wind and gusts, *ie* it is an incidence related to a spatial reference frame rather than to the atmosphere. One would expect an aircraft stabilised through such an 'inertial' incidence feedback to react in an unorthodox manner to turbulence. Although for many reasons not a very practical scheme, we shall consider it briefly.

3 INCIDENCE FEEDBACK

By relaxing longitudinal stability the only aerodynamic term significantly affected is weathercock-stability $m_w = \partial C_m/\partial\alpha$. An active control system sensing incidence and feeding a proportional signal to the elevator (or other pitch control)

$$\eta = K_\alpha \alpha \tag{4}$$

will therefore be able directly to restore the effective $\partial C_m/\partial\alpha$ to the desired value by providing an increment

$$\Delta \frac{\partial C_m}{\partial \alpha} = K_\alpha \frac{\partial C_m}{\partial \eta} \tag{5}$$

where $K_\alpha > 0$ for a stabilising contribution. At the same time the elevator lift generated by this mechanisation will augment the effective lift slope by a positive increment:

$$\Delta \frac{\partial C_L}{\partial \alpha} = K_\alpha \frac{\partial C_L}{\partial \eta} \ . \tag{6}$$

It is convenient to express the elevator lift derivative by introducing the concept of an effective elevator moment arm x_E (negative for conventional rear controls) so that

$$\frac{\partial C_L}{\partial \eta} = \frac{\partial C_m}{\partial \eta} \frac{\ell}{x_E}$$

where ℓ is the reference length, usually taken as the mean chord $\bar{c}$. Hence

$$\Delta \frac{\partial C_L}{\partial \alpha} = K_\alpha \frac{\partial C_m}{\partial \eta} \frac{\ell}{x_E} \ . \tag{7}$$

The effective lift slope of an aircraft stabilised by α-feedback therefore is

$$\frac{\partial C_L}{\partial \alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_a + K_\alpha \frac{\partial C_m}{\partial \eta} \frac{\ell}{x_E} \tag{8}$$

where suffix a denotes the aerodynamic contribution of the basic aircraft. It should be noted that this lift slope is that relevant to stick fixed aircraft dynamics.

Since the manoeuvre margin of an aircraft with a negligible natural m_q contribution is:

$$H_m = - \frac{\partial C_m / \partial \alpha}{\partial C_L / \partial \alpha} .$$

the effect of the elevator lift on the lift slope will also have to be taken into account when assessing the gain K_α required to effect a given increment to the manoeuvre margin. It can be readily shown that:

$$K_\alpha = - \frac{(\partial C_L / \partial \alpha)_a}{m_\eta} \frac{\Delta H_m}{1 + \frac{\ell}{x_E} (H_{ma} + \Delta H_m)}$$

or since the effective manoeuvre margin is $H_m = H_{ma} + \Delta H_m$

$$K_\alpha = - \frac{(\partial C_L / \partial \alpha)_a}{m_\eta} \frac{\Delta H_m}{1 + \frac{\ell}{x_E} H_m} . \qquad (9)$$

The effective lift slope is

$$\frac{\partial C_L}{\partial \alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_a \left(1 - \frac{\Delta H_m}{\frac{x_E}{\ell} + H_m}\right) . \qquad (10)$$

The elevator lift effect is seen to increase the gain K_α required to effect a given increment ΔH_m in the manoeuvre margin over that estimated without this contribution and it also increases the effective lift slope. These effects are of course only significant for aircraft with pitch control surfaces acting at a short moment arm, such as the elevons of a tailless aircraft. Here $(-x_E/\ell)$ can be of the order of unity and with an active control system designed to increase the manoeuvre margin by $\Delta H_m = 0.2$, say from −0.1 to +0.1 the elevator lift effect would increase the effective lift slope by about 22%.

This would evidently be beneficial to the damping of the short period oscillation but also increase the normal acceleration sensitivity of the aircraft to vertical gusts and therefore adversely affect ride comfort.

As long as the servo loop performing the α-stabilisation function has sufficient bandwidth so that system dynamics can be ignored the α-stabilised aircraft will - apart from the minor anomalies observed above - behave exactly as a naturally stable airframe as far as stability, control and gust response are concerned and within the limited scope of the present study nothing more need be added. In practice of course there are other factors limiting the gains that can be safely utilised.

A particularly interesting effect unique to the particular feedback is associated with the fact that most practical locations for the flow direction sensor are well forward of the centre of gravity of the aircraft. Such a sensor sees a gust before it strikes the aerodynamically active parts of the airframe, especially the tailplane which provides normally the dominant aerodynamic pitching moment contribution. This lead in the sensor signal is in one sense useful in that it will compensate for the inevitable transport lag of the signal through the servo loop but since the time lag for the air to pass a given distance along the airframe is inversely proportional to true speed whereas the system lag is independent of speed, ideal compensation can only be expected at one particular speed. Fig 2 shows the time intervals involved for the aircraft to pass a given air-distance d or more importantly the time for a gust to travel a given distance past the aircraft. The consequences of the phenomenon on the immersion phase of gust response are illustrated by an idealised schematic in Fig 3.

Fig 3a shows the development of the pitching moment disturbance ΔM as the aircraft passes through a vertical step gust at two different airspeeds $M = 0.2$ and $M = 0.8$. Case (1) represents a conventional naturally stable aircraft. We note that the destabilising wing first meets the gust followed by the stabilising tailplane. The difference between the low and high speed case is in scale only since a given gust velocity produces aerodynamic forces proportional to speed - assuming flight at the same height. Case (2) gives the corresponding picture for the unstable aircraft stabilised by α-feedback to the stability level of aircraft (1). The incidence sensor is assumed to be near the nose as indicated and we now get as an additional contribution the reaction of the elevator η to the gust signal. At low speeds the elevator reacts before even the wing has reached the gust front so that the initial aircraft reaction is opposite to that of the naturally stable airframe. At high speeds the whole aircraft has passed through the gust before the system lag allows the elevator to respond. The picture is of course rather idealised by assuming step reactions of all the elements involved but this should not greatly affect the general conclusions. Shown is the pitching moment disturbance caused by the gust and not the consequent aircraft response, which of course would tend to remove the disturbance with time. Perhaps the most vivid illustration of the difference between the naturally stable aircraft and the α-stabilised unstable airframe is obtained by integrating Fig 3a to give the disturbance in angular momentum $\int \Delta M dt$ picked up from the gust as illustrated in Fig 3b. As far as the general magnitude of the initial disturbance is concerned there is little to choose between the two aircraft configurations at low speed. However since the reaction of the artificially stabilised aircraft is throughout in the sense to reduce the gust induced incidence, we do in fact get some gust alleviation and a consequent improvement in ride comfort. At high speeds, on the other hand the artificially stabilised unstable airframe suffers substantially more severe pitch disturbance in the sense that will increase normal acceleration response. The results shown here apply of course only to a particular set of assumed aircraft and system characteristics and are shown merely to point at a phenomenon that will need proper assessment in each individual case. Since we are dealing with a high frequency phenomenon the more important consequence may well be in the excitation of structural modes, a subject well outside the scope of the present Report.

α-feedback produces an increment to $\partial C_m/\partial\alpha$ which according to equation (5) is a function (proportional to) only of the gain K_α and the elevator effectiveness and not explicitly of airspeed. There is therefore a good chance that a constant gain will give the desired result over the full flight envelope. M_η will decrease at supersonic speed but in that regime aircraft longitudinal stability increases anyway so that the reduced effectiveness of an α-feedback loop there is likely to be welcome. α-feedback stabilisation therefore would appear to require little if any gain scheduling, an attractive feature.

4 PITCH RATE FEEDBACK

Apart from situations where only minor corrections are required, the use of pitch rate feedback as the only or at least primary pitch stabilisation term is not generally considered a realistic proposition. However it is at least theoretically an effective means of restoring manoeuvre stability and we shall discuss it here at some length mainly as a simple introduction to the more general utilization of gyroscopic feedbacks, in particular that of integral pitch rate.

The proposition of using pitch rate as the principal feedback signal for augmenting deficient incidence stability rests on the fact that the manoeuvre margin (equation (1)) can be enhanced by pitch damping and this in turn can be provided artificially by elevator control of the form

$$\eta = K_q q \quad . \tag{11}$$

Since this method of stabilisation does not directly restore the deficient α-stability derivative $\partial C_m/\partial\alpha$ of the aircraft, as in the case of α-feedback, the possibility exists that an aircraft so augmented may differ from the naturally stable conventional aircraft in some important fashion and we shall therefore examine likely problem areas in detail.

4.1 Manoeuvre margin and manoeuvre control

As distinct from a conventional pitch autostabiliser the pitch rate signal required for the relaxed stability aircraft must not be washed out lest its contribution in prolonged manoeuvres is lost. Within the simplifying assumptions used here throughout we can therefore take equation (11) as fully defining the system control law and with this the increment in pitch damping would be

$$\Delta m_q = \frac{V}{\ell} K_q m_\eta \tag{12}$$

where $K_q > 0$. The corresponding increment in the manoeuvre margin is

$$\Delta H_m = -\frac{\Delta m_q}{\mu} = -m_\eta K_q \frac{g(\rho/2)V}{W/S} \quad . \tag{13}$$

Therefore, to produce a given increment ΔH_m the gain K_q must be adjusted according to

$$K_q = \text{const}\frac{W/S}{\sqrt{\sigma}V_E m_\eta} = f(W,H,V_E,M) \quad .$$

Even if some of these scheduling parameters can be ignored there is clearly an *a priori* need for gain scheduling.

The concept of the manoeuvre margin is derived from small perturbation analysis. In this context H_m is a factor defining directly the 'stiffness' and therefore the undamped frequency ω_n of the short period oscillation:

$$\omega_n^2 = \left(\frac{V}{\ell}\right)^2 \frac{H_m}{\mu} \frac{\partial C_L}{\partial\alpha} \quad . \tag{14}$$

It also determines the elevator angle per g and therefore the stick force gradient relevant to the control of steady increments in incidence or normal acceleration through

$$\Delta\eta = \Delta\alpha \frac{H_m}{m_\eta} \frac{\partial C_L}{\partial \alpha} \quad . \tag{15}$$

The fact that this latter relationship contains m_q as an element of H_m rather than $\partial C_m/\partial\alpha$ alone is a reflection of the kinematic relationship that exists for steady manoeuvre between normal acceleration and pitch rate

$$n - 1 = q \frac{V}{g} \quad . \tag{16}$$

However, equation (16) is only valid for manoeuvres in the vertical plane not substantially departing from level flight. As has been re-emphasised in Ref 1 in all other cases more complex relationships govern the coordination of pitch rate and normal acceleration and therefore α and equation (15) is then only valid as an approximation if the manoeuvre margin is dominated by the $\partial C_m/\partial\alpha$ term. However, we are concerned here with aircraft in which the effective manoeuvre margin is dominated by the m_q term and where $\partial C_m/\partial\alpha$ may even have reversed sign. We must therefore consider more rigorously the individual contributions to the pitching moment balance involved in general manoeuvres. From Ref 1 we quote the relevant kinematic relationships for two specific classes of steady manoeuvres.

In pull up or push over manoeuvres in the vertical plane

$$q = \frac{g}{V}(n - \cos\gamma) \tag{17}$$

where γ is the instantaneous flight path angle. In level coordinated turns

$$q = \frac{g}{V}\, n - \left(\frac{1}{n}\right) \quad . \tag{18}$$

These relationships are shown graphically in Fig 4.

Pitching moment equilibrium requires that

$$\frac{\partial C_m}{\partial C_L} \Delta C_L + m_q q \frac{\ell}{V} + m_\eta K_q q = - m_\eta \eta_p \tag{19}$$

where η_p is the elevator angle applied by the pilot. We ignore the aerodynamic m_q contribution as insignificant, *ie* we assume that the manoeuvre margin of the basic airframe is fully defined by

$$H_{ma} = -\frac{\partial C_m}{\partial C_L} .$$

The contribution of the pitch rate feedback term can be readily shown to be

$$H_{mq} = -m_\eta K_q \frac{V/\ell}{\mu} . \qquad (20)$$

Substituting this into equation (19) gives

$$-H_{ma}\Delta C_L - H_{mq}\mu\frac{\ell}{V}q = -m_\eta \eta_p . \qquad (21)$$

Since $\Delta C_L = C_{L_0}\Delta n$

$$H_{ma}C_{L_0}\Delta n + H_{mq}\frac{\ell}{V}\mu q = m_\eta \eta_p \qquad (22)$$

which reduces to

$$\Delta n C_{L_0}\left(H_{ma} + H_{mq}\frac{V}{g}\frac{q}{\Delta n}\right) = m_\eta \eta_p . \qquad (23)$$

Taking the appropriate relationships between q and Δn such as equations (17) and (18) we can now calculate from equation (23) the relationships between η_p and Δn for any given quasi-steady manoeuvre. The results are shown in normalised form in Figs 5 and 6, where the augmentation contribution to the manoeuvre margin is expressed as the ratio $\Delta H_{mq}/H_m$ where $H_m = H_{ma} + H_{mq}$ is the effective total manoeuvre margin. We note that both in aerobatic manoeuvres in the vertical plane (Fig 5) and in coordinated turns, (Fig 6) the relationships between pilot elevator and therefore stick force and Δn are becoming increasingly irregular as an increasing portion of the effective manoeuvre margin H_m is supplied by q-feedback to the elevator.

The unique and largely linear relationship between stick displacement or force and normal acceleration response Δn of the conventional aircraft is now replaced by a characteristic where pilot stick input is no longer uniquely related to a single aircraft response quantity. Current airworthiness requirements for stick force per g tolerance to nonlinearity etc are not framed with such a phenomenon in mind and cannot be applied unambiguously to this situation. It should be noted that a similar but less extreme characteristic is obtained with a pitch rate demand control even on a basically stable aircraft and during flight trials of the RAE fly-by-wire Hunter of such a system pilots have in fact commented on the odd control behaviour in aerobatic combat manoeuvres. All we can say here is that this phenomenon must be carefully assessed before a design is committed to this form of stability augmentation and that it may be necessary to introduce features into the feel system to restore more normal manoeuvre control.

4.2 Static margin and airspeed stability

The static margin, classically defined as

$$H_n = -\frac{\partial C_m}{\partial C_L} + \frac{m_u}{2C_{L_0}} \qquad (24)$$

is not affected by pitch damping. An aircraft with negative airframe stability ($\partial C_m/\partial\alpha > 0$) is therefore likely to suffer from a negative static margin and in consequence a divergent mode replacing the phugoid. Airworthiness requirements[3,4] are less demanding on static stability than for instance on manoeuvre stability, but present regulations do not permit negative stability. Recent work has furthermore demonstrated that especially at high speeds equation (24) does not satisfactorily define real flight stability. Since the subject is rather complex the interest reader is referred to Ref 2, where it has also been noted that many current military aircraft operate at supersonic speeds with negative static stability without apparent piloting difficulties.

Although this suggests that there are no undisputed criteria for this feature, major instability is clearly unacceptable. One would in fact expect that static stability, *ie* the stability of a mode involving primarily airspeed will be highly desirable in cruise flight conditions especially when operating close to a speed limit and that in such flight cases even neutral stability may be unacceptable.

We note from equation (24), that a contributor to static stability is the m_u derivative and it is therefore conceivable to counteract a deficiency in H_n by a feedback loop to the elevator generating positive m_u. This technique is of course commonly employed in the Mach trimmer. In Appendix C it is shown that a desired increment ΔH_n in the static margin is generated by an elevator control law of the form

$$\Delta\eta = \eta_0 - \frac{4}{3} \frac{\Delta H_n}{m_\eta \rho_0 V_E^2} \frac{W}{S} \tag{25}$$

where η_0 can be chosen arbitrarily for instance to suit trim requirements. The local sensitivity of the elevator to changes in airspeed is

$$\frac{d\eta}{dV_E} = 4 \frac{\Delta H_n}{m_\eta \rho_0 V_E^3} \frac{W}{S} . \tag{26}$$

This term represents a pitching moment sensitivity to fore and aft gusts

$$\frac{\partial C_m}{\partial V_E} = \frac{d\eta}{dV_E} m_\eta = 4 \frac{\Delta H_n}{\rho_0 V_E^3} \frac{W}{S} \tag{27}$$

which is seen to increase rapidly at low speeds. The effect has no equivalent in the naturally stable aircraft, since $\partial C_m/\partial C_L$ does not generate pitching moments in response to a change in airspeed at constant α . Since a positive speed increment produces a positive pitching reaction and hence an increase in incidence, the associated lift increase will add to that fundamentally generated by an increase in airspeed and so make the aircraft in every sense more sensitive to fore and aft turbulence especially at low flying speeds. To suppress this undesirable side effect one may have to consider filtering the speed signal. However, as shown in section 5.3, integral pitch rate feedback largely overcomes this problem.

4.3 Short period oscillation

Pitch damping affects the short period pitching oscillation in two ways.

It affects the 'stiffness', *ie* the undamped frequency of the mode through a term proportional to the manoeuvre margin in the form $\left(H_m(\partial C_L/\partial\alpha)\right)$ and it also is dominant damping term. This implies that if we choose to augment the manoeuvre margin of an aerodynamically unstable airframe by pitch rate feedback to the elevator we will inevitably also increase the damping of the mode. It is generally desirable to have this mode well damped, hence the pitch damping term in the conventional stability augmentation system. However, handling research has shown that there is an upper acceptable limit of damping ratio and we must ensure that this is not exceeded by a too powerful pitch rate feedback.

In the current British flying qualities requirements of AvP 970[4] the following values are specified for level 1 flying qualities of the damping ratio ζ

Flight phase category	Typical application	ζ_{min}	ζ_{max}
A	Combat	0.35	1.30
B	Cruise	0.35	2.00
C	Take off and landing	0.50	1.30

The objection to high damping ratios of course reflects the fact that it leads to sluggish response.

Details of the mathematical derivation are given in Appendix D with the result that if ΔH_{mq} is the increment in the manoeuvre margin generated by pitch rate feedback and H_m the total effective manoeuvre margin the total effective damping ratio is

$$\zeta = \zeta_a + \zeta_q = \zeta_a + \frac{\Delta H_{mq}}{2\left(\frac{i_B}{\mu}\frac{\partial C_L}{\partial \alpha} H_m\right)^{\frac{1}{2}}} \tag{28}$$

where ζ_a is the 'aerodynamic' damping of the basic unaugmented airframe. For a typical example with $H_m = +0.2$, $i_B = 1$, $\partial C_L/\partial\alpha = 5$ and $\mu = 25$ at sea level we get the result shown in Fig 7. Also shown is the total value of ζ, assuming that at sea level the airframe alone produces $\zeta_a = 0.5$. We note that the effect is independent of airspeed and that it increases with altitude. We also observe that for this example the permissible maximum value $\zeta = 1.3$ is quickly exceeded if q-stabilisation is applied as a means of restoring pitch stability to an aerodynamically unstable airframe.

We are therefore discussing a real problem that may limit the range of airframe instability for which pitch rate feedback is an acceptable form of augmentation. For less ambitious cases q-stabilisation is of course in this sense at least ideal since the feedback loop satisfies simultaneously two desirable objectives, enhancing the manoeuvre margin and short period damping.

4.4 Flight near the stall

The manoeuvre margin

$$H_m = -\frac{\partial C_m}{\partial C_L} - \frac{m_q}{\mu} = -\frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu}$$

is a sensible stability criterion only as long as the airframe operates in a region where the lift slope $\partial C_L/\partial\alpha$ has values one normally associates with orderly wing flow, and is sensibly invariant. At or near the stall, however, the lift slope may be seriously reduced, vanish or even reverse sign. The manoeuvre margin is of course essentially associated with manoeuvring forces and such things as stick force per g are not a major concern at the stall. We are, however, still and indeed primarily concerned with pitch stability and for this (see for instance Appendix D) we are interested in the pitch stiffness term which is proportional to $(H_m \times \partial C_L/\partial\alpha)$

$$H_m \frac{\partial C_L}{\partial\alpha} = -\frac{\partial C_m}{\partial\alpha} - \frac{m_q}{\mu}\frac{\partial C_L}{\partial\alpha}$$

or if the pitch damping contribution is generated by feedback control with (equation (13))

$$H_m \frac{\partial C_L}{\partial\alpha} = -\frac{\partial C_m}{\partial\alpha} - m_\eta K_q \frac{\frac{\rho}{2} gV}{W/S}\frac{\partial C_L}{\partial\alpha} \quad . \tag{29}$$

We note from this expression that the pitch rate feedback contribution is directly proportional to the lift slope and will therefore be seriously reduced in a flight regime where $\partial C_L/\partial\alpha$ drops substantially below its normal attached flow value. It is impossible to generalise as this phenomenon depends strongly on the characteristics of the particular airframe but it is quite obvious that q-feedback is likely to fail in its function as an augmentor of incidence stability in those regions of the flight envelope where the lift slope falls significantly below its normal value. This is even more important since frequently in the same regime the $(\partial C_m/\partial\alpha)$ of the airframe will also suffer so that demands on artificial stability augmentation becomes particularly pressing.

In the approach to the stall the aircraft may experience a rapid drag rise and a resultant loss of airspeed in spite of the development of a descending flight path. In this situation pitch attitude may stay constant whilst incidence increases. If an aerodynamically unstable aircraft relying on pitch rate feedback (or integral pitch rate) for stability augmentation is experiencing such a situation, evidently there is no pitch rate to actuate the elevator and the aircraft motion will be governed by its divergent airframe characteristics. This situation is the exact analogue of the superstall case. (See Fig 8.)

In our argument we had assumed that the only form of stability augmentation in operation is pitch rate feedback and as we have seen in section 4.2 this would leave the aircraft with a negative static margin which is the reason why the divergent flight state could develop in spite of positive short period stability. To prevent this problem, it is essential that the q-stabilised aircraft is in addition augmented by the type of airspeed feedback discussed in section 4.2 and what is more that this feedback loop operates down to the stalling speed. This 'speed trim' would operate at a very high gain (see equation (27)) at these low speeds and the reaction to horizontal gust may then become a problem.

4.5 Response to gusts

An aircraft having an aerodynamically unstable airframe and deriving stability from pitch rate feedback to the elevator must be expected to have some unorthodox gust response characteristics. In Appendix E the relevant frequency response in α and q to vertical gusts $\alpha_g = w_g/V_0$ is derived. For an aircraft with the characteristics defined in Table 1, the response in incidence and pitch rate has been evaluated with the results shown in Fig 9. Three cases are compared, all of which have the same effective manoeuvre margin H_m but in one case the stability is assumed to be produced aerodynamically whereas in the two others aerodynamic stability is relaxed and the difference made up by pitch rate feedback to the elevator. Pitch rate response shows the expected trend, being generally attenuated as pitch damping increases. More important, however, is the effect of pitch rate augmentation on the response in incidence which can be taken to be proportional to normal acceleration response, the quantity relevant to ride qualities and structural gust loading. Although at frequencies above that of the aircraft short period mode the response of the aircraft with relaxed stability is slightly attenuated, at lower frequencies the situation is dramatically reversed and the more aerodynamic stability is relaxed and hence pitch rate feedback increased the more vigorous will the aircraft respond. This effect is more vividly illustrated if we calculate the power spectrum ϕ_d of the incidence response to turbulence modelled by the von Karman spectrum

$$\phi_{w_g}(\Omega) = \phi_{w_g}^2 \frac{L}{\pi} \frac{1 + \frac{8}{3}(1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}}$$

where $\Omega = \omega V$ = spatial frequency

and $L = 750$ m .

The resulting response spectra are also shown in Fig 9. The comparative results are virtually insensitive to airspeed etc so that Fig 9 can be taken to give a universally representative picture. The principal effect of q-stabilisation is therefore seen to be the amplification of the response at frequencies below the natural frequency of the aircraft short period mode and a substantial shift of the peak response towards lower frequencies where the naturally stable aircraft shows very little response. The physical reason for this phenomenon is that the strong pitch damping required to achieve the desired manoeuvre stability resists the normal tendency of the aircraft to weathercock into gusts of longer duration and therefore inhibits the consequent alleviation of the normal acceleration response of the aircraft.

It should be noted that the results shown in Figs 9 and 10, which only represent the response in the short period mode, are invalid at very low frequencies where the response would be dominated by the phugoid mode, ignored here.

To complete this study of the gust response of the q-stabilised aircraft, corresponding calculations were also carried out for the short period mode response to fore and aft turbulence, although intuitively one would not expect here this form of stability augmentation to have any adverse effects. This is indeed confirmed by the results shown in Fig 10. There is again some amplification, or more accurately lack of alleviation, of the normal acceleration response at frequencies near the natural frequency of the short period mode, but this effect is much less pronounced than in the response to vertical gusts and need not cause concern in a response that is generally of little practical significance other than at very low speeds. As expected, the enhanced pitch damping constrains pitch response throughout and in this respect the pitch rate stabilised aircraft is superior to its naturally stable counterpart.

4.6 Inertia-coupled rapid rolling

In previous generalised studies of inertia-coupled rapid roll manoeuvres (Refs 5 and 6) the aircraft aerodynamics have been simplified by assuming the frequency of the uncoupled longitudinal motion to be fully defined by the incidence stability derivative M_α, ignoring L_α, which through M_q makes a contribution to the manoeuvre stability of the uncoupled motion. This contribution is relatively small in the conventional aircraft, but when pitch rate feedback is used as the major stabilising effect in an actively controlled design, it will of course become dominant. It was felt prudent therefore to re-analyse stability in rapid rolling with a proper representation of this now important term.

Details of this work are given in Appendix G. The results show that fortunately in roll-coupled states the M_q-contribution to the manoeuvre margin has the same stabilising effect as it has in uncoupled longitudinal motion. The stability of the aircraft in coupled rolling manoeuvres is shown to be fully determined, as far as the longitudinal terms are concerned, by the effective pitch stability expressed in the nomenclature of that discipline as $\omega_\theta \equiv \omega_n$ of normal longitudinal short period stability analysis, and by the overall damping ratio ζ_θ. This therefore permits the results of earlier work such as Ref 5 to be used directly if appropriate values of ω_θ and ζ_θ are taken. Since pitch damping as such has a favourable effect on both the stability of the rolling aircraft according to Fig 16 and on the autorotational tendencies as shown in Fig 17, pitch rate stabilisation is therefore beneficial in both these areas.

Although not formally studied in Appendix G it is self-evident that both q-feedback and normal acceleration feedback do not introduce any new effects and that therefore the aircraft stabilised by either of these two methods will have rapid roll characteristics virtually indistinguishable from those of an aircraft achieving the same level of manoeuvre stability by natural means.

There is, however, one aspect that may cause difficulties when full analysis is made with realistic representation of the transfer function of the stabilising feedback loop. When the two oscillatory aircraft modes become coupled under the influence of rapid rolling, the frequency of the faster of the two modes, normally the longitudinal short period oscillation, is substantially increased and this may cause closed loop stability problems, whatever form of feedback is used, if the servo loop has inadequate bandwidth.

5 INTEGRAL PITCH RATE FEEDBACK

It is widely held that perhaps the most efficient augmentation is obtained from a system based on rate sensors if integral pitch rate is used as the principal feedback to give a form of attitude stabilisation. Such a term would of course inhibit general manoeuvring and for this reason the basic control system must be organised as a pitch rate demand control control as indicated in Fig 11, and only the pitch rate error term $(q - q_0)$ integrated to form the stabilisation signal. As a convenient shorthand we denote the integral $\int(q - q_0)$ as θ when defining equivalent force and moment derivatives such as m_θ and L_θ etc. For small perturbation analysis about an essentially level flight equilibrium condition the implied identity is of course correct and most of the stability analysis to follow is restricted to this situation.

We are concerned with a control law of the form

$$\eta = K_q(q - q_D) + K_\theta \int (q - q_D)dt \qquad (30)$$

where q_D is demanded pitch rate. In addition to integral pitch rate with gain K_θ we also employ direct pitch rate feedback with gain K_q as a means of controlling the damping of the resulting aircraft modes.

Detailed analysis of the stability and gust response characteristics of the aircraft augmented by a control law according to equation (30), is given in Appendix F and discussed in the appropriate sections below. As with the other schemes discussed in this Report the omission of control system dynamics allows us to represent the effects of the feedback loops on the aircraft as equivalent increments to aircraft stability derivatives

$$\Delta M_q = M_\eta K_q \;, \qquad \Delta L_q = L_\eta K_q \;,$$

$$M_\theta = M_\eta K_\theta \;, \qquad L_\theta = L_\eta K_\theta \;.$$

One of the more important conclusions from Appendix F is that the intervention of integral pitch rate feedback fundamentally changes the character of the aircraft response modes. The phugoid of the classical aircraft degenerates into a first order mode, which we define as the airspeed-stability mode. This mode defines the long term 'static stability' of the aircraft. In addition to the pitching moment contributions $\partial C_m/\partial_\alpha$ and M_u usually associated with static stability it now contains powerful drag contributions. The short period motion, on the other hand, is raised to third order. In addition to the familiar short period oscillation we now obtain a first order mode in the form of a strongly damped convergence. Physically this resembles a heave mode. Although stability analysis shows a term strongly resembling a manoeuvre margin

$$H_m^* = - \frac{\partial C_m/\partial\alpha + \partial C_m/\partial\int q}{\partial C_L/\partial\alpha} - \frac{M_q}{\mu} \qquad (31)$$

closer analysis reveals that this term no longer has the unique significance of the classical manoeuvre margin. Steady control characteristics are insensitive to this factor and although it still appears as a necessary condition for stability, it can no longer be treated as a unique stability criterion.

