R. & M. No. 2592
(10,922)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Solution by Lifting-Line Theory of Problems involving Discontinuities

By

V. M. Falkner, B.Sc., A.M.I.Mech.E.,
of the Aerodynamics Division, N.P.L.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1952
PRICE 6s. 6d. NET

The Solution by Lifting-Line Theory of Problems involving Discontinuities

By

V. M. FALKNER, B.Sc., A.M.I.Mech.E., of the Aerodynamics Division, N.P.L.

Reports and Memoranda No. 2592

October, 1947

Summary.—The report, which has been written as a preliminary to a later account of similar work in lifting-plane theory, describes how wing loading problems involving discontinuities are solved by lifting-line theory. The four discontinuities considered are (a) direction of leading or trailing edge, (b) incidence, (c) two-dimensional lift slope and (d) chord. As the effects of the first are of minor importance in lifting-line theory, attention is mainly confined to the last three, the solution being based on the use of a few terms of a Fourier series in conjunction with special functions tabulated elsewhere.

The work is limited to straight unyawed flight and includes lift, induced drag, and pitching, rolling and yawing moments, all with or without deflected landing flaps and ailerons. The method of formation of the equations, and the solutions of a representative range of problems for a hypothetical wing, including loading due to incidence, symmetrical wing twist, uniform roll, and deflected flaps and ailerons, are fully described. An indication is given of how induced drag and yawing moment calculations will later be simplified by the use of special derived functions.

Absolute values of wing properties as given by lifting-line theory are usually too high, but the specification of correction factors for viscosity is beyond the scope of the report.

1. *Introduction.*—The report has been written in order to demonstrate the principles by which problems involving discontinuities are solved by lifting-line theory, preliminary to a later account of similar work in lifting-plane theory. The nature of the discontinuities which are to be treated is first described, and a brief description is included of the loading functions, tabulated elsewhere, which are required to allow satisfactorily for the discontinuities.

The method of solution is demonstrated by calculating symmetrical and anti-symmetrical solutions for a hypothetical wing, a departure from the usual application of lifting-line theory being that the equating points are now spaced at even intervals of the semispan instead of in angular measure, an arrangement which is thought to be generally advantageous as well as coupling usefully with vortex lattice theory.

The scope of the report is limited to straight unyawed flight and includes lift, induced drag, and pitching, rolling and yawing moments, all with or without deflected landing flaps and ailerons. The induced drag and yawing moment have been calculated in the report by numerical integration. It is possible for these to be calculated from a formula which involves certain derived functions the computation of which is not yet complete. These will be published later, but their use will not introduce any new principle, the main purpose being to reduce the work of computation to a minimum.

Although the relative magnitudes of the properties of a straight wing as given by lifting line theory are usually considered to be reasonably accurate, the absolute magnitudes are usually too high on account of the influence of viscosity. This report deals only with the potential solutions obtained by the theory, and no attempt is made to specify correction factors for the effect of viscosity.

(21061) A

2. *Statement of Problem.*—The discontinuities which will be considered in this report, all but one of which are believed to be introduced for the first time, are the following:—(a) discontinuity in direction of leading or trailing edge, such as occurs at the median section of a straight tapered wing, (b) discontinuity of incidence, due either to a sudden change in geometrical incidence or to the deflection of a movable flap, (c) discontinuity of $dC_L/d\alpha$ due to a sudden change in wing profile, and (d) discontinuity of chord.

Since the circulation is continuous, a discontinuity of incidence such as (b) can only be expressed mathematically if a function is introduced by which the discontinuity of incidence is offset by a discontinuity in the induced downwash. Similar considerations apply to (c) and (d), for, with the former, a discontinuity of induced downwash must be introduced in order to satisfy the condition that $dC_L/d\alpha$ multiplied by effective incidence is continuous, and, with the latter, a similar discontinuity must be introduced in order to express the discontinuity of local lift coefficient which follows from continuity of circulation. The functions which are used to represent the discontinuities (b), (c), and (d) are the Multhopp functions, the derivation and tabulated values of which for centre and tip flaps, and centre and tip ailerons, are given in another report[1]. The discontinuity (a) is not of the same severity as (b), (c), and (d) and there are good reasons, supported by a trial calculation, for stating that no special functions are necessary in lifting-line theory, the usual terms of the Fourier series being adequate to cover or smooth over any effects due to this cause.

However, this discontinuity becomes of considerable importance in lifting plane theory, particularly where large angles of sweepback are involved, and some of the functions which it is proposed to use, which involve a discontinuity of rate of change of induced downwash with span, have, therefore, been described and tabulated. Hence, the present work is all based on the neglect of any special effects due to discontinuities of direction of leading or trailing edges, and on the treatment of the three other discontinuities defined above by including in the wing loading Multhopp functions with discontinuities at the appropriate spanwise positions.

2.1. The present work will be based entirely on expressing the loading as a Fourier series with the addition of Multhopp functions, and the subsequent solution of a set of simultaneous equations as described in a paper in *Aircraft Engineering*[2]. An alternative method described by Multhopp is based on the use of factors obtained by the pre-solution of simultaneous equations by an iterative process and is designed to simplify the work. There are three reasons why the latter method has been rejected in the present work:—(a) it has not been possible to devote any time to the consideration of whether this method, which has been demonstrated for a simple discontinuity of incidence only, could be extended effectively to include general discontinuities, (b) for the comprehensive set of solutions described in this report, which are carried out with despatch by trained computers, it is doubtful whether the saving of effort would be appreciable, and (c) it is frequently necessary when discontinuities are present to guard against oscillatory solutions by using more relations than necessary and reducing the number of equations by normalisation.

2.2. Where a single discontinuity of incidence is involved, the amount of Multhopp function to be included in the circulation is known, being, in fact, the value of $K/4sV$ as tabulated per radian of full chord discontinuity. The amount to be included to represent the other discontinuities is initially unknown, and is derived as part of the solution of the problem. In this report the Multhopp function has been left entirely unrestricted, and, where normalisation is used, has been subjected to the same treatment as the other functions.

3. *Specification of Wing Example.*—The method will be described by carrying out a complete hypothetical example as given in Fig. 1. The basic wing has straight taper of $2\frac{1}{2}$ to 1 with the centre section $0 \cdot 25$ span and the tip $0 \cdot 10$ span. From $0 \cdot 5$ semispan to the tip, the chord is increased by $1 \cdot 2$ to 1. Symmetrical flaps, equivalent to 25 per cent hinged flaps, extend from 0 to $0 \cdot 5$ semispan, and ailerons extend from $0 \cdot 5$ semispan to the tip, the flap chord ratio varying

from 25 per cent at the inner end to 35 per cent at the tip. The two-dimensional lift slope is supposed to have a discontinuity at $\eta = 0{\cdot}25$; the assumed values are $7{\cdot}0$ for $\eta = 0$ to $0{\cdot}25$, and $5{\cdot}875$ at $\eta = 0{\cdot}25$ decreasing linearly to $5{\cdot}5$ at $\eta = 1$. Provision is made for a discontinuity of incidence at $\eta = 0{\cdot}25$, and for symmetrical wing twist varying according to the law chord $\times$ twist linear.

4. *Formation of Equations.*—The formula for the circulation for the symmetrical loading will be taken as

$$K/4sV = A_1 \sin\phi + A_3 \sin 3\phi + A_5 \sin 5\phi + A_7 \sin 7\phi + m_1 M_{\text{CF25}} + m_2 M_{\text{CF50}} \quad \ldots \quad (1)$$

and for the anti-symmetrical loading as

$$K/4sV = A_2 \sin 2\phi + A_4 \sin 4\phi + A_6 \sin 6\phi + A_8 \sin 8\phi + n_1 M_{\text{TA25}} + n_2 M_{\text{TA50}} \quad \ldots \quad (2)$$

The first four terms of each series are the usual Fourier terms, whilst the remaining two are the Multhopp functions which must be included in order to allow for the discontinuities which are present at $\eta = 0{\cdot}25$ and $\eta = 0{\cdot}5$. These functions are taken directly from the Tables given in R. & M. 2593[1] and the notation is as follows:—

Suffix CF25 $\equiv$ centre flaps for $\eta^* = 0{\cdot}25$

Suffix TF25 $\equiv$ tip flaps for $\eta^* = 0{\cdot}25$

Suffix CA25 $\equiv$ centre ailerons for $\eta^* = 0{\cdot}25$

Suffix TA25 $\equiv$ tip ailerons for $\eta^* = 0{\cdot}25$, and so on.

The standard equation for the solution of any problem is given in a paper in *Aircraft Engineering*[2]. This is

$$\Sigma A_n \sin n\phi \left[\sin\phi + \frac{a_0 c}{8s} n\right] = -\sin\phi \left[\sum_n^m M - \frac{a_0 c}{8s}(\alpha - G\phi)\right] \quad \ldots \quad (3)$$

where a_0 is the two-dimensional lift slope $dC_L/d\alpha$, c is the chord, s the semispan, α the geometrical incidence at any section, ΣmM and ΣnM the sums of all the Multhopp functions used in the solution under consideration, and $G\phi$ the induced downwash due to the sum of all the Multhopp functions used.

