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Summary

Two flat parallel surfaces, oscillating harmonically about a mean configuration are immersed in a uniform subsonic main stream in a direction parallel to the surfaces. The linearised equations of potential flow are assumed to be valid, so that the upwash on the surfaces can be related to the loading on the surfaces by means of a pair of integral equations. This pair of integral equations is solved, by collocation, for approximations to the loadings in terms of given upwashes and these approximations are used to evaluate generalised airforces. The results are compared with some results obtained by other numerical procedures and with some experimental results.

The procedure has been programmed in 1900 FORTRAN.

*Replaces R.A.E. Technical Report 72180—A.R.C. 34 466

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1. Introduction

When two surfaces such as wing and tailplane in close proximity to each other are oscillating in an airstream, the airforces on either one of the surfaces can be quite different from those which occur on the surface when it is oscillating in isolation, because the oscillation of one surface modifies the aerodynamic flow in the neighbourhood of the other as well as in the neighbourhood of itself. Indeed this modification can cause a lowering of the flutter speed when the wing and tailplane are moved nearer to each other, as was shown by Topp, Rowe and Shattuck.[1]

For flutter calculations of an aeroelastic system, the values of the generalised airforce coefficients at a given frequency of oscillation in a flow of given Mach number for a number of modes of oscillation of the aeroelastic system are required. In this report a method is developed for obtaining these generalised airforce coefficients for two parallel planar surfaces oscillating harmonically in subsonic flow. In the method, the loading on each of the two surfaces is represented approximately by a linear combination of given functions each of which is continuous over the surface, and has the correct behaviour at the edges of the surface. The coefficients in these linear combinations are determined by ensuring that the upwashes corresponding to these approximations to the loading, are the same as the known upwash distributions at sets of points on the two surfaces. These sets of points are the Multhopp[2] points for each of the surfaces. The generalised airforces are then obtained by using the so obtained approximate representations of the loading.

Similar types of method have been used by Laschka and Schmid[3] and by Albano, Perkinson and Rodden[4] to solve this problem, but Laschka and Schmid consider only the particular case of the two surfaces being coplanar. Rodden, Giesing and Kálmán[5] have also considered this problem using the doublet lattice method in which the approximation to the loading by means of continuous functions is abandoned and replaced by discrete loading along certain lines, the strengths of these discrete loads being adjusted to satisfy boundary conditions at sets of points on the surfaces. In the few comparisons that have been made, the results obtained by all these methods generally agree quite well with results obtained from the method of this report.

Experimental work on two rectangular wings oscillating harmonically in subsonic flow has been carried out at O.N.E.R.A.[6] and some results obtained are compared with results obtained from the method of this report.

2. Discussion of Generalised Airforces on the Wing and Tailplane

The wing and tailplane are immersed in an airstream and are assumed to be vibrating in such a way that the position of any point on the surface of either the wing or tailplane is always near to its mean position fixed relative to a certain inertial rectangular cartesian frame of reference C. The airflow at large distances from the wing and tailplane is uniform with speed V relative to the frame C and its density is ρ. The x-axis of the frame of reference is taken parallel to the direction of the uniform velocity of the air at large distances from the wing and tailplane, and positive in the direction of the uniform velocity.

The wing and tailplane are assumed to be very thin and their surfaces nearly plane and orientated in such a way that the surfaces are at small angles to the uniform-flow direction everywhere except in the neighbourhood of their leading edges. Then linearised theory is applicable and the wing and tailplane, including their wakes, may be replaced by flat surfaces of zero thickness in planes parallel to the x-axis.

The positions of these flat surfaces are such that any point on the wing or tailplane is always near to one of these flat surfaces, and the orthogonal projection of the wing on the flat surface in its vicinity will be denoted by W and the orthogonal projection of the tailplane on the flat surface in its vicinity will be denoted by T. The planes of W and T will be taken parallel to each other and at a distance h apart. The surfaces W and T will be assumed to be symmetric about a plane normal to these surfaces. The origin of coordinates of the frame of reference C is taken at an arbitrary point on the line of intersection of the plane of symmetry and the plane containing W, and this line of intersection is automatically the x-axis. The y-axis is taken in the plane of W in a direction perpendicular to the x-axis and positive in the direction which we henceforth call starboard. The z-axis is taken in the plane of symmetry, perpendicular to the plane of W, positive in the direction we call upwards, and forming together with the x and y axes a right-handed orthogonal frame of reference.

There will be a pressure difference across W and across T at any point on either one of them and this pressure difference is called the loading at the point. The upwashes on W and on T are discontinuous across W and across T but can be split up into two parts on each surface, one of which is equal on the two sides of a surface and the other of which is equal but of opposite sign on the two sides of a surface. The part which is equal but of opposite sign is associated with the thickness distribution of the wing or the tailplane and is independent of time. The part which is equal on the two sides of a surface may be split up into two further parts, one of which is time independent and the other of which is time dependent. The time independent part is associated with camber and steady angle of incidence of a mean thickness line in the wing or tailplane, and the time dependent part is associated with vibration of the mean thickness line.

We can separate the aerodynamic problem into three separate problems each one associated with a part of the upwash described in the preceding paragraph. The three solutions can then be superimposed linearly to get the complete solution.

In the first problem, the upwash is equal but of opposite sign on the two sides of the surfaces W and T. This problem can be solved by covering the surfaces W and T by source distributions whose strengths have to be adjusted to give the correct upwash distributions. Since the pressure difference across surface distributions of sources is zero, the loading distribution on W and T must be zero in this problem.

In the second problem, the normal wash is equal on the two sides of the surfaces W and T but is time independent. A loading distribution on W and T is now present but it is time independent.

In the third problem, the upwash is equal on the two sides of the surfaces W and T but time dependent, and consequently the loading is time dependent. We shall be concerned in this report with this problem only and we shall take the time dependence to be harmonic with circular frequency ω. The second problem is the particular case of $\omega = 0$, but we must note that if we take a limit process $\omega \to 0$ then we get the correct loading at $\omega = 0$ only if the camber and steady angle of incidence are taken to be limiting cases of displacements which have harmonic time dependence of circular frequency ω.

We assume that the wing-tailplane system is capable of vibration in a number of modes of displacement that can be numbered $1, 2, 3, \ldots$ etc. In the mode p, the displacement in the direction of the positive z-axis from the mean position of a point on the wing, with abscissae (x, y), is taken to be proportional to $Z_p^{(1)}(x, y)$ and the displacement in the direction of the positive z-axis of a point on the tailplane, with abscissae (x, y), from its mean position is taken to be proportional to $Z_p^{(2)}(x, y)$ with the same constant of proportionality as for the wing.

When the wing-tailplane system is oscillating harmonically about its mean position, with circular frequency ω, in the mode p, we may therefore take the displacement in the direction of the positive z-axis of a point on the wing, with abscissae (x, y), to be

$$Z_p^{(1)}(x, y) b_p \, e^{i\omega t} \tag{1}$$

at time t, and the displacement in the direction of the positive z-axis of a point on the tailplane, with abscissae (x, y), to be

$$Z_p^{(2)}(x, y) b_p \, e^{i\omega t} \tag{2}$$

at time t, where b_p is a measure of the amplitude and phase of the oscillation and it is to be understood that only the real part of a complex number corresponds to the physical quantity concerned. The quantity b_p may be complex, but $Z_p^{(1)}(x, y)$ and $Z_p^{(2)}(x, y)$ must be real.

Since linearised theory is applicable, the loading at the point $(x, y, 0)$ on W can be written in the form

$$L_p^{(1)}(x, y; \nu, M) b_p \, e^{i\omega t} \tag{3}$$

at time t, and the loading at the point (x, y, h) on T can be written in the form

$$L_p^{(2)}(x, y; \nu, M) b_p \, e^{i\omega t} \tag{4}$$

at time t, where

$$\nu = \frac{\omega l}{V} \tag{5}$$

is the frequency parameter, l is a typical length of the wing and tailplane, such as mean chord of W, and M is the Mach number of the uniform flow at large distances, given by

$$M = \frac{V}{a}, \tag{6}$$

where a is the speed of sound in the uniform flow.

For dynamical analyses of the vibration of the wing-tailplane combination we generally apply Lagrange's equations of motion and to do this we need expressions for the generalised airforces that occur. These airforces act on the actual wing and tailplane but within the linearised approximation we can take the loadings described above as acting on W and T to evaluate these generalised airforces. The expression for P_{pq}, the generalised airforce in the mode p due to oscillation in the mode q is then given by

$$P_{pq} = b_q\, e^{i\omega t} \iint\limits_W Z_p^{(1)}(x, y) L_q^{(1)}(x, y; \nu, M)\, dx\, dy + b_q\, e^{i\omega t} \iint\limits_T Z_p^{(2)}(x, y) L_q^{(2)}(x, y; \nu, M)\, dx\, dy. \tag{7}$$

We introduce reduced displacement functions $\zeta_p^{(1)}(x, y)$ and $\zeta_p^{(2)}(x, y)$ for the wing and tailplane by means of the formulae

$$Z_p^{(1)}(x, y) = l\zeta_p^{(1)}(x, y) \tag{8}$$

and

$$Z_p^{(2)}(x, y) = l\zeta_p^{(2)}(x, y), \tag{9}$$

and we introduce reduced loading functions $\lambda_p^{(1)}(x, y; \nu, M)$ and $\lambda_p^{(2)}(x, y; \nu, M)$ for the wing and tailplane by means of the formulae

$$L_p^{(1)}(x, y; \nu, M) = \rho V^2 \lambda_p^{(1)}(x, y; \nu, M) \tag{10}$$

and

$$L_p^{(2)}(x, y; \nu, M) = \rho V^2 \lambda_p^{(2)}(x, y; \nu, M). \tag{11}$$

If we substitute (8), (9), (10) and (11) into (7) we get

$$P_{pq} = \rho V^2 l^3 Q_{pq} b_q\, e^{i\omega t}, \tag{12}$$

where

$$\begin{aligned} Q_{pq} &= Q_{pq}(\nu, M) \\ &= \frac{1}{l^2} \iint\limits_W \zeta_p^{(1)}(x, y) \lambda_q^{(1)}(x, y; \nu, M)\, dx\, dy + \frac{1}{l^2} \iint\limits_T \zeta_p^{(2)}(x, y) \lambda_q^{(2)}(x, y; \nu, M)\, dx\, dy. \end{aligned} \tag{13}$$

Our object is to determine the $Q_{pq}(\nu, M)$ for the wing-tailplane combination oscillating in given modes, at a given frequency parameter, in a subsonic flow of given Mach number.

It is customary, for dynamical analyses, to write Q_{pq} in the form

$$Q_{pq} = Q'_{pq} + i\nu Q''_{pq}, \tag{14}$$

where Q'_{pq} and Q''_{pq} are real quantities.

Since the system of wing and tailplane is assumed to be symmetric about the x, z coordinate plane we can write

$$\zeta_p^{(1)}(x, y) = \zeta_p^{(1,s)}(x, y) + \zeta_p^{(1,a)}(x, y), \tag{15}$$

$$\zeta_p^{(2)}(x, y) = \zeta_p^{(2,s)}(x, y) + \zeta_p^{(2,a)}(x, y), \tag{16}$$

$$\lambda_p^{(1)}(x, y; \nu, M) = \lambda_p^{(1,s)}(x, y; \nu, M) + \lambda_p^{(1,a)}(x, y; \nu, M) \tag{17}$$

and

$$\lambda_p^{(2)}(x, y; \nu, M) = \lambda_p^{(2,s)}(x, y; \nu, M) + \lambda_p^{(2,a)}(x, y; \nu, M), \tag{18}$$

where

$$\zeta_p^{(1,s)}(x, y) = \tfrac{1}{2}\zeta_p^{(1)}(x, y) + \tfrac{1}{2}\zeta_p^{(1)}(x, -y), \tag{19}$$

$$\zeta_p^{(1,a)}(x, y) = \tfrac{1}{2}\zeta_p^{(1)}(x, y) - \tfrac{1}{2}\zeta_p^{(1)}(x, -y), \tag{20}$$

$$\zeta_p^{(2,s)}(x, y) = \tfrac{1}{2}\zeta_p^{(2)}(x, y) + \tfrac{1}{2}\zeta_p^{(2)}(x, -y), \tag{21}$$

$$\zeta_p^{(2,a)}(x, y) = \tfrac{1}{2}\zeta_p^{(2)}(x, y) - \tfrac{1}{2}\zeta_p^{(2)}(x, -y), \tag{22}$$

$$\lambda_p^{(1,s)}(x, y; \nu, M) = \tfrac{1}{2}\lambda_p^{(1)}(x, y; \nu, M) + \tfrac{1}{2}\lambda_p^{(1)}(x, -y; \nu, M), \tag{23}$$

$$\lambda_p^{(1,a)}(x, y; \nu, M) = \tfrac{1}{2}\lambda_p^{(1)}(x, y; \nu, M) - \tfrac{1}{2}\lambda_p^{(1)}(x, -y; \nu, M), \tag{24}$$

$$\lambda_p^{(2,s)}(x, y; \nu, M) = \tfrac{1}{2}\lambda_p^{(2)}(x, y; \nu, M) + \tfrac{1}{2}\lambda_p^{(2)}(x, -y; \nu, M) \tag{25}$$

and

$$\lambda_p^{(2,a)}(x, y; \nu, M) = \tfrac{1}{2}\lambda_p^{(2)}(x, y; \nu, M) - \tfrac{1}{2}\lambda_p^{(2)}(x, -y; \nu, M). \tag{26}$$

The functions $\zeta_p^{(1,s)}(x, y)$, $\zeta_p^{(2,s)}(x, y)$, $\lambda_p^{(1,s)}(x, y; \nu, M)$ and $\lambda_p^{(2,s)}(x, y; \nu, M)$ are even functions of y, whereas the functions $\zeta_p^{(1,a)}(x, y)$, $\zeta_p^{(2,a)}(x, y)$, $\lambda_p^{(1,a)}(x, y; \nu, M)$ and $\lambda_p^{(2,a)}(x, y; \nu, M)$ are odd functions of y. The reduced displacement functions and reduced loading functions have therefore been resolved into symmetric and antisymmetric parts in equations (15), (16), (17) and (18). Further it is seen that the loading function corresponding to a symmetric displacement function is symmetric and the loading function corresponding to an antisymmetric displacement function is antisymmetric in the variable y.

If we substitute from equations (15), (16), (17) and (18) into equation (13) we get

$$Q_{pq} = \frac{1}{l^2}\iint_W \zeta_p^{(1,s)}(x, y)\lambda_q^{(1,s)}(x, y; \nu, M)\,dx\,dy + \frac{1}{l^2}\iint_T \zeta_p^{(2,s)}(x, y)\lambda_q^{(2,s)}(x, y; \nu, M)\,dx\,dy +$$

$$+ \frac{1}{l^2}\iint_W \zeta_p^{(1,a)}(x, y)\lambda_q^{(1,a)}(x, y; \nu, M)\,dx\,dy + \frac{1}{l^2}\iint_T \zeta_p^{(2,a)}(x, y)\lambda_q^{(2,a)}(x, y; \nu, M)\,dx\,dy. \tag{27}$$

It is convenient in dynamical applications to use only modes which are either purely symmetric or purely antisymmetric. If p and q refer to such modes which are not both symmetric or not both antisymmetric then, according to (27),

$$Q_{pq} = 0. \tag{28}$$

Hence any dynamical problem concerned with the symmetric wing-tailplane combination can be considered as two separate problems. In the one all the modes are purely symmetric and in the other all the modes are purely antisymmetric. To cope with either of these problems we write

$$\zeta_p^{(1)}(x, -y) = \kappa\zeta_p^{(1)}(x, y), \tag{29}$$

$$\zeta_p^{(2)}(x, -y) = \kappa\zeta_p^{(2)}(x, y), \tag{30}$$

$$\lambda_p^{(1)}(x, -y; \nu, M) = \kappa\lambda_p^{(1)}(x, y; \nu, M) \tag{31}$$

and

$$\lambda_p^{(2)}(x, -y; \nu, M) = \kappa\lambda_p^{(2)}(x, y; \nu, M). \tag{32}$$

Then for purely symmetric oscillations we put $\kappa = 1$ and for purely antisymmetric oscillations we put $\kappa = -1$.

3. The Integral Equation Relating the Loadings and Upwashes

The boundary condition that the airflow does not penetrate either the wing or the tailplane surface can be transferred to the flat surfaces W and T where it takes the form that the upwash on W and T takes prescribed forms when the vibration of the wing and tailplane is prescribed. We are interested only in the time-dependent contribution to the upwash and in the mode q of oscillation; these are given by

$$W_q^{(1)}(x, y)b_q\,e^{i\omega t} \tag{33}$$

at time t at the point $(x, y, 0)$ of W, and by

$$W_q^{(2)}(x, y)b_q\,e^{i\omega t} \tag{34}$$

at time t, at the point (x, y, h) of T, where

$$W_q^{(1)}(x, y) = V\frac{\partial}{\partial x}Z_q^{(1)}(x, y) + i\omega Z_q^{(1)}(x, y) \tag{35}$$

and

$$W_q^{(2)}(x, y) = V\frac{\partial}{\partial x}Z_q^{(2)}(x, y) + i\omega Z_q^{(2)}(x, y). \tag{36}$$

If we introduce reduced upwash functions $\alpha_q^{(1)}(x, y; v)$ and $\alpha_q^{(2)}(x, y; v)$ for the wing and tailplane by means of the formulae

$$W_q^{(1)}(x, y) = V\alpha_1^{(1)}(x, y; v) \tag{37}$$

and

$$W_q^{(2)}(x, y) = V\alpha_q^{(2)}(x, y; v), \tag{38}$$

then by substituting formulae (37) and (38) into equations (35) and (36) and making use of equations (8) and (9) we get

$$\alpha_q^{(1)}(x, y; v) = \left(l\frac{\partial}{\partial x} + iv\right)\zeta_q^{(1)}(x, y) \tag{39}$$

and

$$\alpha_q^{(2)}(x, y; v) = \left(l\frac{\partial}{\partial x} + iv\right)\zeta_q^{(2)}(x, y). \tag{40}$$

If now we apply linearised potential-flow theory we get the following pair of simultaneous integral equations:

$$\begin{aligned}\alpha_q^{(1)}(x, y; v) = &\frac{1}{4\pi l^2}\iint_W \lambda_q^{(1)}(x_0, y_0; v, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; v, M\right) \exp\left\{-\frac{iv(x - x_0)}{l}\right\} dx_0\, dy_0 + \\ &+ \frac{1}{4\pi l^2}\iint_T \lambda_q^{(2)}(x_0, y_0; v, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; v, M\right) \exp\left\{-\frac{iv(x - x_0)}{l}\right\} dx_0\, dy_0\end{aligned} \tag{41}$$

$$\begin{aligned}\alpha_q^{(2)}(x, y; v) = &\frac{1}{4\pi l^2}\iint_W \lambda_q^{(1)}(x_0, y_0; v, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; v, M\right) \exp\left\{-\frac{iv(x - x_0)}{l}\right\} dx_0\, dy_0 + \\ &+ \frac{1}{4\pi l^2}\iint_T \lambda_q^{(2)}(x_0, y_0; v, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; v, M\right) \exp\left\{-\frac{iv(x - x_0)}{l}\right\} dx_0\, dy_0,\end{aligned} \tag{42}$$

where

$$\begin{aligned}K\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}; v, M\right) = l^2\Bigg[&\int_{\frac{-x+MR}{1-M^2}}^{\infty} e^{-ivu/l}\frac{(u^2 + y^2 - 2z^2)}{(u^2 + y^2 + z^2)^{\frac{5}{2}}}\,du + \exp\left\{-\frac{iv}{l}\left(\frac{-x + MR}{1 - M^2}\right)\right\}\Bigg\{\frac{M(Mx + R)}{R(x^2 + y^2 + z^2)} - \\ &- \frac{z^2M(Mx + R)^3}{R(x^2 + y^2 + z^2)^3} - \frac{z^2M^2(1 - M^2)x}{R^3(x^2 + y^2 + z^2)} - \frac{2z^2M(Mx + R)}{R(x^2 + y^2 + z^2)^2} - \frac{iv}{l}\,\frac{z^2M^2(Mx + R)}{R^2(x^2 + y^2 + z^2)}\Bigg\}\Bigg]\end{aligned} \tag{43}$$

with

$$R = \sqrt{x^2 + (1 - M^2)(y^2 + z^2)}. \tag{44}$$

The formula (43) of the kernel function $K(x/l, y/l, z/l; v, M)$ can be obtained from Ref. 7.

We introduce parametric coordinates ξ_0, η_0 on W by means of the transformation formulae

$$\xi_0 = \frac{1}{c_1(y_0)}[x_0 - x_L^{(1)}(y_0)]$$

and

$$\eta_0 = \frac{1}{s_1}y_0 \tag{45}$$

where s_1 is the semi-span of W, $c_1(y_0)$ is the local chord of W and $x_L^{(1)}(y_0)$ is the x coordinate of the leading edge of W at spanwise position y_0. Then W is the region $0 \leqslant \xi_0 \leqslant 1$, $-1 \leqslant \eta_0 \leqslant 1$ of the ξ_0, η_0 space.

We introduce parametric coordinates ε_0, ζ_0 on T by means of the transformation formulae

$$\varepsilon_0 = \frac{1}{c_2(y_0)}[x_0 - x_L^{(2)}(y_0)]$$

and

$$\zeta_0 = \frac{1}{s_2}y_0 \tag{46}$$

where s_2 is the semi-span of T, $c_2(y_0)$ is the local chord of T and $x_L^{(2)}(y_0)$ is the x coordinate of the leading edge of T at spanwise position y_0. Then T is the region $0 \leqslant \varepsilon_0 \leqslant 1$, $-1 \leqslant \zeta_0 \leqslant 1$ of the ε_0, ζ_0 space.

The pair of integral equations (41) and (42) become, on transforming the integration variables by means of equations (45) and (46),

$$\alpha_q^{(1)}(x, y; \nu) = \frac{1}{4\pi}\frac{s_1}{l}\int_{-1}^{+1}\frac{c_1(y_0)}{l}\,d\eta_0\int_0^1 \lambda_q^{(1)}(x_0, y_0; \nu, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; \nu, M\right)\exp\left\{-\frac{i\nu(x - x_0)}{l}\right\}d\xi_0 +$$
$$+ \frac{1}{4\pi}\frac{s_2}{l}\int_{-1}^{+1}\frac{c_2(y_0)}{l}\,d\zeta_0\int_0^1 \lambda_q^{(2)}(x_0, y_0; \nu, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M\right)\times$$
$$\times \exp\left\{-\frac{i\nu(x - x_0)}{l}\right\}d\varepsilon_0 \tag{47}$$

and

$$\alpha_q^{(2)}(x, y; \nu) = \frac{1}{4\pi}\frac{s_1}{l}\int_{-1}^{+1}\frac{c_1(y_0)}{l}\,d\eta_0\int_0^1 \lambda_q^{(1)}(x_0, y_0; \nu, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M\right)\exp\left\{-\frac{i\nu(x - x_0}{l}\right\}d\xi_0 +$$
$$+ \frac{1}{4\pi}\frac{s_2}{l}\int_{-1}^{+1}\frac{c_2(y_0)}{l}\,d\zeta_0\int_0^1 \lambda_q^{(2)}(x_0, y_0; \nu, M)K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; \nu, M\right)\times$$
$$\times \exp\left\{-\frac{i\nu(x - x_0)}{l}\right\}d\varepsilon_0. \tag{48}$$

4. Approximation to the Loading Functions

The solution of equations (47) and (48) is not unique in general, but if Kutta's condition that the flow at the trailing edges of W and T is smooth is imposed, then the solution becomes unique. The reduced loading distributions $\lambda_q^{(1)}(x_0, y_0; \nu, M)$ and $\lambda_q^{(2)}(x_0, y_0; \nu, M)$ then acquire known behaviours near the edges of W and T.

For the parametric coordinates (ξ_0, η_0) introduced on W let $\xi_i^{(1)}$, $i = 1, 2, \ldots, n_1$, be a set of n_1 distinct points ξ_0 in (0, 1) and let $\eta_j^{(1)}$, $j = 1, 2, \ldots, m_1$, be a set of m_1 distinct points η_0 in $(-1, 1)$. Precise locations of these points will be given later, *see* equations (260) and (261).

Let $h_i^{(1)}(\xi_0)$, $i = 1, 2, \ldots, n_1$ be the set of n_1 interpolation polynomials based on the points $\xi_i^{(1)}$, $i = 1, 2, \ldots, n_1$, and defined by the formulae

$$h_i^{(1)}(\xi_0) = \prod_{\substack{r=1\\ r\neq i}}^{n_1}\left(\frac{\xi_0 - \xi_r^{(1)}}{\xi_i^{(1)} - \xi_r^{(1)}}\right), i = 1, 2, \ldots, n_1. \tag{49}$$

The $h_i^{(1)}(\xi_0)$ are a set of n_1 linearly independent polynomials of degree $(n - 1)$ in ξ_0 which have the property

$$h_i^{(1)}(\xi_r^{(1)}) = \delta_{ir}, \tag{50}$$

where δ_{ir} is Kronecker's delta.

Let $g_j^{(1)}(\eta_0)$, $j = 1, 2, \ldots, m_1$, be the set of m_1 interpolation polynomials based on the points $\eta_j^{(1)}$, $j = 1, 2, \ldots, m_1$, and defined by the formulae

$$g_j^{(1)}(\eta_0) = \prod_{\substack{s=1\\ s\neq j}}^{m_1}\left(\frac{\eta_0 - \eta_s^{(1)}}{\eta_j^{(1)} - \eta_s^{(1)}}\right), j = 1, 2, \ldots, m_1. \tag{51}$$

The $g_j^{(1)}(\eta_0)$ are a set of m_1 linearly independent polynomials of degree $(m_1 - 1)$ in η_0 which have the property

$$g_j^{(1)}(w_s^{(1)}) = \delta_{js}. \tag{52}$$

We take an approximation $\hat{\lambda}_q^{(1)}(x_0, y_0)$ to $\lambda_q^{(1)}(x_0, y_0; \nu, M)$, which is given by the formula

$$\hat{\lambda}_q^{(1)}(x_0, y_0) = \frac{l}{c_1(y_0)}\exp\left(-\frac{i\nu x_0}{l}\right)\sum_{r=1}^{n_1}\sum_{s=1}^{m_1} A_{q;r,s}^{(1)}h_r^{(1)}(\xi_0)g_s^{(1)}(\eta_0)\sqrt{\frac{1 - \xi_0}{\xi_0}}\sqrt{1 - \eta_0^2}. \tag{53}$$

The factor $\sqrt{(1 - \xi_0)/\xi_0}\sqrt{1 - \eta_0^2}$ accounts for the known behaviour of the reduced loading at the edges of W. The factor $l\exp(-i\nu x_0/l)/c_1(y_0)$ is introduced for convenience since by doing so numerical integration work is reduced to some extent. The summation

$$\sum_{r=1}^{n_1}\sum_{s=1}^{m_1} A_{q;r,s}^{(1)}h_r^{(1)}(\xi_0)g_s^{(1)}(\eta_0)$$

is a general double polynomial of degree $(n_1 - 1)$ in ξ_0 and $(m_1 - 1)$ in η_0. The coefficients $A^{(1)}_{q;r,s}$ are, as yet, undetermined.

For the parametric coordinates (ε_0, ζ_0) introduced on T let $\xi^{(2)}_i$, $i = 1, 2, \ldots, n_2$, be a set of n_2 distinct points ε_0 in $(0, 1)$ and let $\eta^{(2)}_j$, $j = 1, 2, \ldots, m_2$, be a set of m_2 distinct points ζ_0 in $(-1, 1)$. Precise locations of these points will be given later, *see* equations (262) and (263).

Let $h^{(2)}_i(\varepsilon_0)$, $i = 1, 2, \ldots, n_2$, be the set of n_2 interpolation polynomials based on the points $\xi^{(2)}_i$, $i = 1, 2, \ldots, n_2$, and defined by the formulae

$$h^{(2)}_i(\varepsilon_0) = \prod_{\substack{r=1\\r\neq i}}^{n_2} \left(\frac{\varepsilon_0 - \xi^{(2)}_r}{\xi^{(2)}_i - \xi^{(2)}_r}\right), \; i = 1, 2, \ldots, n_2. \tag{54}$$

The $h^{(2)}_i(\varepsilon_0)$ are a set of n_2 linearly independent polynomials of degree $(n_2 - 1)$ in ε_0 which have the property

$$h^{(2)}_i(\xi^{(2)}_r) = \delta_{ir}. \tag{55}$$

Let $g^{(2)}_j(\zeta_0)$, $j = 1, 2, \ldots, m_2$, be the set of m_2 interpolation polynomials based on the points $\eta^{(2)}_j$, $j = 1, 2, \ldots, m_2$, and defined by the formulae

$$g^{(2)}_j(\zeta_0) = \prod_{\substack{s=1\\s\neq j}}^{m_2} \left(\frac{\zeta_0 - \eta^{(2)}_s}{\eta^{(2)}_j - \eta^{(2)}_s}\right), \; j = 1, 2, \ldots, m_2. \tag{56}$$

The $g^{(2)}_j(\zeta_0)$ are a set of m_2 linearly independent polynomials of degree $(m_2 - 1)$ in ζ_0 which have the property

$$g^{(2)}_j(\eta^{(2)}_s) = \delta_{js}. \tag{57}$$

We take an approximation $\hat{\lambda}^{(2)}_q(x_0, y_0)$ to $\lambda^{(2)}_q(x_0, y_0; v, M)$, which is given by the formula

$$\hat{\lambda}^{(2)}_q(x_0, y_0) = \frac{l}{c_2(y_0)} \exp\left(-\frac{ivx_0}{l}\right) \sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A^{(2)}_{q;r,s} h^{(2)}_r(\varepsilon_0) g^{(2)}_s(\zeta_0) \sqrt{\frac{1 - \varepsilon_0}{\varepsilon_0}} \sqrt{1 - \zeta_0^2}. \tag{58}$$

The factor $\sqrt{(1 - \varepsilon_0)/\varepsilon_0}\sqrt{1 - \zeta_0^2}$ accounts for the known behaviour of the reduced loading at the edges of T. The factor $l\exp(-ivx_0/l)/c_2(y_0)$ is introduced for convenience since by doing so numerical integration work is reduced to some extent. The summation

$$\sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A^{(2)}_{q;r,s} h^{(2)}_r(\varepsilon_0) g^{(2)}_s(\zeta_0)$$

is a general double polynomial of degree $(n_2 - 1)$ in ε_0 and $(m_2 - 1)$ in ζ_0. The coefficients $A^{(2)}_{q;r,s}$ are, as yet, undetermined.

The choice of locations of the points $\xi^{(1)}_i$, $i = 1, 2, \ldots, n_1$; $\xi^{(2)}_i$, $i = 1, 2, \ldots, n_2$; $\eta^{(1)}_j$, $j = 1, 2, \ldots, m_1$; $\eta^{(2)}_j$, $j = 1, 2, \ldots, m_2$ will have no influence on the accuracy of the approximations (53) and (58), except insofar as good conditioning of matrices, from which the $A^{(1)}_{q;r,s}$ and $A^{(2)}_{q;r,s}$ are obtained later in the analysis, may be affected. The expressions (53) and (58) are always the most general expressions of their kind. The results obtained would be exactly the same irrespective of the location of these points, provided precise numerical values at each stage of the calculation were possible. Since we have to work with a relatively small number of significant figures the results can be different because ill conditioning may lead to loss in accuracy through not retaining a sufficient number of significant figures. The choice of points we make will be discussed near the end of Section 6.

If we substitute the approximations (53) and (58) for $\lambda^{(1)}_q(x_0, y_0; v, M)$ and $\lambda^{(2)}_q(x_0, y_0; v, M)$ into the right hand sides of equations (47) and (48) and denote the resulting functions on the left hand sides by $\hat{\alpha}^{(1)}_q(x, y)$ and $\hat{\alpha}^{(2)}_q(x, y)$ respectively, then we can write

$$\hat{\alpha}^{(1)}_q(x, y) = \left[\sum_{r=1}^{n_1} \sum_{s=1}^{m_1} A^{(1)}_{q;r,s} U^{(1)}_{r,s}(x, y; v, M) + \sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A^{(2)}_{q;r,s} V^{(2)}_{r,s}(x, y; v, M)\right] \exp\left(-\frac{ivx}{l}\right) \tag{59}$$

and

$$\hat{\alpha}^{(2)}_q(x, y) = \left[\sum_{r=1}^{n_1} \sum_{s=1}^{m_1} A^{(1)}_{q;r,s} V^{(1)}_{r,s}(x, y; v, M) + \sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A^{(2)}_{q;r,s} U^{(2)}_{r,s}(x, y; v, M)\right] \exp\left(-\frac{ivx}{l}\right) \tag{60}$$

where

$$U^{(1)}_{r,s}(x, y; v, M) = \frac{1}{4\pi} \frac{s_1}{l} \int_{-1}^{+1} g^{(1)}_s(\eta_0)\sqrt{1 - \eta_0^2}\, d\eta_0 \int_0^1 h^{(1)}_r(\xi_0)\sqrt{\frac{1 - \xi_0}{\xi_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; v, M\right) d\xi_0, \tag{61}$$

$$U_{r,s}^{(2)}(x, y; \nu, M) = \frac{1}{4\pi} \frac{s_2}{l} \int_{-1}^{+1} g_s^{(2)}(\zeta_0)\sqrt{1 - \zeta_0^2}\, d\zeta_0 \int_0^1 h_r^{(2)}(\varepsilon_0)\sqrt{\frac{1 - \varepsilon_0}{\varepsilon_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; \nu, M\right) d\varepsilon_0, \tag{62}$$

$$V_{r,s}^{(1)}(x, y; \nu, M) = \frac{1}{4\pi} \frac{s_1}{l} \int_{-1}^{+1} g_s^{(1)}(\eta_0)\sqrt{1 - \eta_0^2}\, d\eta_0 \int_0^1 h_r^{(1)}(\xi_0)\sqrt{\frac{1 - \xi_0}{\xi_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M\right) d\xi_0 \tag{63}$$

and

$$V_{r,s}^{(2)}(x, y; \nu, M) = \frac{1}{4\pi} \frac{s_2}{l} \int_{-1}^{+1} g_s^{(2)}(\zeta_0)\sqrt{1 - \zeta_0^2}\, d\zeta_0 \int_0^1 h_r^{(2)}(\varepsilon_0)\sqrt{\frac{1 - \varepsilon_0}{\varepsilon_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M\right) d\varepsilon_0. \tag{64}$$

We determine the coefficients $A_{q;r,s}^{(1)}$ and $A_{q;r,s}^{(2)}$ from the sets of linear equations

$$\int_{-1}^{+1} g_j^{(1)}(\eta)\sqrt{1 - \eta^2}\, d\eta \int_0^1 h_i^{(1)}(1 - \xi)\sqrt{\frac{\xi}{1 - \xi}}\{\alpha_q^{(1)}(x, y; \nu) - \hat{\alpha}_q^{(1)}(x, y)\} \times$$
$$\times \exp\left(\frac{i\nu x}{l}\right) d\xi = 0, \qquad \begin{array}{l} i = 1, 2, \ldots, n_1 \\ j = 1, 2, \ldots, m_1 \end{array} \tag{65}$$

and

$$\int_{-1}^{+1} g_j^{(2)}(\zeta)\sqrt{1 - \zeta^2}\, d\zeta \int_0^1 h_i^{(2)}(1 - \varepsilon)\sqrt{\frac{\varepsilon}{1 - \varepsilon}}\{\alpha_q^{(2)}(x, y; \nu) - \hat{\alpha}_q^{(2)}(x, y)\}$$
$$\times \exp\left(\frac{i\nu x}{l}\right) d\varepsilon = 0, \qquad \begin{array}{l} i = 1, 2, \ldots, n_2 \\ j = 1, 2, \ldots, m_2 \end{array} \tag{66}$$

where

$$\xi = \frac{1}{c_1(y)}[x - x_L^{(1)}(y)]$$

and

$$\eta = \frac{1}{s_1} y \tag{67}$$

on W, and

$$\varepsilon = \frac{1}{c_2(y)}[x - x_L^{(2)}(y)]$$

and

$$\zeta = \frac{1}{s_2} y \tag{68}$$

on T. The equations (65) and (66) are obtained in a straightforward manner by applying the Flax variational procedure, in exactly the same manner as was indicated for a single plane wing in Ref. 8.

If we substitute from equations (59) and (60) into equations (65) and (66) we get

$$\theta_{q;i,j}^{(1)} = \sum_{r=1}^{n_1} \sum_{s=1}^{m_1} A_{q;r,s}^{(1)} \psi_{i,j;r,s}^{(1,1)} + \sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A_{q;r,s}^{(2)} \psi_{i,j;r,s}^{(1,2)} \qquad i = 1, 2, \ldots, n_1; \quad j = 1, 2, \ldots, m_1, \tag{69}$$

and

$$\theta_{q;i,j}^{(2)} = \sum_{r=1}^{n_1} \sum_{s=1}^{m_1} A_{q;r,s}^{(1)} \psi_{i,j;r,s}^{(2,1)} + \sum_{r=1}^{n_2} \sum_{s=1}^{m_2} A_{q;r,s}^{(2)} \psi_{i,j;r,s}^{(2,2)} \qquad i = 1, 2, \ldots, n_2; \quad j = 1, 2, \ldots, m_2, \tag{70}$$

where

$$\theta_{q;i,j}^{(1)} = \int_{-1}^{+1} g_j^{(1)}(\eta)\sqrt{1 - \eta^2}\, d\eta \int_0^1 h_i^{(1)}(1 - \xi)\sqrt{\frac{\xi}{1 - \xi}}\alpha_q^{(1)}(x, y; \nu) \exp\left(\frac{i\nu x}{l}\right) d\xi$$
$$i = 1, 2, \ldots, n_1; \quad j = 1, 2, \ldots, m_1, \tag{71}$$

$$\theta_{q;i,j}^{(2)} = \int_{-1}^{+1} g_j^{(2)}(\zeta)\sqrt{1 - \zeta^2}\, d\zeta \int_0^1 h_i^{(2)}(1 - \varepsilon)\sqrt{\frac{\varepsilon}{1 - \varepsilon}}\alpha_q^{(2)}(x, y; \nu) \exp\left(\frac{i\nu x}{l}\right) d\xi$$

$$i = 1,2,\ldots,n_2; \quad j = 1,2,\ldots,m_2 \tag{72}$$

$$\psi^{(1,1)}_{i,j;r,s} = \int_{-1}^{+1} g_j^{(1)}(\eta)\sqrt{1-\eta^2}\,d\eta \int_0^1 h_i^{(1)}(1-\xi)\sqrt{\frac{\xi}{1-\xi}}U^{(1)}_{r,s}(x, y; \nu, M)\,d\xi$$

$$i = 1,2,\ldots,n_1; \quad r = 1,2,\ldots,n_1; \quad j = 1,2,\ldots,m_1; \quad s = 1,2,\ldots,m_1, \tag{73}$$

$$\psi^{(2,2)}_{i,j;r,s} = \int_{-1}^{+1} g_j^{(2)}(\zeta)\sqrt{1-\zeta^2}\,d\zeta \int_0^1 h_i^{(2)}(1-\varepsilon)\sqrt{\frac{\varepsilon}{1-\varepsilon}}U^{(2)}_{r,s}(x, y; \nu, M)\,d\varepsilon$$

$$i = 1,2,\ldots,n_2; \quad r = 1,2,\ldots,n_2; \quad j = 1,2,\ldots,m_2; \quad s = 1,2,\ldots,m_2, \tag{74}$$

$$\psi^{(1,2)}_{i,j;r,s} = \int_{-1}^{+1} g_j^{(1)}(\eta)\sqrt{1-\eta^2}\,d\eta \int_0^1 h_i^{(1)}(1-\xi)\sqrt{\frac{\xi}{1-\xi}}V^{(2)}_{r,s}(x, y; \nu, M)\,d\xi$$

$$i = 1,2,\ldots,n_1; \quad r = 1,2,\ldots,n_2; \quad j = 1,2,\ldots,m_1; \quad s = 1,2,\ldots,m_2, \tag{75}$$

and

$$\psi^{(2,1)}_{i,j;r,s} = \int_{-1}^{+1} g_j^{(2)}(\zeta)\sqrt{1-\zeta^2}\,d\zeta \int_0^1 h_i^{(2)}(1-\varepsilon)\sqrt{\frac{\varepsilon}{1-\varepsilon}}V^{(1)}_{r,s}(x, y; \nu, M)\,d\varepsilon$$

$$i = 1,2,\ldots,n_2; \quad r = 1,2,\ldots,n_1; \quad j = 1,2,\ldots,m_2; \quad s = 1,2,\ldots,m_1. \tag{76}$$

The sets of equations (69) and (70) form together a set of $m_1n_1 + m_2n_2$ linear simultaneous equations for the m_1n_1 coefficients $A^{(1)}_{q;r,s}$, $r = 1,2,\ldots,n_1$; $s = 1,2,\ldots,m_1$, and the m_2n_2 coefficients $A^{(2)}_{q;r,s}$, $r = 1,2,\ldots,n_2$; $s = 1,2,\ldots,m_2$.