5.1 Manoeuvre control

Steady manoeuvring is fundamentally controlled by the manoeuvre demand (rate demand) feature of the system layout, which is of course designed with this end in mind. As far as steady manoeuvring is concerned the addition of an integral error term tightens control over the steady state performance so that we can assume strict correspondence between stick demand (as scaled by the command path gain K_D) and pitch rate. From the discussion in section 4.2 it will be obvious that this will result in the manoeuvring control characteristics there associated with the aerodynamically neutral configuration with $M_\alpha = 0$. The appropriate relationships from Figs 5 and 6 are repeated in Fig 12. Although not displaying the more extreme trends shown in Figs 5 and 6 for the aerodynamically unstable case, we still note substantial departures from the customary tight association of stick input to normal acceleration response. In a pull up, for instance, the pilot must progressively pull the stick back to maintain a given load factor n and the opposite applies in a pull up from a steep dive or loop. Such behaviour has no precedence in the aircraft as we know it today and its acceptability must be tested in flight.

Also, the manoeuvre margin as defined in equation (31) does not affect this relationship, which in a manoeuvre demand system is virtually divorced from design features influencing stability. Of course, short period stability must exist for a discussion of steady state manoeuvres to have practical significance.

Integral pitch rate stabilisation also affects the long term aircraft response to throttle and elevator control. Conventionally we expect the throttle to control in the long term only aircraft vertical velocity and the elevator airspeed. Since $\int q$-feedback imposes a constraint on pitch attitude changes, the simple relationships of classical control theory leading to the above mentioned results are being interfered with and as a consequence each of the two principal longitudinal controls now results in changes in both these flight state variables. For instance it can be readily shown that the steady state response to an increment in thrust produces a change in flight path angle

$$\Delta\gamma = \frac{\Delta T}{W} \frac{1}{1 + \frac{\partial C_L/\partial\alpha}{(L/D)_0} \frac{1}{C_{L0}} \frac{1}{1 + \frac{m_\alpha}{m_\theta}}} \tag{32}$$

and in airspeed

$$\frac{\Delta V_i}{V_{i0}} = \frac{1}{2} \frac{\Delta T}{W} \frac{1}{\frac{C_{L0}}{\partial C_L/\partial\alpha}\left(1 + \frac{m_w}{m_\theta}\right) + \frac{1}{(L/D)_0}} \tag{33}$$

Classically we get of course $\Delta\gamma = \Delta T/W$ and $\Delta V_i = 0$.

The elevator response on the other hand is dictated by the manoeuvre demand feature of the control layout and no unique steady state response in either airspeed or vertical velocity can be assigned to it.

5.2 Short period stability and manoeuvre margin

It is shown in Appendix F that with aircraft augmented by integral pitch rate feedback the short period behaviour is defined by a cubic characteristic equation

$$s^3 + B_2 s^2 + B_1 s + B_0 = 0 \tag{34}$$

with $B_2 = -\mathscr{L}_\alpha \mathscr{M}_q - \mathscr{M}_{\dot\alpha}(1 - \mathscr{L}_q)$

$B_1 = -\mathscr{M}_q \mathscr{L}_\alpha - \mathscr{M}_\theta - \mathscr{M}_\alpha (1 - \mathscr{L}_q) - \mathscr{M}_{\dot\alpha}\mathscr{L}_\theta$

$B_0 = -\mathscr{L}_\alpha \mathscr{M}_\theta + \mathscr{L}_\theta \mathscr{M}_\alpha$

Where $\mathscr{L}_q$ and $\mathscr{L}_\theta$ are the elevator lift terms associated with the active pitching moment feedbacks $\Delta\mathscr{M}_q$ and $\mathscr{M}_\theta$.

The first two terms B_2 and B_1 , are formally identical to the corresponding terms of the classical short period approximation, with the stiffness term B_1 being augmented as expected by $\mathscr{M}_\theta$. This term appears in a form strongly suggesting equivalence with the manoeuvre margin of the classical aircraft and one would expect it to play a similar role. We have already dealt with steady manoeuvre control in section 5.1 and shown there that such features as stick force per g are no longer controlled by the manoeuvre margin which therefore loses in this control situation one of its traditional functions. This leaves its role in stability. Clearly stability demands that $B_1 > 0$ and this implies that a positive manoeuvre margin is a necessary condition for short period stability. In the classical situation a positive value of the pitch stiffness term or manoeuvre margin is for all practical purposes also a sufficient condition, since the only other factor capable of causing instability, namely negative pitch damping ($m_q > 0$), is physically improbable.

However, with integral pitch rate feedback leading to a significant finite value of the last coefficient of the cubic characteristic equation B_0 we have to consider two strongly interacting modes and a more obscure stability situation.

In fact all the numerical work on the examples considered below has shown that the additional real root tends to absorb a substantial portion of the total damping (B_2) in the system at the expense of the short period oscillation, so that this mode may be rendered unstable for even substantially positive values of the stiffness term B_1. This effect can be corrected by an increase in pitch damping, *ie* by increasing K_q. This can be seen from Table 2. Using again the aircraft used previously as a numerical example with the characteristics defined in Table 1, aerodynamic incidence stability $\mathcal{M}_\alpha$ was relaxed and the feedback terms $\mathcal{M}_\theta$ and $\mathcal{M}_q$ adjusted so that the resulting short period oscillation maintained the values (ω_n = 2.5 rad/s and ζ = 0.6) of the datum aircraft. The value of the real root λ_1 resulting from this configuration is also listed and we note how strongly damped this convergence mode is. We also observe that $\mathcal{M}_\theta$ had to be increased much more than would seem necessary to just compensate for the shortfall in $\mathcal{M}_\alpha$ against the datum value -4.33 of the datum aircraft. For instance in case 4 with neutral aerodynamic stability ($\mathcal{M}_\alpha = 0$) the required value of $\mathcal{M}_\theta$ is (-7.0) instead of the expected (-4.33). In the unstable case 5 corresponding values are (-13.5) instead of (-8.66), etc. Also at the same time pitch damping $\mathcal{M}_q$ had to be substantially augmented to maintain ζ at the desired level of 0.6.

We conclude therefore that with $\int q$ feedback there is no longer a term equivalent to the manoeuvre margin of the classical aircraft which expresses readily the fundamental functions normally associated with this parameter. Meaningful studies of the sensitivity of an aircraft so augmented to variations in the CG and the aerodynamic centre must therefore always be conducted by full analysis of the complete systems.

5.3 Static stability

According to Appendix F, when more than a trivial amount of integral pitch rate feedback is introduced, the phugoid oscillation of the classical aircraft degenerates into a first order mode with time constant

$$\lambda_2 = -\mathcal{D}_u + \left(\mathcal{D}_\alpha - \frac{g}{V}\right)\frac{\mathcal{L}_u}{\mathcal{L}_\alpha} + \frac{\mathcal{M}_\alpha}{\mathcal{M}_\theta}\frac{\mathcal{L}_u}{\mathcal{L}_\alpha}\frac{g}{V} - \frac{\mathcal{M}_u}{\mathcal{M}_\theta}\frac{g}{V} \qquad (35)$$

We define this mode as the 'airspeed stability' mode, since it principally controls the ability of the aircraft to maintain a given trimmed speed. We use the term 'airspeed' to distinguish it from the familiar speed-stability mode of the aircraft constrained to a rectilinear flight path.

Using the approximations for the drag and lift derivatives given in Appendix A, equation (35) can be written with more familiar non-dimensional terms as

$$\lambda_2 = -\frac{2}{\hat{t}}\left[C_{D_0} - \frac{C_{L0}}{\partial C_L/\partial\alpha}\left(\frac{\partial C_D}{\partial\alpha} - C_{L_0}\right) + \frac{C_{L0}^2}{m_\theta}\left(-\frac{\partial C_m/\partial\lambda}{\partial C_L/\partial\alpha} + \frac{m_u}{2C_{L0}}\right)\right] . \qquad (36)$$

The static margin of classical analysis appears in the bracket of the last term of this expression, and we note it to be now attenuated by m_θ the integral pitch rate effect. In addition we observe a group of terms mainly reflecting the drag characteristics of the aircraft which numerical examples will show to become dominant and destabilising at low speeds. At high speeds, however, these terms are stabilising and together with the attenuating effect of m_θ on the $m_w = \partial C_m/\partial\alpha$ contribution ensure overall mode stability even when aerodynamically the aircraft is unstable. So we get the important result that at least over a significant part of its flight envelope integral pitch rate feedback confers to the aerodynamically unstable aircraft static stability, a benefit not provided by the other indirect augmentation schemes discussed in this Report.

The drag term $C_{D0} - C_{L0}\,\frac{(\partial C_D/\partial\alpha) - C_{L0}}{\partial C_L/\partial\alpha}$ has been evaluated from wind tunnel data for two typical modern combat aircraft with the result shown in Fig 13a. We note the general trend towards destabilisation as the induced drag factor $\partial C_L/\partial\alpha$ grows with increasing C_L .

Combining these characteristics with the pitching moment characteristic of the aircraft used as a numerical example throughout this Report and defined in Table 1 and as cases 4 and 5 in Table 2 the root λ_2 of the resulting airspeed mode has been calculated with the result shown in Fig 13b. We note that with the exception of the extreme low speed regime the value of this root is extremely small with time constants of the order of over 40 seconds, suggesting near neutral stability.

It may be of some interest to note that the drag term appearing in equation (36) bears a strong resemblance to the term defining the familiar speed stability solutions, which reads

$$\lambda_3 = -\frac{2}{\hat{t}}\left(C_{D0} - C_{L0}\,\frac{\partial C_L}{\partial\alpha}\right) .$$

5.4 Flight near the stall

We have shown in section 5.3 that near the stall the 'airspeed-mode' of the $\int q$-stabilised aircraft may become strongly unstable and this clearly is an undesirable feature. We are again faced potentially with the situation discussed under pitch rate feedback in section 4.4 and this needs careful evaluation in any design when it may indicate the need for some additional feedback loop, such as of incidence.

5.5 Response to gusts

We only consider vertical gusts as the principal disturbance in high speed flight. Results for the same configurations as considered with plain pitch rate feedback have again been calculated with the results shown in Fig 14. These calculations are based on the short period approximations developed in Appendix F and become invalid for very low frequencies where the airspeed mode would become significant. This region is typically $\frac{\omega}{\omega_n} \leqslant 0.1$ and therefore does not affect the practical conclusions.

The general effect of integral pitch rate feedback in the incidence response is very similar to that observed with pitch rate feedback alone. There is strong amplification of the aircraft response at frequencies below the natural frequency of the short period oscillation; this effect is most noticeable in the effect it has on the power spectrum of the incidence response to turbulence modelled on the von Karman spectrum.

It should be noted that with integral pitch rate feedback the process of controlling the resulting mode characteristics is less direct than with pure pitch rate feedback and was in fact obtained by a trial and error approach. The chosen objective was to achieve in all cases short period oscillation characteristics (ω_n = 2.5 rad/s and ζ = 0.6) close to those of the datum aircraft. The actually achieved mode characteristics and the values of the feedbacks expressed in terms of $\mathscr{M}_q$ and $\mathscr{M}_\theta$ are listed in Table 2, cases 4 to 6.

Since it was apparent that the lift slope has a strong influence especially on the heave mode damping λ_1 its effect was studied separately. The original configuration was assumed to have a relatively large lift slope $\partial C_L/\partial\alpha$ = 5.0. Reducing this value to 3.12 typifying a low aspect ratio wing the gust response of the new datum aircraft (case 7) and the unstable configuration with $\mathscr{M}_\alpha$ = +4.33 was evaluated and compared with the corresponding original cases in Fig 15. We note that reducing the lift slope increases somewhat the incidence-response of the naturally stable aircraft but that we now get even more powerful amplification when integral pitch rate is used to stabilise an unstable airframe. Integrating the power spectra from $0 < \omega < 6.0$ rad/s we get the following values of rms response.

Case	$\partial C_L/\partial \alpha$	$\mathcal{M}_\alpha$	$\frac{\sigma_\alpha}{\sigma_{ag}}$	$\frac{\sigma_n}{\sigma_{wg}}$	Increase over corresponding stable case
1	5.0	-4.33	0.384	0.0595	
2	5.0	+4.33	0.437	0.0677	1.13
3	3.12	-4.45	0.505	0.0490	
4	3.12	+4.33	0.652	0.0633	1.29

A column has been added to show the rms normal acceleration σ_n, a more physically meaningful gust response quantity. An interesting comparison is between lines 4 and 1, which shows that the $\int q$-stabilised aircraft with the low lift slope has worse gust response than the naturally stable aircraft with a lift slope 1.6 times higher.

It is emphasised that just looking at rms response may not tell the full story, since the most striking effect is clearly the extension towards lower frequencies of the regime where the aircraft significantly responds to turbulence.

5.6 Inertia coupled rapid rolling

The intervention of integral pitch rate must not cause difficulties in rapid rolling where inertial interactions become important. Unfortunately time did not permit an adequate study of this rather complex phenomenon, briefly considered in Appendix G.

It has been shown there that instead of a quartic as in the conventional case the characteristic equation defining pitch and yaw stability of the rolling aircraft now becomes quintic so that the solution will contain an additional root and no generalised observations on the result of this on overall stability can be offered.

However, an important observation can be made with respect to the autorotation problem. Since a pitching moment feedback of $\int q\,dt$ allows no steady state pitch rate to develop other than that demanded by the pilot, this quantity vital to cross-coupling is suppressed and in Appendix G it is shown that this removes the existence of steady autorotational state to a totally impractical regime at extremely large negative incidence. For all practical purposes therefore integral pitch rate feedback appears to remove this major problem of the modern combat aircraft.

6 NORMAL ACCELERATION FEEDBACK

Since at any given airspeed normal acceleration is to a first order approximation directly proportional to C_L and within the linear range of lift slope to α , normal acceleration feedback to the elevator suggests itself as a plausible substitute for incidence feedback. The normal acceleration required is that for the centre of gravity of the aircraft and if this location is not available for mounting an instrument, signals from two sensors must be combined to allow the necessary correction. A more serious problem is of course the sensitivity of accelerometers to structural mode responses and great attention will have to be paid in the development of such a system to avoid adverse coupling with these modes. This problem is generally appreciated and will not be further discussed here.

We proceed therefore from the assumption that a sufficiently accurate and pure rigid body value of normal acceleration is available for feedback to the elevator. On this basis we shall now cover the same areas considered earlier for the other augmentation concepts.

6.1 Manoeuvre margin and manoeuvre control

The short period mode characteristics of the aircraft stabilised by normal acceleration feedback are analysed in Appendix H. An approximate expression for the gain K_n required to achieve a given increment in the manoeuvre margin is

$$K_n = -\frac{\Delta H_m (W/S)}{m_\eta (\rho_0/2) V_E^2} \quad . \tag{37}$$

As expected, K_n must be scheduled in proportion to the inverse of dynamic pressure and with weight to maintain a given increment ΔH_m .

Another interesting observation is that exactly as with incidence feedback the elevator lift effect increases the effective lift slope, and therefore gust response.

The short period characteristics are largely identical to those of a naturally stable aircraft.

6.2 Static stability

Static stability implies a stable change in pitch trim (down elevator with increasing speed) with airspeed in rectilinear flight (1 g). In the naturally stable aircraft this characteristic is normally produced by the fact that increasing airspeed requires a reduction in C_L and therefore incidence. The pitching moment change associated with this reduction in incidence is in the desired sense if $\partial C_m/\partial\alpha < 0$, *ie* if the aircraft has aerodynamic incidence stability. If the aircraft is aerodynamically unstable, $\partial C_m/\partial\alpha > 0$ and this effect is reversed and therefore destabilising.

If we stabilise the aircraft by a normal acceleration feedback of the normally assumed form

$$\eta = K_n(n - 1)$$

a change in airspeed whilst maintaining $n = 1$ g does not generate any reaction from this stability augmentation loop which therefore makes no contribution to static stability. This is the physical basis of this generally observed fact. If the aerodynamic stability of an airframe is relaxed into the unstable regime and normal acceleration feedback used to restore manoeuvre stability according to equation (37) we are inevitably left with an unacceptable static instability.

This problem could be avoided if instead of basing the stabilising feedback on $\Delta n = n - 1$ we used absolute n instead and derive from it with simultaneous knowledge of aircraft weight and equivalent airspeed

$$C_L = n \frac{W/S}{(\rho/2)V^2} .$$

This allows the generation of a direct equivalent to aerodynamic stability in the form

$$\Delta \frac{\partial C_m}{\partial C_L} = \frac{\partial C_m}{\partial \eta} \frac{d\eta}{dC_L}$$

which is equivalent to a control law

$$\eta = K_n^* n$$

where K_n is scheduled exactly as equation (37). The difficulty with this proposition is in the precision required for scheduling, which must model with great accuracy not only changes in $(\rho/2)V^2$ but also aircraft weight and elevator effectiveness m_η . We have not pursued this idea any further here but this need not imply that it may not be realisable at least in some specific cases.

6.3 Flight near the stall

We note from Appendix H that in common with q-feedback the aircraft stabilised by normal acceleration feedback depends on a healthy positive lift slope for the effectiveness of the feedback and all that was said in section 4.4 again applies. It must be reiterated that consequent loss of effectiveness represents a potentially serious problem that in many cases would require the introduction of incidence sensors, an option which of course the search for alternative feedback strategies was designed to avoid.

If used on an airframe maintaining a healthy lift slope up to the highest incidence that we wish to be safely available to the pilot, then normal acceleration feedback will suffice to maintain manoeuvre stability but again fail to provide static stability, a situation which as discussed in section 4.4 may become unacceptable at very low speeds.

Fig 8b illustrates a divergent approach to the stall. Normal acceleration feedback is not only unable to resist such a divergence but may in fact accelerate it if the effect of reducing airspeed due to the rapidly growing drag leads to a progressive reduction in normal acceleration as the stall is approached. Feedback of airspeed to the elevator so as to restore static stability could be an answer and so would of course be incidence feedback.

6.4 Response to gusts

As long as lags in servo loop dynamics can be ignored the response to vertical gusts of the aircraft artificially stabilised by normal acceleration feedback is materially identical to that of the naturally stable aircraft.

There are, however, anomalies in the reaction to fore and aft gusts. The direct effect of a fore and aft gust is to change airspeed without affecting incidence. The immediate aircraft reaction is a change in lift and therefore normal acceleration but not in pitch. The change in normal acceleration on an n-stabilised aircraft, however, will cause an immediate elevator response and therefore pitching disturbance. The phenomenon is analysed in detail in Appendix J with some typical results shown in Fig 16. As throughout this Report we compare aircraft stabilised to the same effective level of manoeuvre stability, one possessing this stability naturally and two others using an increasing amount of normal acceleration feedback for the purpose. We notice the expected substantial increase in pitch response over practically the full frequency range for which this short period solution is applicable and in addition some increased normal acceleration response. One must deduce that this may lead to some noticeable deterioration of aircraft behaviour at low speed and especially near the ground when the gust sensitivity of an aircraft tends to become dominated by fore and aft turbulence.

7 GUST RESPONSE OF AIRCRAFT USING AIRSPEED-FEEDBACK FOR AUGMENTATING STATIC STABILITY

As discussed earlier stability augmentation by either pitch rate feedback or normal acceleration feedback is capable of restoring the aircraft's manoeuvre margin and hence its short period stability to the required standard but fails to restore the static margin. In section 4.2 an additional loop feeding airspeed to the elevator has been suggested as a possible answer. Again this is an indirect form of stability augmentation where the basic deficiency is in $\partial C_m/\partial\alpha$ and it is possible therefore that this technique results in unusual response properties and the feature likely to show anomalies is the response to fore and aft gusts.

The conventional aircraft which at low speeds has normally no significant m_u term senses a u-gust as a change in airspeed at constant incidence and reacts therefore initially only by an appropriate change in lift, *ie* normal acceleration. There is no change in the pitching moment equilibrium if the aircraft was initially in trim and hence no initial pitch disturbance. Airspeed feedback to the elevator on the other hand reacts to a change in airspeed by an elevator response and hence an immediate pitch disturbance. As we have shown in section 4.2 at high speeds the required gain is very low and is unlikely to cause a significant response, but at low speeds the gain required in this loop to give the same increment in the effective margin becomes large and we have to consider this as a critical case.

We choose for a numerical example an aircraft in the approach configuration at V_0 = 120 knots with a $C_L = 2.0$ and $\partial C_L/\partial\alpha = 5$. Again only considering the short period response the result is shown in Fig 17. As previously we compare a naturally stable aircraft with two relaxed stability configurations in which the manoeuvre margin is restored by q-feedback and the static margin by V-feedback to the values of the naturally stable aircraft. We note the expected increase by a large factor in pitch response by comparison with the conventional aircraft but no substantial increase in normal acceleration response. To put this into perspective we are comparing in Fig 18 the result for the worst example of

Fig 17 with the familiar response of the conventional aircraft to vertical gusts, using the result of Fig 9, converting the α-response to normal acceleration response by the relationship

$$\Delta n = \Delta\alpha \frac{\partial C_L/\partial_\alpha}{C_{L_0}} .$$

This comparison indicates that the strongly amplified pitch rate response to u-gusts of the V-stabilized aircraft is still well below that of the normal aircraft to w-gusts and it would appear therefore that the phenomenon we are here investigating is unlikely to be of primary significance. Also the q-stabilised aircraft, as shown in Fig 9 has of course a strongly attenuated pitch response, so that the total pitch activity of such an aircraft in an atmosphere of simultaneous vertical and horizontal turbulence is bound to be better in the q- and V-stabilised unstable aircraft than in the naturally stable conventional aircraft.

However, in the frequency range below the natural frequency the normal acceleration response to both components of turbulence is increased by the combination of feedback loops under discussion and this aspect could have more serious handling implications.

Airspeed feedback also is a plausible additional feature in the aircraft stabilised by normal acceleration feedback. We have shown earlier, see Fig 16, that normal acceleration feedback itself causes undesirable response amplification to fore and aft gusts.

It is readily apparent that airspeed feedback geared to provide static stability will be in the opposite sense to the elevator reaction to an increase in normal acceleration with increasing airspeed and this should therefore counteract the effects illustrated in Fig 16 and no formal response calculations were therefore made for this case.

8 DERIVED INCIDENCE FEEDBACK

From the kinematic relationship

$$\alpha' = \theta - \gamma \qquad (38)$$

where θ is pitch attitude and γ the flight path angle it would appear that α is a redundant quantity which can be derived from inertial measurements, *ie* from measurements of pitch attitude θ and integrated normal acceleration since

$$\dot{\gamma} = \frac{g}{V}(n - \cos\gamma) \quad .$$

The equations apply of course only for motion purely in the vertical plane and become substantially more complex in general flight where from Ref 7

$$\sin\alpha' = \cos\phi\cos(\chi - \psi)\cos\gamma - \sin\phi\sin(\chi - \psi)\cos\gamma - \cos\phi\cos\theta\sin\gamma \qquad \ldots\ldots (39)$$

where χ and ψ are flight path and aircraft azimuth angle respectively, both measured with respect to a common datum heading. Clearly the quantities on the right hand side of equation (39) can only be obtained with meaningful accuracy from measurements performed to IN standards and this is not an attractive proposition for a basic flight control system demanding multiplexed sensor information for integrity.

Another difficulty is that the angle of attack α' so derived (similar arguments apply to sideslip which could be similarly computed) is true incidence only in the absence of wind and fails to register any contributions from changes in windspeed or turbulence. This causes two problems. First the effects of windspeed and of any longer term changes in that quantity would have to be compensated for by some updating routine, which would be difficult to implement. There is no way one could even conceive of such a correction technique operating instantaneously other than with the aid of airflow sensors and therefore turbulence will simply not be seen by a derived incidence sensor. This must cause some strange gust response characteristics. The frequency response to vertical turbulence has been calculated for the three aircraft configurations used throughout this Report with the results shown in Fig 19. The most bizarre behaviour is displayed at the lower frequencies where the aircraft stabilized by this so called derived incidence feedback is seeking an anomalous trim state.

There is no real incentive to the exploitation of this scheme, presenting clearly immense difficulties first in the generation of the basic feedback quantity and then in the provision of additional terms to constrain its gust response behaviour. We shall therefore not pursue this option any further.

9 THE AIRCRAFT WITH RELAXED DIRECTIONAL STABILITY

Although most of the literature concerned with active control has concentrated on relaxed longitudinal stability as the primary objective, perhaps equally impressive performance gain can be had from reduction in fin size.

However, this is not the place for making the case for this area of active control application. As in the longitudinal area our aim is to consider what alternatives there are for effective loop feedback strategies as indicated in Fig 23.

The lateral stability problem is intrinsically more complex than the longitudinal one; there are three aircraft modes of almost equal significance, each being powerfully affected by n_v , the directional stability derivative. Full analysis of this complex problem must be left for another paper and we shall here only consider some elementary issues.

The most straightforward solution is again direct augmentation of the relaxed directional stability derivative $n_v \simeq (\partial C_n/\partial\beta)$ *ie* feedback of sideslip to the rudder. This would essentially, although not precisely restore to the aircraft normal behaviour. If the sensor integrity problem can be solved satisfactorily, this would again be the option one would choose. If an acceptable solution cannot be found in this direction then we have again to search for alternatives.

The equivalent to pitch rate in the longitudinal case would be yaw rate. However, the most superficial analysis shows this to be no real alternative. In the Dutch roll mode n_r makes a stabilising contribution exactly in the same way that m_q enters into longitudinal manoeuvre stability. In practice, however, this effect is far less powerful than the equivalent pitch damping contribution to longitudinal stability, as it operates via the lateral lift slope $\partial C_y/\partial\beta$ which is substantially smaller than the vertical lift slope $\partial C_L/\partial\alpha$ applicable in the pitch case. Consequently the gains required for synthetic yaw damping to procure a significant increment in effective directional stability would be excessive. They would inevitably constrain manoeuvres in the yaw plane in a totally unacceptable manner. Even with conventional yaw dampers this effect has to be encountered by subjecting this feedback loop to washout, an option not permissible if directional stability is to be augmented. In the longitudinal case the corresponding problem is overcome by operating the pitch control loop as pitch rate demand system. This solution too is not available in the directional case since it would convert the rudder into a yaw rate demand controller, a function which is clearly incompatible with the proper role of this control for instance in a crosswind approach. Yaw rate is, of course, principally controlled by banking and this could not be reconciled with a separate and powerful separate yaw controller.

Having to dismiss rate feedback as quite impracticable the only serious alternative to sideslip feedback is therefore lateral acceleration feedback to the rudder.

Theoretically there is as a third option a scheme analogous to that discussed in section 8 for the longitudinal case, using inertially derived sideslip.

$$\sin\beta = \sin\phi \sin\theta \cos(\chi - \psi) \cos\gamma + \cos\phi \sin(\chi - \psi) \cos\gamma - \sin\phi \cos\theta \sin\gamma \quad \ldots\ldots (40)$$

However, for the same reasons as with derived incidence this alternative must be dismissed as impractical.

This leaves us with sideslip feedback and lateral acceleration feedback as the only real options.

9.1 Sideslip feedback to the rudder

Directional stability is augmented by sideslip feedback with gain

$$K_\beta = \frac{d\zeta}{d\beta}$$

as

$$\Delta n_v = K_\beta n_\zeta \quad . \qquad (41)$$

As rudder power n_ζ is likely to drop in supersonic flight and as also n_v is normally reducing in that same regime there will be a need to schedule K_β as a function of Mach number, but there is no fundamental reason to expect any additional scheduling requirements in the normal flight regime except perhaps to compensate for major stores effects.

Rudder power also tends to reduce dramatically in the high incidence regime, where a healthy value of n_v is essential for the maintenance of spin resistance and this is likely to require an α-schedule.

Apart from a yawing moment n_ζ, rudder application also generates a rolling moment given an increment to ℓ_v

$$\Delta\ell_v = K_\beta \ell_\zeta \qquad (42)$$

and a sideforce

$$\Delta y_v = K_\beta y_\zeta \quad . \qquad (43)$$

Since $K_\beta > 1$ and normally $\ell_\zeta > 0$, $y_\zeta > 0$, these secondary contributions will all be in the sense to increase $(-\ell_v)$ and $(-y_v)$ which are normally seen as desirable effects although an excess of $(-\ell_v)$ is increasing aircraft gust sensitivity, but this effect can be compensated by reducing the dihedral of the basic aircrame. There are therefore no obvious adverse side effects, in fact all those one finds appear to be favourable. If the problems of sensor performance and reliability can be solved, sideslip feedback would appear the ideal solution for the aircraft with relaxed directional stability.