It is usual to separate the symmetrical and anti-symmetrical solutions, and, for the solutions investigated here, the equations become:—

Symmetrical.

$$A_1 \sin\phi \left[\sin\phi + \frac{a_0 c}{8s}\right] + A_3 \sin 3\phi \left[\sin\phi + \frac{3\, a_0 c}{8s}\right]$$

$$+ A_5 \sin 5\phi \left[\sin\phi + \frac{5\, a_0 c}{8s}\right] + A_7 \sin 7\phi \left[\sin\phi + \frac{7\, a_0 c}{8s}\right]$$

$$= -\sin\phi \left[m_1 M_{\text{CF25}} + m_2 M_{\text{CF50}} - \frac{a_0 c}{8s}(\alpha - G\phi)\right] \quad \ldots \quad (4)$$

where $$G\phi = m_1 \left[\eta = 0 \text{ to } 0{\cdot}25\right] + m_2 \left[\eta = 0 \text{ to } 0{\cdot}50\right].$$

Anti-symmetrical.

$$A_2 \sin 2\phi \left[\sin \phi + \frac{2\, a_0 c}{8s}\right] + A_4 \sin 4\phi \left[\sin \phi + \frac{4\, a_0 c}{8s}\right]$$

$$+ A_6 \sin 6\phi \left[\sin \phi + \frac{6\, a_0 c}{8s}\right] + A_8 \sin 8\phi \left[\sin \phi + \frac{8\, a_0 c}{8s}\right]$$

$$= - \sin \phi \left[n_1 M_{TA25} + n_2 M_{TA50} - \frac{a_0 c}{8s} (\alpha - G\phi)\right] \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where $$G\phi = n_1 \left[\eta = 0 \cdot 25 \text{ to } 1\right] + n_2 \left[\eta = 0 \cdot 5 \text{ to } 1\right].$$

By transferring to the left hand side the terms in m_1 and m_2 or n_1 and n_2, the right hand side is left as $\frac{a_0 c}{8s} \alpha \sin \phi$, and there will be six unknowns, *i.e.*, A_1, A_3, A_5, A_7, m_1, m_2 for the symmetrical solution and A_2, A_4, A_6, A_8, n_1, n_2 for the anti-symmetrical solution. The coefficient of m_1 will be

$$\sin \phi \left[M_{CF25} + \frac{a_0 c}{8s}\right] \text{ for } 0 \leqslant \eta \leqslant \eta^*_{\text{inner}} \text{ and}$$

$$\sin \phi \left[M_{CF25}\right] \qquad \text{for } \eta^*_{\text{outer}} \leqslant \eta \leqslant 1,$$

and there are similar expressions for m_2, n_1, and n_2.

An alternative expression to equation (1), which includes the function known as P which will be used to compensate for discontinuity of slope of leading or trailing edge, is

$$K/4sV = A_1 \sin \phi + A_3 \sin 3\phi + A_5 \sin 5\phi + pP + m_1 M_{CF25} + m_2 M_{CF50} \,. \quad \ldots \quad (6)$$

4.1. Six equations would normally be sufficient to determine the six unknowns, but, in this work, it is usually advisable to take more than six equations and reduce by normalisation. One reason for this is that, unless the stabilising influence of a least squares solution is present, a solution containing Multhopp functions sometimes develops a tendency to oscillate. Another reason is that it is doubtful whether the wing can be represented adequately by six conditions only. It will be noted that at each point of discontinuity there will be two equations, corresponding to the inner and outer edges of the discontinuity respectively. There will be, in the present work, inner and outer values for a_0 and $G\phi$ at $\eta^* = 0 \cdot 25$, and inner and outer values for c and $G\phi$ at $\eta^* = 0 \cdot 5$.

Where there is a simple discontinuity of incidence only, the value of the corresponding m coefficient is known, being, in fact, unity per radian of discontinuity. Where other discontinuities are present the m coefficient is unknown, and in the present work, because of this unknown contribution and because a least squares solution is involved, all coefficients of the M functions have been taken as unknown.

5. *General Formulae.*—The lift coefficients and rolling moments due to Multhopp functions are defined in R. & M. 2593[1], from which it is deduced that the total lift coefficient for the circulation (1) is

$$C_L = \frac{4s^2}{S} \left[\pi\, A_1 + m_1 (\pi - 2\phi_1^* + \sin 2\phi_1^*) + m_2 (\pi - 2\phi_2^* + \sin 2\phi_2^*)\right] \quad \ldots \quad (7)$$

where ϕ_1^* and ϕ_2^* are angular measures corresponding to $\eta^* = 0 \cdot 25$ and $\eta^* = 0 \cdot 50$ respectively.

The local lift coefficient for $C_L = 1$ is related to the circulation thus:—

$$C_{LL} = \frac{4\,(K/4sV)}{(c/2s)(dC_L/d\alpha)} \quad \ldots \quad (8)$$

The rolling moment coefficient for the circulation (2) is

$$C_l = -\frac{s^2}{S}\left[\pi A_2 + \tfrac{4}{3}n_1\,(1 - \eta_1{}^{*2})^{3/2} + \tfrac{4}{3}n_2\,(1 - \eta_2{}^{*2})^{3/2}\right] \quad \ldots \quad (9)$$

where $\eta_1{}^* = 0 \cdot 25$ and $\eta_2{}^* = 0 \cdot 50$.

The value of C_{m0}, referred to the mean chord $\bar{c}$, is

$$-\frac{32s^4}{S^2}\int_{-1}^{1} \frac{K}{4sV}\,\frac{x_g}{2s}\,d\eta \quad \ldots \quad (10)$$

where $K/4sV$ is the circulation at zero lift, and $x_g/2s$ the distance back from datum of the local centre of pressure.

The general expression for induced drag is

$$C_{Di} = \frac{8s^2}{S}\int_{-1}^{1}\left(\frac{w}{V}\right)\left(\frac{K}{4sV}\right)d\eta, \quad \ldots \quad (11)$$

and for induced yawing moment

$$C_n = \frac{4s^2}{S}\int_{-1}^{1}\left(\frac{w}{V}\right)\left(\frac{K}{4sV}\right)\eta\,d\eta. \quad \ldots \quad (12)$$

The formula for w/V due to the Fourier terms is

$$w/V = \frac{\Sigma n\,A_n \sin n\phi}{\sin\phi}.$$

The evaluation of C_{m0} will usually require numerical integration because of irregularities in x_g, but it is possible to avoid this process for C_{Di} and C_n by expressing the integrals in terms of functions derived from the Multhopp functions by integration. These functions are being calculated and will be published as soon as possible, but, as no new principle will be involved, examples are now given of how the drag and yawing moment are derived directly by numerical integration.

5.1. Lifting-line solutions are based on the treatment of each strip as if the relative chordwise distribution were purely two-dimensional. The geometrical incidence to be used in the formation of the equations when deflected flaps are under consideration is therefore the incidence of the equivalent straight line aerofoil with the same lift. This equivalent incidence per radian flap deflection is the quantity a_2/a_1, given by Glauert in R. & M. 1095[3]:—

$$a_2/a_1 = 1 - 2/\pi\,[\cos^{-1}\sqrt{E} - \sqrt{(E(1 - E))}] \quad \ldots \quad (13)$$

where E is the flap/chord ratio.

The location of the local centre of pressure (C.P.) on the chord follows from the same potential theory.

The two-dimensional lift is

$$C_L = 2\pi\,(\alpha' + R_1\eta') \quad \ldots \quad (14)$$

and the moment about the leading edge

$$C_m = -\frac{\pi}{2}\alpha' + R_2\eta' \quad \ldots \quad (15)$$

where α' is the incidence of the main wing profile, η' the deflection of the flap, and

$$R_1 = 1 - \frac{\theta^*}{\pi} + \frac{\sin\theta^*}{\pi} \quad \ldots \quad (16)$$

and

$$R_2 = \tfrac{1}{2}\theta^* - \sin\theta^* + \tfrac{1}{4}\sin 2\theta^* - \frac{\pi}{2} \quad \ldots \quad (17)$$

θ^* being the angular measure at the hinge, related to the flap/chord ratio E by

$$E = 0{\cdot}5\,(1 + \cos\theta^*). \quad \ldots \quad (18)$$

For the three-dimensional wing, the circulation would first be calculated for a given flap deflection, usually per radian, from which it would follow that

$$C_{LL} = \frac{Lift}{\frac{1}{2}\rho V^2 c} = \frac{8s}{c}\,\frac{K}{4sV}\,. \quad \ldots \quad (19)$$

The position of the local C.P. on the section is $-C_m/C_{LL}$, which, after substituting the relations (14) and (15) reduces to

$$0{\cdot}25 + \frac{R_5\eta'}{C_{LL}} \quad \text{or}$$

$$0{\cdot}25 + \frac{R_5\eta'}{4\,(K/4sV)}\,\frac{c}{2s} \quad \ldots \quad (20)$$

where

$$R_5 = \tfrac{1}{2}\sin\theta^* - \tfrac{1}{4}\sin 2\theta^*. \quad \ldots \quad (21)$$

If η' were zero, the local C.P. would be at $0{\cdot}25$ chord. Since $R_5 = \frac{1}{2}\sin\theta^*\,(1 - \cos\theta^*)$, which is positive for $0 \leqslant \theta^* \leqslant \pi$; the local C.P. for positive lift coupled with positive flap deflection is always to the rear of $0{\cdot}25$ chord.

The value of C_{m0} can be calculated either by the direct use of relation (10), in which $x_g/2s$ is the distance of the local C.P. behind datum, or, alternatively, by separating the loading into two components. The loading on any section can be regarded as made up of C_{m0}, the local moment coefficient for zero local lift, together with the local lift force acting at $0{\cdot}25$ chord. The alternative calculation for overall C_{m0}, is, therefore, by the use of relation (10), $x_g/2s$ being taken as the distance of the quarter chord behind datum, with the addition of the integral of the local moments for zero local lift. Now it can easily be shown from relations (14) and (15), or (20), that the local moment coefficient $C_{m0} = M_0/\frac{1}{2}\rho V^2 c^2 = -R_5\eta'$ and it follows that the

$$\text{additional } C_{m0} = M/\tfrac{1}{2}\rho V^2 \bar{c} S = \frac{1}{\bar{c}S}\int_{-s}^{s} - R_5\eta' c^2\,dy \quad \text{or}$$

$$C_{m0} = -\frac{8s^4}{S^2}\int_{-1}^{1} R_5\eta'(c/2s)^2\,d\eta. \quad \ldots \quad (22)$$

As the centre of pressure of any additional loading due to incidence is also at the quarter-chord, it follows that the loading system under any conditions can be represented by applying the local lift force at the quarter-chord, and adding the moment defined by equation (22).

