C.P. No. 289
(18,038)
A.R.C. Technical Report

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By

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1956

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The Effects of Convex Surface Curvature on Boundary Layer Separation in Supersonic Flow

- By -
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26th November, 1955

SUMMARY

The effects of convex surface curvature on boundary layer separation in supersonic flow have been investigated experimentally and theoretically. The radius of curvature used in the experiments was about 3 times the distance from the leading edge of the surface on which the boundary layer was formed to the separation point. In accordance with theory, the surface curvature greatly reduced the separation pressure coefficient for laminar layers, but for turbulent layers (for which no theory of the effects of curvature has yet been developed) there was little effect.

1. Introduction and Description of Apparatus

A considerable amount of information is available[1] concerning boundary layer separation from flat surfaces in supersonic flow under conditions in which separation takes place well upstream of the shock wave or other agency provoking it, so that the process is governed by the equilibrium between the rate of growth of the boundary layer and the local pressure distribution. The present investigation is concerned with separation under similar conditions from curved convex surfaces whose generators are at right angles to the flow. This is of interest in connection with the flow past aerofoils.

Surface curvature affects separation in two different ways. On the one hand, the pressure gradients associated with the curvature well upstream of the separation region affect the upstream boundary layer velocity profiles and hence indirectly affect separation. On the other hand, there is also a direct effect of curvature arising from the curving away of the wall from underneath the separating boundary layer. In the present paper we are largely concerned with cases where the wall is flat upstream of the region of pressure increase which immediately precedes separation. In these circumstances only the second, direct, effect of curvature is operative.

It would be expected that surface curvature would have little direct effect on separation if, over the length of the compression region upstream of the separation point, the change of slope of the wall were small compared with the external-flow deflection caused by the thickening of the boundary layer. The latter deflection is roughly 2° for laminar layers at Reynolds numbers in the range 10^5 to 10^6, and 10° for turbulent ones at all Reynolds numbers [1,2]. The length of the compression region up to separation is of the order of ten boundary layer thicknesses for laminar layers and one for turbulent layers. Hence with turbulent layers the flat-plate separation data should be approximately applicable for

curvatures/

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curvatures much sharper in terms of boundary layer thickness than with laminar layers, and the present experiment confirms these expectations.

The experimental arrangement was as follows: Two plates, one with a curved portion and the other, for comparison, flat, as shown in Fig.1, were used as the surfaces on which the test boundary layers were formed. In terms of boundary layer thickness the curvature of the curved plate is of the same order as that over the rear half of a 10% thick biconvex aerofoil. Thus the plate has a rather sharper effective curvature than the thin aerofoils which are used at transonic and supersonic speeds. The curved and flat plates were supported in the 2.6" × 1.5" blow-down wind tunnel in the same way as the flat plate in the experiments reported in Ref.2, the tunnel wall boundary layer being ducted underneath. The experiment was performed at free stream Mach numbers of 2 and 3. It was not possible to use a lower Mach number because of the blockage of the plates, but it was considered that rough extrapolation of the results to the region of $M = 1.4$, (the sort of Mach number which typically occurs locally in transonic flow past aerofoils,) should be permissible. Separation was provoked at various positions on the upper surfaces of the plates either by means of a shock wave generated by a wedge held in the mainstream[2], or by spoilers attached to the surface. Figs. 2 to 4 are typical schlieren photographs of the flow showing these arrangements for causing separation. There were pressure tappings along the plates so that the pressure distributions over the surfaces could be determined.

The results for laminar and turbulent layers are now discussed separately.

2. Results for Laminar Layers

Figs. 5 and 6 show pressure distributions for the curved and flat plate respectively with various separation-provoking arrangements at a Mach number of 3. The stagnation pressure was fairly low (1.67 atmospheres) so that the flow was laminar at separation, and the curves show the characteristic "laminar foot"[2]. The lowest curves of Figs. 5 and 6 represent the empty tunnel distributions. These differ from the theoretical inviscid flow distributions, sketched in Fig.1, partly because of experimental imperfections but principally because of the presence of the boundary layer. Thus for the curved plate the corner formed at the downstream end of the curved part itself causes an upstream laminar separation, so that the minimum pressure occurs about 1.8" downstream of hole 1, and not at the corner, which is about 2.4" downstream of hole 1. Also, the pressures at hole 1 and for a short distance downstream are increased due to the effect, transmitted through the laminar boundary layer, of the relatively high pressures on the upstream inclined portions of the test surfaces. (See Fig.1)

