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Tests on Probes Designed to Measure Stagnation Pressure Directly in a Supersonic Stream

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Summary

The geometry is given of a probe capable of measuring stagnation pressure directly in a supersonic flow, by means of an isentropic compression. Tests have shown that such a probe can indicate the local stagnation pressure to within 0·1 per cent for Mach numbers up to 2·2. Above $M = 2{\cdot}2$ the pressure recovery begins to fall from unity and precalibration becomes necessary.

* Replaces R.A.E. Technical Report 73123—A.R.C. 35 091.

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1. Introduction

At supersonic speeds, local stagnation pressure is usually determined indirectly from measurements using both pitot and static tubes, together with an estimated value of the specific heat ratio, γ. The stagnation pressure is obtained by multiplying the pitot reading by the Rayleigh correction factor, F, to allow for the loss in total pressure across the normal shock at the mouth of the pitot. F is normally expressed as a function of Mach number and γ, and is determined using the measured static and pitot pressures.

This indirect method can lead to an appreciable uncertainty in the deduced value of stagnation pressure, which is due partly to the sensitivity of the relevant equations to small errors in the measured quantities, and partly to the well known difficulties inherent in measuring static pressure accurately.

Errors in the measured quantities are most likely to occur in flows which are non-uniform and it is in such cases that local values of stagnation pressure are most commonly required. An example occurs in surveys across an under-expanded turbojet exhaust, where the problem is compounded not only by strong radial gradients in Mach number and flow angle, but also by the difficulty in evaluating the specific heat ratio γ, which depends on temperature and gas composition and can vary locally between wide limits.

In the case of supersonic wind-tunnel tests, it is often possible to measure stagnation pressure directly by means of a pitot installed upstream of the working section in a subsonic part of the tunnel circuit. This possibility is excluded whenever non-isentropic processes occur between the pitot and the test section. Obvious examples are propulsion experiments involving heat addition and flow surveys near models where shocks or viscous regions are present. Another factor casting doubt on the use of an upstream pitot is the possibility of condensation shocks. Although their existence is often neglected it should be noted that condensation shocks are extremely difficult to avoid at Mach numbers above 2 and even if the air is dried to a frost point of $-25°C$, the loss in stagnation pressure is still typically 1 per cent.

The difficulties associated with accurate measurement of stagnation pressure would be circumvented by a probe which was capable of directly recovering the full stagnation pressure when placed in a supersonic flow. In addition an independent means of measuring stagnation pressure would provide an alternative method of calculating local Mach number if the probe were used in combination with either an ordinary pitot tube or a static tube.

Ideally such a probe should have a pressure recovery of unity for a wide range of values of M, γ and flow angle; that is, the instrument should always sense the full stagnation pressure. However, even if the probe were to recover only a known fraction of the stagnation pressure, provided this recovery factor were less sensitive to M and γ than that of a pitot tube, the instrument would still be superior in accuracy.

To achieve such behaviour, the discontinuous normal shock in front of an ordinary pitot must be replaced by an isentropic or near-isentropic compression. Thus, the required probe must consist of a compression surface placed in the flow and a tube mounted so as to sample a streamline which has been decelerated as nearly isentropically as possible to a Mach number near unity. This principle has been described[1] and used[2] previously with sensing tubes mounted on very simple compression surfaces such as cones or wedges.

The need for a more suitable compression surface has been considered by Goodyer,[3] who found a simple shape which theoretically gives a fully isentropic compression for a range of flow angles at all Mach numbers up to $M = [(\gamma + 3)/2]^{\frac{1}{2}}$, ($= 1{\cdot}483$ for air). Initial tests were promising and led to the development of a considerably improved probe design.[4]

This Report describes a wind-tunnel evaluation of a set of probes which were originally manufactured to resolve uncertainties in the stagnation pressure and Mach number distribution at the inlet plane of a model intake rig for the 3 ft × 4 ft supersonic tunnel at R.A.E., Bedford.

2. Development of the Probe Geometry

2.1. Design Considerations

The desired probe design consists of a sensing tube mounted so as to sample a streamline which has been decelerated by a fixed geometry compression surface, and we require that the pressure recovery, defined as the ratio of the sensed pressure to the true stagnation pressure, should be as near unity as possible for a range of Mach numbers and flow orientations.

The simplest possible compression surface is the wedge and this has been used previously in a probe design.[2] However the wedge angle required to maximise the pressure recovery varies quite rapidly with Mach number at low supersonic speeds, so that when a design of fixed geometry is considered, it cannot act very efficiently over a wide range of Mach numbers or orientations. In addition, below a certain Mach number the shock will detach and the pressure recovery will revert approximately to that of a conventional pitot with a normal shock. A conical compression surface gives slightly better recoveries due to the additional compression field between the shock and the cone surface but suffers from the same limitations.

Large improvements in pressure recovery can be made by replacing the single shock with a number of weaker ones. This is of direct relevance to the design of external compression intakes for engines, where it is normal practice to bring all the waves to a common focus at a point just outside the cowl lip, in order to achieve uniform flow over the whole intake plane. The limiting case is a surface of continuous curvature which in two-dimensional flow will be a reversed streamline of a Prandtl–Meyer corner expansion. Such an isolated surface immersed in a uniform flow will not necessarily be capable of giving fully isentropic compression down to a local Mach number of unity, as might be expected from a simple reversal of Prandtl–Meyer duct flow. In the absence of a physical wall to represent the corner streamline, the incident compression fan will not be cancelled. Consequently the boundary conditions at the wave focus are different with an isolated compression surface.

Connors[5] has considered the problem of an isolated segment of a reversed Prandtl–Meyer streamline which declerates the flow from free-stream conditions to some lower Mach number through a fan of waves centred to a single focus. His flow model at the wave focus consisted of the isentropic compression fan, a reflected compression or expansion wave, a shock wave and a vortex sheet; all of which radiated from a single point. Above a Mach number of about 2 it was impossible to find a solution giving equal pressures and flow directions across the vortex sheet unless the compression fan terminated before the flow had been decelerated fully to sonic velocity. A tube mounted downstream of the fan, so as to sample a compressed streamline, would thus still generate a normal shock which would limit the pressure recovery. A similar compression limit is encountered for the axisymmetric case of waves focussed in a ring.

The flow model analysed by Connors is only relevant to compression surfaces which focus the waves to a single point, but for our purposes this is not necessary. The optimised pressure recovery solution for a system of $(n - 1)$ oblique shocks followed by a normal shock (in our case at the mouth of the sensing tube) has been known for many years[6,7] and similar solutions are also available for the axisymmetric case.[8] This analysis gives the required set of wedge or cone element angles at the design Mach number, but does not consider conditions at any shock intersection. It is tacitly assumed that the solution is valid at least close to the compression surface. The geometry is not fixed uniquely since the length of each element can be varied arbitrarily. Rather than adjust the elements to give centred waves it may be preferable to arrange them so that the pressure gradient along the compression surface is uniform, thus reducing the likelihood of boundary layer separation.

A computation of the optimised two-dimensional solution giving a uniform pressure gradient for the case $n = 17$ and $M = 2{\cdot}455$, gave a pressure recovery of 0·9963 and showed that the curvature of the compression polygon did not vary greatly along its length, giving an approximation to a circular arc profile.

The possible probe designs considered so far can only be optimised for maximum pressure recovery at a single Mach number and are inherently sensitive to changes in flow direction. A different approach to the compression surface design has been suggested by Goodyer,[3] who considered the flow near a swept cyclinder as sketched in Fig. 1. Results based on his simple analysis for the pressure recovery of an infinite swept cylinder are given in the Appendix, and it is found that up to a Mach number of $[(\gamma + 3)/2]^{\frac{1}{2}}$ the pressure recovery is identically unity for a finite range of flow directions, Λ, relative to the cyclinder. At higher Mach numbers the recovery is below unity but can be maximised by a suitable choice of the sweepback angle to give recoveries higher than those which can be achieved by either the single-wedge or single-cone probes. The optimum sweep is a very weak function of M and γ so that if a fixed geometry design is used over a range of Mach numbers, it can give very similar results to designs whose geometry is optimised at each Mach number.

A practical probe design must of course be of finite length, and a shock will arise from the upstream tip, but as shown by Fig. 2 the influence of the tip only extends downstream for a few probe diameters. By analogy with the multi-shock configurations already mentioned, substantial increases in recovery can be realised by curving the tip towards the free-stream direction, thus replacing the bow shock with a system of weak compression waves. Figure 2 also shows that although a favourable pressure gradient exists near the tip of the straight cylinder, increasing the curvature of the tip gives a progressively more unfavourable gradient, until eventually the possibility of boundary layer separation may have to be considered.

2.2 Details of the Probes Tested

The probe designs tested* are shown in Fig. 3 and all have cylindrical compression surfaces which have been curved through a 48 degree sector of a circular arc. The included angle at the tip of each probe is 7 degrees with a nominal tip thickness of 0·1 mm. The circular arc profile was chosen merely on the grounds of ease of manufacture, although as discussed in Section 2.1, such a curve does give an approximately uniform pressure gradient for the simpler case of two-dimensional flow.

* The design of these and similar probes is the subject of British Patent Application No. 1308080 (1973).

Probes 1 to 6, including the unmodified version of Probe 5, are all nominally identical, whereas Probes 7 and 8 are respectively 2/3 and 1/3 scale models of Probes 1 to 6, but retain the same size sensing tubes.

Various modifications were made to Probe 5 to explore the effects of changing the size and position of the sensing tube.

3. Test Procedure and Presentation of Results

The facility used was the R.A.E. No. 8 supersonic wind tunnel, which has a 23 cm square working section. The tests were made at Mach numbers of 1·51, 1·86, 2·10 and 2·45, using a stagnation temperature of 30°C and atmospheric stagnation pressure. Except during tests when humidity was intentionally varied, the air was predried very efficiently to frost points in the range −44°C to −53°C.

Values of pressure recovery, R, defined as the ratio of the pressure sensed by the probe, P_p, to the stagnation pressure in the tunnel contraction, H_0, were measured for each probe over a wide range of pitch and yaw angles. These angles could be repeated to an accuracy of ±0·05 degress and the values shown have had small corrections applied to account for the slight flow direction asymmetries in the tunnel. Sensitive instrumentation was used so that errors in the measured values of R due to instrument errors alone were below 0·02 per cent. This high accuracy was obtained by measuring H_0 using a conventional self-balancing capsule manometer and then recording the small pressure difference $\Delta p = (H_0 - P_p)$ on a sensitive oil manometer. R is then given by $R = (1 - \Delta p/H_0)$, and because $\Delta p \ll H_0$, the resulting accuracy is considerably higher than that which could be achieved if P_p were measured absolutely. Absolute values of P_p were also monitored with a capsule manometer and used for checking purposes.

All experimental results for R plotted as full lines are for the measured values of P_p/H_0, whereas strictly speaking $R = P_p/H$, where H is the local value of stagnation pressure in the working section. Slight differences between H and H_0 could arise if condensation shocks occurred upstream of the model or if non-uniformities in stagnation pressure were present across the tunnel contraction.

In order to minimise any possible errors from the latter cause, the different probes were arranged to sample approximately the same streamlines when mounted in the tunnel. Additional experiments involving pitot traverses just upstream of the liner throat verified that there was no measurable gradient in stagnation pressure across the width of the tunnel near its centre-line and that the stagnation pressure in the throat region was within ±0·02 per cent of H_0.

Subsidiary experiments were made to evaluate the loss in stagnation pressure due to condensation shocks, and the measured losses are shown in Fig. 4. During these tests the humidity was varied by admitting controlled quantities of atmospheric air upstream of the tunnel working section. For the range of frost points used during the probe calibrations no condensation shocks occurred at $M = 1{\cdot}51$ or 1·86, so that for these Mach numbers the measured value H_0 is the same as the true local stagnation pressure, H, in the working section. At $M = 2{\cdot}10$ and 2·45 however, it proved impossible to avoid weak condensation shocks in spite of the efficient drying plant. At the higher Mach numbers, pressure recoveries are therefore shown in the relevant figures both as the measured values of P_p/H_0, plotted using full lines, and as values of P_p/H, corrected using Fig. 4 and plotted as dashed lines. At $M = 2{\cdot}45$ the experimental errors in measuring frost point lead to uncertainties in the corrected value of R of up to ±0·1 per cent.

4. Comparison of Probe Performance with Alternative Stagnation Pressure Probes

4.1. Comparison of Pressure Recoveries

Figure 5 shows a comparison of the pressure recoveries achieved experimentally for Probes 1 to 6 with theoretical values for alternative probe designs. The alternative designs considered each consist of a sensing tube mounted on a fixed-geometry compression surface which has been optimised to give the maximum possible pressure recovery at $M = 2{\cdot}5$.

Both the wedge and cone probes have limiting Mach numbers below which the bow-shock detaches, reducing the pressure recovery to that of a normal shock for a sensing tube mounted flush with the surface. The precise variation of R with Mach number close to this discontinuity would in fact depend somewhat on the sensing tube location. Changes in probe orientation will also cause changes in the pressure recovery so that the cone or wedge probes would be unsuitable for use over a wide range of Mach number and incidence.

The upper limits of pressure recovery consistent with Connors[5] flow model of an isentropic fan of waves centred to a single focus, as discussed in Section 2, are also shown for both the two-dimensional and axisymmetric case. These limits are not for fixed geometries but represent designs which have been optimised continuously as Mach number is varied.