6. *Equations*.—The constants for the wing derived from the specification of section 3 are given in Table 1 and it will be noted that there are inner and outer values for $a_0c/8s$ at $\eta^* = 0 \cdot 25$ and $0 \cdot 50$. Three sets of equations are required, *i.e.*, (a) symmetrical equations for incidence (b) symmetrical equations for zero lift and (c) anti-symmetrical equations. The derivation of these is from the formulae of section 4, the values of the Multhopp functions being taken from R. & M. 2593[1], and the values of sines of multiple angles from Table 1.

The equations are formed for $\eta = 0, 0 \cdot 15, 0 \cdot 25$ (2), $0 \cdot 35, 0 \cdot 5$ (2), $0 \cdot 7$ and $0 \cdot 9$, giving a total of nine for six variables.

The symmetrical equations for the plain incidence solution are given in Table 4 before and after normalisation by the process of which an example is given in Table 8 of R. & M. 2591[4].

The symmetrical equations for wing twist, which are given in Table 5, are derived from the previous equations by substituting the condition for zero lift. This condition, that the right hand of relation (7) must be zero, reduces to

$$A_1 + 0 \cdot 31496 m_1 + 0 \cdot 60900 m_2 = 0.$$

Hence, to convert the incidence equations to zero lift, the following transformations are used:—

A_1 omitted:

A_3, A_5, A_7, as before:

m_1 (zero lift) $= m_1 - 0 \cdot 31496 A_1$:

m_2 (zero lift) $= m_2 - 0 \cdot 60900 A_1$.

The revised incidence is α_0 plus wing twist. Hence the α_0 column is the same as for the original constant column, and the new constant column will be $-$ $(a_0c/8s) \sin \phi \times$ (twist). In Table 5, the twist used in calculating the constant column refers to the $c\theta$ linear figures of Table 2.

The anti-symmetrical equations of Table 6 are derived from relation (2), and the constant column refers to the solution for uniform roll, for which the geometrical incidence for V/ws unity at the tip is η.

7. *Solutions*.—The three sets of equations described are all that are necessary for the solution of any straight flight problem. Once they have been normalised and solved by elimination, by the process described in R. & M. 2591[4] the solution for any other problem which involves a change in the constant column can easily be obtained by the process described also in the same report.

We now proceed to describe the representative solutions which have been obtained, together with the derivation of the corresponding constant column.

7.1. *Plain Wing, Incidence Solution*.—This solution is given in Table 7. The symmetrical equations of Table 4 are used, the constant column being $-$ $(a_0c/8s) \sin \phi$. The circulation per radian (column 2) is calculated from the sines of multiple angles given in Table 1, and the Multhopp functions given in R. & M. 2593[1]. The quantity in column 3 is the distance back of the quarter-chord from datum in terms of the span. The aerodynamic centre is obtained by dividing the integral of column 4 by the integral of column 2. Because of irregularities in the circulation, the standard Simpson factors are not applicable, and the integrating column 5 is built up from the components described in Appendix 1. The local lift coefficient is derived from relation (8), and the geometrical mean quarter chord from an integration of columns 6 and 8.

The induced drag is calculated by the process given in Table 8. The three separate components of the induced downwash are tabulated, the two corresponding to centre flaps being derived from the coefficients m_1 and m_2 given by the solution of the equations. That part of the induced downwash due to the Fourier terms is calculated from $w/V = \Sigma nA_n \sin n\phi/\sin \phi$ using Table 1. The total w/V is obtained by summation and the induced drag obtained by integrating (w/V) $(K/4sV)$ as in relation (11). The same integrating factors are used as in Table 7, and, after the result is divided by $C_L{}^2$ per radian, lead to $C_{Di} = 0{\cdot}0608C_L{}^2$, the minimum being $0{\cdot}0601C_L{}^2$.

7.2. *Symmetrical Wing Twist, Linear Product of Chord and Twist.*—This solution is given in Table 9, the equations used being the symmetrical equations for zero lift of Table 5, the constant column being $-$ $(a_0c/8s)$ $\sin \phi \times$ twist. In the Table are given the circulation per radian twist at the tip; the integral of the product of circulation and distance back of quarter chord from datum, which, after a suitable factor defined in relation 10, gives C_{m0} per radian; and the local lift coefficient obtained as previously.

7.3. *Fifty per cent Span Flaps: Zero Lift.*—This solution, given in Table 10, applies to hinged flaps or their equivalent. The symmetrical equations for zero lift of Table 5 are used, the constant column being $-$ (a_2/a_1) $(a_0c/8s)$ $\sin \phi$ from $\eta = 0$ to $0{\cdot}5$ inner, and zero from $0{\cdot}5$ outer to tip. The ratio a_2/a_1 is given by formula (13), numerical values being included in Table 2. Since the flap/chord ratio is a constant $0{\cdot}25$, $a_2/a_1 = 0{\cdot}60900$. The solution follows the same lines as previously, excepting that, because of the deflected flap, the local C.P. is no longer at $0{\cdot}25$ chord. The position of the local C.P. on the chord is given by relation 20, leading to $[(0{\cdot}25 + 0{\cdot}16238\ c/2s)/(K/4sV)]$ chord over the flap span.

7.4. *Discontinuity of Incidence at* $\eta = 0{\cdot}25$.—This solution is given in Table 11 and follows the same pattern as previous solutions. It should be noted that the coefficient m_1, which should be unity for one radian discontinuity, solves by the least squares process to $0{\cdot}9833$.

7.5. *Uniform roll.*—The solution for uniform roll applicable to $V/ws =$ unity at the tip, given in Table 12, is obtained by using the anti-symmetrical equations, Table 6, with constant column $-$ $(a_0c/8s)$ $\eta \sin \phi$. The solution gives rolling moment, circulation, and the local lift coefficient for unit $(-\ C_l)$.

7.6. *Aileron from* $\eta = 0{\cdot}5$ *to* $1{\cdot}0$, *Wing at Zero Incidence.*—This solution is given in Table 13, the anti-symmetrical equations of Table 6 being used with constant column zero from $\eta = 0$ to $0{\cdot}5$ inner, and $-$ (a_2/a_1) $(a_0c/8s)$ $\sin \phi$ from $\eta = 0{\cdot}5$ outer to tip. The value of flap/chord ratio E varies from $0{\cdot}25$ to $0{\cdot}33$ and the corresponding values of a_2/a_1 are given in Table 2. The solution is calculated in the same way as the previous solution.

7.7. Half span distributions of circulation for six of these solutions are plotted in Fig. 2.

8. *Composite Solutions.*—In order to demonstrate the method for finding a composite solution, the calculation of yawing moment, and a suitable specification of the wing loading for use in aeroelastic problems, a composite solution for flaps and ailerons at zero lift has been calculated. The specification is for 50 per cent span flaps with ΔC_L due to flaps $= 1$, and 50 per cent ailerons deflected to give rolling moment coefficient $+ 0{\cdot}1$. It follows from the separate solutions that the incidence for zero lift is $-$ $0{\cdot}2275$ radians, the aileron deflection is $-$ $0{\cdot}3138$ radians, and the composite circulation is $0{\cdot}6135$ (flaps) minus $0{\cdot}3138$ (ailerons). In Table 14, the circulation for port and starboard half wings, chord, R_5 for the flap/chord ratios from Table 2, and η' the flap deflection, are given.

It has been shown in section 5.1 that the force system on any section is made up of the lift acting at the quarter-chord, with the addition of a moment defined by

$$C_m = \frac{M dy}{\frac{1}{2}\rho V^2 \bar{c} S} = -\ \frac{8s^4}{S^2} R_5 \eta'\ (c/2s)^2\ d\eta .$$

The last column of Table 14 gives the numerical values of the local C_m.

9. *Conclusion.*—In conclusion, the writer acknowledges the valuable help received from Miss W. M. Tafe, who was responsible for computing the solutions given in the report.

REFERENCES

No.	*Author*	*Title, etc.*
1	V. M. Falkner	Tables of Multhopp and Other Functions for use in Lifting-line and Lifting-plane Theory. R. & M. 2593. February, 1948.
2	V. M. Falkner	Glauert Loading of Wings with Discontinuities of Incidence. *Aircraft Engineering.* September, 1946.
3	H. Glauert	Theoretical Relationships for an Aerofoil with Hinged Flap. R. & M. 1095.
4	V. M. Falkner	The Solution of Lifting-plane Problems by Vortex Lattice Theory. R. & M. 2591. September, 1947.

APPENDIX I

Modified Simpson factors

The standard Simpson factors of 1, 4, 1 require modification when the curve to be integrated has an infinite slope. It is shown in R. & M. 2591[4] that the factors 0·800, 4·525, 0·675* will give a reliable value for the integral when used over the intervals at the wing tip which include an infinite slope at the boundary, the starred value being used at the infinity.

In this report, since it is occasionally necessary to cover an odd number of intervals, the three-eighths rule is also used, factors 1·125, 3·375, 3·375, 1·125. A modification for infinite slope calculated by a similar method to the above gives 0·655, 4·454, 2·520, and 1·371.

At a discontinuity, the exact magnitudes of the factors are difficult to calculate exactly, and it has been found that mean values between the above sets give reliable results.

Hence, for any given integration, the factors are made up from a combination of the following to suit the positions of the discontinuities.

Case	Standard factors	Modified for infinite slope	Modified for discontinuity
Two interval	1	0·675*	0·838†
	4	4·525	4·262
	1	0·800	0·900
Three interval	1·125	0·655*	0·890†
	3·375	4·454	3·914
	3·375	2·520	2·948
	1·125	1·371	1·248

* To be used at the infinity.
† To be used at the discontinuity.