The wing-tailplane configuration has been assumed to be symmetric about the xz coordinate plane and the relations (29), (30), (31) and (32) are valid since we shall consider only purely symmetric or purely anti-symmetric oscillations. We take the loading points $\eta_j^{(1)}$ and $\eta_j^{(2)}$ to be symmetrically distributed about zero. The number of equations and unknowns in (69) and (70) can then be reduced. The process for doing this is slightly different for even and odd values of m_1 and m_2. We shall consider here only the case of both m_1 and m_2 being even, and we shall order the points $\eta_j^{(1)}$ and $\eta_j^{(2)}$ such that

$$\eta^{(1)}_{m_1-s+1} = -\eta^{(1)}_s, \qquad s = 1,2,\ldots,\frac{m_1}{2},$$

and

$$\eta^{(2)}_{m_2-s+1} = -\eta^{(2)}_s, \qquad s = 1,2,\ldots,\frac{m_2}{2}. \tag{77}$$

It then follows that

$$g^{(1)}_{m_1-s+1}(\eta_0) = g_s^{(1)}(-\eta_0), \qquad s = 1,2,\ldots,\frac{m_1}{2},$$

$$g^{(1)}_{m_2-s+1}(\zeta_0) = g_s^{(2)}(-\zeta_0), \qquad s = 1,2,\ldots,\frac{m_2}{2}. \tag{78}$$

Then, since $K(x/l, y/l, z/l; \nu, M)$ is an even function of y/l we get immediately from equations (61), (62), (63) and (64) that

$$U^{(1)}_{r,s}(x, -y; \nu, M) = U^{(1)}_{r,m_1-s+1}(x, y; \nu, M), \tag{79}$$

$$U^{(2)}_{r,s}(x, -y; \nu, M) = U^{(2)}_{r,m_2-s+1}(x, y; \nu, M), \tag{80}$$

$$V^{(1)}_{r,s}(x, -y; \nu, M) = V^{(1)}_{r,m_1-s+1}(x, y; \nu, M) \tag{81}$$

and

$$V^{(2)}_{r,s}(x, -y; \nu, M) = V^{(2)}_{r,m_2-s+1}(x, y; \nu, M). \tag{82}$$

Further, it follows from equations (73), (74), (75) and (76) that

$$\psi^{(1,1)}_{i,m_1-j+1;r,m_1-s+1} = \psi^{(1,1)}_{i,j;r,s}, \tag{83}$$

$$\psi^{(1,1)}_{i,j;r,m_1-s+1} = \psi^{(1,1)}_{i,m_1-j+1;r,s}, \tag{84}$$

$$\psi^{(2,2)}_{i,m_2-j+1;r,m_2-s+1} = \psi^{(2,2)}_{i,j;r,s}, \tag{85}$$

$$\psi^{(2,2)}_{i,j;r,m_2-s+1} = \psi^{(2,2)}_{i,m_2-j+1;r,s}, \tag{86}$$

$$\psi^{(1,2)}_{i,m_1-j+1;r,m_2-s+1} = \psi^{(1,2)}_{i,j;r,s}, \tag{87}$$

$$\psi^{(1,2)}_{i,j;r,m_2-s+1} = \psi^{(1,2)}_{i,m_1-j+1;r,s}, \tag{88}$$

$$\psi^{(2,1)}_{i,m_2-j+1;r,m_1-s+1} = \psi^{(2,1)}_{i,j;r,s} \tag{89}$$

and

$$\psi^{(2,1)}_{i,j;r,m_1-s+1} = \psi^{(2,1)}_{i,m_2-j+1;r,s}. \tag{90}$$

The reduced upwash functions $\alpha^{(1)}_q(x, y; \nu)$ and $\alpha^{(2)}_q(x, y; \nu)$ satisfy the relations

$$\alpha^{(1)}_q(x, -y; \nu) = \kappa\alpha^{(1)}_q(x, y; \nu) \tag{91}$$

and

$$\alpha^{(2)}_q(x, -y; \nu) = \kappa\alpha^{(2)}_q(x, y; \nu), \tag{92}$$

which follow from equations (29), (30), (39) and (40).

Therefore, from equations (71) and (72) we get

$$\theta^{(1)}_{q;i,m_1-j+1} = \kappa\theta^{(1)}_{q;i,j}, \tag{93}$$

and

$$\theta^{(2)}_{q;i,m_2-j+1} = \kappa\theta^{(2)}_{q;i,j}. \tag{94}$$

In view of the relationships (79) to (90) and (93) and (94) it follows that the solutions $A^{(1)}_{q;r,s}$, $r = 1, 2, \ldots, n_1$; $s = 1, 2, \ldots, m_1$, and $A^{(2)}_{q;r,s}$, $r = 1, 2, \ldots, n_2$; $s = 1, 2, \ldots, m_2$ of equations (69) and (70) must satisfy the relations

$$A^{(1)}_{q;r,m_1-s+1} = \kappa A^{(1)}_{q;r,s}, \tag{95}$$

$$A^{(2)}_{q;r,m_2-s+1} = \kappa A^{(2)}_{q;r,s}. \tag{96}$$

It now follows that the set of $m_1n_1 + m_2n_2$ linear simultaneous equations (69) and (70) may be replaced by the set of $\frac{1}{2}(m_1n_1 + m_2n_2)$ linear simultaneous equations

$$\theta^{(1)}_{q;i,j} = \sum_{r=1}^{n_1}\sum_{s=1}^{\frac{1}{2}m_1} A^{(1)}_{q;r,s}[\psi^{(1,1)}_{i,j;r,s} + \kappa\psi^{(1,1)}_{i,j;r,m_1-s+1}] + \sum_{r=1}^{n_2}\sum_{s=1}^{\frac{1}{2}m_2} A^{(2)}_{q;r,s}[\psi^{(1,2)}_{i,j;r,s} + \kappa\psi^{(1,2)}_{i,j;r,m_2-s+1}] \quad \begin{array}{l} i = 1, 2, \ldots, n_1, \\ j = 1, 2, \ldots, \frac{1}{2}m_1, \end{array} \tag{97}$$

and

$$\theta^{(2)}_{q;i,j} = \sum_{r=1}^{n_1}\sum_{s=1}^{\frac{1}{2}m_1} A^{(1)}_{q;r,s}[\psi^{(2,1)}_{i,j;r,s} + \kappa\psi^{(2,1)}_{i,j;r,m_1-s+1}] + \sum_{r=1}^{n_2}\sum_{s=1}^{\frac{1}{2}m_2} A^{(2)}_{q;r,s}[\psi^{(2,2)}_{i,j;r,s} + \kappa\psi^{(2,2)}_{i,j;r,m_2-s+1}] \quad \begin{array}{l} i = 1, 2, \ldots, n_2, \\ j = 1, 2, \ldots, \frac{1}{2}m_2. \end{array} \tag{98}$$

The set of equations (97) and (98) may be written as the single matrix equation

$$\begin{bmatrix} \Theta^{(1)}_q \\ \Theta^{(2)}_q \end{bmatrix} = \begin{bmatrix} \Psi^{(1,1)} & \Psi^{(1,2)} \\ \Psi^{(2,1)} & \Psi^{(2,2)} \end{bmatrix} \begin{bmatrix} A^{(1)}_q \\ A^{(2)}_q \end{bmatrix}. \tag{99}$$

The column submatrix $\Theta^{(1)}_q$ consists of the $\frac{1}{2}m_1n_1$ elements $\theta^{(1)}_{q;i,j}$, $i = 1, 2, \ldots, n_1$; $j = 1, 2, \ldots, \frac{1}{2}m_1$. The column submatrix $\Theta^{(2)}_q$ consists of the $\frac{1}{2}m_2n_2$ elements $\theta^{(2)}_{q;i,j}$, $i = 1, 2, \ldots, n_2$; $j = 1, 2, \ldots, \frac{1}{2}m_2$. The column submatrix $A^{(1)}_q$ consists of the $\frac{1}{2}m_1n_1$ elements $A^{(1)}_{q;r,s}$, $r = 1, 2, \ldots, n_1$; $s = 1, 2, \ldots, \frac{1}{2}m_1$. The column submatrix $A^{(2)}_q$ consists of the $\frac{1}{2}m_2n_2$ elements $A^{(2)}_{q;r,s}$, $r = 1, 2, \ldots, n_2$; $s = 1, 2, \ldots, \frac{1}{2}m_2$.

We must arrange the elements of $\Theta^{(1)}_q$, $\Theta^{(2)}_q$, $A^{(1)}_q$ and $A^{(2)}_q$ in some prescribed order. It is convenient to use the following ordering:

$\theta^{(1)}_{q;i,j}$ is the $n_1(\frac{1}{2}m_1 - j) + i$'th element of $\Theta^{(1)}_q$, $\theta^{(2)}_{q;i,j}$ is the $n_2(\frac{1}{2}m_2 - j) + i$'th element of $\Theta^{(2)}_q$,

$A^{(1)}_{q;r,s}$ is the $n_1(\frac{1}{2}m_1 - s) + r$'th element of $A^{(1)}_q$ and $A^{(2)}_{q;r,s}$ is the $n_2(\frac{1}{2}m_2 - s) + r$'th element of $A^{(2)}_q$.

The arrangement of elements in $\Psi^{(1,1)}$, $\Psi^{(2,2)}$, $\Psi^{(1,2)}$ and $\Psi^{(2,1)}$ must correspond to the arrangement of elements in $\Theta_q^{(1)}$, $\Theta_q^{(2)}$, $A_q^{(1)}$ and $A_q^{(2)}$. The submatrix $\Psi^{(1,1)}$ is a square matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_1n_1$ with the element

$$\psi^{(1,1)}_{i,j;r,s} + \kappa\psi^{(1,1)}_{i,j;r,m_1-s+1} \qquad \begin{array}{ll} i = 1,2,\ldots,n_1; & r = 1,2,\ldots,n_1; \\ j = 1,2,\ldots,\frac{1}{2}m_1; & s = 1,2,\ldots,\frac{1}{2}m_1, \end{array}$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and $n_1(\frac{1}{2}m_1 - s) + r$'th column. The submatrix $\Psi^{(2,2)}$ is a square matrix of order $\frac{1}{2}m_2n_2 + \frac{1}{2}m_2n_2$ with the element

$$\psi^{(2,2)}_{i,j;r,s} + \kappa\psi^{(2,2)}_{i,j;r,m_2-s+1} \qquad \begin{array}{ll} i = 1,2,\ldots,n_2; & r = 1,2,\ldots,n_1; \\ j = 1,2,\ldots,\frac{1}{2}m_2; & s = 1,2,\ldots,\frac{1}{2}m_2, \end{array}$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and $n_2(\frac{1}{2}m_2 - s) + r$'th column. The submatrix $\Psi^{(1,2)}$ is a rectangular matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_2n_2$ with the element

$$\psi^{(1,2)}_{i,j;r,s} + \kappa\psi^{(1,2)}_{i,j;r,m_1-s+1} \qquad \begin{array}{ll} i = 1,2,\ldots,n_1; & r = 1,2,\ldots,n_2; \\ j = 1,2,\ldots,\frac{1}{2}m_1; & s = 1,2,\ldots,\frac{1}{2}m_2, \end{array}$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and $n_2(\frac{1}{2}m_2 - s) + r$'th column. The submatrix $\Psi^{(2,1)}$ is a rectangular matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}m_1n_1$ with the element

$$\psi^{(2,1)}_{i,j;r,s} + \kappa\psi^{(2,1)}_{i,j;r,m_1-s+1} \qquad \begin{array}{ll} i = 1,2,\ldots,n_2; & r = 1,2,\ldots,n_1; \\ j = 1,2,\ldots,\frac{1}{2}m_2; & s = 1,2,\ldots,\frac{1}{2}m_1, \end{array}$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and $n_1(\frac{1}{2}m_1 - s) + r$'th column.

5. Approximation to the Generalised Airforces

If we transform the integration variables x, y in formula (13) to (ξ, η) over W and to (ε, ζ) over T according to equations (67) and (68) we get

$$Q_{pq} = \frac{s_1}{l}\int_{-1}^{+1}\frac{c_1(y)}{l}\,d\eta\int_0^1 \zeta_p^{(1)}(x,y)\lambda_q^{(1)}(x,y;\nu,M)\,d\xi + \frac{s_2}{l}\int_{-1}^{+1}\frac{c_2(y)}{l}\,d\zeta\int_0^1 \zeta_p^{(2)}(x,y)\lambda_q^{(2)}(x,y;\nu,M)\,d\varepsilon. \quad (100)$$

If we substitute the approximation $\hat{\lambda}_q^{(1)}(x, y)$ to $\lambda_q^{(1)}(x, y; \nu, M)$ from equation (53) and the approximation $\hat{\lambda}_q^{(2)}(x, y)$ to $\lambda_q^{(2)}(x, y; \nu, M)$ from equation (58) into formula (100) we get an approximation $\hat{Q}_{pq}$ to Q_{pq}, given explicitly by

$$\hat{Q}_{pq} = \sum_{r=1}^{n_1}\sum_{s=1}^{m_1} A^{(1)}_{q;r,s}\chi^{(1)}_{p;r,s} + \sum_{r=1}^{n_2}\sum_{s=1}^{m_2} A^{(2)}_{q;r,s}\chi^{(2)}_{p;r,s} \quad (101)$$

where

$$\chi^{(1)}_{p;r,s} = \frac{s_1}{l}\int_{-1}^{+1} g_s^{(1)}(\eta)\sqrt{1-\eta^2}\,d\eta\int_0^1 h_r^{(1)}(\xi)\sqrt{\frac{1-\xi}{\xi}}\zeta_p^{(1)}(x,y)\exp\left(-\frac{i\nu x}{l}\right)d\xi$$
$$r = 1,2,\ldots,n_1; s = 1,2,\ldots,m_1, \quad (102)$$

and

$$\chi^{(2)}_{p;r,s} = \frac{s_2}{l}\int_{-1}^{+1} g_s^{(2)}(\zeta)\sqrt{1-\zeta^2}\,d\zeta\int_0^1 h_r^{(2)}(\varepsilon)\sqrt{\frac{1-\varepsilon}{\varepsilon}}\zeta_p^{(2)}(x,y)\exp\left(-\frac{i\nu x}{l}\right)d\varepsilon$$
$$r = 1,2,\ldots,n_2;\quad s = 1,2,\ldots,m_2. \quad (103)$$

It follows, in view of equations (29), (30) and (78) that

$$\chi^{(1)}_{p;r,m_1-s+1} = \kappa\chi^{(1)}_{p;r,s}, \quad (104)$$

and

$$\chi^{(2)}_{p;r,m_2-s+1} = \kappa\chi^{(2)}_{p;r,s}. \quad (105)$$

If we substitute equations (95), (96), (104) and (105) into equation (101) we get

$$\hat{Q}_{pq} = 2\sum_{r=1}^{n_1}\sum_{s=1}^{\frac{1}{2}m_1} A^{(1)}_{q;r,s}\chi^{(1)}_{p;r,s} + 2\sum_{r=1}^{n_2}\sum_{s=1}^{\frac{1}{2}m_2} A^{(2)}_{q;r,s}\chi^{(2)}_{p;r,s}, \tag{106}$$

since $\kappa = +1$ or $\kappa = -1$, and then expressing this equation in matrix form we get

$$[\hat{Q}_{pq}] = 2[\chi_p^{(1)}, \chi_p^{(2)}]\begin{bmatrix} A_q^{(1)} \\ A_q^{(2)} \end{bmatrix}, \tag{107}$$

where $[\hat{Q}_{pq}]$ is a matrix of the one element $\hat{Q}_{pq}$. The row matrix $\chi_p^{(1)}$ is a row matrix of $\frac{1}{2}m_1n_1$ elements with the element

$$\chi^{(1)}_{p;r,s} \qquad r = 1, 2, \ldots, n_1; s = 1, 2, \ldots, \tfrac{1}{2}m_1$$

in the $n_1(s-1)+r$'th column. The row matrix $\chi_p^{(2)}$ is a row matrix of $\frac{1}{2}m_2n_2$ elements with the element

$$\chi^{(2)}_{p;r,s} \qquad r = 1, 2, \ldots, n_2; s = 1, 2, \ldots, \tfrac{1}{2}m_2,$$

in the $n_2(s-1)+r$'th column.

From equations (99) and (107) we get finally

$$[\hat{Q}_{pq}] = [\chi_p^{(1)}, \chi_p^{(2)}]\begin{bmatrix} \frac{1}{2}\Psi^{(1,1)} & \frac{1}{2}\Psi^{(1,2)} \\ \frac{1}{2}\Psi^{(2,1)} & \frac{1}{2}\Psi^{(2,2)} \end{bmatrix}^{-1}\begin{bmatrix} \Theta_q^{(1)} \\ \Theta_q^{(2)} \end{bmatrix}, \tag{108}$$

an equation that can be used to determine the approximation $\hat{Q}_{pq}$ to the generalised airforce coefficient Q_{pq}, since all the elements of the matrices on its right hand side can be evaluated.

If we consider k modes of oscillation then there are k^2 generalised airforce coefficients Q_{pq}, $p = 1, 2, \ldots, k$; $q = 1, 2, \ldots, k$.

Let $[\hat{Q}]$ be the square matrix of order $k \times k$ with the element

$$\hat{Q}_{p,q}, \qquad p = 1, 2, \ldots, k; q = 1, 2, \ldots, k,$$

in the p'th row and q'th column.

Let $\chi^{(1)}$ be the matrix obtained by arranging the row matrices $\chi_p^{(1)}$, $p = 1, 2, \ldots, k$ consecutively beneath each other and let $\chi^{(2)}$ be the matrix obtained by arranging the row matrices $\chi_p^{(2)}$, $p = 1, 2, \ldots, k$, consecutively beneath each other.

Let $\Theta^{(1)}$ be the matrix obtained by arranging the column matrices $\Theta_q^{(1)}$, $q = 1, 2, \ldots, k$, consecutively alongside each other and let $\Theta^{(2)}$ be the matrix obtained by arranging the column matrices $\Theta_q^{(2)}$, $q = 1, 2, \ldots, k$ consecutively alongside each other.

Then we may write

$$[\hat{Q}] = [\chi^{(1)}, \chi^{(2)}]\begin{bmatrix} \frac{1}{2}\Psi^{(1,1)} & \frac{1}{2}\Psi^{(1,2)} \\ \frac{1}{2}\Psi^{(2,1)} & \frac{1}{2}\Psi^{(2,2)} \end{bmatrix}^{-1}\begin{bmatrix} \Theta^{(1)} \\ \Theta^{(2)} \end{bmatrix} \tag{109}$$

6. Numerical Integration

6.1. Formulae for $\Psi^{(i,j)}$

To evaluate the elements in the matrices occurring in formula (109) certain integrals must be evaluated and we shall now discuss their numerical evaluation.

We obtain the elements of the matrices $\Psi^{(1,1)}$, $\Psi^{(1,2)}$, $\Psi^{(2,1)}$ and $\Psi^{(2,2)}$ from the formulae (73), (74), (75) and (76). The functions $U^{(1)}_{r,s}(x, y; \nu, M)$, $U^{(2)}_{r,s}(x, y; \nu, M)$, $V^{(1)}_{r,s}(x, y; \nu, M)$ and $V^{(2)}_{r,s}(x, y; \nu, M)$ occurring in these formulae are not known explicitly but have to be obtained from the convolution integrals in equations (61), (62), (63) and (64) respectively. Since the evaluation of these convolution integrals is rather lengthy it is not practical to obtain values of $U^{(1)}_{r,s}(x, y; \nu, M)$, $U^{(2)}_{r,s}(x, y; \nu, M)$, $V^{(1)}_{r,s}(x, y; \nu, M)$ and $V^{(2)}_{r,s}(x, y; \nu, M)$ at a very large number of points (x, y) on the surfaces W and T. For this reason the number of integration points used for the numerical evaluation of the integrals in formulae (73), (74), (75) and (76) must be restricted in number. The selection of the points is made so that the numerical values of the integrals are as close as possible to the actual values and to this end we use the techniques of Gaussian integration.

Let $l_n(\sigma)$ be a polynomial of degree n in σ which satisfies the relations

$$\int_0^1 \sigma^r l_n(\sigma)\sqrt{\frac{1-\sigma}{\sigma}}\,d\sigma = 0, \qquad r = 0, 1, 2, \ldots, n-1. \tag{110}$$

Let the zeros of $l_n(\sigma)$ be denoted by $\sigma_I^{(n)}$, $I = 1, 2, \ldots, n$. All these zeros are in $0 \leqslant \sigma \leqslant 1$ and are given by (*see* Ref. 9)

$$\sigma_I^{(n)} = \tfrac{1}{2} - \tfrac{1}{2}\cos\left(\frac{2I - 1}{2n + 1}\pi\right), \qquad I = 1, 2, \ldots, n. \tag{111}$$

Let $\tilde{h}_I^{(n)}(\sigma)$, $I = 1, 2, \ldots, n$, be the set of n interpolation polynomials based on the points $\sigma_I^{(n)}$, $I = 1, 2, \ldots, n$, and defined by the formulae

$$\tilde{h}_I^{(n)}(\sigma) = \prod_{\substack{P=1\\P\neq I}}^{n} \left(\frac{\sigma - \sigma_P^{(n)}}{\sigma_I^{(n)} - \sigma_P^{(n)}}\right), \qquad I = 1, 2, \ldots, n. \tag{112}$$

The $\tilde{h}_I^{(n)}(\sigma)$ are a set of n linearly independent polynomials of degree $(n - 1)$ in σ which have the properties

$$\tilde{h}_I^{(n)}(\sigma_P^{(n)}) = \delta_{IP} \tag{113}$$

and

$$\int_0^1 \tilde{h}_I^{(n)}(\sigma)\tilde{h}_P^{(n)}(\sigma)\sqrt{\frac{1-\sigma}{\sigma}}\, d\sigma = \delta_{IP}\tilde{H}_I^{(n)}, \tag{114}$$

where (*see* Appendix 3 of Ref. 9)

$$\begin{aligned}\tilde{H}_I^{(n)} &= \int_0^1 [\tilde{h}_I^{(n)}(\sigma)]^2\sqrt{\frac{1-\sigma}{\sigma}}\, d\sigma \\ &= \int_0^1 \tilde{h}_I^{(n)}(\sigma)\sqrt{\frac{1-\sigma}{\sigma}}\, d\sigma \\ &= \frac{2\pi}{2n+1}(1 - \sigma_I^{(n)})\end{aligned} \tag{115}$$

and δ_{IP} is Kronecker's delta.

Let $\gamma_m(\mu)$ be a polynomial of degree m in μ which satisfies the relations

$$\int_{-1}^{+1} \mu^s \gamma_m(\mu)\sqrt{1 - \mu^2}\, d\mu = 0, \qquad s = 0, 1, 2, \ldots, m - 1. \tag{116}$$

Let the zeros of $\gamma_m(\mu)$ be denoted by $\mu_J^{(m)}$, $J = 1, 2, \ldots, m$. All these zeros are in $-1 \leqslant \mu \leqslant 1$ and are given by (*see* Ref. 9)

$$\mu_J^{(m)} = \cos\left(\frac{J}{m+1}\pi\right), \qquad J = 1, 2, \ldots, m. \tag{117}$$

Let $\tilde{g}_J^{(m)}(\mu)$, $J = 1, 2, \ldots, m$, be the set of m interpolation polynomials based on the points $\mu_J^{(m)}$, $J = 1, 2, \ldots, m$, and defined by the formulae

$$\tilde{g}_J^{(m)}(\mu) = \prod_{\substack{Q=1\\Q\neq J}}^{m} \left(\frac{\mu - \mu_Q^{(m)}}{\mu_J^{(m)} - \mu_Q^{(m)}}\right), \qquad J = 1, 2, \ldots, m. \tag{118}$$

The $\tilde{g}_J^{(m)}(\mu)$ are a set of m linearly independent polynomials of degree $(m - 1)$ in μ which have the properties

$$\tilde{g}_J^{(m)}(\mu_Q^{(m)}) = \delta_{JQ} \tag{119}$$

and

$$\int_{-1}^{+1} \tilde{g}_J^{(m)}(\mu)\tilde{g}_Q^{(m)}(\mu)\sqrt{1-\mu^2}\, d\mu = \delta_{JQ}\tilde{G}_J^{(m)}, \tag{120}$$

where (*see* Appendix 3 of Ref. 9)

$$\begin{aligned}\tilde{G}_J^{(m)} &= \int_{-1}^{+1} [\tilde{g}_J^{(m)}(\mu)]^2\sqrt{1-\mu^2}\, d\mu \\ &= \int_{-1}^{1} \tilde{g}_J^{(m)}(\mu)\sqrt{1-\mu^2}\, d\mu\end{aligned}$$

$$= \frac{\pi}{m+1}[1 - (\mu_J^{(m)})^2]. \tag{121}$$

We define the numbers $H_{i,I}^{(1,n)}$, $H_{i,I}^{(2,n)}$, $G_{j,J}^{(1,m)}$ and $G_{j,J}^{(2,m)}$ by means of the formulae

$$H_{i,I}^{(1,n)} = \int_0^1 h_i^{(1)}(\xi)\tilde{h}_I^{(n)}(\xi)\sqrt{\frac{1-\xi}{\xi}}\,d\xi \qquad \begin{array}{l} i = 1, 2, \ldots, n_1; \\ I = 1, 2, \ldots, n, \end{array} \tag{122}$$

$$H_{i,I}^{(2,n)} = \int_0^1 h_i^{(2)}(\varepsilon)\tilde{h}_I^{(n)}(\varepsilon)\sqrt{\frac{1-\varepsilon}{\varepsilon}}\,d\varepsilon \qquad \begin{array}{l} i = 1, 2, \ldots, n_2; \\ I = 1, 2, \ldots, n \end{array} \tag{123}$$

$$G_{j,J}^{(1,m)} = \int_{-1}^{+1} g_j^{(1)}(\eta)\tilde{g}_J^{(m)}(\eta)\sqrt{1-\eta^2}\,d\eta \qquad \begin{array}{l} j = 1, 2, \ldots, m_1; \\ J = 1, 2, \ldots, m, \end{array} \tag{124}$$

and

$$G_{j,J}^{(2,m)} = \int_{-1}^{+1} g_j^{(2)}(\zeta)\tilde{g}_J^{(m)}(\zeta)\sqrt{1-\zeta^2}\,d\zeta \qquad \begin{array}{l} j = 1, 2, \ldots, m_2; \\ J = 1, 2, \ldots, m. \end{array} \tag{125}$$

If

$$n \geqslant n_1,\ n \geqslant n_2,\ m \geqslant m_1 \quad \text{and} \quad m \geqslant m_2, \tag{126}$$

the formulae (122), (123), (124) and (125) give respectively

$$H_{i,I}^{(1,n)} = h_i^{(1)}(\sigma^{(n)})\tilde{H}_I^{(n)} \qquad i = 1, 2, \ldots, n_1; \quad I = 1, 2, \ldots, n, \tag{127}$$

$$H_{i,I}^{(2,n)} = h_i^{(2)}(\sigma^{(n)})\tilde{H}_I^{(n)} \qquad i = 1, 2, \ldots, n_2; \quad I = 1, 2, \ldots, n, \tag{128}$$

$$G_{j,J}^{(1,m)} = g_j^{(1)}(\mu_J^{(m)})\tilde{G}_J^{(m)} \qquad j = 1, 2, \ldots, m_1; \quad J = 1, 2, \ldots, m \tag{129}$$

and

$$G_{j,J}^{(2,m)} = g_j^{(2)}(\mu_J^{(m)})\tilde{G}_J^{(m)} \qquad j = 1, 2, \ldots, m_2; \quad J = 1, 2, \ldots, m. \tag{130}$$

In order to apply Gaussian integration techniques to the evaluation of the integrals in equations (73), (74), (75) and (76) we introduce the further set of points $\bar{\sigma}_I^{(n)}$, $I = 1, 2, \ldots, n$, in (0, 1), defined by

$$\bar{\sigma}_I^{(n)} = 1 - \sigma_{n-I+1}^{(n)}. \tag{131}$$

On the surface W we choose the N_1 integration points ξ given by

$$\xi = \bar{\sigma}_I^{(N_1)}, \qquad I = 1, 2, \ldots, N_1, \tag{132}$$

and the M_1 integration points η given by

$$\eta = \mu_J^{(M_1)}, \qquad J = 1, 2, \ldots, M_1. \tag{133}$$

To the point $(\xi, \eta) \equiv (\bar{\sigma}_I^{(N_1)}, \mu_J^{(M_1)})$ in the transformed variables on W there corresponds the point $(x, y) \equiv (\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)})$ in the original coordinates, where

$$y_{1;J}^{(M_1)} = s_1\mu_J^{(M_1)}$$

and

$$\bar{x}_{1;I,J}^{(N_1,M_1)} = c_1(y_{1;J}^{(M_1)})\bar{\sigma}_I^{(N_1)} + x_L^{(1)}(y_{1;J}^{(M_1)}). \tag{134}$$

On the surface T we choose N_2 integration points ε given by

$$\varepsilon = \bar{\sigma}_I^{(N_2)}, \qquad I = 1, 2, \ldots, N_2, \tag{135}$$

and M_2 integration points ζ given by

$$\zeta = \mu_J^{(M_2)}, \qquad J = 1, 2, \ldots, M_2. \tag{136}$$

To the point $(\varepsilon, \zeta) \equiv (\bar{\sigma}_I^{(N_2)}, \mu_J^{(M_2)})$ in the transformed variables on T there corresponds the point

$$(x, y) \equiv (\bar{x}_{2;I,J}^{(N_2,M_2)}, y_{2;J}^{(M_2)})$$

in the original coordinates, where

$$y_{2;J}^{(M_2)} = s_2\mu_J^{(M_2)}$$

and

$$\bar{x}_{2;I,J}^{(N_2,M_2)} = c_2(y_{2;J}^{(M_2)})\bar{\sigma}_I^{(N_2)} + x_L^{(2)}(y_{2;J}^{(M_2)}). \tag{137}$$

The points (132) and (133) are the Gaussian points of integration for the evaluation of equations (73) and (75) and the points (135) and (136) are the Gaussian points of integration for the evaluation of equations (74) and (76). If $N_2 = N_1$ and $M_2 = M_1$ these sets of points become identical to each other.

If we approximate to $U^{(1)}_{r,s}(x, y; v, M)$ by the double polynomial of degree $(N_1 - 1)$ in ξ and degree $(M_1 - 1)$ in η

$$U^{(1)}_{r,s}(x, y; v, M) = \sum_{I=1}^{N_1} \sum_{J=1}^{M_1} U^{(1)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M)\tilde{h}^{(N_1)}_{N_1-I+1}(1-\xi)\tilde{g}^{(M_1)}_J(\eta) \tag{138}$$

substitute this into equation (73), integrate and make use of the relations (122), (124), (127) and (129), we get

$$\psi^{(1,1)}_{i,j;r,s} = \sum_{I=1}^{N_1} \sum_{J=1}^{M_1} h^{(1)}_i(\sigma^{(N_1)}_{N_1-I+1})\tilde{H}^{(N_1)}_{N_1-I+1}g^{(1)}_j(\mu^{(M_1)}_J)\tilde{G}^{(M_1)}_J U^{(1)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M)$$

$$i = 1, 2, \ldots, n_1; \quad r = 1, 2, \ldots, n_1;$$

$$j = 1, 2, \ldots, m_1; \quad s = 1, 2, \ldots, m_1. \tag{139}$$

The formula (139) is exact if $g^{(1)}_j(\eta)h^{(1)}_i(1-\xi)U^{(1)}_{r,s}(x, y; v, M)$ is a double polynomial of degree $\leqslant(2N_1 - 1)$ in ξ and $\leqslant(2M_1 - 1)$ in η. Otherwise it is only an approximate formula.
Similarly we get

$$\psi^{(2,2)}_{i,j;r,s} = \sum_{I=1}^{N_2} \sum_{J=1}^{M_2} h^{(2)}_i(\sigma^{(N_2)}_{N_2-I+1})\tilde{H}^{(N_2)}_{N_2-I+1}g^{(2)}_j(\mu^{(M_2)}_J)\tilde{G}^{(M_2)}_J U^{(2)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M)$$

$$i = 1, 2, \ldots, n_2; \quad r = 1, 2, \ldots, n_2;$$

$$j = 1, 2, \ldots, m_2; \quad s = 1, 2, \ldots, m_2, \tag{140}$$

$$\psi^{(1,2)}_{i,j;r,s} = \sum_{I=1}^{N_1} \sum_{J=1}^{M_1} h^{(1)}_i(\sigma^{(N_1)}_{N_1-I+1})\tilde{H}^{(N_1)}_{N_1-I+1}g^{(1)}_j(\mu^{(M_1)}_J)\tilde{G}^{(M_1)}_J V^{(2)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M)$$

$$i = 1, 2, \ldots, n_1; \quad r = 1, 2, \ldots, n_2;$$

$$j = 1, 2, \ldots, m_1; \quad s = 1, 2, \ldots, m_2, \tag{141}$$

and

$$\psi^{(2,1)}_{i,j;r,s} = \sum_{I=1}^{N_2} \sum_{J=1}^{M_2} h^{(2)}_i(\sigma^{(N_2)}_{N_2-I+1})\tilde{H}^{(N_2)}_{N_2-I+1}g^{(2)}_j(\mu^{(M_2)}_J)\tilde{G}^{(M_2)}_J V^{(1)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M)$$

$$i = 1, 2, \ldots, n_2; \quad r = 1, 2, \ldots, n_1;$$

$$j = 1, 2, \ldots, m_2; \quad s = 1, 2, \ldots, m_1. \tag{142}$$

The formula (140) is exact if $g^{(2)}_j(\zeta)h^{(2)}_i(1-\varepsilon)U^{(2)}_{r,s}(x, y; v, M)$ is a double polynomial of degree $\leqslant(2N_2 - 1)$ in ε and $\leqslant(2M_2 - 1)$ in ζ, the formula (141) is exact if $g^{(1)}_j(\eta)h^{(1)}_i(1-\xi)V^{(2)}_{r,s}(x, y; v, M)$ is a double polynomial of degree $\leqslant(2N_1 - 1)$ in ξ and $\leqslant(2M_1 - 1)$ in η, and the formula (142) is exact if $g^{(2)}_j(\zeta)h^{(2)}_i(1-\varepsilon)V^{(1)}_{r,s}(x, y; v, M)$ is a double polynomial of degree $\leqslant(2N_2 - 1)$ in ε and $\leqslant(2M_2 - 1)$ in ζ. Otherwise the formulae are only approximate ones.

If we take M_1 and M_2 to be even numbers, take into account the properties in equations (79), (80), (81) and (82) and note that

$$\mu^{(m)}_{m-J+1} = -\mu^{(m)}_J, \tag{143}$$

then from formulae (139), (140), (141) and (142) we obtain

$$\psi^{(1,1)}_{i,j;r,s} + \kappa\psi^{(1,1)}_{i,j;r,m_1-s+1} = \sum_{I=1}^{N_1} \sum_{J=1}^{\frac{1}{2}M_1} h^{(1)}_i(\sigma^{(N_1)}_{N_1-I+1})\tilde{H}^{(N_1)}_{N_1-I+1}\{g^{(1)}_j(\mu^{(M_1)}_J) + \kappa g^{(1)}_j(-\mu^{(M_1)}_J)\}\tilde{G}^{(M_1)}_J \times$$
$$\times \{U^{(1)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) + \kappa U^{(1)}_{r,m_1-s+1}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M)\}$$

$$i = 1, 2, \ldots, n_1; \quad r = 1, 2, \ldots, n_1;$$

$$j = 1, 2, \ldots, \tfrac{1}{2}m_1; \quad s = 1, 2, \ldots, \tfrac{1}{2}m_1, \tag{144}$$

$$\psi^{(2,2)}_{i,j;r,s} + \kappa\psi^{(2,2)}_{i,j;r,m_2-s+1} = \sum_{I=1}^{N_2} \sum_{J=1}^{\frac{1}{2}M_2} h^{(2)}_i(\sigma^{(N_2)}_{N_2-I+1})\tilde{H}^{(N_2)}_{N_2-I+1}\{g^{(2)}_j(\mu^{(M_2)}_J) + \kappa g^{(2)}_j(-\mu^{(M_2)}_J)\}\tilde{G}^{(M_2)}_J \times$$
$$\times \{U^{(2)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) + \kappa U^{(2)}_{r,m_2-s+1}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M)\}$$

$$i = 1,2,\ldots,n_2; \quad r = 1,2,\ldots,n_2;$$
$$j = 1,2,\ldots,\tfrac{1}{2}m_2; \quad s = 1,2,\ldots,\tfrac{1}{2}m_2, \tag{145}$$

$$\psi^{(1,2)}_{i,j;r,s} + \kappa\psi^{(1,2)}_{i,j;r,m_2-s+1} = \sum_{I=1}^{N_1}\sum_{J=1}^{\frac{1}{2}M_1} h_i^{(1)}(\sigma^{(N_1)}_{N_1-I+1})\tilde{H}^{(N_1)}_{N_1-I+1}\{g_j^{(1)}(\mu_J^{(M_1)}) + \kappa g_j^{(1)}(-\mu_J^{(M_1)})\}\tilde{G}_J^{(M_1)} \times$$
$$\times \{V^{(2)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) + \kappa V^{(2)}_{r,m_2-s+1}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M)\}$$
$$i = 1,2,\ldots,n_1; \quad r = 1,2,\ldots,n_2;$$
$$j = 1,2,\ldots,\tfrac{1}{2}m_1; \quad s = 1,2,\ldots,\tfrac{1}{2}m_2, \tag{146}$$

and

$$\psi^{(2,1)}_{i,j;r,s} + \kappa\psi^{(2,1)}_{i,j;r,m_1-s+1} = \sum_{I=1}^{N2}\sum_{J=1}^{\frac{1}{2}M_2} h_i^{(2)}(\sigma^{(N_2)}_{N_2-I+1})\tilde{H}^{(N_2)}_{N_2-I+1}\{g_j^{(2)}(\mu_J^{(M_2)}) + \kappa g_j^{(2)}(-\mu_J^{(M_2)})\}\tilde{G}_J^{(M_2)} \times$$
$$\times \{V^{(1)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) + \kappa V^{(1)}_{r,m_1-s+1}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M)\}$$
$$i = 1,2,\ldots,n_2; \quad r = 1,2,\ldots,n_1;$$
$$j = 1,2,\ldots,\tfrac{1}{2}m_2; \quad s = 1,2,\ldots,\tfrac{1}{2}m_1. \tag{147}$$

We can now write

$$\begin{bmatrix} \frac{1}{2}\Psi^{(1,1)} & \frac{1}{2}\Psi^{(1,2)} \\ \frac{1}{2}\Psi^{(2,1)} & \frac{1}{2}\Psi^{(2,2)} \end{bmatrix} = \begin{bmatrix} L^{(1)} & 0 \\ 0 & L^{(2)} \end{bmatrix} \begin{bmatrix} U^{(1)} & V^{(2)} \\ V^{(1)} & U^{(2)} \end{bmatrix}. \tag{148}$$

The matrix appearing on the left-hand side of equation (148) occurs in formula (108) and has elements which are one half of those in a related matrix appearing in formula (99).