9.2 Lateral acceleration feedback to the rudder

Sideslip generates a sideforce via the y_v derivative and this force can be sensed as a lateral acceleration n_y. This mechanism suggests lateral acceleration feedback to the rudder as an alternative to direct sideslip feedback. Hence we consider stabilisation of the aircraft with relaxed directional stability through a control loop operating on

$$\zeta = \frac{d\zeta}{dn_y} n_y = K_y n_y \quad . \tag{44}$$

However, in addition to sideslip, the rudder itself generates a relatively important sideforce and we have to take this into account, *ie*

$$n_y = \frac{(\rho/2)V^2}{W/S} (y_v \beta + y_\zeta \zeta) \quad . \tag{45}$$

Since

$$\Delta n_v = n_\zeta K_y \frac{dn_y}{d\beta}$$

we arrive after some algebraic manipulation at a relationship between the desired increment in directional stability Δn_v and the gain K_y as

$$K_y = \frac{\dfrac{1}{y_v} \dfrac{W/S}{(\rho/2)V^2}}{\dfrac{n_\zeta}{\Delta n_v} + \dfrac{y_\zeta}{y_v}} \quad . \tag{46}$$

As in the corresponding longitudinal scheme the acceleration feedback gain must be scheduled in inverse proportion to dynamic pressure and in proportion to weight if a fixed increment in n_v is to be achieved. The rudder sideforce term acts as a feedforward and increases the effective gain thereby reducing K_y for a given Δn_v by a quite significant amount. This term can cause loop stability problems at high frequencies in a real system and may need controlling by suitable filters. The relationship between n_ζ and y_ζ is

$$n_\zeta = - y_\zeta \frac{\ell_R}{\ell} \qquad (47)$$

where ℓ_R is the rudder sideforce moment arm and ℓ the reference length used in nondimensionalising the lateral derivatives. With this relationship we can write equation (46) as

$$K_y = \frac{1}{y_v} \frac{\Delta n_v}{n_\zeta} \frac{\frac{W/S}{(\rho/2)V^2}}{1 - \frac{\Delta n_v}{y_v} \frac{\ell}{\ell_R}} . \qquad (48)$$

There will be increments to ℓ_v and to y_v from this loop closure identical to those discussed with β-feedback and these are again generally beneficial. Equation (45) gives

$$\Delta n_v = \frac{K_y n_\zeta y_v}{\frac{W/S}{(\rho/2)V^2} - y_\zeta K_y} . \qquad (49)$$

Δn_v is obviously proportional to y_v and this means that the effectiveness of a lateral acceleration feedback loop depends on y_v maintaining a reasonably consistent value over the flight envelope. There are two areas where this might not be necessarily so. The first is the effect of stores on combat aircraft which can make a very substantial contribution to the overall sideforce derivative and this effect may have to be compensated for by an appropriate schedule.

More important and more potentially problematic, however, will be the behaviour of the sideforce derivative at high angles of attack. This aerodynamic feature has received a good deal of attention recently, see for instance Refs 8 and 9, with the conclusion that sideforce characteristics become erratic at high incidence and may in fact lead to sign reversal in y_v, which would of course turn a stabilising lateral acceleration feedback into a destabilising agent.

In this context one must also consider that the aircraft exploiting relaxed directional stability for performance enhancement is clearly featuring a substantially smaller fin than todays conventional designs. This means that a much larger portion of the proportionally smaller sideforce derivative y_v will be provided by the fuselage and other 'non-lifting' aircraft components, such as external stores, and will be more at the mercy therefore of sometimes capricious body-aerodynamics.

We may conclude that lateral acceleration feedback is likely to prove effective over major parts of the flight envelope, depending on powerful and well matched scheduling, but that it may become of dubious value in the high incidence regime. Combat aircraft expected to operate in this area are likely to require alternative (*ie* sideslip) feedback to restore acceptable spin resistance characteristics.

Considerable effort will also be required in the development of an acceleration feedback system to avoid coupling with structural modes, an area outside the limited scope of the present enquiry. This problem is likely to be more serious in the lateral case by comparison with the equivalent longitudinal normal acceleration feedback scheme, since the rigid body sideforce derivative providing the desired equal signal content is relatively smaller.

10 FLIGHT AT VANISHING DYNAMIC PRESSURE

Certain types of military aircraft may require to be zoomed into regions of vanishing dynamic pressure and one must be assured, before such manoeuvres are undertaken, that a relaxed stability design is as capable of recovering from such extreme flight conditions as its naturally stable counterpart. It is generally feared that the aircraft stabilised by automatic control is likely to be in serious difficulties in this condition, although no proper studies have been published on this subject.

It is difficult to visualise any meaningful analysis of this situation without a detailed mathematical model of the aircraft's aerodynamics covering almost the full global range of incidence and sideslip, since the possibility must exist of the aircraft falling back into the normal dynamic pressure region in almost any attitude.

The problem then resolves into one of spinning and spin recovery, a subject far outside the scope of these present studies. It is certain that in this regime there will be no significant positive and indeed very likely a negative lift slope and it is obvious therefore that normal acceleration feedback and pitch rate feedback will be of no avail or even detrimental. Aircraft with relaxed directional stability are likely to suffer similarly if relying on lateral acceleration feedback but more seriously, the reduced fin size, which is of course the *raison d'être* of this scheme is by itself a severe handicap in spin recovery.

Limitations of control power and authority will also come into play in flight at large values of α and β and so that one must conclude that once departed by whatever mechanism the aircraft with relaxed airframe stability is bound to be in a much more perilous state than a conventional design.

However, there are less extreme zoom manoeuvres in which dynamic pressure drops below the value allowing 1 g flight but not sufficiently for aerodynamics to be suspended altogether. If there is enough control to contain the aircraft within a healthy range of α and β then the artificially stabilised aircraft need not necessarily be worse off. The critical points are then

(a) whether the gain schedules required for q and Δn stabilisation are carried far enough to cover this extreme range and

(b) whether the aircraft has enough control power to respond without saturation to these extreme gain demands.

At first sight it would seem that neither of these requirements are likely to be satisfied in any realistic design, but closer analysis shows these fears to be not necessarily justified.

For this we look first at the aircraft stabilised (say for pitch stability) by α-feedback to the elevator. We know that this form of feedback operates essentially with constant gain, *ie* the elevator angle demanded is the same for a given aircraft incidence, whatever the airspeed. Therefore if the system is designed to cover a given incidence range in the normal regime of airspeed it will do so also at much lower speeds.

The gain for a normal acceleration feedback loop has to vary as the inverse of dynamic pressure and would therefore assume very large values of very low airspeeds. When expressed, however, in relation to increments in incidence corresponding to a given increment in Δn we find that the gain now is exactly the same as that required for incidence feedback, *ie* it is independent of airspeed. Clearly the amount of elevator to be applied by the automatic control system to provide a given increment in aircraft stability does not substantially depend on the choice of sensors from which the information driving the control loop is derived.

We find a similar result with respect to pitch rate feedback. So we need only make sure that point (a) above is satisfied, (b) is then automatically also satisfied for a system designed to cope with the normal flight regime down to the stall.

So we can conclude that excursion into low dynamic pressure regimes are as safe for the automatically stabilised aircraft as for the naturally stable configuration *if* the pilot concentrates on maintaining incidence and sideslip within the normal range and does not permit dynamic pressure to become so low that aerodynamic control is no longer effective.

If, however, the aircraft departs into the spin region, the relaxed stability aircraft may be in serious trouble, especially if stabilised by 'indirect' feedback. There may well be a firm requirement here for a reliable spin and departure prevention system.

11 TAKE OFF AND LANDING

The constraint imposed by undercarriage contact with the ground fundamentally changes the kinematics and dynamics of the aircraft during the ground roll and during touchdown and take-off phase of flight. It is therefore necessary to ensure that augmentor control laws developed for free flight are compatible with ground-borne operations.

When all the wheels are firmly on the ground and lift is substantially smaller than aircraft weight longitudinal aerodynamics are of little significance but there are occasions when the pilot wishes to hold the nosewheel off the ground, for instance in the latter stages of take off or during the early landing roll over rough surfaces and of course during a controlled rotation to lift off. In these situations pitch stability becomes relevant to allow the pilot positive control of attitude.

The relative disposition of the main wheels in relation to the centre of gravity is governed by factors largely unaffected by aerodynamic stability and should therefore be the same for the naturally stable and the actively stabilised aircraft. The aerodynamic centre of the latter configuration will, however, be further forward as indicated in Fig 24. The relaxed stability design is therefore inherently less stable than its conventional counterpart. The relevant reference point for stability analysis is of course the main undercarriage as the pitch pivot, which makes it very likely that we will finish up with an unstable situation with the aerodynamic centre being forward of the main wheels. This can give rise to piloting difficulties unless the aircraft's augmentation system makes an appropriate stabilising contribution. We shall consider again the three principal feedback concepts under discussion.

(a) Incidence feedback

When the main wheels are on the ground and oleo response is ignored, incidence equals pitch attitude and incidence feedback to the pitch control therefore provides a stabilising increment to pitch attitude stiffness. This should restore the characteristics to those of the naturally stable aircraft, but one has to take account of the fact that the position error on the incidence sensor may be significantly affected by ground effect and the large sideslips possible in this situation so that the admissibility of this form of stabilisation on the ground is dubious.

(b) Pitch rate and integral pitch rate feedback

Since main wheel contact ($L < W$) essentially suppresses the heave response of the aircraft, augmented pitch damping will not be able to make a contribution to pitch attitude stability and be restricted to a damping contribution which will only constrain the divergence to be expected if the configuration is aerodynamically unstable. If the resulting instability should prove unacceptable, additional feedbacks must be considered necessary for a mode which will only be engaged during periods of ground contact. Pitch attitude or integral pitch rate would seem promising candidates. Clearly with integral pitch rate forming the principal augmentation loop the problem vanishes.

(c) Normal acceleration feedback

A normal accelerometer located at the centre of gravity and hence close to the main wheels of the aircraft will measure during the ground run, apart from high frequency reactions to uneven ground, principally a component of earth gravity

$$n = \cos\theta - \frac{\dot{V}}{g}\sin\theta$$

A stability augmentation system operating on such a sensor therefore makes a negative contribution to pitch stability during the ground roll, when the aircraft is accelerating. Again the need for some additional stabilisation term must be considered for aircraft aerodynamically unstable in this situation.

The aircraft with relaxed weathercock stability presents a potential problem for directional control. Directional controllability and therefore stability is a more fundamental requirement[10] than pitch stability as it is needed during all ground-borne operations. The ground contact of the undercarriage presents no kinematic constraint, merely a resistance to tyre slip and we must use the centre of gravity as our reference point. With a tricycle design the nosewheel, unless freely castoring, makes a destabilising contribution whereas the main wheels, being aft of the centre of gravity, are stabilising. The more adverse situation therefore arises with the nose wheels down and locked and in particular during braking when weight is 'transferred' forward on to the nosewheels. A full assessment of this problem must take account of tyre and undercarriage characteristics and is outside the scope of the present paper.

With all wheels on the ground we can approximately expect the undercarriage contributions to cancel, *ie* to produce neutral directional stability so that the issue at least at the high speeds may well be decided by the aerodynamic term n_v and again the contribution from an augmentation system may become significant. Feedback of sideslip is clearly as beneficial as it is in free flight but it will be difficult to provide adequate aerodynamic control power at the lower airspeeds. The contribution of a lateral acceleration feedback is less obvious and has been analysed in Appendix K with the conclusion that it does in fact both stabilise and damp the yawing motion on the ground and that this contribution is larger than the equivalent effect in free flight. Again, however, any form of active augmentation makes increasing demands on aerodynamic control power as airspeed reduces and these may become difficult to satisfy.

12 CONCLUSIONS AND DISCUSSIONS

This study has considered the repercussions on a wide range of flying qualities of aircraft in which deficient longitudinal and directional aerodynamic stability is augmented by alternative control feedbacks. In the longitudinal case, feasible techniques were found to be feed back of incidence, pitch rate, integral pitch rate or normal acceleration to the elevator. In the directional case there are only two viable alternatives, feedback of sideslip or lateral acceleration to the rudder.

In each case one can distinguish between methods which *directly* augment the deficient aerodynamic aircraft derivatives m_w or n_v and *indirect* techniques which provide the desired stabilising effect by some alternative route.

Direct feedbacks demand sensing of flow direction, α or β, and operating the elevator or rudder respectively in response to these sensor signals. They depend on sensors mounted external to the airframe and thus are vulnerable to damage. In addition they are subject to substantial position errors for which it might be difficult to compensate. On the other hand, aircraft stabilised by such feedbacks are restored in every important respect to normal flying qualities and moreover the feedback loop gain needs only a minimum of scheduling.

The principal alternative indirect feedbacks for longitudinal stability augmentation are pitch rate, integral pitch rate or normal acceleration. Pitch rate and normal acceleration feedback both fail to restore static stability which therefore demands an additional feedback, preferably of airspeed to the elevator. Integral pitch rate feedback, on the other hand, provides static stability except at very low airspeeds. Pitch-rate based feedback introduces abnormal steady control characteristics, especially in aerobatic manoeuvres, and an exaggerated gust response. The real significance of these phenomena to the pilot cannot be fully assessed by theroetical analysis alone, to which the present study is restricted, but must be investigated in flight or at least in a simulator. Many of these problems appear to be amenable to study on ground-based simulators, but there are others in which the restricted motion cues of simulators may present a barrier to faithful reproduction. This applies for instance to the study of abnormal stick control characteristics in aerobatic manoeuvres as well as to the effect on ride quality of the shift to lower frequencies and increased magnitude of the response to vertical gusts.

Normal acceleration feedback appears to offer more acceptable flying characteristics in these areas but it shares with pitch rate feedback the possibility of serious loss in efficiency in the stall region, because the effective feedback gain with both techniques depends on lift slope maintaining a healthy value. Lift slope is almost certain to decrease after the onset of flow separation and in configurations where this effect becomes substantial the stabilisation loop will become proportionally less effective and the aircraft tend to revert to the stability characteristics of the unaugmented airframe.

In the lateral plane, lateral acceleration feedback would appear a generally satisfactory alternative to sideslip feedback, but again the scheme could fail at high incidence if in this regime the aircraft's sideforce characteristics as expressed in the derivative y_v change significantly from normal values. In fact, there are indications that this appears to happen with modern combat aircraft shapes. There is further a danger that rudder power itself may vanish in this area and this would of course prevent directional stability augmentation from functioning, irrespective of the feedback policy adopted.

All the phenomena considered will increase in severity in proportion to the degree by which aerodynamic stability is relaxed on the basic airframe, *ie* on the amount of incremental stability the active control system has to supply.

Taking this observation into consideration one arrives at the conclusion that, for longitudinal stability augmentaion, pure pitch rate feedback might be acceptable even without further auxiliary feedbacks (of airspeed for instance) for airframes configured to no worse than neutral pitch stability. If negative airframe stability is to be controlled, normal acceleration feedback might offer a viable solution, provided static stability is corrected by an additional speed-trim loop. Of the indirect methods integral pitch rate as the primary stabilising term offers perhaps the best overall performance. The most critical area might, however, turn out to be the stall regime and if this presents a major problem then only resort to direct incidence feedback would be acceptable.

For augmenting relaxed directional stability, lateral acceleration feedback appears to be a generally viable alternative to direct sideslip feedback, but virtually insurmountable difficulties can arise in flight at high incidence which appears to present generally the critical design case for any form of directional stability augmentation.

Acceleration feedback will generally demand careful 'detuning' to avoid adverse coupling with structural modes and this problem is almost certainly more severe in the lateral case where the lateral acceleration level associated with the rigid body motion is inevitably much lower than the corresponding normal acceleration signal required for longitudinal motion stabilisation and a serious signal to noise ratio problem will tend to arise especially at low airspeeds. These matters have not been studied analytically in this Report and must be left to appropriate specialised studies. To complete the study of extreme flight conditions, we have considered flight into the regime of vanishing dynamic pressure as might occur in a zoom manoeuvre. Contrary to superficial expectation it was shown that irrespective of the feedback scheme adopted these can be performed with acceptable safety provided the pilot concentrates on maintaining incidence and sideslip within the normal range and does not permit dynamic pressure to drop so low that aerodynamic controls become totally ineffective. However, if a departure into the spin regime is allowed to develop, the relaxed stability configuration is almost inevitably worse off than its naturally stable counterpart. This observation applies irrespective of the dynamic pressure at which departure occurs and this makes the provision of effective departure prevention an absolute necessity.

Finally, we have briefly explored possible problem areas in control of the ground roll during take off and landing. The only area giving some concern was found to be control of pitch attitude when the nosewheel is off the ground. The aircraft with significantly relaxed longitudinal stability may be unstable in this situation and of the three feedback types considered only direct incidence feedback is shown to make a positive contribution.

Appendix A

THE EQUATIONS OF MOTION

For small perturbation analysis we only consider near level flight as the equilibrium condition so that $L = W$ and the state variables α, ζ, q, u are defined as increments with respect to the steady state values.

To allow clearer representation of the aerodynamic force coefficient directly in terms of lift and drag we use wind axes for the force equations:

$$\sum L = mV\dot{\gamma} = mV(\dot{\theta} - \dot{\alpha}) \tag{A-1}$$

since $\gamma = \theta - \alpha$ in level flight

$$\sum D + mg\gamma = -m\dot{u} = \sum D + mg(\theta - \alpha) \ . \tag{A-2}$$

For the general case including $\int q$-feedback to the elevator, represented by the derivatives $M_\theta = \partial M/\partial \int q$ and $L_\theta = \partial L/\partial\theta$ we get the equations of motion

$$\begin{bmatrix} \mathscr{L}_\alpha + s & \mathscr{L}_\theta - \mathscr{L}_q s - s & \mathscr{L}_u \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot{\alpha}} s & \mathscr{M}_\theta + \mathscr{M}_q s - s^2 & \mathscr{M}_u \\ \mathscr{D}_\alpha - \frac{g}{V_0} & \frac{g}{V_0} & \mathscr{D}_u + s \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \\ \hat{u} \end{bmatrix} = -\alpha_g \begin{bmatrix} \mathscr{L}_\alpha \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot{\alpha}} s \\ \mathscr{D}_\alpha \end{bmatrix} - u_g \begin{bmatrix} \mathscr{L}_u \\ \mathscr{M}_u \\ \mathscr{D}_u \end{bmatrix} \tag{A-3}$$

$\alpha_g = w_g/V_0$ and $\hat{u}_g = u_\theta/V_0$ represent the gust components normal to and tangential to the flight path respectively. $\alpha = w/V_0$ and $\hat{u} = \Delta V/V_0$ on the other hand are values of these state variables of aircraft motion in relation to a steady atmosphere.

The concise derivatives denoted by curly letters are related to the dimensional derivatives M_α, L_α etc and to the nondimensional derivatives as

$$\mathscr{M}_\alpha = \frac{M_\alpha}{B} = \frac{\partial M/\partial\alpha}{B} = \frac{m_w}{i_B}\frac{1}{\hat{t}}\frac{V_0}{\ell}$$

$$\mathscr{M}_\theta = \frac{M_\theta}{B} = \frac{\partial M/\partial\int q}{B} = \frac{m_\theta}{i_B}\frac{1}{\hat{t}}\frac{V_0}{\ell} = K_\theta \frac{m_\eta}{i_B}\frac{1}{\hat{t}}\frac{V_0}{\ell}$$

$$\mathscr{M}_q = \frac{M_q}{B} = \frac{\partial M/\partial q}{B} = \frac{m_q}{i_B}\frac{1}{\hat{t}} = K_q \frac{m_\eta}{i_B}\frac{1}{\hat{t}}\frac{V_0}{\ell}$$

$$\mathscr{M}_{\dot{\alpha}} = \frac{M_{\dot{\alpha}}}{B} = \frac{\partial M/\partial \dot{\alpha}}{B} = \frac{m_{\dot{w}}}{i_B}\frac{1}{\hat{t}}$$

$$\mathscr{L}_\alpha = \frac{L_\alpha}{mV_0} = \frac{\partial L/\partial \alpha}{mV_0} = \frac{\partial C_L}{\partial \alpha}\frac{1}{\hat{t}}$$

$$\mathscr{D}_\alpha = \frac{D_\alpha}{mV_0} = \frac{\partial D/\partial \alpha}{mV_0} = \frac{\partial C_D}{\partial \alpha}\frac{1}{\hat{t}}$$

$$\mathscr{L}_u = \frac{L_u}{mV_0} = \frac{\partial L/\partial u}{mV_0} = \left(2C_{L_0} + \frac{\partial C_L}{\partial \hat{u}}\right)\frac{1}{\hat{t}}$$

$$\mathscr{D}_u = \frac{D_u}{mV_0} = \frac{\partial D/\partial u}{mV_0} = \left(2C_{D_0} + \frac{\partial C_D}{\partial \hat{u}}\right)\frac{1}{\hat{t}}$$

$$\mathscr{L}_q = \frac{L_q}{mV_0} = \frac{\partial L/\partial q}{mV_0} = \qquad = - K_q m_\eta \frac{\ell}{\ell_E}\frac{V_0}{\hat{t}}$$

$$\mathscr{L}_\theta = \frac{L_\theta}{mV} = \frac{\partial L/\partial \int q}{mV_0} = \qquad = - K_\theta m_\eta \frac{\ell}{\ell_E}\frac{1}{\hat{t}}$$

since $M = - L\ell_E$ for elevator generated pitching moments.

$$\hat{t} = \frac{W/S}{g(\rho/2)V_0}$$

which in level flight reduces to

$$\hat{t} = C_L \frac{V_0}{g}$$

we can write

$$\mathscr{L}_u{}^* = 2 \frac{g}{V_0}$$

$$\mathscr{D}_u{}^* = 2 \frac{C_{D0}}{C_{L0}} \frac{g}{V_0}$$

$$\mathscr{L}_\alpha = \frac{\partial C_L}{\partial \alpha} \frac{1}{C_{L0}} \frac{g}{V_0}$$

$$\mathscr{D}_\alpha = \frac{\partial C_D}{\partial \alpha} \frac{1}{C_{L0}} \frac{g}{V_0} \quad .$$

With the exception of flight at very low airspeeds it is usually permissible to distinguish in the solution of equation (A-3) modes or groups of modes widely separated in frequency and in particular to approximate the short period motion by assuming airspeed to be constant, *ie* $\hat{u} = 0$ and equation (A-3) then reduces to

$$\begin{bmatrix} \mathscr{L}_\alpha + s & \mathscr{L}_\theta - \mathscr{L}_q s - s^2 \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot{\alpha}} s & \mathscr{M}_\theta + \mathscr{M}_q s - s^2 \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \end{bmatrix} = - \alpha_g \begin{bmatrix} \mathscr{L}_\alpha \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot{\alpha}} s \end{bmatrix} - \hat{u}g \begin{bmatrix} \mathscr{L}_u \\ \mathscr{M}_u \end{bmatrix} . \qquad \text{(A-4)}$$

* If $\partial C_L/\partial u$ and $\partial C_D/\partial u$ are negligible.

Appendix B

THE MANOEUVRE MARGIN OF THE CONVENTIONAL AIRCRAFT AND OF CONFIGURATIONS STABILISED BY AUTOMATIC FEEDBACK CONTROL

In conventional aircraft design the manoeuvre margin is a dominant longitudinal stability criterion. Classical stability theory defines it as

$$H_m = -\frac{\partial C_m}{\partial C_L} - \frac{m_q}{\mu} \tag{B-1}$$

where $\mu = m/\frac{1}{2}\rho s\ell$ is the relative density parameter.

Incidence stability $\partial C_m/\partial\alpha$ is contained in the $\partial C_m/\partial C_L$ term, the centre of gravity margin, which can be written as

$$\frac{\partial C_m}{\partial C_L} = \frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} \tag{B-2}$$

and therefore

$$H_m = -\frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu} \tag{B-3}$$

The classical manoeuvre margin applies equally to steady control and to the stability of the short period oscillation. If, however, the stability of the short period oscillatory mode is generated or augmented by a feedback loop with frequency dependent performance, then the contribution of loop to the closed loop stability of the system will depend on the frequency of the resulting modes and only complete analysis of the total system will be able to predict the outcome. We can therefore immediately note that in this case there will be no direct relationship between the frequency of the SPO and the manoeuvre margin as defined in equation (A-3) when the terms are read to represent the steady state contributions say to $\partial C_m/\partial\alpha$ or to m_q of the feedback loops. However, there is still the important question as to whether it is still possible to define a 'manoeuvre margin' which signifies, as in the classical aircraft, the amount of centre of gravity movement or shift in the aerodynamic centre the aircraft can tolerate before the longitudinal SPO becomes divergent and also where there is no longer a positive relationship between pilot's control and steady state response.

Let us consider as an example an aircraft with augmented pitch damping $M_q = M_{q_a} + \Delta M_q$ where M_{q_a} is the 'natural' pitch damping of the airframe and

ΔM_q the stabiliser contribution. We can represent the effect of the feedback loop as generating a complex pitch damping derivative

$$\Delta M_q(s) = M_{q_0} G(s) \tag{B-4}$$

where M_{q_0} is the steady state (s = 0) contribution and G(s) the dynamic transfer function of the control loop representing sensor dynamics, filters and actuator dynamics. As long as there is no wash-out, which in this application would be totally inappropriate, G(s) will be unity for s = 0 , *ie* for zero frequency, however complex the polynomials making up the transfer function

$$G(s) = \frac{1 + a_1 s + a_2 s^2 \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 \ldots + b_m s^m} = \frac{1 + N(s)}{1 + M(s)} . \tag{B-5}$$

The eigenvalues of the aircraft augmented by such a feedback term are given by

$$\begin{bmatrix} (\mathscr{L}_\alpha + s) & -1 \\ \mathscr{M}_\alpha & \mathscr{M}_{q_a} + \mathscr{M}_{q_0} \frac{1 + N(s)}{1 + M(s)} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = 0 \tag{B-6}$$

giving the characteristic equation

$$s^2 - s\left\{\mathscr{M}_{q_a} + \mathscr{M}_{q_0} \frac{1 + N(s)}{1 + M(s)} - \mathscr{L}_\alpha\right\} - \mathscr{M}_\alpha - \mathscr{L}_\alpha \mathscr{M}_{q_a} - \mathscr{L}_\alpha \mathscr{M}_{q_0} \frac{1 + N(s)}{1 + M(s)} = 0 .$$

Multiplied by (1 + M(s)) this gives

$$s^2\{1 + M(s)\} - s\left\{\mathscr{M}_{q_a}\left(1 + M(s)\right) + \mathscr{M}_{q_0}\left(1 + N(s)\right) - \mathscr{L}_\alpha\left(1 + M(s)\right)\right\}$$

$$- \mathscr{L}_\alpha \mathscr{M}_{q_0}\left(1 + N(s)\right) - (\mathscr{M}_\alpha + \mathscr{L}_\alpha \mathscr{M}_{q_a})\left(1 + M(s)\right) - \mathscr{L}_\alpha \mathscr{M}_{q_0}\left(1 + N(s)\right) = 0$$

or separating frequency-dependent terms

$$s^2\left(1 + M(s)\right) - s\left\{(\mathscr{M}_{q_a} - \mathscr{L}_\alpha)\left(1 + M(s)\right) + \mathscr{M}_{q_0}\left(1 + N(s)\right)\right\} - \mathscr{L}_\alpha \mathscr{M}_{q_0}\left(1 + N(s)\right)$$

$$- (\mathscr{M}_\alpha + \mathscr{L}_\alpha \mathscr{M}_{q_a})M(s) - \mathscr{L}_\alpha \mathscr{M}_{q_0} N(s) - (\mathscr{M}_\alpha + \mathscr{L}_\alpha \mathscr{M}_{q_a} + \mathscr{L}_\alpha \mathscr{M}_{q_0}) = 0$$

...... (B-7)

The order of this characteristic equation depends of course on the order of the polynomials N(s) and M(s) and there will be a proportionate number of additional response modes in the augmented aircraft. The total stability of this system depends on all the factors in equation (B-7) and can only be determined by numerical evaluation of this equation. However, we note that the absolute term

$$-\left\{\mathcal{M}_\alpha + \mathcal{L}_\alpha(\mathcal{M}_{q_a} + \mathcal{M}_{q_0})\right\} \tag{B-8}$$

is exactly that of classical stability analysis of the conventional aircraft and is directly proportional to the manoeuvre margin of equation (B-3). It is a necessary, although not sufficient, stability condition that this term be positive and we arrive at the important conclusion that, however complex the transfer function of the pitch damping augmentation loop, stability requires a positive value of the manoeuvre margin and must vanish when the manoeuvre margin vanishes. Therefore in the aircraft augmented by pitch rate feedback the manoeuvre margin plays the same dominating and unique role as in the classical aircraft, although there are of course additional stability conditions to be satisfied in this case.

It can be readily shown that we get the same result if we consider augmentation of $\partial C_m/\partial\alpha$ by a term

$$\Delta M_\alpha(s) = \mathcal{M}_{\alpha_0}\frac{1 + N(s)}{1 + M(s)} \tag{B-9}$$

or by a nominally identical normal acceleration feedback.

The introduction of integral pitch rate feedback and the associated necessity to design the control system as a pitch rate demand control, fundamentally alters the situation. As far as steady manoeuvring is concerned the integral pitch rate loop acts as an integral error term with the result that it tightens the steady state performance, *ie* it ensures that the steady state of aircraft response assumes exactly the form dictated by the demand. A given stick displacement commands the pitch rate q_D assigned to it and there is a strict correspondence therefore between stick displacement or force and aircraft steady state pitching response. In the scheme of Fig 11 this relationship is fully determined by the command path gain K_D. The other system and airframe characteristics have no effect on this relationship, and this applies therefore too, to a quantity defining a form of manoeuvre margin or pitch stiffness which according to Appendix F is now

$$H_m^* = -\frac{\partial C_m/\partial\alpha + \partial C_m/\partial\theta}{\partial C_L/\partial\alpha} + \frac{m_q}{\mu} \quad . \tag{B-10}$$

Clearly, the steady state performance controlled by K_D is only of practical significance if the equilibrium state defined by it is stable, *ie* if the closed loop behaviour of the total system is stable. In the classical aircraft and as we had shown above with augmentation not involving significant integral terms short period longitudinal stability is satisfied by a positive manoeuvre margin. As shown in Appendix F for the aircraft augmented by an integral pitch rate feedback $\partial C_m/\partial\int q$ the manoeuvre margin equation (B-10) is again a stability criterion, but now only a necessary but not sufficient stability condition. In no sense therefore does it now play the dominant role that we normally associate with it.