TABLE 1

Sines of Multiple Angles for Regular Values of η

η	ϕ deg	ϕ radn	sin ϕ	sin 2ϕ	sin 3ϕ	sin 4ϕ	sin 5ϕ
0	90·000	1·5708	1·00000	0	− 1·00000	0	1·00000
0·05	87·134	1·5208	0·99875	0·09987	− 0·98876	− 0·19875	0·96889
0·10	84·261	1·4706	0·99499	0·19900	− 0·95519	− 0·39004	0·87718
0·15	81·373	1·4202	0·98869	0·29661	− 0·89970	− 0·56652	0·72975
0·20	78·463	1·3694	0·97980	0·39192	− 0·82303	− 0·72113	0·53458
0·25	75·522	1·3181	0·96825	0·48412	− 0·72618	− 0·84722	0·30258
0·30	72·542	1·2661	0·95394	0·57236	− 0·61052	− 0·93868	0·04732
0·35	69·513	1·2132	0·93675	0·65572	− 0·47774	− 0·99014	− 0·21536
0·40	66·422	1·1593	0·91652	0·73321	− 0·32994	− 0·99717	− 0·46779
0·45	63·256	1·1040	0·89303	0·80373	− 0·16968	− 0·95643	− 0·69112
0·50	60·000	1·0472	0·86603	0·86603	0·00000	− 0·86603	− 0·86603
0·55	56·633	0·9884	0·83516	0·91868	0·17538	− 0·72576	− 0·97372
0·60	53·130	0·9273	0·80000	0·96000	0·35200	− 0·53760	− 0·97712
0·65	49·458	0·8632	0·75993	0·98791	0·52436	− 0·30625	− 0·92248
0·70	45·573	0·7954	0·71414	0·99980	0·68558	− 0·03999	− 0·74157
0·75	41·410	0·7227	0·66144	0·99216	0·82680	0·24804	− 0·45474
0·80	36·870	0·6435	0·60000	0·96000	0·93600	0·53760	− 0·07584
0·85	31·788	0·5548	0·52678	0·89553	0·99562	0·79702	0·35932
0·90	25·842	0·4510	0·43589	0·78460	0·97639	0·97291	0·77484
0·95	18·195	0·3176	0·31225	0·59328	0·81497	0·95517	0·99986
1·00	0	0	0	0	0	0	0

η	sin 6ϕ	sin 7ϕ	sin 8ϕ	sin 9ϕ	sin 10ϕ	sin 11ϕ
0	0	− 1·00000	0	1·00000	0	− 1·00000
0·05	0·29564	− 0·93932	− 0·38957	0·90037	0·47961	− 0·85240
0·10	0·56547	− 0·76409	− 0·71829	0·62043	0·84238	− 0·45195
0·15	0·78544	− 0·49412	− 0·93368	0·21401	0·99788	0·08535
0·20	0·93496	− 0·16059	− 0·99920	− 0·23909	0·90356	0·60051
0·25	0·99850	0·19668	− 0·90017	− 0·64676	0·57679	0·93515
0·30	0·96706	0·53292	− 0·64731	− 0·92131	0·09452	0·97802
0·35	0·83939	0·80293	− 0·27734	− 0·99707	− 0·42061	0·70264
0·40	0·62294	0·96614	0·14997	− 0·84616	− 0·82690	0·18464
0·45	0·33443	0·99210	0·55846	− 0·48949	− 0·99900	− 0·40961
0·50	0·00000	0·86603	0·86603	0·00000	− 0·86603	− 0·86603
0·55	− 0·34533	0·59385	0·99857	0·50458	− 0·44354	− 0·99247
0·60	− 0·65894	0·20639	0·90661	0·88154	0·15124	− 0·70005
0·65	− 0·89298	− 0·23838	0·58308	0·99638	0·71222	− 0·07049
0·70	− 0·99820	− 0·65592	0·07992	0·76780	0·99500	0·62520
0·75	− 0·93015	− 0·94048	− 0·48058	0·21962	0·81000	0·99539
0·80	− 0·65894	− 0·97847	− 0·90661	− 0·47210	0·15124	0·71409
0·85	− 0·18618	− 0·67582	− 0·96272	− 0·96080	− 0·67064	− 0·17929
0·90	0·42180	− 0·01560	− 0·44987	− 0·79418	− 0·97964	− 0·96918
0·95	0·94455	0·79480	0·56556	0·27976	− 0·03400	− 0·34437
1·00	0	0	0	0	0	0

TABLE 2

Table of Constants for Wing

η	a_0	$c/2s$	$a_0c/8s$	Twist θ linear	Twist $c\theta$ linear
0	7·000	0·2500	0·43750	0	0
0·15	7·000	0·2275	0·39812	0·15	0·06593
0·25 inner	7·000	0·2125	0·37188	0·25	0·11765
0·25 outer	5·875	0·2125	0·31211	0·25	0·11765
0·35	5·825	0·1975	0·28761	0·35	0·17722
0·50 inner	5·750	0·1750	0·25156	0·50	0·28571
0·50 outer	5·750	0·2100	0·30188	0·50	0·28571
0·70	5·650	0·1740	0·24578	0·70	0·48276
0·90	5·550	0·1380	0·19148	0·90	0·78261
1·00	5·500	0·1200	0·16500	1·00	1·00000

Function	η_1^* (0·25)	η_2^* (0·50)
ϕ^* radians	1·318116	1·047198
$\sin 2\phi^*$	0·484123	0·866025
$\pi/2 - \phi^* + \frac{1}{2}\sin 2\phi^*$ ..	0·494742	0·956611
$4/3\ (1 - \eta^{*2})^{3/2}$	1·210307	0·866025

E	a_2/a_1	R_5
0·25	0·60900	0·64952
0·26		0·64918
0·27	0·63048	0·64818
0·28		0·64656
0·29	0·65090	0·64434
0·30		0·64156
0·31	0·67036	0·63824
0·32		0·63441
0·33	0·68892	0·63008
0·34		0·62530
0·35	0·70666	0·62006

Aspect ratio = 5·298

$64s^4/S^2 = 112{\cdot}28 \qquad \pi s^2/S = 4{\cdot}161$

E = flap/chord ratio

$$a_2/a_1 = 1 - \frac{2}{\pi}\left[\cos^{-1}\sqrt{E} - \sqrt{\left(E\,(1 - E)\right)}\right].$$

TABLE 3

Table of Distances for Wing in Terms of Span

η	Datum to leading edge	chord
0	0	0·2500
0·05	0·0019	0·2425
0·10	0·0038	0·2350
0·15	0·0056	0·2275
0·20	0·0075	0·2200
0·25	0·0094	0·2125
0·30	0·0112	0·2050
0·35	0·0131	0·1975
0·40	0·0150	0·1900
0·45	0·0169	0·1825
0·50	0·0188	0·1750
0·50	0·0188	0·2100
0·55	0·0206	0·2010
0·60	0·0225	0·1920
0·65	0·0244	0·1830
0·70	0·0262	0·1740
0·75	0·0281	0·1650
0·80	0·0300	0·1560
0·85	0·0319	0·1470
0·90	0·0338	0·1380
0·95	0·0356	0·1290
1·00	0·0375	0·1200

TABLE 4

Symmetrical Equations

Before normalisation

η	A_1	A_3	A_5	A_7	m_1	m_2	1	
0	1·4375	− 2·3125	3·1875	− 4·0625	0·9268	1·1900	− 0·4375	
0·15	1·3711	− 1·9641	2·1741	− 1·8656	0·8461	1·1214	− 0·3936	
0·25 i	1·2976	− 1·5133	0·8556	0·7024	0·7245	1·0433	− 0·3601	
0·25 o	1·2397	− 1·3831	0·7652	0·6201	0·3644	0·9855	− 0·3022	
0·35	1·1469	− 0·8597	− 0·5114	2·3687	0·2696	0·8839	− 0·2694	= 0
0·50 i	0·9679	0·0000	− 1·8393	2·2750	0·1870	0·6589	− 0·2179	
0·50 o	1·0114	0·0000	− 2·0572	2·5800	0·1870	0·4411	− 0·2614	
0·70	0·6855	0·9951	− 1·4409	− 1·5969	0·1036	0·2210	− 0·1755	
0·90	0·2735	0·9865	1·0796	− 0·0277	0·0328	0·0684	− 0·0835	

After normalisation

A_1	A_3	A_5	A_7	m_1	m_2	1	
10·987	− 9·730	4·482	− 0·292	4·644	8·092	− 2·938	
− 9·730	16·111	− 13·924	7·485	− 5·502	− 8·369	2·722	
4·482	− 13·924	27·323	− 24·362	4·712	5·062	− 1·551	= 0
− 0·292	7·485	− 24·362	40·856	− 3·229	− 1·207	0·546	
4·644	− 5·502	4·712	− 3·229	2·387	3·636	− 1·293	
8·092	− 8·369	5·062	− 1·207	3·636	6·197	− 2·177	

TABLE 5

Symmetrical Equations. Zero Lift

Before normalisation

η	A_3	A_5	A_7	m_1	m_2	α_0	1	
0	− 2·3125	3·1875	− 4·0625	0·4740	0·3146	− 0·4375	0	
0·15	− 1·9641	2·1741	− 1·8656	0·4143	0·2864	− 0·3936	− 0·0260	
0·25 i	− 1·5133	0·8556	0·7024	0·3158	0·2531	− 0·3601	− 0·0424	
0·25 o	− 1·3831	0·7652	0·6201	− 0·0261	0·2305	− 0·3022	− 0·0356	
0·35	− 0·8597	− 0·5114	2·3687	− 0·0916	0·1854	− 0·2694	− 0·0477	= 0
0·50 i	0·0000	− 1·8393	2·2750	− 0·1178	0·0694	− 0·2179	− 0·0622	
0·50 o	0·0000	− 2·0572	2·5800	− 0·1316	− 0·1748	− 0·2614	− 0·0747	
0·70	0·9951	− 1·4409	− 1·5969	− 0·1123	− 0·1965	− 0·1755	− 0·0847	
0·90	0·9865	1·0796	− 0·0277	− 0·0533	− 0·0982	− 0·0835	− 0·0653	