It is possible to define a foot pressure ratio p_T/p_o, where p_o is the pressure at the point where the pressure begins to rise or at the pressure minimum, and p_T is the pressure at the "top" of the foot, defined as the pressure maximum position or the point of inflexion in the distribution, as in Fig. 7. The ratio p_T/p_o is greater than p_s/p_o, where p_s is the pressure at the separation point, because separation occurs somewhere on the relatively steep upstream part of the foot. With convex surface curvature the dead-air region near the wall downstream of separation thickens more rapidly than with a flat wall, so that the pressure gradient would be expected to fall off more sharply[2], and $p_T - p_s$ should be reduced. However, as shown in the Appendix, theory suggests that an analogous effect also operates upstream of separation. Accordingly one might expect both p_T/p_o and p_s/p_o to be reduced by curvature in roughly the same proportion. In other words, if p_T/p_o is found experimentally to decrease markedly, it is reasonable to

infer/

infer a similar variation in p_s/p_o. Thus the foot pressure ratio p_T/p_o, which can easily be determined experimentally, can be taken as a rough measure of the separation pressure ratio p_s/p_o, whose precise experimental determination is very difficult. In Fig. 8 the foot pressure ratio is plotted against the position of O, the point where the pressure is p_o. It can be seen that the flat plate results decrease slightly and quite smoothly with distance from the leading edge: this is an effect of the increasing Reynolds number[1, 2]. The values for the curved plate, by contrast, are much less than those for the flat plate when the pressure rise in the vicinity of separation at the upstream end of the laminar foot takes place entirely on the curved part of the surface. Where it takes place entirely on the upstream flat part the results agree with those for the flat plate, as would be expected. There is an intermediate region where some of the sharp pressure rise at the upstream end of the laminar foot is on the curved portion of the surface and some is on the flat.

Similar results were obtained at a Mach number of 2, as can be seen from Figs. 9 and 10. No results for the flat plate at this Mach number are presented as they were made difficult to interpret by certain experimental imperfections. If these had been absent, however, there can be no doubt that the results would have been similar to those obtained at $M = 3$. (See also Ref.2)

These results agree with the theory presented in the Appendix. This predicts that the curving away of the surface from underneath the boundary layer upsets the equilibrium between the layer and the external flow in such a way that, compared with the flat surface case, the pressure gradients in the vicinity of separation are slightly increased and the pressure coefficients at separation, (proportional to $p_s/p_o - 1$, and probably roughly proportional to $p_T/p_o - 1$,) are considerably reduced. The predicted reduction is about 50% at Mach numbers of 2 and 3 with the configuration of the present experiments, and this agrees reasonably with the experimental results. The minimum separation pressure coefficient should occur when the point O coincides with the junction J of the flat and curved parts of the surface. When O is downstream of J the favourable pressure gradients between J and O should, due to their effects on the boundary layer velocity profiles and also to the increase of Mach number, lead to some increase in the ratio p_T/p_o, but it is difficult to estimate the magnitude of this effect. However the experimental results of Figs. 8 and 10 show that it must be small.

3. Results for Turbulent Layers

Figs. 11 and 12 show pressure distributions for the curved and flat plate respectively with various separation-provoking arrangements at a Mach number of 3. They correspond to Figs. 5 and 6 which are for laminar layers. For the distributions of Figs. 11 and 12 the boundary layer flow was turbulent because the stagnation pressure was high (7.8 atmospheres) and also transition-promoting strips were attached to the surface in the neighbourhood of hole 1. The pressure distributions all show a "kink" between an initial region of sharply rising pressure and a region of reduced pressure gradients. This kink always occurs with turbulent layers when there is a considerable extent of separated flow[1,2], and a kink pressure p_K may then be defined as the intersection of the maximum and minimum slope tangents, as in Fig. 13. The reduced pressure gradients downstream of separation, giving rise to the kink, are due to the thickening of the dead-air region near the wall. With convex surface curvature the dead-air region must thicken more rapidly than with a flat wall, so that the pressure gradients should diminish more sharply downstream of separation. However, the kink is so close in position to the separation point that p_K/p_s (which is only a little greater than 1) will probably not be greatly affected by surface curvature. Hence p_K may be taken as a convenient measure of p_s, which is more difficult to measure experimentally. As in the laminar cases, a position O can be defined