The submatrix $U^{(1)}$ is a rectangular matrix of order $\frac{1}{2}M_1N_1 \times \frac{1}{2}m_1n_1$ with the element

$$U^{(1)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) + \kappa U^{(1)}_{r,m_1-s+1}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) \qquad \begin{array}{ll} I = 1,2,\ldots,N_1; & r = 1,2,\ldots,n_1; \\ J = 1,2,\ldots,\frac{1}{2}M_1; & s = 1,2,\ldots,\frac{1}{2}m_1, \end{array}$$

in the $N_1(\frac{1}{2}M_1 - J) + I$'th row and $n_1(\frac{1}{2}m_1 - s) + r$'th column. The submatrix $U^{(2)}$ is a rectangular matrix of order $\frac{1}{2}M_2N_2 \times \frac{1}{2}m_2n_2$ with the element

$$U^{(2)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) + \kappa U^{(2)}_{r,m_2-s+1}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) \qquad \begin{array}{ll} I = 1,2,\ldots,N_2; & r = 1,2,\ldots,n_2; \\ J = 1,2,\ldots,\frac{1}{2}M_2; & s = 1,2,\ldots,\frac{1}{2}m_2, \end{array}$$

in the $N_2(\frac{1}{2}M_2 - J) + I$'th row and $n_2(\frac{1}{2}m_2 - s) + r$'th column. The submatrix $V^{(2)}$ is a rectangular matrix of order $\frac{1}{2}M_1N_1 \times \frac{1}{2}m_2n_2$ with the element

$$V^{(2)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) + \kappa V^{(2)}_{r,m_2-s+1}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; v, M) \qquad \begin{array}{ll} I = 1,2,\ldots,N_1; & r = 1,2,\ldots,n_2; \\ J = 1,2,\ldots,\frac{1}{2}M_1; & s = 1,2,\ldots,\frac{1}{2}m_2, \end{array}$$

in the $N_1(\frac{1}{2}M_1 - J) + I$'th row and $n_2(\frac{1}{2}m_2 - s) + r$'th column. The submatrix $V^{(1)}$ is a rectangular matrix of order $\frac{1}{2}M_2N_2 \times \frac{1}{2}m_1n_1$ with the element

$$V^{(1)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) + \kappa V^{(2)}_{r,m_1-s+1}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; v, M) \qquad \begin{array}{ll} I = 1,2,\ldots,N_2; & r = 1,2,\ldots,n_1; \\ J = 1,2,\ldots,\frac{1}{2}M_2; & s = 1,2,\ldots,\frac{1}{2}m_1. \end{array}$$

in the $N_2(\frac{1}{2}M_2 - J) + I$'th row and $n_1(\frac{1}{2}m_1 - s) + r$'th column. The submatrix $L^{(1)}$ is a rectangular matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}M_1N_1$ with the element

$$\tfrac{1}{2}h_i^{(1)}(\sigma^{(N_1)}_{N_1-I+1})\tilde{H}^{(N_1)}_{N_1-I+1}\{g_j^{(1)}(\mu_J^{(M_1)}) + \kappa g_j^{(1)}(-\mu_J^{(M_1)})\}\tilde{G}_J^{(M_1)} \qquad \begin{array}{ll} i = 1,2,\ldots,n_1; & I = 1,2,\ldots,N_1; \\ j = 1,2,\ldots,\frac{1}{2}m_1; & J = 1,2,\ldots,\frac{1}{2}M_1, \end{array}$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and $N_1(\frac{1}{2}M_1 - J) + I$'th column. The submatrix $L^{(2)}$ is a rectangular matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}M_2N_2$ with the element

$$\tfrac{1}{2}h_i^{(2)}(\sigma^{(N_2)}_{N_2-I+1})\tilde{H}^{(N_2)}_{N_2-I+1}\{g_j^{(2)}(\mu_J^{(M_2)}) + \kappa g_j^{(2)}(-\mu_J^{(M_2)})\}\tilde{G}_J^{(M_2)} \qquad \begin{array}{ll} i = 1,2,\ldots,n_2; & I = 1,2,\ldots,N_2; \\ j = 1,2,\ldots,\frac{1}{2}m_2; & J = 1,2,\ldots,\frac{1}{2}M_2, \end{array}$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and $N_2(\frac{1}{2}M_2 - J) + I$'th column.

6.2. Equivalent Reduced Upwashes and Displacements

For the evaluation of the integrals in equations (71) and (72) we note that the functions $\alpha_q^{(1)}(x, y; v)$ and $\alpha_q^{(2)}(x, y; v)$ are known explicitly and we can, possibly, carry out these integrations analytically. Otherwise we can carry out the integrations numerically to as high a precision as we desire. Also for the evaluation of the integrals in equations (102) and (103) we note that the functions $\zeta_p^{(1)}(x, y)$ and $\zeta_p^{(2)}(x, y)$ are known explicitly and we can, possibly, carry out these integrals analytically. Otherwise we can carry out the integrations numerically to as high a precision as we desire.

Let us write

$$\alpha_{q;i,j}^{(1,e)} = \frac{1}{\tilde{H}_i^{(n_1)}\tilde{G}_j^{(m_1)}} \exp\left\{-\frac{iv}{l}\bar{x}_{1;i,j}^{(n_1,m_1)}\right\}\theta_{q;i,j}^{(1)}, \tag{149}$$

$$\alpha_{q;i,j}^{(2,e)} = \frac{1}{\tilde{H}_i^{(n_2)}\tilde{G}_j^{(m_2)}} \exp\left\{-\frac{iv}{l}\bar{x}_{2;i,j}^{(n_2,m_2)}\right\}\theta_{q;i,j}^{(2)}, \tag{150}$$

$$\zeta_{p;r,s}^{(1,e)} = \frac{l}{s\tilde{H}_r^{(n_1)}\tilde{G}_s^{(m_1)}} \exp\left\{\frac{iv}{l}x_{1;r,s}^{(n_1,m_1)}\right\}\chi_{p;r,s}^{(1)} \tag{151}$$

and

$$\zeta_{p;r,s}^{(2,e)} = \frac{l}{s\tilde{H}_r^{(n_2)}\tilde{G}_s^{(m_2)}} \exp\left\{\frac{iv}{l}x_{2;r,s}^{(n_2,m_2)}\right\}\chi_{p;r,s}^{(2)}, \tag{152}$$

where

$$\begin{aligned} y_{1;s}^{(m)} &= s_1\mu_s^{(m)} \\ x_{1;r,s}^{(m,n)} &= c_1(y_{1;s}^{(m)})\sigma_r^{(n)} + x_L^{(1)}(y_{1;s}^{(m)}) \end{aligned} \tag{153}$$

and

$$\begin{aligned} y_{2;s}^{(m)} &= s_2\mu_s^{(m)} \\ x_{2;r,s}^{(m,n)} &= c_2(y_{2;s}^{(m)})\sigma_r^{(n)} + x_L^{(2)}(y_{2;s}^{(m)}). \end{aligned} \tag{154}$$

If $\alpha_q^{(1)}(x, y; v)$, $\alpha_q^{(2)}(x, y; v)$, $\zeta_p^{(1)}(x, y)$ and $\zeta_p^{(2)}(x, y)$ are sufficiently smooth we can apply Gaussian numerical integration techniques to evaluate $\theta_{q;i,j}^{(1)}$, $\theta_{q;i,j}^{(2)}$, $\chi_{p;r,s}^{(1)}$, $\chi_{p;r,s}^{(2)}$ from equations (71), (72), (102) and (103) respectively. To evaluate numerically the integral in equation (71), we approximate to $\alpha_q^{(1)}(x, y; v)\exp(ivx/l)$ by the double polynomial of degree $(\bar{N}_1 - 1)$ in ξ and $(\bar{M}_1 - 1)$ in η

$$\alpha_q^{(1)}(x, y; v)\exp\left(\frac{ivx}{l}\right) = \sum_{I=1}^{\bar{N}_1}\sum_{J=1}^{\bar{M}_1} \alpha_q^{(1)}(\bar{x}_{1;I,J}^{(\bar{N}_1,\bar{M}_1)}, y_{1;J}^{(\bar{M}_1)}; v)\exp\left\{\frac{iv}{l}\bar{x}_{1;I,J}^{(\bar{N}_1,\bar{M}_1)}\right\}\tilde{h}_{\bar{N}_1-I+1}^{(\bar{N}_1)}(1-\xi)g_J^{(\bar{M}_1)}(\eta) \tag{155}$$

and substitute into equation (71) to get $\theta_{q;i,j}^{(1)}$. Then from equation (149) we get

$$\alpha_{q;i,j}^{(1,e)} = \sum_{I=1}^{\bar{N}_1}\sum_{J=1}^{\bar{M}_1} h_i^{(1)}(\sigma_{\bar{N}_1-I+1}^{(\bar{N}_1)})\frac{\tilde{H}_{\bar{N}_1-I+1}^{(\bar{N}_1)}}{\tilde{H}_i^{(n_1)}}g_j^{(1)}(\mu_J^{(\bar{M}_1)})\frac{\tilde{G}_J^{(\bar{M}_1)}}{\tilde{G}_j^{(m_1)}}\exp\left\{\frac{iv}{l}[\bar{x}_{1;I,J}^{(\bar{N}_1,\bar{M}_1)} - \bar{x}_{1;i,j}^{(n_1,m_1)}]\right\}\alpha_q^{(1)}(\bar{x}_{1;I,J}^{(\bar{N}_1,\bar{M}_1)}, y_{1;J}^{(\bar{M}_1)}; v)$$

$$i = 1, 2, \ldots, n_1; \quad j = 1, 2, \ldots, m_1. \tag{156}$$

The formula (156) is exact if $g^{(1)}(\eta)h_i^{(1)}(1-\xi)\alpha_q^{(1)}(x, y; v)\exp(ivx/l)$ is a double polynomial of degree $\leqslant(2\bar{N}_1 - 1)$ in ξ and $\leqslant(2\bar{M}_1 - 1)$ in η. Otherwise it is only an approximate formula.

Similarly we get

$$\alpha_{q;i,j}^{(2,e)} = \sum_{I=1}^{\bar{N}_2}\sum_{J=1}^{\bar{M}_2} h_i^{(2)}(\sigma_{\bar{N}_2-I+1}^{(\bar{N}_2)})\frac{\tilde{H}_{\bar{N}_2-I+1}^{(\bar{N}_2)}}{\tilde{H}_i^{(n_2)}}g_j^{(2)}(\mu_J^{(\bar{M}_2)})\frac{\tilde{G}_J^{(\bar{M}_2)}}{\tilde{G}_j^{(m_2)}}\exp\left\{\frac{iv}{l}[\bar{x}_{2;I,J}^{(\bar{N}_2,\bar{M}_2)} - \bar{x}_{2;i,j}^{(n_2,m_2)}]\right\}\alpha_q^{(2)}(\bar{x}_{2;I,J}^{(\bar{N}_2,\bar{M}_2)}, y_{2;J}^{(\bar{M}_2)}; v)$$

$$i = 1, 2, \ldots, n_2; \quad j = 1, 2, \ldots, m_2. \tag{157}$$

The formula (157) is exact if $g_j^{(2)}(\zeta)h_i^{(2)}(1-\varepsilon)\alpha_q^{(2)}(x, y; v)\exp(ivx/l)$ is a double polynomial of degree $\leqslant(2\bar{N}_2 - 1)$ in ε and $\leqslant(2\bar{M}_2 - 1)$ in ζ. Otherwise it is only an approximate formula.

To evaluate numerically the integral in equation (102) we approximate to $\zeta_p^{(1)}(x, y)\exp(-ivx/l)$ by the double polynomial of degree $(\bar{N}_1 - 1)$ in ξ and $(\bar{M}_1 - 1)$ in η,

$$\zeta_p^{(1)}(x, y)\exp\left(-\frac{ivx}{l}\right) = \sum_{I=1}^{\bar{N}_1}\sum_{J=1}^{\bar{M}_1} \zeta_p^{(1)}(x_{1;I,J}^{(\bar{N}_1,\bar{M}_1)}, y_{1;J}^{(\bar{M}_1)})\exp\left\{-\frac{iv}{l}x_{1;I,J}^{(\bar{N}_1,\bar{M}_1)}\right\}\tilde{h}_i^{(\bar{N}_1)}(\xi)\tilde{g}_j^{(\bar{M}_1)}(\eta) \tag{158}$$

and substitute into equation (102) to get $\chi^{(1)}_{p;r,s}$. Then, from equation (151) we get

$$\zeta^{(1,e)}_{p;r,s} = \sum_{I=1}^{\bar{N}_1} \sum_{J=1}^{\bar{M}_1} h^{(1)}_r(\sigma_I^{(\bar{N}_1)}) \frac{\tilde{H}_I^{(\bar{N}_1)}}{\tilde{H}_r^{(n_1)}} g^{(1)}_s(\mu_J^{(\bar{M}_1)}) \frac{\tilde{G}_J^{(\bar{M}_1)}}{\tilde{G}_s^{(m_1)}} \exp\left\{ -\frac{iv}{l}[x^{(\bar{N}_1,\bar{M}_1)}_{1;I,J} - x^{(n_1,m_1)}_{1;I,J}]\right\} \zeta^{(1)}_p(x^{(\bar{N}_1,\bar{M}_1)}_{1;I,J}, y^{(\bar{M}_1)}_{1;J})$$

$$r = 1, 2, \ldots, n_1; \quad s = 1, 2, \ldots, m_1. \tag{159}$$

The formula (159) is exact if $g^{(1)}_s(\eta)^{h(1)}_r(\xi)\zeta^{(1)}_p(x, y)\exp(-ivx/l)$ is a double polynomial of degree $\leqslant(2\bar{N}_1 - 1)$ in ξ and $\leqslant(2\bar{M}_2 - 1)$ in η. Otherwise it is only an approximate formula.

Similarly we get

$$\zeta^{(2,e)}_{p;r,s} = \sum_{I=1}^{\bar{N}_2} \sum_{J=1}^{\bar{M}_2} h^{(2)}_r(\sigma_I^{(\bar{N}_2)}) \frac{\tilde{H}_I^{(\bar{N}_2)}}{\tilde{H}_r^{(n_2)}} g^{(2)}_s(\mu_J^{(\bar{M}_2)}) \frac{\tilde{G}_J^{(\bar{M}_2)}}{\tilde{G}_s^{(m_2)}} \exp\left\{ -\frac{iv}{l}[x^{(\bar{N}_2,\bar{M}_2)}_{2;I,J} - x^{(n_2,m_2)}_{2;r,s}]\right\} \zeta^{(2)}_p(x^{(\bar{N}_2,\bar{M}_2)}_{2;I,J}, y^{(\bar{M}_2)}_{2;J})$$

$$r = 1, 2, \ldots, n_2; \quad s = 1, 2, \ldots, m_2. \tag{160}$$

The formula (160) is exact if $g^{(2)}_s(\zeta)h^{(2)}_r(\varepsilon)\zeta^{(2)}_p(x, y)\exp(-ivx/l)$ is a double polynomial of degree $\leqslant(2\bar{N}_1 - 1)$ in ε and $(2\bar{M}_2 - 1)$ in ζ. Otherwise it is only an approximate formula.

If we take

$$\begin{aligned} \bar{N}_1 &= n_1, \\ \bar{N}_2 &= n_2, \\ \bar{M}_1 &= m_1, \\ \bar{M}_2 &= m_2 \end{aligned} \tag{161}$$

and

in formulae (156), (157), (159) and (160) they respectively reduce to

$$\alpha^{(1,e)}_{q;i,j} = \alpha^{(1)}_q(\bar{x}^{(n_1,m_1)}_{1;i,j}, y^{(m_1)}_{1;j}; v), \tag{162}$$

$$\alpha^{(2,e)}_{q;i,j} = \alpha^{(2)}_q(\bar{x}^{(n_2,m_2)}_{2;i,j}, y^{(m_2)}_{2;j}; v), \tag{163}$$

$$\zeta^{(1,e)}_{p;r,s} = \zeta^{(1)}_p(x^{(n_1,m_1)}_{1;i,j}, y^{(m_1)}_{1;j}) \tag{164}$$

and

$$\zeta^{(2,e)}_{p;r,s} = \zeta^{(2)}_p(x^{(n_2,m_2)}_{2;i,j}, y^{(m_2)}_{2;j}). \tag{165}$$

We call $\alpha^{(1,e)}_{q;1,j}$ the equivalent reduced upwash in the mode q at the upwash point $(\bar{x}^{(n_1,m_1)}_{1;i,j}, y^{(m_1)}_{1;j}, 0)$ on the wing, $\alpha^{(2,e)}_{q,i,j}$ the equivalent reduced upwash in the mode q at the upwash point $(\bar{x}^{(n_2,m_2)}_{2;i,j}, y^{(m_2)}_{2;j}, h)$ on the tailplane, $\zeta^{(1,e)}_{p;r,s}$ the equivalent reduced displacement in the mode p at the displacement point $(x^{(n_1,m_1)}_{1;i,j}, y^{(m_1)}_{1;j}, 0)$ on the wing and $\zeta^{(2,e)}_{p;r,s}$ the equivalent reduced displacement in the mode p at the displacement point $(x^{(n_2,m_2)}_{2;i,j}, y^{(m_2)}_{2;j}, h)$ on the tailplane. If the formulae (156), (157), (159) and (160) are of acceptable accuracy when the conditions (161) are satisfied, then formulae (162), (163), (164) and (165) show that the equivalent reduced upwashes can be taken equal to the actual reduced upwashes at the upwash points and the equivalent reduced displacements can be taken equal to the actual reduced displacements at the displacement points.

6.3. Formula for generalised Airforce Coefficient Matrix

Let $\alpha^{(1,e)}_q$ be the column matrix of $\frac{1}{2}m_1n_1$ elements with the element

$$\alpha^{(1,e)}_{q;i,j} \qquad i = 1, 2, \ldots, n_1; \quad j = 1, 2, \ldots, \tfrac{1}{2}m_1,$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row.

Let $\alpha^{(2,e)}_q$ be the column matrix of $\frac{1}{2}m_2n_2$ elements with the element

$$\alpha^{(2,e)}_{q;i,j} \qquad i = 1, 2, \ldots, n_2; \quad j = 1, 2, \ldots, \tfrac{1}{2}m_2,$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row.

Let $\zeta^{(1,e)}_p$ be the row matrix of $\frac{1}{2}m_1n_1$ elements with the element

$$\zeta^{(1,e)}_{p;r,s} \qquad r = 1, 2, \ldots, n_1; \quad s = 1, 2, \ldots, \tfrac{1}{2}m_1,$$

in the $n_1(\frac{1}{2}m_1 - s) + r$'th column.

Let $\zeta^{(2,e)}_p$ be the row matrix of $\frac{1}{2}m_2n_2$ elements with the element

$$\zeta^{(2,e)}_{p;r,s} \qquad r = 1, 2, \ldots, n_2; \quad s = 1, 2, \ldots, \tfrac{1}{2}m_2,$$

in the $n_2(\frac{1}{2}m_2 - s) + r$'th column.

Let $D^{(1)}$ be the diagonal matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_1n_1$ with the element

$$\tilde{H}_i^{(n_1)}\tilde{G}_j^{(m_1)} \exp\left\{\frac{iv}{l}\bar{x}_{1;i,j}^{(n_1,m_1)}\right\} \qquad i = 1,2,\ldots,n_1; \quad j = 1,2,\ldots,\tfrac{1}{2}m_1,$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and column.

Let $D^{(2)}$ be the diagonal matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}m_2n_2$ with the element

$$\tilde{H}_i^{(n_2)}\tilde{G}_j^{(m_2)} \exp\left\{\frac{iv}{l}\bar{x}_{2;i,j}^{(n_2,m_2)}\right\} \qquad i = 1,2,\ldots,n_2; \quad j = 1,2,\ldots,\tfrac{1}{2}m_2,$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and column.

Let $B^{(1)}$ be the diagonal matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_1n_1$ with the element

$$\frac{s_1}{l}\tilde{H}_r^{(n_1)}\tilde{G}_s^{(m_1)} \exp\left\{-\frac{iv}{l}x_{1;r,s}^{(n_1,m_1)}\right\} \qquad r = 1,2,\ldots,n_1; \quad s = 1,2,\ldots,\tfrac{1}{2}m_1,$$

in the $n_1(\frac{1}{2}m_1 - s) + r$'th row and column.

Let $B^{(2)}$ be the diagonal matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}m_2n_2$ with the element

$$\frac{s_2}{l}\tilde{H}_r^{(n_2)}\tilde{G}_s^{(m_2)} \exp\left\{-\frac{iv}{l}x_{2;r,s}^{(n_2,m_2)}\right\} \qquad r = 1,2,\ldots,n_2; \quad s = 1,2,\ldots,\tfrac{1}{2}m_2,$$

in the $n_2(\frac{1}{2}m_2 - s) + r$'th row and column. Then from equations (149) and (150) we get

$$\begin{bmatrix}\Theta_q^{(1)}\\ \Theta_q^{(2)}\end{bmatrix} = \begin{bmatrix}D^{(1)} & 0\\ 0 & D^{(2)}\end{bmatrix}\begin{bmatrix}\alpha_q^{(1,e)}\\ \alpha_q^{(2,e)}\end{bmatrix}, \tag{166}$$

and from equations (151) and (152) we get

$$[\chi_p^{(1)}, \chi_p^{(2)}] = [\zeta_p^{(1,e)}, \zeta_p^{(2,e)}]\begin{bmatrix}B^{(1)} & 0\\ 0 & B^{(2)}\end{bmatrix}. \tag{167}$$

The matrices denoted by [0] are matrices with zero elements and with order appropriate to the position they occupy.

If we substitute from equations (148), (166) and (167) into equation (108) we get

$$[\hat{Q}_{pq}] = [\zeta_p^{(1,e)}, \zeta_p^{(2,e)}]\begin{bmatrix}B^{(1)} & 0\\ 0 & B^{(2)}\end{bmatrix}\begin{bmatrix}L^{(1)} & 0\\ 0 & L^{(2)}\end{bmatrix}\begin{bmatrix}U^{(1)} & V^{(2)}\\ V^{(1)} & U^{(2)}\end{bmatrix}^{-1}\begin{bmatrix}D^{(1)} & 0\\ 0 & D^{(2)}\end{bmatrix}\begin{bmatrix}\alpha_q^{(1,e)}\\ \alpha_q^{(2,e)}\end{bmatrix}. \tag{168}$$

Now let $\zeta^{(1,e)}$ be the matrix obtained by arranging the row matrices $\zeta_p^{(1,e)}$, $p = 1,2,\ldots,k$ consecutively beneath each other and let $\zeta^{(2,e)}$ be the matrix obtained by arranging the row matrices $\zeta_p^{(2,e)}$, $p = 1,2,\ldots,k$, consecutively beneath each other. Let $\alpha^{(1,e)}$ be the matrix obtained by arranging the column matrices $\alpha_q^{(1,e)}$, $q = 1,2,\ldots,k$ consecutively alongside each other and let $\alpha^{(2,e)}$ be the matrix obtained by arranging the column matrices $\alpha_q^{(2,e)}$, $q = 1,2,\ldots,k$, consecutively alongside each other. Then we get immediately from equation (168)

$$[\hat{Q}] = [\zeta^{(1,e)}, \zeta^{(2,e)}]\begin{bmatrix}B^{(1)} & 0\\ 0 & B^{(2)}\end{bmatrix}\left\{\begin{bmatrix}L^{(1)} & 0\\ 0 & L^{(2)}\end{bmatrix}\begin{bmatrix}U^{(1)} & V^{(2)}\\ V^{(1)} & U^{(2)}\end{bmatrix}\right\}^{-1}\begin{bmatrix}D^{(1)} & 0\\ 0 & D^{(2)}\end{bmatrix}\begin{bmatrix}\alpha^{(1,e)}\\ \alpha^{(2,e)}\end{bmatrix}. \tag{169}$$

If, in formulae (139), (140), (141) and (142) we take

$$\begin{aligned} N_1 &= n_1,\\ N_2 &= n_2,\\ M_1 &= m_1\\ \text{and} \qquad M_2 &= m_2 \;, \end{aligned} \tag{170}$$

then the matrices $L^{(1)}$ and $L^{(2)}$ of formula (148) reduce to diagonal matrices.

The matrix $L^{(1)}$ becomes a diagonal matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_1n_1$ with the element

$$\tilde{H}_i^{(n_1)}\tilde{G}_j^{(m_1)} \qquad i = 1,2,\ldots,n_1; \quad j = 1,2,\ldots,\tfrac{1}{2}m_1, \tag{171}$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and column

The matrix $L^{(2)}$ becomes a diagonal matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}m_2n_2$ with the element

$$\tilde{H}_i^{(n_2)}\tilde{G}_j^{(m_2)} \qquad i = 1, 2, \ldots, n_2; \quad j = 1, 2, \ldots, \tfrac{1}{2}m_2, \tag{172}$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and column.

The formula (169) then reduces to

$$\hat{Q} = [\zeta^{(1,e)}, \zeta^{(2,e)}]\begin{bmatrix} B^{(1)} & 0 \\ 0 & B^{(2)} \end{bmatrix}\begin{bmatrix} U^{(1)} & V^{(2)} \\ V^{(1)} & U^{(2)} \end{bmatrix}^{-1}\begin{bmatrix} E^{(1)} & 0 \\ 0 & E^{(2)} \end{bmatrix}\begin{bmatrix} \alpha^{(1,e)} \\ \alpha^{(2,e)} \end{bmatrix} \tag{173}$$

where $E^{(1)}$ is the diagonal matrix of order $\frac{1}{2}m_1n_1 \times \frac{1}{2}m_1n_1$ with the element

$$\exp\left\{\frac{iv}{l}\bar{x}_{1;i,j}^{(n_1,m_1)}\right\} \qquad i = 1, 2, \ldots, n_1; \quad j = 1, 2, \ldots, \tfrac{1}{2}m_1, \tag{174}$$

in the $n_1(\frac{1}{2}m_1 - j) + i$'th row and column and $E^{(2)}$ is the diagonal matrix of order $\frac{1}{2}m_2n_2 \times \frac{1}{2}m_2n_2$ with the element

$$\exp\left\{\frac{iv}{l}\bar{x}_{2;i,j}^{(n_2,m_2)}\right\} \qquad i = 1, 2, \ldots, n_2; \quad j = 1, 2, \ldots, \tfrac{1}{2}m_2, \tag{175}$$

in the $n_2(\frac{1}{2}m_2 - j) + i$'th row and column.

6.4. Evaluation of $U_{r,s}^{(1)}$, $U_{r,s}^{(2)}$

The methods of evaluation of $U_{r,s}^{(1)}(\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M)$, $U_{r,s}^{(2)}(\bar{x}_{2;I,J}^{(N_2,M_2)}, y_{2;J}^{(M_2)}; v, M)$, $V_{r,s}^{(2)}(\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M)$ and $V_{r,s}^{(1)}(\bar{x}_{2;I,J}^{(N_1,M_1)}, y_{2;J}^{(M_1)}; v, M)$ are still to be discussed. We consider the evaluation of $U_{r,s}^{(1)}(\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M)$ first. We can rewrite equation (61) in the form

$$U_{r,s}^{(1)}(x, y; v, M) = \int_{-1}^{+1} \frac{g_s^{(1)}(\eta_0)\sqrt{1-\eta_0^2}}{(\eta-\eta_0)^2} I_r^{(1)}(\xi, \eta, \eta_0; v, M)\, d\eta_0 \tag{176}$$

where

$$I_r^{(1)}(\xi, \eta, \eta_0; v, M) = \frac{1}{4\pi}\frac{s_1}{l}(\eta-\eta_0)^2 \int_0^1 h_r^{(1)}(\xi_0)\sqrt{\frac{1-\xi_0}{\xi_0}}\, K\left(\frac{x-x_0}{l}, \frac{y-y_0}{l}, 0; v, M\right) d\xi_0. \tag{177}$$

We have immediately from the definition of $K(x/l, y/l, z/l; v, M)$ given in equation (43) that

$$\lim_{y\to 0}\frac{y^2}{l^2}K\left(\frac{x}{l}, \frac{y}{l}, 0; v, M\right) = \begin{cases} 2 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{178}$$

and therefore it follows that

$$I_r^{(1)}(\xi, \eta, \eta; v, M) = \frac{1}{2\pi}\frac{l}{s_1}\int_0^{\xi} h_r^{(1)}(\xi_0)\sqrt{\frac{1-\xi_0}{\xi_0}}\, d\xi_0. \tag{179}$$

The function $I_r^{(1)}(\xi, \eta, \eta_0; v, M)$ is finite for any η_0 in the range $(-1, 1)$. For $\eta_0 = \eta$ its numerical value is obtained from formula (179) but for $\eta_0 \neq \eta$ the complete formula (177) must be used and numerical integration carried out. For $z = 0$ we can write, from equation (43),

$$\frac{y^2}{l^2}K\left(\frac{x}{l}, \frac{y}{l}, 0; v, M\right) = y^2\int_{X_1}^{\infty} e^{-ivu/l}\frac{du}{(u^2+y^2)^{\frac{3}{2}}} + \frac{M(Mx+R_1)}{R_1(x^2+y^2)}y^2\exp\left(-\frac{iv}{l}X_1\right) \tag{180}$$

where

$$R_1 = \sqrt{x^2 + (1-M^2)y^2} \tag{181}$$

and

$$X_1 = \frac{-x + MR_1}{(1-M^2)}. \tag{182}$$

We can further write

$$y^2\int_{X_1}^{\infty} e^{-ivu/l}\frac{du}{(u^2+y^2)^{\frac{3}{2}}} = -\frac{\pi}{2}\left(\frac{iv|y|}{l}\right)\left[\mathbf{H}_{-1}\left(\frac{iv|y|}{l}\right) + \frac{2i}{\pi}K_1\left(\frac{v|y|}{l}\right) - I_1\left(\frac{v|y|}{l}\right)\right]$$

$$-\int_0^{X_1/|y|} e^{-iv|y|v/l}\frac{dv}{(v^2+1)^{\frac{3}{2}}} \tag{183}$$

where I_1 and K_1 are modified Bessel functions of the first order and respectively of the first and second kinds and $\mathbf{H}_{-1}$ is a Struve function of order -1.

The numerical evaluation of $y^2/(l^2)K(x/l, y/l, 0; \nu, M)$ for $u \neq 0$ can now be carried out using equations (180) and (183), so the integrand in the integral on the right-hand side of equation (177) can be evaluated and the numerical evaluation of that integral carried out.

However we observe that when y is very small, $X_1/|y|$ can become very large and the numerical evaluation of the integral on the right-hand side of equation (183) can become a lengthy process. In this case we can evaluate the integral on the left-hand side of equation (183) by a different but more rapid process.

If $X_1/|y|$ is very large and positive we write

$$y^2\int_{X_1}^{\infty} e^{-i\nu u/l}\frac{du}{(u^2+y^2)^{\frac{3}{2}}} = \int_{X_1/|y|}^{\infty} e^{-i\nu|y|v/l}\frac{dv}{(v^2+1)^{\frac{3}{2}}} = \sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)!}{2^{2n}(n!)^2}\left(\frac{\nu y}{l}\right)^{2n+2}\hat{I}_{2n+3}\left(\frac{\nu X_1}{l}\right) \tag{185}$$

where

$$\hat{I}_n(\alpha) = \int_\alpha^\infty e^{-iv}\frac{dv}{v^n}. \tag{186}$$

The rapidly convergent series (185) must be summed to get the value of the integral. The $\hat{I}_n(\alpha)$ satisfy the recurrence relation

$$\hat{I}_{n+1}(\alpha) = \frac{1}{n}\frac{1}{\alpha^n}e^{-i\alpha} - \frac{i}{n}\hat{I}_n(\alpha) \tag{187}$$

which enables the $\hat{I}_n(\alpha)$, $n > 3$ to be obtained from a stable process from $\hat{I}_3(\alpha)$.

We evaluate $\hat{I}_3(\alpha)$ from

$$\hat{I}_3(\alpha) = \frac{1}{2}\left(\gamma + \frac{i\pi}{2}\right) + \frac{1}{2}\log\alpha + \frac{1}{2\alpha^2} - \frac{i}{\alpha} - \frac{3}{4} + \sum_{n=1}^{\infty}\frac{(-i\alpha)^n}{n(n+2)!} \tag{188}$$

for α positive, except when α is very large and positive, in which case we use the asymptotic expansion

$$\hat{I}_3(\alpha) \sim \left[\frac{1}{(-i\alpha)^3} + \frac{3}{(-i\alpha)^4} + \frac{3\cdot 4}{(-i\alpha)^5} + \ldots + \frac{n!}{2(-i\alpha)^{n+1}} + \ldots\right]e^{-i\alpha}. \tag{189}$$

If $X_1/|y|$ is very large and negative we write

$$\begin{aligned} y^2\int_{X_1}^{\infty} e^{-i\nu u/l}\frac{du}{(u^2+y^2)^{\frac{3}{2}}} &= \int_{-\infty}^{\infty} e^{-i\nu|y|v/l}\frac{dv}{(v^2+1)^{\frac{3}{2}}} - \int_{-\infty}^{X_1/|y|} e^{-i\nu|y|v/l}\frac{dv}{(v^2+1)^{\frac{3}{2}}} \\ &= 2\frac{\nu|y|}{l}K_1\left(\frac{\nu|y|}{l}\right) - \int_{-X_1/|y|}^{\infty} e^{i\nu|y|v/l}_{\;y}\frac{dv}{(v^2+1)^{\frac{3}{2}}}, \end{aligned} \tag{190}$$

and we can evaluate the integral on the right-hand side of equation (190) in a similar way to the way the integral in equation (185) was evaluated, the only difference being that the sign of i must be changed.

In the numerical evaluation of the integral on the right-hand side of formula (177) one should note the rapid change in values of $(\eta-\eta_0)^2K((x-x_0)/l, (y-y_0)/l, 0; \nu, M)$ when y_0 is near to y and x_0 passes through x. A number of subintervals in the range $(0, 1)$ of ξ_0 are taken, which together cover the whole range $(0, 1)$, and over each of these subintervals a Gaussian numerical integration is carried out. These subintervals should be smaller where $(\eta-\eta_0)^2K((x-x_0)/l, (y-y_0)/l, 0; \nu, M)$ has a rapid change in its values than they are elsewhere, so that the subintervals can be larger as x_0 moves away from x. Also when y_0 is far from y fewer subintervals can be taken than when y_0 is near to y.

The integral on the right-hand side of equation (176) is an improper integral, the integrand at $\eta_0 = \eta$ having a singularity which is dealt with according to Hadamard's method of finite part integration. We could approximate to $I_r^{(1)}(\xi, \eta, \eta_0; \nu, M)$ for η_0 in $(-1, 1)$ by means of a polynomial in η_0 in order to perform this integration. We note, however, that in the neighbourhood of $\eta_0 = \eta$ the function $I_r^{(1)}(\xi, \eta, \eta_0; \nu, M)$ may be developed into a series of the form

$$I_r^{(1)}(\xi, \eta, \eta_0; \nu, M) = \sum_{p=0}^{\infty} E_{r;p}^{(1)}(\xi, \eta; \nu, M)(\eta-\eta_0)^p + (\eta-\eta_0)^2\log|\eta-\eta_0|\sum_{p=0}^{\infty} F_{r,p}^{(1)}(\xi, \eta; \nu, M)(\eta-\eta_0)^p \tag{191}$$

and that an improvement in the accuracy of the integration can be effected by subtacting the lowest order logarithmic term from $I_r^{(1)}(\xi, \eta, \eta_0; v, M)$ before making the polynomial approximation. To do this the coefficient $F_{r,0}^{(1)}(\xi, \eta; v, M)$ of the lowest order logarithmic term must be known, and, according to Ref. 9 this is given by

$$F_{r,0}^{(1)}(\xi, \eta; v, M) = \frac{1}{4\pi}\frac{s_1 l}{c_1^2(y)}\left\{-(1 - M^2)\frac{d}{d\xi}\left[h_r^{(1)}(\xi)\sqrt{\frac{1-\xi}{\xi}}\right] + 2\frac{ivc_1(y)}{l}h_r^{(1)}(\xi)\sqrt{\frac{1-\xi}{\xi}} + \right.$$
$$\left. + v^2\frac{c_1^2(y)}{l^2}\int_0^{\xi} h_r^{(1)}(u)\sqrt{\frac{1-u}{u}}\,du\right\}. \tag{192}$$

We now write

$$g_s^{(1)}(\eta_0)I_r^{(1)}(\xi, \eta, \eta_0; v, M) = g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)(\eta - \eta_0)^2 \log|\eta - \eta_0| +$$
$$+ [g_s^{(1)}(\eta_0)I_r^{(1)}(\xi, \eta, \eta_0; v, M) - g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)(\eta - \eta_0)^2 \log|\eta - \eta_0|] \tag{193}$$

and approximate to the expression in square brackets by means of a polynomial of degree $(\bar{m}_1 - 1)$ in η_0. We take this polynomial to be the interpolation polynomial with the same values as the expression in square brackets has at the points $\eta_0 = \mu_Q^{(\bar{m}_1)}$, $Q = 1, 2, \ldots, \bar{m}_1$, where $\mu_J^{(m)}$ is defined by formula (117), and on doing this we get

$$g_s^{(1)}(\eta_0)I_r^{(1)}(\xi, \eta, \eta_0; v, M) = g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)(\eta - \eta_0)^2 \log|\eta - \eta_0| + \sum_{Q=1}^{\bar{m}_1} [g_s^{(1)}(\mu_Q^{(\bar{m}_1)})I_r^{(1)}(\xi, \eta, \mu_Q^{(\bar{m}_1)}; v, M) -$$
$$- g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)(\eta - \mu_Q^{(\bar{m}_1)})^2 \log|\eta - \mu_Q^{(\bar{m}_1)}|]\tilde{g}_Q^{(\bar{m}_1)}(\eta_0) \tag{194}$$

where the interpolation polynomial $\tilde{g}_J^{(m)}(\mu)$, $J = 1, 2, \ldots, m$, has been defined in formula (118).