After normalisation

A_3	A_5	A_7	m_1	m_2	α_0	1	
16·111	− 13·924	7·485	− 2·437	− 2·444	2·722	0·0568	
− 13·924	27·323	− 24·362	3·300	2·333	− 1·551	0·2240	
7·485	− 24·362	40·856	− 3·137	− 1·029	0·546	− 0·3135	= 0
− 2·437	3·300	− 3·137	0·552	0·367	− 0·367	0·0113	
− 2·444	2·333	− 1·029	0·367	0·416	− 0·388	− 0·0034	
2·722	− 1·551	0·546	− 0·367	− 0·388	0·793	0·1025	

TABLE 6

Antisymmetrical Equations

Before normalisation

η	A_2	A_4	A_6	A_8	n_1	n_2	1	
0	0	0	0	0	0	0	0	
0·15	0·5294	− 1·4623	2·6528	− 3·8968	0·1093	0·0435	− 0·0590	
0·25 i	0·8288	− 2·0806	3·1947	− 3·5496	0·2136	0·0742	− 0·0900	
0·25 o	0·7710	− 1·8780	2·8367	− 3·1192	0·5158	0·0742	− 0·0756	
0·35	0·9914	− 2·0666	2·2348	− 0·8979	0·5708	0·1082	− 0·0943	= 0
0·50 i	1·1857	− 1·6214	0·0000	2·4929	0·5495	0·1911	− 0·1089	
0·50 o	1·2729	− 1·7957	0·0000	2·8415	0·5931	0·4525	− 0·1307	
0·70	1·2055	− 0·0679	− 2·1849	0·2142	0·4456	0·3945	− 0·1229	
0·90	0·6425	1·1692	0·6685	− 0·8852	0·1958	0·1819	− 0·0751	

After normalisation

A_2	A_4	A_6	A_8	n_1	n_2	1	
7·437	− 9·534	6·251	− 2·038	3·268	1·644	− 0·7495	
− 9·534	21·490	− 19·542	10·603	− 4·510	− 1·517	0·9423	
6·251	− 19·542	35·505	− 33·592	2·869	0·064	− 0·6510	= 0
− 2·038	10·603	− 33·592	53·439	− 0·329	0·924	0·2671	
3·268	− 4·510	2·869	− 0·329	1·540	0·705	− 0·3253	
1·644	− 1·517	0·064	0·924	0·705	0·455	− 0·1672	

TABLE 7

Plain Wing, Incidence Solution

Equations: Symmetrical, Table 4.

Constant column: $-(a_0c/8s)\sin\phi$, as given in Table 4.

Solution:

$$K/4sV = 0\cdot3207\sin\phi - 0\cdot0077\sin3\phi + 0\cdot0087\sin5\phi + 0\cdot0003\sin7\phi + 0\cdot1171\,M_{CF25} - 0\cdot1536\,M_{CF50}.$$

Local aerodynamic centre 0·25 chord:

$$dc_L/d\alpha = 4\cdot395.$$

(1) η	(2) $K/4sV$	(3) $x_g/2s$	(4) (2) × (3)	(5) Factors	(6) $c/2s$	(7) C_{LL}	(8) (3) × (6)	(9) Factors
0	0·279	0·0625	0·0174	1·125	0·2500	1·02	0·01562	1
0·05	0·278	0·0625	0·0174	3·375	0·2425	1·04	0·01516	4
0·10	0·275	0·0626	0·0172	3·375	0·2350	1·07	0·01471	2
0·15	0·272	0·0625	0·0170	2·025	0·2275	1·09	0·01422	4
0·20	0·265	0·0625	0·0166	4·262	0·2200	1·10	0·01375	2
0·25	0·254	0·0625	0·0159	1·676	0·2125	1·09	0·01328	4
0·30	0·244	0·0624	0·0152	4·262	0·2050	1·08	0·01279	2
0·35	0·236	0·0625	0·0148	2·148	0·1975	1·09	0·01234	4
0·40	0·227	0·0625	0·0142	2·948	0·1900	1·09	0·01188	2
0·45	0·220	0·0625	0·0138	3·914	0·1825	1·10	0·01141	4
0·50	0·217	0·0626	0·0136	0·890	0·1750	1·13	0·01096	1
0·50	0·217	0·0713	0·0155	0·838	0·2100	0·94	0·01497	1
0·55	0·213	0·0708	0·0151	4·262	0·2010	0·96	0·01423	4
0·60	0·206	0·0705	0·0145	1·9	0·1920	0·98	0·01354	2
0·65	0·197	0·0702	0·0138	4	0·1830	0·98	0·01285	4
0·70	0·186	0·0697	0·0130	2	0·1740	0·97	0·01213	2
0·75	0·175	0·0694	0·0121	4	0·1650	0·97	0·01145	4
0·80	0·161	0·0690	0·0111	2	0·1560	0·94	0·01076	2
0·85	0·145	0·0686	0·0099	4	0·1470	0·90	0·01008	4
0·90	0·124	0·0683	0·0085	1·8	0·1380	0·82	0·00943	2
0·95	0·092	0·0678	0·0062	4·525	0·1290	0·65	0·00875	4
1·00	0	0·0675	0	0·675	0·1200	0	0·00810	1

Aerodynamic centre $= \int$ column 4/$\int$ column 2 = 0·06531 span behind datum

= 0·346 $\bar{c}$ behind datum.

Geometrical mean quarter-chord $= \int$ column 8/$\int$ column 6

= 0·06558 span behind datum = 0·347 $\bar{c}$ behind datum.

TABLE 8

Plain Wing, Induced Drag

η	$K/4sV$	w/V M_{CF25}	w/V M_{CF50}	w/V Fourier	Total w/V	$\left(\frac{w}{V}\right)\left(\frac{K}{4sV}\right)$	Factors
0	0·279	0·1171	− 0·1536	0·3852	0·3487	0·0973	1·125
0·05	0·278	0·1171	− 0·1536	0·3838	0·3473	0·0965	3·375
0·10	0·275	0·1171	− 0·1536	0·3796	0·3431	0·0944	3·375
0·15	0·272	0·1171	− 0·1536	0·3728	0·3363	0·0915	2·025
0·20	0·265	0·1171	− 0·1536	0·3635	0·3270	0·0867	4·262
0·25	0·254	0·1171	− 0·1536	0·3520	0·3155	0·0801	0·838
0·25	0·254	0	− 0·1536	0·3520	0·1984	0·0504	0·838
0·30	0·244	0	− 0·1536	0·3388	0·1852	0·0452	4·262
0·35	0·236	0	− 0·1536	0·3243	0·1707	0·0403	2·148
0·40	0·227	0	− 0·1536	0·3090	0·1554	0·0353	2·948
0·45	0·220	0	− 0·1536	0·2938	0·1402	0·0308	3·914
0·50	0·217	0	− 0·1536	0·2793	0·1257	0·0273	0·890
0·50	0·217	0	0	0·2793	0·2793	0·0606	0·838
0·55	0·213	0	0	0·2666	0·2666	0·0568	4·262
0·60	0·206	0	0	0·2569	0·2569	0·0529	1·9
0·65	0·197	0	0	0·2513	0·2513	0·0495	4
0·70	0·186	0	0	0·2514	0·2514	0·0468	2
0·75	0·175	0	0	0·2589	0·2589	0·0453	4
0·80	0·161	0	0	0·2757	0·2757	0·0444	2
0·85	0·145	0	0	0·3040	0·3040	0·0441	4
0·90	0·124	0	0	0·3462	0·3462	0·0429	1·8
0·95	0·092	0	0	0·4050	0·4050	0·0373	4·525
1·00	0	0	0	0·4836	0·4836	0	0·675

$$C_{Di} = \frac{21 \cdot 19}{(4 \cdot 395)^2} \, \frac{3 \cdot 323}{60} \, C_L{}^2 = 0 \cdot 0608 \, C_L{}^2 .$$

$$\frac{1}{\pi A} = 0 \cdot 0601 .$$

TABLE 9

Symmetrical Wing Twist, Chord × Twist Linear

Equations: Symmetrical, zero lift, Table 5.

Constant column: $-(a_0c/8s)\sin\phi \times$ twist, Table 2.