as/

as the point where the pressure begins to rise or where the pressure is a minimum. In Fig. 14, the kink pressure ratio p_K/p_o, where p_o is the pressure at O, is plotted against the position of O. There is considerable experimental scatter but there are no obvious systematic differences between the results for the flat and curved plates. This is true also at a Mach number of 2, as can be seen from Figs. 15, 16 and 17. The Mach number M_o at O increases as separation is made to occur further downstream on the curved part of the curved surface, and one would expect this increase to result in an increase in p_K/p_o, since on a flat surface p_K/p_o increases with Mach number, especially at the lower Mach numbers. (c.f. Refs. 1 and 2.) Thus the scatter of results in Figs. 14 and 17 may possibly be concealing a tendency for separation to occur at a slightly lower pressure ratio p_s/p_o on the curved surface than it would occur on a flat surface with the same upstream Mach number M_o. However, the variation in M_o is between about 3.1 and 3.6 in Fig.11 and 2 and 2.25 in Fig.15, and such variations in M_o would, on a flat surface, produce variations of p_K/p_o no greater than the scatter of the results of Figs.14 and 17. Accordingly, the suggestion that the curvature may, on the average, be reducing p_K/p_o should be treated with caution. The noteworthy conclusion to be drawn from Figs. 14 and 17 is that with turbulent flow, in contrast with laminar flow, surface curvature of the amount used in the experiments has at the most only a small effect on the pressure coefficient at separation. This is understandable since, as was pointed out in the Introduction, the flat-plate separation data would be expected to be applicable for much sharper curvatures with turbulent than with laminar layers. It is, however, quite possible that, with turbulent layers, effects qualitatively similar to those observed with laminar separations would be produced if the curvature were made sufficiently sharp.

4. Conclusions

The experimental results for laminar layers confirm the theoretical prediction of the Appendix that convex surface curvature reduces the separation pressure ratio. Comparison with the theory shows that for Reynolds numbers in the range 10^5 to 10^6 and Mach numbers in the range 1.5 to 4 appreciable differences from the flat plate results should occur if the radius of curvature of the surface is less than about 20 times the distance from the leading edge of the boundary layer. In the present experiments the radius of curvature was only about 3 times the distance from the leading edge of the boundary layer, and the effects on laminar separation were accordingly large. However this order of curvature has been found to have small effects on turbulent layers at Mach numbers of 2 and 3, and this is probably true also at lower Mach numbers. Hence it is reasonable to infer that with turbulent boundary layers on aerofoils, the flat plate data for separation should be roughly applicable both in transonic and supersonic flow so long as the radius of curvature of the surface under the shock wave exceeds about 3 times the distance from the leading edge to the shock (and even sharper curvatures might well be permissible). In actual experiments on aerofoils in transonic flow[3], it is found that the kink pressure ratio p_K/p_o depends to some extent on the aerofoil shape as well as on the local upstream Mach number M_o. However the results cannot be correlated in any simple way with the local surface curvature, and furthermore, the variations in p_K/p_o for any given M_o are quite small - no bigger than the experimental scatter of the results of the present investigation shown in Figs. 14 and 17.

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References

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2	G. E. Gadd, D. W. Holder and J. D. Regan	An experimental investigation of the interaction between shock waves and boundary layers. Proc.Roy.Soc. A Vol.226, p.227, (1954)
3	H. H. Pearcey	Some effects of shock-induced separation of turbulent boundary layers in transonic flow past aerofoils. Paper No. 9 presented at the Symposium on Boundary Layer Effects in Aerodynamics, N.P.L. March - April, 1955.
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APPENDIX/

APPENDIX

Theoretical Analysis of the Direct Effect of Surface Curvature on Laminar Separation

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It is assumed that, as in Fig.18, the surface is flat up to the position O where the pressure begins to increase. Hence the boundary layer profile at O is the same as on a flat plate, and only the direct effect of curvature on separation arises. (c.f. Introduction.) For simplicity, the curvature is assumed to be constant downstream of O.

The analysis proceeds exactly as in Ref.4 as far as equation (14). In place, however, of assuming that the inclination α of the external flow at s to its direction at O is given by

$$\alpha = \frac{2\Delta z}{X}$$

we must write

$$\alpha = \frac{2\Delta z}{X} - mX$$

where m is a constant depending on the curavture.

Hence

$$C_{ps} = \frac{2\alpha}{(M_{EO}^2 - 1)^{\frac{1}{2}}} = \frac{5.84[I_W(\gamma - 1)C_{ps}]^{\frac{1}{2}}M_{EO}}{(M_{EO}^2 - 1)^{\frac{1}{2}}\, X \left(\frac{\partial u}{\partial z}\right)_{WO}} - \frac{2mX}{(M_{EO}^2 - 1)^{\frac{1}{2}}}$$