If we substitute the approximate expression (194) for $g_s^{(1)}(\eta_0)I_r^{(1)}(\xi, \eta, \eta_0; v, M)$ into equation (176) we get

$$U_{r,s}^{(1)}(x, y; v, M) = g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)\int_{-1}^{+1} \log|\eta - \eta_0|\sqrt{1 - \eta_0^2}\,d\eta_0 + \sum_{Q=1}^{\bar{m}_1} [g_s^{(1)}(\mu_Q^{(\bar{m}_1)})I_r^{(1)}(\xi, \eta, \mu_Q^{(\bar{m}_1)}; v, M) -$$
$$- g_s^{(1)}(\eta)F_{r,0}^{(1)}(\xi, \eta; v, M)(\eta - \mu_Q^{(\bar{m}_1)})^2 \log|\eta - \mu_Q^{(\bar{m}_1)}|]\int_{-1}^{+1}\frac{\tilde{g}_Q^{(\bar{m}_1)}(\eta_0)\sqrt{1-\eta_0^2}}{(\eta - \eta_0)^2}\,d\eta_0 \tag{195}$$

and therefore

$$U_{r,s}^{(1)}(\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M) = g_s^{(1)}(\mu_J^{(M_1)})F_{r,0}^{(1)}(\bar{\sigma}_I^{(N_1)}, \mu_J^{(M_1)}; v, M)\,\times$$
$$\times\left[\int_{-1}^{+1}\log|\mu_J^{(M_1)} - \eta_0|\sqrt{1-\eta_0^2}\,d\eta_0 - \sum_{Q=1}^{\bar{m}_1}(\mu_J^{(M_1)} - \mu_Q^{(\bar{m}_1)})^2\log|\mu_J^{(M_1)} - \mu_Q^{(\bar{m}_1)}|\,\times\right.$$
$$\left.\times\int_{-1}^{+1}\frac{\tilde{g}_Q^{(\bar{m}_1)}(\eta_0)\sqrt{1-\eta_0^2}}{(\mu_J^{(M_1)} - \eta_0)^2}\,d\eta_0\right] + \sum_{Q=1}^{\bar{m}_1} g_s^{(1)}(\mu_Q^{(\bar{m}_1)})I_r^{(1)}(\bar{\sigma}_I^{(N_1)}, \mu_J^{(M_1)}, \mu_Q^{(\bar{m}_1)}; v, M)\,\times$$
$$\times\int_{-1}^{+1}\frac{\tilde{g}_Q^{(\bar{m}_1)}(\eta_0)\sqrt{1-\eta_0^2}}{(\mu_J^{(M_1)} - \eta_0)^2}\,d\eta_0. \tag{196}$$

Now, we know that, (*see* Ref. 2 or Ref. 9),

$$\int_{-1}^{+1}\frac{\tilde{g}_Q^{(m)}(\eta_0)\sqrt{1-\eta_0^2}}{(\mu_J^{(m)} - \eta_0)^2}\,d\eta_0 = \begin{cases} -\frac{\pi}{2}(m+1) & Q = J \\ \frac{2\pi}{(m+1)}\frac{\{1 - (\mu_Q^{(m)})^2\}}{(\mu_J^{(m)} - \mu_Q^{(m)})^2} & (-1)^{Q+J} = -1 \\ 0 & (-1)^{Q+J} = 1, Q \neq J. \end{cases} \tag{197}$$

We can make use of formula (197) in equation (196) if

$$\bar{m}_1 + 1 = q_1(M_1 + 1) \tag{198}$$

for some positive integer q_1, for then all the $\mu_J^{(M_1)}$, $J = 1, 2, \ldots, M_1$, are included among the $\mu_Q^{(\bar{m}_1)}$, $Q = 1, 2, \ldots, \bar{m}_1$. We shall henceforth take $\bar{m}_1$ to be given by the relation (198) for some q_1, and we shall evaluate $U_{r,s}^{(1)}(\bar{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M)$ from formula (196). To do this we need the result

$$\int_{-1}^{+1}\log|\eta - \eta_0|\sqrt{1-\eta_0^2}\,d\eta_0 = \frac{\pi}{4}(2\eta^2 - 1) - \frac{\pi}{2}\log_e 2. \tag{199}$$

The technique leading to the formula (196) for evaluating $U^{(1)}_{r,s}(\bar{x}^{(N_1,M_1)}_{1;I,J}, y^{(M_1)}_{1;J}; \nu, M)$ was first advocated in Ref. 10 for isolated wings oscillating at low frequency and later applied, in Ref. 11, to isolated wings oscillating at general frequency.

We note that

$$\int_{-1}^{+1} \frac{\gamma_m(\eta_0)}{(\mu^{(m)}_J - \eta_0)^2} \sqrt{1 - \eta_0^2}\, d\eta_0 = 0, \qquad J = 1, 2, \ldots, m, \tag{200}$$

where $\gamma_m(\eta_0)$ is obtained from relations (116). It follows that the formula (196) is exact provided that the expression in square brackets in formula (193) is a polynomial of degree $\leqslant \bar{m}_1$ in η_0.

The formula analogous to (196) for evaluation $U^{(2)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}, \nu, M)$ is

$$\begin{aligned} U^{(2)}_{r,s}(\bar{x}^{(N_2,M_2)}_{2;I,J}, y^{(M_2)}_{2;J}; \nu, M) &= g^{(2)}_s(\mu^{(M_2)}_J) F^{(2)}_{r,0}(\bar{\sigma}^{(N_2)}_I, \mu^{(M_2)}_J; \nu, M) \times \\ &\times \left[\int_{-1}^{+1} \log|\mu^{(M_2)}_J - \zeta_0| \sqrt{1 - \zeta_0^2}\, d\zeta_0 - \sum_{Q=1}^{\bar{m}_2} (\mu^{(M_2)}_J - \mu^{(\bar{m}_2)}_Q)^2 \log|\mu^{(M_2)}_J - \mu^{(\bar{m}_2)}_Q| \times \right. \\ &\times \left. \int_{-1}^{+1} \frac{\bar{g}^{(\bar{m}_2)}_Q(\zeta_0)\sqrt{1 - \zeta_0^2}}{(\mu^{(M_2)}_J - \zeta_0)^2} d\zeta_0 \right] + \sum_{Q=1}^{\bar{m}_2} g^{(2)}_s(\mu^{(\bar{m}_2)}_Q) I^{(2)}_r(\bar{\sigma}^{(N_2)}_I, \mu^{(M_2)}_J, \mu^{(\bar{m}_2)}_Q; \nu, M) \times \\ &\times \int_{-1}^{+1} \frac{\bar{g}^{(\bar{m}_2)}_Q(\zeta_0)\sqrt{1 - \zeta_0^2}}{(\mu^{(M_2)}_J - \zeta_0)^2} d\zeta_0 \end{aligned} \tag{201}$$

where

$$I^{(2)}_r(\varepsilon, \zeta, \zeta_0; \nu, M) = \frac{1}{4\pi} \frac{s_2}{l} (\zeta - \zeta_0)^2 \int_0^1 h^{(2)}_r(\varepsilon_0) \sqrt{\frac{1 - \varepsilon_0}{\varepsilon_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, 0; \nu, M \right) d\varepsilon_0 \tag{202}$$

and

$$\begin{aligned} F^{(2)}_{r,0}(\varepsilon, \zeta; \nu, M) = \frac{1}{4\pi} \frac{s_2 l}{c_2^2(y)} \Bigg\{ &-(1 - M^2) \frac{d}{d\varepsilon}\left[h^{(2)}_r(\varepsilon) \sqrt{\frac{1 - \varepsilon}{\varepsilon}} \right] + 2i\nu \frac{c_2(y)}{l} h^{(2)}_r(\varepsilon) \sqrt{\frac{1 - \varepsilon}{\varepsilon}} + \\ &+ \frac{\nu^2 c_2^2(y)}{l^2} \int_0^\varepsilon h^{(2)}_r(u) \sqrt{\frac{1 - u}{u}}\, du \Bigg\}. \end{aligned} \tag{203}$$

We take

$$\bar{m}_2 + 1 = q_2(M + 1). \tag{204}$$

6.5. Evaluation of $V^{(2)}_{r,s}$

We can rewrite formula (64) for $V^{(2)}_{r,s}(x, y; \nu, M)$ in the form

$$V^{(2)}_{r,s}(x, y; \nu, M) = \int_{-1}^{+1} g^{(2)}_s(\zeta_0) \sqrt{1 - \zeta_0^2} J^{(2)}_r(\xi, \eta, \zeta_0; \nu, M)\, d\zeta_0 \tag{205}$$

where

$$J^{(2)}_r(\xi, \eta, \zeta_0; \nu, M) = \frac{1}{4\pi} \frac{s_2}{l} \int_0^1 h^{(2)}_r(\varepsilon_0) \sqrt{\frac{1 - \varepsilon_0}{\varepsilon_0}} K\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M \right) d\varepsilon_0. \tag{206}$$

The function $J^{(2)}_r(\xi, \eta, \zeta_0; \nu, M)$ is finite for any ζ_0 in the range $(-1, 1)$ since the tailplane is assumed not to be in front of the wing in the plane of the wing. To evaluate $K(x/l, y/l, h/l; \nu, M)$ from equation (43) we need to evaluate

$$l^2 \int_X^\infty e^{-i\nu u/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du \tag{207}$$

where

$$X = \frac{-x + MR}{(1 - M^2)} \tag{208}$$

and

$$R = \sqrt{x^2 + (1 - M^2)(y^2 + h^2)}. \tag{209}$$

We can write

$$l^2 \int_X^\infty e^{-ivu/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du$$

$$= -\frac{\pi l^2}{2(y^2 + h^2)} \left\{\frac{iv}{l}\sqrt{y^2 + h^2}\right\} \left\{\mathbf{H}_{-1}\left(\frac{iv}{l}\sqrt{y^2 + h^2}\right) - \frac{2i}{\pi} K_1\left(\frac{v}{l}\sqrt{y^2 + h^2}\right) - I_1\left(\frac{v}{l}\sqrt{y^2 + h^2}\right)\right\} -$$

$$- \frac{\pi l^2 h^2}{2(y^2 + h^2)^2} \left\{\frac{iv}{l}\sqrt{y^2 + h^2}\right\}^2 \left\{\mathbf{H}_{-2}\left(\frac{iv}{l}\sqrt{y^2 + h^2}\right) - \frac{2}{\pi} K_2\left(\frac{v}{l}\sqrt{y^2 + h^2}\right) + iI_2\left(\frac{v}{l}\sqrt{y^2 + h^2}\right)\right\} -$$

$$- l^2 \int_0^X e^{-ivu/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du, \tag{210}$$

where I_1 and I_2 are modified Bessel functions of the first kind and of the first and second orders respectively, K_1 and K_2 are modified Bessel functions of the second kind of the first and second orders respectively and $\mathbf{H}_{-1}$ and $\mathbf{H}_{-2}$ are Struve functions of orders -1 and -2 respectively.

The numerical evaluation of the integral (207) can now be carried out using the formula (210) so that $K(x/l, y/l, h/l; v, M)$ can be determined from formula (43) and then the numerical evaluation of the integral on the right-hand side of equation (206) can be carried out using Gaussian integration over a number of subintervals.

However we observe that when $X/\sqrt{y^2 + h^2}$ is very large the numerical evaluation of the integral on the right-hand side of equation (210) can become a lengthy process. In this case we can evaluate the integral (207) by a different more rapid process.

We take $X/\sqrt{y^2 + h^2}$ to be very large and positive and write

$$l^2 \int_X^\infty e^{-ivu/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du = v^2 \sum_{n=0}^\infty (-1)^n \frac{(2n+1)!}{2^{2n}(n!)^2} \left(\frac{vr}{l}\right)^{2n} \hat{I}_{2n+3}\left(\frac{vX}{l}\right) -$$

$$- v^2 \frac{h^2}{r^2} \sum_{n=0}^\infty (-1)^n \frac{(2n+3)!}{2^{2n+1} n!(n+1)!} \left(\frac{vr}{l}\right)^{2n+2} \hat{I}_{2n+5}\left(\frac{vX}{l}\right), \tag{211}$$

where

$$r = \sqrt{y^1 + h^2}. \tag{212}$$

The quantities $\hat{I}_n(\alpha)$ can be evaluated as before and the rapidly convergent series in equation (211) summed to get the value of the integral.

We could consider the case of $X/\sqrt{y^2 + h^2}$ being very large and negative but this is not likely to occur in a practical wing-tailplane configuration.

We now approximate to $g_s^{(2)}(\zeta_0) J_r^{(2)}(\xi, \eta, \zeta_0; v, M)$ by means of the interpolation polynomial of degree $(\overline{m}_2 - 1)$ in ζ_0

$$g_s^{(2)}(\zeta_0) J_r^{(2)}(\xi, \eta, \zeta_0; v, M) = \sum_{Q=1}^{\overline{m}_2} g_s^{(2)}(\mu_Q^{(\overline{m}_2)}) J_r^{(2)}(\xi, \eta, \mu_Q^{(\overline{m}_2)}; v, M) \tilde{g}_Q^{(\overline{m}_2)}(\zeta_0), \tag{213}$$

where the interpolation polynomial $\tilde{g}_J^{(m)}(\mu)$, $J = 1, 2, \ldots, m$, has been defined in formula (118).

If we substitute the approximation formula (213) for $g_s^{(2)}(\zeta_0) J_r^{(2)}(\xi, \eta, \zeta_0, v, M)$ into (205) we get

$$V_{r,s}^{(2)}(x, y; v, M) = \sum_{Q=1}^{\overline{m}_2} \tilde{G}_Q^{(\overline{m}_2)} g_s^{(2)}(\mu_Q^{(\overline{m}_2)}) J_r^{(2)}(\xi, \eta, \mu_Q^{(\overline{m}_2)}; v, M) \tag{214}$$

and this formula can be used for evaluation $V_{r,s}^{(2)}(\overline{x}_{1;I,J}^{(N_1,M_1)}, y_{1;J}^{(M_1)}; v, M)$.

The formula (214) is exact provided that $g_s^{(2)}(\zeta_0) J_r^{(2)}(\xi, \eta, \zeta_0; v, M)$ is a polynomial of degree $\leqslant (2\overline{m}_2 - 1)$ in ζ_0.

6.6. Evaluation of $V_{r,s}^{(1)}$

The function $V_{r,s}^{(1)}(x, y; v, M)$ is defined by formula (63). We note that if the tailplane is in the plane of the wing, behind the wing, the kernel function $K((x - x_0)/l, (y - y_0)/l, h/l; v, M)$ can become infinite as $y_0 \to y$.

If the tailplane is nearly in the plane of the wing the kernel function can become very large as $y_0 \to y$ and this feature can lead to difficulties with subsequent numerical integration. We can separate the kernel into two parts, one of which is well behaved for all values of h/l and the other of which becomes infinite if $h/l = 0$, or becomes very large if $h/l \ll 1$ when $y_0 \to y$. This second part will occur in integrals which can be evaluated analytically so that no numerical difficulties are experienced.

Thus we can write, from equation (43),

$$K\left(\frac{x}{l}, \frac{y}{l}, \frac{h}{l}; \nu, M\right) = \bar{K}\left(\frac{y}{l}, \frac{h}{l}; \nu\right) + K^*\left(\frac{x}{l}, \frac{y}{l}, \frac{h}{l}; \nu, M\right) \tag{215}$$

where

$$\bar{K}\left(\frac{y}{l}, \frac{h}{l}; \nu\right) = l^2 \int_{-\infty}^{\infty} e^{-i\nu u/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du$$

$$= \frac{2l^2}{(y^2 + h^2)}\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right) K_1\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right) - \frac{2l^2h^2}{(y^2 + h^2)^2}\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right)^2 K_2\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right)$$

$$= -2\nu^2 K_0\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right) - \frac{d}{dy}\left[\frac{2l^2 y}{(y^2 + h^2)}\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right) K_1\left(\frac{\nu}{l}\sqrt{y^2 + h^2}\right)\right] \tag{216}$$

and

$$K^*\left(\frac{x}{l}, \frac{y}{l}, \frac{h}{l}; \nu, M\right) = l^2\Bigg[-\int_{-\infty}^{X} e^{-i\nu u/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du +$$

$$+ \exp\left(-\frac{i\nu X}{l}\right)\Bigg\{\frac{M(Mx + R)}{R(x^2 + y^2 + h^2)} - \frac{h^2 M(Mx + R)^3}{R(x^2 + y^2 + h^2)^3} - \frac{h^2 M^2(1 - M^2)x}{R^3(x^2 + y^2 + h^2)} -$$

$$- \frac{2h^2 M(Mx + R)}{R(x^2 + y^2 + h^2)^2} - \frac{i\nu}{l}\frac{h^2 M^2(Mx + R)}{R^2(x^2 + y^2 + h^2)}\Bigg\}\Bigg]. \tag{217}$$

In the above, K_0 is the modified Bessel function of the second kind and zero order and X and R are defined in formulae (208) and (209).

The function $K^*((x - x_0)/l, (y - y_0)/l, h/l; \nu, M)$ is well behaved as $y_0 \to y$ when x is a point on the tailplane and x_0 is a point on the wing.

We now write for $V_{r,s}^{(1)}(x, y; \nu, M)$ given in formula (63)

$$V_{r,s}^{(1)}(x, y; \nu, M) = \bar{V}_{r,s}^{(1)}(y; \nu) + \overset{*}{V}{}_{r,s}^{(1)}(x, y; \nu, M), \tag{218}$$

where

$$\bar{V}_{r,s}^{(1)}(y; \nu) = \frac{1}{4\pi}\frac{s_1}{l}\int_0^1 h_r^{(1)}(\xi_0)\sqrt{\frac{1 - \xi_0}{\xi_0}}\, d\xi_0 \int_{-1}^{+1} g_s^{(1)}(\eta_0)\sqrt{1 - \eta_0^2}\,\bar{K}\left(\frac{y - y_0}{l}, \frac{h}{l}; \nu\right) d\eta_0, \tag{219}$$

$$\overset{*}{V}{}_{r,s}^{(1)}(x, y; \nu, M) = \int_{-1}^{+1} g_s^{(1)}(\eta_0)\sqrt{1 - \eta_0^2}\,\overset{*}{J}{}_r^{(1)}(\varepsilon, \zeta, \eta_0; \nu, M)\, d\eta_0, \tag{220}$$

and

$$\overset{*}{J}{}_r^{(1)}(\varepsilon, \zeta, \eta_0; \nu, M) = \frac{1}{4\pi}\frac{s_1}{l}\int_0^1 h_r^{(1)}(\xi_0)\sqrt{\frac{1 - \xi_0}{\xi_0}}\, K^*\left(\frac{x - x_0}{l}, \frac{y - y_0}{l}, \frac{h}{l}; \nu, M\right) d\xi_0. \tag{221}$$

To evaluate $K^*(x/l, y/l, h/l; \nu, M)$ from equation (217) we need to evaluate

$$l^2 \int_{-\infty}^{X} e^{-i\nu u/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du. \tag{222}$$

We can write (*cf.* equation (210))

$$l^2 \int_{-\infty}^{X} e^{-i\nu u/l} \frac{(u^2 + y^2 - 2h^2)}{(u^2 + y^2 + h^2)^{\frac{5}{2}}} du$$

$$= \frac{\pi l^2}{2(y^2+h^2)}\frac{iv}{l}\sqrt{y^2+h^2}\left\{\mathbf{H}_{-1}\left(\frac{iv}{l}\sqrt{y^2+h^2}\right)-\frac{2i}{\pi}K_1\left(\frac{v}{l}\sqrt{y^2+h^2}\right)-I_1\left(\frac{v}{l}\sqrt{y^2+h^2}\right)\right\}+$$

$$+\frac{\pi l^2h^2}{2(y^2+h^2)^2}\left\{\frac{iv}{l}\sqrt{y^2+h^2}\right\}^2\left\{\mathbf{H}_{-2}\left(\frac{iv}{l}\sqrt{y^2+h^2}\right)+\frac{2}{\pi}K_2\left(\frac{v}{l}\sqrt{y^2+h^2}\right)+iI_2\left(\frac{v}{l}\sqrt{y^2+h^2}\right)\right\}+$$

$$+l^2\int_0^X e^{-ivu/l}\frac{(u^2+y^2-2h^2)}{(u^2+y^2+h^2)^{\frac{5}{2}}}\,du. \tag{223}$$

The numerical evaluation of the integral (222) can now be carried out using the formula (223) so that $K^*(z/l, y/l, h/l; v, M)$ can be determined from formula (217) and then the numerical evaluation of the integral on the right-hand side of (221) can be carried out using Gaussian integration over a number of subintervals.

Again we observe that when $X/\sqrt{y^2+h^2}$ is very large the numerical evaluation of the integral on the right-hand side of (223) can become a lengthy process. In this case we can evaluate the integral (222) by a different more rapid process.

We take $X/\sqrt{y^2+h^2}$ to be very large and negative and write

$$l^2\int_{-\infty}^X e^{-ivu/l}\frac{(u^2+y^2-2h^2)}{(u^2+y^2+h^2)^{\frac{5}{2}}}\,du = v^2\sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)!}{2^{2n}(n!)^2}\left(\frac{vr}{l}\right)^{2n}\bar{I}_{2n+3}\left(-\frac{vX}{l}\right)-$$

$$-v^2\frac{h^2}{r^2}\sum_{n=0}^{\infty}(-1)^n\frac{(2n+3)!}{2^{2n+1}n!(n+1)!}\left(\frac{vr}{l}\right)^{2n+2}\bar{I}_{2n+5}\left(-\frac{vX}{l}\right) \tag{224}$$

where $\bar{I}_n(\alpha)$ is the complex conjugate of $\hat{I}_n(\alpha)$, and these can be evaluated as before.

We could consider the case of $X/\sqrt{y^2+h^2}$ being very large and positive but this is not likely to occur in a practical wing-tailplane configuration.

We now approximate to $g_s^{(1)}(\eta_0)\mathring{J}_r^{(1)}(\varepsilon, \zeta, \eta_0; v, M)$ by means of the interpolation polynomial of degree $(\bar{m}_1 - 1)$ in η_0,

$$g_s^{(1)}(\eta_0)\mathring{J}_r^{(1)}(\varepsilon,\zeta,\eta_0;v,M) = \sum_{Q=1}^{\bar{m}_1} g_s^{(1)}(\mu_Q^{(\bar{m}_1)})\mathring{J}_r^{(1)}(\varepsilon,\zeta,\mu_Q^{(\bar{m}_1)};v,M)\tilde{g}_Q^{(\bar{m}_1)}(\eta_0), \tag{225}$$

where the interpolation polynomial $\tilde{g}_J^{(m)}(\mu)$, $J = 1, 2, \ldots, m$, has been defined in formula (118).

If we substitute the approximation formula (225) for $g_s^{(1)}(\eta_0)\mathring{J}_r^{(1)}(\varepsilon, \zeta, \eta_0; v, M)$ into (220) we get

$$\mathring{V}_{r,s}^{(1)}(x,y;v,M) = \sum_{Q=1}^{\bar{m}_1}\tilde{G}_Q^{(\bar{m}_1)}g_s^{(1)}(\mu_Q^{(\bar{m}_1)})\mathring{J}_r^{(1)}(\varepsilon,\zeta,\mu_Q^{(\bar{m}_1)};v,M), \tag{226}$$

and this formula can be used for evaluating $\mathring{V}_{r,s}^{(1)}(\bar{x}_{2;I,J}^{(N_2,M_2)}, y_{2;J}^{(M_2)}; v, M)$.

The formula (226) is exact provided that $g_s^{(1)}(\eta_0)\mathring{J}_r^{(1)}(\varepsilon, \zeta, \eta_0; v, M)$ is a polynomial of degree $\leqslant (2\bar{m}_1 - 1)$ in η_0.

6.7. Evaluation of M_s

If

$$n_1 \leqslant N_1 \tag{227}$$

then the formula

$$\int_0^1 h_r^{(1)}(\xi_0)\sqrt{\frac{1-\xi_0}{\xi_0}}\,d\xi_0 = \sum_{I=1}^{N_1} h_r^{(1)}(\sigma_I^{(N_1)})\tilde{H}_I^{(N_1)}, \tag{228}$$

where the $\sigma_I^{(n)}$ are given in equation (111) and the $\tilde{H}_I^{(n)}$ are given in equation (115), is exact and may be used in equation (219).

In equation (219) we also need to evaluate

$$M_s\left(\frac{y}{l},\frac{h}{l};v\right) = \int_{-1}^{+1} g_s^{(1)}(\eta_0)\sqrt{1-\eta_0^2}\,\bar{K}\left(\frac{y-y_0}{l},\frac{h}{l}v\right)d\eta_0. \tag{229}$$

We shall write, in the integrand of the integral on the right-hand side of equation (229) (assuming $s_2 \leqslant s_1$),

and

$$\eta_0 = \cos\phi_0$$
$$y/s_1 = \cos\phi \tag{230}$$

and put

$$\chi = \frac{(y - y_0)^2 + h^2}{s_1^2}. \tag{231}$$

Until now the positions of the points $\eta_j^{(1)}, j = 1, 2, \ldots, n_1$ in $(-1, 1)$, introduced in Section 4 has not been specified. If we take these points to be given by the formulae

$$\eta_j^{(1)} = \cos\left(\frac{j\pi}{m_1 + 1}\right), \qquad j = 1, 2, \ldots, m_1, \tag{232}$$

then we can use the expansion

$$g_s^{(1)}(\eta_0) = \frac{2}{(m_1 + 1)} \frac{\sin\phi_s}{\sin\phi_0} \sum_{j=1}^{m_1} \sin j\phi_s \sin j\phi_0 \tag{233}$$

where

$$\phi_s = \frac{s\pi}{(m_1 + 1)}. \tag{234}$$

If we use the formula (216) and the expansion (233) in equation (229) we get

$$\begin{aligned} M_s\left(\frac{y}{l}, \frac{h}{l}; \nu\right) &= -\frac{4\nu^2}{(m_1+1)} \sin\phi_s \sum_{j=1}^{m_1} \sin j\phi_s \int_0^\pi \sin j\phi_0 \sin\phi_0 K_0\left(\frac{\nu s_1}{l}\sqrt{\chi}\right) d\phi_0 + \frac{l^2}{s_1^2} \frac{4}{(m_1+1)} \times \\ &\quad \times \sin\phi_s \sum_{j=1}^{m_1} \sin j\phi_s \int_0^\pi \sin j\phi_0 \frac{d}{d\phi_0}\left\{\frac{1}{\chi}\left(\frac{\nu s_1\sqrt{\chi}}{l}\right)(\cos\phi_0 - \cos\phi) K_1\left(\frac{\nu s_1\sqrt{\chi}}{l}\right)\right\} \mathrm{d}\phi_0 \\ &= -\frac{4\nu^2}{(m_1+1)} \sin\phi_s \sum_{j=1}^{m_1} \sin j\phi_s \int_0^\pi \sin j\phi_0 \sin\phi_0 K_0\left(\frac{\nu s_1}{l}\sqrt{\chi}\right) d\phi_0 - \frac{l^2}{s_1^2} \frac{4}{(m_1+1)} \times \\ &\quad \times \sin\phi_s \sum_{j=1}^{m_1} j \sin j\phi_s \int_0^\pi \frac{1}{\chi}\left(\frac{\nu s_1\sqrt{\chi}}{l}\right)(\cos\phi_0 - \cos\phi) \cos j\phi_0 K_1\left(\frac{\nu s_1\sqrt{\chi}}{l}\right) d\phi_0. \end{aligned} \tag{235}$$

We shall write the modified Bessel functions of the second kind of order zero and one respectively in the forms

$$K_0(x) = -\left\{\gamma + \log\left(\frac{x}{2}\right)\right\} + K_{0,1}(x) \tag{236}$$

and

$$K_1(x) = \frac{1}{x} + \frac{x}{2}\left\{\gamma - \frac{1}{2} + \log\left(\frac{x}{2}\right)\right\} + xK_{1,1}(x) \tag{237}$$

where γ is Euler's constant and

$$K_{0,1}(x) = \sum_{r=1}^{\infty} c_r \left(\frac{1}{2^r r!}\right)^2 x^{2r} - \left\{\gamma + \log\left(\frac{x}{2}\right)\right\} \sum_{r=1}^{\infty} \left(\frac{1}{2^r r!}\right)^2 x^{2r}, \tag{238}$$

$$K_{1,1}(x) = -\sum_{r=1}^{\infty} d_r \frac{1}{2^{2r+1} r!(r+1)!} x^{2r} + \left\{\gamma + \log\left(\frac{x}{2}\right)\right\} \sum_{r=1}^{\infty} \frac{1}{2^{2r+1} r!(r+1)!} x^{2r}, \tag{239}$$

$$c_r = \sum_{s=1}^{r} \frac{1}{s} \tag{240}$$

and

$$d_r = c_r + \frac{1}{2(r+1)}. \tag{241}$$

Then we can write

$$M_s\left(\frac{y}{l},\frac{h}{l};\nu\right) = \frac{4\nu^2}{(m_1+1)}\left\{\gamma + \log\left(\frac{\nu s_1}{2l}\right)\right\}\sin\phi_s\sum_{j=1}^{m_1}\sin j\phi_s\int_0^\pi \sin j\phi_0 \sin\phi_0\, d\phi_0 + \frac{2\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1}\times$$

$$\times \sin j\phi_s\int_0^\pi \log(\chi)\sin j\phi_0\sin\phi_0\,d\phi_0 - \frac{l^2}{s_1^2}\frac{4}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\sin j\phi_s\int_0^\pi \frac{(\cos\phi_0-\cos\phi)}{\chi}\times$$

$$\times\cos j\phi_0\,d\phi_0 - \frac{2\nu^2}{(m_1+1)}\sin\phi_s\left\{\gamma - \frac{1}{2} + \log\left(\frac{\nu s_1}{2l}\right)\right\}\sum_{j=1}^{m_1} j\sin j\phi_s\int_0^\pi(\cos\phi_0-\cos\phi)\times$$

$$\times\cos j\phi_0\,d\phi_0 - \frac{\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\sin j\phi_s\int_0^\pi \log(\chi)(\cos\phi_0-\cos\phi)\cos j\phi_0\,d\phi_0 -$$

$$-\frac{4\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1}\sin j\phi_s\int_0^\pi K_{0,1}\left(\frac{\nu s_1}{l}\sqrt{\chi}\right)\sin j\phi_0\sin\phi_0\,d\phi_0 - \frac{4\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\times$$

$$\times\sin j\phi_s\int_0^\pi K_{1,1}\left(\frac{\nu s_1}{l}\sqrt{\chi}\right)(\cos\phi_0-\cos\phi)\cos j\phi_0\,d\phi_0$$

$$=\frac{2\pi\nu^2}{(m_1+1)}\left\{\gamma+\log\left(\frac{\nu s_1}{2l}\right)\right\}\sin^2\phi_s - \frac{\pi\nu^2}{(m_1+1)}\left\{\gamma-\frac{1}{2}+\log\left(\frac{\nu s_1}{2l}\right)\right\}\sin^2\phi_s +$$

$$+\frac{2\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1}\sin j\phi_s\int_0^\pi\log(\chi)\sin j\phi_0\sin\phi_0\,d\phi_0 - \frac{\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\times$$

$$\times\sin j\phi_s\int_0^\pi \log(\chi)(\cos\phi_0-\cos\phi)\cos j\phi_0\,d\phi_0 - \frac{l^2}{s_1^2}\frac{4}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\times$$

$$\times\sin j\phi_s\int_0^\pi\frac{(\cos\phi_0-\cos\phi)}{\chi}\cos j\phi_0\,d\phi_0 - \frac{4\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1}\sin j\phi_s\int_0^\pi K_{0,1}\left(\frac{\nu s_1}{l}\sqrt{\chi}\right)\times$$

$$\times\sin j\phi_0\sin\phi_0\,d\phi_0 - \frac{4\nu^2}{(m_1+1)}\sin\phi_s\sum_{j=1}^{m_1} j\sin j\phi_s\int_0^\pi K_{1,1}\left(\frac{\nu s_1}{l}\sqrt{\chi}\right)\times$$

$$\times(\cos\phi_0-\cos\phi)\cos j\phi_0\,d\phi_0. \tag{242}$$

In formula (242) the last two integrals are to be evaluated numerically. The range of integration is divided into a large number of subranges over each of which a Gaussian numerical integration is carried out. This process should have good accuracy even though logarithmic functions occur in formulae (238) and (239) for $K_{0,1}(x)$ and $K_{1,1}(x)$ because the logarithms do not lead to infinite values of these functions in the integrands.

The other integrals in formula (242) can be evaluated analytically but the present writer has not succeeded in obtaining general formulae valid for all j. They are evaluated by using a combination of reduction formulae, numerical integration and analytical integration as shown below.

Let us define

$$N_j\left(\frac{y}{l},\frac{h}{l}\right) = \int_0^\pi\frac{(\cos\phi_0-\cos\phi)}{\chi}\cos j\phi_0\,d\phi_0 \tag{243}$$

and

$$L_j\left(\frac{y}{l},\frac{h}{l}\right) = \int_0^\pi \log(\chi)\cos j\phi_0\,d\phi_0. \tag{244}$$

Then

$$\int_0^\pi\log(\chi)\sin j\phi_0\sin\phi_0\,d\phi_0 = \frac{1}{2}\left\{L_{j-1}\left(\frac{y}{l},\frac{h}{l}\right) - L_{j+1}\left(\frac{y}{l},\frac{h}{l}\right)\right\} \tag{245}$$

and

$$\int_0^\pi\log(\chi)(\cos\phi_0-\cos\phi)\cos j\phi_0\,d\phi_0 = \frac{1}{2}\left\{L_{j-1}\left(\frac{y}{l},\frac{h}{l}\right)+L_{j+1}\left(\frac{y}{l},\frac{h}{l}\right)\right\} - \cos\phi L_j\left(\frac{y}{l},\frac{h}{l}\right). \tag{246}$$

Now, for $j \neq 0$,

$$L_j\left(\frac{y}{l}, \frac{h}{l}\right) = \frac{1}{j}\int_0^\pi \log(\chi)\frac{d}{d\phi_0}(\sin j\phi_0)\,d\phi_0$$
$$= -\frac{1}{j}\int_0^\pi \sin j\phi_0 \frac{d}{d\phi_0}\{\log(\chi)\}\,d\phi_0$$
$$= \frac{2}{j}\int_0^\pi \frac{\sin j\phi_0}{\chi}(\cos\phi_0 - \cos\phi)\sin\phi_0\,d\phi_0$$
$$= \frac{1}{j}\left\{N_{j-1}\left(\frac{y}{l}, \frac{h}{l}\right) - N_{j+1}\left(\frac{y}{l}, \frac{h}{l}\right)\right\}. \tag{247}$$

To evaluate $N_j(y/l, h/l)$ we write, instead of formula (243),

$$N_j\left(\frac{y}{l}, \frac{h}{l}\right) = \int_0^\pi \frac{(\cos\phi_0 - \cos\phi)}{\chi}(\cos j\phi_0 - \cos j\phi)\,d\phi_0 + \cos j\phi\, N_0\left(\frac{y}{l}, \frac{h}{l}\right)$$
$$= \int_0^\pi \frac{(\cos\phi_0 - \cos\phi)^2}{\chi}\,\frac{\sin\frac{1}{2}j(\phi_0+\phi)}{\sin\frac{1}{2}(\phi_0+\phi)}\,\frac{\sin\frac{1}{2}j(\phi_0-\phi)}{\sin\frac{1}{2}(\phi_0-\phi)}\,d\phi_0 + \cos j\phi N_0\left(\frac{y}{l}, \frac{h}{l}\right). \tag{248}$$

Now, $(\cos\phi_0 - \cos\phi)^2/\chi$ is a continuous function, even as $h \to 0$, and it is multiplied by a continuous function in the integrand of the integral in equation (248). The integral can therefore be evaluated numerically by Gaussian integration to a high degree of accuracy. We note that

$$\frac{\sin n\theta}{\sin\theta} = \begin{cases} 2\{\cos(n-1)\theta + \cos(n-3)\theta + \ldots + \cos\theta\} & n \text{ even} \\ 2\{\cos(n-1)\theta + \cos(n-2)\theta + \ldots + \cos 2\theta\} + 1 & n \text{ odd.} \end{cases} \tag{249}$$

It now only remains to determine $N_0(y/l, h/l)$ and $L_0(y/l, h/l)$. Tractable expressions for these are determined analytically in the appendix and the results are:

$$N_0\left(\frac{y}{l}, \frac{h}{l}\right) = \frac{\pi}{2\sqrt{2}}\,\frac{H}{\sqrt{1+H^2}}\,\frac{(u^4-1)}{\sqrt{u^4+H^2}}\,\frac{\sqrt{\left\{\sqrt{\dfrac{u^4+H^2}{1+H^2}} + \dfrac{u^2-H^2}{1+H^2}\right\}}}{\left\{u^2 + \sqrt{\dfrac{u^4+H^2}{1+H^2}}\right\}} \tag{250}$$

and

$$L_0\left(\frac{y}{l}, \frac{h}{l}\right) = 2\pi\log\left[\frac{\sqrt{1+H^2}}{2(1+u^2)}\left(\sqrt{\frac{u^4+H^2}{1+H^2}} + 1 + \sqrt{2}\frac{H(1+u^2)}{(1+H^2)}\,\frac{1}{\sqrt{\left\{\sqrt{\dfrac{u^4+H^2}{1+H^2}} + \dfrac{u^2-H^2}{1+H^2}\right\}}}\right)\right], \tag{251}$$

where

$$u = \sqrt{\frac{1-y/s_1}{1+y/s_1}}, \qquad H = \frac{h/s_1}{1+y/s_1}. \tag{252}$$

As a matter of interest the analytical formulae for the first few $N_j(y/l, h/l)$ are:

$$N_1\left(\frac{y}{l}, \frac{h}{l}\right) = \pi t_1(\phi) + N_0\left(\frac{y}{l}, \frac{h}{l}\right)\cos\phi - \frac{h^2}{s_1^2}P_0\left(\frac{y}{l}, \frac{h}{l}\right)t_1(\phi), \tag{253}$$

$$N_2\left(\frac{y}{l}, \frac{h}{l}\right) = \pi t_2(\phi) + N_0\left(\frac{y}{l}, \frac{h}{l}\right)\left(\cos 2\phi - \frac{2h^2}{s_1^2}t_1(\phi)\right) - \frac{2h^2}{s_1^2}P_0\left(\frac{y}{l}, \frac{h}{l}\right)t_2(\phi), \tag{254}$$

$$N_3\left(\frac{y}{l}, \frac{h}{l}\right) = \pi t_3(\phi) - 4\pi\frac{h^2}{s_1^2}t_1(\phi) + N_0\left(\frac{y}{l}, \frac{h}{l}\right)\left(\cos 3\phi - \frac{6h^2}{s_1^2}t_2(\phi)\right) - \frac{h^2}{s_1^2}P_0\left(\frac{y}{l}, \frac{h}{l}\right)\left(3t_3(\phi) - \frac{4h^2}{s_1^2}t_1(\phi)\right), \tag{255}$$

$$N_4\left(\frac{y}{l}, \frac{h}{l}\right) = \pi t_4(\phi) - 12\pi\frac{h^2}{s_1^2}t_2(\phi) + N_0\left(\frac{y}{l}, \frac{h}{l}\right)\left(\cos 4\phi - \frac{h^2}{s_1^2}[12t_3(\phi) + 4t_1(\phi)] + \frac{8h^4}{s_1^4}t_1(\phi)\right) -$$
$$- \frac{h^2}{s_1^2}P_0\left(\frac{y}{l}, \frac{h}{l}\right)\left(4t_4(\phi) - 16\frac{h^2}{s_1^2}t_2(\phi)\right) \tag{256}$$

and

$$N_5\left(\frac{y}{l},\frac{h}{l}\right) = \pi t_5(\phi) - \pi\frac{h^2}{s_1^2}[24t_3(\phi) + 12t_1(\phi)] + 16\pi\frac{h^4}{s_1^4}t_1(\phi)$$
$$+ N_0\left(\frac{y}{l},\frac{h}{l}\right)\left(\cos 5\phi - \frac{h^2}{s_1^2}[20t_4(\phi) + 10t_2(\phi)] + 40\frac{h^4}{s_1^4}t_2(\phi)\right)$$
$$- \frac{h^2}{s_1^2}P_0\left(\frac{y}{l},\frac{h}{l}\right)\left(5t_5(\phi) - \frac{h^2}{s_1^2}[40t_3(\phi) + 20t_1(\phi)] + 16\frac{h^4}{s_1^4}t_1(\phi)\right) \tag{257}$$

where

$$t_r(\phi) = \frac{\sin r\phi}{\sin \phi} \tag{258}$$

and

$$P_0\left(\frac{y}{l},\frac{h}{l}\right) = \int_0^\pi \frac{1}{\chi}\,d\phi_0 = \frac{\pi}{2\sqrt{2}}\frac{s_1}{h}\sqrt{\frac{1 + H^2}{u^4 + H^2}}\sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}\right\}\left\{1 + \sqrt{\frac{u^4 + H^2}{1 + H^2}}\right\}}. \tag{259}$$

6.8. Location of Loading Points

Once the points $\xi_i^{(1)}$, $i = 1, 2, \ldots, n_1$; $\eta_j^{(1)}$, $j = 1, 2, \ldots, m_1$; $\xi_i^{(2)}$, $i = 1, 2, \ldots, n_2$; and $\eta_j^{(2)}$, $j = 1, 2, \ldots, m_2$, have been specified, all the elements of the matrices appearing on the right-hand side of formula (169) or the simplified formula (173) may be evaluated so that the matrix $\hat{Q}$ of the generalised airforce coefficients may be obtained. We specify these points by means of the formulae

$$\xi_i^{(1)} = \frac{1}{2}\left(1 - \cos\frac{(2i-1)\pi}{2n_1 + 1}\right) \qquad i = 1, 2, \ldots, n_1, \tag{260}$$

$$\eta_j^{(1)} = \cos\frac{j\pi}{m_1 + 1} \qquad j = 1, 2, \ldots, m_1, \tag{261}$$

$$\xi_i^{(2)} = \frac{1}{2}\left(1 - \cos\frac{(2i-1)\pi}{2n_2 + 1}\right) \qquad i = 1, 2, \ldots, n_2 \tag{262}$$

and

$$\eta_j^{(2)} = \cos\frac{j\pi}{m_2 + 1} \qquad j = 1, 2, \ldots, m_2. \tag{263}$$

As mentioned earlier, the location of these points is not crucial to the accuracy of the final results for $\hat{Q}_{pq}$, but we note that we have used the locations (261) already in formula (232).