Solution:

$$K/4sV = -0\cdot0086\sin\phi + 0\cdot0441\sin 3\phi + 0\cdot0018\sin 5\phi + 0\cdot0043\sin 7\phi - 0\cdot0061\,M_{CF25} + 0\cdot0172\,M_{CF50}.$$

$\alpha_0 = -0\cdot2745.$

(1) η	(2) $K/4sV$ per radn	(3) $x_g/2s$	(4) (2) × (3)	(5) Factors	(6) $c/2s$	(7) C_{LL} per radn
0	− 0·0452	0·0625	− 0·00282	1·125	0·2500	− 0·723
0·05	− 0·0445	0·0625	− 0·00278	3·375	0·2425	− 0·734
0·10	− 0·0425	0·0626	− 0·00266	3·375	0·2350	− 0·723
0·15	− 0·0391	0·0625	− 0·00244	2·025	0·2275	− 0·688
0·20	− 0·0346	0·0625	− 0·00216	4·262	0·2200	− 0·629
0·25	− 0·0291	0·0625	− 0·00182	1·676	0·2125	− 0·548
0·30	− 0·0230	0·0624	− 0·00144	4·262	0·2050	− 0·449
0·35	− 0·0165	0·0625	− 0·00103	2·148	0·1975	− 0·334
0·40	− 0·0100	0·0625	− 0·00062	2·948	0·1900	− 0·210
0·45	− 0·0037	0·0625	− 0·00023	3·914	0·1825	− 0·081
0·50	0·0022	0·0626	0·00014	0·890	0·1750	0·050
0·50	0·0022	0·0713	0·00016	0·838	0·2100	0·042
0·55	0·0077	0·0708	0·00055	4·262	0·2010	0·153
0·60	0·0134	0·0705	0·00094	1·9	0·1920	0·279
0·65	0·0189	0·0702	0·00133	4	0·1830	0·413
0·70	0·0244	0·0697	0·00170	2	0·1740	0·561
0·75	0·0298	0·0694	0·00207	4	0·1650	0·722
0·80	0·0351	0·0690	0·00242	2	0·1560	0·900
0·85	0·0399	0·0686	0·00274	4	0·1470	1·086
0·90	0·0429	0·0683	0·00293	1·8	0·1380	1·244
0·95	0·0400	0·0678	0·00271	4·525	0·1290	1·240
1·00	0	0·0675	0	0·675	0·1200	0

$$c_{m0} = \frac{64s^4}{S^2}\int \text{column 4} = -112\cdot28\,\frac{0\cdot00492}{60} = -0\cdot0092.$$

TABLE 10

Fifty per cent Span Flaps. Zero Lift

Equations: Symmetrical, zero lift, Table 5.

Constant column: $-(a_2/a_1)(a_0c/8s)\sin\phi$ from $\eta = 0$ to $0\cdot5$ inner, and zero from $0\cdot5$ outer to tip: the flap/chord ratio is $0\cdot25$ for which $a_2/a_1 = 0\cdot60900$ (*see* Table 2).

Solution:

$$K/4sV = -0\cdot3676\sin\phi + 0\cdot0376\sin 3\phi - 0\cdot0095\sin 5\phi - 0\cdot0045\sin 7\phi + 0\cdot0293\, M_{CF25} + 0\cdot5884\, M_{CF50}.$$

$$\alpha_0 = -0\cdot3708.$$

$$R_5 = \tfrac{1}{2}\sin\Theta^* - \tfrac{1}{4}\sin 2\Theta^* = 0\cdot6495.$$

Local centre of pressure $= \left[0\cdot25 + \dfrac{0\cdot16238\ c/2s}{K/4sV}\right]$ chord for $\eta = 0$ to $0\cdot5$ inner.

(1) η	(2) $K/4sV$	(3) $c/2s$	(4) Local C.P.	(5) $x_g/2s$	(6) (2) × (5)	(7) Factors	(8) C_{LL}
0	0·0447	0·2500	1·158	0·290	0·01295	1·125	0·715
0·05	0·0445	0·2425	1·135	0·277	0·01233	3·375	0·734
0·10	0·0439	0·2350	1·119	0·267	0·01171	3·375	0·747
0·15	0·0428	0·2275	1·113	0·259	0·01108	2·025	0·752
0·20	0·0410	0·2200	1·121	0·254	0·01042	4·262	0·746
0·25	0·0381	0·2125	1·156	0·255	0·00972	1·676	0·717
0·30	0·0344	0·2050	1·218	0·261	0·00897	4·262	0·671
0·35	0·0301	0·1975	1·315	0·273	0·00821	2·148	0·610
0·40	0·0240	0·1900	1·536	0·307	0·00736	2·948	0·505
0·45	0·0141	0·1825	2·352	0·446	0·00629	3·914	0·309
0·50	− 0·0078	0·1750	− 3·393	− 0·575	0·00448	0·890	− 0·178
0·50	− 0·0078	0·2100	0·25	0·071	− 0·00056	0·838	− 0·149
0·55	− 0·0290	0·2010	0·25	0·071	− 0·00205	4·262	− 0·577
0·60	− 0·0369	0·1920	0·25	0·070	− 0·00260	1·9	− 0·769
0·65	− 0·0400	0·1830	0·25	0·070	− 0·00281	4	− 0·874
0·70	− 0·0405	0·1740	0·25	0·070	− 0·00282	2	− 0·931
0·75	− 0·0398	0·1650	0·25	0·069	− 0·00276	4	− 0·965
0·80	− 0·0388	0·1560	0·25	0·069	− 0·00268	2	− 0·995
0·85	− 0·0379	0·1470	0·20	0·069	− 0·00260	4	− 1·031
0·90	− 0·0365	0·1380	0·25	0·068	− 0·00249	1·800	− 1·058
0·95	− 0·0321	0·1290	0·25	0·068	− 0·00218	4·525	− 0·995
1·00	0	0·1200	0·25	0·068	0	0·675	0

$$c_{m0} = -112\cdot28\,\frac{0\cdot21284}{60} = -0\cdot3983.$$

TABLE 11

Discontinuity of Incidence at $\eta = 0{\cdot}25$. *Zero Lift*

Equations: Symmetrical, zero lift, Table 5.

Constant column: $-(a_0c/8s)\sin\phi$ from $\eta = 0$ to $0{\cdot}25$ inner, and zero from $0{\cdot}25$ outer to tip.

Solution:

$$K/4sV = -0{\cdot}3372\sin\phi + 0{\cdot}0486\sin 3\phi - 0{\cdot}0166\sin 5\phi + 0{\cdot}0069\sin 7\phi + 0{\cdot}9833\,M_{CF25} + 0{\cdot}0452\,M_{CF50}.$$

$\alpha_0 = -0{\cdot}3264$. $c_{m0} = 0{\cdot}01416$.

Local centre of pressure 0·25 chord.

η	$K/4sV$	C_{LL}	η	$K/4sV$	C_{LL}
0	0·1048	1·68	0·50	— 0·0359	— 0·68
0·05	0·1034	1·71	0·55	— 0·0390	— 0·78
0·10	0·0988	1·68	0·60	— 0·0406	— 0·85
0·15	0·0899	1·58	0·65	— 0·0418	— 0·91
0·20	0·0736	1·34	0·70	— 0·0425	— 0·98
0·25	0·0361	0·68	0·75	— 0·0425	— 1·03
0·30	— 0·0013	— 0·03	0·80	— 0·0413	— 1·06
0·35	— 0·0174	— 0·35	0·85	— 0·0378	— 1·03
0·40	— 0·0264	— 0·56	0·90	— 0·0313	— 0·91
0·45	— 0·0318	— 0·70	0·95	— 0·0207	— 0·64
0·50	— 0·0359	— 0·82	1·00	0	0

TABLE 12

Uniform Roll: 1 *Radian at Tip* $\equiv$ *Unit* $V/\omega s$

Equations: Anti-symmetrical, Table 6.

Constant column: $-(a_0c/8s)\,\eta\sin\phi$.

Solution:

$$K/4sV = 0{\cdot}0986\sin 2\phi + 0{\cdot}0027\sin 4\phi + 0{\cdot}0035\sin 6\phi - 0{\cdot}0006\sin 8\phi - 0{\cdot}0210\,M_{TA25} + 0{\cdot}0533\,M_{TA50}.$$

$C_l = -0{\cdot}438$.

Local centre of pressure 0·25 chord.

η	$K/4sV$ per radn	C_{LL} per unit $(-C_l)$	η	$K/4sV$ per radn	C_{LL} per unit $(-C_l)$
0	0	0	0·50	0·0859	3·74
0·05	0·0109	0·41	0·55	0·0924	4·20
0·10	0·0215	0·84	0·60	0·0971	4·62
0·15	0·0318	1·28	0·65	0·1007	5·03
0·20	0·0413	1·72	0·70	0·1031	5·42
0·25	0·0497	2·14	0·75	0·1041	5·76
0·30	0·0574	2·56	0·80	0·1030	6·03
0·35	0·0649	3·00	0·85	0·0988	6·14
0·40	0·0720	3·46	0·90	0·0893	5·91
0·45	0·0787	3·94	0·95	0·0697	4·94
0·50	0·0859	4·48	1·00	0	0

TABLE 13

Aileron from $\eta = 0\cdot5$ to $1\cdot0$, Wing at Zero Incidence

Equations: Anti-symmetrical, Table 6.

Constant column: Zero from $\eta = 0$ to $0\cdot5$ inner:
$-(a_2/a_1)(a_0c/8s)\sin\phi$ for $\eta = 0\cdot5$ outer to tip.
The value of E varies from $0\cdot25$ at $\eta = 0\cdot5$ to $0\cdot33$ at $\eta = 0\cdot9$: the corresponding values of a_2/a_1 are given in Table 1.

Solution:

$$K/4sV = -0\cdot1016\sin 2\phi - 0\cdot0006\sin 4\phi + 0\cdot0082\sin 6\phi$$
$$- 0\cdot0015\sin 8\phi + 0\cdot0105\, M_{TA25} + 0\cdot6317\, M_{TA50}.$$

$C_l = -0\cdot319.$

η	$K/4sV$	C_{LL} per unit $(-C_l)$	η	$K/4sV$	C_{LL} per unit $(-C_l)$
0	0	0	0·50	0·0541	3·23
0·05	0·0026	0·13	0·55	0·0760	4·75
0·10	0·0051	0·27	0·60	0·0839	5·49
0·15	0·0074	0·41	0·65	0·0871	5·97
0·20	0·0096	0·55	0·70	0·0875	6·31
0·25	0·0119	0·70	0·75	0·0860	6·54
0·30	0·0144	0·88	0·80	0·0829	6·67
0·35	0·0176	1·12	0·85	0·0775	6·62
0·40	0·0227	1·50	0·90	0·0686	6·24
0·45	0·0318	2·19	0·95	0·0525	5·11
0·50	0·0541	3·88	1·00	0	0

TABLE 14

Composite Leading for Flaps and Ailerons. Zero Lift

Specification: ΔC_L due to flaps = 1.
Rolling moment due to ailerons + 0·1.