Also as before

$$C_{ps} = \frac{\mu_W \left(\frac{\partial u}{\partial z}\right)_{WO}^2 X}{[12\, I_W\, (\gamma - 1)C_{ps}]^{\frac{1}{2}}\, \gamma M_{EO}^3\, p_o}$$

and

$$\mu_W\left(\frac{\partial u}{\partial z}\right)_{WO} = 0.332\, \frac{\rho_{EO}\, u_{EO}^2}{\sqrt{R_o}}$$

From these equations and the assumption that viscosity is proportional to absolute temperature it can be shown that

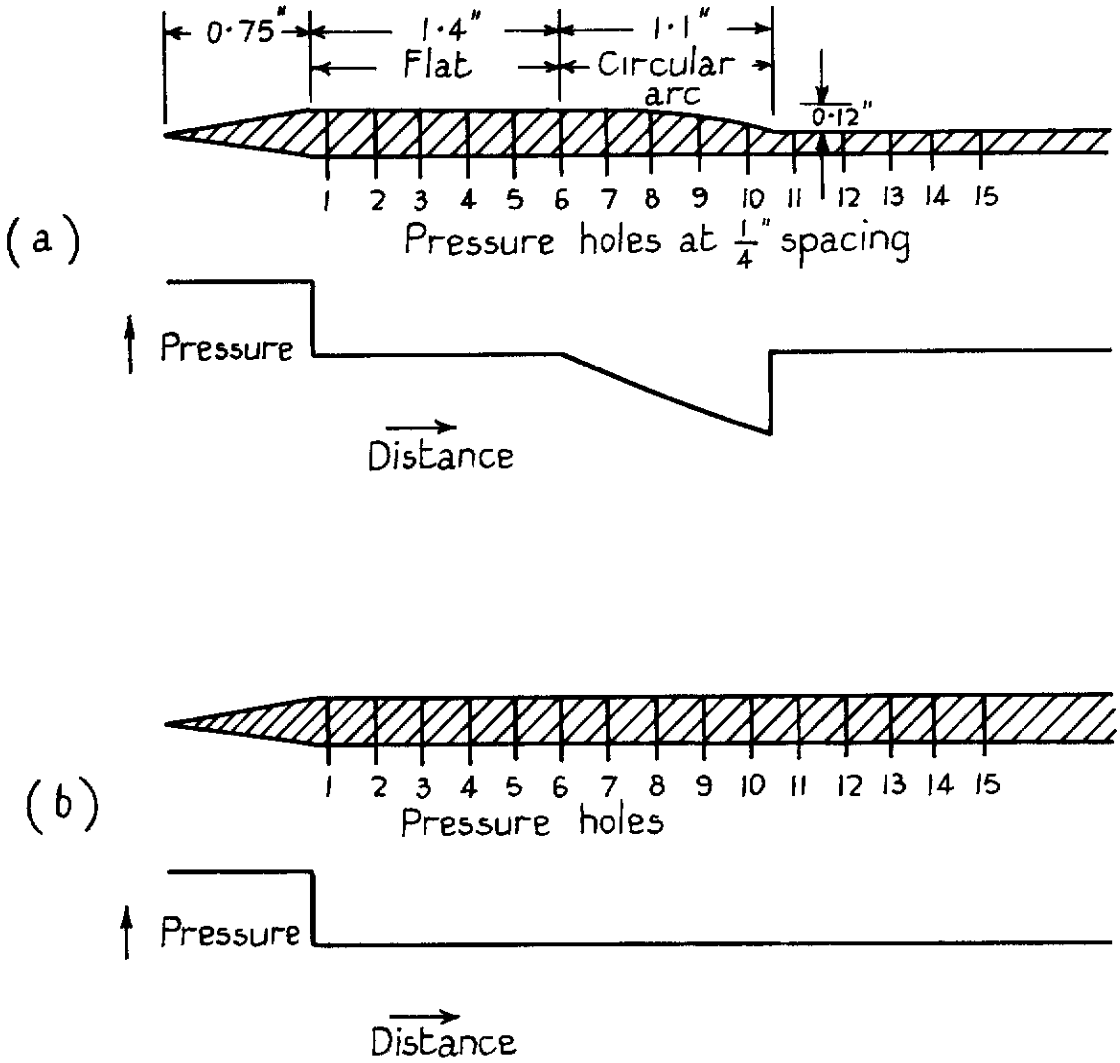
$$C_{ps}\left[1 + 63\left(\frac{T_{WO}}{T_{EO}}\right)^{\frac{3}{2}} \frac{mx_o\, C_{ps}^{\frac{1}{2}}}{(M_{EO}^2 - 1)^{\frac{1}{2}}}\right]^{\frac{1}{2}} = \frac{0.75}{[(M_{EO}^2 - 1)R_o]^{\frac{1}{4}}}$$

and

$$\frac{X}{x_o} = 31.3\left(\frac{T_{WO}}{T_{EO}}\right)^2 C_{ps}^{\frac{3}{2}}$$