Appendix C

STATIC STABILITY AUGMENTATION BY AIRSPEED FEEDBACK TO THE ELEVATOR

The static margin

$$H_n = -\frac{\partial C_m}{\partial C_L} + \frac{m_u}{2C_{L0}}$$

can be augmented by speed feedback to the elevator $d\eta/dV$ giving

$$\Delta H_n = \Delta \frac{\partial C_m}{\partial(u/V_0)} \frac{1}{2C_{L0}} = \frac{\partial C_m}{\partial \eta} \frac{d\eta}{dV} \frac{V_0}{2C_{L0}} \tag{C-1}$$

or

$$\Delta H_n = m_\eta \frac{d\eta}{dV_E} V_E^3 \frac{\rho_0}{4(W/S)} .$$

The gain required to produce a given increment in H_n is therefore

$$K_V = \frac{d\eta}{dV_E} = \frac{4(W/S)}{\rho_0 V_E^3 m_\eta} \Delta H_n . \tag{C-2}$$

A control law satisfying this requirement is

$$\Delta\eta = \eta_0 - \frac{4}{3} \frac{W/S}{\rho_0 m_\eta V_E^3} \Delta H_n \tag{C-3}$$

where η_0 is an arbitrary constant chosen for instance to accommodate trim requirements.

Appendix D

SHORT PERIOD STABILITY OF THE AIRCRAFT WITH PITCH RATE FEEDBACK

Making the usual assumption of constant airspeed the short period motion of the aircraft is classically defined (see Appendix A) by

$$\begin{bmatrix} \mathcal{L}_\alpha + s & \mathcal{L}_q - 1 \\ \mathcal{M}_\alpha + \mathcal{M}_{\dot\alpha} s & \mathcal{M}_q - s \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = 0 \quad . \tag{D-1}$$

If the aircraft is stabilised by q-feedback to the elevator the effect of this can be expressed by an appropriate increment to the effective pitch damping derivative $\mathcal{M}_q$ and to the $\mathcal{L}_q$ term. If ℓ_E is the effective elevator moment arm we get

$$\Delta \frac{\partial C_L}{\partial q(\ell/V)} = - \Delta m_q \frac{\ell}{\ell_E} \tag{D-2}$$

where from equation (12)

$$\Delta m_q = \frac{V}{\ell} K_q m_\eta \quad .$$

Therefore $\Delta \mathcal{L}_q$ can be related to $\Delta \mathcal{M}_q$ as

$$\Delta \mathcal{L}_q = - \Delta \mathcal{M}_q \frac{1}{i_B} \frac{\ell}{\ell_E} \frac{\ell}{V} \quad . \tag{D-3}$$

Equation (C-1) defines a quadratic

$$s^2 + s2\zeta\omega_n + \omega_n^2 = 0 \tag{D-4}$$

where the undamped frequency is

$$\omega_n = \frac{V}{\ell} \left[- \frac{m_w}{\mu i_B} \left(1 - \frac{C_{Lq}}{\mu} \right) - \frac{\partial C_L/\partial \alpha}{\mu^2} \frac{m_q}{i_B} \right]^{\frac{1}{2}} \tag{D-5}$$

with the relative density parameter

$$\mu = \frac{W/S}{\frac{1}{2}\rho g \ell} \quad .$$

Since the manoeuvre margin is

$$H_m = -\frac{m_w}{\partial C_L/\partial\alpha}\left(1 - \frac{C_{Lq}}{\mu}\right) - \frac{m_q}{\mu} \quad . \tag{D-6}$$

If one assumes that virtually all pitch damping is produced by the tailplane this expression can be reduced to

$$H_m \cong -\frac{m_w}{\partial C_L/\partial\alpha}\left(1 + \frac{m_q}{\mu}\frac{\ell}{\ell_E}\right) - \frac{m_q}{\mu} \tag{D-7}$$

and we can express the frequency (C-5) in terms of the manoeuvre margin as

$$\omega_n = \frac{V}{\ell}\left[\frac{\partial C_L/\partial\alpha}{\mu i_B} H_m\right]^{\frac{1}{2}} \tag{D-8}$$

and the damping ratio as

$$\zeta = \frac{1}{2}\frac{\frac{\partial C_L}{\partial\alpha} - \frac{m_q + m_{\dot{\alpha}}}{i_B}}{\left(\mu\frac{\partial C_L/\partial\alpha}{i_B} H_m\right)^{\frac{1}{2}}} \quad . \tag{D-9}$$

If we express the manoeuvre margin H_m as the sum of the aerodynamic airframe contribution H_{ma} and the contribution H_{mq} of the stability augmentation system so that

$$H_m = H_{ma} + H_{mq} \tag{D-10}$$

and ignore the elevator lift term in (C-7) then

$$H_{mq} = -\Delta\frac{m_q}{\mu} \quad .$$

We can also express the damping ratio (C-9) as the sum of a natural airframe contribution and the contribution from pitch rate feedback as

$$\zeta = \zeta_a + \zeta_q = \zeta_a + \frac{\Delta H_{mq}}{2\left(\frac{i_B}{\mu}\frac{\partial C_L}{\partial\alpha} H_m\right)^{\frac{1}{2}}} \quad . \tag{D-11}$$

Equations (D-8), (D-10) and (D-11) will be particularly useful for a discussion of the effect of pitch rate feedback on the dynamics of the short period pitching oscillation.

Appendix E

RESPONSE TO GUSTS OF AIRCRAFT STABILISED BY PITCH RATE FEEDBACK TO THE ELEVATOR

E.1 Vertical gusts

The short period response of the aircraft to vertical gusts w_g , *ie* to $\alpha_g = w_g/V_0$ is defined by

$$\begin{bmatrix} \mathscr{L}_\alpha + s & \mathscr{L}_q - 1 \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot\alpha} s & \mathscr{M}_q - s \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = -\alpha_g \begin{bmatrix} \mathscr{L}_\alpha \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot\alpha} s \end{bmatrix} . \qquad \text{(E-1)}$$

Again we express the contribution of pitch rate feedback of appropriate increments to $\mathscr{M}_g$ and $\mathscr{L}_q$ as in Appendix D, equation (D-1), which is the classical description of the conventional aircraft therefore also applies to the aircraft augmented by pitch rate stabilisation over the frequency range where we can ignore the servo loop transfer function. (E-1) yields the frequency response functions in α and q

$$\frac{\alpha}{\alpha_q} = \frac{\omega\left[\mathscr{L}_\alpha - \mathscr{M}_{\dot\alpha}(1 - \mathscr{L}_q)\right] + i\left[\mathscr{L}_\alpha \mathscr{M}_q + \mathscr{M}_\alpha(1 - \mathscr{L}_q)\right]}{-B_2\omega + i\left[B_1 - \omega^2\right]} \qquad \text{(E-2)}$$

where $B_2 = \mathscr{L}_\alpha - \mathscr{M}_q - \mathscr{M}_{\dot\alpha}(1 - \mathscr{L}_q)$

$B_1 = -\mathscr{M}_q \mathscr{L}_\alpha - \mathscr{M}_\alpha(1 - \mathscr{L}_q)$.

Since total aircraft incidence $\sum\alpha = \alpha + \alpha_g$

$$\frac{\sum\alpha}{\alpha_g}(\omega) = 1 + \frac{\alpha}{\alpha_g} . \qquad \text{(E-3)}$$

Pitch rate response is given by

$$\frac{q}{\alpha_g}(\omega) = \omega \frac{\mathscr{M}_\alpha + i\omega\mathscr{M}_{\dot\alpha}}{B_2\omega + i(\omega^2 - B_1)} . \qquad \text{(E-4)}$$

E.2 Response to fore and aft gusts u_g

If u_g is the gust velocity component in the direction of the aircraft flight path, $\hat{u}_g = u_g/V_0$, the short period response to such gusts is given by

$$\begin{bmatrix} \mathcal{L}_\alpha + s & \mathcal{L}_q - 1 \\ \mathcal{M}_\alpha + \mathcal{M}_{\dot{\alpha}} s & \mathcal{M}_q - s \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = -\hat{u}_g \begin{bmatrix} \mathcal{L}_u \\ \mathcal{M}_u \end{bmatrix} . \tag{E-5}$$

which yields the frequency response functions

$$\frac{\alpha}{\hat{u}_g}(\omega) = \frac{\mathcal{L}_u \mathcal{M}_q + \mathcal{M}_\alpha(1 - \mathcal{L}_q) - i\omega\mathcal{L}_u}{B_1 - \omega^2 + i\omega B_2} \tag{E-6}$$

$$\frac{q}{\hat{u}_g}(\omega) = \frac{\mathcal{L}_\alpha \mathcal{M}_u - \mathcal{M}_\alpha \mathcal{L}_u + i\omega(\mathcal{M}_u - \mathcal{L}_u \mathcal{M}_{\dot{\alpha}})}{B_1 - \omega^2 + i\omega B_2} . \tag{E-7}$$

Now of course $\sum\alpha = \alpha$ and $\dfrac{\sum\alpha}{\hat{u}_g} = \dfrac{\alpha}{\hat{u}_g}$.

For the aircraft defined in Table 1 and for the three stability configuration listed as cases 1 to 3 in Table 2 these response functions were evaluated with the results shown in Figs 9 and 10. For the u_g calculations the α-response results have been converted into corresponding normal acceleration using the relationship

$$\Delta n = \frac{\partial C_L/\partial\alpha}{C_{L_0}} \alpha .$$

Appendix F

STABILITY AND GUST RESPONSE OF AIRCRAFT STABILISED BY INTEGRAL PITCH RATE FEEDBACK

F.1 Stability

We consider the pitch rate demand system illustrated in Fig 11 where the error between demanded q_D and actual pitch rate q is used to activate the pitch control according to

$$\eta = K_q(q - q_D) + K_\theta \int (q - q_D)dt \tag{F-1}$$

the notation K_θ and the subsequent use of the symbol θ is only used as a shorthand for $\int(q - q_D)$ and does not imply reference to true pitch attitude. For small perturbation analysis of aircraft response about an essentially level flight condition with controls, *ie* $q_D = 0$, fixed in the trimmed position, $\int(q - q_D)dt$ does of course equate with θ and this is the case principally studied here.

We can express the control law (F-1) in terms of equivalent derivatives

$$\begin{aligned} \Delta M_q &= M_\eta K_q \\ \Delta L_q &= L_\eta K_q \\ M_\theta &= M_\eta K_\theta \\ L_\theta &= L_\eta K_\theta \ . \end{aligned}$$

If ℓ_E is the effective elevator moment arm the lift derivatives L_θ and ΔL_q can be expressed as

$$\Delta L_q = - \Delta M_q \frac{1}{\ell_E} = - M_\eta \frac{K_q}{\ell_E} \tag{F-2}$$

$$L_\theta = - M_\theta \frac{1}{\ell_E} = - M_\eta \frac{K_\theta}{\ell_E} \ . \tag{F-3}$$

The terms ΔM_q and ΔL_q have to be treated as increments to the corresponding contributions from the aerodynamics of the airframe (suffix a) so that

$$M_q = M_{q_a} + \Delta M_q$$

$$L_q = L_{q_a} + \Delta L_q \quad .$$

With the nomenclature of Appendix A the equations of motion of the aircraft are

$$\begin{bmatrix} \mathcal{L}_\alpha + s & \mathcal{L}_\theta + \mathcal{L}_q - s^2 & \mathcal{L}_u \\ \mathcal{M}_\alpha + \mathcal{M}_{\dot\alpha} s & \mathcal{M}_\theta + \mathcal{M}_q s - s^2 & \mathcal{M}_u \\ \mathcal{D}_\alpha - \frac{g}{V_0} & \frac{g}{V_0} & \mathcal{D}_u + s \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \\ \hat{u} \end{bmatrix} = - \alpha_g \begin{bmatrix} \mathcal{L}_\alpha \\ \mathcal{M}_\alpha + \mathcal{M}_{\dot\alpha} s \\ \mathcal{D}_\alpha \end{bmatrix} - u_g \begin{bmatrix} \mathcal{L}_u \\ \mathcal{M}_u \\ \mathcal{D}_u \end{bmatrix}$$

...... (F-4)

The eigenvalues of this system are the roots of the polynomial

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 \qquad \text{(F-5)}$$

with

$$A_3 = \mathcal{L}_\alpha + \mathcal{D}_u - \mathcal{M}_q - \mathcal{M}_{\dot\alpha}(1 - \mathcal{L}_q)$$

$$A_2 = - \mathcal{M}_q(\mathcal{L}_\alpha + \mathcal{D}_u) + \mathcal{L}_\alpha \mathcal{D}_u - \mathcal{L}_u\left(\mathcal{D}_\alpha - \frac{g}{V_0}\right) - \mathcal{M}_\theta - \mathcal{M}_\alpha(1 - \mathcal{L}_g)$$
$$- \mathcal{M}_{\dot\alpha}\mathcal{D}_u(1 - \mathcal{L}_q) + \mathcal{M}_{\dot\alpha}\mathcal{L}_\theta$$

$$A_1 = \mathcal{D}_u\{- \mathcal{M}_\theta - \mathcal{L}_\alpha \mathcal{M}_q - \mathcal{M}_\alpha(1 - \mathcal{L}_q)\} - \mathcal{L}_\alpha \mathcal{M}_\theta + \mathcal{L}_\theta \mathcal{M}_\alpha$$
$$+ \left(\mathcal{D}_\alpha - \frac{\theta}{V_0}\right)\{\mathcal{M}_u(1 - \mathcal{L}_q) + \mathcal{L}_u \mathcal{M}_q\} + \mathcal{M}_u \frac{g}{V_0} - \mathcal{M}_{\dot\alpha}\left\{\mathcal{L}_u \frac{g}{V_0} - \mathcal{L}_\theta \mathcal{D}_u\right\}$$

$$A_0 = - \mathcal{M}_\theta\left\{\mathcal{L}_\alpha \mathcal{D}_u - \mathcal{L}_u\left(\mathcal{D}_\alpha - \frac{g}{V_0}\right)\right\} - \mathcal{M}_\alpha\left(\mathcal{L}_u \frac{g}{V_0} - \mathcal{L}_\theta \mathcal{D}_u\right)$$
$$+ \mathcal{M}_u\left\{\mathcal{L}_\alpha \frac{g}{V_0} + \mathcal{L}_\theta\left(\mathcal{D}_\alpha - \frac{g}{V_0}\right)\right\}$$

We note that the introduction of integral pitch rate feedback ($\mathcal{M}_\theta$ and $\mathcal{L}_\theta$) has not raised the order of the characteristic equation by comparison with that of the classical aircraft, which is identical to the above when $\mathcal{M}_\theta$ and $\mathcal{L}_\theta$ are omitted.

However, unless insignificantly small integral pitch rate terms are considered, we find that the polynomial (F-5) no longer splits readily into two quadratics, which is normally the case in the classical situation. Whereas classically A_0 and $A_1 \ll A_3$ and A_2 now only A_0 is small allowing the approximation

$$\lambda_2 = -\frac{A_0}{A_1} .$$

The first order mode corresponding to this root is evidently a degenerate form of the classical phugoid. If the integral feedback term $\mathscr{M}_\theta$ is other than negligible, A_1 is dominated by the $\mathscr{L}_\alpha \mathscr{M}_\theta$ term and ignoring other minor terms we can get the approximate solution

$$\lambda_2 \simeq \mathscr{L}_u\left(1 - \frac{g/V_0}{\mathscr{L}_\alpha}\right) - \mathscr{D}_u - \frac{g/V_0}{\mathscr{M}_\theta}\left\{\mathscr{M}_\alpha \frac{\mathscr{L}_u}{\mathscr{L}_\alpha} + \mathscr{M}_u\right\} . \qquad \text{(F-6)}$$

Using only major terms for the $\mathscr{L}_u$ and $\mathscr{D}_u$ derivatives this can be expressed in more familiar nondimensional form as

$$\lambda_2 \simeq -\frac{2}{\hat{t}}\left[C_{D0} - \frac{C_{L0}}{\partial C_L/\partial\alpha}\left(\frac{\partial C_D}{\partial\alpha} - C_{L0}\right) + \frac{C_{L0}^2}{m_\theta}\left(-\frac{\partial C_m/\partial\alpha}{\partial C_L/\partial\alpha} + \frac{m_u}{2C_{L0}}\right)\right] . \qquad \text{(F-7)}$$

We observe the last term in the bracket to be the static margin of equation (2), containing only airframe contributions, but this term is divided by the integral pitch rate derivative and clearly becomes less important the greater m_θ , *ie* K_θ. The remaining terms in this expression are essentially controlled by the drag characteristics of the aircraft and are in fact related to the familiar speed stability mode, which is defined by

$$\lambda_V = -\frac{2}{\hat{t}}\left(C_{D0} - C_{L0}\frac{\partial C_D/\partial\alpha}{\partial C_L/\partial\alpha}\right) .$$

The expression

$$\left[C_{D0} - C_{L0}\frac{\partial C_D/\partial\alpha - C_{L0}}{\partial C_L/\partial\alpha}\right]$$

has been evaluated numerically for two modern combat aircraft with the results presented in Fig 13a. Taking the pitching moment characteristics of the aircraft defined in Table 1, and as cases 4 and 5 of Table 2, equation (F-7) has been used to calculate the root λ_2 of the mode corresponding to static stability of the classical aircraft. We note from Fig 13b that except at the extreme low speed end, this root is very small, defining convergent, or divergent modes with time constants of the order of 40 seconds or more.

These examples can be taken as typically representative and this applies particularly to the trend of a stable characteristic at high speeds to change to instability at low speeds with a sharp deterioration as the stalling regime is approached.

For an analysis of the short period characteristics of this case we make the usual assumption of constant airspeed, reducing (F-4) to

$$\begin{bmatrix} \mathscr{L}_\alpha + s & \mathscr{L}_\theta + \mathscr{L}_q s - s^2 \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot\alpha} s & \mathscr{M}_\theta + \mathscr{M}_q s - s^2 \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \end{bmatrix} = - \alpha_g \begin{bmatrix} \mathscr{L}_\alpha \\ \mathscr{M}_\alpha + \mathscr{M}_{\dot\alpha} s \end{bmatrix} - \hat{u}_g \begin{bmatrix} \mathscr{L}_u \\ \mathscr{M}_u \end{bmatrix} . \qquad \text{...... (F-8)}$$

The eigenvalues are the root of the cubic

$$s^3 + B_2 s^2 + B_1 s + B_0 = 0 \qquad \text{(F-9)}$$

with

$$B_2 = - \mathscr{L}_\alpha \mathscr{M}_q - \mathscr{M}_{\dot\alpha}(1 - \mathscr{L}_q)$$

$$B_1 = - \mathscr{M}_q \mathscr{L}_\alpha - \mathscr{M}_\theta - \mathscr{M}_\alpha (1 - \mathscr{L}_q) - \mathscr{M}_{\dot\alpha} \mathscr{L}_\theta$$

$$B_0 = - \mathscr{L}_\alpha \mathscr{M}_\theta + \mathscr{L}_\theta \mathscr{M}_\alpha \quad .$$

The short period motion therefore is third order and in addition to a complex pair of roots defining a short period oscillation we now obtain an aperiodic mode, which turns out to be a heavily damped subsidence. This mode, being strongly affected by the lift slope term $\mathscr{L}_\alpha$ in B_0 can be visualised as a predominant heave mode. (If $\mathscr{M}_\theta \to \infty$, it will become a pure heave mode with $\lambda_1 = - \mathscr{L}_\alpha$.) It is interesting to note that B_1 takes a form analogous to the stiffness term of the classical solution, which reads

$$- \mathscr{M}_q \mathscr{L}_\alpha - \mathscr{M}_\alpha (1 - \mathscr{L}_q)$$

The principal difference is the addition of $(- \mathscr{M}_\theta)$ which therefore appears as a direct addition to the aerodynamic pitch stability term $(- \mathscr{M}_\alpha)$.

We can derive from B_1 an equivalent 'manoeuvre margin'

$$H^*_m = -\frac{\partial C_m/\partial\alpha + \partial C_m/\partial\theta}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu} \quad . \qquad \text{(F-10)}$$

This margin in the form of B_1 is clearly an important stability criterion as $B_1 > 0$ is a necessary stability criterion according to the rules of elementary algebra. However, because of the inevitable coupling between the modes defined by equation (F-9) this stiffness term no longer directly defines the period of the short period oscillation as in the classical case nor does it necessarily define a limit of stability. Indeed numerical work quickly shows that the short period oscillation may become divergent for very substantially positive value of the 'manoeuvre margin' H^*_m unless $\mathscr{M}_q$ is sufficiently enhanced. This reflects the fact that the first order mode has a strong tendency to extract damping from the system at the expense of the oscillatory mode.

A.2 Gust response

From equation (F-8) we can derive short period approximations to the gust response function. As we have seen from Fig 13 the static stability mode ignored by this treatment has, at least in the normal speed range, a very long time constant and will only intervene therefore at the extreme low frequency and of the response spectrum.

The response to vertical gusts α_g is given by the frequency response functions

$$\frac{\alpha}{\alpha_g}(\omega) = \frac{\omega^2[\mathscr{L}_\alpha - \mathscr{M}_{\dot\alpha}(1-\mathscr{L}_q)] + \mathscr{L}_\alpha\mathscr{M}_\theta - \mathscr{L}_\theta\mathscr{M}_\alpha + i\omega[\mathscr{L}_\alpha\mathscr{M}_q(1-\mathscr{L}_q) - \mathscr{L}_\theta\mathscr{M}_{\dot\alpha}]}{B_0 - B_2\omega^2 + i\omega[B_1 - \omega^2]}$$

...... (F-11)

$$\frac{\sum\alpha}{\alpha} = 1 + \frac{\alpha}{\alpha_g} \qquad \text{(F-12)}$$

$$\frac{q}{\alpha_g} = \omega^2 \frac{\mathscr{M}_\alpha + i\omega\mathscr{M}_{\dot\alpha}}{B_2\omega - B_0 + i[\omega^2 - B_1]} \quad . \qquad \text{(F-13)}$$

Using the aircraft with characteristics defined in Table 1 we have evaluated these expressions numerically for configurations 4 to 8 of Table 2 with the results given in Fig 14.

Appendix G

STABILITY IN RAPID ROLL MANOEUVRES OF THE AIRCRAFT STABILISED BY PITCH RATE AND INTEGRAL PITCH RATE FEEDBACK

G.1 Pitch rate feedback

Following the procedure introduced by Phillips we assume airspeed to be constant and the roll freedom to be constrained to steady roll rate p_0, the full six degrees of aircraft motion are reduced to four. Expressing the effect of pitch rate feedback to the elevator by an appropriate increase in the m_q derivative but ignoring the elevator lift effect and minor aerodynamic terms, we arrive at the conventional looking set of equations

$$\left.\begin{aligned} \mathscr{M}_\alpha \alpha + \mathscr{M}_q q + p_0 r \frac{C - A}{B} - \dot{q} &= 0 \\ \mathscr{N}_\beta \beta + \mathscr{N}_r r + p_0 q \frac{A - B}{C} - \dot{r} &= 0 \\ \mathscr{L}_\alpha \alpha + p_0 \beta - q + \dot{\alpha} &= 0 \\ p_0 \alpha - r - \dot{\beta} &= 0 \end{aligned}\right\} . \qquad \text{(G-1)}$$

Introducing the operator $s = d/p_o d\tau$ and using the approximation $C = B + A$ this leads to

$$\begin{bmatrix} \dfrac{\mathscr{M}_\alpha}{p_0^2} & 0 & \left(\dfrac{\mathscr{M}_q}{p_0} - s\right) & 1 \\ 0 & \dfrac{\mathscr{N}_\beta}{p_0^2} & -\dfrac{B - A}{B + A} & \left(\dfrac{\mathscr{N}_r}{p_0} - s\right) \\ \left(\dfrac{\mathscr{L}_\alpha}{p_0} + s\right) & +1 & -1 & 0 \\ 1 & -s & 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \dfrac{q}{p_0} \\ \dfrac{r}{p_0} \end{bmatrix} = 0 . \qquad \text{(G-2)}$$

The eigenvalues are defined by the quartic

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 . \qquad \text{(G-3)}$$

with

$$A_3 = -\frac{\mathcal{M}_q}{p_0} + \frac{\mathcal{L}_\alpha}{p_0} - \frac{\mathcal{N}_r}{p_0}$$

$$A_2 = 1 + \frac{B - A}{B + A} - \frac{\mathcal{M}_\alpha}{p_0}\left(\frac{\mathcal{M}_q}{p_0} + \frac{\mathcal{N}_r}{p_0}\right) + \frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} - \frac{\mathcal{M}_\alpha}{p_0^2} + \frac{\mathcal{N}_\beta}{p_0^2}$$

$$A_1 = -\frac{\mathcal{M}_q}{p_0} - \frac{\mathcal{N}_r}{p_0} + \frac{\mathcal{L}_\alpha}{p_0}\frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} + \frac{\mathcal{L}_\alpha}{p_0}\left(\frac{B - A}{B + A} + \frac{\mathcal{N}_\beta}{p_0^2}\right) + \frac{\mathcal{M}_\alpha}{p_0^2}\frac{\mathcal{N}_r}{p_0} - \frac{\mathcal{N}_\beta}{p_0^2}\frac{\mathcal{M}_q}{p_0}$$

$$A_0 = \frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} + \frac{B - A}{B + A} - \frac{\mathcal{N}_\beta}{p_0^2}\left(1 + \frac{\mathcal{L}_\alpha}{p_0}\frac{\mathcal{M}_\alpha}{p_0^2}\right) + \frac{\mathcal{M}_\alpha}{p_0^2}\left(\frac{B - A}{B + A} - \frac{\mathcal{N}_\beta}{p_0^2}\right) .$$

...... (G-4)

We can identify the aerodynamic terms in this equation with the stability parameters of the uncoupled longitudinal and directional (not lateral) short period oscillations as

$$\left.\begin{aligned} \mathcal{N}_\beta &= \omega_\psi^2 \\ -\mathcal{M}_\alpha - \mathcal{L}_\alpha\mathcal{M}_q &= \omega_\theta^2 \\ -\mathcal{N}_r &= 2\zeta_\psi\omega_\psi \\ -\mathcal{M}_q + \mathcal{L}_\alpha &= 2\zeta_\theta\omega_\theta \end{aligned}\right\} . \qquad \text{(G-5)}$$

We further define the contributions of $\mathcal{M}_q$ and $\mathcal{L}_\alpha$ to the longitudinal damping ratio as

$$\zeta_\theta = \zeta_q + \zeta_\alpha .$$

With these terms the coefficients of the stability quartic become

$$\left.\begin{aligned}
A_3 &= 2\zeta_\theta\left(\frac{\omega_\theta}{p_0}\right) + 2\zeta_\psi\left(\frac{\omega_\psi}{p_0}\right) \\
A_2 &= 1 + \frac{B - A}{B + A} + \left(\frac{\omega_\theta}{p_0}\right)^2 + 4\zeta_\psi\zeta_\theta\left(\frac{\omega_\psi}{p_0}\right)\left(\frac{\omega_\theta}{p_0}\right) + \left(\frac{\omega_\psi}{p_0}\right)^2 \\
A_1 &= 2\zeta_\theta\left(\frac{\omega_\theta}{p_0}\right)\left(\frac{\omega_\psi}{p_0}\right)^2 + 2\zeta_\alpha\left(\frac{\omega_\theta}{p_0}\right)\left(1 - \frac{B - A}{B + A}\right) \\
&\quad + 2\zeta_\psi\left(\frac{\omega_\psi}{p_0}\right) + 2\zeta_\theta\left(\frac{\omega_\theta}{p_0}\right) - \boxed{2\zeta_\alpha\left(\frac{\omega_\theta}{p_0}\right)\left(1 - \frac{B - A}{B + B}\right)} \\
A_0 &= \frac{B - A}{B + A}\left(1 - \left(\frac{\omega_\theta}{p_0}\right)^2\right) - \left(\frac{\omega_\psi}{p_0}\right)^2\left(1 - \left(\frac{\omega_\theta}{p_0}\right)^2\right) \\
&\quad + 4\zeta_\psi(\zeta_\theta + \zeta_\alpha)\left(\frac{\omega_\psi}{p_0}\right)\left(\frac{\omega_\theta}{p_0}\right) + \boxed{4\zeta_\alpha(\zeta_\theta - \zeta_\alpha)\,\frac{B - A}{B + A}\left(\frac{\omega_\theta}{p_0}\right)^2}
\end{aligned}\right\} . \qquad \text{(G-6)}$$

With the exception of the boxed terms these equations are identical to the solution given in Ref 5. In the original work $\omega_\theta^2 = -\mathcal{M}_\alpha$ stood for pitch stability only but if the term is taken to mean what it implies, namely the actual undamped frequency of the short period mode as defined in (G-5) then the solution of (G-6) will give the correct answer.

As in previous analysis the stability boundaries for the coupled motion are fully defined by conditions at which the absolute term A_0 in the stability quartic vanishes. Solving for (ω_θ/p_0) we get

$$\left(\frac{\omega_\theta}{p_0}\right)^2\left\{\left(\frac{\omega_\psi}{p_0}\right)^2 - \frac{B - A}{B + A}\,(1 - 4\zeta_\alpha\zeta_q)\right\} + \left(\frac{\omega_\theta}{p_0}\right)4\zeta_\alpha\zeta_q\left(\frac{\omega_\psi}{p_0}\right) + \frac{B - A}{B + A} - \left(\frac{\omega_\psi}{p_0}\right)^2 = 0$$

or

$$\left(\frac{\omega_\theta}{p_0}\right)_{A_0=0} = -\frac{D}{2E} \pm \sqrt{\left(\frac{D}{2E}\right)^2 - \frac{\frac{B - A}{B + A} - \left(\frac{\omega_\psi}{p_0}\right)^2}{E}} \qquad \text{(G-7)}$$

where $D = 4\zeta_\alpha\zeta_q\left(\frac{\omega_\psi}{p_0}\right)$

$$E = \left(\frac{\omega_\psi}{p_0}\right)^2 - \frac{B - A}{B + A}(1 - 4\zeta_\alpha\zeta_q) \quad .$$

For some typical values of the parameters ζ_θ , ζ_ψ and the ratio of roll to pitch inertia A/B equation (G-7) has been evaluated with the result presented in Fig 20. The calculations were carried out both with and without the ζ_α term, *ie* with the $\partial C_L/\partial\alpha$ effect included or ignored and we note that this term provides some additional stability. Otherwise the result is identical to the classical solution of Ref 5, indicating a substantial widening of the stable regime as ζ_θ is increased to the large values associated with the type of pitch rate feedback considered here.