The theoretical pressure recovery for a long straight cylinder probe, of the type shown in Fig. 1, does not suffer from any discontinuities. As shown in Fig. 5 it gives a higher pressure recovery than either the cone or wedge, and in addition there is a range of Mach numbers for which R is identically unity. Moreover in this range of Mach number, the pressure recovery remains unity for a finite range of incidences (sweep angles), as discussed in the Appendix.

Although an infinitely long cylinder hardly makes a practical probe design it does indicate the type of geometry that is required, and with the further isentropic compression which can be gained by curving the cylinder axis, the experimentally measured pressure recoveries shown in Fig. 5 for Probes 1 to 6 are virtually unity for a wide range of Mach numbers. The experimental results are shown in more detail on the greatly enlarged vertical scale of Fig. 6. Probes 7 and 8 which had progressively smaller compression surfaces in relation to the sensing tube diameter gave somewhat lower pressure recoveries.

As predicted for the infinite cylindrical probe, the results for the probes tested show that at low Mach numbers R remains close to unity for a wide range of pitch angles. In addition R is remarkably insensitive to yaw as indicated by Fig. 7. This figure shows the range of pitch angles at zero yaw and the range of yaw angles at constant pitch for which the pressure recovery is always above 0·99. Thus at $M = 2{\cdot}0$, Probe 1 has a total pitch (i.e. cylinder sweep) range of 20 degrees and a total yaw range of 33 degrees before R falls below 0·99.

4.2. Comparison of Sensitivity to Errors

In order to deduce the local value of stagnation pressure in an unknown supersonic stream, one conventional method is to use measurements of the pitot pressure downstream of a normal shock, P_{pit}, and the local static pressure, P_s, together with an estimate of the specific heat ratio, γ.

Generalisation to measurements using a probe of pressure recovery $R = P_p/H = R(M, \gamma)$, in addition to the local static pressure, leads to the result that the percentage errors in deduced stagnation pressure for a specified error in either P_s or γ are given by

$$\frac{(\delta H/H)}{(\delta P_s/P_s)} = \frac{S\left(\dfrac{\partial R}{\partial M}\right)}{S\left(\dfrac{\partial R}{\partial M}\right) - R\left(\dfrac{\partial S}{\partial M}\right)}, \text{ for no error in } \gamma \text{ or } P_p.$$

$$\frac{(\delta H/H)}{(\delta \gamma/\gamma)} = \gamma\left[\frac{\left(\dfrac{\partial S}{\partial M}\right)\left(\dfrac{\partial R}{\partial \gamma}\right) - \left(\dfrac{\partial S}{\partial \gamma}\right)\left(\dfrac{\partial R}{\partial M}\right)}{S\left(\dfrac{\partial R}{\partial M}\right) - R\left(\dfrac{\partial S}{\partial M}\right)}\right], \text{ for no error in } P_s \text{ or } P_p$$

where $S = P_s/H = S(M, \gamma)$.

These relations have been evaluated numerically for $\gamma = 1{\cdot}4$ and are shown in Fig. 8 for the case of the conventional pitot, the infinite straight cylinder probe, and for the mean experimental results of Probes 1 to 6.

In sipersonic flowfields where flow directions and Mach number gradients exist, it is difficult to measure P_s to accuracies of the order of 2 or 3 per cent, and similar errors in estimates of γ are likely when traversing a turbojet exhaust. The marked superiority of Probes 1 to 6 in error sensitivity is evident from Fig. 8, and in fact if it were known in advance that the local Mach numbers did not exceed $M = 2{\cdot}2$, the local stagnation pressure could be read directly using the probe, making subsidiary measurements of static pressure or γ superfluous. The comparison shown in Fig. 8 for $\gamma = 1{\cdot}4$ becomes even more unfavourable for the conventional pitot tube at lower values of γ, which occur in turbojet exhausts.

5. Discussion of Detailed Calibration Results

5.1. Variation of Pressure Recovery with Pitch and Yaw

The effects of pitch and yaw on pressure recovery are shown in detail for a range of Mach numbers in Figs. 9 to 17. Figures 10 and 11 show a plateau region in which R is independent of α. This plateau is similar in nature to that predicted in the Appendix for an infinite cylinder. The upper incidence limit of the plateau occurs when M_L becomes supersonic, so that a normal shock appears at the mouth of the sensing tube, whereas the lower limit corresponds to an increase in the strength of the compression surface bow shock.

Probes 1 to 6 which are of nominally identical design all give closely similar results below $M = 2{\cdot}1$, with pressure recoveries of virtually unity for a wide range of pitch angles. However at the highest Mach number tested, $M = 2{\cdot}45$, Figs. 13 to 15 show that there are differences of up to 1 per cent in the pressure recovery between individual probes. Subsidiary experiments indicated that these differences were caused by manufacturing errors in the compression surface rather than in the sensing tube or its location. Thus manufacturing errors in the geometry of the compression surface which are insignificant at low supersonic speeds become important at higher speeds and may make individual calibrations mandatory if the probes are to be used beyond about $M = 2{\cdot}2$.

Probes 7 and 8, which had progressively smaller compression surfaces but the same diameter sensing tube as the other probes, showed generally poorer pressure recoveries, particularly at higher Mach numbers.

The tips of all the probes were truncated at an angle of 7 degrees so that at $\alpha = 7$ degrees the upstream end of the compression surface was aligned with the free stream flow vector, and Figs. 13 and 14 show that the optimum pitch angle for Probes 1 to 6 occurred near $\alpha = 7$ degrees. However this correspondence may be little more than fortuitous since Fig. 16 shows that Probes 7 and 8 have significantly higher optimum pitch angles, and as discussed in Section 5.2, the optimum orientation is dependent on both the size and positioning of the sensing tube.

The somewhat surprising insensitivity to yaw angle is shown in Fig. 17 and implies that lateral errors in positioning the sensing tube around the circumference of the compression surface are not important.

5.2. Possible Improvements in the Probe Design

A limited parametric study was undertaken at $M = 2{\cdot}45$ to investigate the effects of systematic changes in the probe design. At each condition tested a range of pitch angles was covered to establish the angle which gave the highest pressure recovery, and these optimum angles with the corresponding pressure recoveries are summarised in Figs. 18 to 20. Unfortunately during these tests the frost point could not be monitored so exact corrections for tunnel condensation losses were not known. Such corrections would however increase the value of R shown by between 0·002 and 0·004.

The effect of axial changes in the position of the sensing tube is shown in Fig. 18, together with a sketch of the flowfield. For a constant pitch angle, as the sensing tube is moved rearwards it senses streamlines which pass through the compression fan at progressively increasing distances from the probe tip. Eventually, for large values of l, the sampled streamline will not traverse the compression region but will pass directly through the shock, thus losing the advantages of the curved tip. The nominal design of all the probes had $l = 0$ but Fig. 18 shows that substantial improvements could be made by moving the sensing tube slightly further aft. In fact, if the pressure recoveries shown were corrected for the tunnel condensation losses then the pressure recovery would be very close to unity near $l/D = 0{\cdot}45$.

Changing the height of the sensing tube above the compression surface will also affect the compression history of the sampled streamline. No evidence of boundary-layer separation was detected and the thickness of the viscous region was small compared with the nominal 1·1 mm sensing tube, since the highest pressure recoveries achieved with this tube occurred when it was flush with the surface. Figure 19 shows that when a smaller diameter sensing tube was used, there was a decrease in recovery close to the surface.

There is an advantage in having a flush mounted tube in that such a design is considerably easier to reproduce accurately. Figure 20 shows that for flush tubes the best results are achieved with a sensing tube diameter of 0·1 to 0·15 times the stem diameter, D.

A potentially important parameter which was not varied is the flow turning angle of the curved tip and it is very likely that a pressure recovery of unity could be realised for a usable pitch range up to $M = 2{\cdot}5$ by optimising the turning angle in conjunction with the sensing-tube size and position.

Earlier tests[4] have shown that the probe can be used to sense flow direction, in addition to stagnation pressure, by means of pressures sensed at holes in the compression surface. The versatility of the probe would be increased further if it incorporated a means of measuring Mach number. At supersonic speeds one possible solution may be to attach a conventional pitot to the underside of the compression surface with its tip positioned to sense the undisturbed stream. Provided that the pitot did not significantly reduce the stagnation pressure recovery, Mach number could be deduced from the measured pitot and stagnation pressures, although these quantities would necessarily be on spatially separated streamlines.

6. Conclusions

When a conventional pitot tube is used to deduce stagnation pressure at supersonic speeds, evaluation of the normal shock correction which must be applied involves a knowledge of the specific heat ratio and the local static pressure, both of which are inherently difficult to measure accurately.

The design of a probe has been given which is capable of measuring the full local stagnation pressure directly when placed in either a subsonic or a supersonic stream. In supersonic flow the probe decelerates a portion of the steam to a subsonic velocity by means of a fan of compression waves generated by a curved cylindrical surface, and a pitot tube samples the stagnation pressure in the locally subsonic region.

Tests have shown that such a probe can record a pressure within 0·1 per cent of the true stagnation pressure for Mach numbers up to 2·2 and it is moreover remarkably insensitive to both pitch and yaw. Above $M = 2{\cdot}2$ the pressure recovery, defined as the ratio of the indicated pressure to the true stagnation pressure, begins to fall from unity and calibration is necessary.

There is a potential for further improvements in the geometry of the probe which could extend its useful range to higher Mach numbers.

APPENDIX

Pressure Recovery for a Sensing Tube Mounted on an Infinite Straight Swept Cylinder

Figure 1 shows the flowfield over a swept cylinder with a flush mounted sensing tube. The freestream velocity can be resolved into crossflow and axial components V_c and V_a. If the crossflow Mach number component M_c is supersonic then a shock will exist upstream of the cylinder. After passing through this shock a streamline sampled by the sensing tube will undergo a region of isentropic compression during which V_c is reduced to zero whilst V_a remains constant. During this compression the static temperature rises and the axial Mach number component immediately upstream of the sensing tube is given, for subsonic or supersonic M_c, by

$$M_A = M \sin \Lambda \left[1 + \frac{(\gamma - 1)}{2} M^2 \cos^2 \Lambda\right]^{-\frac{1}{2}}.$$

When $M_A > 1$ in addition to $M_c > 1$, a normal shock will also occur just upstream of the sensing tube, and the pressure recovery is given by the product of two pitot correction factors,

$$R = \left[\frac{(\gamma + 1)M_c^2}{(\gamma - 1)M_c^2 + 2}\right]^{\gamma/(\gamma-1)} \left[\frac{(\gamma + 1)}{2\gamma M_c^2 - \gamma + 1}\right]^{1/(\gamma-1)} \left[\frac{(\gamma + 1)M_A^2}{(\gamma - 1)M_A^2 + 2}\right]^{\gamma/(\gamma-1)} \left[\frac{(\gamma + 1)}{2\gamma M_A^2 - \gamma + 1}\right]^{1/(\gamma-1)}.$$

This is valid when

$$\cos^{-1}\left[\frac{2(M^2 - 1)}{(\gamma + 1)M^2}\right]^{\frac{1}{2}} \leqslant \Lambda \leqslant \cos^{-1}\left(\frac{1}{M}\right)$$

where

$$M_c^2 = M^2 \cos^2 \Lambda$$

and

$$M_A^2 = M^2 \sin^2 \Lambda \left[1 + \frac{(\gamma - 1)}{2} M^2 \cos^2 \Lambda\right]^{-1}.$$

If $M_A < 1$,

$$\left(\text{i.e. } \Lambda < \cos^{-1}\left[\frac{2(M^2 - 1)}{(\gamma + 1)M^2}\right]^{\frac{1}{2}}\right),$$

then the last two terms in the expression for R must be omitted. If $M_c < 1$, [i.e. $\Lambda > \cos^{-1}(1/M)$], then the first two terms should be omitted.