In the present experiments the surface is not flat all the way upstream of O. However when O coincides with the junction J between the flat and curved portions of the surface the boundary layer profiles at O should be close to the flat plate form, and we can assume the effective values $R_o \doteqdot 0.6 \times 10^6$ at $M_{EO} = 3$, $R_o \doteqdot 0.4 \times 10^6$ at $M_{EO} = 2$, and $mx_o \doteqdot \frac{1}{3}$. Hence the curvature should reduce C_{ps} from about 0.016 to 0.008 at $M_{EO} = 3$, and from about 0.023 to about 0.011 at $M_{EO} = 2$. In both cases there is thus about a 50% reduction in C_{ps}, in quite good agreement with the observed reductions in $(p_T - p_o)/p_o$.

FIG 1.



Shapes of (a) curved plate and (b) flat plate with corresponding theoretical inviscid empty tunnel pressure distributions

FIGS. 2-4.

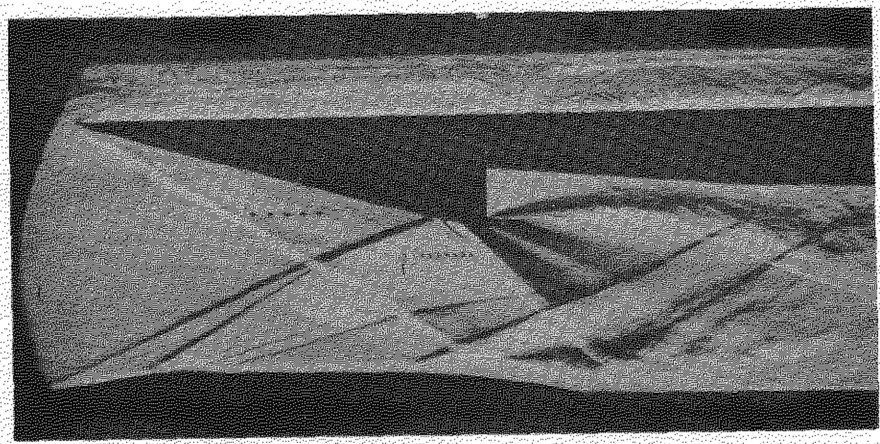


FIG. 2. Mach number about 3, 7·8 atmospheres stagnation pressure, turbulent boundary layer with transition-promoting strips.

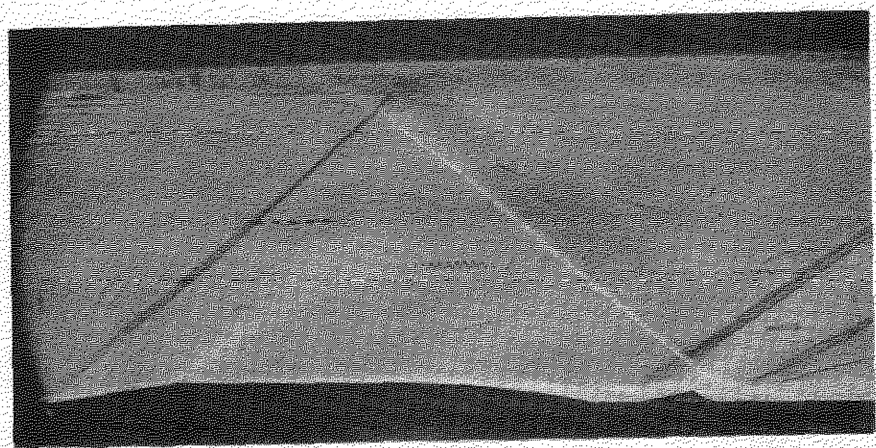


FIG. 3. Mach number about 2, 0·67 atmospheres stagnation pressure, laminar boundary layer.