Incidence for zero lift: − 0·2275 radn

Aileron deflection: − 0·3138 radn

Composite $K/4sV$ = 0·6135 (flaps) − 0·3138 (ailerons).

$$\text{Local } C_m = \frac{M dy}{\frac{1}{2}\rho v^2 \bar{c} S} = -\frac{8s^4}{S} R_5 \eta' \left(\frac{c}{2s}\right)^2 d\eta = B d\eta .$$

η	$K/4sV$ starboard	$K/4sV$ port	$c/2s$	R_5	η'	B
0	0·0274	0·0274	0·2500	0·6495	0·6135	− 0·350
0·05	0·0265	0·0281	0·2425	0·6495	0·6135	− 0·329
0·10	0·0253	0·0285	0·2350	0·6495	0·6135	− 0·309
0·15	0·0240	0·0286	0·2275	0·6495	0·6135	− 0·290
0·20	0·0222	0·0282	0·2200	0·6495	0·6135	− 0·271
0·25	0·0197	0·0271	0·2125	0·6495	0·6135	− 0·253
0·30	0·0166	0·0256	0·2050	0·6495	0·6135	− 0·235
0·35	0·0130	0·0240	0·1975	0·6495	0·6135	− 0·218
0·40	0·0076	0·0218	0·1900	0·6495	0·6135	− 0·202
0·45	− 0·0013	0·0187	0·1825	0·6495	0·6135	− 0·186
0·50	− 0·0218	0·0122	0·1750	0·6495	0·6135	− 0·171
0·50	− 0·0218	0·0122	0·2100	0·6495	$\pm$ 0·3138	$\mp$ 0·126
0·55	− 0·0416	0·0060	0·2010	0·6492	$\pm$ 0·3138	$\mp$ 0·116
0·60	− 0·0489	0·0037	0·1920	0·6482	$\pm$ 0·3138	$\mp$ 0·105
0·65	− 0·0518	0·0028	0·1830	0·6466	$\pm$ 0·3138	$\mp$ 0·095
0·70	− 0·0523	0·0027	0·1740	0·6443	$\pm$ 0·3138	$\mp$ 0·086
0·75	− 0·0514	0·0026	0·1650	0·6416	$\pm$ 0·3138	$\mp$ 0·077
0·80	− 0·0498	0·0022	0·1560	0·6382	$\pm$ 0·3138	$\mp$ 0·068
0·85	− 0·0476	0·0010	0·1470	0·6344	$\pm$ 0·3138	$\mp$ 0·060
0·90	− 0·0439	− 0·0009	0·1380	0·6301	$\pm$ 0·3138	$\mp$ 0·053
0·95	− 0·0362	− 0·0032	0·1290	0·6253	$\pm$ 0·3138	$\mp$ 0·046
1·00	0	0	0·1200	0·6201	$\pm$ 0·3138	$\mp$ 0·039

TABLE 15

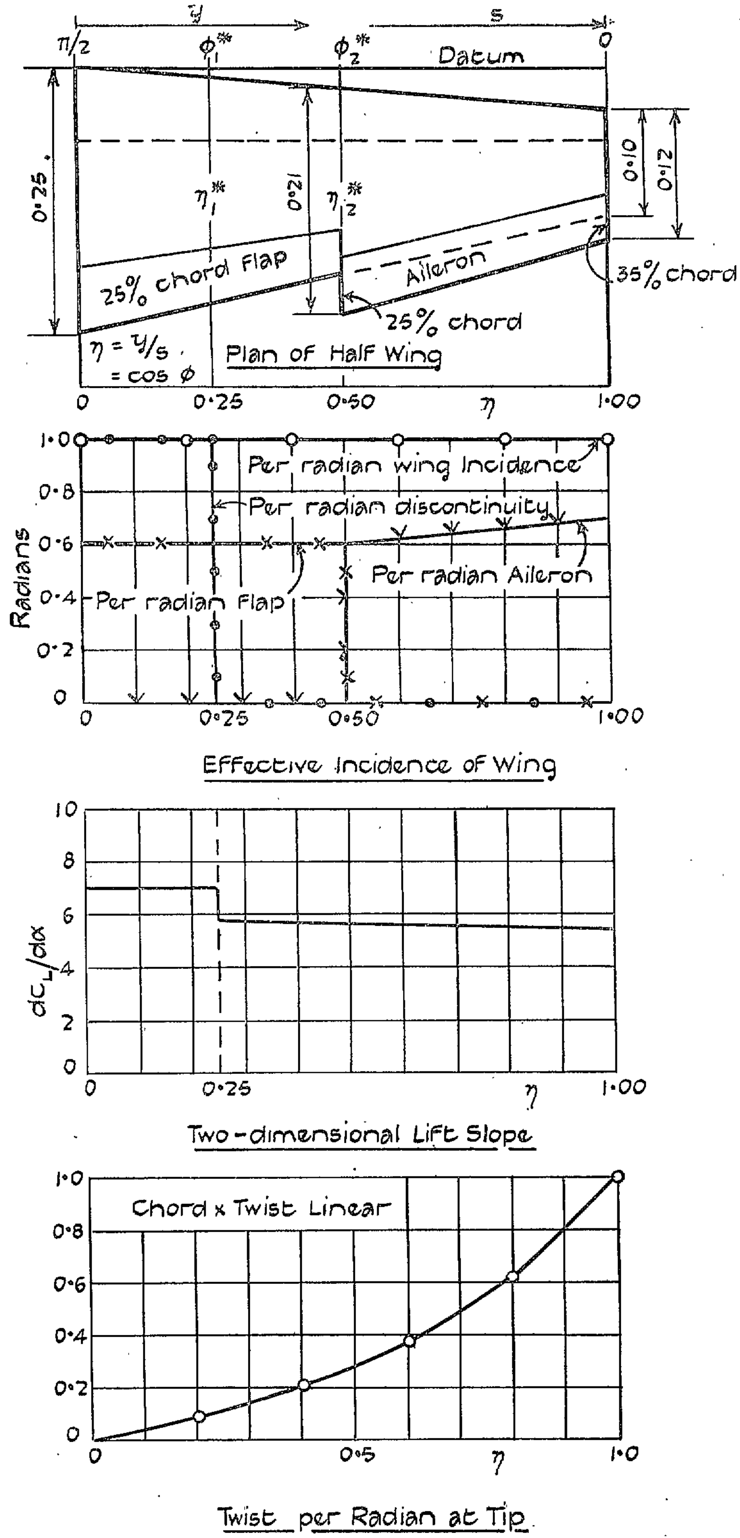
Yawing Moment with Deflected Flaps and Ailerons. Zero Lift

η	$K/4sV$ Symml.	w/V Antisymmetrical				$\left(\frac{w}{V}\right)\left(\frac{K}{4sV}\right)$	$K/4sV$ Antisymml	w/V Symmetrical				$\left(\frac{w}{V}\right)\left(\frac{K}{4sV}\right)$	Integrating Factors
		M_{TA25}	M_{TA50}	Fourier	Total			M_{CF25}	M_{CF50}	Fourier	Total		
0	0·0274	0	0	0	0	0	0	0·0180	0·3610	—0·3045	0·0745	0	0
0·05	0·0273	0	0	0·0002	0·0002	0·00001	—0·0008	0·0180	0·3610	—0·3041	0·0749	—0·00006	0·169
0·10	0·0269	0	0	0·0010	0·0010	0·00003	—0·0016	0·0180	0·3610	—0·3028	0·0762	—0·00012	0·338
0·15	0·0263	0	0	0·0029	0·0029	0·00008	—0·0023	0·0180	0·3610	—0·3003	0·0787	—0·00018	0·304
0·20	0·0252	0	0	0·0064	0·0064	0·00016	—0·0030	0·0180	0·3610	—0·2963	0·0827	—0·00025	0·852
0·25	0·0234	0	0	0·0118	0·0118	0·00028	—0·0037	0·0180	0·3610	—0·2904	0·0886	—0·00033	0·210
0·25	0·0234	—0·0033	0	0·0118	0·0085	0·00020	—0·0037	0	0·3610	—0·2904	0·0706	—0·00026	0·210
0·30	0·0211	—0·0033	0	0·0193	0·0160	0·00034	—0·0045	0	0·3610	—0·2820	0·0790	—0·00036	1·279
0·35	0·0185	—0·0033	0	0·0289	0·0256	0·00047	—0·0055	0	0·3610	—0·2706	0·0904	—0·00050	0·752
0·40	0·0147	—0·0033	0	0·0403	0·0370	0·00054	—0·0071	0	0·3610	—0·2559	0·1051	—0·00075	1·179
0·45	0·0087	—0·0033	0	0·0532	0·0499	0·00043	—0·0100	0	0·3610	—0·2376	0·1234	—0·00123	1·761
0·50	—0·0048	—0·0033	0	0·0668	0·0635	—0·00030	—0·0170	0	0·3610	—0·2157	0·1453	—0·00247	0·445
0·50	—0·0048	—0·0033	—0·1982	0·0668	—0·1347	0·00065	—0·0170	0	0	—0·2157	—0·2157	0·00367	0·419
0·55	—0·0178	—0·0033	—0·1982	0·0804	—0·1211	0·00216	—0·0238	0	0	—0·1908	—0·1908	0·00454	2·344
0·60	—0·0226	—0·0033	—0·1982	0·0930	—0·1085	0·00245	—0·0263	0	0	—0·1638	—0·1638	0·00431	1·140
0·65	—0·0245	—0·0033	—0·1982	0·1036	—0·0979	0·00240	—0·0273	0	0	—0·1364	—0·1364	0·00372	2·600
0·70	—0·0248	—0·0033	—0·1982	0·1112	—0·0903	0·00224	—0·0275	0	0	—0·1111	—0·1111	0·00306	1·400
0·75	—0·0244	—0·0033	—0·1982	0·1149	—0·0866	0·00211	—0·0270	0	0	—0·0916	—0·0916	0·00247	3·000
0·80	—0·0238	—0·0033	—0·1982	0·1139	—0·0876	0·00208	—0·0260	0	0	—0·0824	—0·0824	0·00214	1·600
0·85	—0·0233	—0·0033	—0·1982	0·1081	—0·0934	0·00218	—0·0243	0	0	—0·0898	—0·0898	0·00218	3·400
0·90	—0·0224	—0·0033	—0·1982	0·0976	—0·1039	0·00233	—0·0215	0	0	—0·1215	—0·1215	0·00261	1·620
0·95	—0·0197	—0·0033	—0·1982	0·0836	—0·1179	0·00232	—0·0165	0	0	—0·1872	—0·1872	0·00309	4·299
1·00	0	—0·0033	—0·1982	0·0680	—0·1335	0	0	0	0	—0·2985	—0·2985	0	0·675