When the free stream Mach number satisfies $M < [(\gamma + 3)/2]^{\frac{1}{2}}$, it is possible by a suitable choice of Λ, to make both M_A and M_c subsonic. In this case the pressure recovery is identically unity for a finite range of values of Λ given by:

$$R = 1{\cdot}000 \text{ for } \cos^{-1}\left(\frac{1}{M}\right) \leqslant \Lambda \leqslant \cos^{-1}\left[\frac{2(M^2 - 1)}{(\gamma + 1)M^2}\right]^{\frac{1}{2}}.$$

At $M \doteq [(\gamma + 3)/2]^{\frac{1}{2}}$, (= 1·483 for air), R reaches unity at a single value of Λ given by:

$$R = 1{\cdot}000 \text{ at } \Lambda = \cos^{-1}[2/(\gamma + 3)]^{\frac{1}{2}}, \text{ (= 47·61 degrees for air).}$$

For $M > [(\gamma + 3)/2]^{\frac{1}{2}}$ the sweepback angle can be optimised to give maximum pressure recovery and this occurs when $M_A = M_c$ and is given by:

$$R_{\text{opt}} = \left[\frac{(\gamma + 1) M^2 \cos^2 \Lambda_{\text{opt}}}{(\gamma - 1)M^2 \cos^2 \Lambda_{\text{opt}} + 2}\right]^{2\gamma/(\gamma-1)} \left[\frac{(\gamma + 1)}{2\gamma M^2 \cos^2 \Lambda_{\text{opt}} - \gamma + 1}\right]^{2/(\gamma-1)},$$

where the optimum value of Λ is given by:

$$\cos^2\Lambda_{opt} = \left[\frac{4}{(\gamma - 1)^2 M^4} + \frac{2}{(\gamma - 1)M^2}\right]^{\frac{1}{2}} - \frac{2}{(\gamma - 1)M^2}.$$

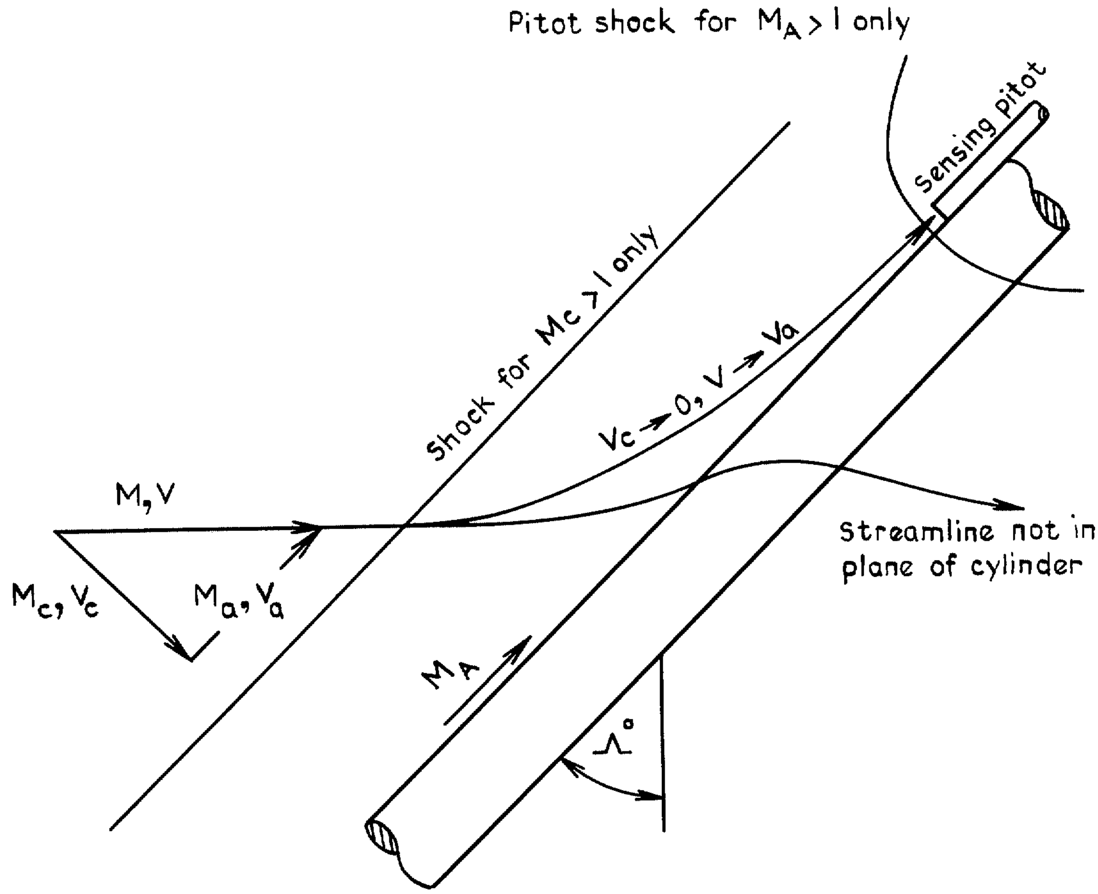
This optimum sweepback is in fact very insensitive to both M and γ and remains in the range between 45 and 55 degrees for all values of M up to $M = 4$, at any value of γ from 1·1 to 1·4.

LIST OF SYMBOLS

a, b, c, d, e, f, D	Geometric parameters defined on Fig. 3
F	$= H/P_{pit}$
h	Height of underside of sensing tube above compression surface
H	True local stagnation pressure in working section, downstream of any possible condensation shocks
H_0	Value of stagnation pressure recorded in tunnel contraction (upstream of any possible condensation shocks)
l	Axial distance of mouth of sensing tube from its nominal position
M	Free-stream Mach number
M_a	Component of free-stream M in cylinder axial direction ($= M \sin \Lambda$)
M_A	Axial Mach number close to cylinder surface
M_c	Cross flow component of free-stream M normal to cylinder surface
n	Number of shock waves from compression polygon, including final normal shock
P	Local static pressure on compression surface
P_p	Pressure recorded by probe sensing tube
P_{pit}	Pitot pressure downstream of a normal shock
P_s	Free-stream static pressure
R	Pressure recovery ($= P_p/H_0$; but after correction for humidity effects, $R = P_p/H$. *See* Section 3)
S	$= P_s/H$
T_{stag}	Stagnation temperature
V	Free-stream velocity
V_a	Component of V in cylinder's axial direction
V_c	Cross-flow component of V normal to cylinder surface
α	Pitch angle with zero and direction as defined on Fig. 3
α_{opt}	Value of α which gives maximum pressure recovery
β	Yaw angle, positive for model nose to port
γ	Ratio of specific heats
Λ	Sweep angle of cylinder
Ω	Specific humidity (mass of water per unit mass of air)

REFERENCES

No.	Author(s)	Title, etc.
1	J. A. F. Hill	On the calibration of supersonic wind tunnels. *J. Aero. Sci.*, *22*, 6, pp. 441–443 (1955).
2	J. H. Nichols, M. W. Davis and C. L. Garner	Initial aerodynamic calibration results for the A.E.D.C.-P.W.T. 16-foot supersonic tunnel. Report No. A.E.D.C.-T.D.R.-62-55 (1962).
3	M. J. Goodyer	A new stagnation pressure probe having a high pressure recovery in supersonic flow. *Instrumentation in the Aerospace Industry*, Vol. 18, pp. 93–103, Pittsburg, Instrument Society of America (1972).
4	M. J. Goodyer	A new probe for the direct measurement of stagnation pressure in supersonic flow. R.A.E. Technical Report 73122.
5	J. F. Connors and R. C. Meyer	Design criteria for axisymmetric and two-dimensional supersonic inlets and exits. N.A.C.A. T.N. 3589 (1956).
6	K. L. Oswatitsch	Forschungen und Entwicklungen des Heereswaffenamtes, Bericht No. 1005 (1944). Available as "Pressure recovery for missiles with reaction propulsion at high supersonic speeds. (The efficiency of shock diffusers)". N.A.C.A. T.M. 1140 (1947).
7	R. Hermann	*Supersonic inlet diffusers and an introduction to internal aerodynamics*, pp. 193–225. U.S.A., Minneapolis–Honeywell Regulator Company (1956).
8	A. P. Pudovecv	A diagram for the optimal design of a shock system in axisymmetric supersonic diffusers with external compression. R.A.E. Library Translation 1259 (1967).



$$R = \frac{\text{Pressure sensed}}{\text{Freestream stagnation pressure}}$$

$$R = 1{\cdot}000 \text{ for } M \leqslant \sqrt{\frac{\gamma+3}{2}}, \quad (M \leqslant 1{\cdot}483 \text{ for air}),$$

for a range of sweep angles given by:-

$$\cos^{-1}\left(\frac{1}{M}\right) \leqslant \Lambda \leqslant \cos^{-1}\left[\frac{2(M^2-1)}{(\gamma+1)M^2}\right]^{\frac{1}{2}}$$

FIG. 1. Supersonic flow over an infinite swept cylinder.

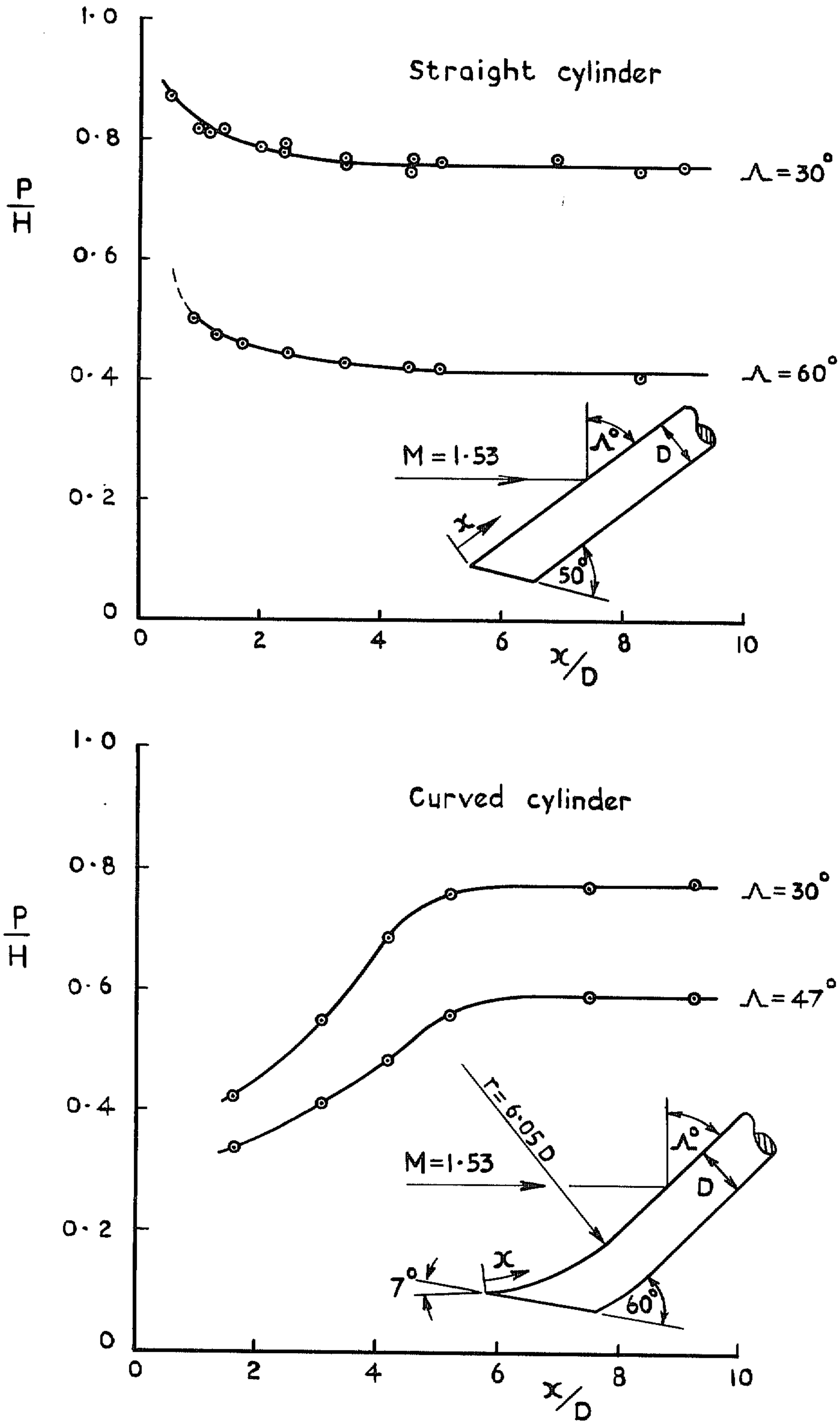
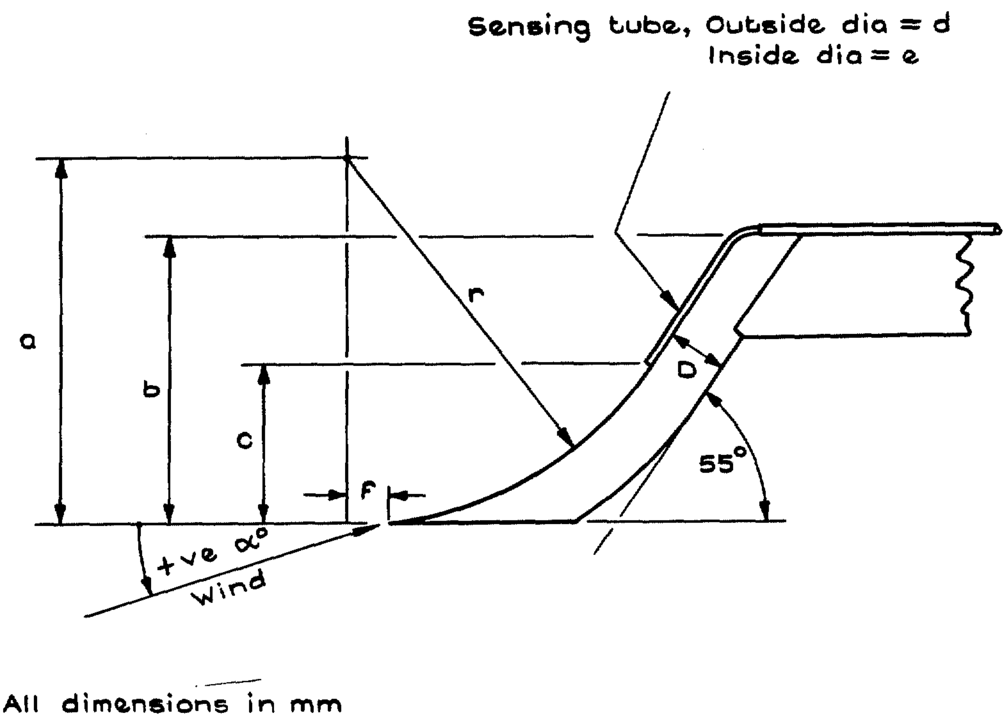


FIG. 2. Static pressure distributions along straight and curved cylinders.