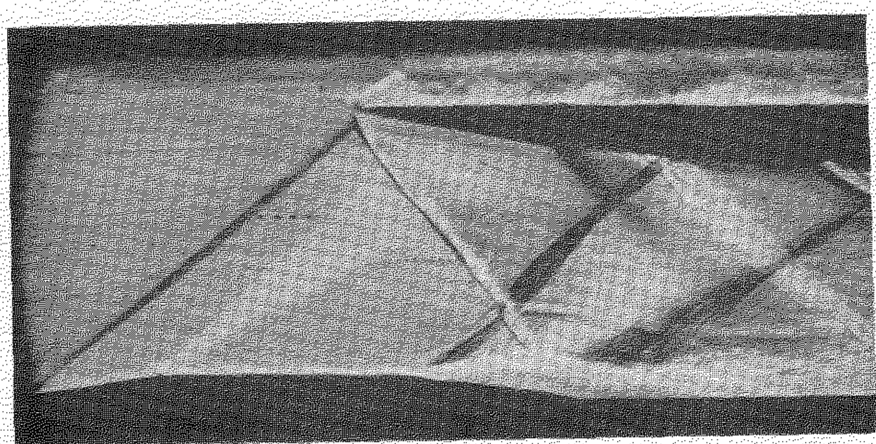
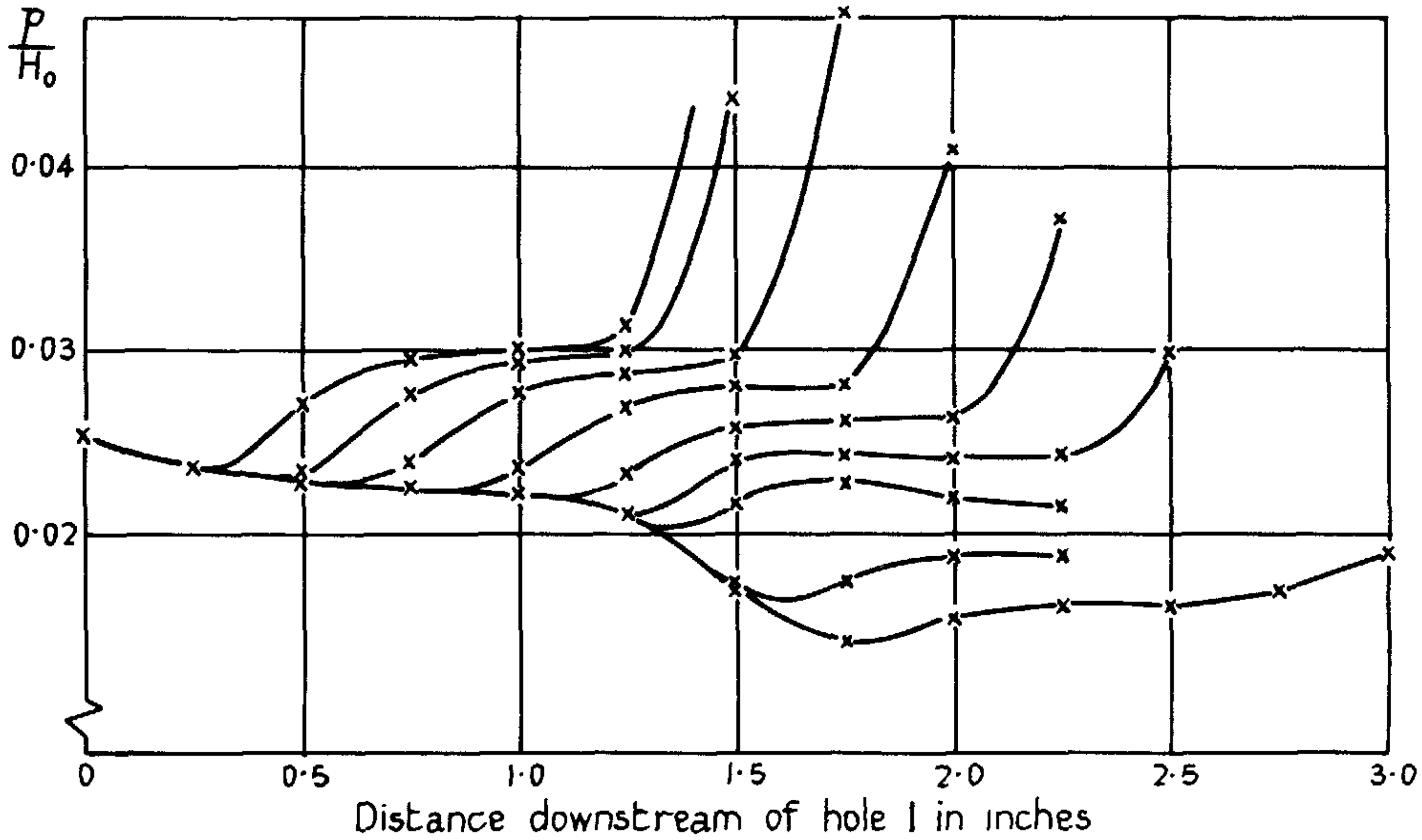
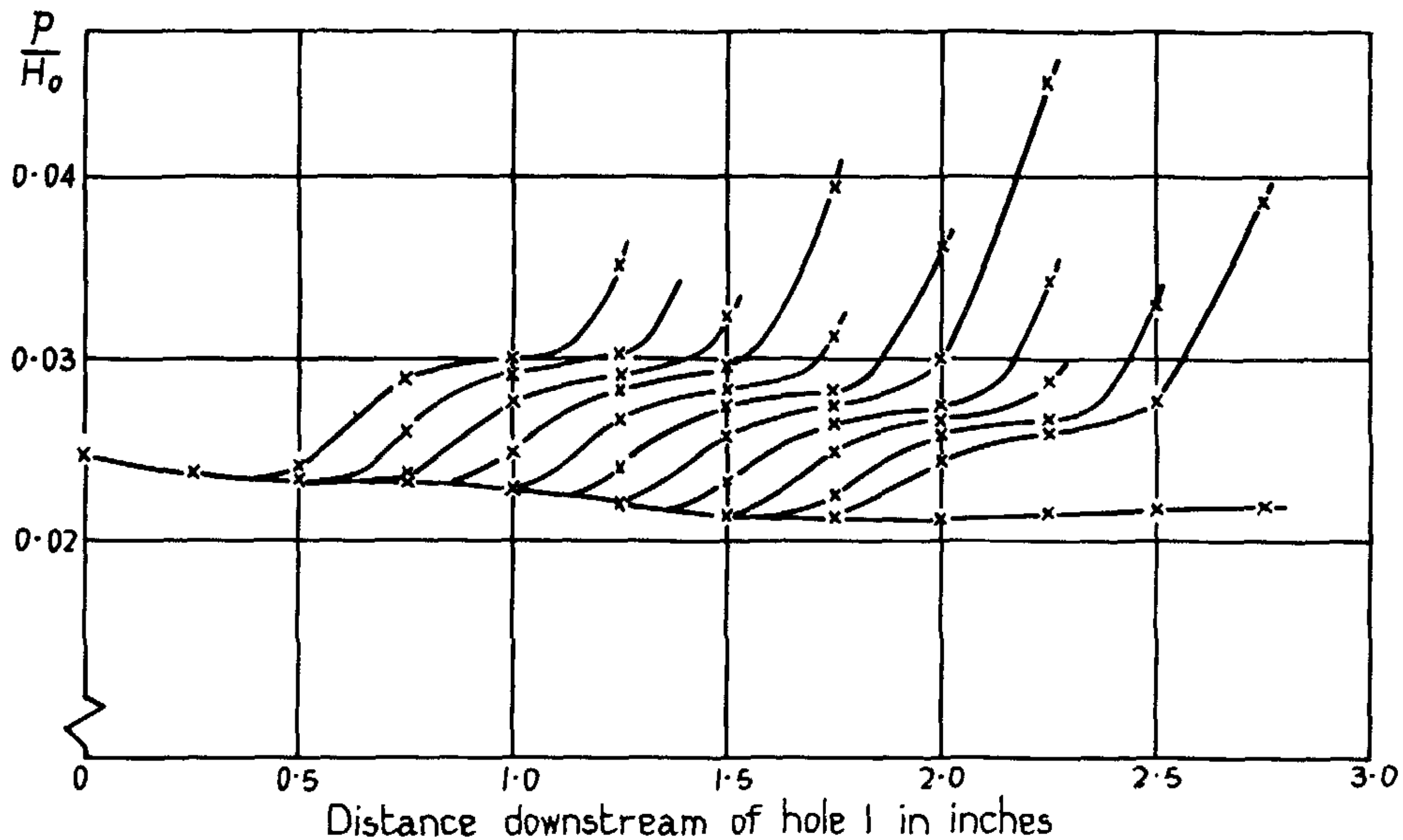