A program has been written in 1900 FORTRAN (*see* Ref. 15) to evaluate the generalised airforce coefficients from formula (173), and taking $q_1 = 1$ in equation (198) and $q_2 = 1$ in equation (204). Results obtained using this program are described in the next section.

7. Examples

Example 1

As a first example we shall consider the wing-tailplane configuration of Laschka and Schmid (*see* Ref. 3). The wing and tailplane are swept-back tapered wings as shown in Fig. 1. The typical length l of the configuration is taken to be the root chord OA of the wing and its value can be taken to be unity. The configuration is immersed in a subsonic flow of free-stream Mach number $M = 0$ and is assumed to be oscillating with a frequency parameter $v = 1{\cdot}0$ in one of four symmetric modes of oscillation defined by

$$\zeta_1^{(1)}(x, y) = 1 \qquad \zeta_1^{(2)}(x, y) = 0, \tag{264}$$

$$\zeta_2^{(1)}(x, y) = 0 \qquad \zeta_2^{(2)}(x, y) = 1, \tag{265}$$

$$\zeta_3^{(1)}(x, y) = x/l \qquad \zeta_3^{(2)}(x, y) = 0 \tag{266}$$

and

$$\zeta_4^{(1)}(x, y) = 0 \qquad \zeta_4^{(2)}(x, y) = \frac{x}{l} - \frac{3}{2} \tag{267}$$

where the origin of coordinates is at the apex of the wing.

The configuration with $h \neq 0$ is obtained from the configuration when the wing and tailplane are coplanar simply by translating the tailplane a distance h in the direction of the z-axis.

We present results for the elements $\hat{Q}_{pq}$ of the matrix $[\hat{Q}]$ of the approximations to the generalised airforce coefficients Q_{pq} (equation 13) for a number of values of h, the vertical separation of the planes of the wing and tailplane, and for a selection of combinations of the integers m_1, n_1, m_2, n_2. We write the quantities $\hat{Q}_{pq}$ in the form

$$\hat{Q}_{pq} = \hat{Q}'_{pq} + iv\hat{Q}''_{pq}. \tag{268}$$

Laschka and Schmid[3] obtained values of $\hat{Q}'_{pq}$ and $\hat{Q}''_{pq}$ for $h = 0$ only, using an integral equation approach. The values of m_1, n_1, m_2, n_2 used for obtaining these values are not quoted in Ref. 3.

Mykytow, Olsen and Pollock have also reported results for the wing-tailplane configuration of Laschka and Schmid with $h = 0$ in Ref. 13. Results have been obtained by the integral equation approach of Albano, Perkinson and Rodden,[4] by the doublet-lattice method of Albano and Rodden[12] and by the later doublet-lattice method of Kálmán, Rodden and Giesing.[5] In the application of the integral equation method there were 15 collocation positions across the total span of the wing and tailplane and there were 3 collocation points at each spanwise position. In the application of both doublet-lattice methods an array of 9 boxes along the semi-span and 5 boxes along the chord of the wing, and 6 boxes along the semi-span and 5 boxes along the chord of the tailplane were used. All the results have been converted to the notation of the present report and are given in Table 1 together with results from the present theory with $m_1 = 10, n_1 = 3, m_2 = 10, n_2 = 3$. The results are seen to be in generally good agreement although there are a few instances of large discrepancies, for example in $\hat{Q}'_{11}$ and $\hat{Q}'_{33}$ where the results from the doublet-lattice methods differ by up to 10 per cent from the results from the present method.

Results from the present theory are given in Table 2 for a number of values of h and for a selection of combinations of the integers m_1, n_1, m_2, n_2 so that convergence of the results with increase of the values of the integers m_1, n_1, m_2, n_2 may be observed. With $n_1 = n_2 = 2$, the values of the $\hat{Q}_{ij}$ seem to be converging as m_1 and m_2 are increased. With $n_1 = n_2 = 3$ only the values $m_1 = m_2 = 10$ of m_1 and m_2 are considered, and in each case the value of $\hat{Q}_{ij}$ is not greatly different from the corresponding value when $m_1 = 10, n_1 = 2, m_2 = 10, n_2 = 2$.

We note that when $h = 0$, airforces on the tailplane caused by wing motion only, e.g. $\hat{Q}_{21}$, are comparable in magnitude with airforces on the wing caused by wing motion only, e.g. $\hat{Q}_{11}$. This is due to the influence of the wake from the wing which, in inviscid linearised theory, is carried downstream with undiminished strength in the plane of the wing. The actual values are $\hat{Q}_{21} = 0{\cdot}6671 + i0{\cdot}1921$ ($m_1 = 10, n_1 = 3, m_2 = 10, n_2 = 3$) and $\hat{Q}_{11} = 0{\cdot}5305 - i1{\cdot}947$ ($m_1 = 10, n_1 = 3, m_2 = 10, n_2 = 3$). As h increases from zero the magnitude of $\hat{Q}_{21}$ decreases, that is the interference effects of the wing on the tailplane decrease, and for $h = 2$ we have $\hat{Q}_{21} = 0{\cdot}03655 + i0{\cdot}01088$ ($m_1 = 14, n_1 = 2, m_2 = 14, n_1 = 2$). The magnitude of $\hat{Q}_{11}$ also decreases as h increases, but only to a limited extent.

We also note that airforces on the wing caused by tailplane motion only, e.g. $\hat{Q}_{12}$, are of small magnitude compared with airforces on the wing caused by wing motion only, e.g. $\hat{Q}_{11}$. At $h = 0$ we have $\hat{Q}_{12} = -0{\cdot}01824 - i0{\cdot}06486$ ($m_1 = 10, n_1 = 3, m_2 = 10, n_1 = 3$) and at $h = 2$ we have $\hat{Q}_{12} = 0{\cdot}002399 + i0{\cdot}009830$ ($m_1 = 14, n_1 = 2, m_2 = 14, n_1 = 2$). The magnitudes of both these values are small compared with the value of $\hat{Q}_{11}$ for $h = 0$, which was given above.

Example 2

As a second example we shall consider the wing-tailplane configuration shown in Fig. 2. This is the configuration which A.G.A.R.D. specified for calculation of generalised airforces and for which some preliminary results appeared in Refs. 5, 13 and 14. The wing and tailplane are again swept-back tapered wings. The semi-spans of the wing and tailplane are both of unit length and the typical length l is taken equal to the semi-span. The origin of coordinates is at the apex of the wing.

The configuration is immersed in a subsonic flow of free-stream Mach number $M = 0{\cdot}8$ and is oscillating with a frequency parameter ν in the range 0 to 1·5 in one of two antisymmetric modes of oscillation, defined by

$$\zeta_1^{(1)}(x, y) = y(x - 2{\cdot}25|y| - 0{\cdot}85), \qquad \zeta_1^{(2)}(x, y) = y, \tag{269}$$

$$\zeta_2^{(1)}(x, y) = y|y| \quad \text{and} \quad \zeta_2^{(2)}(x, y) = (x - 3{\cdot}35)\,\mathrm{sgn}\, y. \tag{270}$$

The configuration with $h \neq 0$ is obtained from the configuration when the wing and tailplane are coplanar simply by translating the tailplane a distance h in the direction of the z-axis.

Mykytow, Olsen and Pollock have reported results for the A.G.A.R.D. wing-tailplane configuration in Ref. 13. Results have been obtained by the integral equation approach of Albano, Perkinson and Rodden[4] when $h = 0$ and $h = 0{\cdot}6$ and by the doublet-lattice method of Albano and Rodden[12] when $h = 0$. In the application of the integral equation method there were 15 collocation positions across the total span of the wing and of the tailplane, and there were 4 collocation points at each spanwise position. In the application of the doublet-lattice method an array of 8 boxes along the semi-span and 6 boxes along the chord of the wing and of 8 boxes along the semi-span and 4 boxes along the chord of the tailplane were used. Four modes of oscillation were used but these could be combined together in pairs to give the modal shapes (269) and (270). The results given in Ref. 13 could then be combined appropriately to give the corresponding generalised airforces. The results are given in Table 3 together with the results from the present theory with $m_1 = 16$, $n_1 = 4$, $m_2 = 16$ and $n_2 = 4$. Also included are the results of the later doublet-lattice method of Rodden, Giesing and Kálmán, which are reported in Ref. 5 in the same form as those given in Ref. 13. Only the case $h = 0$, from this reference, are quoted in Table 3 since for $h = 0{\cdot}6$ the tailplane was moved aft. The results, again, are seen to be in generally good agreement with each other, although in most cases the results from the integral equation approach are nearer to the results from the present theory than are those from either of the two doublet-lattice methods.

Tables 4a, 4b, 4c, 4d give results for the generalised force coefficients for $(\nu, h) = (0, 0), (1{\cdot}5, 0), (0, 0{\cdot}6)$ and $(1{\cdot}5, 0{\cdot}6)$ for a selection of combinations of values of m_1, n_1, m_2 and n_2, so that the nature of convergence of results may be observed when m_1, n_1, m_2 and n_2 are increased. With $n_1 = n_2 = 3$, the values of $\hat{Q}_{ij}$ seem to be converging as m_1 and m_2 are increased. With $n_1 = n_2 = 4$ the values of $\hat{Q}_{ij}$ seem to be converging as m_1 and m_2 are increased but to values that are different, but not greatly different, from the values of $\hat{Q}_{ij}$ when $n_1 = n_2 = 3$. With $n_1 = n_2 = 5$ only the values $m_1 = 12$, $m_2 = 12$ of m_1 and m_2 were considered and in each case the value of $\hat{Q}_{ij}$ was not greatly different from the corresponding value when $m_1 = 12$, $n_1 = 4$, $m_2 = 12$ and $n_2 = 4$.

Table 5 gives results with $m_1 = 16$, $n_1 = 4$, $m_2 = 16$ and $n_2 = 4$, for $\nu = 0$ and $\nu = 1{\cdot}5$ and a number of values of h in the range 0 to 0·6.

Table 6 gives results with $m_1 = 16$, $n_1 = 4$, $m_2 = 16$ and $n_2 = 4$, for $h = 0$ and $h = 0{\cdot}6$ and a number of values of ν in the range 0 to 1·5.

Example 3

As a third and final example we shall consider the wing and tailplane to be identical rectangles of chord $c = 0{\cdot}098$ metres and semi-span $s = 0{\cdot}1515$ metres as shown in Fig. 3. The reason for choosing this example is that experimental work has been carried out at O.N.E.R.A. (*see* Ref. 6) to estimate the values of lift and pitching moment on a configuration of two rectangular wings in tandem oscillating harmonically in heave and in pitch, and so theoretical values could be compared with experimentally determined values.

The leading edge of the tailplane is at a distance $c\lambda$ metres downstream of the trailing edge of the wing and the plane of the tailplane is at a distance CH metres from the plane of the wing. The position of the tailplane relative to that of the wing is then characterised by the non-dimensional separation parameter λ and height parameter H.

Models of a half-wing and half-tailplane were mounted on a wall and immersed in a subsonic flow. Either the wing or the tailplane could be excited in heave or in pitch about the mid-chord line and measurements of total lift or total pitching moment could be made on either surface. The wall acts as a reflecting plate so that the values of lift and moment relevant to a complete wing and tail are obtained by doubling the values obtained experimentally for the half-wing and half-tailplane.

The models may therefore oscillate in one of the following four symmetric modes

$$\zeta_1^{(1)}(x, y) = 1 \qquad \zeta_1^{(2)}(x, y) = 0, \tag{271}$$

$$\zeta_2^{(1)}(x, y) = 0 \qquad \zeta_2^{(2)}(x, y) = 1, \tag{272}$$

$$\zeta_3^{(1)}(x, y) = \frac{x}{c} - \frac{1}{2} \qquad \zeta_3^{(2)}(x, y) = 0 \tag{273}$$

and

$$\zeta_4^{(1)}(x, y) = 0 \qquad \zeta_4^{(2)}(x, y) = \frac{x}{c} - \lambda + \frac{3}{2}, \tag{274}$$

where the origin of coordinates is at the centre of the leading edge of the wing.

Approximate values $\hat{Q}_{ij}$ were evaluated with $m_1 = 6$, $n_1 = 2$, $m_2 = 6$ and $n_2 = 2$, for a number of values of Mach number M and frequency parameter ν. The low values of m_1, n_1, m_2 and n_2 were considered adequate for the high semi-span to chord ratio of wing and tailplane and the low values of ν taken, bearing in mind that the values of $\hat{Q}_{ij}$ were only required for comparison with experimental results.

In the experimental work either the wing or the tailplane was restrained elastically in pitch about the mid-chord line, and it could be excited by applying an external pitching moment which was harmonically time dependent. The other surface was restrained to be at rest, but both the total lifting force and pitching moment induced on it, as a result of the vibration of the first surface, could be measured.

With an airflow of Mach number M relative to the wing and tailplane one of the surfaces was excited in pitch by applying an harmonically time-dependent external pitching moment and the frequency of the harmonic excitation was adjusted, without changing its amplitude, until resonance in the motion of that surface was observed. The aerodynamic pitching moment acting on the excited surface could then be deduced from measurements made of the response of that surface and also the lifting force and pitching moment on the surface not being excited were measured at the resonance frequency. The resonance frequency changed significantly with change of Mach number but not significantly with change of λ or H at a fixed Mach number. In the numerical calculations, therefore, the frequency parameter was taken to be dependent on M only with a value close to that obtaining at resonance in any of the tests at that Mach number. This dependence is given by the following set of associated numbers.

$M = 0{\cdot}30$	$0{\cdot}45$	$0{\cdot}65$	$0{\cdot}80$
$\nu = 0{\cdot}3856$	$0{\cdot}2436$	$0{\cdot}1513$	$0{\cdot}1112$

The particular quantities $\hat{Q}_{ij}$ that could be compared with experimentally determined values from these tests were $\hat{Q}_{33}$, $\hat{Q}_{34}$, $\hat{Q}_{43}$, $\hat{Q}_{44}$, $\hat{Q}_{14}$ and $\hat{Q}_{23}$. The $\hat{Q}_{ij}$ are expressed as in formula (268). Numerical values of $\hat{Q}'_{ij}$ and $\hat{Q}''_{ij}$ for $H = 0$ and $H = \frac{1}{8}$ were obtained at a number of values of λ from $\lambda = 0$ to $\lambda = \infty$ and are recorded in Tables 7 to 18. The values at $\lambda = \infty$ were obtained by applying a computer program for a single isolated surface, or by deducing that the values were zero, as appropriate.

Graphs of these numerical values have been drawn in Figs. 4 to 15 and also on these graphs are shown the experimental values obtained by O.N.E.R.A. The O.N.E.R.A. values were obtained by personal communication to the present author. They were expressed in a different notation but could easily be adapted for our notation simply by multiplying by proportionality factors.

When $H = 0$, $\lambda = 0$ the tailplane leading edge abuts on the wing trailing edge. The analytical conditions at the tailplane leading edge and wing trailing edge assumed in the theory are then not correct and the results obtained may not be good approximations to the actual linearised values. Otherwise the analytical conditions are correct, and we may expect the results to be good approximations to the actual linearised values except when both λ and H are very near to zero, in which cases the values of $n_1 = 2$ and $n_2 = 2$ taken in the calculations would not be high enough.

There is a good deal of qualitative agreement between the numerical and experimental results, as can be seen from Figs. 4 to 15, but these are discrepancies in the magnitudes of the results. The best comparisons are for the direct aerodynamic moment coefficients $\hat{Q}_{33}$ and $\hat{Q}_{44}$, as we might expect because the interaction of one surface on the other is dominated by the action of a surface on itself. There is less interaction of the tailplane on the wing than of the wing on the tailplane and in conformity with this we find that the numerical and experimental values are closer to each other in the case of $\hat{Q}_{33}$ than in the case of $\hat{Q}_{44}$. Furthermore comparison of experimental and theoretical values is no worse for $H = 0$ than it is for $H = \frac{1}{8}$, so that the theoretical model does not appear to be invalidated when the tailplane lies in the wake from the wing.

8. Conclusions

A method has been developed for calculating generalised airforce coefficients on two parallel lifting surfaces using continuous functions as approximations to the loading. Results have been obtained for a number of configurations and comparisons of the results have been made with results obtained by other workers. The comparisons show generally good agreement.

The results were obtained using a computer program in 1900 FORTRAN.[15] This computer program was constructed to use only the lowest value unity of the integration parameters q_1 and q_2. An improvement in accuracy of the results for given values of m_1, n_1, m_2 and n_2 should result if higher values than unity for q_1 and q_2 were taken.

Comparison of O.N.E.R.A. experimental results[6] for two rectangular wings oscillating harmonically in subsonic flow has been made with results obtained using the method of this report. There is qualitative agreement in the behaviour of the results for the different cases considered, but there are discrepancies in the actual magnitudes between the experimental and theoretical values.

LIST OF SYMBOLS

a	Speed of sound in undisturbed main stream
$A^{(1)}_{q;r,s}$	Coefficients appearing in equation (53)
$A^{(2)}_{q;r,s}$	Coefficients appearing in equation (58)
$A^{(1)}_q, A^{(2)}_q$	Column matrices defined below equation (99)
b_p	Generalised coordinate for mode p. *See* equations (1) and (2)
$B^{(1)}, B^{(2)}$	Diagonal matrices defined above equation (166)
$c_1(y)$	Chord of wing at spanwise station y
$c_2(y)$	Chord of tailplane at spanwise station y
$D^{(1)}, D^{(2)}$	Diagonal matrices defined above equation (166)
$E^{(1)}, E^{(2)}$	Diagonal matrices defined below equation (173)
$E^{(1)}_{r,p}(\xi, \eta; \nu, M)$	Expansion coefficient appearing in expansion (191)
$F^{(1)}_{r,p}(\xi, \eta; \nu, M)$	Expansion coefficient appearing in expansion (191)
$F^{(1)}_{r,0}(\xi, \eta; \nu, M)$	Defined by formula (192)
$F^{(2)}_{r,0}(\varepsilon, \zeta; \nu, M)$	Defined by formula (203)
$g^{(1)}_j(\eta_0)$	Interpolation polynomials defined by formula (51)
$g^{(2)}_j(\zeta_0)$	Interpolation polynomials defined by formula (56)
$\tilde{g}^{(m)}_j(\mu)$	Interpolation polynomials defined by formula (118)
$\tilde{G}^{(m)}_j$	Quantities defined by formula (121)
$G^{(1,m)}_{j,J}$	Quantities defined by formula (124)
$G^{(2,m)}_{j,J}$	Quantities defined by formula (125)
h	Vertical separation of wing and tailplane
$h^{(1)}_i(\xi_0)$	Interpolation polynomials defined by formula (49)
$h^{(2)}_i(\varepsilon_0)$	Interpolation polynomials defined by formula (54)
H	Either the quantity defined in formula (252) or the height parameter in Example 3
$\tilde{h}^{(n)}_i(\sigma)$	Interpolation polynomi ls defined by formula (112)
$\tilde{H}^{(n)}_i$	Quantities defined by formula (115)
$H^{(1,n)}_{i,I}$	Quantities defined by formula (122)
$H^{(2,n)}_{i,I}$	Quantities defined by formula (123)
$I^{(1)}_r(\xi, \eta, \eta_0; \nu, M)$	Chordwise integral defined by formula (177)
$I^{(2)}_r(\varepsilon, \zeta, \zeta_0; \nu, M)$	Chordwise integral defined by formula (202)
$\hat{I}_n(\alpha)$	Exponential integral defined by formula (186)
$\bar{I}_n(\alpha)$	Complex conjugate of $\hat{I}_n(\alpha)$
$J^{(2)}_r(\xi, \eta, \zeta_0; \nu, M)$	Chordwise integral defined by formula (206)
$\mathring{J}^{(1)}_r(\varepsilon, \zeta, \eta_0; \nu, M)$	Chordwise integral defined by formula (221)
$K\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}; \nu, M\right)$	Kernel function defined by formula (43)
$\bar{K}\left(\frac{y}{l}, \frac{h}{l}; \nu\right)$	Function defined by formula (216)

$K^*\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}; \nu, M\right)$	Modified kernel function defined by formula (217)
$K_{0,1}(x)$	Function defined by formula (238)
$K_{1,1}(x)$	Function defined by formula (239)
l	Typical length of wing and tailplane
$L_p^{(1)}(x, y; \nu, M)$	*See* equation (3)
$L_p^{(2)}(x, y; \nu, M)$	*See* equation (4)
$l_n(\sigma)$	*See* equation (110)
$L^{(1)}, L^{(2)}$	Rectangular matrices defined below equation (148)
$L_j\left(\frac{y}{l}, \frac{h}{l}\right)$	Function defined by formula (244)
M	Mach number. *See* equation (6)
m_1	Number of spanwise basis functions appearing in equation (53). (Taken to be even in this report)
m_2	Number of spanwise basis functions appearing in equation (58). (Taken to be even in this report)
M_1	Number of spanwise integration points used in the evaluation of $\psi_{i,j;r,s}^{(1,1)}$ and $\psi_{i,j;r,s}^{(1,2)}$. (Taken to be even in this report)
M_2	Number of spanwise integration points used in the evaluation of $\psi_{i,j;r,s}^{(2,2)}$ and $\psi_{i,j;r,s}^{(2,1)}$ (Taken to be even in this report)
$\overline{M}_1$	Number of spanwise integration points used in the evaluation of $\theta_{q;i,j}^{(1)}$
$\overline{M}_2$	Number of spanwise integration points used in the evaluation of $\theta_{q;i,j}^{(2)}$
$\overline{m}_1$	Number of spanwise integration points used in the evaluation of $U_{r,s}^{(1)}(x, y; \nu, M)$ from formula (176). This is related to M_1 by means of formula (198). Also it is the number of spanwise integration points used to evaluate $\overset{*}{V}{}_{r,s}^{(1)}(x, y; \nu, M)$ from formula (220)
$\overline{m}_2$	Number of spanwise integration points used in the evaluation of $U_{r,s}^{(2)}(x, y; \nu, M)$ from formula (62). This is related to M_2 by means of formula (204). Also it is the number of spanwise integration points used to evaluate $V_{r,s}^{(2)}(x, y; \nu, M)$ from formula 205)
$M_s\left(\frac{y}{l}, \frac{h}{l}; \nu\right)$	Function defined by formula (229)
n_1	Number of chordwise basis functions appearing in equation (53)
n_2	Number of chordwise basis functions appearing in equation (58)
N_1	Number of chordwise integration points used to evaluate $\psi_{i,j;r,s}^{(1,1)}$ and $\psi_{i,j;r,s}^{(1,2)}$
N_2	Number of chordwise integration points used in the evaluation of $\psi_{i,j;r,s}^{(2,2)}$ and $\psi_{i,j;r,s}^{(2,1)}$
$\overline{N}_1$	Number of chordwise integration points used to evaluate $\theta_{q;i,j}^{(1)}$
$\overline{N}_2$	Number of chordwise integration points used to evaluate $\theta_{q;i,j}^{(2)}$
$N_j\left(\frac{y}{l}, \frac{h}{l}\right)$	Function defined by formula (243)
P_{pq}	Generalised aerodynamic force, defined in equation (7)
$P_0\left(\frac{y}{l}, \frac{h}{l}\right)$	Function defined by formula (259)

q_1, q_2	Parameters determining number of integration points. *See* equations (198) and (204)
Q_{pq}	Generalised aerodynamic force coefficient. *See* equation (13)
Q'_{pq}, Q''_{pq}	Defined in equation (14)
$\hat{Q}_{pq}$	Approximation to generalised aerodynamic force coefficient. *See* equation (101). Also a matrix consisting of one element in formulae (107) and (108)
$\hat{Q}_{pq}, \hat{Q}''_{pq}$	Defined in equation (268)
$\hat{Q}$	Square matrix defined above equation (109)
R	Defined in formula (44), or (209)
R_1	Defined in formula (181)
s_1	Semi-span of wing
s_2	Semi-span of tailplane
$t_r(\phi)$	Function defined in formula (258)
t	Time
T	Orthogonal projection of tailplane onto the plane $z = 0$
u	Quantity defined in formula (252)
$U^{(1)}_{r,s}(x, y; \nu, M)$	Function defined by formula (61)
$U^{(2)}_{r,s}(x, y; \nu, M)$	Function defined by formula (62)
$U^{(1)}, U^{(2)}$	Rectangular matrices defined below equation (148)
V	Speed of free stream
$V^{(1)}_{r,s}(x, y; \nu, M)$	Function defined by formula (63)
$V^{(2)}_{r,s}(x, y; \nu, M)$	Function defined by formula (64)
$\bar{V}^{(1)}_{r,s}(y; \nu)$	Function defined by formula (219)
$\overset{*}{V}{}^{(1)}_{r,s}(x, y; \nu, M)$	Function defined by formula (220)
$V^{(1)}, V^{(2)}$	Rectangular matrices defined below equation (148)
W	Orthogonal projection of the wing onto the plane $z = 0$
$W^{(1)}_q(x, y)$	Upwash function on the wing in mode q, defined in equation (35)
$W^{(2)}_q(x, y)$	Upwash function on the tailplane in mode q, defined in equation (36)
x, y, z	Cartesian coordinates with respect to the mean wing-tailplane configuration
x_0, y_0, z_0	Cartesian coordinates with respect to the mean wing-tailplane configuration
$x^{(1)}_L(y)$	x-coordinate of the leading edge of W at spanwise station y
$x^{(2)}_L(y)$	x-coordinate of the leading edge of T at spanwise station y
$x^{(m,n)}_{1;r,s}$	Numbers defined by formula (153)
$x^{(m,n)}_{2;r,s}$	Numbers defined by formula (154)
$\bar{x}^{(N_1,M_1)}_{1;I,J}$	Numbers defined by formula (134)
$\bar{x}^{(N_2,M_2)}_{2;I,J}$	Numbers defined by formula (137)
X	Defined in formula (208)
X_1	Defined in formula (182)
$y^{(m)}_{1;s}$	Numbers defined by formula (153)
$y^{(m)}_{2;s}$	Numbers defined by formula (154)

$Z_p^{(1)}(x, y)$	Displacement of the wing in mode p in the direction of the positive z-axis
$Z_p^{(2)}(x, y)$	Displacement of the tailplane in mode p in the direction of the positive z-axis
$\alpha_q^{(1)}(x, y; v)$	Reduced upwash function on the wing in mode q, defined in equation (37)
$\alpha_q^{(2)}(x, y; v)$	Reduced upwash function on the tailplane in mode q, defined in equation (38)
$\hat{\alpha}_q^{(1)}(x, y)$	Approximation to $\alpha_q^{(1)}(x, y; v)$. *See* equation (59)
$\hat{\alpha}_q^{(2)}(x, y)$	Approximation to $\alpha_q^{(2)}(x, y; v)$. *See* equation (60)
$\alpha_{q;i,j}^{(1,e)}$	Numbers defined by formula (149)
$\alpha_{q,i,j}^{(2,e)}$	Numbers defined by formula (150)
$\alpha_q^{(1,e)}$, $\alpha_q^{(2,e)}$	Column matrices defined above equation (166)
$\alpha^{(1,e)}$, $\alpha^{(2,e)}$	Rectangular matrices defined above equation (169)
γ	Euler's constant
$\gamma_m(\mu)$	*See* equation (116)
ε_0	Parametric coordinate on T, defined in formula (46)
ε	Parametric coordinate on T, defined in formula (68)
ζ_0	Parametric coordinate on T, defined in formula (46)
ζ	Parametric coordinate on T, defined in formula (68)
$\zeta_p^{(1)}(x, y)$	Reduced displacement function on the wing in mode p, defined in equation (8)
$\zeta_p^{(2)}(x, y)$	Reduced displacement function on the tailplane in mode p, defined in equation (9)
$\zeta_p^{(1,s)}(x, y)$	Symmetric contribution to $\zeta_p^{(1)}(x, y)$. *See* equation (19)
$\zeta_p^{(1,a)}(x, y)$	Antisymmetric contribution to $\zeta_p^{(1)}(x, y)$. *See* equation (20)
$\zeta_p^{(2,s)}(x, y)$	Symmetric contribution to $\zeta_p^{(2)}(x, y)$. *See* equation (21)
$\zeta_p^{(2,a)}(x, y)$	Antisymmetric contribution to $\zeta_p^{(2)}(x, y)$. *See* equation (22)
$\zeta_{p;r,s}^{(1,e)}$	Numbers defined by formula (151)
$\zeta_{p;r,s}^{(2,e)}$	Numbers defined by formula (152)
$\zeta_p^{(1,e)}$, $\zeta_p^{(2,e)}$	Row matrices defined above equation (166)
$\zeta^{(1,e)}$, $\zeta^{(2,e)}$	Rectangular matrices defined above equation (169)
η_0	Parametric coordinate on W, defined in formula (45)
η	Parametric coordinate on W, defined in formula (67)
$\eta_j^{(1)}$	Set of m_1 distinct loading points in $(-1, 1)$. *See* formula (261)
$\eta_j^{(2)}$	Set of m_2 distinct loading points in $(-1, 1)$. *See* formula (263)
$\theta_{q;i,j}^{(1)}$	Numbers defined by formula (71)
$\theta_{q;i,j}^{(2)}$	Numbers defined by formula (72)
$\Theta_q^{(1)}$, $\Theta_q^{(2)}$	Column matrices defined below equation (99)
$\Theta^{(1)}$, $\Theta^{(2)}$	Rectangular matrices defined above equation (109)
κ	$\kappa = +1$ for symmetric oscillation, $\kappa = -1$ for antisymmetric oscillation
λ	The separation parameter in Example 3
$\lambda_p^{(1)}(x, y; v, M)$	Reduced loading function for the wing in mode p, defined in equation (10)
$\lambda_p^{(2)}(x, y; v, M)$	Reduced loading function for the tailplane in mode p, defined in equation (11)

$\lambda_p^{(1,s)}(x, y; \nu, M)$	Symmetric contribution to $\lambda_p^{(1)}(x, y; \nu, M)$. *See* equation (23)
$\lambda_p^{(1,a)}(x, y; \nu, M)$	Antisymmetric contribution to $\lambda_p^{(1)}(x, y; \nu, M)$. *See* equation (24)
$\lambda_p^{(2,s)}(x, y; \nu, M)$	Symmetric contribution to $\lambda_p^{(2)}(x, y; \nu, M)$. *See* equation (25)
$\lambda_p^{(2,a)}(a, y; \nu, M)$	Antisymmetric contribution to $\lambda_p^{(2)}(x, y; \nu, M)$. *See* equation (26)
$\bar{\lambda}_q^{(1)}(x_0, y_0)$	Approximation to $\lambda_q^{(1)}(x_0, y_0; \nu, M)$. *See* equation (53)
$\bar{\lambda}_q^{(2)}(x_0, y_0)$	Approximation to $\lambda_q^{(2)}(x_0, y_0; \nu, M)$. *See* equation (58)
$\mu_J^{(m)}$	Numbers defined in formula (117)
$\nu = \dfrac{\omega l}{V}$	Frequency parameter
ξ_0	Parametric coordinate on W, defined in formula (45)
ξ	Parametric coordinate on W, defined in formula (67)
$\xi_i^{(1)}$	Set of n_1 distinct loading points in (0, 1). *See* formula (260)
$\xi_i^{(2)}$	Set of n_2 distinct loading points in (0, 1). *See* formula (262)
ρ	Density of the air in the free stream
$\sigma_I^{(n)}$	Numbers defined in formula (111)
$\bar{\sigma}_I^{(n)}$	Numbers defined in formula (131)
χ	Defined in formula (231)
$\chi_{p;r,s}^{(1)}$	Numbers defined by formula (102)
$\chi_{p;r,s}^{(2)}$	Numbers defined by formula (103)
$\chi_p^{(1)}, \chi_p^{(2)}$	Row matrices defined below equation (107)
$\chi^{(1)}, \chi^{(2)}$	Rectangular matrices defined above equation (109)
$\psi_{i,j;r,s}^{(1,1)}$	Numbers defined by formula (73)
$\psi_{i,j;r,s}^{(2,2)}$	Numbers defined by formula (74)
$\psi_{i,j;r,s}^{(1,2)}$	Numbers defined by formula (75)
$\psi_{i,j;r,s}^{(2,1)}$	Numbers defined by formula (76)
$\Psi^{(1,1)}, \Psi^{(1,2)}$	Rectangular matrices defined below equation (99)
$\Psi^{(2,1)}, \Psi^{(2,2)}$	Rectangular matrices defined below equation (99)
ω	Circular frequency of oscillation

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APPENDIX

Evaluation of Some Integrals

In this Appendix we obtain analytical expressions for the integrals (259), and (243) and (244) with $j = 0$,

$$P_0\left(\frac{y}{l}, \frac{h}{l}\right) = \int_0 \frac{1}{\chi} d\phi_0 \tag{A-1}$$

$$N_0\left(\frac{y}{l}, \frac{h}{l}\right) = \int_0^{h} \frac{(\cos\phi_0 - \cos\phi)}{\chi} d\phi_0 \tag{A-2}$$

and

$$L_0\left(\frac{y}{l}, \frac{h}{l}\right) = \int_0^{\pi} \log(\chi)\, d\phi_0 \tag{A-3}$$

where

$$\chi = \frac{(y - y_0)^2 + h^2}{s_1^2} \tag{A-4}$$

$$y_0 = s_1 \cos\phi_0 \tag{A-5}$$

and

$$y = s_1 \cos\phi. \tag{A-6}$$

The analytical expressions to be obtained have been quoted in formulae (259), (250) and (251) of the main text.

If we put

$$u_0 = \tan\frac{\phi_0}{2} \tag{A-7}$$

and

$$u = \tan\frac{\phi}{2} = \sqrt{\frac{1 - y/s_1}{1 + y/s_1}} \tag{A-8}$$

in the integrands of the integrals on the right-hand sides of (A-1), (A-2) and (A-3) we get,

$$\begin{aligned} P_0\left(\frac{y}{l}, \frac{h}{l}\right) &= \frac{1}{2}(1 + u^2)^2 \int_0^{\infty} \frac{(1 + u_0^2)\, du_0}{[(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2]} \\ &= \frac{1}{4}(1 + u^2)^2 \int_{-\infty}^{\infty} \frac{(1 + u_0^2)\, du_0}{[(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2]}, \end{aligned} \tag{A-9}$$

$$\begin{aligned} N_0\left(\frac{y}{l}, \frac{h}{l}\right) &= (1 + u^2) \int_0^{\infty} \frac{(u^2 - u_0^2)\, du_0}{[(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2]} \\ &= \frac{1}{2}(1 + u^2) \int_{-\infty}^{\infty} \frac{(u^2 - u_0^2)\, du_0}{[(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2]} \end{aligned} \tag{A-10}$$

and

$$\begin{aligned} L_0\left(\frac{y}{l}, \frac{h}{l}\right) &= 2\int_0^{\infty} \log\left[\frac{4\{(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2\}}{(1 + u_0^2)^2(1 + u^2)^2}\right] \frac{du_0}{1 + u_0^2} \\ &= \int_{-\infty}^{\infty} \log\left[\frac{(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2}{1 + H^2}\right] \frac{du_0}{1 + u_0^2} - 2\int_{-\infty}^{\infty} \log(1 + u_0^2) \frac{du_0}{1 + u_0^2} + \\ &\quad + \left\{\log(1 + H^2) - 2\log\left(\frac{1 + u^2}{2}\right)\right\} \int_{-\infty}^{\infty} \frac{du_0}{1 + u_0^2} \end{aligned} \tag{A-11}$$

where

$$H = \frac{h/s_1}{1 + y/s_1}. \tag{A-12}$$

Now, we can write

$$(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2 = (1 + H^2)[(u_0 - \alpha)^2 + \beta^2][(u_0 + \alpha)^2 + \beta^2] \tag{A-13}$$

where

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}\right\}} \tag{A-14}$$

and

$$\beta = \frac{1}{\sqrt{2}}\sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} - \frac{(u^2 - H^2)}{1 + H^2}\right\}}. \tag{A-15}$$

Then we get

$$\begin{aligned}
P_0\left(\frac{y}{l}, \frac{h}{l}\right) &= \frac{(1 + u^2)^2(\alpha^2 + \beta^2 + 1)}{16(\alpha^2 + \beta^2)(1 + H^2)} \int_{-\infty}^{+\infty} \left\{\frac{1}{[(u_0 - \alpha)^2 + \beta^2]} + \frac{1}{[(u_0 + \alpha)^2 + \beta^2]}\right\} du_0 + \\
&\quad + \frac{(1 + u^2)^2(\alpha^2 + \beta^2 - 1)}{16\alpha(\alpha^2 + \beta^2)(1 + H^2)} \int_{-\infty}^{\infty} \left\{\frac{(u_0 - \alpha)}{[(u_0 - \alpha)^2 + \beta^2]} - \frac{(u_0 + \alpha)}{[(u_0 + \alpha)^2 + \beta^2]}\right\} du_0 \\
&= \frac{(1 + u^2)^2(\alpha + \beta^2 + 1)}{16\beta(\alpha^2 + \beta^2)(1 + H^2)}\left[\tan^{-1}\left(\frac{u_0 - \alpha}{\beta}\right) + \tan^{-1}\left(\frac{u_0 + \alpha}{\beta}\right)\right]_{-\infty}^{\infty} + \\
&\quad + \frac{(1 + u^2)^2(\alpha^2 + \beta^2 - 1)}{32\alpha(\alpha^2 + \beta^2)(1 + H^2)}\left[\log\left\{\frac{(u_0 - \alpha)^2 + \beta^2}{(u_0 + \alpha)^2 + \beta^2}\right\}\right]_{-\infty}^{+\infty} \\
&= \frac{\pi(1 + u^2)^2(\alpha^2 + \beta^2 + 1)}{8\beta(\alpha^2 + \beta^2)(1 + H^2)} \\
&= \frac{\pi}{4\sqrt{2}} \frac{\sqrt{1 + H^2}}{H} \frac{(1 + u^2)}{\sqrt{u^4 + H^2}} \sqrt{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}}\left\{1 + \sqrt{\frac{u^4 + H^6}{1 + H^2}}\right\} \\
&= \frac{\pi}{2\sqrt{2}} \frac{s_1}{h} \frac{\sqrt{1 + H^2}}{\sqrt{u^4 + H^2}} \sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}\right\}}\left\{1 + \quad \frac{u^4 + H^2}{1 + H^2}\right\}
\end{aligned} \tag{A-16}$$

and

$$\begin{aligned}
N_0\left(\frac{y}{l}, \frac{h}{l}\right) &= \frac{(1 + u^2)(u^2 - \alpha^2 - \beta^2)}{8(\alpha^2 + \beta^2)(1 + H^2)} \int_{-\infty}^{\infty} \left\{\frac{1}{[(u_0 - \alpha)^2 + \beta^2]} + \frac{1}{[(u_0 + \alpha)^2 + \beta^2]}\right\} du_0 - \\
&\quad - \frac{(1 + u^2)(u^2 + \alpha^2 + \beta^2)}{8\alpha(\alpha^2 + \beta^2)(1 + H^2)} \int_{-\infty}^{\infty} \left\{\frac{(u_0 - \alpha)}{[(u_0 - \alpha)^2 + \beta^2]} - \frac{(u_0 + \alpha)}{[(u_0 + \alpha)^2 + \beta^2]}\right\} du_0 \\
&= \frac{\pi(1 + u^2)(u^2 - \alpha^2 - \beta^2)}{4\beta(\alpha^2 + \beta^2)(1 + H^2)} \\
&= \frac{\pi}{2\sqrt{2}} \frac{H}{\sqrt{1 + H^2}} \frac{(u^4 - 1)}{\sqrt{u^4 + H^2}} \frac{\sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}\right\}}}{\left\{u^2 + \sqrt{\frac{u^2 + H^2}{1 + H^2}}\right\}}
\end{aligned} \tag{A-17}$$