Probe	a	b	c	d	e	f	r	D
1, 2, 3, 4, 6, 5 (unmodified)	56·7	43·8	24·0	1·09	0·71	6·96	57·2	9·53
5	56·7	43·8	Variable	1·09	0·71	6·96	57·2	9·53
5	56·7	43·8	Variable	0·50	0·25	6·96	57·2	9·53
5	56·7	43·8	28·3	1·00	0·60	6·96	57·2	9·53
7	37·8	29·2	16·0	1·09	0·71	4·64	38·1	6·35
8	18·9	14·6	8·0	1·09	0·71	2·32	19·1	3·18

FIG. 3. Details of probes tested.

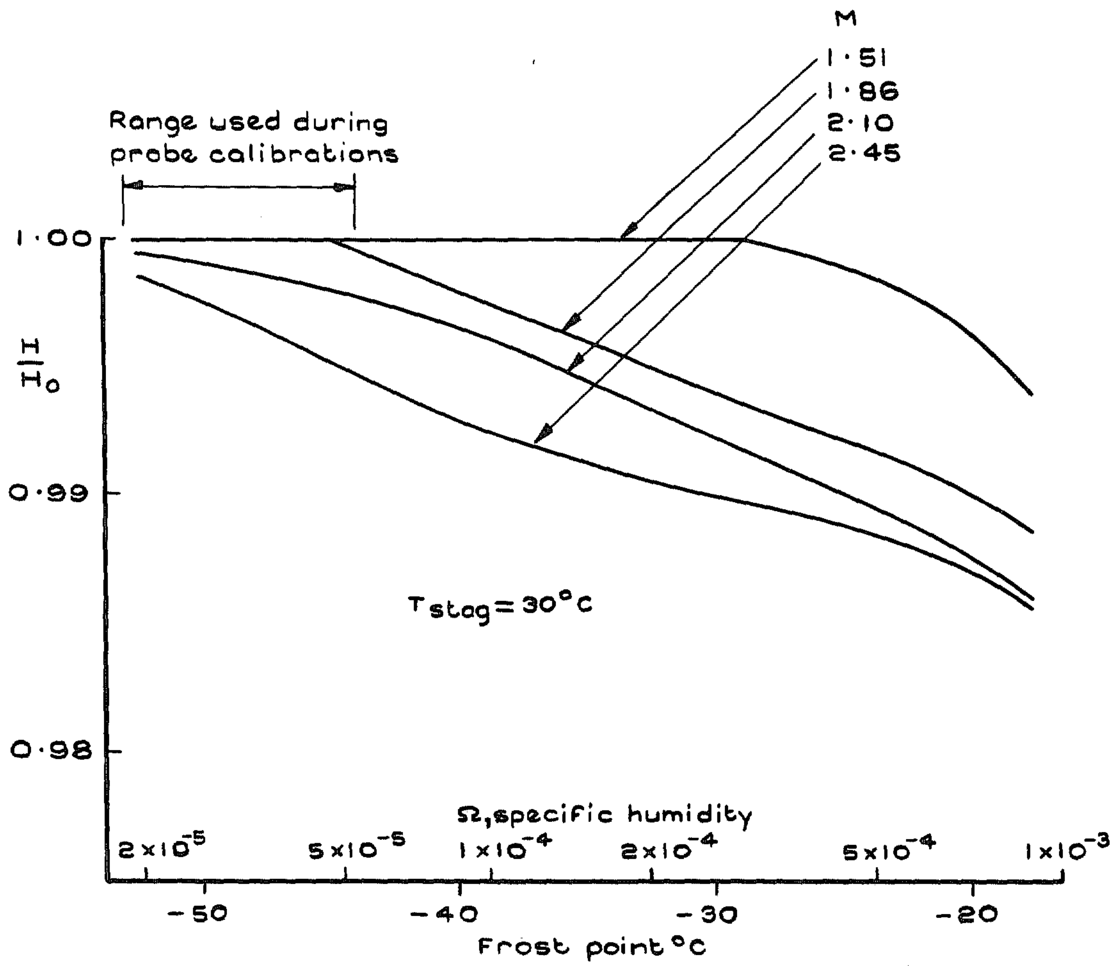


FIG. 4. Measured loss in stagnation pressure due to condensation shocks.

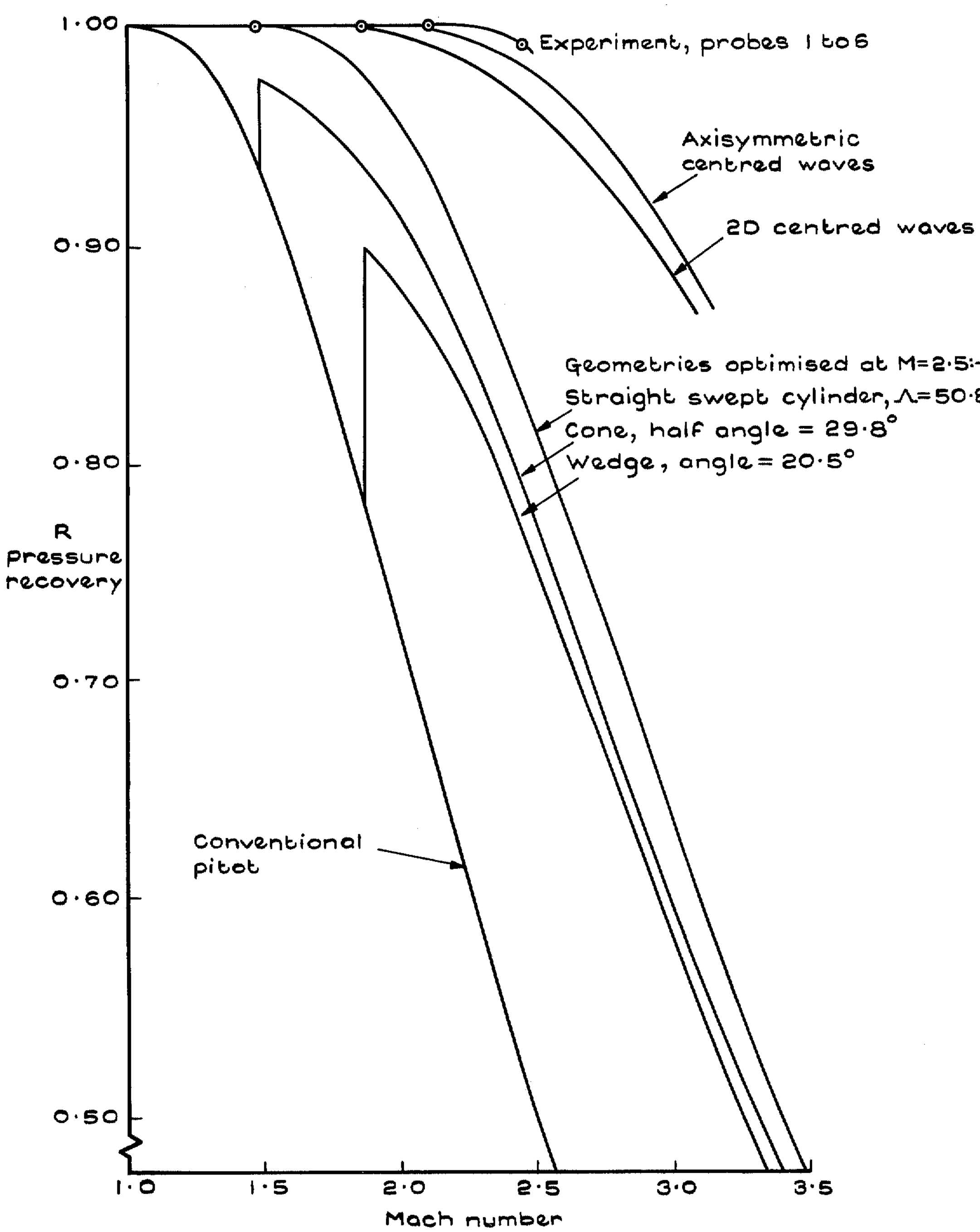


FIG. 5. Comparison of pressure recovery for different probe geometries, $\gamma = 1{\cdot}4$.

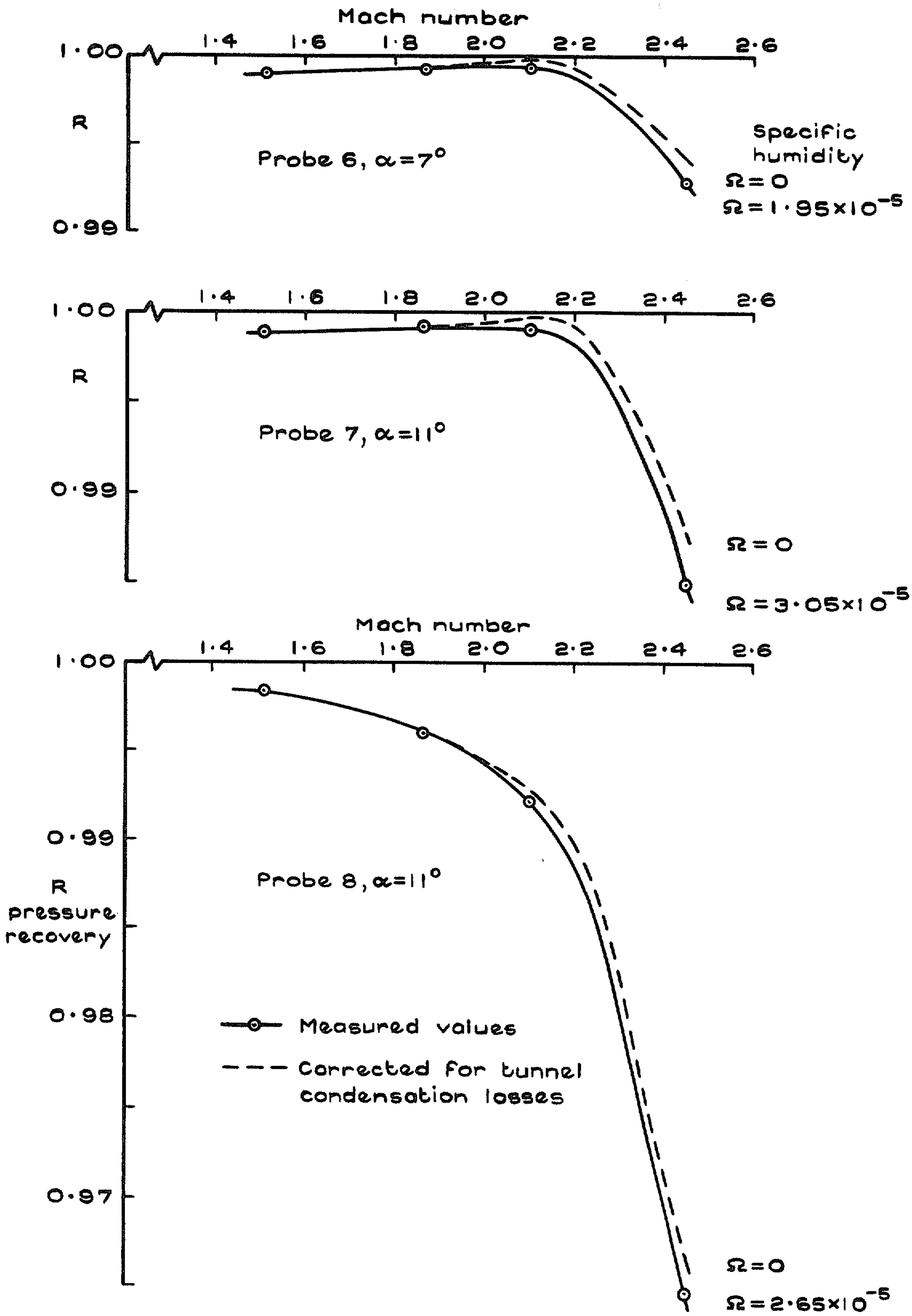
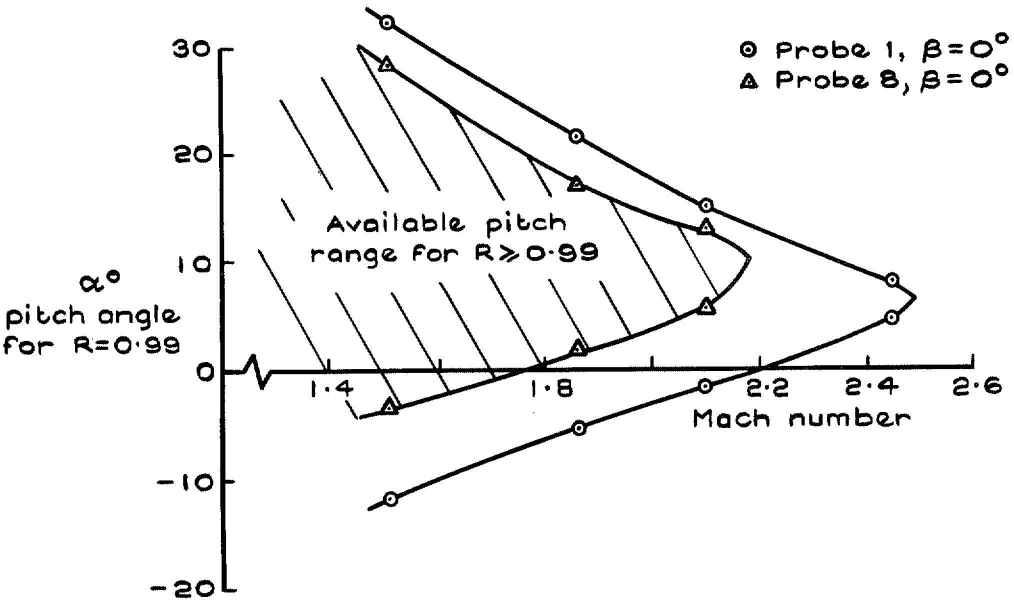


FIG. 6. Variation of pressure recovery with Mach number.