FIG. 4. Mach number about 2, 6 atmospheres stagnation pressure, turbulent boundary layer with transition free.



Pressure distributions on curved plate with a laminar layer at a free stream Mach number of about 3. Stagnation pressure H_0 = 1·67 atmospheres

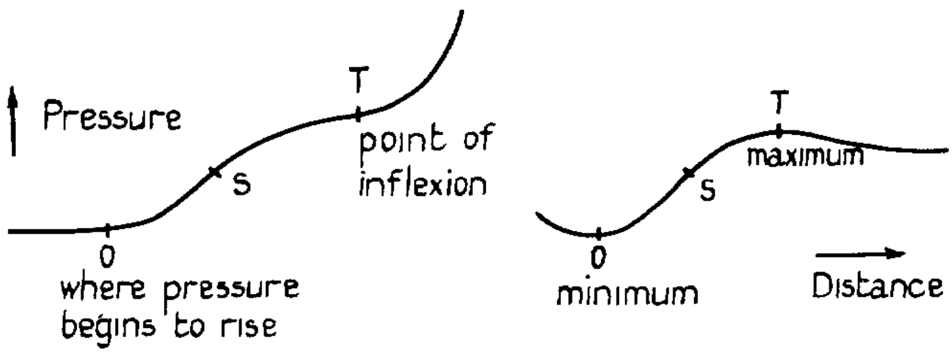
FIG. 5



Pressure distributions on flat plate with a laminar layer at a free stream Mach number of about 3. Stagnation pressure H_0 = 1·67 atmospheres