In order to obtain an analytical expression for $L_0(y/l, h/l)$ using equation (A-11), we must obtain an analytical expression for the integral I, where

$$\begin{aligned}
I &= \int_{-\infty}^{\infty} \log\left[\frac{(u_0^2 - u^2)^2 + H^2(1 + u_0^2)^2}{1 + H^2}\right]\frac{du_0}{1 + u_0^2} \\
&= \int_{-\infty}^{\infty} \log\{[(u_0 - \alpha)^2 + \beta^2][(u_0 + \alpha)^2 + \beta^2]\}\frac{du_0}{1 + u_0^2}.
\end{aligned} \tag{A-18}$$

We do this by resorting to the calculus of residues in the complex u_0 plane.
Let us put

$$u_0 - \alpha - i\beta = r_1 e^{i\theta_1} \qquad -\pi \leqslant \theta_1 < \pi, \tag{A-19}$$

$$u_0 + \alpha - i\beta = r_2 e^{i\theta_2} \qquad -\pi \leqslant \theta_2 < \pi, \tag{A-20}$$

$$u_0 - \alpha + i\beta = r_3 e^{i\theta_3} \qquad -\pi \leqslant \theta_3 < \pi \tag{A-21}$$

and

$$u_0 + \alpha + i\beta = r_4 e^{i\theta_4} \qquad -\pi \leqslant \theta_4 < \pi \tag{A-22}$$

where $r_1, r_2, r_3, r_4, \theta_1, \theta_2, \theta_3, \theta_4$ are real numbers. Let us, further, take

$$\log(u_0 - \alpha - i\beta) = \log r_1 + i\theta_1, \tag{A-23}$$

$$\log(u_0 + \alpha - i\beta) = \log r_2 + i\theta_2, \tag{A-24}$$

$$\log(u_0 - \alpha + i\beta) = \log r_3 + i\theta_3 \tag{A-25}$$

and

$$\log(u_0 + \alpha + i\beta) = \log r_4 + i\theta_4. \tag{A-26}$$

Then

$$\begin{aligned}
&\log\{[(u_0 - \alpha)^2 + \beta^2][(u_0 + \alpha)^2 + \beta^2]\} \\
&\quad = \log\{(u_0 - \alpha - i\beta)(u_0 + \alpha - i\beta)(u_0 - \alpha + i\beta)(u_0 + \alpha + i\beta)\} \\
&\quad = \log\{(u_0 - \alpha - i\beta)(u_0 + \alpha - i\beta)\} + \log\{(u_0 - \alpha + i\beta)(u_0 + \alpha + i\beta)\}.
\end{aligned} \tag{A-27}$$

By taking the contour integral of the function

$$\frac{1}{1 + u_0^2}\log\{(u_0 - \alpha - i\beta)(u_0 + \alpha - i\beta)\}$$

around a contour consisting of the real axis and infinite semicircle, centre origin, in the lower half-plane $\mathrm{Im}(u_0) < 0$, and noting that the only singularity of the function within the contour is a simple pole at $u_0 = -i$, we get, on applying the calculus of residues

$$\int_{-\infty}^{\infty} \log\{(u_0 - \alpha - i\beta)(u_0 + \alpha - i\beta)\}\frac{du_0}{1 + u_0^2} = \pi \log[\alpha^2 + (1 + \beta)^2] - i\pi^2. \tag{A-28}$$

By taking the contour integral of the function

$$\frac{1}{1 + u_0^2}\log\{(u_0 - \alpha + i\beta)(u_0 + \alpha + i\beta)\}$$

around a contour consisting of the real axis and infinite semicircle, centre origin, in the upper half-plane $\mathrm{Im}(u_0) > 0$, and noting that the only singularity of the function within the contour is a simple pole at $u = +i$, we get, on applying the calculus of residues

$$\int_{-\infty}^{\infty} \log\{(u_0 - \alpha + i\beta)(u_0 + \alpha + i\beta)\}\frac{du_0}{1 + u_0^2} = \pi \log[\alpha^2 + (1 + \beta)^2] + i\pi^2. \tag{A-29}$$

Therefore, on noting equation (A-27), we get from equations (A-18), (A-28) and (A-29),

$$I = 2\pi \log[\alpha^2 + (1 + \beta)^2]. \tag{A-30}$$

Further, we have

$$\begin{aligned}
\int_{-\infty}^{\infty} \log(1 + u_0^2)\frac{du_0}{1 + u_0^2} &= \int_{-\pi/2}^{\pi/2} \log(\sec^2\psi_0)\, d\psi_0 \\
&= -4\int_0^{\pi/2} \log(\cos\psi_0)\, d\psi_0 \\
&= -2\int_0^{\pi/2} \log(\cos\psi_0 \sin\psi_0)\, d\psi_0
\end{aligned}$$

$$= \pi \log 2 - 2\int_0^{\pi/2} \log(\sin 2\psi_0)\, d\psi_0$$

$$= \pi \log 2 - \int_0^{\pi} \log(\sin \psi_0)\, d\psi_0$$

$$= \pi \log 2 - 2\int_0^{\pi/2} \log(\sin \psi_0)\, d\psi_0$$

$$= \pi \log 2 + \frac{1}{2}\int_{-\infty}^{\infty} \log(1 + u_0^2)\frac{du_0}{1 + u_0^2}, \qquad \text{(A-31)}$$

and therefore

$$\int_{-\infty}^{\infty} \log(1 + u_0^2)\frac{du_0}{1 + u_0^2} = 2\pi \log 2. \qquad \text{(A-32)}$$

Also

$$\int_{-\infty}^{\infty} \frac{du_0}{1 + u_0^2} = \pi. \qquad \text{(A-33)}$$

If we substitute from equations (A-30), (A-32) and (A-33) into equation (A-11) we get

$$L_0\left(\frac{y}{l}, \frac{h}{l}\right) = 2\pi \log\left\{\frac{[\alpha^2 + (1 + \beta)^2]\sqrt{1 + H^2}}{2(1 + u^2)}\right\}$$

$$= 2\pi \log\left[\frac{\sqrt{1 + H^2}}{2(1 + u^2)}\left(\sqrt{\frac{u^4 + H^2}{1 + H^2}} + 1 + \frac{\sqrt{2}H(1 + u^2)}{(1 + H^2)}\,\frac{1}{\sqrt{\left\{\sqrt{\frac{u^4 + H^2}{1 + H^2}} + \frac{u^2 - H^2}{1 + H^2}\right\}}}\right)\right]. \qquad \text{(A-34)}$$

The requisite analytical expressions have now all been obtained and are given in formulae (A-16), (A-17) and (A-34).

TABLE 1

Values of Generalised Airforce Coefficients for the Wing Tailplane Configuration of Laschka and Schmid for $h = 0$, $\nu = 1{\cdot}0$, $M = 0$ as Obtained by Different Workers

	Laschka Schmid	Albano Perkinson Rodden	Davies	Albano Rodden	Giesing Kálmán Rodden
$\hat{Q}'_{11}$	0·515	0·516	0·531	0·498	0·478
$\hat{Q}'_{12}$	−0·012	−0·018	−0·018	−0·016	−0·016
$\hat{Q}'_{13}$	−1·559	−1·561	−1·568	−1·591	−1·587
$\hat{Q}'_{14}$	−0·077	−0·076	−0·078	−0·080	−0·080
$\hat{Q}''_{11}$	−1·929	−1·933	−1·947	−1·966	−1·952
$\hat{Q}''_{12}$	−0·066	−0·064	−0·065	−0·068	−0·068
$\hat{Q}''_{13}$	−2·508	−2·512	−2·544	−2·547	−2·519
$\hat{Q}''_{14}$	−0·024	−0·019	−0·019	−0·025	−0·025
$\hat{Q}'_{21}$	0·659	0·655	0·667	0·668	0·699
$\hat{Q}'_{22}$	0·140	0·140	0·144	0·131	0·131
$\hat{Q}'_{23}$	0·785	0·789	0·813	0·808	0·813
$\hat{Q}'_{24}$	−0·825	−0·828	−0·832	−0·864	−0·864
$\hat{Q}''_{21}$	0·175	0·183	0·192	0·184	0·189
$\hat{Q}''_{22}$	−0·889	−0·892	−0·897	−0·925	0·926
$\hat{Q}''_{23}$	−0·525	−0·512	−0·516	−0·519	−0·518
$\hat{Q}''_{24}$	−0·752	−0·753	−0·762	−0·775	0·775
$\hat{Q}'_{31}$	0·437	0·436	0·448	0·412	0·404
$\hat{Q}'_{32}$	−0·008	−0·011	−0·011	−0·010	−0·010
$\hat{Q}'_{33}$	−0·845	−0·847	−0·843	−0·911	−0·908
$\hat{Q}'_{34}$	−0·059	−0·058	−0·059	−0·061	−0·061
$\hat{Q}''_{31}$	−1·204	−1·206	−1·207	−1·259	−1·255
$\hat{Q}''_{32}$	−0·052	−0·051	−0·051	−0·053	−0·053
$\hat{Q}''_{33}$	−1·879	−1·882	−1·898	−1·913	−1·901
$\hat{Q}''_{34}$	−0·022	−0·018	−0·018	−0·022	−0·022
$\hat{Q}'_{41}$	0·278	0·276	0·280	0·292	0·292
$\hat{Q}'_{42}$	0·079	0·079	0·081	0·073	0·074
$\hat{Q}'_{43}$	0·334	0·336	0·341	0·353	0·355
$\hat{Q}'_{44}$	−0·329	−0·330	−0·330	−0·359	−0·359
$\hat{Q}''_{41}$	0·069	0·073	0·075	0·074	0·076
$\hat{Q}''_{42}$	−0·338	−0·373	−0·373	−0·399	−0·399
$\hat{Q}''_{43}$	−0·219	−0·214	−0·217	−0·227	−0·227
$\hat{Q}''_{44}$	−0·378	−0·375	−0·378	−0·360	−0·381

TABLE 2

Values of Generalised Airforce Coefficients for the Wing-Tailplane Configuration of Laschka and Schmid $\nu = 1{\cdot}0$, $M = 0$

	m_1	n_1	m_2	n_2	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{13}$	$\hat{Q}'_{14}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{13}$	$\hat{Q}''_{14}$
$h = 0$	6	2	4	2	0·5391	−0·01815	−1·573	−0·07757	−1·962	−0·06422	−2·573	−0·02007
	6	2	6	2	0·5398	−0·01787	−1·572	−0·07857	−1·961	−0·06531	−2·573	−0·02045
	8	2	6	2	0·5276	−0·01776	−1·566	−0·07803	−1·947	−0·06485	−2·548	−0·02029
	8	2	8	2	0·5272	−0·01767	−1·566	−0·07747	−1·947	−0·06438	−2·548	−0·02001
	10	2	8	2	0·5156	−0·01757	−1·560	−0·07695	−1·934	−0·06394	−2·524	−0·01985
	10	2	10	2	0·5152	−0·01751	−1·561	−0·07629	−1·934	−0·06337	−2·524	−0·01960
	12	2	10	2	0·5076	−0·01742	−1·556	−0·07589	−1·925	−0·06304	−2·507	−0·01949
	12	2	12	2	0·5073	−0·01736	−1·556	−0·07545	−1·925	−0·06267	−2·507	−0·01934
	14	2	12	2	0·5032	−0·01729	−1·552	−0·07518	−1·920	−0·06245	−2·496	−0·01929
	14	2	14	2	0·5031	−0·01726	−1·552	−0·07497	−1·920	−0·06229	−2·496	−0·01921
	16	2	14	2	0·5012	−0·01721	−1·550	−0·07479	−1·917	−0·06214	−2·491	−0·01918
	16	2	16	2	0·5012	−0·01719	−1·550	−0·07473	−1·917	−0·06211	−2·490	−0·01915
	10	3	10	3	0·5305	−0·01824	−1·568	−0·07789	−1·947	−0·06486	−2·544	−0·01923
$h = 0{\cdot}25$	10	2	10	2	0·4920	−0·01625	−1·577	−0·06074	−1·932	−0·04901	−2·496	−0·01313
	12	2	12	2	0·4845	−0·01612	−1·572	−0·06006	−1·923	−0·04846	−2·479	−0·01295
	14	2	14	2	0·4805	−0·01603	−1·568	−0·05966	−1·917	−0·04814	−2·469	−0·01285
	16	2	16	2	0·4787	−0·01598	−1·566	−0·05946	−1·914	−0·04799	−2·463	−0·01280
	10	3	10	3	0·5064	−0·01667	−1·585	−0·06168	−1·944	−0·04988	−2·515	−0·01290
$h = 0{\cdot}5$	10	2	10	2	0·4746	−0·01249	−1·594	−0·03237	−1·932	−0·02362	−2·479	−0·002686
	12	2	12	2	0·4673	−0·01241	−1·588	−0·03201	−1·923	−0·02333	−2·462	−0·002605
	14	2	14	2	0·4635	−0·01235	−1·584	−0·03179	−1·918	−0·02317	−2·452	−0·002562
	10	3	10	3	0·4885	−0·01247	−1·602	−0·03255	−1·945	−0·02381	−2·497	−0·002723
$h = 1{\cdot}0$	10	2	10	2	0·4649	−0·004469	−1·603	0·002400	−1·931	0·005385	−2·468	0·006994
	12	2	12	2	0·4578	−0·004457	−1·597	0·002355	−1·922	0·005333	−2·452	0·006955
	14	2	14	2	0·4541	−0·004449	−1·593	0·002332	−1·917	0·005305	−2·442	0·006933
	10	3	10	3	0·4788	−0·004332	−1·611	0·002569	−1·944	0·005507	−2·486	0·006855
$h = 2{\cdot}0$	10	2	10	2	0·4651	0·002426	−1·601	0·01173	−1·930	0·009999	−2·467	0·003800
	12	2	12	2	0·4580	0·002411	−1·595	0·01161	−1·921	0·009892	−2·451	0·003753
	14	2	14	2	0·4543	0·002399	−1·591	0·01154	−1·915	0·009830	−2·441	0·003728

TABLE 2 (continued)

	m_1	n_1	m_2	n_2	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}'_{23}$	$\hat{Q}'_{24}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$	$\hat{Q}''_{23}$	$\hat{Q}''_{24}$
$h = 0$	6	2	4	2	0·6813	0·1436	0·8336	−0·8448	0·1971	−0·9106	−0·5275	−0·7700
	6	2	6	2	0·6768	0·1476	0·8293	−0·8381	0·1965	−0·9047	−0·5243	−0·7725
	8	2	6	2	0·6730	0·1475	0·8269	−0·8381	0·1968	−0·9047	−0·5192	−0·7724
	8	2	8	2	0·6701	0·1439	0·8201	−0·8337	0·1927	−0·8988	−0·5200	−0·7658
	10	2	8	2	0·6667	0·1438	0·8160	−0·8337	0·1917	−0·8988	−0·5167	−0·7657
	10	2	10	2	0·6664	0·1404	0·8101	−0·8297	0·1884	−0·8936	−0·5169	−0·7591
	12	2	10	2	0·6612	0·1403	0·8052	−0·8298	0·1867	−0·8936	−0·5151	−0·7590
	12	2	12	2	0·6592	0·1380	0·8013	−0·8267	0·1847	−0·8899	−0·5147	−0·7542
	14	2	12	2	0·6575	0·1379	0·7975	−0·8268	0·1833	−0·8899	−0·5139	−0·7541
	14	2	14	2	0·6562	0·1369	0·7954	−0·8249	0·1824	−0·8877	−0·5133	−0·7512
	16	2	14	2	0·6554	0·1367	0·7930	−0·8249	0·1814	−0·8877	−0·5132	−0·7512
	16	2	16	2	0·6547	0·1361	0·7922	−0·8238	0·1812	−0·8865	−0·5125	−0·7495
	10	3	10	3	0·6671	0·1444	0·8131	−0·8319	0·1921	−0·8973	−0·5160	−0·7621
$h = 0{\cdot}25$	10	2	10	2	0·4350	0·1308	0·5730	−0·8332	0·1523	−0·8917	−0·2918	−0·7479
	12	2	12	2	0·4318	0·1286	0·5672	−0·8301	0·1499	−0·8880	−0·2904	−0·7432
	14	2	14	2	0·4299	0·1274	0·5635	−0·8282	0·1484	−0·8858	−0·2896	−0·7403
	16	2	16	2	0·4289	0·1268	0·5614	−0·8271	0·1476	−0·8847	−0·2891	−0·7386
	10	3	10	3	0·4370	0·1344	0·5746	−0·8355	0·1545	−0·8955	−0·2915	−0·7506
$h = 0{\cdot}5$	10	2	10	2	0·2860	0·1233	0·3906	−0·8373	0·1092	−0·8915	−0·1769	−0·7399
	12	2	12	2	0·2840	0·1212	0·3869	−0·8341	0·1076	−0·8878	−0·1761	−0·7353
	14	2	14	2	0·2827	0·1200	0·3845	−0·8322	0·1066	−0·8855	−0·1756	−0·7325
	10	3	10	3	0·2874	0·1267	0·3917	−0·8397	0·1106	−0·8953	−0·1768	−0·7425
$h = 1{\cdot}0$	10	2	10	2	0·1325	0·1190	0·1849	−0·8393	0·05191	−0·8908	−0·07889	−0·7351
	12	2	12	2	0·1315	0·1170	0·1832	−0·8361	0·05115	−0·8871	−0·07854	−0·7306
	14	2	14	2	0·1310	0·1159	0·1821	−0·8341	0·05071	−0·8849	−0·07832	−0·7278
	10	3	10	3	0·1332	0·1224	0·1856	−0·8417	0·05265	−0·8946	−0·07881	−0·7377
$h = 2{\cdot}0$	10	2	10	2	0·03700	0·1191	0·04905	−0·8388	0·01119	−0·8903	−0·02565	−0·7349
	12	2	12	2	0·03672	0·1171	0·04857	−0·8356	0·01099	−0·8866	−0·02553	−0·7304
	14	2	14	2	0·03655	0·1160	0·04827	−0·8336	0·01088	−0·8844	−0·02544	−0·7276

TABLE 2 (continued)

	m_1	n_1	m_2	n_2	$\hat{Q}'_{31}$	$\hat{Q}'_{32}$	$\hat{Q}'_{33}$	$\hat{Q}'_{34}$	$\hat{Q}''_{31}$	$\hat{Q}''_{32}$	$\hat{Q}''_{33}$	$\hat{Q}''_{34}$
$h = 0$	6	2	4	2	0·4544	−0·01089	−0·8502	−0·05797	−1·221	−0·04959	−1·911	−0·01869
	6	2	6	2	0·4548	−0·01065	−0·8494	−0·05865	−1·220	−0·05035	−1·911	−0·01898
	8	2	6	2	0·4422	−0·01081	−0·8600	−0·05853	−1·220	−0·05014	−1·899	−0·01868
	8	2	8	2	0·4419	−0·01077	−0·8603	−0·05808	−1·220	−0·04975	−1·899	−0·01844
	10	2	8	2	0·4308	−0·01086	−0·8680	−0·05788	−1·220	−0·04949	−1·885	−0·01819
	10	2	10	2	0·4305	−0·01083	−0·8684	−0·05737	−1·220	−0·04905	−1·885	−0·01797
	12	2	10	2	0·4231	−0·01086	−0·8717	−0·05716	−1·219	−0·04883	−1·873	−0·01781
	12	2	12	2	0·4229	−0·01083	−0·8720	−0·05683	−1·219	−0·04854	−1·873	−0·01768
	14	2	12	2	0·4186	−0·01082	−0·8725	−0·05664	−1·217	−0·04837	−1·865	−0·01759
	14	2	14	2	0·4185	−0·01080	−0·8726	−0·05648	−1·217	−0·04824	−1·865	−0·01753
	16	2	14	2	0·4162	−0·01078	−0·8719	−0·05634	−1·215	−0·04812	−1·860	−0·01748
	16	2	16	2	0·4162	−0·01077	−0·8718	−0·05629	−1·215	−0·04809	−1·860	−0·01746
	10	3	10	3	0·4480	−0·01132	−0·8426	−0·05912	−1·207	−0·05073	−1·898	−0·01804
$h = 0·25$	10	2	10	2	0·4123	−0·01028	−0·8825	−0·04503	−1·219	−0·03742	−1·864	−0·01224
	12	2	12	2	0·4050	−0·01027	−0·8857	−0·04460	−1·217	−0·03702	−1·853	−0·01212
	14	2	14	2	0·4008	−0·01024	−0·8861	−0·04432	−1·215	−0·03678	−1·845	−0·01190
	16	2	16	2	0·3985	−0·01021	−0·8852	−0·04416	−1·213	−0·03665	−1·839	−0·01185
	10	3	10	3	0·4289	−0·01048	−0·8574	−0·04590	−1·206	−0·03828	−1·876	−0·01229
$h = 0·5$	10	2	10	2	0·3989	−0·008112	−0·8962	−0·02284	−1·220	−0·01712	−1·851	−0·002984
	12	2	12	2	0·3917	−0·008109	−0·8991	−0·02264	−1·218	−0·01693	−1·840	−0·002882
	14	2	14	2	0·3876	−0·008094	−0·8994	−0·02250	−1·216	−0·01681	−1·832	−0·002825
	10	3	10	3	0·4149	−0·007933	−0·8716	−0·02271	−1·207	−0·01709	−1·863	−0·003110
$h = 1·0$	10	2	10	2	0·3918	−0·002982	−0·9027	0·003005	−1·220	0·004949	−1·844	0·005435
	12	2	12	2	0·3847	−0·002979	−0·9056	0·002953	−1·218	0·004900	−1·833	0·005407
	14	2	14	2	0·3807	−0·002978	−0·9058	0·002927	−1·216	0·004873	−1·825	0·005391
	10	3	10	3	0·4078	−0·002775	−0·8782	0·003370	−1·207	0·005208	−1·856	0·005316
$h = 2·0$	10	2	10	2	0·3921	0·001611	−0·9016	0·008609	−1·219	0·007430	−1·843	0·003038
	12	2	12	2	0·3850	0·001614	−0·9045	0·008535	−1·217	0·007360	−1·832	0·002994
	14	2	14	2	0·3810	0·001612	−0·9047	0·008486	−1·215	0·007316	−1·824	0·002969

TABLE 2 (concluded)

	m_1	n_1	m_2	n_2	$\hat{Q}'_{41}$	$\hat{Q}'_{42}$	$\hat{Q}'_{43}$	$\hat{Q}'_{44}$	$\hat{Q}''_{41}$	$\hat{Q}''_{42}$	$\hat{Q}''_{43}$	$\hat{Q}''_{44}$
$h = 0$	6	2	4	2	0·2885	0·08347	0·3527	−0·3375	0·07883	−0·3819	−0·2239	−0·3823
	6	2	6	2	0·2854	0·08384	0·3478	−0·3328	0·07629	−0·3769	−0·2229	−0·3827
	8	2	6	2	0·2831	0·08390	0·3480	−0·3327	0·07735	−0·3768	−0·2187	−0·3828
	8	2	8	2	0·2841	0·08131	0·3472	−0·3343	0·07547	−0·3770	−0·2214	−0·3806
	10	2	8	2	0·2825	0·08130	0·3465	−0·3343	0·07568	−0·3770	−0·2190	−0·3806
	10	2	10	2	0·2832	0·07904	0·3459	−0·3355	0·07438	−0·3771	−0·2209	−0·3780
	12	2	10	2	0·2820	0·07900	0·3442	−0·3355	0·07395	−0·3771	−0·2197	−0·3779
	12	2	12	2	0·2821	0·07752	0·3435	−0·3359	0·07316	−0·3768	−0·2206	−0·3759
	14	2	12	2	0·2814	0·07748	0·3422	−0·3359	0·07272	−0·3768	−0·2200	−0·3758
	14	2	14	2	0·2812	0·07664	0·3416	−0·3357	0·07230	−0·3764	−0·2202	−0·3743
	16	2	14	2	0·2809	0·07661	0·3408	−0·3357	0·07203	−0·3764	−0·2200	−0·3743
	16	2	16	2	0·2806	0·07617	0·3403	−0·3354	0·07185	−0·3759	−0·2198	−0·3733
	10	3	10	3	0·2801	0·08144	0·3414	−0·3295	0·07445	−0·3725	−0·2174	−0·3776
$h = 0{\cdot}25$	10	2	10	2	0·1852	0·07464	0·2379	−0·3373	0·05666	−0·3765	−0·1304	−0·3730
	12	2	12	2	0·1845	0·07316	0·2364	−0·3377	0·05580	−0·3762	−0·1303	−0·3709
	14	2	14	2	0·1840	0·07231	0·2352	−0·3375	0·05524	−0·3757	−0·1301	−0·3694
	16	2	16	2	0·1836	0·07186	0·2343	−0·3371	0·05489	−0·3752	−0·1300	−0·3684
	10	3	10	3	0·1832	0·07695	0·2347	−0·3313	0·05649	−0·3719	−0·1284	−0·3726
$h = 0{\cdot}5$	10	2	10	2	0·1214	0·07141	0·1599	−0·3390	0·03960	−0·3763	−0·08115	−0·3695
	12	2	12	2	0·1210	0·06997	0·1589	−0·3393	0·03903	−0·3760	−0·08116	−0·3674
	14	2	14	2	0·1207	0·06914	0·1581	−0·3391	0·03866	−0·3755	−0·08108	−0·3660
	10	3	10	3	0·1201	0·07370	0·1577	−0·3330	0·03942	−0·3717	−0·07992	−0·3691
$h = 1{\cdot}0$	10	2	10	2	0·05618	0·06966	0·07542	−0·3397	0·01888	−0·3759	−0·03641	−0·3675
	12	2	12	2	0·05600	0·06823	0·07496	−0·3400	0·01861	−0·3756	−0·03643	−0·3654
	14	2	14	2	0·05584	0·06742	0·07460	−0·3398	0·01843	−0·3751	−0·03641	−0·3640
	10	3	10	3	0·05554	0·07196	0·07444	−0·3337	0·01883	−0·3713	−0·03580	−0·3671
$h = 2{\cdot}0$	10	2	10	2	0·01562	0·06971	0·02021	−0·3395	0·004205	−0·3757	−0·01131	−0·3674
	12	2	12	2	0·01557	0·06868	0·02008	−0·3398	0·004125	−0·3754	−0·01131	−0·3653
	14	2	14	2	0·01552	0·06747	0·01997	−0·3396	0·004075	−0·3749	−0·01130	−0·3639

TABLE 3

Values of Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration as Obtained by Different Workers

	Davies	Albano Perkinson Rodden	Albano Rodden	Giesing Kálmán Rodden
		$\nu = 0 \quad h = 0$		
$\hat{Q}'_{11}$	0·4403	0·4425	0·4554	0·4401
$\hat{Q}'_{12}$	−0·6202	−0·6121	−0·6655	−0·6557
$\hat{Q}'_{21}$	−0·1046	−0·1054	−0·1107	−0·1044
$\hat{Q}'_{22}$	−0·1759	−0·1954	−0·2237	−0·2126
$\hat{Q}''_{11}$	−0·5425	−0·5420	−0·6052	−0·5875
$\hat{Q}''_{12}$	−0·4611	−0·4476	−0·4557	−0·4735
$\hat{Q}''_{21}$	−0·5127	−0·5166	−0·5784	−0·5553
$\hat{Q}''_{22}$	−0·6649	−0·6398	−0·6538	−0·6758
		$\nu = 1{\cdot}5 \quad h = 0$		
$\hat{Q}'_{11}$	1·106	−1·1208	1·0215	1·0231
$\hat{Q}'_{12}$	−0·07195	−0·0568	−0·0436	−0·0757
$\hat{Q}'_{21}$	0·3716	0·3688	0·3122	0·3337
$\hat{Q}'_{22}$	0·5367	0·5104	0·4876	0·4943
$\hat{Q}''_{11}$	−0·7583	−0·7633	−0·7768	−0·7929
$\hat{Q}''_{12}$	−0·6101	−0·5928	−0·6047	−0·6310
$\hat{Q}''_{21}$	−0·6220	−0·6268	−0·6718	−0·6645
$\hat{Q}''_{22}$	−0·7780	−0·7524	−0·7415	−0·7813

	Davies	Albano Perkinson Rodden	Davies	Albano Perkinson Rodden
	$\nu = 0 \quad h = 0{\cdot}6$		$\nu = 1{\cdot}5 \quad h = 0{\cdot}6$	
$\hat{Q}'_{11}$	0·1470	0·1490	0·5713	0·5814
$\hat{Q}'_{12}$	−0·6402	−0·6312	−0·3558	−0·3450
$\hat{Q}'_{21}$	−0·2404	−0·2405	0·1262	0·1226
$\hat{Q}'_{22}$	−0·1619	−0·1817	0·4568	−0·4278
$\hat{Q}''_{11}$	−0·5492	−0·5505	−0·6274	−0·6382
$\hat{Q}''_{12}$	−0·6181	−0·6070	−0·7180	−0·7053
$\hat{Q}''_{21}$	−0·5308	−0·5329	−0·5989	−0·6026
$\hat{Q}''_{22}$	−0·7565	−0·7306	−0·8729	−0·8464

TABLE 4a

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$\nu = 0,\ M = 0{\cdot}8,\ h = 0$

m_1	n_1	m_2	n_2	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
4	2	4	2	0·4701	−0·6531	−0·1243	−0·1213	−0·5102	−0·5287	−0·4895	−0·7065
8	3	8	3	0·4554	−0·6246	−0·1128	−0·1557	−0·5332	−0·4787	−0·5099	−0·6835
10	3	8	2	0·4314	−0·5932	−0·09012	−0·1987	−0·5149	−0·4790	−0·5192	−0·6734
10	3	10	3	0·4450	−0·6244	−0·1065	−0·1686	−0·5385	−0·4698	−0·5109	−0·6727
12	3	12	3	0·4344	−0·6235	−0·1014	−0·1800	−0·5420	−0·4653	−0·5111	−0·6645
14	3	14	3	0·4263	−0·6222	−0·09750	−0·1889	−0·5444	−0·4625	−0·5120	−0·6584
16	3	16	3	0·4198	−0·6213	−0·09475	−0·1951	−0·5463	−0·4606	−0·5122	−0·6538
18	3	18	3	0·4163	−0·6207	−0·09319	−0·1988	−0·5482	−0·4583	−0·5125	−0·6506
20	3	20	3	0·4153	−0·6202	−0·09258	−0·2006	−0·5500	−0·4560	−0·5129	−0·6486
10	4	10	4	0·4456	−0·6213	−0·1091	−0·1612	−0·5314	−0·4730	−0·5121	−0·6746
12	4	12	4	0·4466	−0·6212	−0·1085	−0·1654	−0·5372	−0·4657	−0·5126	−0·6713
14	4	14	4	0·4442	−0·6207	−0·1068	−0·1705	−0·5404	−0·4627	−0·5127	−0·6681
16	4	16	4	0·4403	−0·6202	−0·1046	−0·1759	−0·5425	−0·4611	−0·5127	−0·6649
12	5	12	5	0·4389	−0·6206	−0·1057	−0·1712	−0·5395	−0·4640	−0·5136	−0·6647

TABLE 4b

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$\nu = 1{\cdot}5,\ M = 0{\cdot}8,\ h = 0$

m_1	n_1	m_2	n_2	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
4	2	4	2	1·128	−0·04327	0·3058	0·5396	−0·6116	−0·5990	−0·5328	−0·6398
8	3	8	3	1·154	−0·07985	0·3670	0·5535	−0·7485	−0·6172	−0·6146	−0·7826
10	3	8	2	1·104	−0·01444	0·3718	0·4505	−0·6986	−0·5858	−0·6332	−0·7178
10	3	10	3	1·111	−0·08074	0·3756	0·5342	−0·7442	−0·6152	−0·6154	−0·7715
12	3	12	3	1·068	−0·08279	0·3786	0·5116	−0·7380	−0·6168	−0·6136	−0·7629
14	3	14	3	1·037	−0·08516	0·3784	0·4944	−0·7342	−0·6177	−0·6132	−0·7569
16	3	16	3	1·013	−0·08732	0·3769	0·4818	−0·7322	−0·6183	−0·6125	−0·7527
18	3	18	3	0·9977	−0·08793	0·3745	0·4752	−0·7319	−0·6178	−0·6124	−0·7503
20	3	20	3	0·9896	−0·08752	0·3717	0·4732	−0·7330	−0·6166	−0·6127	−0·7493
10	4	10	4	1·144	−0·08091	0·3689	0·5528	−0·7546	−0·6113	−0·6224	−0·7842
12	4	12	4	1·113	−0·07524	0·3690	0·5512	−0·7574	−0·6092	−0·6219	−0·7830
14	4	14	4	1·120	−0·07265	0·3702	0·5451	−0·7581	−0·6093	−0·6217	−0·7809
16	4	16	4	1·106	−0·07195	0·3716	0·5367	−0·7583	−0·6101	−0·6220	−0·7780
12	5	12	5	1·110	−0·07567	0·3650	0·5391	−0·7576	−0·6116	−0·6233	−0·7787

TABLE 4c

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$\nu = 0,\ M = 0{\cdot}8,\ h = 0{\cdot}6$

m_1	n_1	m_2	n_2	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
4	2	4	2	0·1789	−0·6788	−0·2412	−0·1092	−0·5519	−0·6838	−0·5385	−0·7934
8	3	8	3	0·1599	−0·6499	−0·2439	−0·1390	−0·5549	−0·6297	−0·5371	−0·7715
10	3	8	2	0·1506	−0·6488	−0·2388	−0·1784	−0·5510	−0·6090	−0·5484	−0·7416
10	3	10	3	0·1505	−0·6457	−0·2413	−0·1544	−0·5503	−0·6253	−0·5333	−0·7641
12	3	12	3	0·1410	−0·6426	−0·2395	−0·1671	−0·5460	−0·6227	−0·5294	−0·7580
14	3	14	3	0·1337	−0·6405	−0·2382	−0·1768	−0·5428	−0·6201	−0·5268	−0·7533
16	3	16	3	0·1281	−0·6389	−0·2372	−0·1834	−0·5407	−0·6183	−0·5244	−0·7496
18	3	18	3	0·1250	−0·6379	−0·2368	−0·1874	−0·5397	−0·6165	−0·5229	−0·7471
20	3	20	3	0·1240	−0·6372	−0·2367	−0·1893	−0·5394	−0·6147	−0·5223	−0·7455
10	4	10	4	0·1536	−0·6439	−0·2412	−0·1446	−0·5529	−0·6229	−0·5362	−0·7636
12	4	12	4	0·1534	−0·6422	−0·2414	−0·1500	−0·5520	−0·6200	−0·5345	−0·7614
14	4	14	4	0·1507	−0·6411	−0·2410	−0·1559	−0·5507	−0·6188	−0·5326	−0·7589
16	4	16	4	0·1470	−0·6402	−0·2404	−0·1619	−0·5492	−0·6181	−0·5308	−0·7565
12	5	12	5	0·1447	−0·6419	−0·2419	−0·1557	−0·5503	−0·6185	−0·5328	−0·7559

TABLE 4d

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$\nu = 1{\cdot}5,\ M = 0{\cdot}8,\ h = 0{\cdot}6$

m_1	n_1	m_2	n_2	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{21}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
4	2	4	2	0·5679	−0·3555	0·09538	0·4878	−0·5329	−0·7096	−0·5453	−0·7178
8	3	8	3	0·5869	−0·3595	0·1253	0·4954	−0·6280	−0·7291	−0·6042	−0·8790
10	3	8	2	0·5186	−0·3645	0·05802	0·3483	−0·5978	−0·6424	−0·5948	−0·7443
10	3	10	3	0·5646	−0·3651	0·1298	0·4594	−0·6157	−0·7215	−0·5936	−0·8660
12	3	12	3	0·5406	−0·3706	0·1309	0·4262	−0·6040	−0·7163	−0·5835	−0·8536
14	3	14	3	0·5214	−0·3741	0·1301	0·4020	−0·5961	−0·7120	−0·5767	−0·8446
16	3	16	3	0·5073	−0·3763	0·1290	0·3847	−0·5913	−0·7089	−0·5717	−0·8385
18	3	18	3	0·4987	−0·3773	0·1273	0·3752	−0·5889	−0·7063	−0·5689	−0·8351
20	3	20	3	0·4950	−0·3775	0·1253	0·3716	−0·5882	−0·7042	−0·5679	−0·8336
10	4	10	3	0·5873	−0·3558	0·1244	0·4883	−0·6353	−0·7241	−0·6070	−0·8813
12	4	12	3	0·5857	−0·3550	0·1253	0·4812	−0·6337	−0·7211	−0·6043	−0·8794
14	4	14	3	0·5796	−0·3550	0·1260	0·4700	−0·6307	−0·7195	−0·6016	−0·8764
16	4	16	3	0·5713	−0·3558	0·1262	0·4568	−0·6274	−0·7180	−0·5989	−0·8729
12	5	12	5	0·5685	−0·3574	0·1183	0·4639	−0·6318	−0·7204	−0·6026	−0·8738

TABLE 5

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$M = 0{\cdot}8\ (m_1 = 16, n_1 = 4, m_2 = 16, n_2 = 4)$

v	h	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
0	0	0·4403	−0·6202	−0·1046	−0·1759	−0·5425	−0·4611	−0·5127	−0·6649
	0·01	0·4199	−0·6242	−0·1162	−0·1786	−0·5467	−0·4742	−0·5176	−0·6733
	0·04	0·3786	−0·6312	−0·1386	−0·1827	−0·5533	−0·4981	−0·5268	−0·6886
	0·1	0·3223	−0·6387	−0·1674	−0·1839	−0·5586	−0·5263	−0·5354	−0·7067
	0·2	0·2591	−0·6423	−0·1976	−0·1774	−0·5588	−0·5562	−0·5378	−0·7256
	0·3	0·2163	−0·6420	−0·2158	−0·1702	−0·5560	−0·5780	−0·5359	−0·7380
	0·4	0·1859	−0·6410	−0·2273	−0·1655	−0·5531	−0·5949	−0·5338	−0·7464
	0·5	0·1636	−0·6404	−0·2350	−0·1630	−0·5508	−0·6079	−0·5320	−0·7523
	0·6	0·1470	−0·6402	−0·2404	−0·1619	−0·5492	−0·6181	−0·5308	−0·7565
1·5	0	1·106	−0·07195	0·3716	0·5367	−0·7583	−0·6101	−0·6220	−0·7780
	0·01	1·064	−0·09692	0·3471	0·5235	−0·7532	−0·6188	−0·6233	−0·7853
	0·04	0·9767	−0·1473	0·2986	0.5001	−0·7389	−0·6351	−0·6253	−0·7994
	0·1	0·8590	−0·2096	0·2390	0·4774	−0·7141	−0·6546	−0·6241	−0·8169
	0·2	0·7394	−0·2679	0·1856	0·4660	−0·6821	−0·6754	−0·6162	−0·8359
	0·3	0·6687	−0·3020	0·1585	0·4638	−0·6601	−0·6906	−0·6090	−0·8494
	0·4	0·6235	−0·3253	0·1430	0·4623	−0·6451	−0·7022	−0·6041	−0·8594
	0·5	0·5928	−0·3425	0·1331	0·4600	−0·6348	−0·7112	−0·6010	−0·8670
	0·6	0·5713	−0·3558	0·1262	0·4568	−0·6274	−0·7180	−0·5989	−0·8729