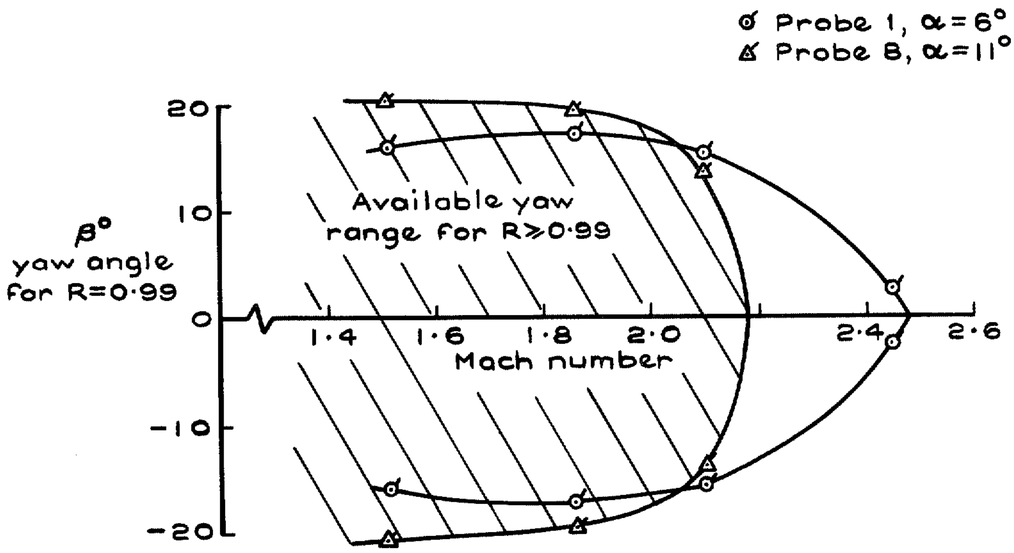


FIG. 7. Available pitch and yaw ranges for pressure recovery above 0·99.

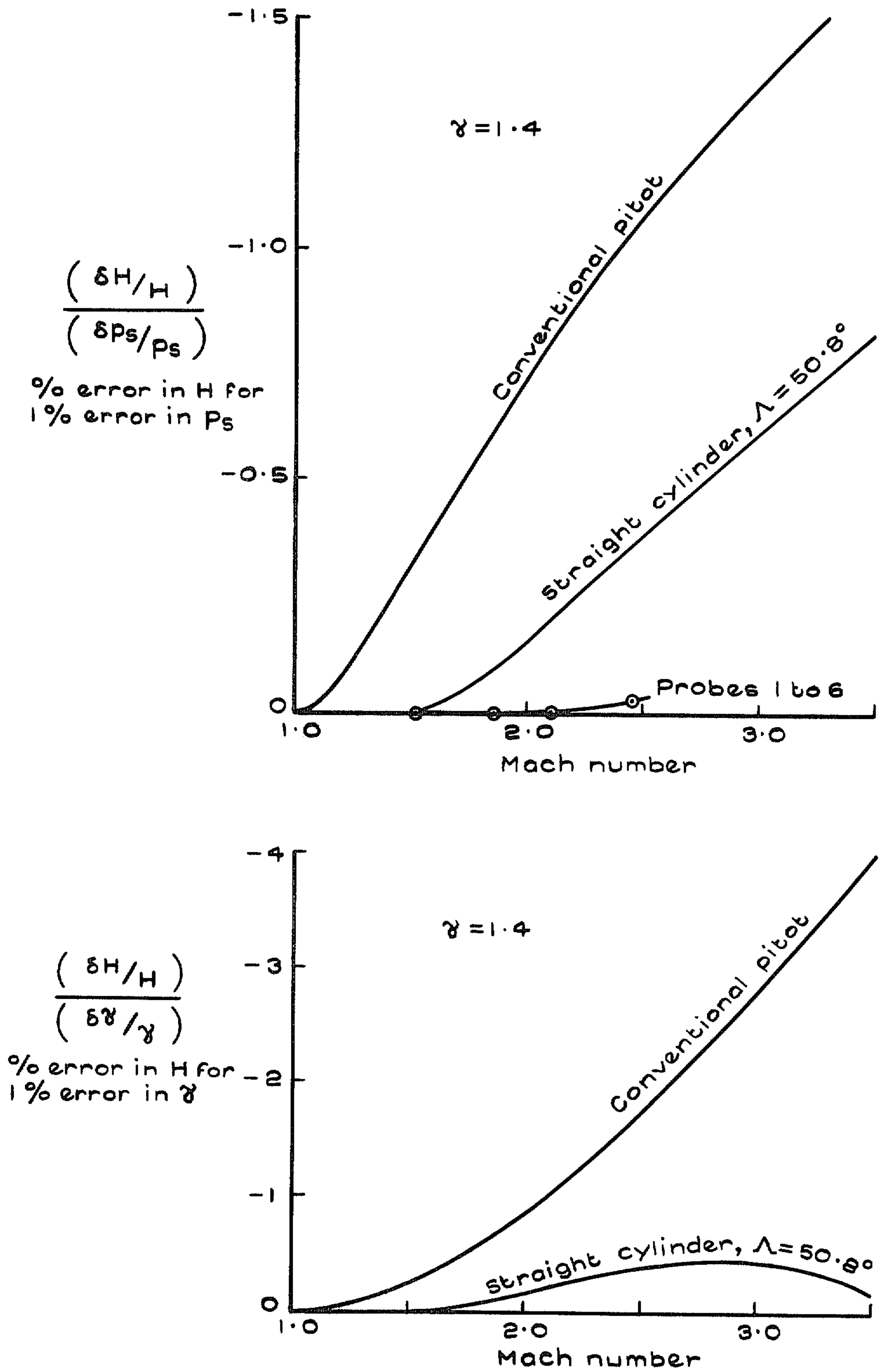


FIG. 8. Comparison of error sensitivity of various stagnation pressure probes.

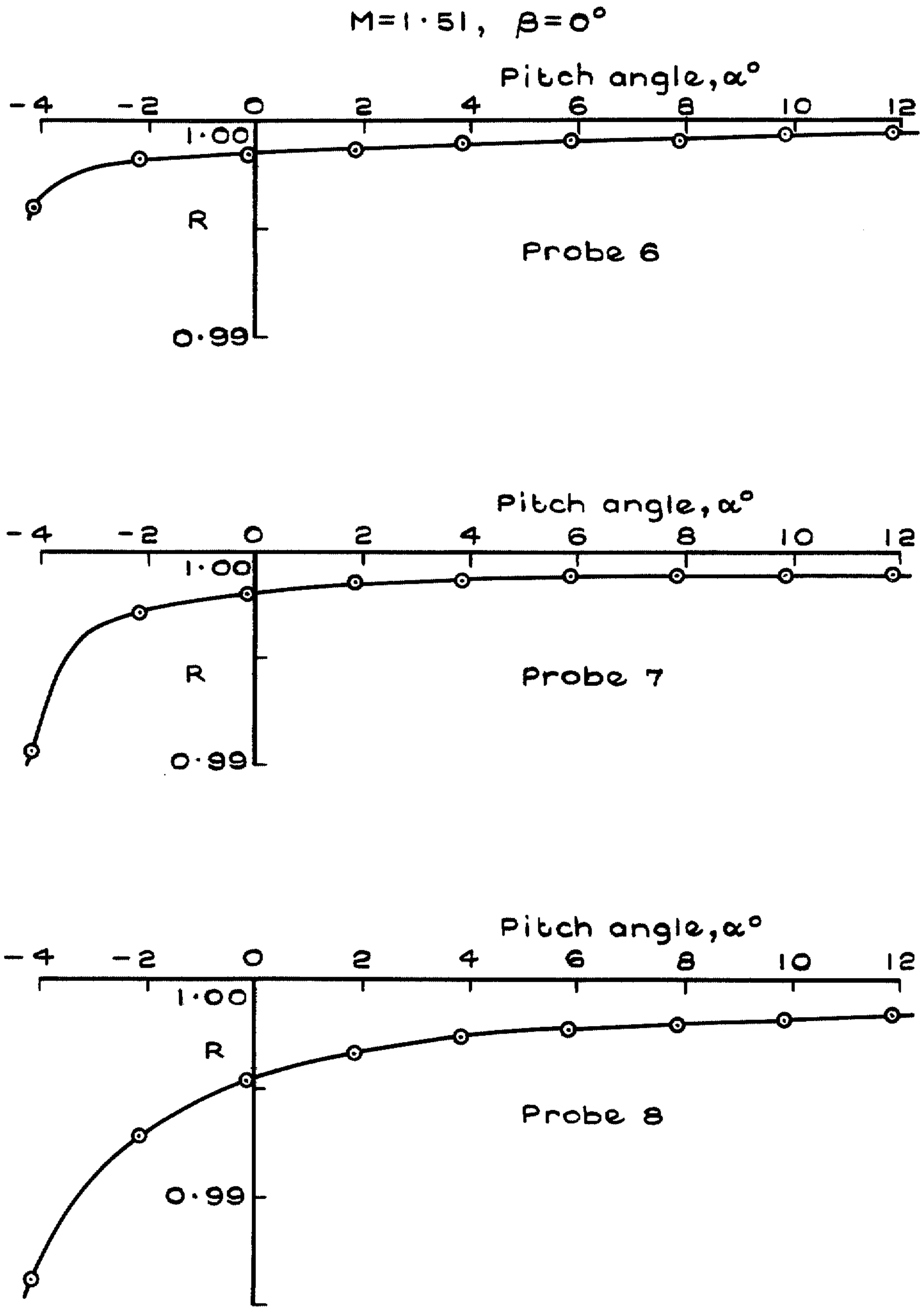


FIG. 9. Effect of pitch on pressure recovery at $M = 1{\cdot}51$.

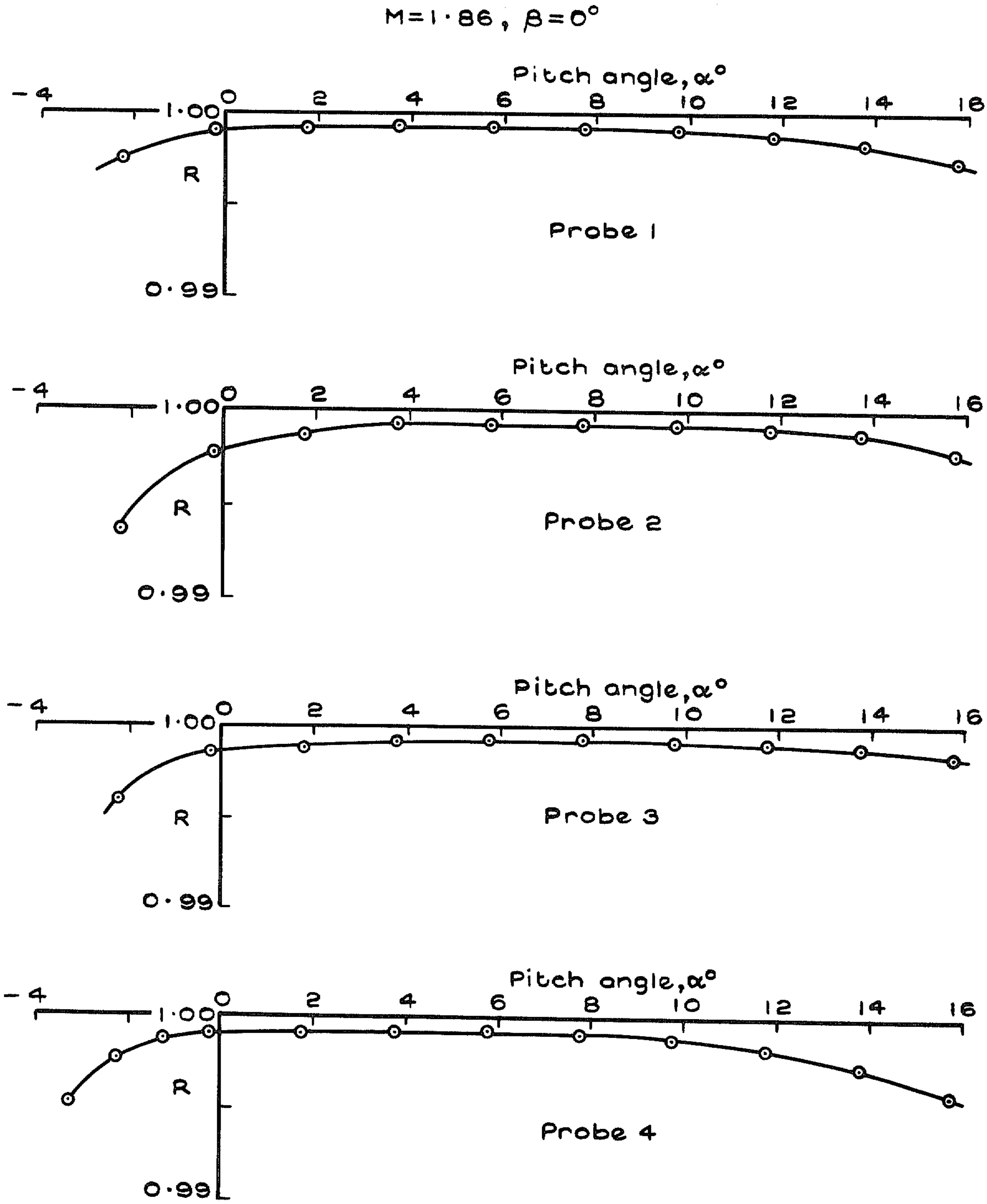


FIG. 10. Effect of pitch on pressure recovery at $M = 1{\cdot}86$.