Although these results show a welcome benefit from pitch rate feedback, there is one aspect that may need more careful study in a rigorous treatment of aircraft stability with a realistic representation of the elevator actuator transfer function.

The two basic oscillatory aircraft modes experience a change in their frequencies as roll rate increases and causes them increasingly to couple. Fig 21 shows results obtained for the simple case where all damping terms are ignored and is derived from the data of Ref 7. Although of course not directly applicable to our case which is distinguished by exceptionally high longitudinal damping it gives trends which are generally valid. In this graph ω_1 is the frequency of the mode associated with the uncoupled longitudinal short period motion and ω_2 is associated with the directional oscillation. We note that the frequency of the short period mode is significantly increased as roll rate increases and this in turn may affect the closed loop response of the pitch rate feedback loop operating now above its basic design frequency. It is not the intention here to pursue this problem further but merely to indicate the possibility of a problem.

In the treatment of inertia-coupled auto-rotation the $\partial C_L/\partial\alpha$ term was always included as a primary agent through which this phenomenon arises.

Nevertheless, it was necessary to rearrange the algebra of the solution of Ref 5, where ω_θ was defined as only representing m_w as the possibility of an aircraft deriving a major portion of its manoeuvre margin from pitch damping was then not anticipated.

As in the treatment of basic stability in autorotational conditions we define ω_θ now as the real undamped frequency of the short period oscillation irrespective of its origin. We also completed the representation of the analysis by including damping in yaw expressed as an equivalent damping ratio ζ_ψ of the directional oscillation.

The expression for the self-sustained steady rolling conditions now reads

$$\left(\frac{p}{\omega_\psi}\right)^2 = \frac{E}{2} \pm \sqrt{\left(\frac{F}{2}\right)^2 - \left(\frac{\omega_\theta}{\omega_\psi}\right)\frac{B+A}{B-A}(1 + 2\zeta_\psi\alpha_K)} \qquad \text{(G-8)}$$

with

$$F = \frac{B+A}{B-A} + \left(\frac{\omega_\theta}{\omega_\psi}\right)^2(1 - 4\zeta_\alpha\zeta_q) - \frac{\omega_\theta}{\omega_\psi}\left(4\zeta_q\zeta_\psi\frac{B+A}{B-A} + 2\zeta_\alpha\alpha_K\right)$$

and

$$\alpha_K = \frac{\alpha_0}{\omega_\psi}\frac{\ell_v}{\ell_p}\frac{b}{2V}$$

assuming the lateral derivatives ℓ_v and ℓ_p are non-dimensionalised with b/2 as the reference length. Equation (A-6) only has real solutions for values of α_K below some critical value which is readily obtained from the terms under the square root in equation (A-6). Therefore self-sustained autorotation with real values of p from equation (G-8) can only exist if

$$\alpha_K\zeta_\alpha \leqslant \frac{1}{2}\left\{\frac{\frac{B+A}{B-A}}{\left(\frac{\omega_\theta}{\omega_\psi}\right)} + \left(\frac{\omega_\theta}{\omega_\psi}\right)(1 - 4\zeta_\alpha\zeta_q) - 4\frac{B+A}{B-A}\zeta_q\zeta_\psi - \left(\frac{B+A}{B-A}\right)^{\frac{1}{2}}2\right\} . \qquad \text{(G-9)}$$

For some typical values of the relevant parameters this expression has been evaluated with the result shown in Fig 22. We note that with increasing pitch damping ζ_q the autorotational regime is pushed toward negative incidence, *ie* away from the practically important flight range.

We can conclude therefore that the aircraft stabilised by pitch rate feedback will show more favourable behaviour in rapid rolling, than the naturally stable aircraft having the same effective manoeuvre margin, *ie* ω_θ .

G.2 Integral pitch rate feedback

With the introduction of an M_θ term the equations of motion become

$$\begin{bmatrix} \frac{\mathcal{M}_\alpha}{p_0^2} & 0 & \left(\frac{\mathcal{M}_\theta}{p_0^2} + \frac{\mathcal{M}_q}{p_0} s - s^2\right) & 1 \\ 0 & \frac{\mathcal{N}_\beta}{p_0^2} & -\frac{B-A}{B+A} s & \left(\frac{\mathcal{N}_r}{p_0} - s\right) \\ \frac{\mathcal{L}_\alpha}{p_0} + s & +1 & -s & 0 \\ -1 & -s & 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \frac{\int q}{p_0} \\ \frac{r}{p_0} \end{bmatrix} = 0 . \quad \text{(G-10)}$$

This leads to a characteristic equation

$$s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \quad \text{(G-11)}$$

with

$$A_4 = \frac{\mathcal{L}_\alpha}{p_0} - \frac{\mathcal{M}_q}{p_0} - \frac{\mathcal{N}_r}{p_0}$$

$$A_3 = -\frac{\mathcal{M}_\alpha}{p_0^2} + \frac{\mathcal{N}_\beta}{p_0^2} - \frac{\mathcal{L}_\alpha}{p_0}\left(\frac{\mathcal{M}_q}{p_0} + \frac{\mathcal{N}_r}{p_0}\right) - \frac{\mathcal{M}_\theta}{p_0^2} + \frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} + \frac{B-A}{B+A} - 1$$

$$A_2 = \frac{\mathcal{L}_\alpha}{p_0}\left(\frac{\mathcal{N}_\beta}{p_0^2} - \frac{\mathcal{M}_\theta}{p_0^2} + \frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} + \frac{B-A}{B+A}\right) + \frac{\mathcal{M}_q}{p_0} + \frac{\mathcal{N}_r}{p_0} + \frac{\mathcal{M}_\alpha}{p_0^2}\frac{\mathcal{N}_r}{p_0} - \frac{\mathcal{N}_\beta}{p_0^2}\frac{\mathcal{M}_q}{p_0} + \frac{\mathcal{M}_\theta}{p_0^2}\frac{\mathcal{N}_r}{p_0}$$

$$A_1 = -\frac{\mathcal{M}_\alpha}{p_0^2}\left(\frac{\mathcal{N}_\beta}{p_0^2} - \frac{B-A}{B+A}\right) - \frac{\mathcal{L}_\alpha}{p_0}\left(\frac{\mathcal{N}_\beta}{p_0^2}\frac{\mathcal{M}_\theta}{p_0^2} - \frac{\mathcal{M}_\theta}{p_0^2}\frac{\mathcal{N}_r}{p_0}\right) - \frac{\mathcal{N}_\beta}{p_0^2}\left(\frac{\mathcal{M}_\theta}{p_0^2} - 1\right)$$
$$+ \frac{\mathcal{M}_\theta}{p_0^2} - \frac{\mathcal{M}_q}{p_0}\frac{\mathcal{N}_r}{p_0} - \frac{B-A}{B+A}$$

$$A_0 = -\frac{\mathcal{M}_\theta}{p_0^2}\left(\frac{\mathcal{L}_\alpha}{p_0}\frac{\mathcal{N}_\beta}{p_0^2} + \frac{\mathcal{N}_r}{p_0}\right) .$$

Compared with (G-6) the order of this polynomial is increased by one so that we get an additional real root as we did in the analysis of the non-rolling longitudinal motion in Appendix F. There was insufficient time to evaluate this rather forbidding equation numerically and no observations on the consequent effect on stability of the rolling states can be offered at this stage.

The analysis of possible autorotational states, however, is more straightforward. It consists of the search for physically real steady roll states in the absence of a roll control input. The obvious effect of an integral pitch rate term $M_\theta = \partial M/\partial \int q$ is to inhibit the possibility of a steady state pitch rate and we solve the equations of motion with

$$\dot{q} = q = \dot{r} = \dot{\beta} = \dot{\alpha} = 0$$

and get

$$L_\beta \beta + L_p p = 0 \tag{G-12}$$

$$N_\beta \beta + N_r r = 0 \tag{G-13}$$

$$M_\alpha \alpha + M_\theta \int q + prB = 0 \tag{G-14}$$

$$-\frac{L_\alpha}{mv} \alpha - p\beta = 0 \tag{G-15}$$

$$-r - p(\alpha + \alpha_0) = 0 \tag{G-16}$$

where α_0 is the incidence of the principal inertia axis in the non-rolling trimmed flight state.

By successive substitution we eliminate the variables r, α and β using equations (G-16), (G-15), (G-13) and finally (G-12) and get a solution

$$p^2 = -\frac{g}{\ell} \frac{\partial C_L/\partial \alpha}{C_{L_0}} \frac{n_v}{n_r} \left(\frac{\ell_v}{n_v} \frac{n_r}{\ell_p} \alpha_0 - 1 \right) .$$

Autorotational equilibrium roll rates are defined by position values of the right-hand side which implies that they exist if

$$\left(\frac{\ell_v}{n_v} \frac{n_r}{\ell_p} \alpha_0 - 1 \right) > 0 .$$

It will be readily seen that this requires for any realistic set of the other aerodynamic quantities involved absurdly high negative values of α_0 and we conclude that the aircraft stabilised by integral pitch rate feedback has no autorotational problem of practical significance. It is interesting to note that this result is unaffected by the magnitude of M_θ provided it is finite.

Appendix H

SHORT PERIOD STABILITY AND MANOEUVRE CONTROL OF THE AIRCRAFT WITH NORMAL ACCELERATION FEEDBACK

We assume a control law of the form

$$\eta = K_n \Delta n \tag{H-1}$$

The kinematic relationship

$$\Delta n = \frac{v}{g}(q - \dot{\alpha}) \tag{H-2}$$

allows us to express this in terms of familiar state variables as

$$\eta = K_n \frac{v}{g}(g - \dot{\alpha})$$

the 'normal acceleration' defined in equation (H-2) is an acceleration normal to the flight path. An accelerometer mounted in the aircraft on the other hand measures an acceleration in a body fixed reference frame. It should be noted that this reference frame does not coincide (except at one specific flight condition) with the stability axes used in the conventional stability analysis. We shall ignore in the following analysis the inaccuracies incurred, which will be unimportant at flight at low incidence but may become significant at large incidence.

We write the equations of motions approximately as

$$\begin{bmatrix} \mathcal{L}_\alpha + s - \mathcal{L}_\eta K_n \frac{V_0}{g} s & -1 + \mathcal{L}_\eta K_n \frac{V_0}{g} \\ \mathcal{M}_\alpha + \mathcal{M}_{\dot{\alpha}} s - \mathcal{M}_\eta K_n \frac{V_0}{g} s & \mathcal{M}_q + \mathcal{M}_\eta K_n \frac{V_0}{g} - s \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = - \eta_p \begin{bmatrix} \mathcal{L}_\eta \\ \mathcal{M}_\eta \end{bmatrix} \tag{H-3}$$

where η_p is the elevator applied by the pilot.

The feedback control effects can be more simple expressed in the form of equivalent derivatives

$$\mathcal{M}_\eta \frac{V_0}{g} K_n = \mathcal{M}_n$$

and

$$\mathcal{L}_\eta \frac{V_0}{g} K_n = \mathcal{L}_n$$

and since

$$\frac{\partial C_L}{\partial \eta} = - m_\eta \frac{\ell}{\ell_E}$$

where ℓ_E is the effective elevator moment arm, negative for a rear controls we can write

$$\mathscr{L}_n = -\mathscr{M}_\eta i_B \frac{\ell}{V} \frac{\ell}{\ell_E} .$$

Steady state manoeuvring response can be calculated from equation (H-2) with $s \approx 0$ as

$$\frac{\alpha}{\eta_p} = - \frac{\mathscr{M}_\eta\left(1 - \mathscr{M}_n i_B \frac{\ell^2}{V\ell_E}\right) - \mathscr{L}_\eta(\mathscr{M}_q - \mathscr{M}_n)}{\mathscr{M}_\alpha\left(1 - \mathscr{M}_n i_B \frac{\ell^2}{V\ell_E}\right) - \mathscr{L}_\alpha(\mathscr{M}_q + \mathscr{M}_n)} \tag{H-4}$$

and

$$\frac{q}{\eta_p} = - \frac{\mathscr{M}_\eta \mathscr{L}_\alpha + \mathscr{M}_\alpha \mathscr{L}_\eta}{\mathscr{M}_\alpha\left(1 - \mathscr{M}_n i_B \frac{\ell^2}{V\ell_E}\right) + (\mathscr{M}_q + \mathscr{M}_n)\mathscr{L}_\alpha} . \tag{H-5}$$

The normal acceleration response is given by

$$\Delta n = \mathscr{L}_\alpha \alpha \frac{V}{g} + \mathscr{L}_\eta \eta \frac{V}{g}$$

where $\eta = \eta_p + K_n \Delta n$.

Therefore

$$\frac{\Delta n}{\eta_p} = \frac{V}{g} \frac{\mathscr{L}_\alpha\left(\frac{\alpha}{\eta_p}\right) + \mathscr{L}_\eta}{1 - \mathscr{M}_n i_B \frac{\ell^2}{V\ell_E}} \tag{H-6}$$

with α/η_p from equation (H-4) this gives finally the steady response relationship

$$\frac{\Delta n}{\eta_p} = \frac{V}{g}\mathscr{L}_\alpha \frac{-\mathscr{M}_\eta + \mathscr{L}_\eta \dfrac{\mathscr{M}_q + \mathscr{M}_n}{1 - \mathscr{M}_n i_B \dfrac{\ell^2}{V\ell_E}}}{\mathscr{M}_\alpha\left(1 - \mathscr{M}_n i_B \dfrac{\ell^2}{V\ell_E}\right) - \mathscr{L}_\alpha(\mathscr{M}_q + \mathscr{M}_n)} + \frac{\mathscr{L}_\eta}{1 - \mathscr{M}_n i_B \dfrac{\ell^2}{V\ell_E}} \quad . \tag{H-7}$$

The characteristic equation defining the stability of the short period oscillation is again a quadratic

$$s^2 + 2\zeta\omega_n s + \omega^2 = 0$$

where

$$2\zeta\omega_n = \frac{\mathscr{L}_\alpha}{1 - \mathscr{M}_n i_B \dfrac{\ell^2}{V_0\ell_E}} - \mathscr{M}_q - \mathscr{M}_{\dot\alpha} \tag{H-8}$$

and

$$\omega_n^2 = -\mathscr{M}_\alpha - \frac{\mathscr{L}_\alpha}{1 - \mathscr{M}_n i_B \dfrac{\ell^2}{V_0\ell_E}}(\mathscr{M}_q + \mathscr{M}_n) \quad . \tag{H-9}$$

We note that the elevator lift effect $(\mathscr{M}_n i_B(\ell^2/V\ell_E))$ amplifies the effective lift slope in a similar manner as we had seen with incidence feedback whereas the pitching moment feedback $\mathscr{M}_n$ adds to the stiffness term ω_n^2. Again, as with pitch rate feedback, this term is multiplied by the lift slope and therefore becomes ineffective if $\partial C_L/\partial\alpha$ vanishes. The ω_n^2 term is proportional to the manoeuvre margin which reads

$$H_m = -\frac{m_w}{\partial C_L/\partial\alpha}\left(1 - \frac{m_\eta}{\mu}\frac{V^2}{\ell_E g}K_n\right) - \frac{m_q}{\mu} - \frac{m_\eta}{\mu}\frac{V^2}{\ell g}K_n \quad . \tag{H-10}$$

Ignoring the elevator lift term we can write approximately

$$\begin{aligned} H_m &= H_{ma} + \Delta H_{mn} \\ &= \left(-\frac{m_w}{\partial C_L/\partial\alpha} - \frac{m_q}{\mu}\right) + \left(-\frac{m_\eta}{\mu}\frac{V^2}{\ell g}K_n\right) \quad . \end{aligned} \tag{H-11}$$

Therefore

$$\Delta H_m \doteq -\frac{m_\eta}{\mu}\frac{V^2}{\ell g}K_n = -m_\eta\frac{(\rho/2)V^2}{W/S}K_n \cdot \qquad (H\text{-}12)$$

Appendix J

RESPONSE TO u-GUSTS OF AIRCRAFT STABILISED BY NORMAL ACCELERATION FEEDBACK TO THE ELEVATOR

The pitching moment disturbance of an aircraft with normal acceleration feedback to the elevator $d\eta/dn$ to a u gust u_g is

$$\Delta C_m = \frac{\partial C_m}{\partial \eta} \frac{d\eta}{dn} \frac{dn}{du} u_g \quad . \tag{J-1}$$

The equivalent contribution to the manoeuvre margin of the feedback term is

$$H_{m_n} = -\Delta \frac{\partial C_m}{\partial C_L} = -\frac{\partial C_m}{\partial \eta} \frac{d\eta}{dn} \frac{dn}{\partial C_L} \quad .$$

Therefore we can write equation (J-1) as

$$\Delta C_m = -H_{m_n} \frac{dC_L}{\partial u} u_g$$

and since

$$\frac{dC_L}{du} = 2\frac{C_{L0}}{V_0}$$

normal acceleration feedback generates an effective

$$m_u = \frac{\partial C_m}{\partial \frac{u}{V_0}} = H_{m_n} 2C_{L0} \quad . \tag{J-2}$$

The corresponding pitch sensitivity of the naturally stable aircraft and of the n-stabilised aircraft to vertical gusts w_g is

$$\Delta C_m = \frac{\partial C_m}{\partial \alpha} \frac{w_g}{V_0} = \frac{\partial C_m}{\partial C_L} \frac{\partial C_L}{\partial \alpha} \frac{w_g}{V_0}$$

or

$$\frac{\partial C_m}{\partial \left(\frac{w_g}{V_0}\right)} = -H_m \frac{\partial C_L}{\partial \alpha} \quad . \tag{J-3}$$

Comparing equations (J-2) and (J-3) we can express the pitch sensitivity of the n-stabilised aircraft to u-gusts to the familiar one of the aircraft to w-gusts

$$\frac{(\partial C_m/\partial u_g)\ \text{u-stabilised}}{(\partial C_m/\partial w_g)\ \text{naturally stable}} = 2\,\frac{C_{L0}}{\partial C_L/\partial\alpha}\,\frac{H_{m_n}}{H_m} \tag{J-4}$$

We note that this ratio will become unity in the approach condition for aircraft with efficient high lift devices if $H_{m_n} \simeq H_m$, *ie* if all the effective manoeuvre margin is supplied by the n-feedback, and even greater for an aerodynamically unstable aircraft where $H_{m_n} > H_m$. This means that at low speeds the aircraft deriving a large measure of manoeuvre stability from normal acceleration feed-back will be as sensitive in pitch to fore and aft gusts as to vertical gusts. This could become embarrassing since at low altitudes in the approach u_g gusts predominate over w_g gusts and the pilot may be faced with a novel handling problem.

If we represent the effective incidence stability of the aircraft by $\mathscr{M}_\alpha$ and the pitch sensitivity contribution of the n-feedback as an equivalent derivative we can write by only retaining major terms

$$\begin{bmatrix} \mathscr{L}_\alpha + s & -1 \\ \mathscr{M}_\alpha & \mathscr{M}_q - s \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} = \left(\frac{u_g}{V_0}\right) \begin{bmatrix} -\mathscr{L}_u \\ -\mathscr{M}_u \end{bmatrix} \tag{J-5}$$

where we can express $\mathscr{M}_u$ as in equation (J-4) as

$$\mathscr{M}_u = \mathscr{M}_\alpha \frac{H_{m_n}}{H_m}\,\frac{C_{L0}}{\partial C_L/\partial\alpha} = \mathscr{M}_\alpha F \ . \tag{J-6}$$

From equation (J-5) we get the frequency response functions

$$\frac{\alpha}{u_g/V_0}(\omega) = \frac{F - \dfrac{\mathscr{L}_u \mathscr{M}_q}{\omega_n^2} + i\,\dfrac{\mathscr{L}_u}{\omega_n}\,\dfrac{\omega}{\omega_n}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 - i2\zeta\left(\dfrac{\omega}{\omega_n}\right)} \tag{J-7}$$

and

$$\frac{q}{u_g/V_0}(\omega) = \frac{-\mathscr{L}_\alpha F + \mathscr{L}_u - i\frac{\omega}{\omega_n}F\omega_n}{1 - \left(\frac{\omega}{\omega_n}\right)^2 - i2\zeta\left(\frac{\omega}{\omega n}\right)} \quad . \tag{J-8}$$

$$\frac{\alpha}{u/V_0}(\omega)$$

$$= \frac{\left(\frac{\mathscr{L}_u \mathscr{M}_q}{\omega_n^2} - F\right)\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) - \frac{\mathscr{L}_u}{\omega_n}\left(\frac{\omega}{\omega_n}\right)^2 2\zeta - i\frac{\omega}{\omega_n}\left\{\frac{\mathscr{L}_u}{\omega_n}\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + 2\zeta\left(\frac{\mathscr{L}_u \mathscr{M}_q}{\omega_n^2} - F\right)\right\}}{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}$$

...... (J-9)

where $\mathscr{L}_u = \frac{2g}{V_0}$

$$\mathscr{L}_\alpha = \frac{\partial C_L/\partial\alpha}{C_{L0}}\frac{a}{V_0}$$

$$\mathscr{M}_q = -2\zeta_q\omega_n$$

$$F = \frac{H_{m_n}}{H_m}\frac{C_{L0}}{\partial C_L/\partial\alpha}$$

and ζ_q is the portion of damping ratio ζ provided by m_q .

Normal acceleration response we get from

$$\Delta n = \Delta\alpha\frac{\rho}{2}V_0^2\frac{S}{W}\frac{\partial C_L}{\partial\alpha} + \frac{\rho}{s}\frac{S}{W}2V_0^2\frac{u_g}{V_0}C_{L0} \tag{J-10}$$

so

$$\frac{\Delta n}{u_g/V_0} = \frac{1}{C_{L0}}\left(\frac{\Delta\alpha}{u_g/V_0}\frac{\partial C_L}{\partial\alpha} + 2C_{L0}\right)$$

or

$$\frac{\Delta n}{u_g/V_0} = \left(\frac{\Delta\alpha}{u_g/V_0}\right)\frac{\partial C_L/\partial\alpha}{C_{L0}} + 2 \quad .$$

Pitch rate response is

$$\frac{q}{u_g/V_0}$$

$$= \frac{(\mathcal{L}_u - \mathcal{L}_\alpha F)\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + 2\zeta F\omega_n\left(\frac{\omega}{\omega_n}\right)^2 + i\,\frac{\omega}{\omega_n}\left\{(\mathcal{L}_u - \mathcal{L}_\alpha F)2\zeta - F\omega_n\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)\right\}}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2} \; .$$

...... (J-11)

As an example equations (J-9) and (J-11) have been evaluated numerically for an aircraft defined by

$$V_0 = 200 \text{ ft/s}$$
$$C_{L0} = 2.0$$
$$\partial C_L/\partial\alpha = 5$$
$$\omega_n = 1$$
$$\zeta = 0.5$$
$$\zeta_q = 0.3 \; .$$

The results are shown in Fig 16 for the three conditions considered throughout this Report.

(a) A naturally stable aircraft without feedback augmentation.

(b) An aerodynamically neutrally stable airframe stabilised by n-feedback to the standard of stability of the datum aircraft.

(c) An unstable aircraft with a manoeuvre margin having a negative value equal to the positive value of that of the n-stabilised aircraft.

Appendix K

DIRECTIONAL STABILITY DURING GROUND ROLL OF AIRCRAFT WITH LATERAL ACCELERATION FEEDBACK TO THE RUDDER

In the absence of wind the motion of the aircraft over the runway surface is defined by the terms shown in Fig 20 where

y = lateral displacement from desired track
χ = azimuth angle of track
ψ = aircraft heading
$\sigma = \psi - \chi$ = tyre slip angle
$\beta = \chi - \psi = -\sigma$ = sideslip.

For stability analysis we consider only major airframe contributions

$N_\beta = \partial N/\partial\beta$ = aerodynamic weathercock stability
$N_\sigma = \partial N/\partial\sigma$ = track stability contribution of undercarriage
$Y_\beta = \partial Y/\partial\beta$ = aerodynamic sideforce derivative
$Y_\sigma = \partial Y/\partial\sigma$ = combined tyre sideforce derivative.

Lateral acceleration feedback to the rudder is represented by a term

$$N_{\ddot{y}} = \frac{\partial N}{\partial \ddot{y}} \qquad \text{(K-1)}$$

$\ddot{y}$ is related to track angle χ by $\ddot{y} = V\dot{\chi}$.

Hence the equations of motion

$$N_\beta\beta + N_\sigma\sigma + N_{\ddot{y}}V\ddot{\chi} - C\ddot{\psi} = 0$$
$$Y_\beta\beta + Y_\sigma\sigma - \dot{\chi}mV \quad . \qquad \text{(K-2)}$$

Dividing the yawing moment equation by C and the sideforce equation by mV this gives

$$\begin{bmatrix} -\mathcal{N}_\beta + \mathcal{N}_\sigma - s^2 & \mathcal{N}_\beta - \mathcal{N}_\sigma + s\frac{N_{\ddot{y}}}{m} \\ -\mathcal{Y}_\beta + \mathcal{Y}_\sigma & \mathcal{Y}_\beta - \mathcal{Y}_\sigma - s \end{bmatrix} \begin{bmatrix} \psi \\ \chi \end{bmatrix} = 0 \quad . \qquad \text{(K-3)}$$

The characteristic equation is

$$s^3 + s^2\{\mathcal{Y}_\sigma - \mathcal{Y}_\beta\} + s\left\{\mathcal{N}_\beta - \mathcal{N}_\sigma + \frac{N_{\ddot{y}}}{m}(\mathcal{Y}_\beta - \mathcal{Y}_\sigma)\right\} = 0 \quad . \qquad \text{(K-4)}$$

This cubic has a zero root representing lack of absolute track stability. The directional stability mode of principal interest, is defined by the remaining quadratic

$$s^2 + s\{\mathcal{Y}_\sigma - \mathcal{Y}_\beta\} + \left\{\mathcal{N}_\beta - \mathcal{N}_\sigma + \frac{N_{\ddot{y}}}{m}\left(\mathcal{Y}_\beta - \mathcal{Y}_\sigma\right)\right\} = 0 . \qquad \text{(K-5)}$$

By definition $\mathcal{Y}_\beta < 0$ and $\mathcal{Y}_\sigma > 0$ also $\mathcal{N}_\beta > 0$ and $\mathcal{N}_\sigma < 0$ for stabilising contributions and of course $N_{\ddot{y}}$ will be negative. Therefore, the two sideforce contributions are seen to provide damping of this second order mode and lateral acceleration feedback makes a positive contribution to the yaw stiffness. This contribution is in fact reinforced on the ground by the tyre sideforce effect $\mathcal{Y}_\sigma$ whereas in free flight only the aerodynamic sideforce factors this term.

It may be of interest to consider the situation in the presence of wind. If V_w is the headwind component and V groundspeed the kinematic relationship for β becomes

$$\beta = -\psi + \frac{\chi}{1 + \frac{V_w}{V}} \simeq -\psi + \chi\left(1 - \frac{V_w}{V}\right) . \qquad \text{(K-6)}$$

With this relationship introduced into (K-2) we get the stability cubic

$$s^3 + s^2\left\{\mathcal{Y}_\sigma + \mathcal{Y}_\beta\left(1 - \frac{V_w}{V}\right)\right\} + s\left\{\mathcal{N}_\beta - \mathcal{N}_\sigma + \mathcal{N}_{\ddot{y}}(\mathcal{Y}_\beta - \mathcal{Y}_\sigma)\right\} + \frac{V_w}{V}\left\{\mathcal{N}_\beta\mathcal{Y}_\sigma - \mathcal{N}_\sigma\mathcal{Y}_\beta\right\} = 0 .$$

...... (K-7)

There is now a finite absolute term $V_w/V\{\mathcal{N}_\beta\mathcal{Y}_\sigma - \mathcal{N}_\sigma\mathcal{Y}_\beta\}$ and thus a finite real root defining track stability. This root will be stable (for a positive headwind V_w) if

$$\mathcal{N}_\beta\mathcal{Y}_\sigma > \mathcal{N}_\sigma\mathcal{Y}_\beta .$$

It is interesting to note that aerodynamic stability $\mathcal{N}_\beta$ stabilises this mode whereas undercarriage generated stability $\mathcal{N}_\sigma$ destabilises it.