C_n: First part $10 \cdot 596 \frac{0 \cdot 05064}{60} = 0 \cdot 00894$

Second part $10 \cdot 596 \frac{0 \cdot 06138}{60} = 0 \cdot 01084$

Total: 0·01978.



y
s
π/2
ϕ_1^*
ϕ_2^*
Datum
0
0·25
η_1^*
0·21
η_2^*
0·10
0·12
25% chord Flap
Aileron
35% chord
25% chord
η = y/s
= cos φ
Plan of Half Wing
0
0·25
0·50
η
1·00
1·0
0·8
0·6
0·4
0·2
0
Radians
Per radian wing Incidence
Per radian discontinuity
Per radian Aileron
Per radian Flap
0
0·25
0·50
1·00
Effective Incidence of Wing
10
8
6
4
2
0
$dc_L/d\alpha$
0
0·25
η
1·00
Two-dimensional Lift Slope
1·0
0·8
0·6
0·4
0·2
0
Chord x Twist Linear
0
0·5
η
1·0
Twist per Radian at Tip

FIG. 1.

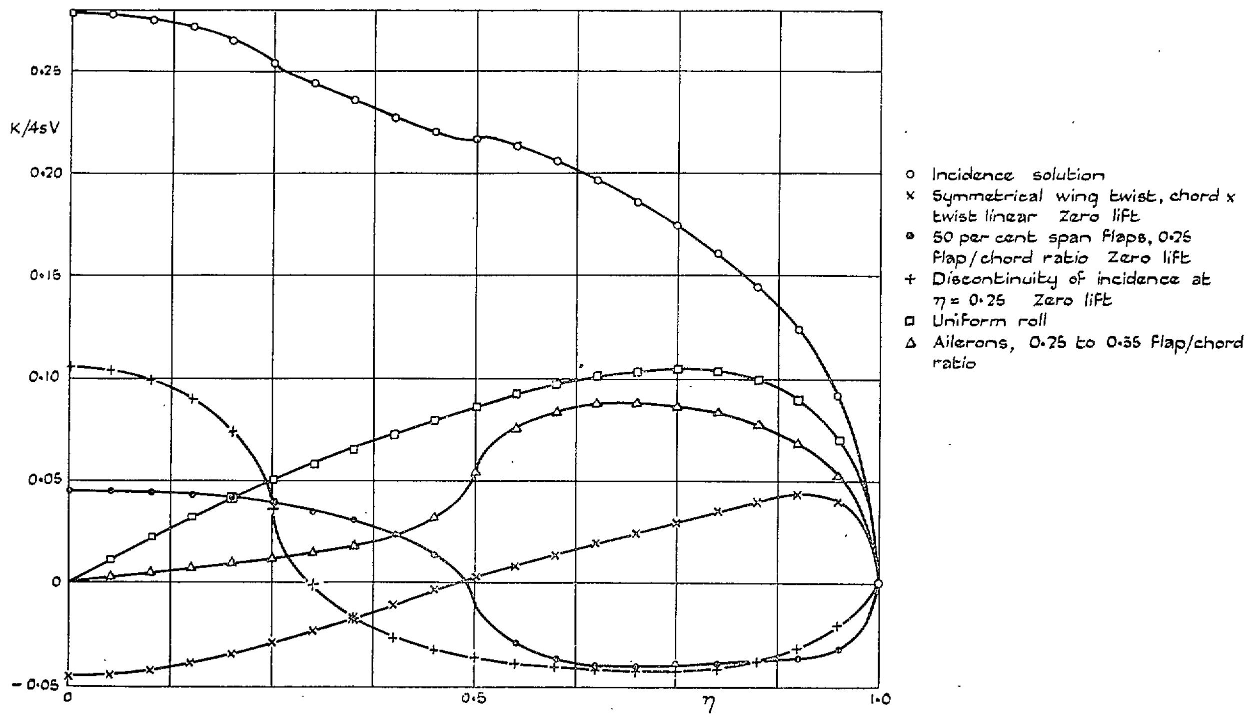


FIG. 2. Half span distribution of circulation for various solutions.

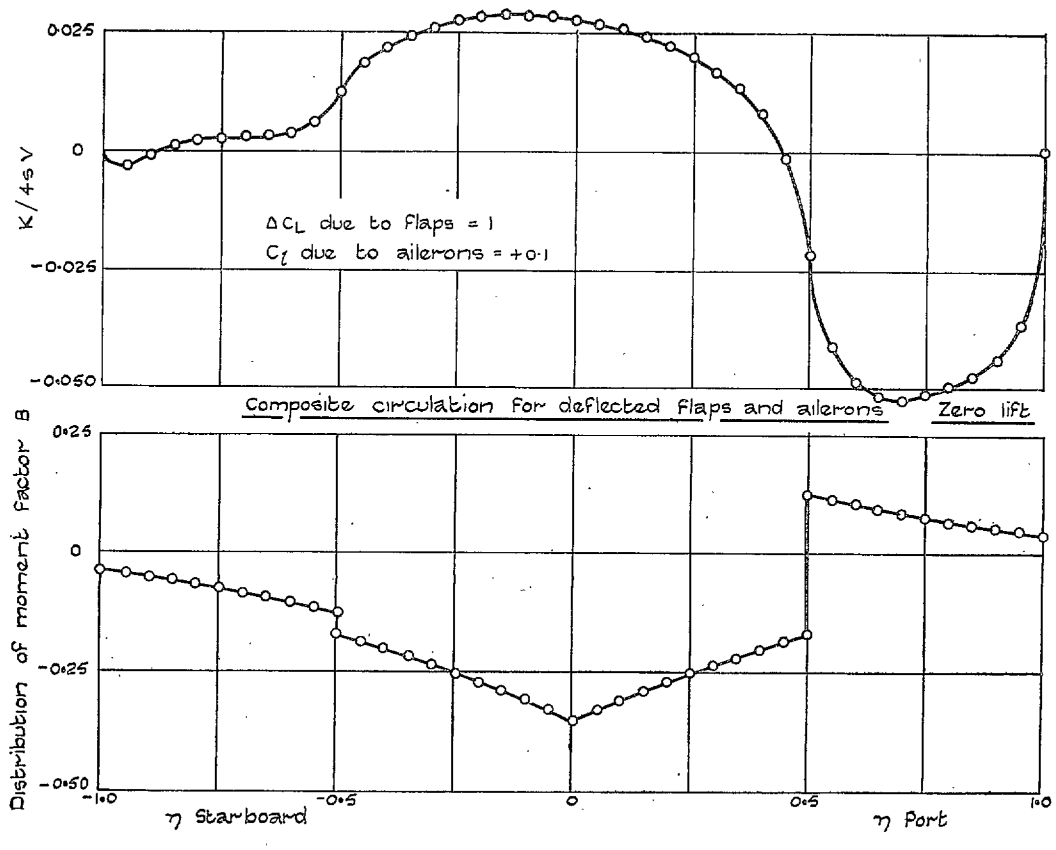


FIG. 3. Moment coefficient about quarter-chord. Deflected flaps and ailerons.

(21061) Wt. 15-680 K9 2/52 F. M. & S. PRINTED IN GREAT BRITAIN

R. & M. No. 2592
(10,922)
A.R.C. Technical Report

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)—

1934–35 Vol. I. Aerodynamics. *Out of print.*
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40*s.* (40*s.* 8*d.*)

1935–36 Vol. I. Aerodynamics. 30*s.* (30*s.* 7*d.*)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30*s.* (30*s.* 7*d.*)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s.* (50*s.* 10*d.*)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 10*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s.* (61*s.*)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50*s.* (51*s.*)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s.* (30*s.* 9*d.*)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (50*s.* 11*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 2*d.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s.* (51*s.*)

Certain other reports proper to the 1940 *volume will subsequently be included in a separate volume.*

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933–34	1*s.* 6*d.* (1*s.* 8*d.*)
1934–35	1*s.* 6*d.* (1*s.* 8*d.*)
April 1, 1935 to December 31, 1936.	4*s.* (4*s.* 4*d.*)
1937	2*s.* (2*s.* 2*d.*)
1938	1*s.* 6*d.* (1*s.* 8*d.*)
1939–48	3*s.* (3*s.* 2*d.*)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 R. & M. No. 2600. 2*s.* 6*d.* (2*s.* 7½*d.*)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 4½d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 1½*d.*)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1*s.* (1*s.* 1½*d.*)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 4½*d.*)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 4½*d.*)

Prices in brackets include postage.

Obtainable from
HER MAJESTY'S STATIONERY OFFICE
York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST
or through any bookseller.

S.O. Code No. 23-2592