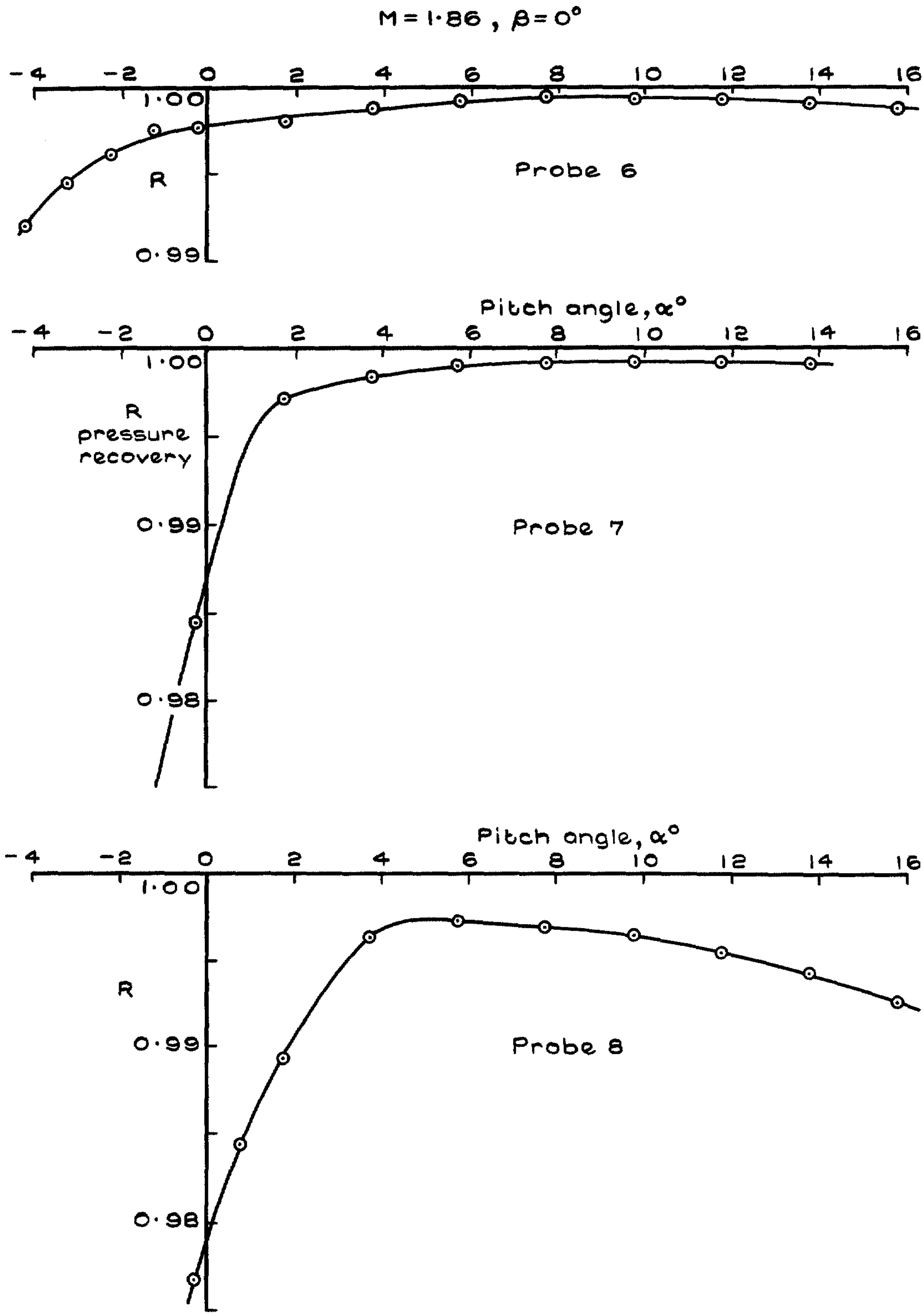


FIG. 11. Effect of pitch on pressure recovery at $M = 1{\cdot}86$.

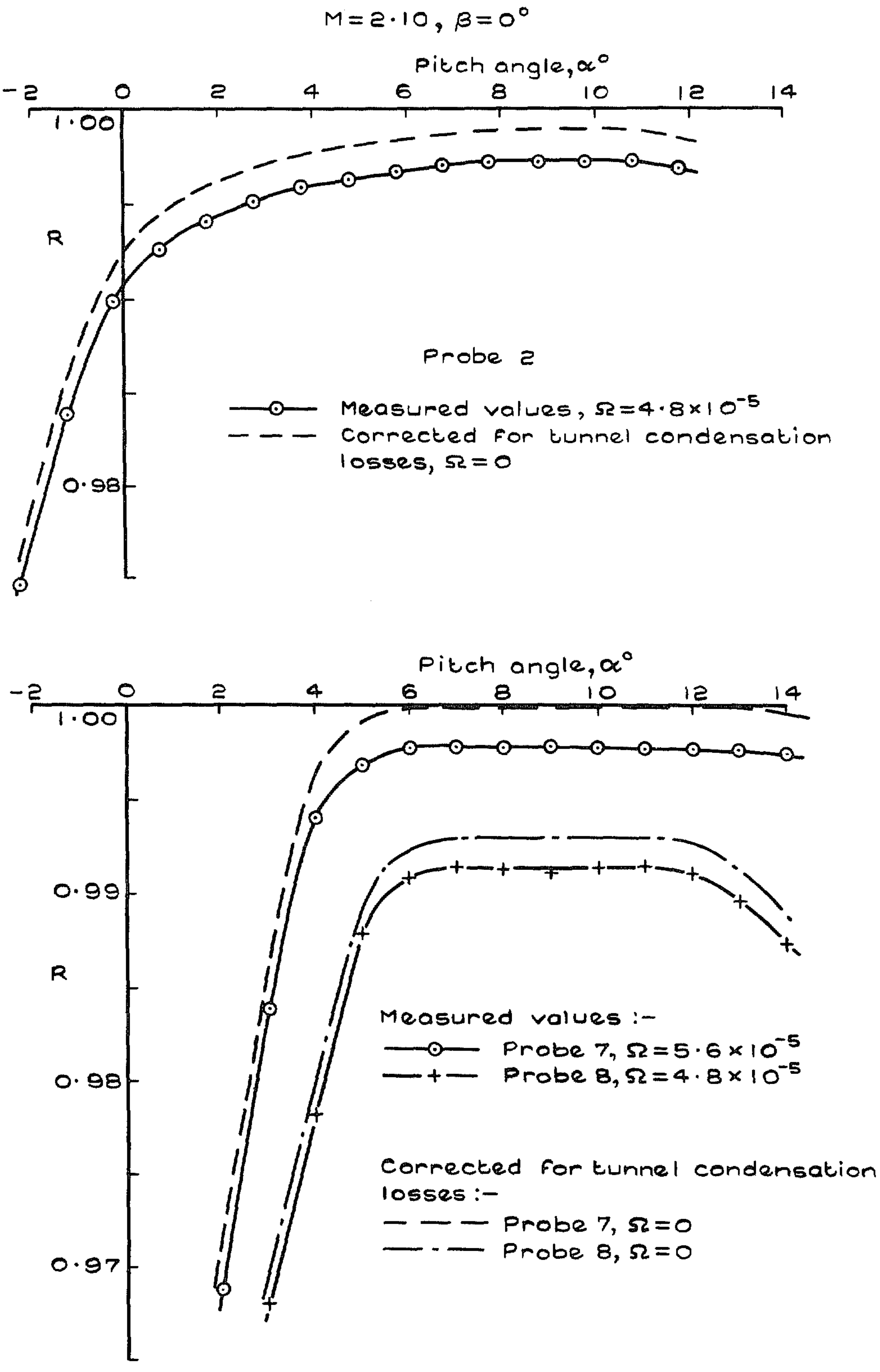


FIG. 12. Effect of pitch on pressure recovery at $M = 2{\cdot}10$.

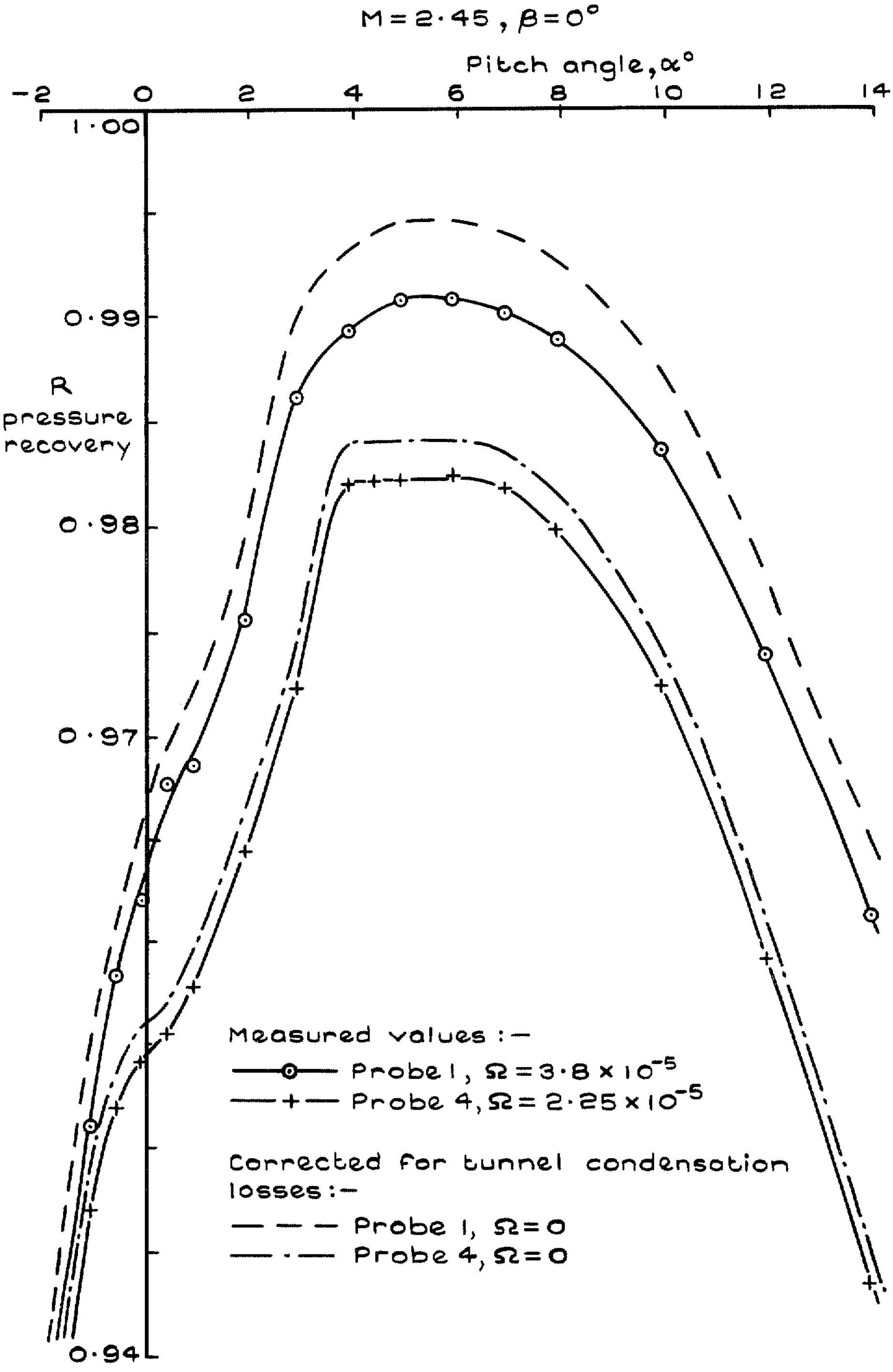


FIG. 13. Effect of pitch on pressure recovery at $M = 2{\cdot}45$.