Table 1

PROPERTIES OF THE AIRCRAFT ASSUMED AS AN EXAMPLE IN THE GUST RESPONSE CALCULATIONS

$W/S = 96\ lb/ft^2$	$V = 800$ ft/s	H = sea level
$\mu = 250$	$\hat{t} = 3.133$	$\ell = 10$ ft
	$i_B = 2.0$	

Manoeuvre margin $H_m = \pm 0.1$ (when possible)

$\frac{\partial C_L}{\partial \alpha} = 5$	$C_L = 0.127$
$m_q = -7.52$	$C_D = 0.0235$
$m_{\dot{w}} = -1.253$	$\frac{\partial C_D}{\partial \alpha} = 0.08$
$m_w = -0.339$	
$\mathscr{L}_\alpha = 1.6$	$\mathscr{D}_\alpha = 0.025$
$\mathscr{L}_u = 0.0811$	$\mathscr{D}_u = 0.015$
$\mathscr{M}_\alpha = -4.33$	
$\mathscr{M}_q = -1.2$	
$\mathscr{M}_{\dot{\alpha}} = -0.2$	
$\omega_n = 2.5$ rad/s	
$\zeta = 0.60$	

Boxed values are for the basic aircraft assumed to be naturally stable and to have satisfactory short period dynamics

Table 2

CASES CONSIDERED IN GUST RESPONSE CALCULATIONS

Case	Aerodynamic		Feedbacks		Mode characteristics			Remarks
					Short period oscillation		Real root	
	$\mathscr{L}_\alpha$	$\mathscr{M}_\alpha$	$\mathscr{M}_q^*$	$\mathscr{M}_\theta$	ω_n rad/s	ζ	λ_1 s^{-1}	
1	1.6	-4.33	-1.2	-	2.5	0.60	-	Datum case
2	1.6	0	-3.91	-	2.5	1.101	-	q feedback
3	1.6	+4.33	-6.6125	-	2.5	1.6425	-	
4	1.6	0	-3.0	-7.0	2.54	0.603	-1.73	$\int q$ feedback
5	1.6	+4.33	-4.7	-13.5	2.49	0.603	-3.50	
6	1.6	+8.66	-6.7	-21.0	2.50	0.623	-5.38	
7	1.0	-4.45	-1.8	-	2.50	0.60	-	Naturally stable reduced $\partial C_L/\partial\alpha$
8	1.0	+4.33	-3.9	-13.1	2.535	0.604	-2.04	unstable

* $\mathscr{M}_q$ in this column contains basic aerodynamic contribution as well as system generated pitch damping

LIST OF SYMBOLS

A	inertia in roll
A_i	coefficient of stability polynomial
B	inertia in pitch
B_i	coefficient of stability polynomial
C	inertia in yaw
C_L	lift coefficient
C_ℓ	rolling moment coefficient
C_D	drag coefficient
C_m	pitching moment coefficient
C_n	yawing moment coefficient
C_y	sideforce coefficient
D	drag
D	damping factor in equation (G-7)
$D_u = \frac{\partial D}{\partial \hat{u}}$, $D_\alpha = \frac{\partial D}{\partial \alpha}$	dimensional drag derivatives
$\mathscr{D}_u = \frac{D_u}{mV_0}$, $\mathscr{D}_\alpha = \frac{D_\alpha}{mV_0}$	concise drag derivatives
E	directional stability factor in equation (G-7)
F	factor in equation (G-8)
g	gravitational acceleration
H_m	manoeuvre margin
H_m^*	apparent manoeuvre margin of aircraft with integral pitch rate feedback
H_n	static margin
$K_n = \frac{\partial \eta}{\partial \Delta n}$, $K_q = \frac{\partial \eta}{\partial q}$, $K_V = \frac{\partial \eta}{\partial V}$, $K_\alpha = \frac{\partial \eta}{\partial \alpha}$, $K_\theta = \frac{\partial \eta}{\partial \int (q - q_D)}$	elevator control feedback gains

LIST OF SYMBOLS (continued)

K_D	command feed-forward gain
$K_y = \frac{\partial\zeta}{\partial n_y}$, $K_\beta = \frac{\partial\zeta}{\partial\beta}$	rudder control feedback gains
ℓ	reference length for non-dimensionalising moment coefficients
ℓ_E	elevator moment arm } positive for rear surfaces
ℓ_R	rudder moment arm } positive for rear surfaces
L	lift
L	rolling moment
$L_q = \frac{\partial L}{\partial q}$, $L_\alpha = \frac{\partial L}{\partial \alpha}$, $L_u = \frac{\partial L}{\partial \hat{u}}$, $L_\theta = \frac{\partial L}{\partial \int (q - q_D)}$, $L_\eta = \frac{\partial L}{\partial \eta}$	dimensional lift derivatives
$\ell_p = \frac{\partial C_\ell}{\partial (p\ell/V_0)}$, $\ell_v = \frac{\partial C_\ell}{\partial \beta}$, $\ell_\zeta = \frac{\partial C_\ell}{\partial \zeta}$	non-dimensional rolling moment derivatives
$\mathcal{L}_\alpha = \frac{L_\alpha}{mV_0}$, $\mathcal{L}_u = \frac{L_u}{mV_0}$, $\mathcal{L}_\theta = \frac{L_\theta}{mV_0}$	concise lift derivatives
m	aircraft mass
M	pitching moment

LIST OF SYMBOLS (continued)

$M_q = \frac{\partial M}{\partial q}$, $M_\alpha = \frac{\partial M}{\partial \alpha}$, $M_u = \frac{\partial M}{\partial \hat{u}}$, $M_\theta = \frac{\partial M}{\partial \int (q - q_D)}$	dimensional pitching moment derivatives
$m_w = \frac{\partial C_m}{\partial \alpha}$, $m_q = \frac{\partial C_m}{\partial (q\ell/V_0)}$, $m_u = \frac{\partial C_m}{\partial \hat{u}}$, $m_\eta = \frac{\partial C_m}{\partial \eta}$, $m_\theta = \frac{\partial C_m}{\partial \int (q - q_D)}$	non-dimensional pitching moment derivatives
$\mathscr{M}_u = \frac{M_u}{B}$, $\mathscr{M}_q = \frac{M_q}{B}$, $\mathscr{M}_\alpha = \frac{M_\alpha}{B}$, $\mathscr{M}_\theta = \frac{M_\theta}{B}$	concise pitching moment derivatives
N	yawing moment
n	normal acceleration (normal to flight path)
n_y	lateral acceleration
$N_r = \frac{\partial N}{\partial r}$, $N_\beta = \frac{\partial N}{\partial \beta}$, $N_\zeta = \frac{\partial N}{\partial \zeta}$	dimensional yawing moment derivatives

LIST OF SYMBOLS (continued)

$n_v = \dfrac{\partial C_n}{\partial \beta}$, $n_r = \dfrac{\partial C_n}{\partial (r\ell/V_0)}$, $n_\zeta = \dfrac{\partial C_n}{\partial \zeta}$	non-dimensional yawing moment derivatives
$\mathcal{N}_r = \dfrac{N_r}{C}$, $\mathcal{N}_\beta = \dfrac{N_\beta}{C}$	concise yawing moment derivatives
p	rate of roll
p_0	steady rate of roll
q	rate of pitch
q_D	demanded pitch rate
r	rate of yaw
$s = \dfrac{d}{dt}$	operator
$s = \dfrac{d}{p_0 dt}$	operator used in analysis of rapid roll manoeuvres (Appendix G)
S	wing area
T	engine thrust
t	time
$\hat{t} = \dfrac{m}{\rho S V_0}$	aerodynamic unit of time
u	increment in airspeed
$\hat{u} = \dfrac{u}{V_0}$	non-dimensionalised airspeed increment
u_g	head on gust velocity
$\hat{u}_g = \dfrac{u_g}{V_0}$	non-dimensionalised value of u_g
V	airspeed
V_0	datum airspeed
V_E	equivalent airspeed
V_i	indicated airspeed
W	aircraft weight
w_g	vertical gust velocity (positive for up-gust)

LIST OF SYMBOLS (continued)

$y_v = \frac{\partial C_y}{\partial \beta}$, $y_\zeta = \frac{\partial C_y}{\partial \zeta}$	non-dimensional sideforce derivatives
α	angle of incidence
α'	aircraft incidence with respect to an inertial reference frame
$\alpha_g = \frac{w_g}{V_0}$	gust induced incidence
α_K	incidence factor in rapid roll analysis (equation (G-9))
α_0	incidence of the principal inertia axis in trimmed non-rolling flight
β	angle of sideslip
γ	flight path climb angle
ζ	rudder angle
ζ	damping ratio
ζ_α	contribution from lift slope derivative to damping of longitudinal SPO
ζ_q	contribution from pitch damping to damping of longitudinal SPO
ζ_θ	longitudinal damping ratio in rapid roll analysis (Appendix G)
ζ_ψ	lateral damping ratio in rapid roll analysis (Appendix G)
θ	pitch attitude
$\theta\int(q - q_D)dt$	shorthand in integral pitch rate augmentation analysis
κ	see Fig 22
λ_1	root of heave mode
λ_2	root of airspeed stability mode
$\mu = \frac{m}{(\rho/2)s\ell}$	relative density parameter
ρ	air density
ρ_0	air density at sea level
$\sigma = \frac{\rho}{\rho_0}$	relative air density
σ_i	rms of variable i
σ_α	rms incidence
$\sigma_{\alpha g}$	rms gust induced incidence

LIST OF SYMBOLS (concluded)

σ_n	rms normal acceleration increment
σ	tyre slip angle
ϕ	angle of bank
$\phi(\omega)$	power spectral density distribution
χ	flight path azimuth angle
ψ	heading angle
ω	angular frequency
ω_n	undamped frequency of the longitudinal short period oscillation
$\omega_\theta = \omega_n$	as used in rapid roll analysis (Appendix G)
ω_ψ	undamped frequency of the lateral oscillation in rapid roll analysis
Ω	spatial frequency

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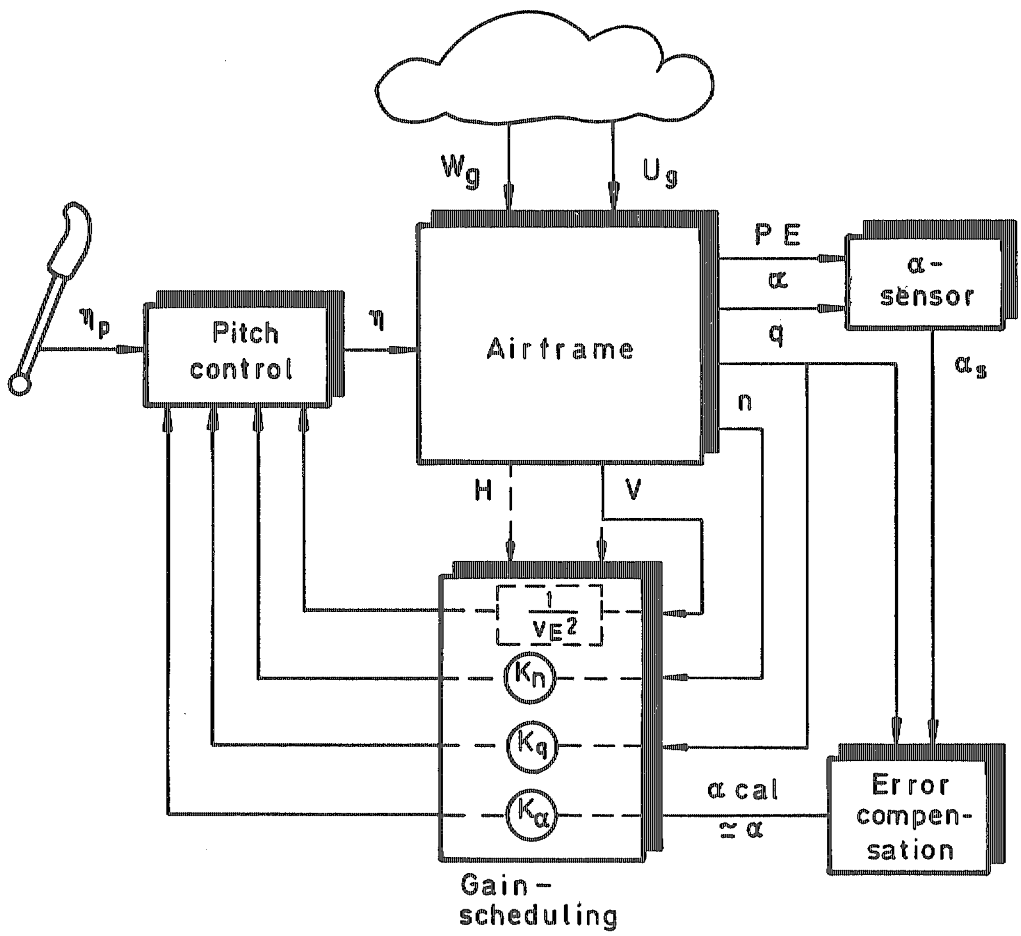


Fig 1 Feedbacks considered for augmenting longitudinal stability

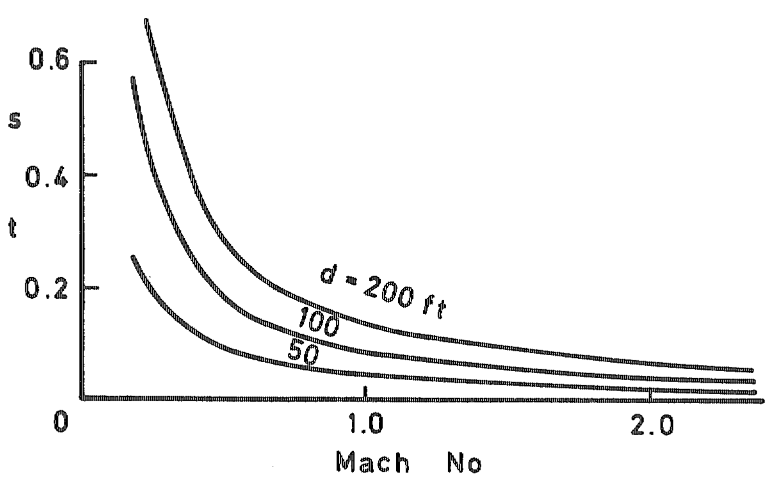


Fig 2 Time Δt for air to pass distance d along aircraft

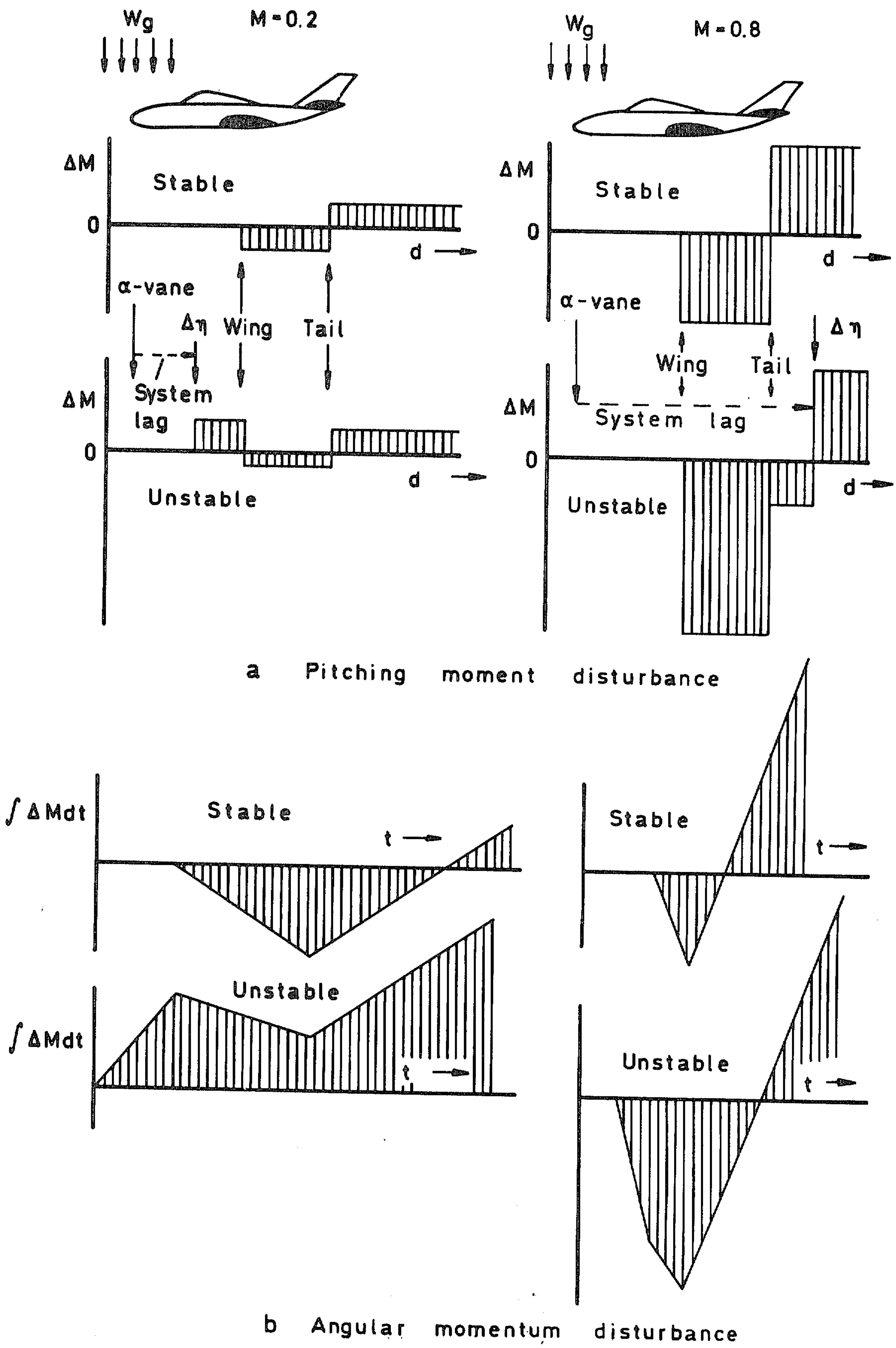


Fig 3a&b Pitch disturbance during penetration of step-down gust of naturally stable and α-stabilised aircraft

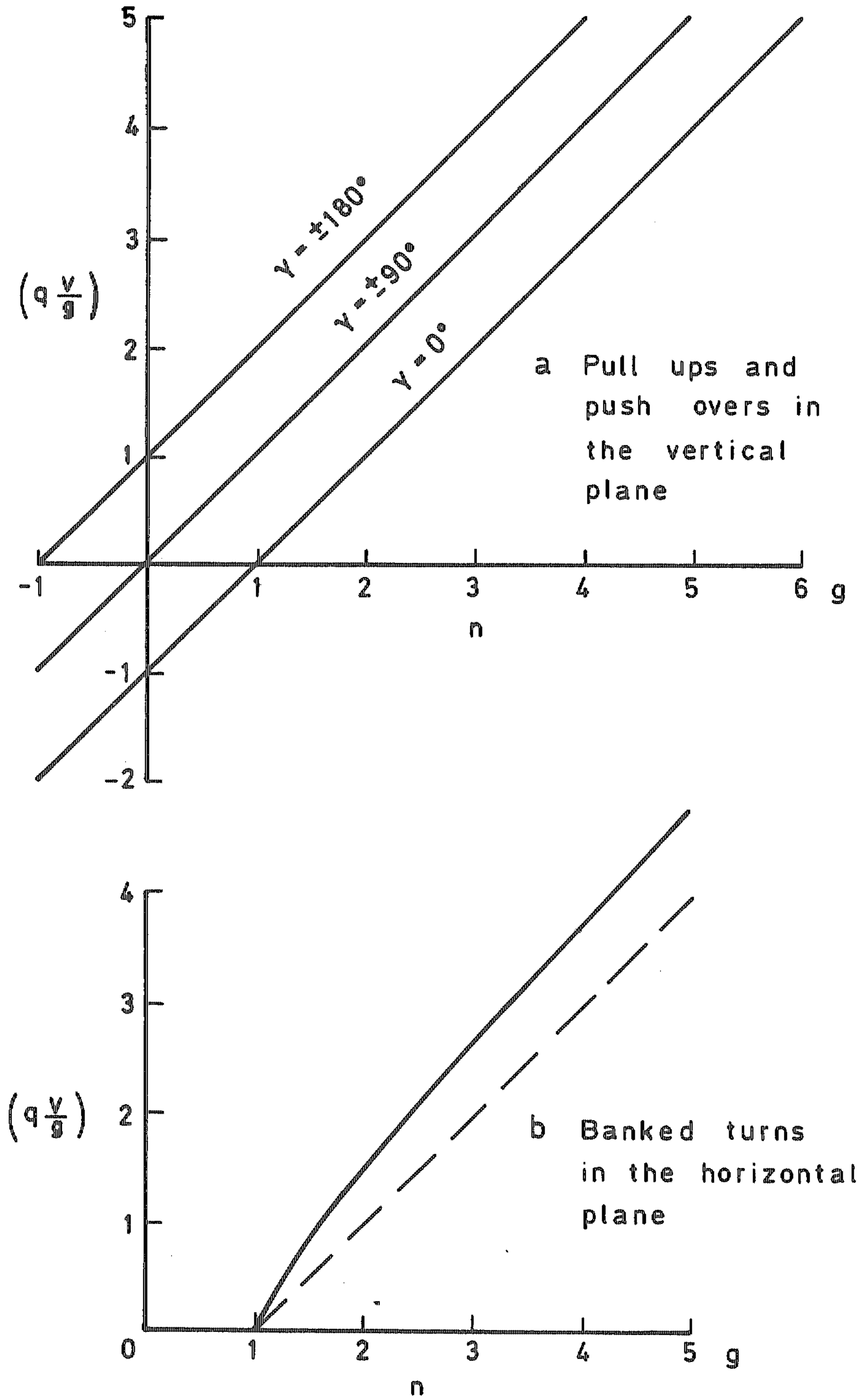


Fig 4 Relationship between pitch rate q and normal acceleration in steady manoeuvres

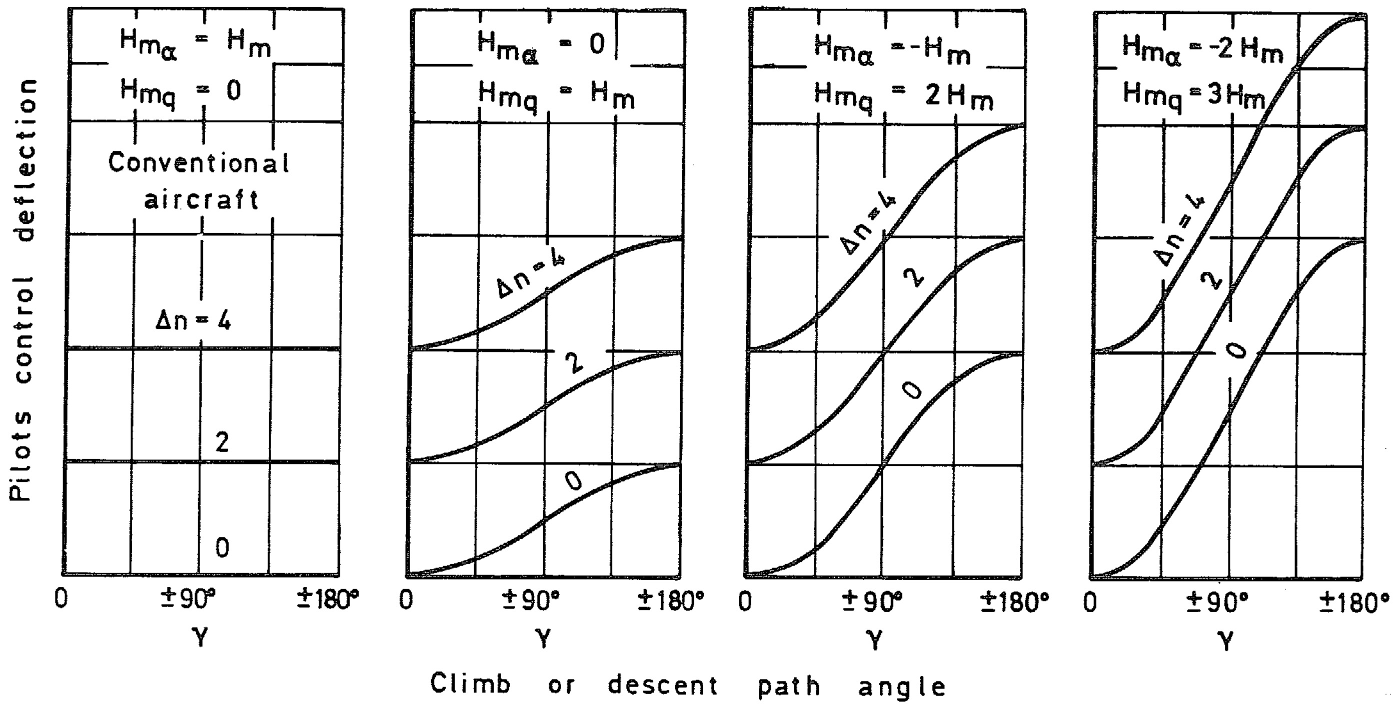


Fig 5 Variation of pilot's pitch control required to pull a given normal acceleration as a function of flight path angle γ of aircraft with various degrees of q-stabilisation

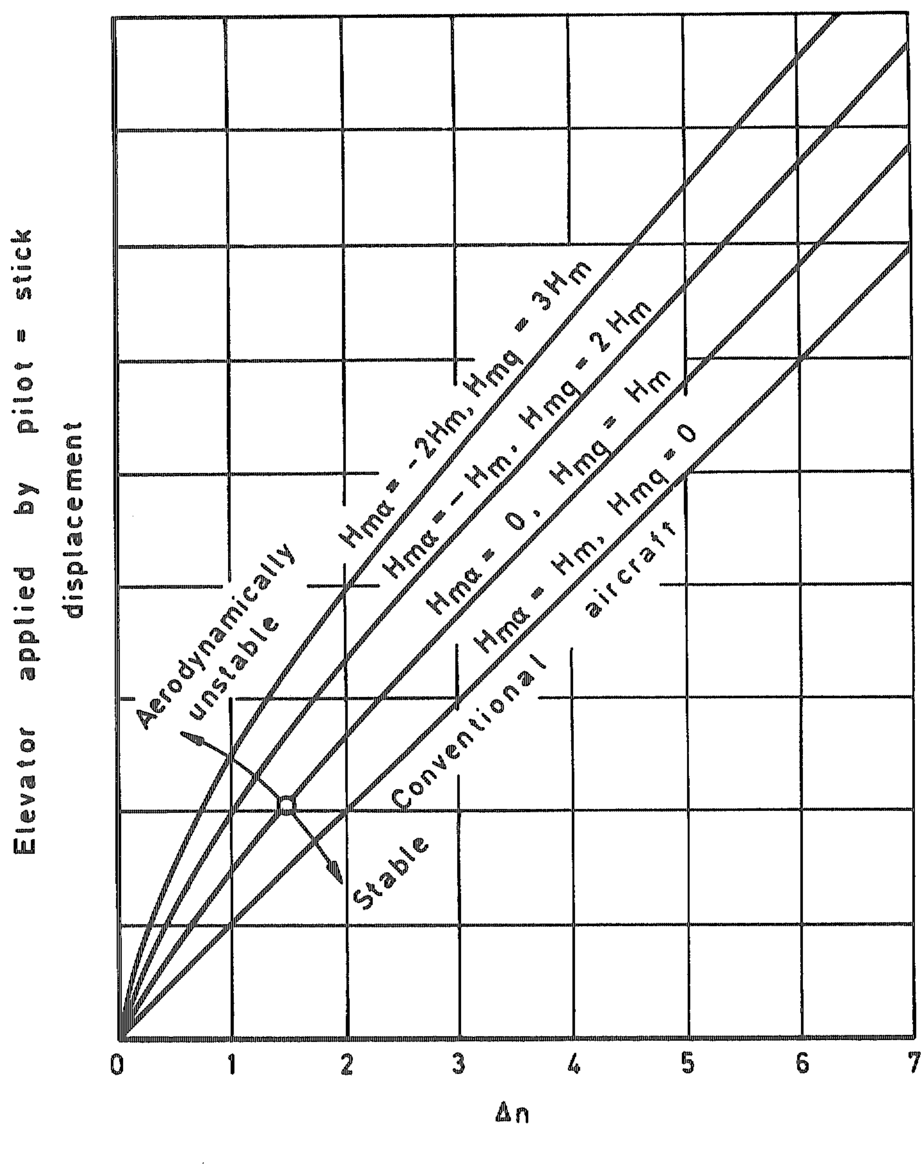


Fig 6 The relationship between pilot's stick displacement and normal acceleration in coordinated steady banked turns in the horizontal plane