TABLE 6

Generalised Airforce Coefficients for the Agard Wing-Tailplane Configuration

$M = 0{\cdot}8\ (m_1 = 16, n_1 = 4, m_2 = 16, n_2 = 4)$

h	v	$\hat{Q}'_{11}$	$\hat{Q}'_{12}$	$\hat{Q}'_{21}$	$\hat{Q}'_{22}$	$\hat{Q}''_{11}$	$\hat{Q}''_{12}$	$\hat{Q}''_{21}$	$\hat{Q}''_{22}$
0	0	0·4403	−0·6202	−0·1046	−0·1759	−0·5425	−0·4611	−0·5127	−0·6649
	0·1	0·4436	−0·6176	−0·1025	−0·1728	−0·5433	−0·4617	−0·5130	−0·6653
	0·2	0·4533	−0·6099	−0·09631	−0·1635	−0·5457	−0·4637	−0·5142	−0·6666
	0·4	0·4925	−0·5794	−0·07141	−0·1264	−0·5557	−0·4714	−0·5188	−0·6716
	0·6	0·5576	−0·5290	−0·02959	−0·06420	−0·5734	−0·4840	−0·5270	−0·6802
	0·8	0·6475	−0·4591	0·02946	0·02351	−0·5998	−0·5017	−0·5391	−0·6927
	1·0	0·7596	−0·3702	0·1060	0·1373	−0·6350	−0·5246	−0·5557	−0·7097
	1·2	0·8895	−0·2629	0·1998	0·2775	−0·6788	−0·5535	−0·5775	−0·7320
	1·5	1·106	−0·07195	0·3716	0·5367	−0·7583	−0·6101	−0·6220	−0·7780
0·6	0	0·1470	−0·6402	−0·2404	−0·1619	−0·5492	−0·6181	−0·5308	−0·7565
	0·1	0·1490	−0·6388	−0·2388	−0·1592	−0·5495	−0·6185	−0·5311	−0·7569
	0·2	0·1548	−0·6346	−0·2341	−0·1511	−0·5504	−0·6198	−0·5318	−0·7580
	0·4	0·1779	−0·6179	−0·2154	−0·1187	−0·5533	−0·6248	−0·5348	−0·7628
	0·6	0·2164	−0·5905	−0·1840	−0·06443	−0·5608	−0·6331	−0·5400	−0·7709
	0·8	0·2701	−0·5528	−0·1395	0·01235	−0·5702	−0·6446	−0·5473	−0·7830
	1·0	0·3387	−0·5053	−0·08154	0·1121	−0·5824	−0·6597	−0·5573	−0·8001
	1·2	0·4217	−0·4493	−0·009303	0·2347	−0·5977	−0·6793	−0·5705	−0·8234
	1·5	0·5713	−0·3558	0·1262	0·4568	−0·6274	−0·7180	−0·5989	−0·8729

TABLE 7

Values of $\hat{Q}_{33}$ for $H = 0(m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2)$ for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}3$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{33}$		
0	1·3078	1·3700	1·5315	1·7779
$\frac{1}{32}$	1·3048	1·3646	1·5218	1·7618
$\frac{1}{16}$	1·3047	1·3631	1·5175	1·7531
$\frac{1}{8}$	1·3087	1·3656	1·5167	1·7462
$\frac{3}{16}$	1·3146	1·3711	1·5200	1·7453
$\frac{1}{4}$	1·3208	1·3772	1·5248	1·7470
$\frac{3}{8}$	1·3322	1·3890	1·5349	1·7526
$\frac{1}{2}$	1·3414	1·3989	1·5439	1·7586
$\frac{3}{4}$	1·3543	1·4136	1·5578	1·7685
1	1·3622	1·4232	1·5671	1·7755
$\frac{3}{2}$	1·3699	1·4339	1·5780	1·7833
2	1·3724	1·4389	1·5834	1·7868
∞	1·3875	1·4379	1·5843	1·7826
		$\hat{Q}''_{33}$		
0	−0·2297	−0·3000	−0·4905	−0·9106
$\frac{1}{32}$	−0·2647	−0·3438	−0·5570	−1·0172
$\frac{1}{16}$	−0·2898	−0·3752	−0·6043	−1·0922
$\frac{1}{8}$	−0·3241	−0·4182	−0·6693	−1·1961
$\frac{3}{16}$	−0·3479	−0·4481	−0·7151	−1·2697
$\frac{1}{4}$	−0·3666	−0·4716	−0·7511	−1·3277
$\frac{3}{8}$	−0·3958	−0·5087	−0·8077	−1·4176
$\frac{1}{2}$	−0·4190	−0·5380	−0·8520	−1·4865
$\frac{3}{4}$	−0·4547	−0·5832	−0·9189	−1·5884
1	−0·4810	−0·6168	−0·9678	−1·6614
$\frac{3}{2}$	−0·5168	−0·6632	−1·0348	−1·7602
2	−0·5390	−0·6933	−1·0785	−1·8239
∞	−0·6304	−0·7602	−1·1859	−1·9466

TABLE 8

Values of $\hat{Q}_{33}$ for $H = \frac{1}{8}$ $(m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2)$ for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{33}$		
0	1·2955	1·3551	1·5104	1·7501
$\frac{1}{32}$	1·3026	1·3619	1·5156	1·7502
$\frac{1}{16}$	1·3076	1·3666	1·5188	1·7494
$\frac{1}{8}$	1·3156	1·3741	1·5239	1·7488
$\frac{3}{16}$	1·3223	1·3808	1·5289	1·7501
$\frac{1}{4}$	1·3283	1·3868	1·5339	1·7523
$\frac{3}{8}$	1·3382	1·3974	1·5432	1·7577
$\frac{1}{2}$	1·3460	1·4059	1·5510	1·7628
$\frac{3}{4}$	1·3566	1·4181	1·5626	1·7711
1	1·3630	1·4260	1·5703	1·7768
$\frac{3}{2}$	1·3691	1·4347	1·5792	1·7832
2	1·3711	1·4388	1·5836	1·7861
∞	1·3875	1·4379	1·5843	1·7826
		$\hat{Q}''_{33}$		
0	−0·3601	−0·4557	−0·6929	−1·1675
$\frac{1}{32}$	−0·3573	−0·4553	−0·7032	−1·2065
$\frac{1}{16}$	−0·3613	−0·4626	−0·7215	−1·2487
$\frac{1}{8}$	−0·3755	−0·4832	−0·7604	−1·3230
$\frac{3}{16}$	−0·3905	−0·5035	−0·7948	−1·3826
$\frac{1}{4}$	−0·4042	−0·5216	−0·8242	−1·4314
$\frac{3}{8}$	−0·4276	−0·5521	−0·8720	−1·5080
$\frac{1}{2}$	−0·4467	−0·5767	−0·9096	−1·5666
$\frac{3}{4}$	−0·4763	−0·6147	−0·9662	−1·6524
1	−0·4980	−0·6427	−1·0071	−1·7132
$\frac{3}{2}$	−0·5273	−0·6811	−1·0626	−1·7946
2	−0·5454	−0·7058	−1·0984	−1·8468
∞	−0·6304	−0·7602	−1·1859	−1·9466

TABLE 9

Values of $\hat{Q}_{34}$ for $H = 0$ $(m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2)$ for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $v = 0{\cdot}3856$	$M = 0{\cdot}45$ $v = 0{\cdot}2436$	$M = 0{\cdot}65$ $v = 0{\cdot}1513$	$M = 0{\cdot}80$ $v = 0{\cdot}1112$
		$\hat{Q}'_{34}$		
0	0·1181	0·1125	0·0810	0·0183
$\frac{1}{32}$	0·1224	0·1197	0·0929	0·0367
$\frac{1}{16}$	0·1227	0·1219	0·0982	0·0468
$\frac{1}{8}$	0·1178	0·1191	0·0996	0·0549
$\frac{3}{16}$	0·1100	0·1125	0·0957	0·0558
$\frac{1}{4}$	0·1017	0·1047	0·0899	0·0539
$\frac{3}{8}$	0·0860	0·0895	0·0775	0·0471
$\frac{1}{2}$	0·0727	0·0762	0·0660	0·0397
$\frac{3}{4}$	0·0526	0·0558	0·0480	0·0271
1	0·0390	0·0417	0·0353	0·0180
$\frac{3}{2}$	0·0228	0·0246	0·0198	0·0070
2	0·0141	0·0152	0·0113	0·0013
∞	0·0000	0·0000	0·0000	0·0000
		$\hat{Q}''_{34}$		
0	−0·3541	−0·4860	−0·7625	−1·1550
$\frac{1}{32}$	−0·3099	−0·4317	−0·6834	−1·0339
$\frac{1}{16}$	−0·2779	−0·3923	−0·6264	−0·9476
$\frac{1}{8}$	−0·2347	−0·3384	−0·5477	−0·8274
$\frac{3}{16}$	−0·2062	−0·3021	−0·4931	−0·7427
$\frac{1}{4}$	−0·1852	−0·2747	−0·4511	−0·6767
$\frac{3}{8}$	−0·1549	−0·2342	−0·3876	−0·5765
$\frac{1}{2}$	−0·1331	−0·2043	−0·3401	−0·5016
$\frac{3}{4}$	−0·1026	−0·1616	−0·2716	−0·3944
1	−0·0820	−0·1321	−0·2239	−0·3198
$\frac{3}{2}$	−0·0561	−0·0940	−0·1614	−0·2213
2	−0·0407	−0·0706	−0·1222	−0·1586
∞	0·0000	0·0000	0·0000	0·0000

TABLE 10

Values of $\hat{Q}_{34}$ for $H = \frac{1}{8}$ ($m_1 = 6,\ n_1 = 2, m_2 = 6, n_2 = 2$)
for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{34}$		
0	0·1469	0·1492	0·1228	0·0571
$\frac{1}{32}$	0·1373	0·1399	0·1159	0·0572
$\frac{1}{16}$	0·1306	0·1335	0·1119	0·0586
$\frac{1}{8}$	0·1196	0·1233	0·1052	0·0596
$\frac{3}{16}$	0·1098	0·1140	0·0983	0·0580
$\frac{1}{4}$	0·1007	0·1051	0·0912	0·0549
$\frac{3}{8}$	0·0847	0·0891	0·0777	0·0472
$\frac{1}{2}$	0·0715	0·0757	0·0659	0·0396
$\frac{3}{4}$	0·0518	0·0553	0·0477	0·0269
1	0·0384	0·0413	0·0351	0·0179
$\frac{3}{2}$	0·0225	0·0243	0·0196	0·0069
2	0·0140	0·0151	0·0112	0·0012
∞	0·0000	0·0000	0·0000	0·0000
		$\hat{Q}''_{34}$		
0	−0·2401	−0·3663	−0·6382	−1·0410
$\frac{1}{32}$	−0·2455	−0·3675	−0·6241	−0·9892
$\frac{1}{16}$	−0·2405	−0·3574	−0·5987	−0·9327
$\frac{1}{8}$	−0·2215	−0·3286	−0·5446	−0·8323
$\frac{3}{16}$	−0·2015	−0·3006	−0·4970	−0·7514
$\frac{1}{4}$	−0·1839	−0·2762	−0·4568	−0·6854
$\frac{3}{8}$	−0·1555	−0·2368	−0·4929	−0·5831
$\frac{1}{2}$	−0·1338	−0·2066	−0·3442	−0·5062
$\frac{3}{4}$	−0·1030	−0·1630	−0·2738	−0·3964
1	−0·0821	−0·1328	−0·2251	−0·3206
$\frac{3}{2}$	−0·0560	−0·0941	−0·1617	−0·2213
2	−0·0407	−0·0706	−0·1222	−0·1584
∞	0·0000	0·0000	0·0000	0·0000

TABLE 11

Values of $\hat{Q}_{43}$ for $H = 0$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$)
for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{43}$		
0	−0·5680	−0·6077	−0·7267	−0·9295
$\frac{1}{32}$	−0·6636	−0·7064	−0·8335	−1·0459
$\frac{1}{16}$	−0·7281	−0·7736	−0·9059	−1·1242
$\frac{1}{8}$	−0·8043	−0·8553	−0·9952	−1·2222
$\frac{3}{16}$	−0·8435	−0·9006	−1·0470	−1·2806
$\frac{1}{4}$	−0·8639	−0·9277	−1·0798	−1·3188
$\frac{3}{8}$	−0·8763	−0·9544	−1·1169	−1·3635
$\frac{1}{2}$	−0·8692	−0·9627	−1·1345	−1·3862
$\frac{3}{4}$	−0·8275	−0·9547	−1·1438	−1·4025
1	−0·7670	−0·9311	−1·1377	−1·4017
$\frac{3}{2}$	−0·6171	−0·8626	−1·1071	−1·3812
2	−0·4438	−0·7776	−1·0648	−1·3500
		$\hat{Q}''_{43}$		
0	0·0175	0·0252	0·0572	0·1432
$\frac{1}{32}$	0·0925	0·1102	0·1739	0·3245
$\frac{1}{16}$	0·1705	0·1974	0·2897	0·4970
$\frac{1}{8}$	0·3185	0·3619	0·5037	0·8076
$\frac{3}{16}$	0·4516	0·5100	0·6944	1·0804
$\frac{1}{4}$	0·5719	0·6445	0·8670	1·3249
$\frac{3}{8}$	0·7846	0·8842	1·1735	1·7523
$\frac{1}{2}$	0·9703	1·0961	1·4432	2·1208
$\frac{3}{4}$	1·2868	1·4656	1·9116	2·7461
1	1·5510	1·7867	2·3196	3·2799
$\frac{3}{2}$	1·9676	2·3372	3·0324	4·2007
2	2·2640	2·8022	3·6623	5·0133

TABLE 12

**Values of $\hat{Q}_{43}$ for $H = \frac{1}{8}$ $(m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2)$
for Onera Wing-Tailplane Configuration**

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{43}$		
0	−0·7202	−0·7419	−0·8183	−0·9462
$\frac{1}{32}$	−0·6977	−0·7327	−0·8226	−0·9725
$\frac{1}{16}$	−0·6985	−0·7379	−0·8378	−1·0018
$\frac{1}{8}$	−0·7103	−0·7570	−0·8696	−1·0493
$\frac{3}{16}$	−0·7203	−0·7731	−0·8935	−1·0817
$\frac{1}{4}$	−0·7252	−0·7838	−0·9100	−1·1036
$\frac{3}{8}$	−0·7232	−0·7934	−0·9284	−1·1288
$\frac{1}{2}$	−0·7109	−0·7933	−0·9358	−1·1405
$\frac{3}{4}$	−0·6701	−0·7790	−0·9352	−1·1458
1	−0·6175	−0·7554	−0·9255	−1·1407
$\frac{3}{2}$	−0·4926	−0·6953	−0·8957	−1·1194
2	−0·3511	−0·6243	−0·8590	−1·0920
		$\hat{Q}''_{43}$		
1	0·1894	0·2002	0·2510	0·3636
$\frac{1}{32}$	0·2114	0·2314	0·3019	0·4601
$\frac{1}{16}$	0·2489	0·2756	0·3667	0·5683
$\frac{1}{8}$	0·3367	0·3768	0·5064	0·7848
$\frac{4}{16}$	0·4269	0·4796	0·6433	0·9868
$\frac{1}{4}$	0·5139	0·5784	0·7725	1·1721
$\frac{3}{8}$	0·6739	0·7608	1·0076	1·5008
$\frac{1}{2}$	0·8170	0·9253	1·2176	1·7871
$\frac{3}{4}$	1·0639	1·2151	1·5855	2·2768
1	1·2715	1·4686	1·9079	2·6980
$\frac{3}{2}$	1·6003	1·9051	2·4743	3·4299
2	1·8342	2·2748	2·9769	4·0799

TABLE 13

Values of $\hat{Q}_{44}$ for $H = 0$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$) for Onera Wing-Tailplane Configuration

λ	$M = 0.30$, $\nu = 0.3856$	$M = 0.45$, $\nu = 0.2436$	$M = 0.65$, $\nu = 0.1513$	$M = 0.80$, $\nu = 0.1112$
		$\hat{Q}'_{44}$		
0	0·6909	0·7371	0·8579	1·0547
$\frac{1}{32}$	0·8237	0·8693	0·9941	1·1950
$\frac{1}{16}$	0·9181	0·9631	1·0895	1·2921
$\frac{1}{8}$	1·0409	1·0854	1·2139	1·4187
$\frac{3}{16}$	1·1167	1·1616	1·2921	1·4991
$\frac{1}{4}$	1·1683	1·2140	1·3465	1·5552
$\frac{3}{8}$	1·2342	1·2821	1·4177	1·6283
$\frac{1}{2}$	1·2745	1·3244	1·4620	1·6731
$\frac{3}{4}$	1·3197	1·3730	1·5131	1·7232
1	1·3428	1·3991	1·5403	1·7490
$\frac{3}{2}$	1·3632	1·4239	1·5666	1·7727
2	1·3703	1·4344	1·5779	1·7822
∞	1·3875	1·4379	1·5843	1·7826
		$\hat{Q}''_{44}$		
0	−0·0355	−0·0590	−0·1085	−0·2102
$\frac{1}{32}$	−0·0610	−0·0946	−0·1702	−0·3242
$\frac{1}{16}$	−0·0883	−0·1302	−0·2280	−0·4262
$\frac{1}{8}$	−0·1386	−0·1933	−0·3267	−0·5968
$\frac{3}{16}$	−0·1806	−0·2451	−0·4068	−0·7349
$\frac{1}{4}$	−0·2158	−0·2885	−0·4738	−0·8503
$\frac{3}{8}$	−0·2726	−0·3584	−0·5814	−1·0339
$\frac{1}{2}$	−0·3169	−0·4131	−0·6649	−1·1736
$\frac{3}{4}$	−0·3824	−0·4938	−0·7862	−1·3707
1	−0·4280	−0·5504	−0·8696	−1·5021
$\frac{3}{2}$	−0·4860	−0·6232	−0·9755	−1·6649
2	−0·5199	−0·6672	−1·0392	−1·7611
∞	−0·6304	−0·7602	−1·1859	−1·9466

TABLE 14

Values of $\hat{Q}_{44}$ for $H = \frac{1}{8}$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$) for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{44}$		
0	1·0968	1·1195	1·1928	1·3232
$\frac{1}{32}$	1·0868	1·1168	1·2090	1·3677
$\frac{1}{16}$	1·0998	1·1351	1·2402	1·4157
$\frac{1}{8}$	1·1424	1·1842	1·3039	1·4964
$\frac{3}{16}$	1·1829	1·2282	1·3552	1·5553
$\frac{1}{4}$	1·2159	1·2635	1·3948	1·5988
$\frac{3}{8}$	1·2629	1·3138	1·4498	1·6572
$\frac{1}{2}$	1·2936	1·3468	1·4852	1·6935
$\frac{3}{4}$	1·3291	1·3857	1·5266	1·7344
1	1·3475	1·4067	1·5487	1·7554
$\frac{3}{2}$	1·3638	1·4268	1·5700	1·7747
2	1·3695	1·4351	1·5792	1·7823
∞	1·3875	1·4379	1·5843	1·7826
		$\hat{Q}''_{44}$		
0	−0·2487	−0·3049	−0·4197	−0·6148
$\frac{1}{32}$	−0·2289	−0·2886	−0·4190	−0·6571
$\frac{1}{16}$	−0·2268	−0·2914	−0·4385	−0·7161
$\frac{1}{8}$	−0·2425	−0·3168	−0·4943	−0·8381
$\frac{3}{16}$	−0·2657	−0·3486	−0·5513	−0·9478
$\frac{1}{4}$	−0·2893	−0·3795	−0·6032	−1·0428
$\frac{3}{8}$	−0·3315	−0·4336	−0·6905	−1·1960
$\frac{1}{2}$	−0·3665	−0·4777	−0·7595	−1·3127
$\frac{3}{4}$	−0·4191	−0·5437	−0·8600	−1·4766
1	−0·4560	−0·5900	−0·9288	−1·5850
$\frac{3}{2}$	−0·5029	−0·6495	−1·0156	−1·7184
2	−0·5303	−0·6852	−1·0675	−1·7966
∞	−0·6304	−0·7602	−1·1859	−1·9466

TABLE 15

Values of $\hat{Q}_{14}$ for $H = 0$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$) for Onera Wing-Tailplane Configuration

λ	$M = 0.30$ $\nu = 0.3856$	$M = 0.45$ $\nu = 0.2436$	$M = 0.65$ $\nu = 0.1513$	$M = 0.80$ $\nu = 0.1112$
		$\hat{Q}'_{14}$		
0	−3·0222	−3·0165	−2·9480	−2·7610
$\frac{1}{32}$	−2·5093	−2·5086	−2·4426	−2·2673
$\frac{1}{16}$	−2·1369	−2·1423	−2·0837	−1·9222
$\frac{1}{8}$	−1·6407	−1·6548	−1·6081	−1·4653
$\frac{3}{16}$	−1·3256	−1·3437	−1·3027	−1·1698
$\frac{1}{4}$	−1·1056	−1·1249	−1·0863	−0·9598
$\frac{3}{8}$	−0·8142	−0·8326	−0·7957	−0·6797
$\frac{1}{2}$	−0·6278	−0·6443	−0·6087	−0·5030
$\frac{3}{4}$	−0·4043	−0·4171	−0·3857	−0·2987
1	−0·2785	−0·2886	−0·2615	−0·1893
$\frac{3}{2}$	−0·1496	−0·1559	−0·1353	−0·0829
2	−0·0891	−0·0929	−0·0765	−0·0360
∞	0·0000	0·0000	0·0000	0·0000
		$\hat{Q}''_{14}$		
0	−0·0521	0·5109	1·6823	3·5719
$\frac{1}{32}$	0·1476	0·6758	1·7918	3·5796
$\frac{1}{16}$	0·2648	0·7626	1·8233	3·5128
$\frac{1}{8}$	0·3767	0·8268	1·7921	3·3101
$\frac{3}{16}$	0·4166	0·8309	1·7182	3·0912
$\frac{1}{4}$	0·4274	0·8126	1·6335	2·8814
$\frac{3}{8}$	0·4149	0·7540	1·4659	2·5090
$\frac{1}{2}$	0·3871	0·6897	1·3150	2·1999
$\frac{3}{4}$	0·3257	0·5734	1·0708	1·7312
1	0·2723	0·4804	0·8886	1·3995
$\frac{3}{2}$	0·1948	0·3497	0·6427	0·9673
2	0·1450	0·2657	0·4875	0·6992
∞	0·0000	0·0000	0·0000	0·0000

TABLE 16

Values of $\hat{Q}_{14}$ for $H = \frac{1}{8}$ $(m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2)$ for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{14}$		
0	−1·5328	−1·6893	−1·9237	−2·0953
$\frac{1}{32}$	−1·6087	−1·7248	−1·8692	−1·9208
$\frac{1}{16}$	−1·5648	−1·6536	−1·7399	−1·7239
$\frac{1}{8}$	−1·3767	−1·4354	−1·4611	−1·3840
$\frac{3}{16}$	−1·1838	−1·2282	−1·2272	−1·1283
$\frac{1}{4}$	−1·0200	−1·0563	−1·0421	−0·9352
$\frac{3}{8}$	−0·7746	−0·8016	−0·7760	−0·6684
$\frac{1}{2}$	−0·6055	−0·6271	−0·5980	−0·4966
$\frac{3}{4}$	−0·3947	−0·4099	−0·3811	−0·2957
1	−0·2735	−0·2848	−0·2590	−0·1876
$\frac{3}{2}$	−0·1477	−0·1544	−0·1343	−0·0823
2	−0·0883	−0·0922	−0·0760	−0·0357
∞	0·0000	0·0000	0·0000	0·0000
		$\hat{Q}''_{14}$		
0	0·4746	0·9467	2·0315	3·9061
$\frac{1}{32}$	0·4841	0·9609	2·0314	3·8311
$\frac{1}{16}$	0·4911	0·9621	2·0045	3·7193
$\frac{1}{8}$	0·4978	0·9448	1·9160	3·4622
$\frac{3}{16}$	0·4933	0·9124	1·8117	3·2068
$\frac{1}{4}$	0·4808	0·8733	1·7066	2·9704
$\frac{3}{8}$	0·4448	0·7913	1·5127	2·5633
$\frac{1}{2}$	0·4053	0·7141	1·3462	2·2340
$\frac{3}{4}$	0·3333	0·5849	1·0857	1·7456
1	0·2756	0·4862	0·8962	1·4057
$\frac{3}{2}$	0·1954	0·3513	0·6448	0·9680
2	0·1449	0·2661	0·4879	0·6986
∞	0·0000	0·0000	0·0000	0·0000

TABLE 17

Values of $\hat{Q}_{23}$ for $H = 0$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$) for Onera Wing-Tailplane Configuration

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{23}$		
0	1·9745	2·1014	2·4799	3·1000
$\frac{1}{32}$	2·2888	2·4247	2·8265	3·4718
$\frac{1}{16}$	2·5048	2·6488	3·0656	3·7262
$\frac{1}{8}$	2·7668	2·9293	3·3691	4·0527
$\frac{3}{16}$	2·9077	3·0916	3·5518	4·2542
$\frac{1}{4}$	2·9858	3·1933	3·6726	4·3905
$\frac{3}{8}$	3·0438	3·3028	3·8173	4·5584
$\frac{1}{2}$	3·0317	3·3467	3·8936	4·6510
$\frac{3}{4}$	2·9044	3·3417	3·9509	4·7317
1	2·7034	3·2748	3·9476	4·7481
$\frac{3}{2}$	2·1856	3·0532	3·8647	4·7048
2	1·5759	2·7644	3·7324	4·6169
		$\hat{Q}''_{23}$		
0	−0·3366	−0·3345	−0·3081	−0·1469
$\frac{1}{32}$	−0·5902	−0·6121	−0·6623	−0·6457
$\frac{1}{16}$	−0·8509	−0·8960	−1·0181	−1·1345
$\frac{1}{8}$	−1·3441	−1·4332	−1·6843	−2·0382
$\frac{3}{16}$	−1·7892	−1·9203	−2·2872	−2·8515
$\frac{1}{4}$	−2·1943	−2·3667	−2·8401	−3·5935
$\frac{3}{8}$	−2·9168	−3·1713	−3·8368	−4·9168
$\frac{1}{2}$	−3·5548	−3·8924	−4·7286	−6·0815
$\frac{3}{4}$	−4·6545	−5·1664	−6·3029	−8·0982
1	−5·5818	−6·2865	−7·6941	−9·8517
$\frac{3}{2}$	−7·0554	−8·2253	−10·1536	−12·9246
2	−8·1094	−9·8746	−12·3477	−15·6708

TABLE 18

Values of $\hat{Q}_{23}$ for $H = \frac{1}{8}$ ($m_1 = 6, n_1 = 2, m_2 = 6, n_2 = 2$)
for Onera Rectangular Planforms

λ	$M = 0{\cdot}30$ $\nu = 0{\cdot}3856$	$M = 0{\cdot}45$ $\nu = 0{\cdot}2436$	$M = 0{\cdot}65$ $\nu = 0{\cdot}1513$	$M = 0{\cdot}80$ $\nu = 0{\cdot}1112$
		$\hat{Q}'_{23}$		
0	2·4223	2·5178	2·7537	3·1319
$\frac{1}{32}$	2·3836	2·4949	2·7742	3·2208
$\frac{1}{16}$	2·3926	2·5185	2·8299	3·3197
$\frac{1}{8}$	2·4434	2·5940	2·9465	3·4846
$\frac{3}{16}$	2·4873	2·6594	3·0378	3·6018
$\frac{1}{4}$	2·5135	2·7063	3·1040	3·6848
$\frac{3}{8}$	2·5224	2·7574	3·1860	3·7883
$\frac{1}{2}$	2·4926	2·7723	3·2274	3·8438
$\frac{3}{4}$	2·3679	2·7448	3·2500	3·8866
1	2·1936	2·6774	3·2338	3·8872
$\frac{3}{2}$	1·7621	2·4835	3·1520	3·8392
2	1·2615	2·2419	3·0372	3·7619
		$\hat{Q}''_{23}$		
0	−0·9285	−0·9388	−0·9642	−0·9153
$\frac{1}{32}$	−0·9990	−1·0274	−1·1101	−1·1809
$\frac{1}{16}$	−1·1156	−1·1638	−1·3043	−1·4874
$\frac{1}{8}$	−1·3998	−1·4870	−1·7351	−2·1165
$\frac{3}{16}$	−1·6983	−1·8225	−2·1662	−2·7172
$\frac{1}{4}$	−1·9894	−2·1492	−2·5790	−3·2785
$\frac{3}{8}$	−2·5327	−2·7613	−3·3434	−4·2948
$\frac{1}{2}$	−3·0251	−3·3221	−4·0384	−5·1988
$\frac{3}{4}$	−3·8865	−4·3254	−5·2782	−6·7793
1	−4·6198	−5·2150	−6·3827	−8·1662
$\frac{3}{2}$	−5·7925	−6·7644	−8·3493	−10·6192
2	−6·6337	−8·0884	−10·1135	−12·8280

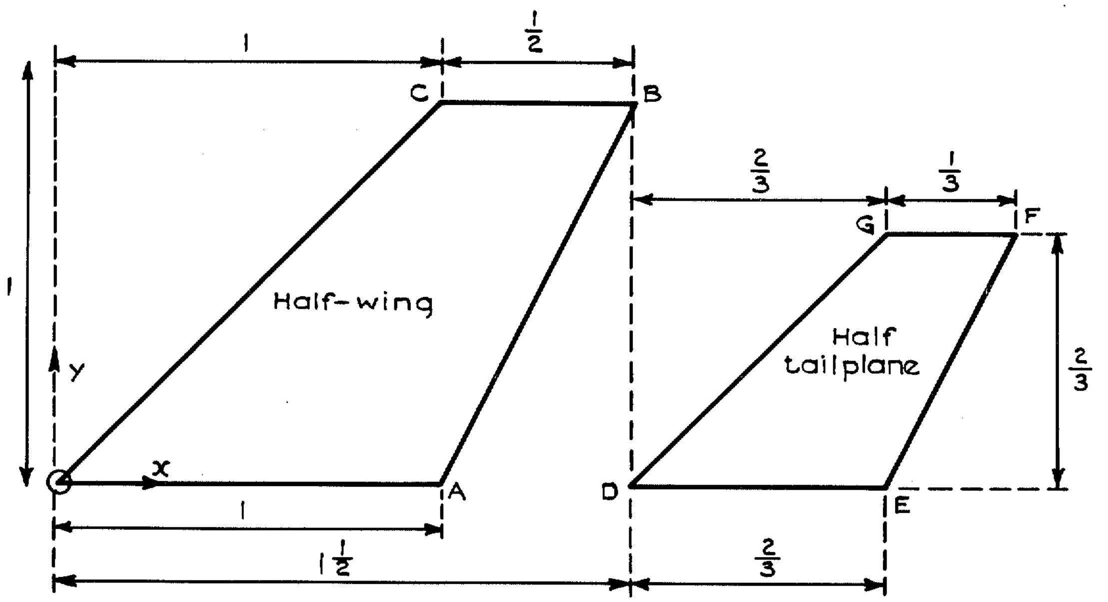


FIG. 1. Wing-tailplane configuration of Laschka and Schmid.

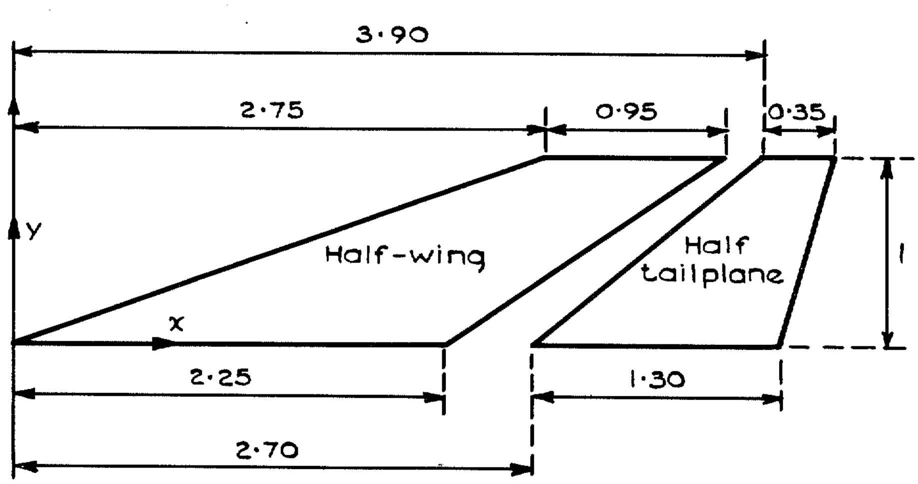


FIG. 2. Agard wing-tailplane configuration.

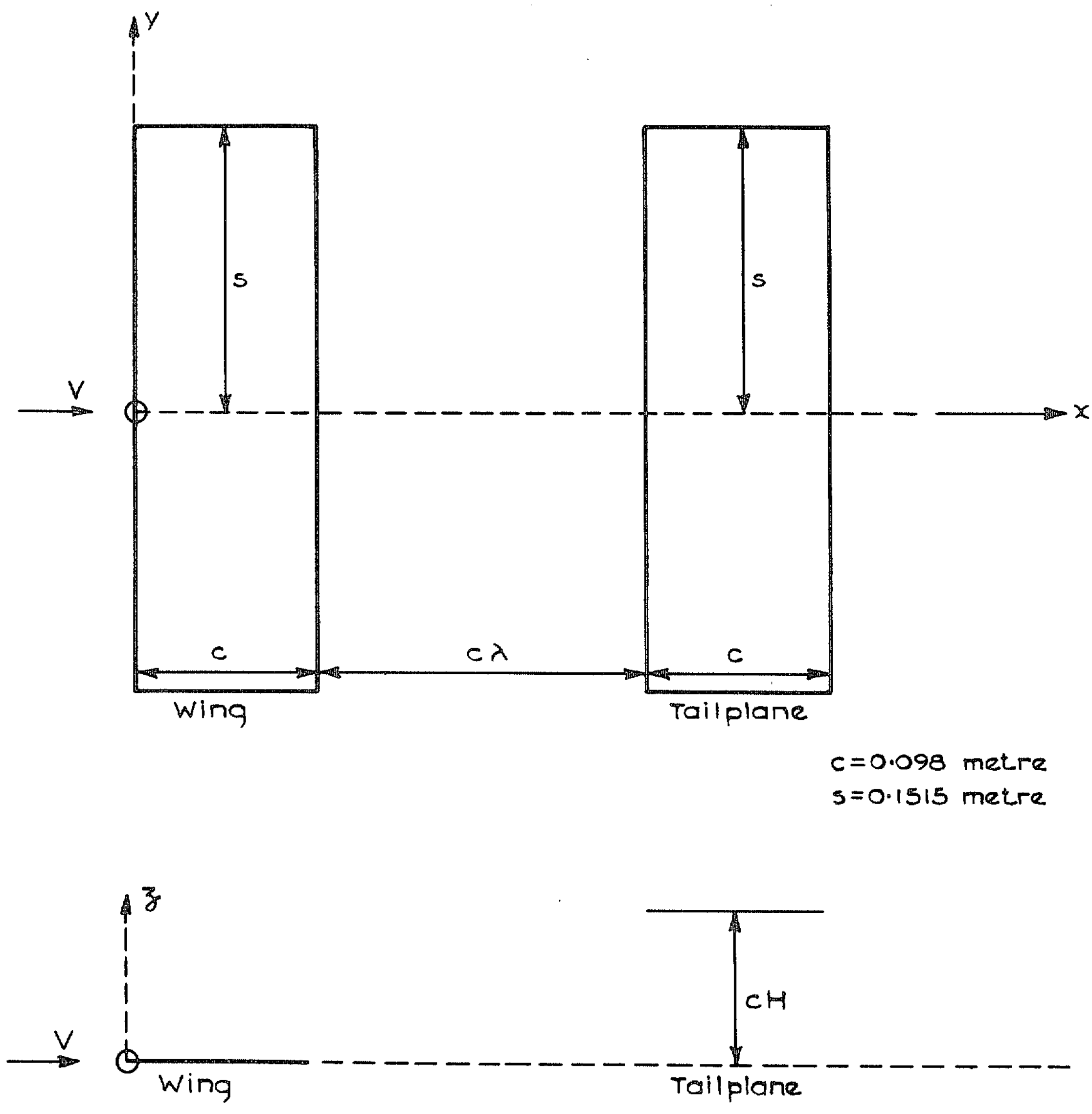


FIG. 3. Plan and side views of Onera wing-tailplane configuration.

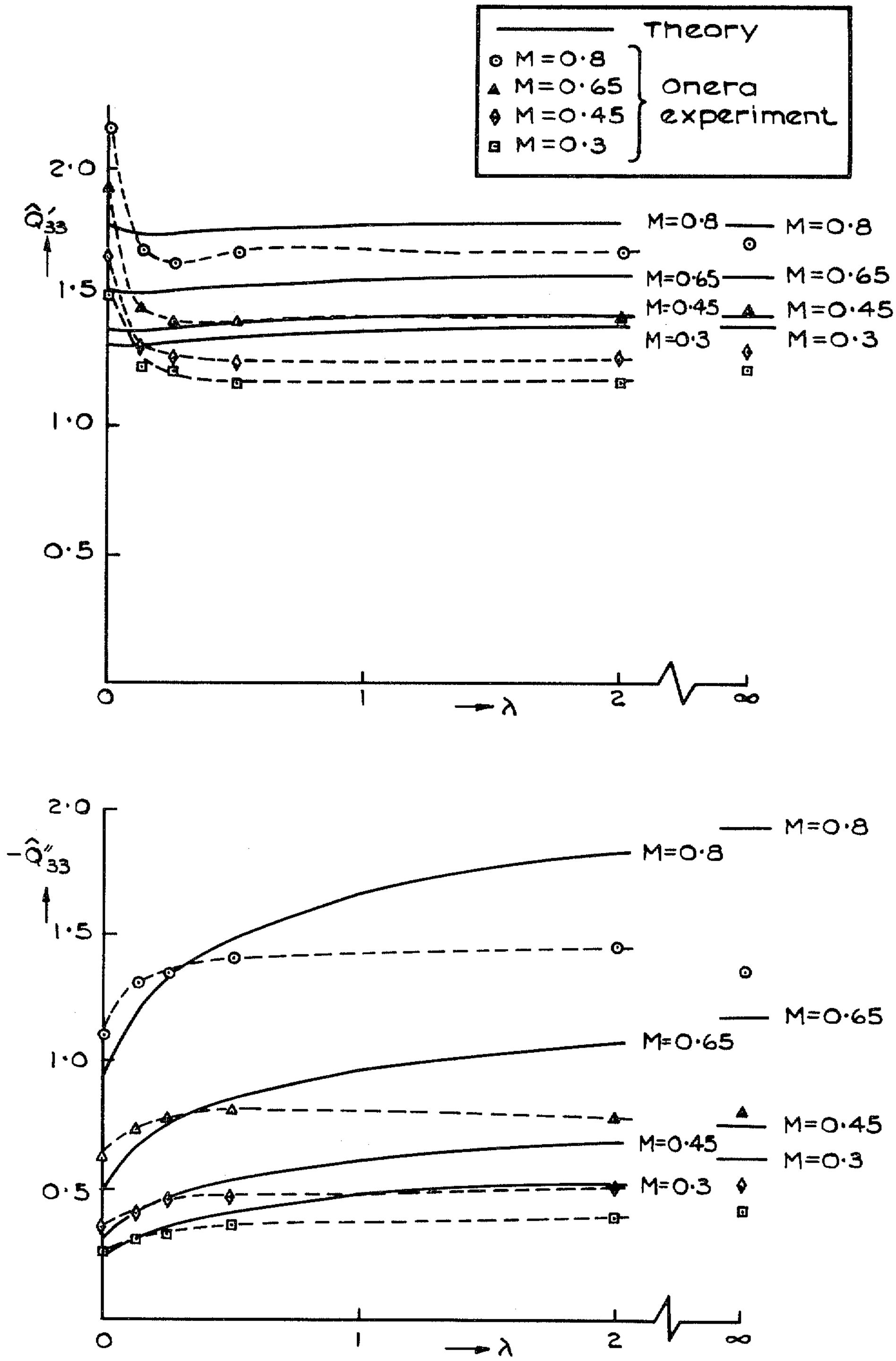


FIG. 4. Values of $\hat{Q}_{33}$ for $H = 0$ for Onera wing-tailplane configuration.