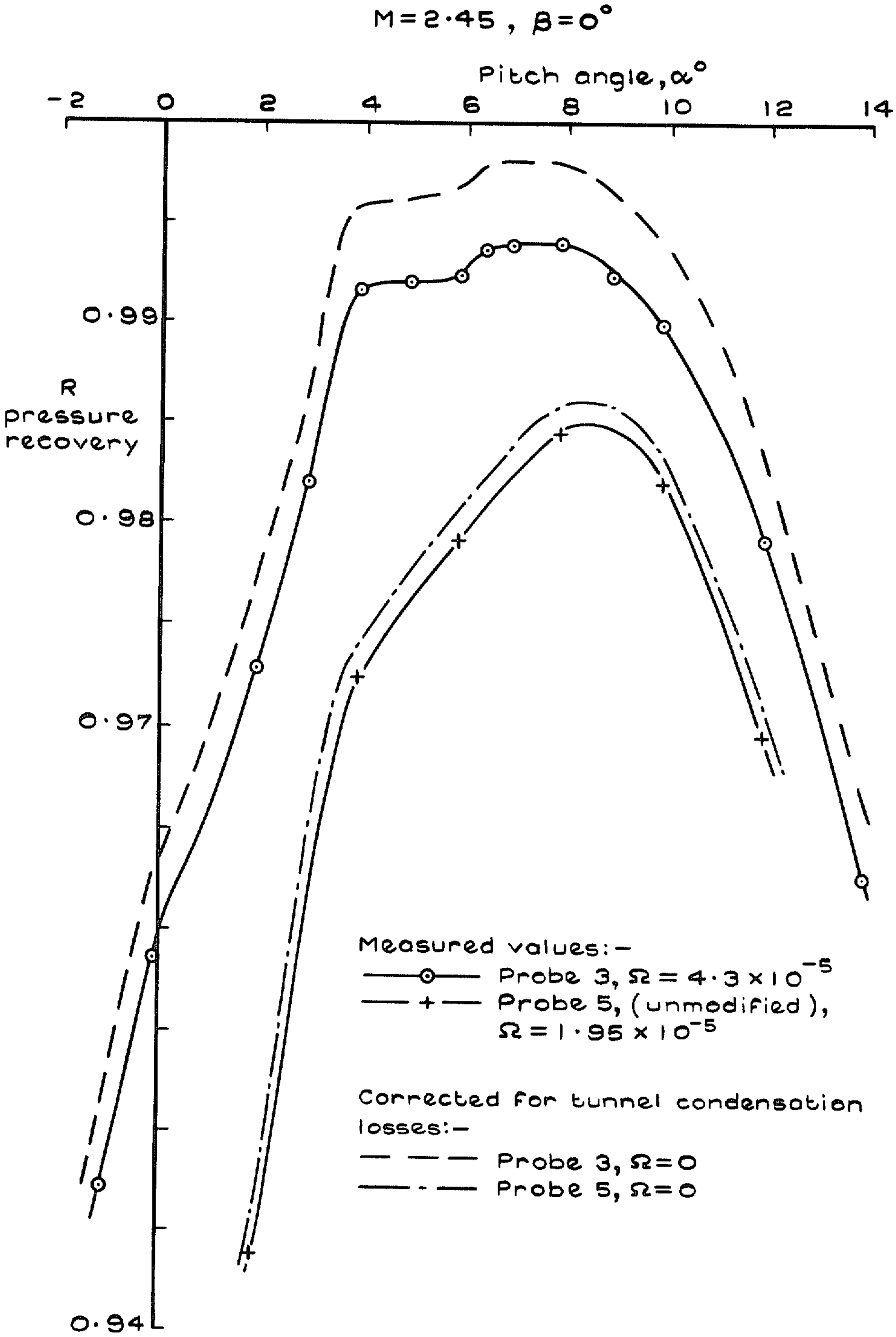


FIG. 14. Effect of pitch on pressure recovery at $M = 2{\cdot}45$.

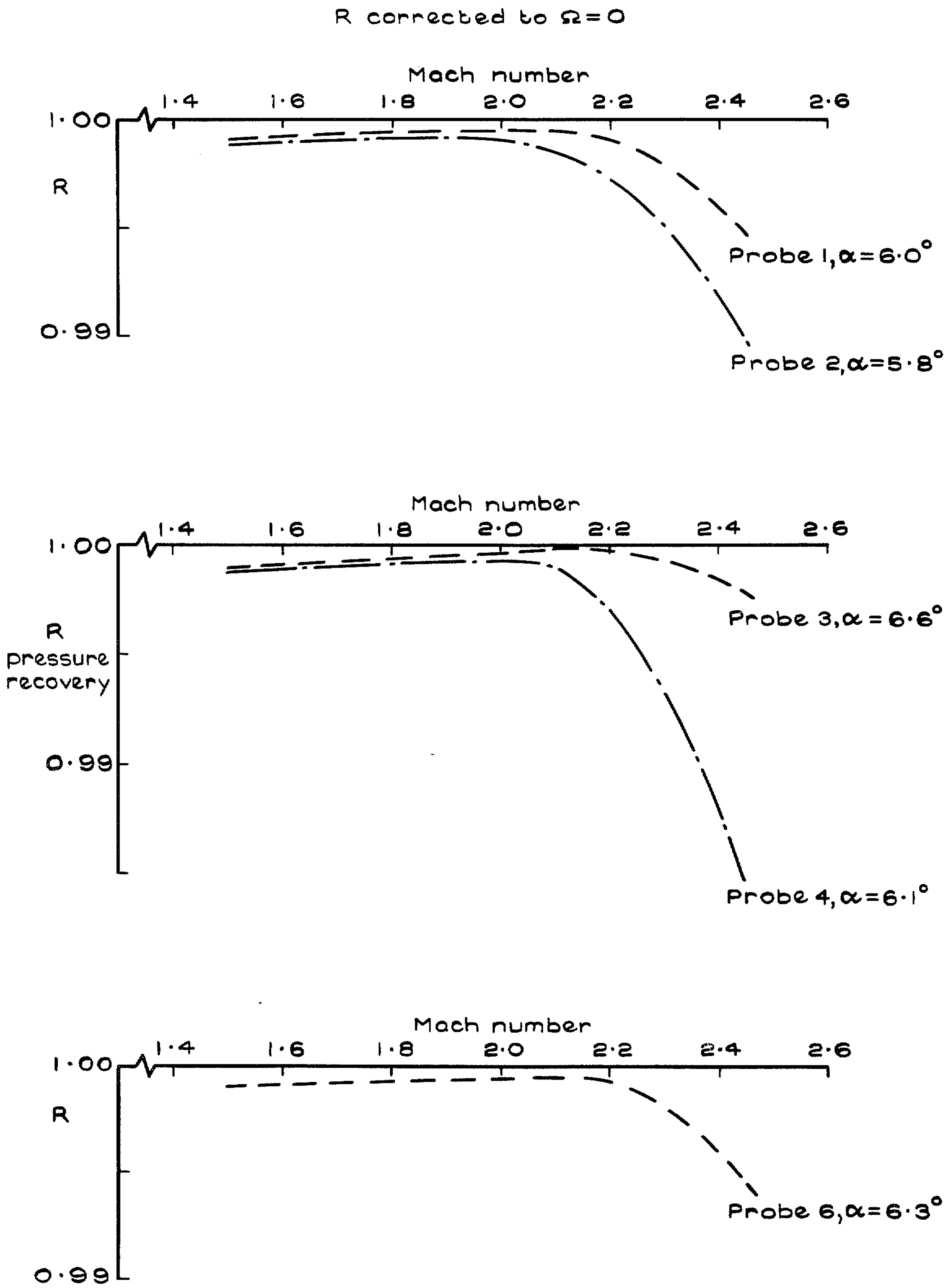


FIG. 15. Comparison of pressure recoveries for nominally similar probes.

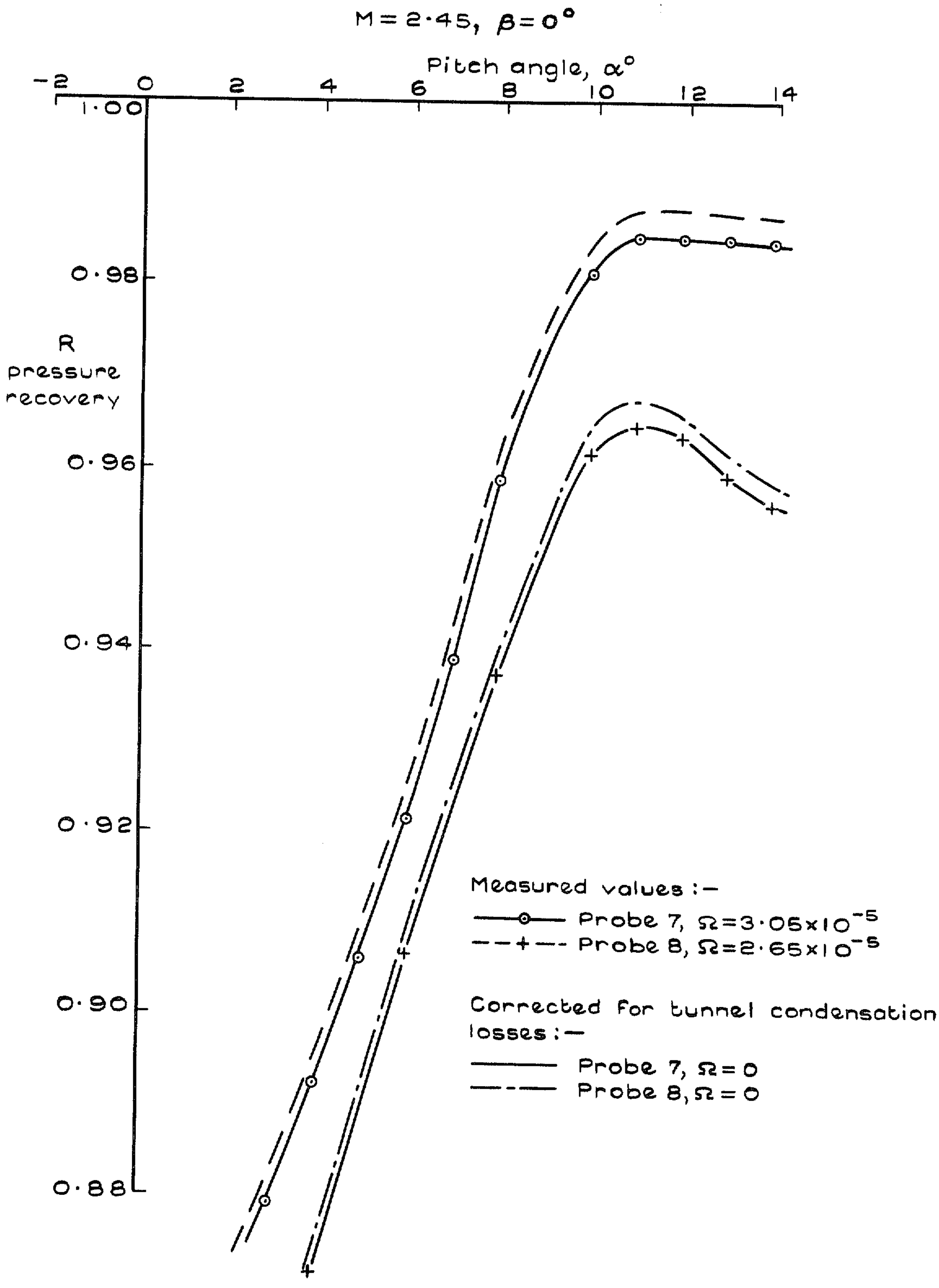


FIG. 16. Effect of pitch on pressure recovery at $M = 2{\cdot}45$.

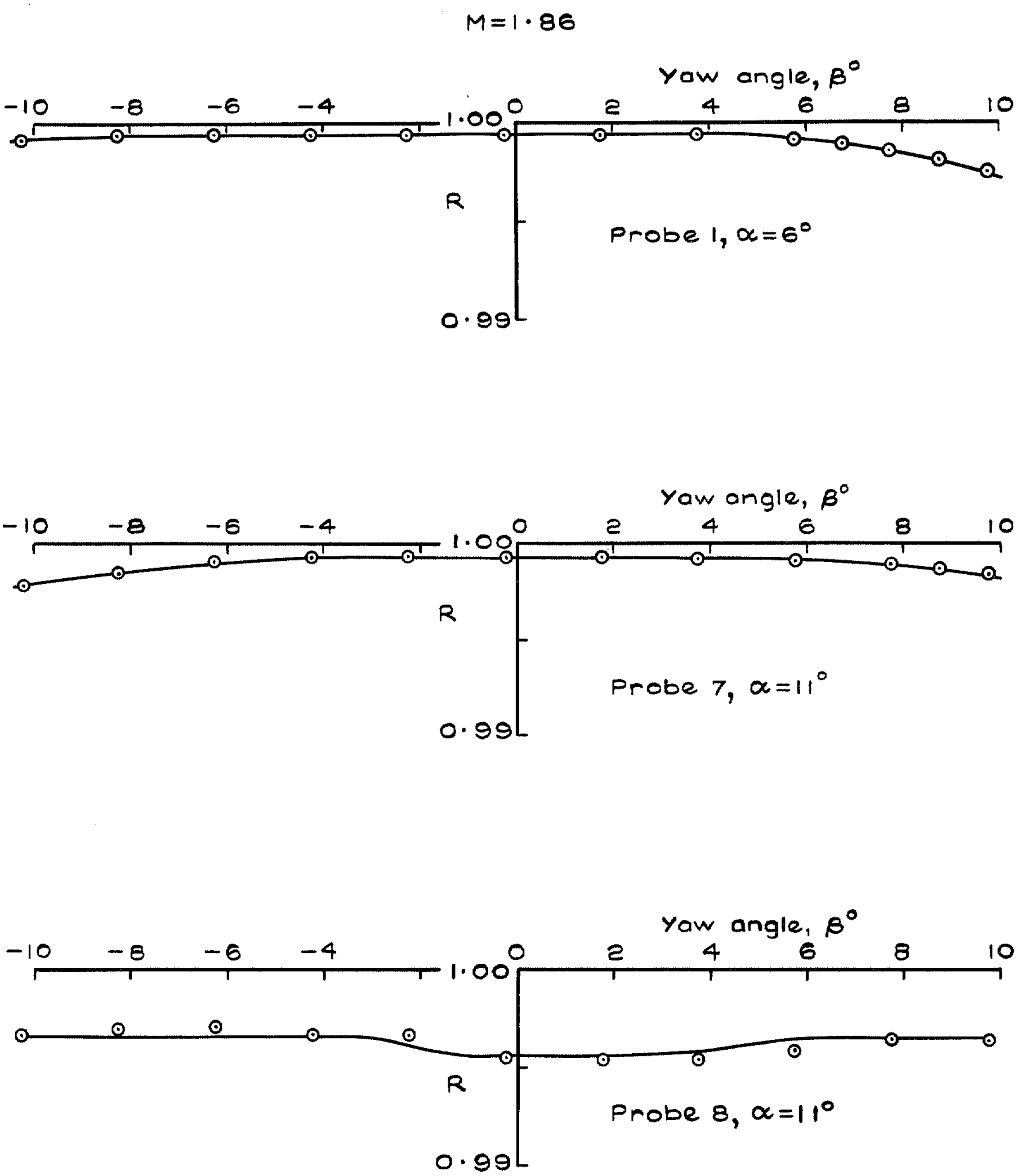


FIG. 17. Effect of yaw on pressure recovery at $M = 1{\cdot}86$.