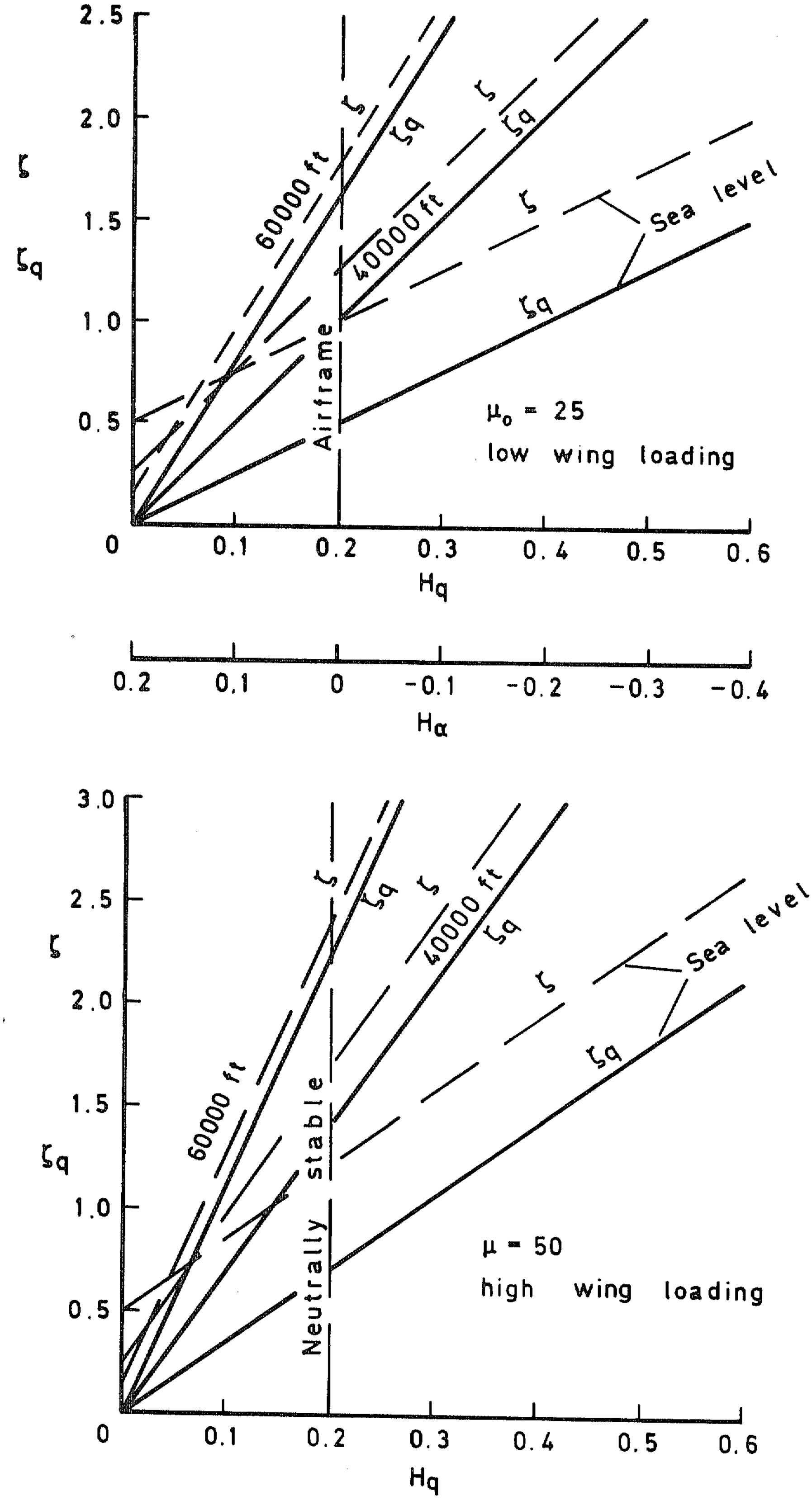


Fig 7 Effect on short period damping ratio ζ of pitch rate feedback for incidence stability augmentation

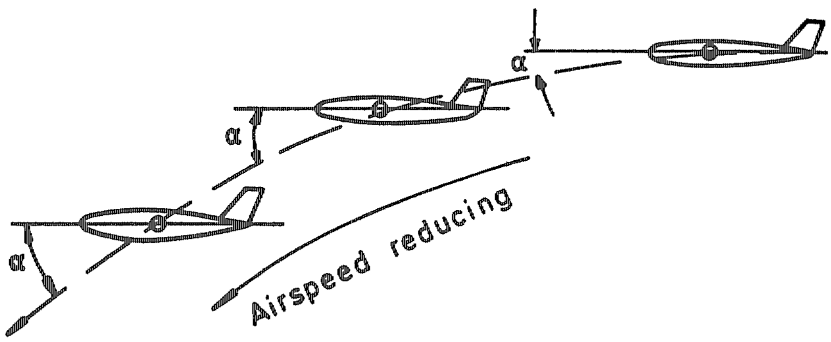


Fig 8a Approach to the stall in flight at constant pitch attitude. This divergence will not be resisted by pitch rate feedback

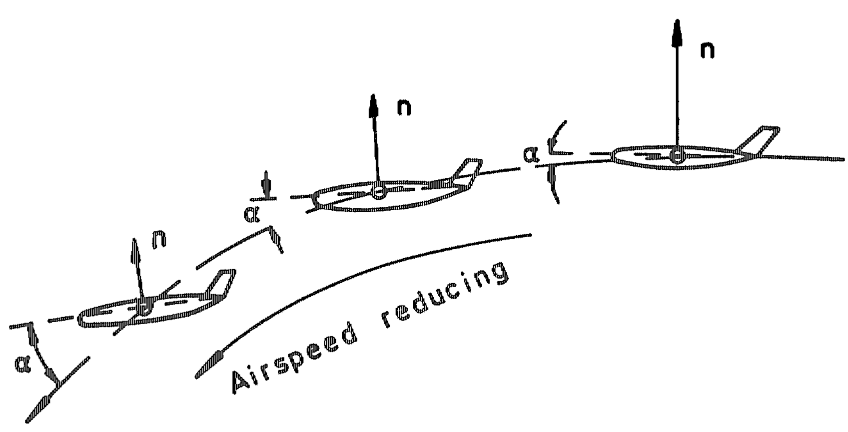


Fig 8b Approach to stall with reducing normal acceleration. This divergence will be accelerated by normal acceleration feedback

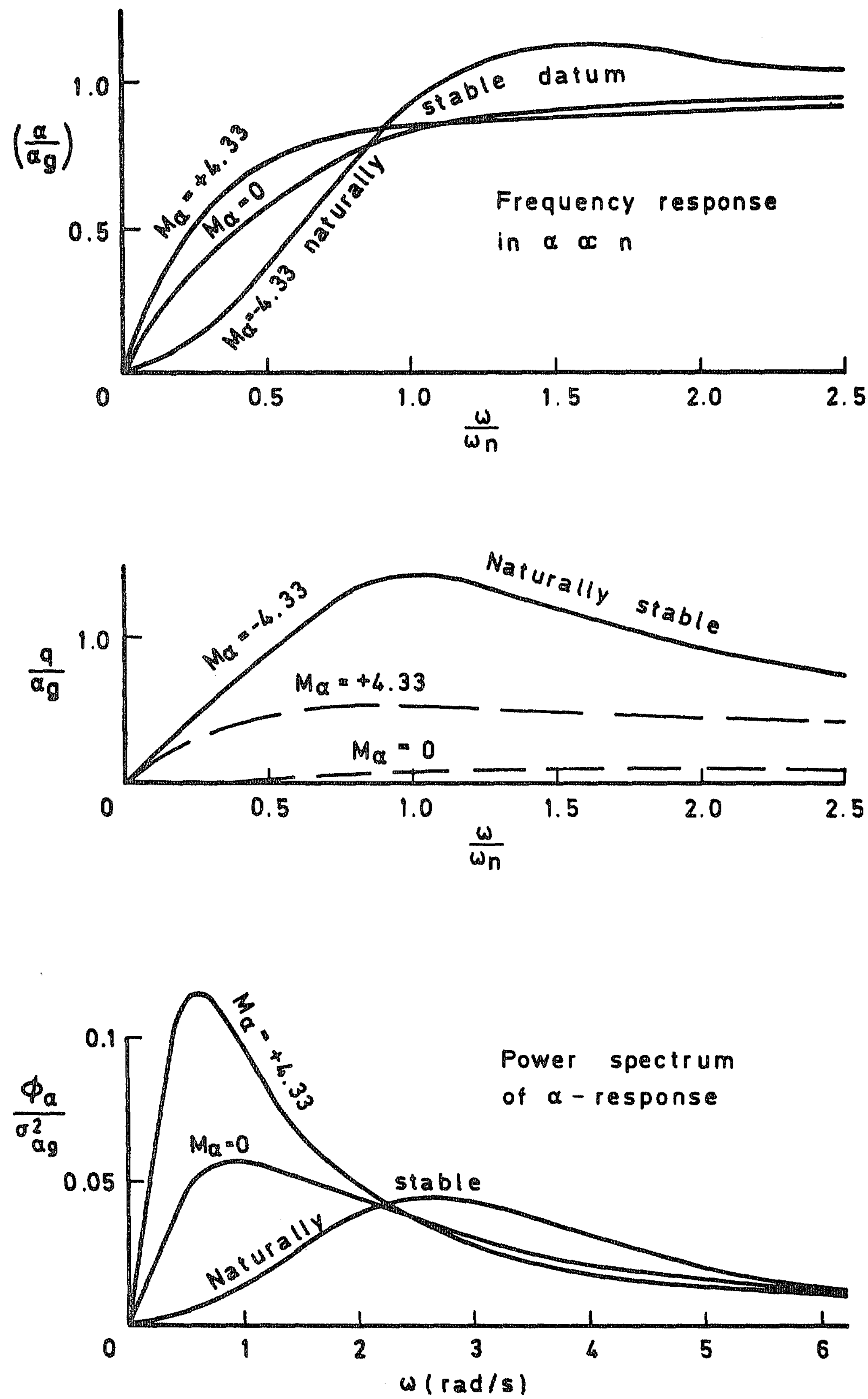


Fig 9 Response to vertical gusts α_g of aircraft stabilised by q-feedback to the elevator so as to maintain manoeuvre margin of datum (naturally stable) design

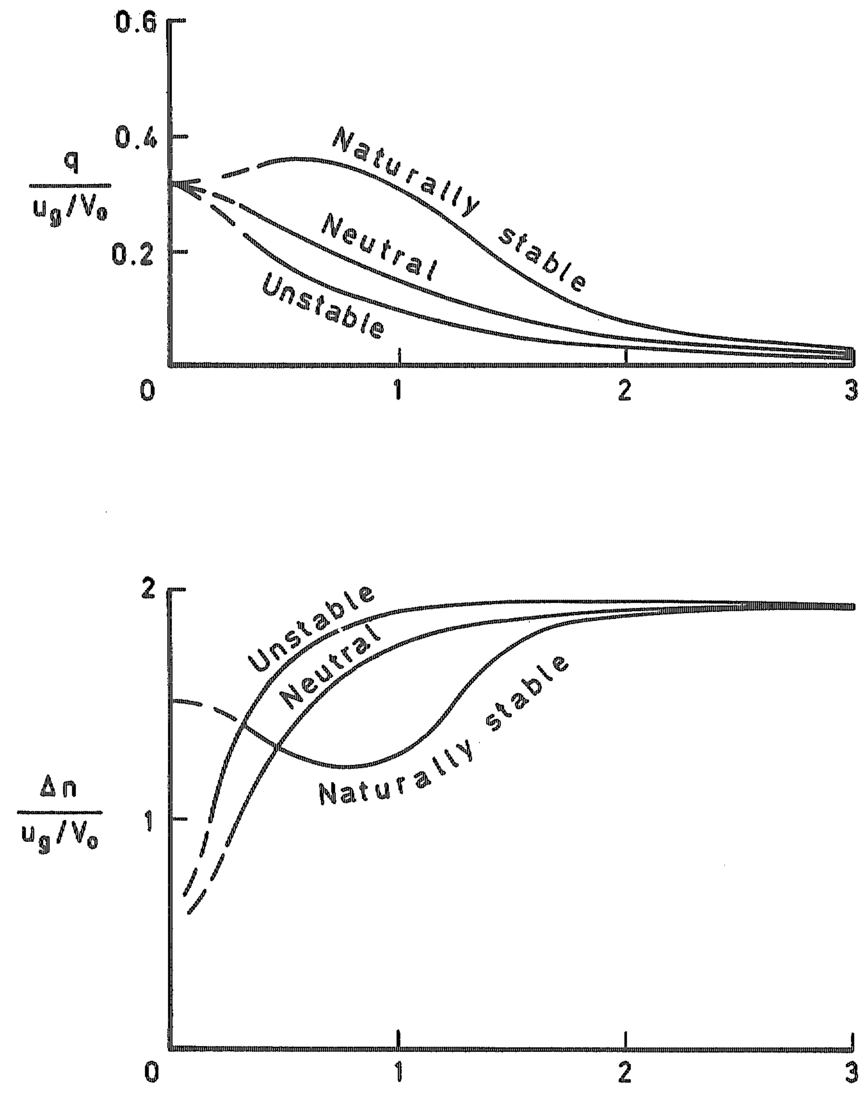


Fig 10 Frequency response in q and Δn to fore and aft gusts u_g of aircraft stabilised by pitch rate feedback

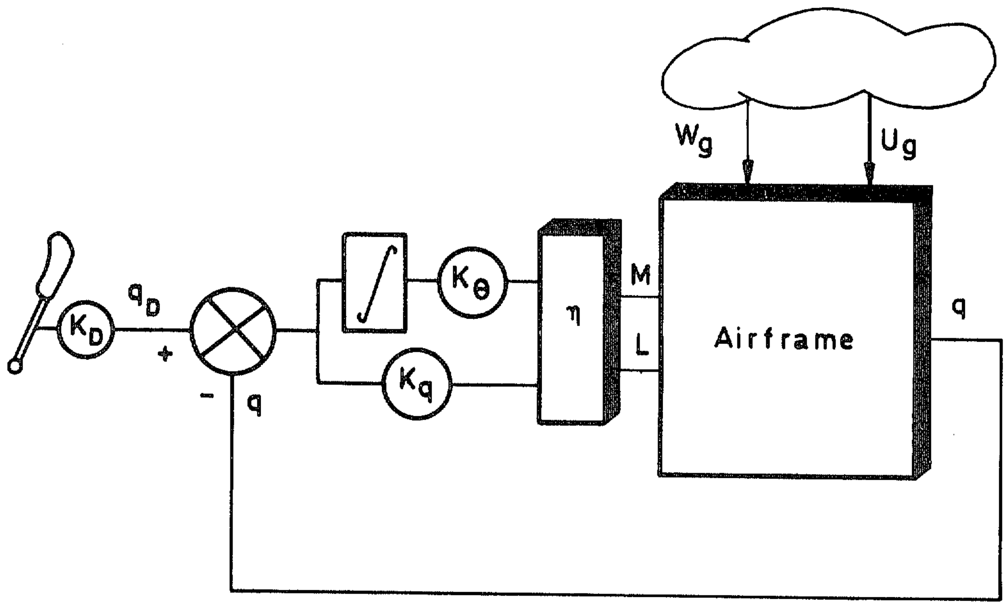


Fig 11 Block diagram of the control system considered for integral pitch rate feedback

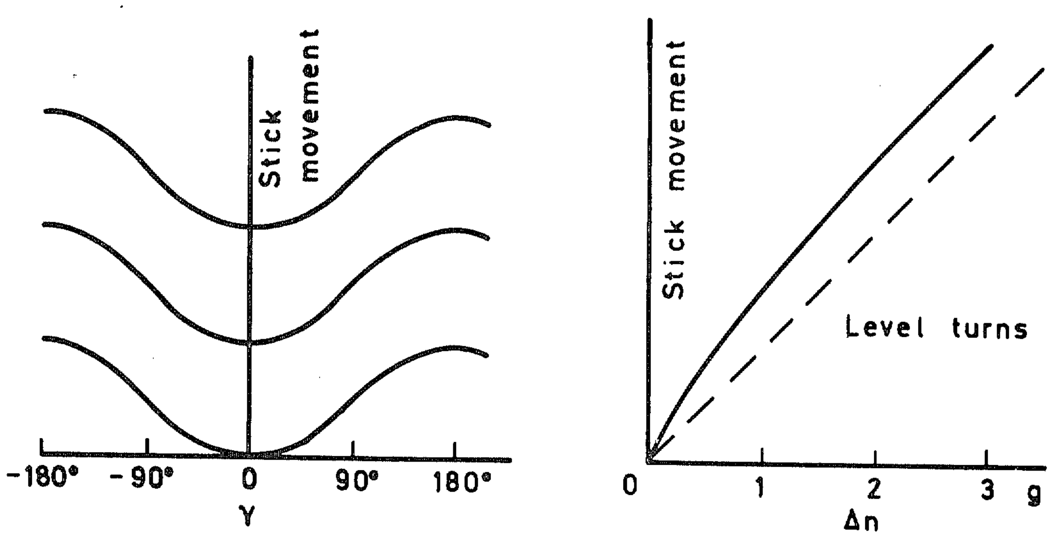
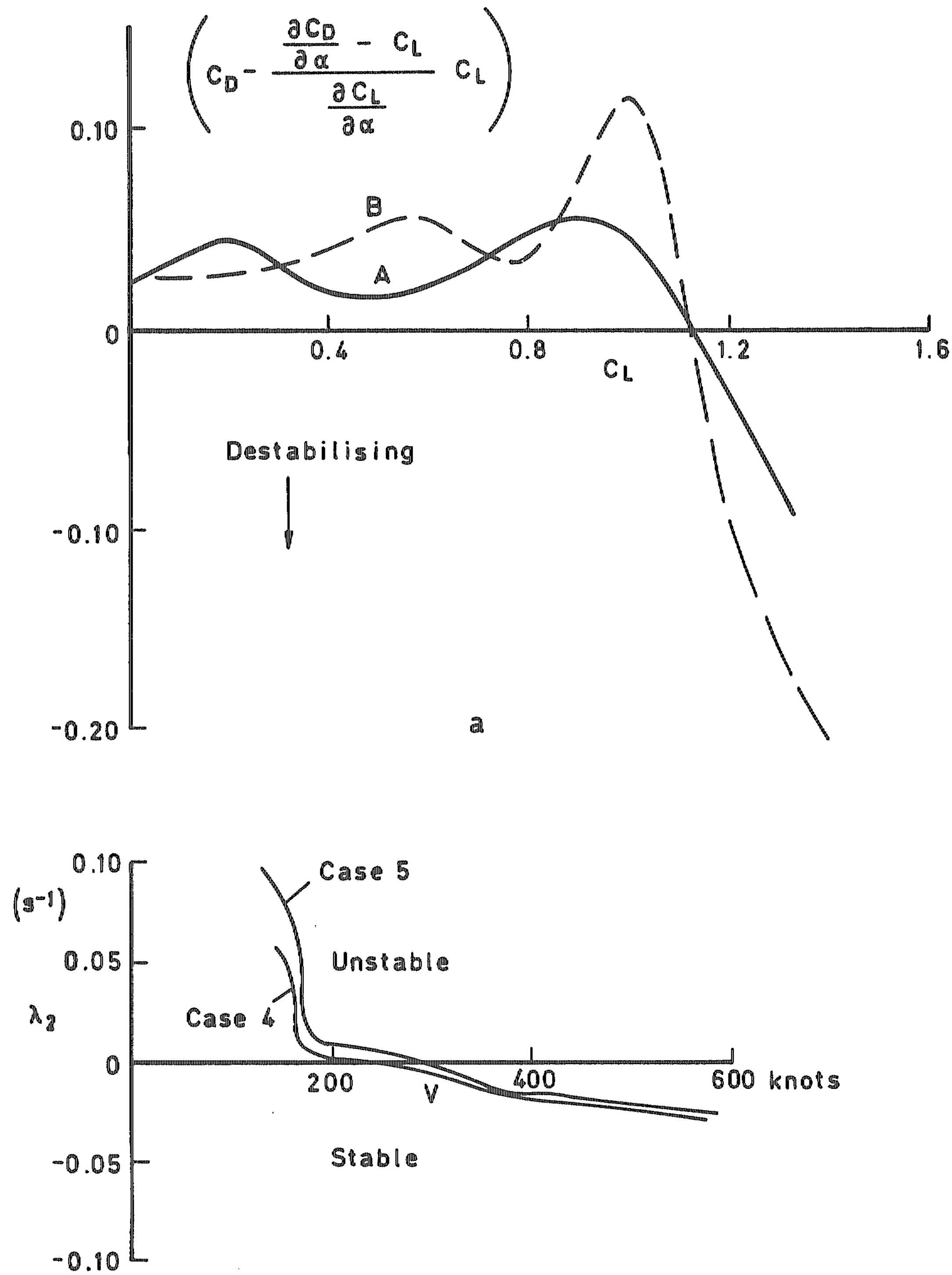


Fig 12 Stick movement (or force) in steady manoeuvring of aircraft augmented by $\int q$ feedback



b Drag characteristics of A above together with moment chacteristics of cases 4 and 5 of Table II

Fig 13 Static stability of the unstable aircraft augmented by $\int q$ feedback

Fig 14

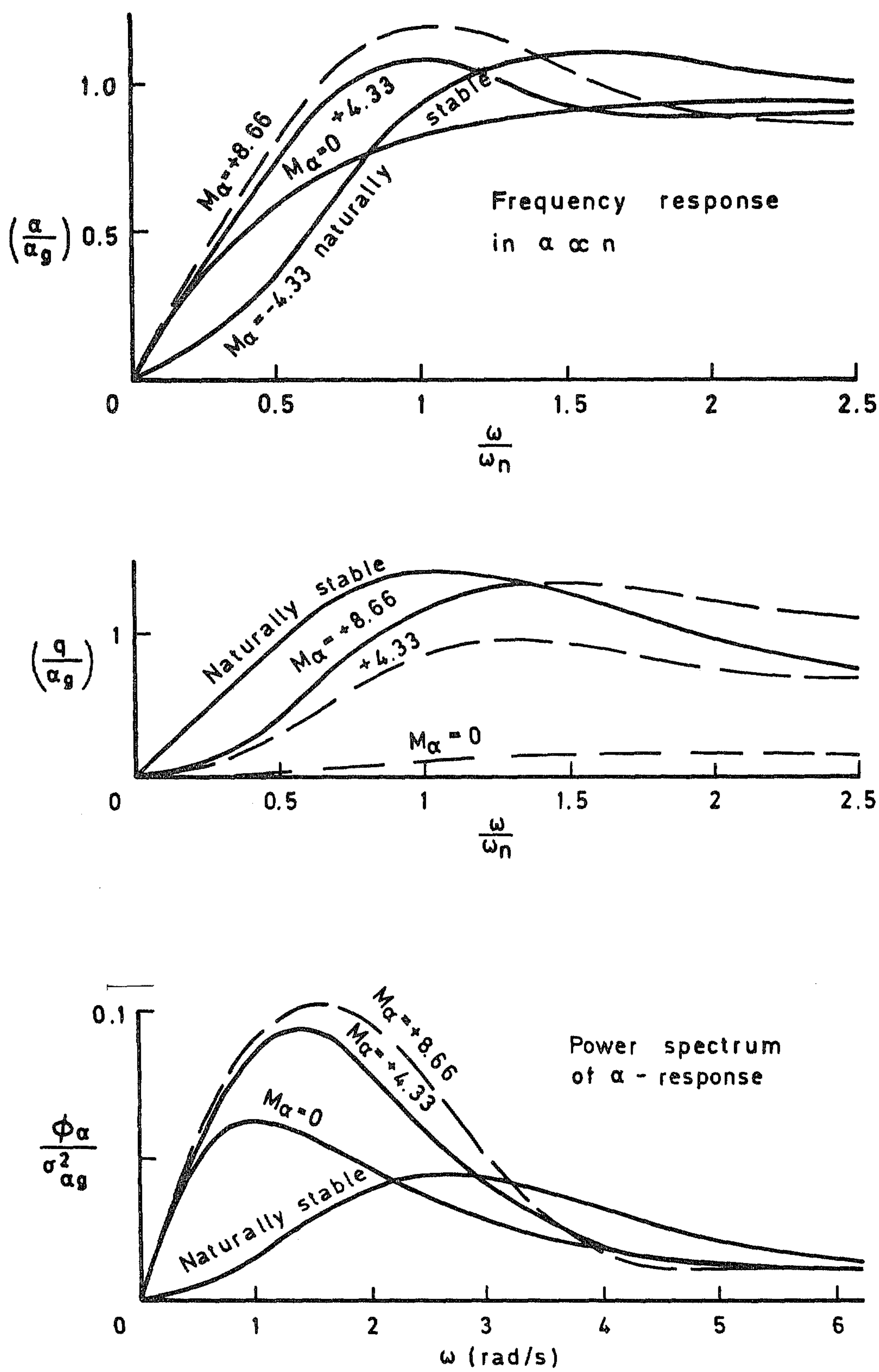


Fig 14 Response to vertical gusts α_g of aircraft stabilised by $\int q + q$ feedback to the elevator so as to maintain short period oscillation characteristics of datum (naturally stable) design

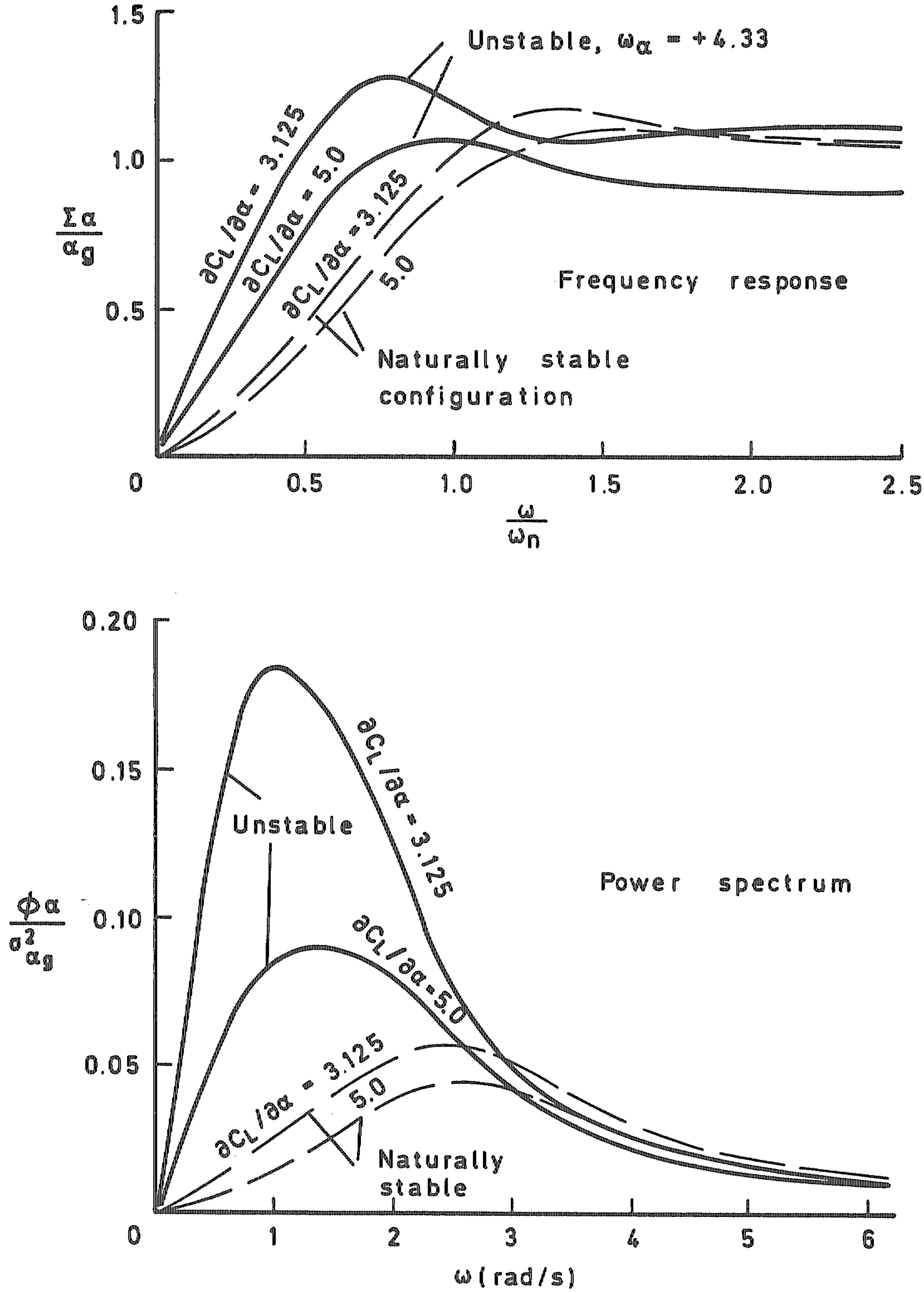


Fig 15 The effect of differences in lift slope $\partial C_L/\partial\alpha$ on response to vertical gusts α_g of naturally stable aircraft and design stabilised by integral pitch rate feedback