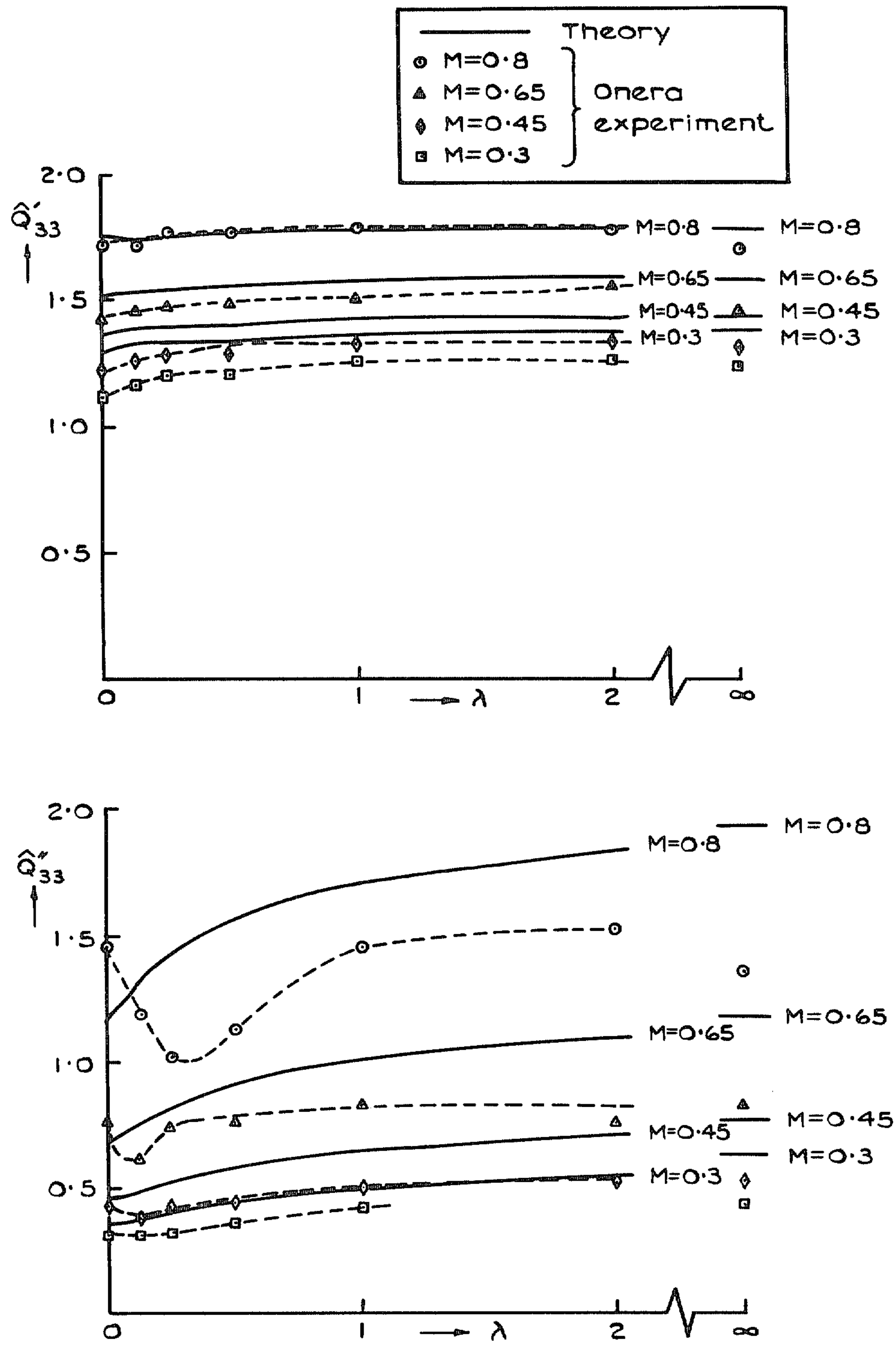
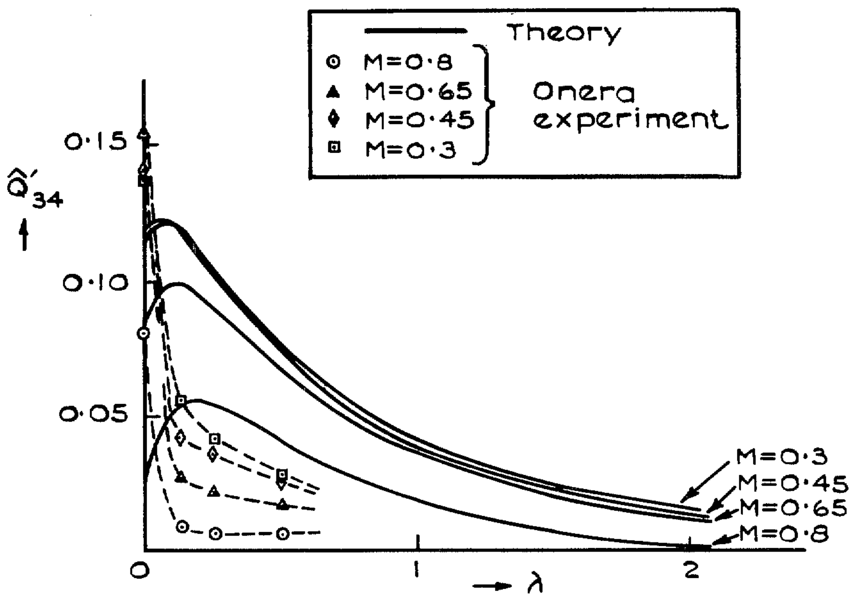


FIG. 5. Values of $\hat{Q}_{33}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.



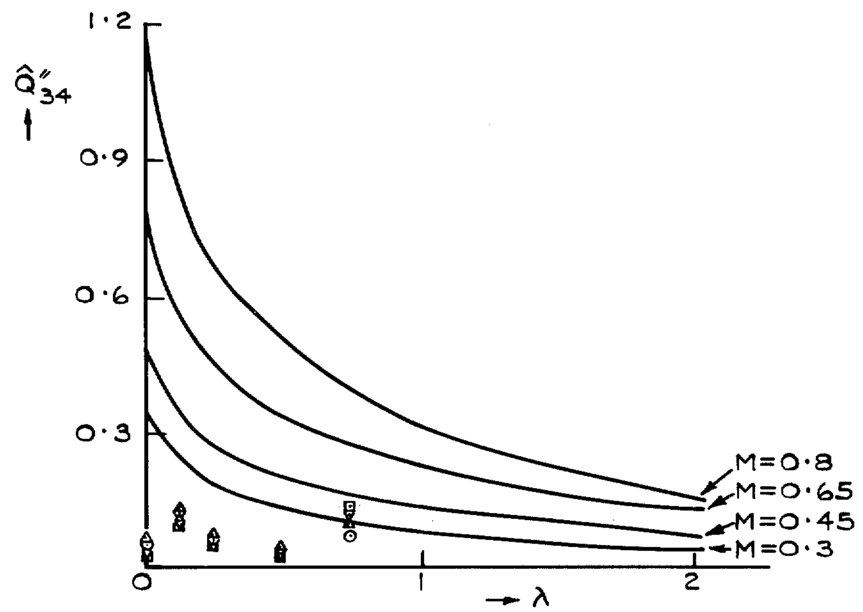
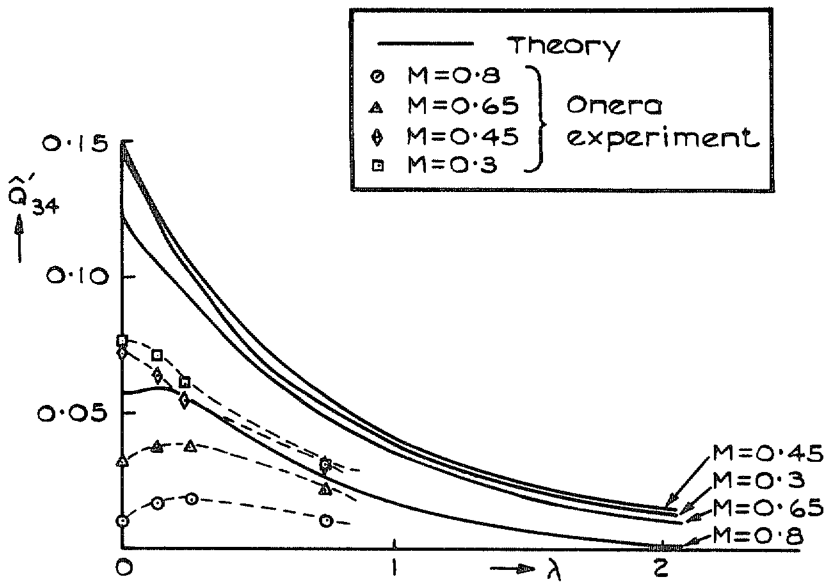


FIG. 6. Values of $\hat{Q}_{34}$ for $H = 0$ for Onera wing-tailplane configuration.



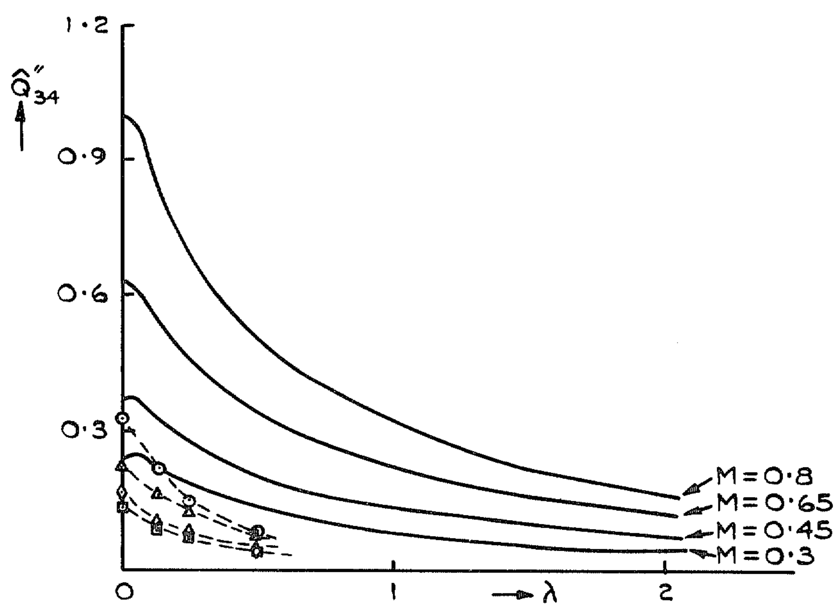
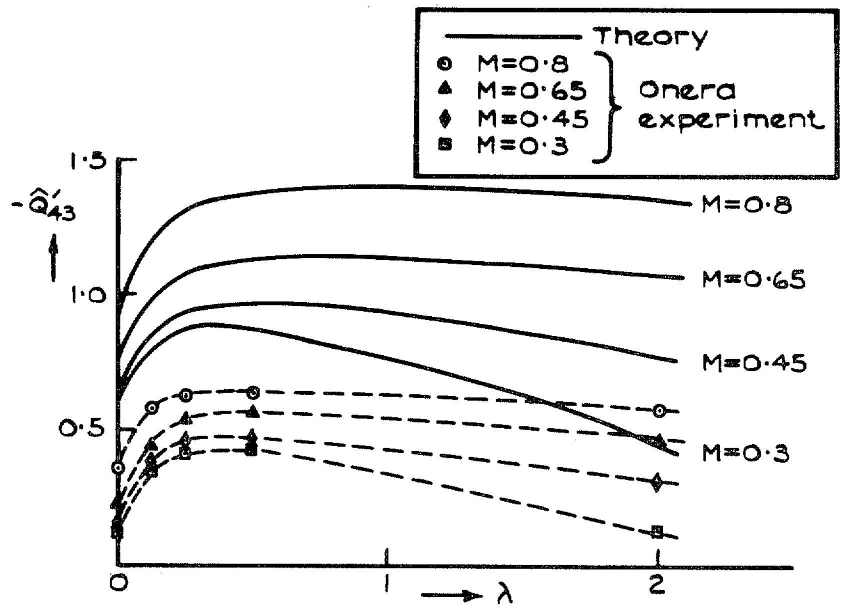


FIG. 7. Values of $\hat{Q}_{34}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.



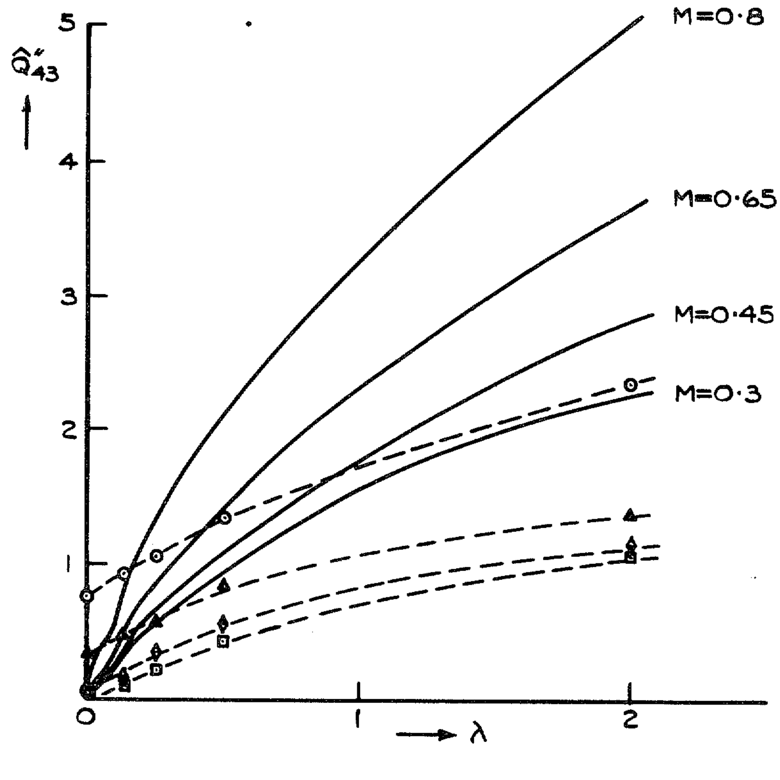


FIG. 8. Values of $\hat{Q}_{43}$ for $H = 0$ for Onera wing-tailplane configuration.

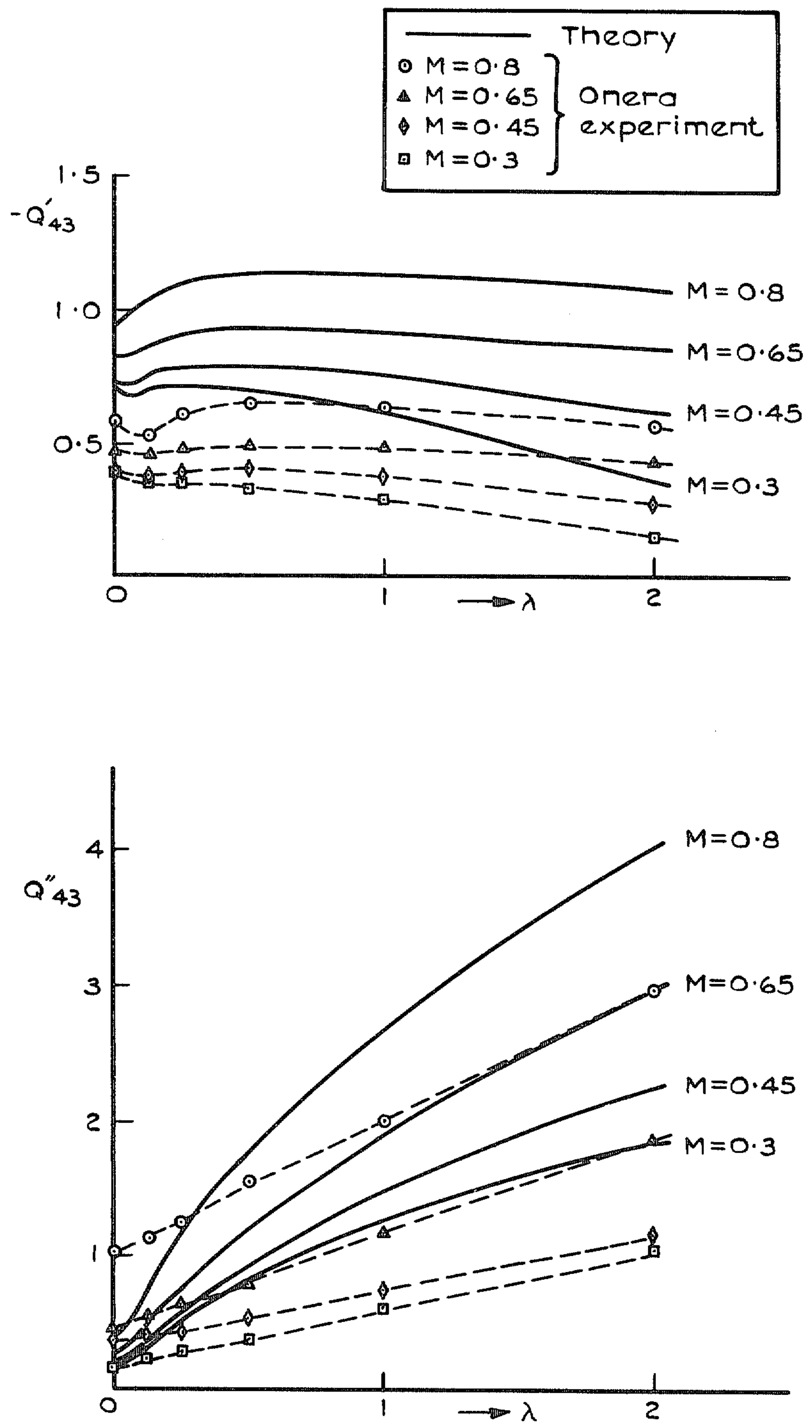


FIG. 9. Values of $\hat{Q}_{43}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.

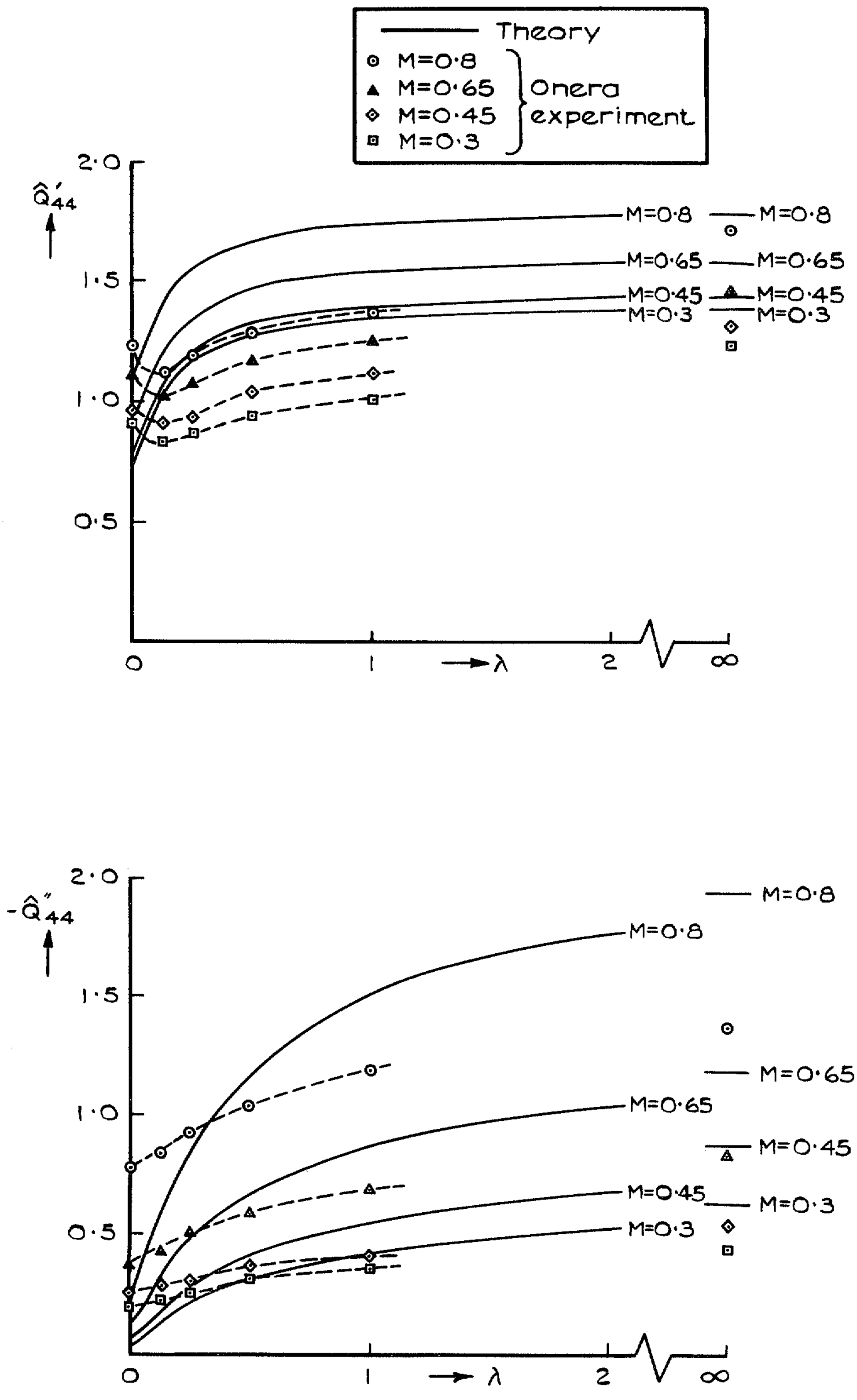


FIG. 10. Values of $\hat{Q}_{44}$ for $H = 0$ for Onera wing-tailplane configuration.

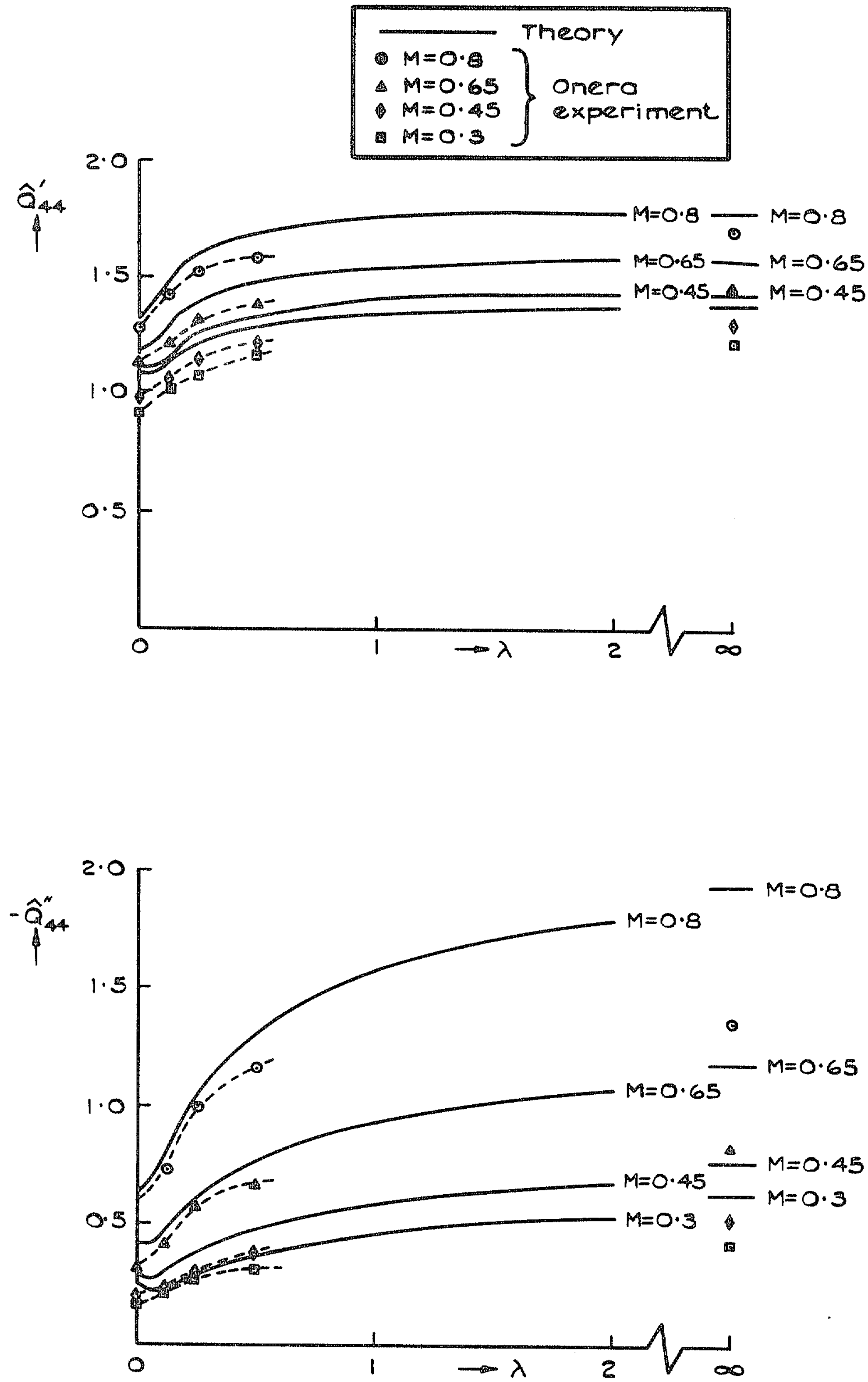


FIG. 11. Values of $\hat{Q}_{44}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.

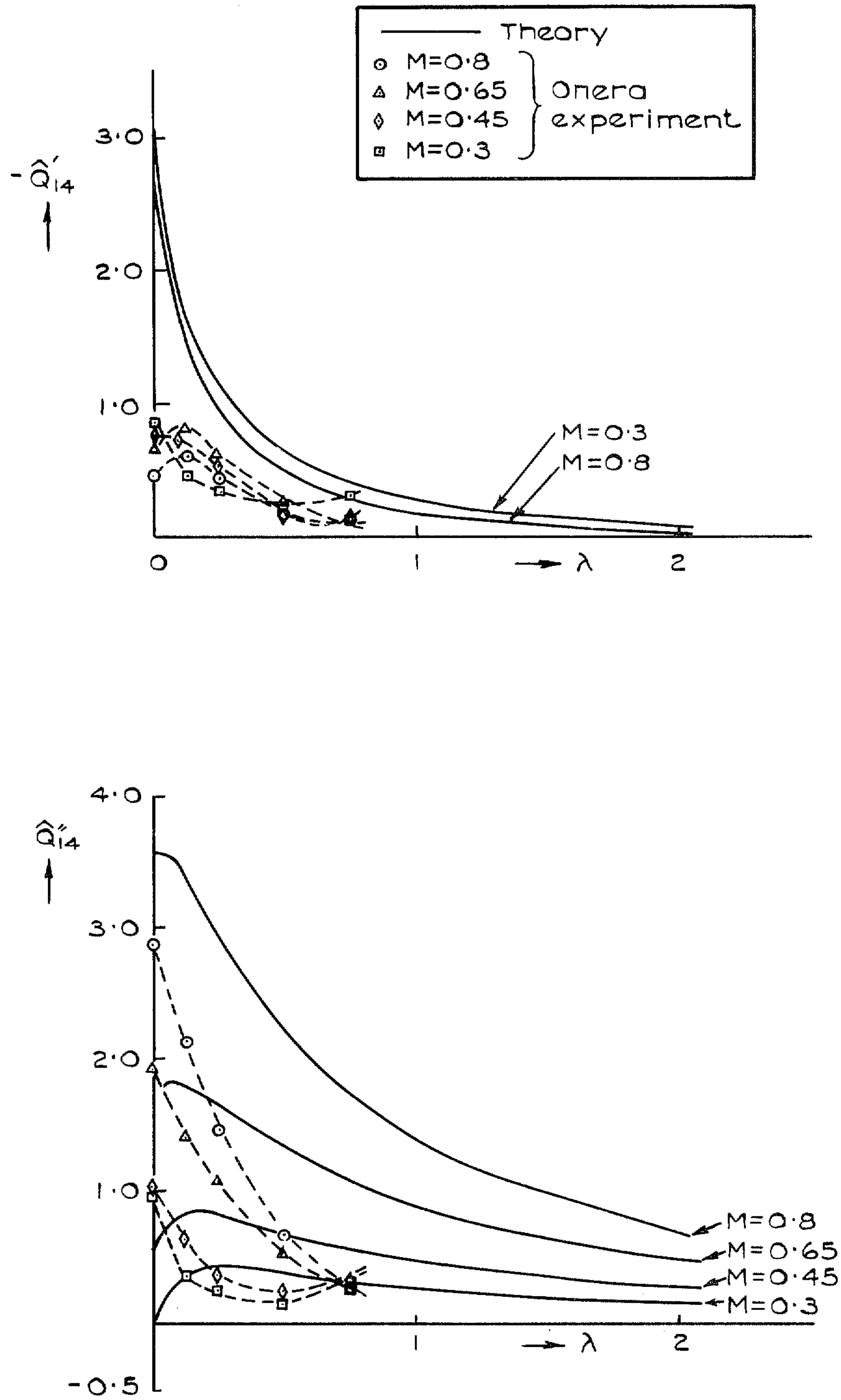


FIG. 12. Values of $\hat{Q}_{14}$ for $H = 0$ for Onera wing-tailplane configuration.

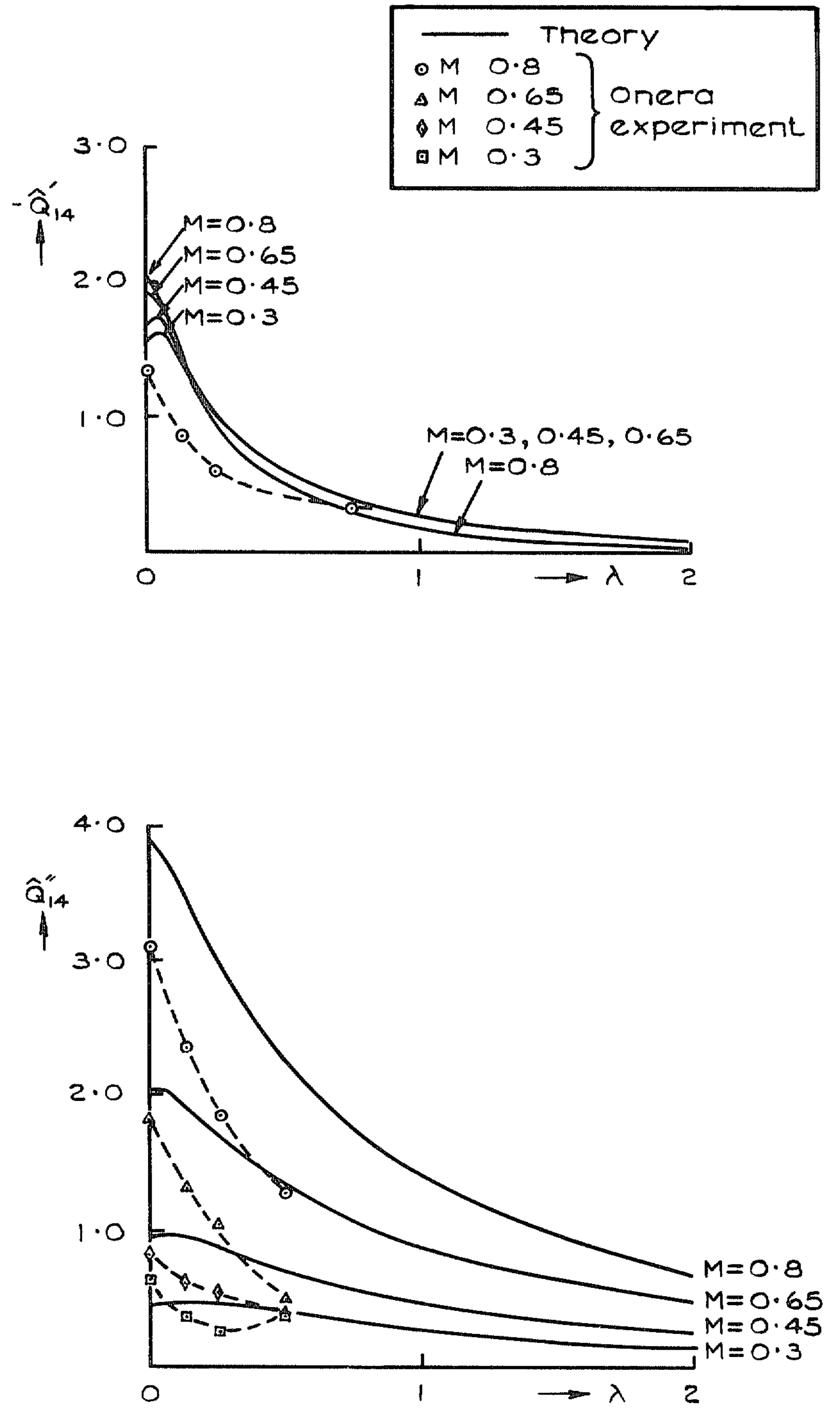


FIG. 13. Values of $\hat{Q}_{14}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.

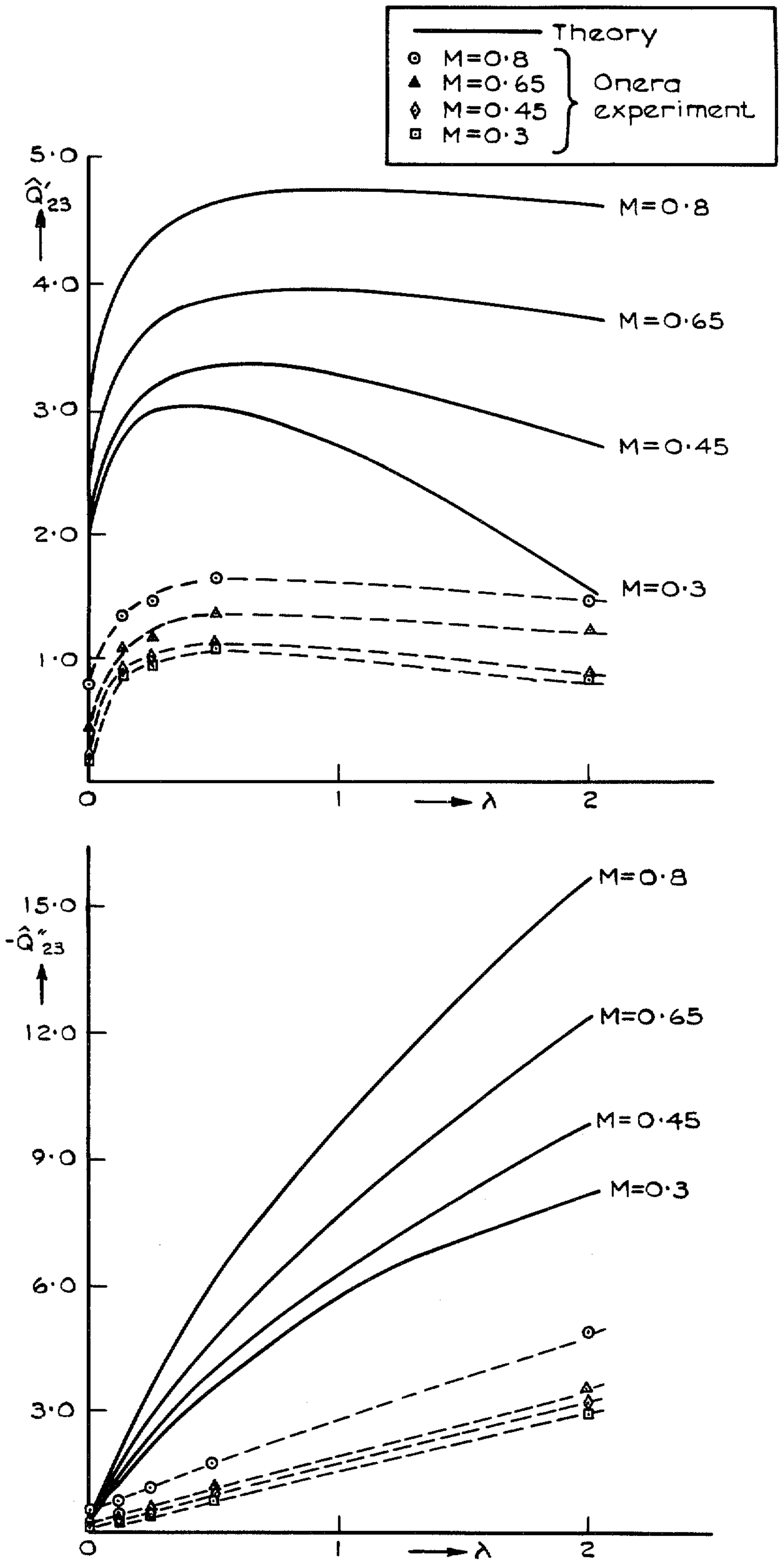


FIG. 14. Values of $\hat{Q}_{23}$ for $H = 0$ for Onera wing-tailplane configuration.

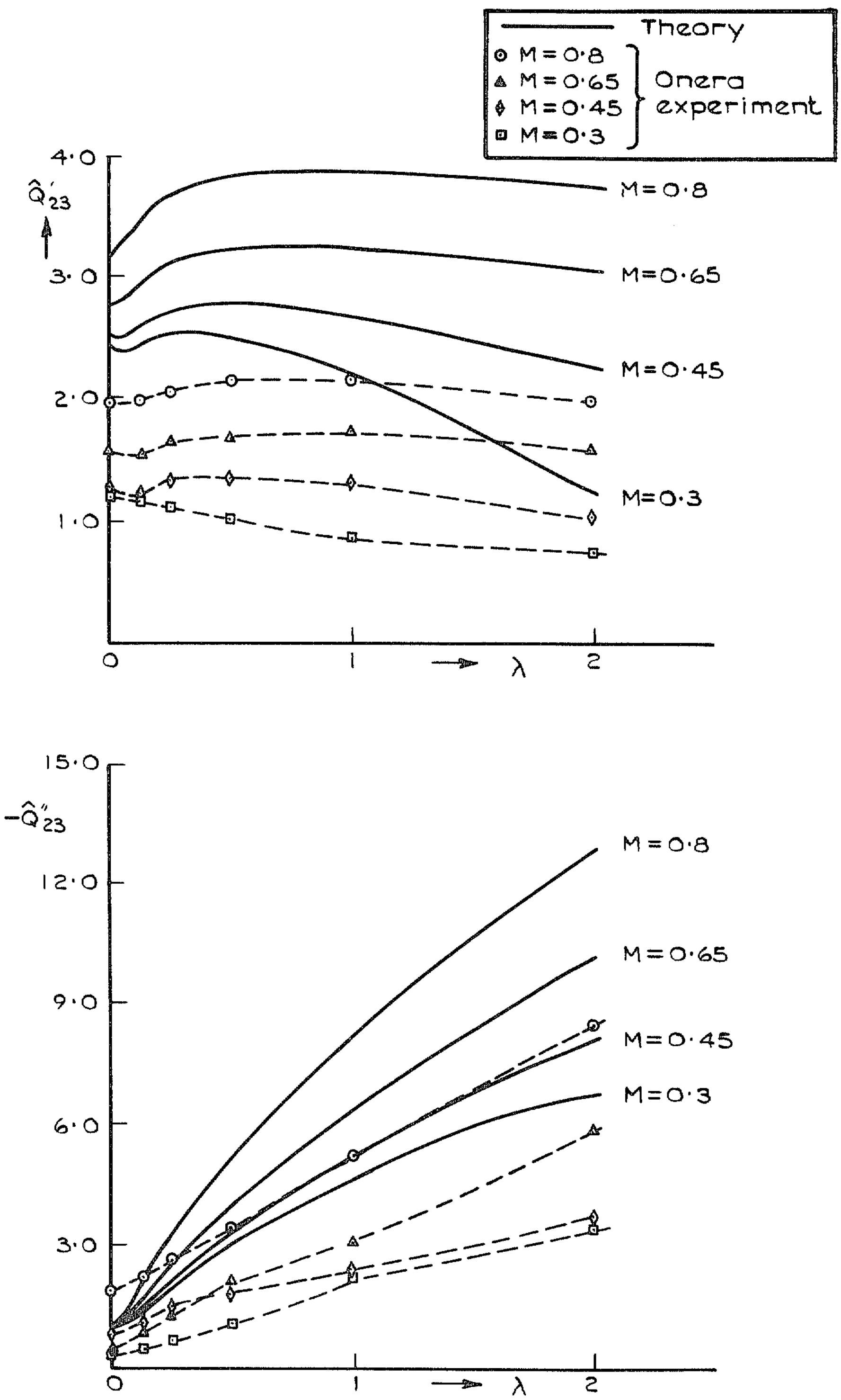


FIG. 15. Values of $\hat{Q}_{23}$ for $H = \frac{1}{8}$ for Onera wing-tailplane configuration.

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