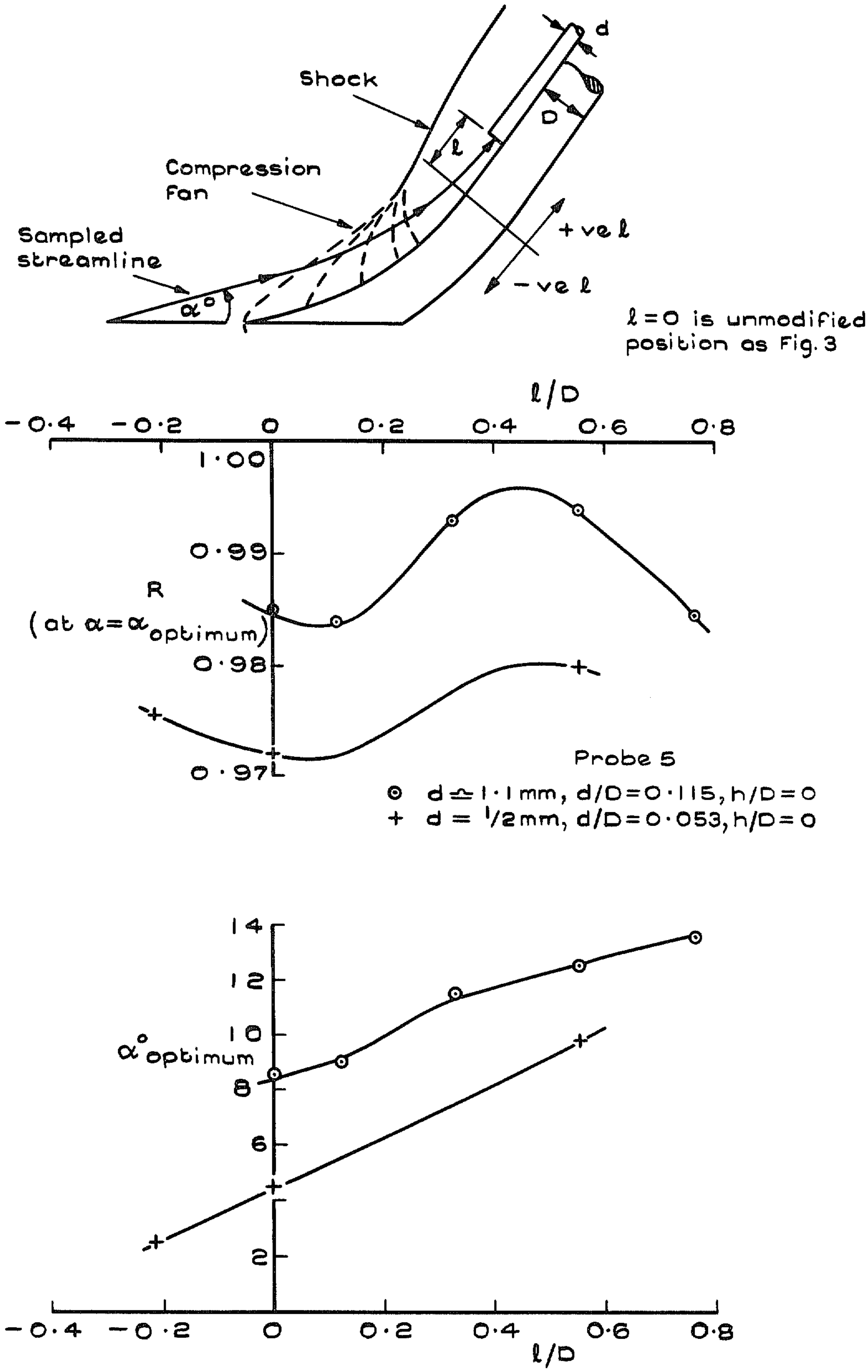


FIG. 18. Effect of moving sensing tube axially, $M = 2{\cdot}45$.

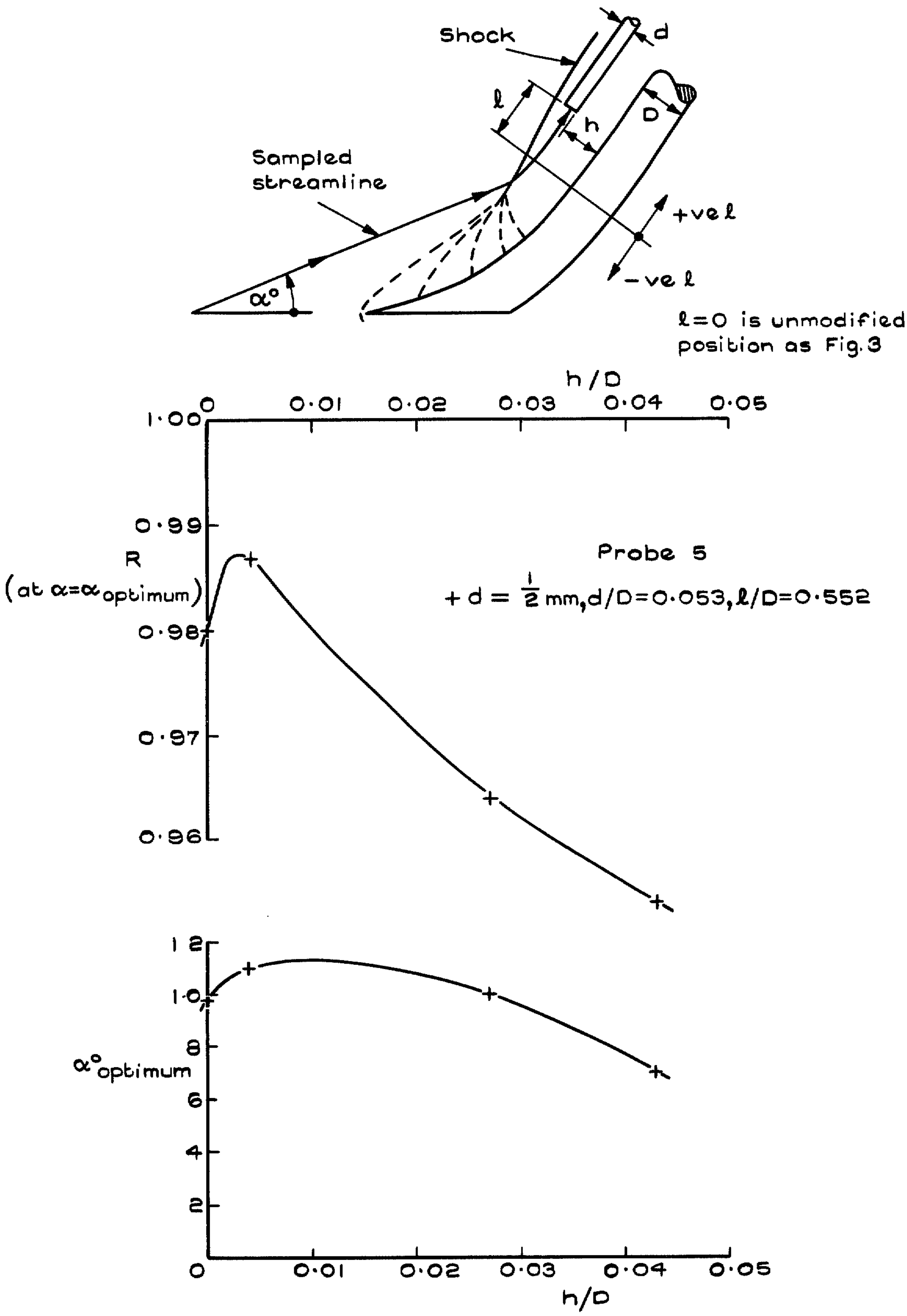


FIG. 19. Effect of changing height of sensing tube, $M = 2{\cdot}45$.

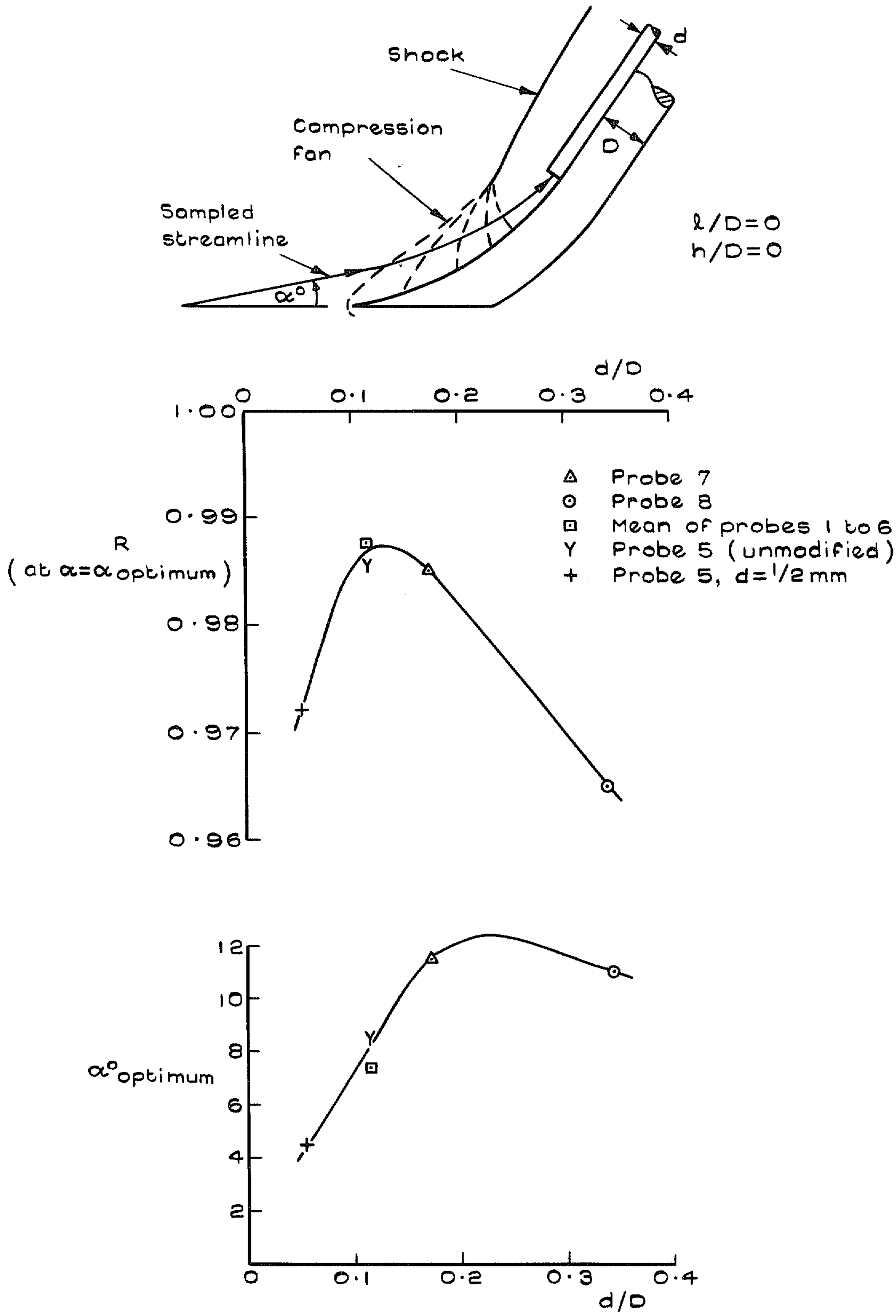


FIG. 20. Effect of changing ratio of sensing tube diameter to stem diameter, $M = 2{\cdot}45$.

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