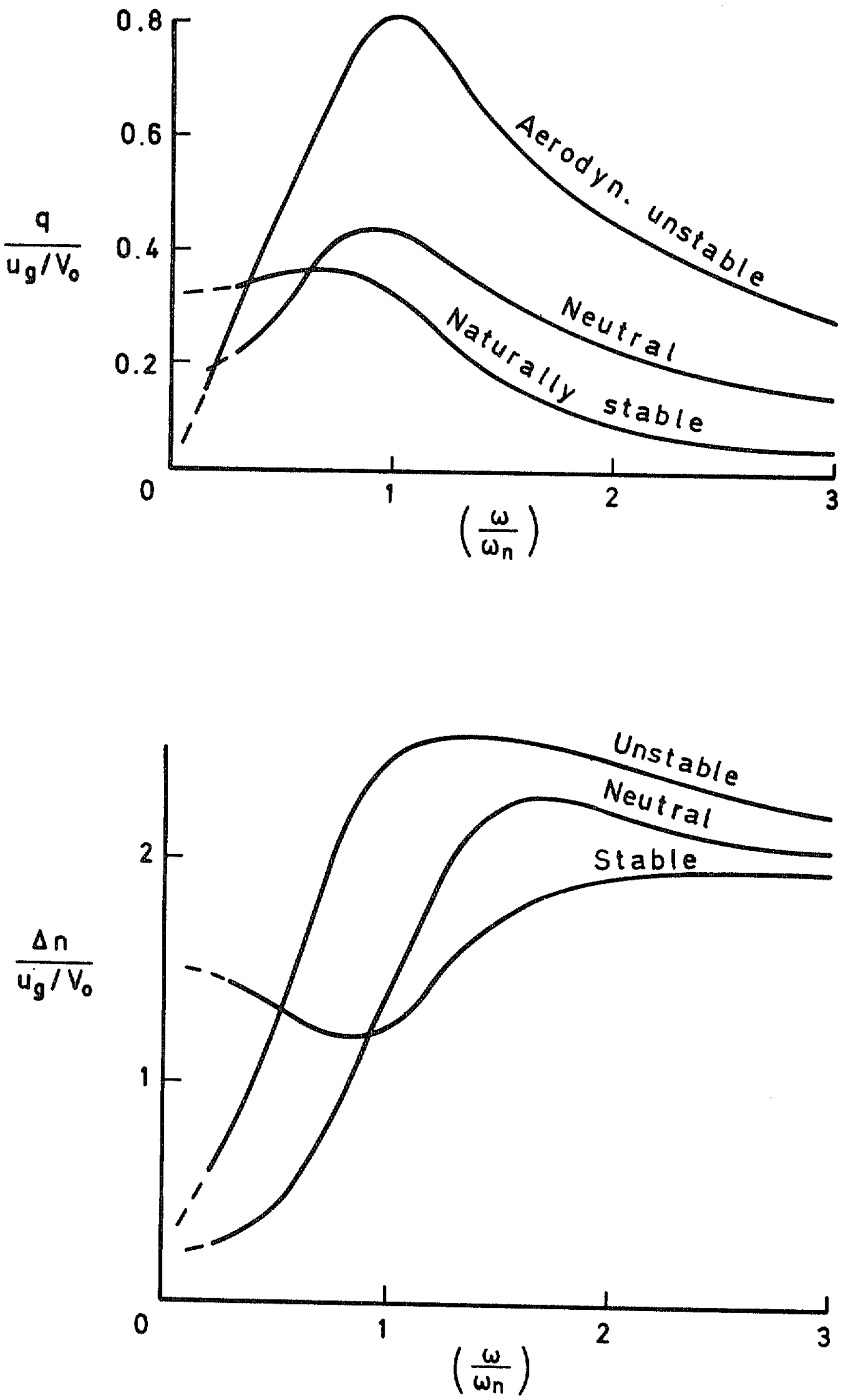
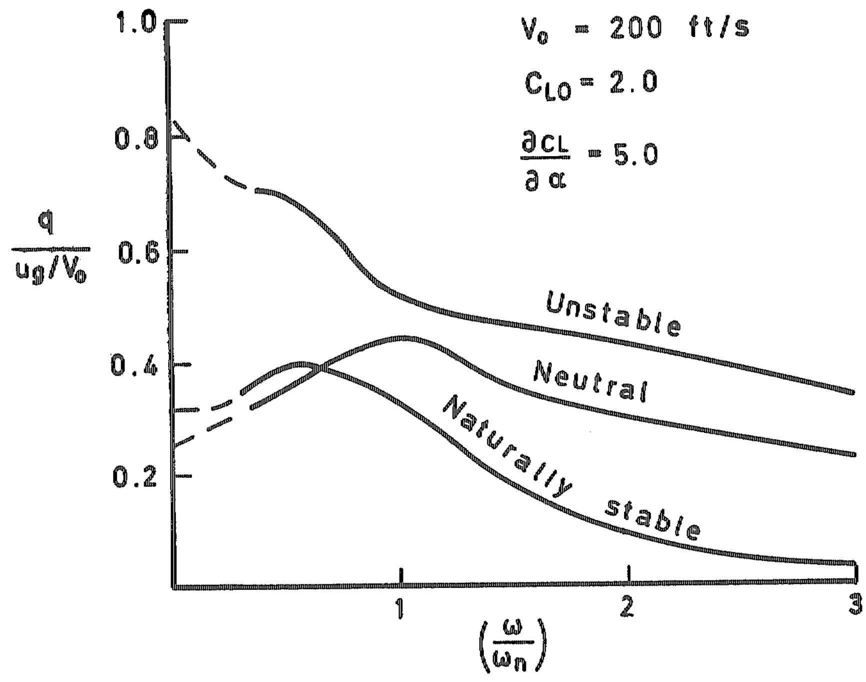


Fig 16 Frequency response in q and Δn to fore and aft gust u_g of aircraft stabilised by normal acceleration feedback



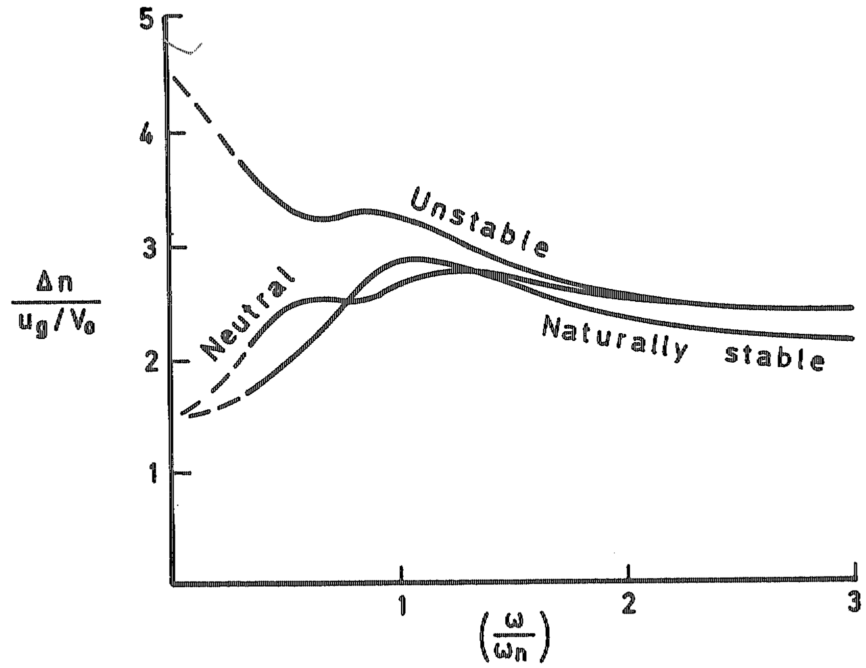


Fig 17 Frequency response to fore and aft gusts u_g of aircraft with q-feedback for manoeuvre stability and V-feedback for static stability

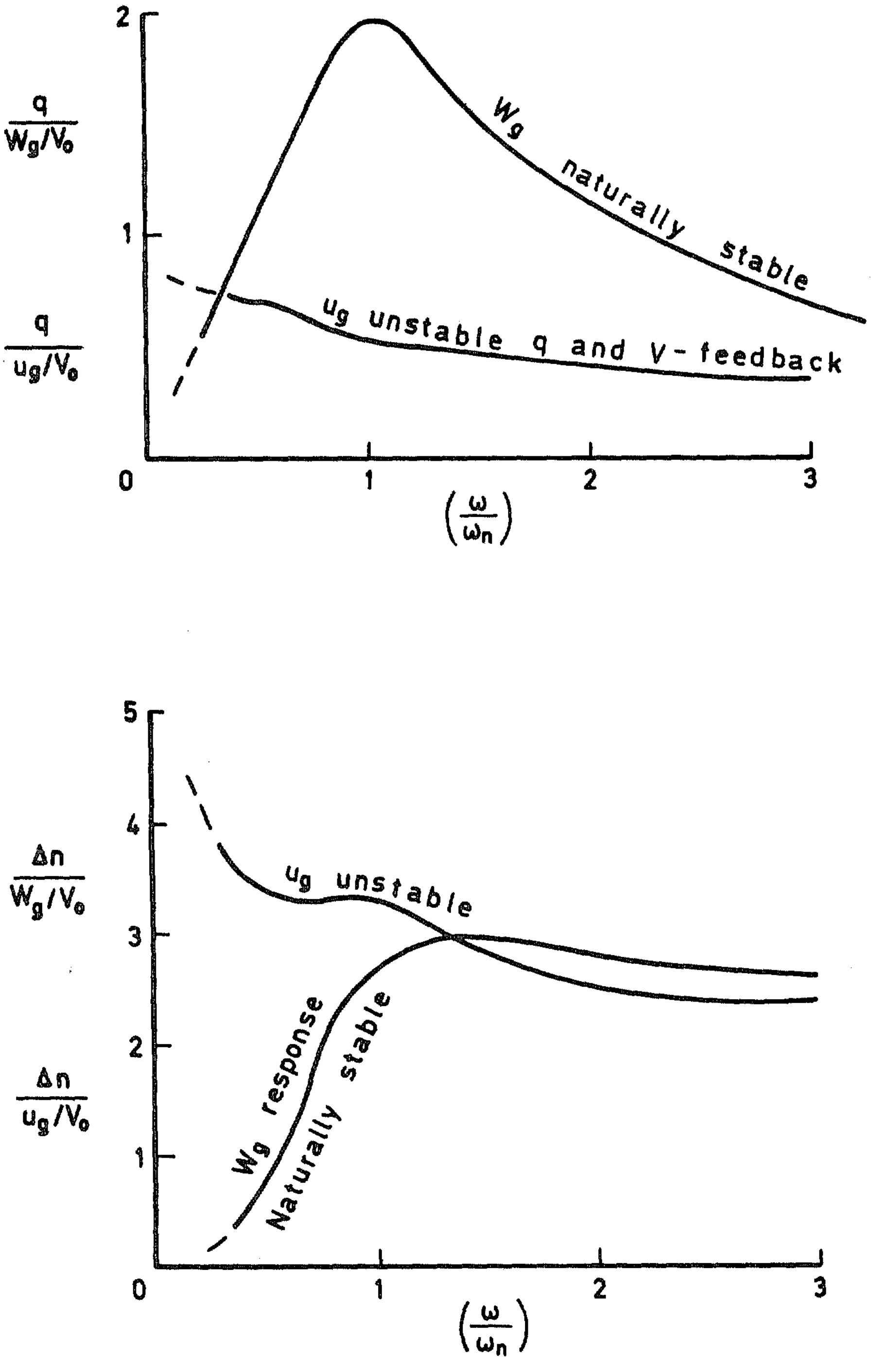
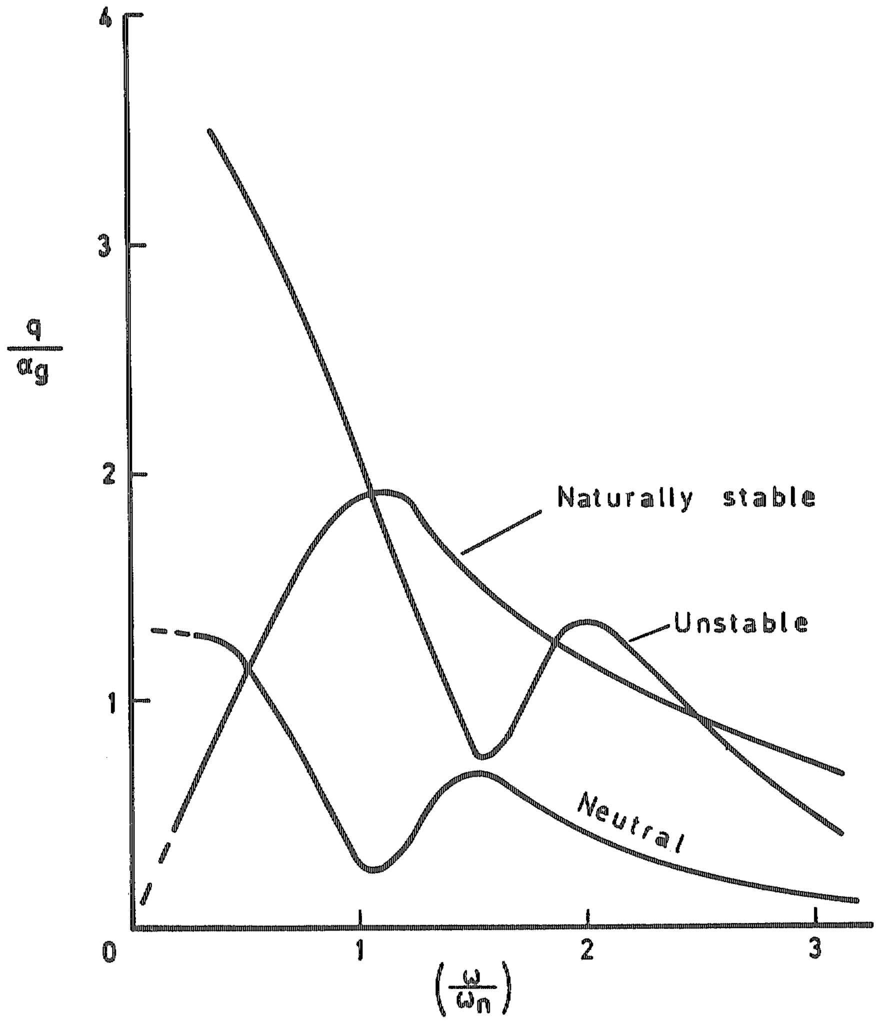


Fig 18 Comparison of u_g response of unstable configuration of Fig 17 with familiar response to vertical gusts of conventional aircraft in the same flight condition



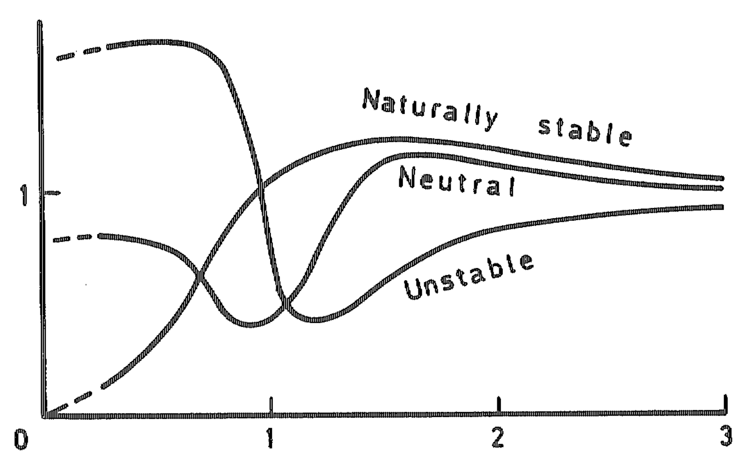


Fig 19 Frequency response in q and α to vertical gusts α_g of aircraft stabilised by 'derived' incidence feedback

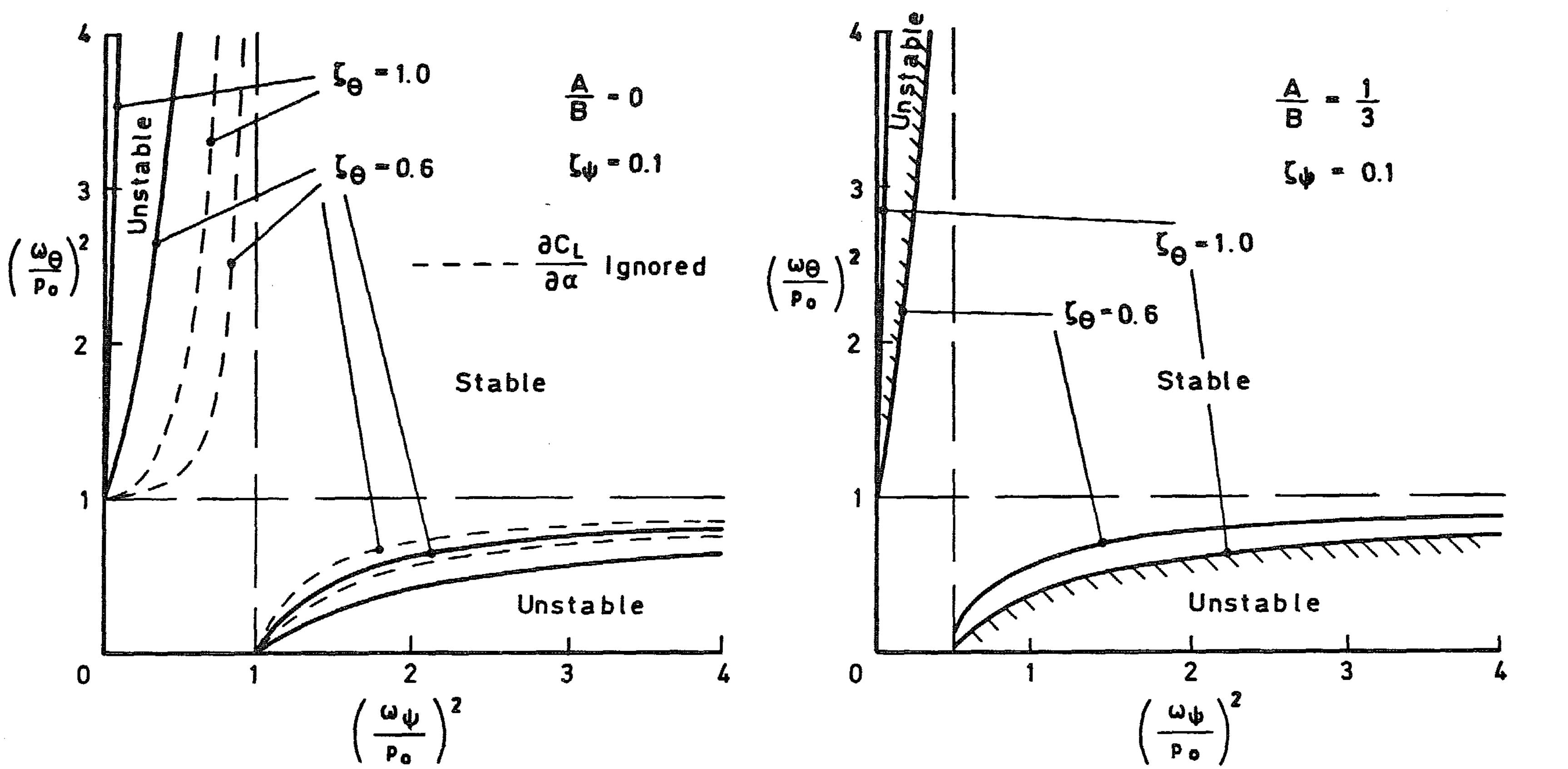


Fig 20 Stability boundaries of the aircraft in inertia-coupled rolling motion re-evaluated with fully accounting for $\partial C_L/\partial\alpha$ effects

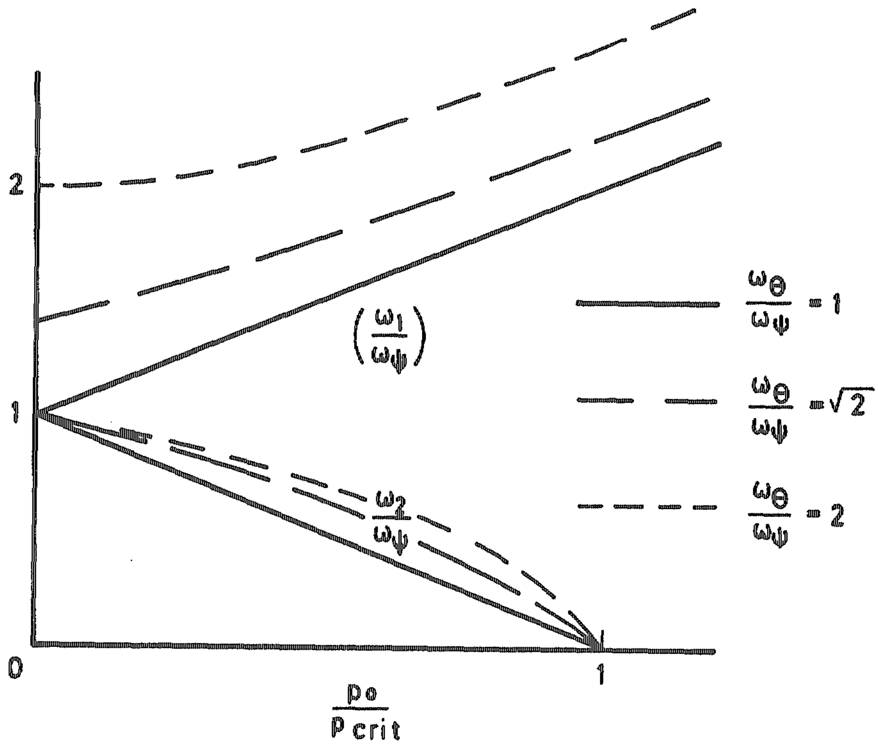


Fig 21 Variation of the frequencies of the two oscillatory modes with increasing roll rate (simplified analysis ignoring pitch and yaw damping)

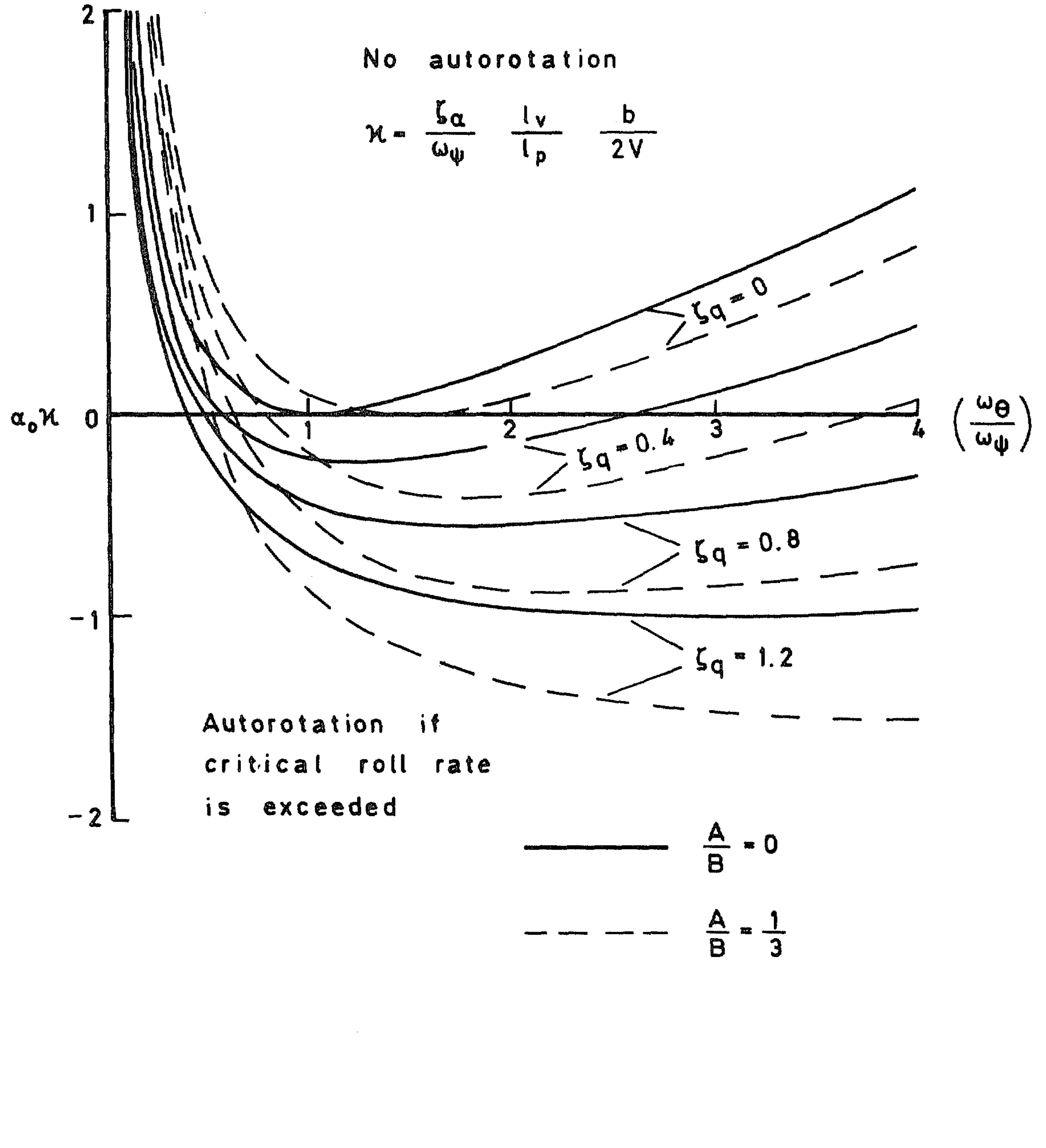


Fig 22 Effect of pitch damping ζ_q on autorotation boundary example with $\zeta_\psi = 0.1$, $\zeta_\alpha = 0.2$

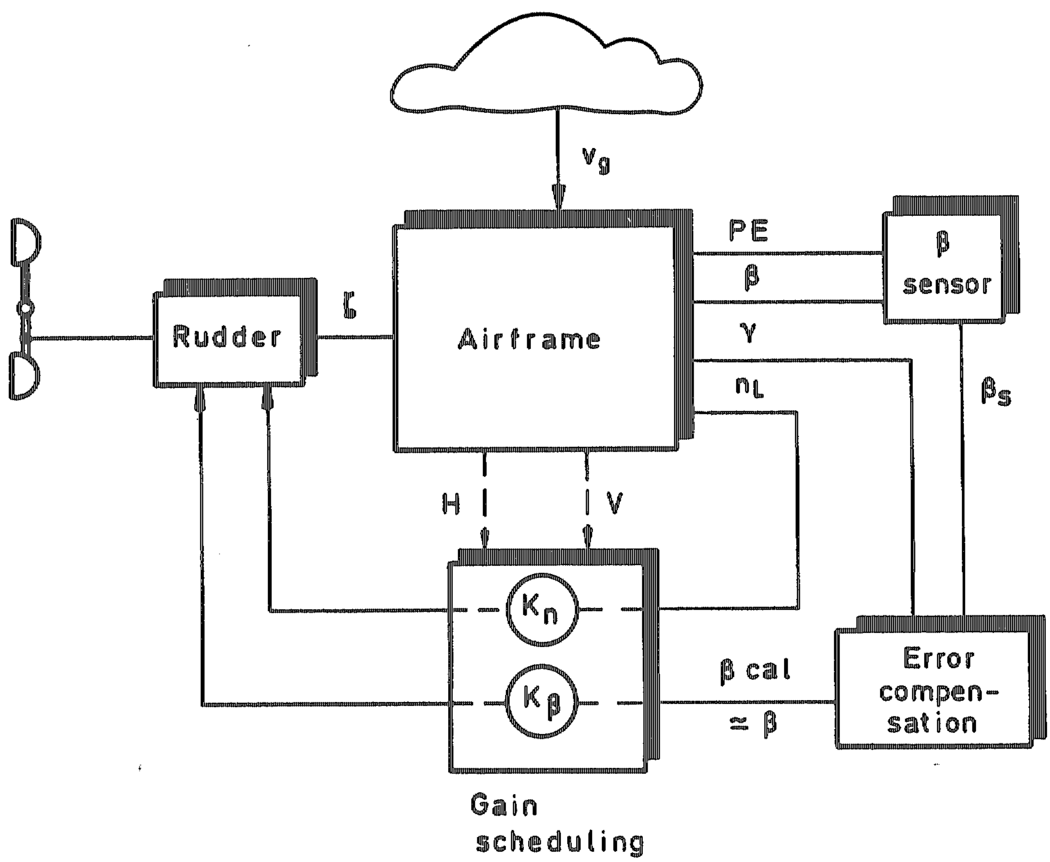


Fig 23 Feedbacks considered for augmenting directional stability

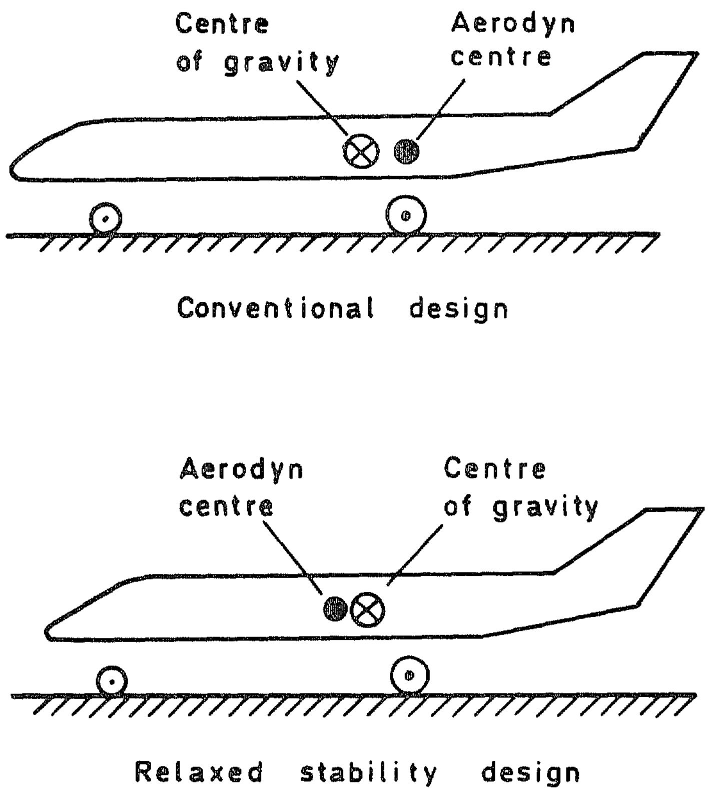


Fig 24 Design features relevant to pitch stability during ground roll

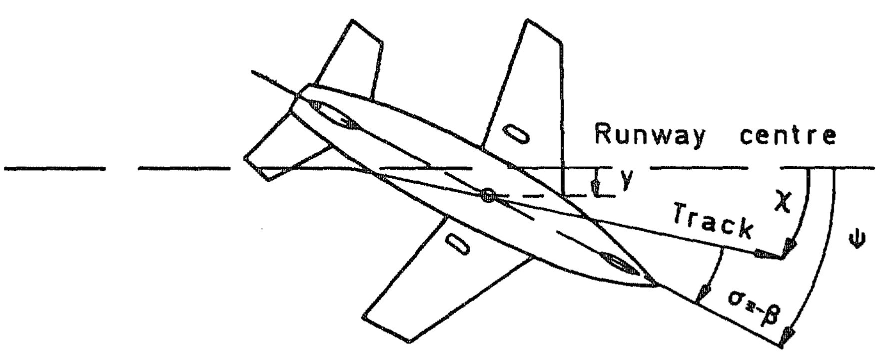


Fig 25 Definitions of terms relevant to directional stability during the ground roll

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