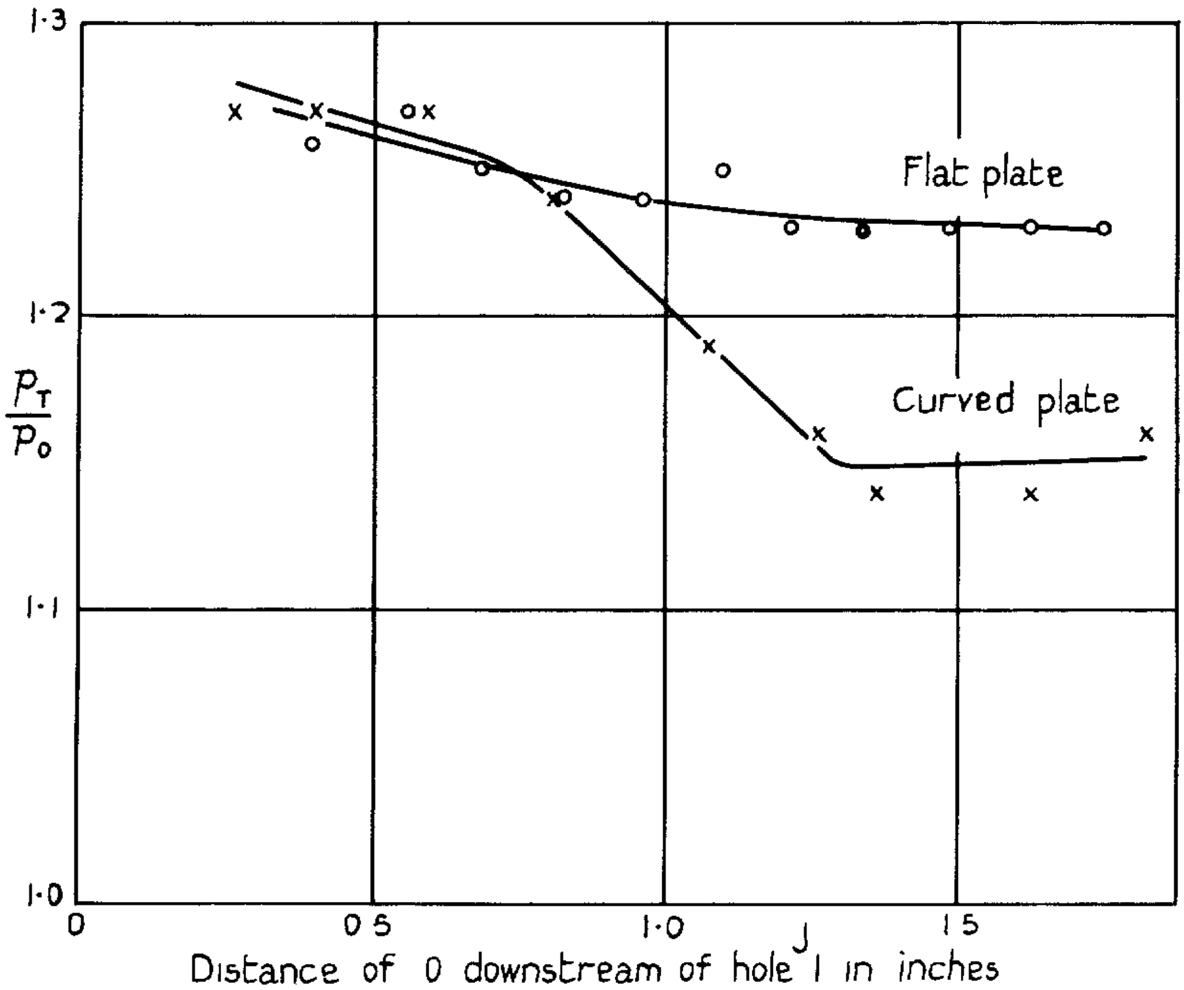
FIG 6.

FIG. 7.



Definition of points 0 and T on laminar foot
S = separation point

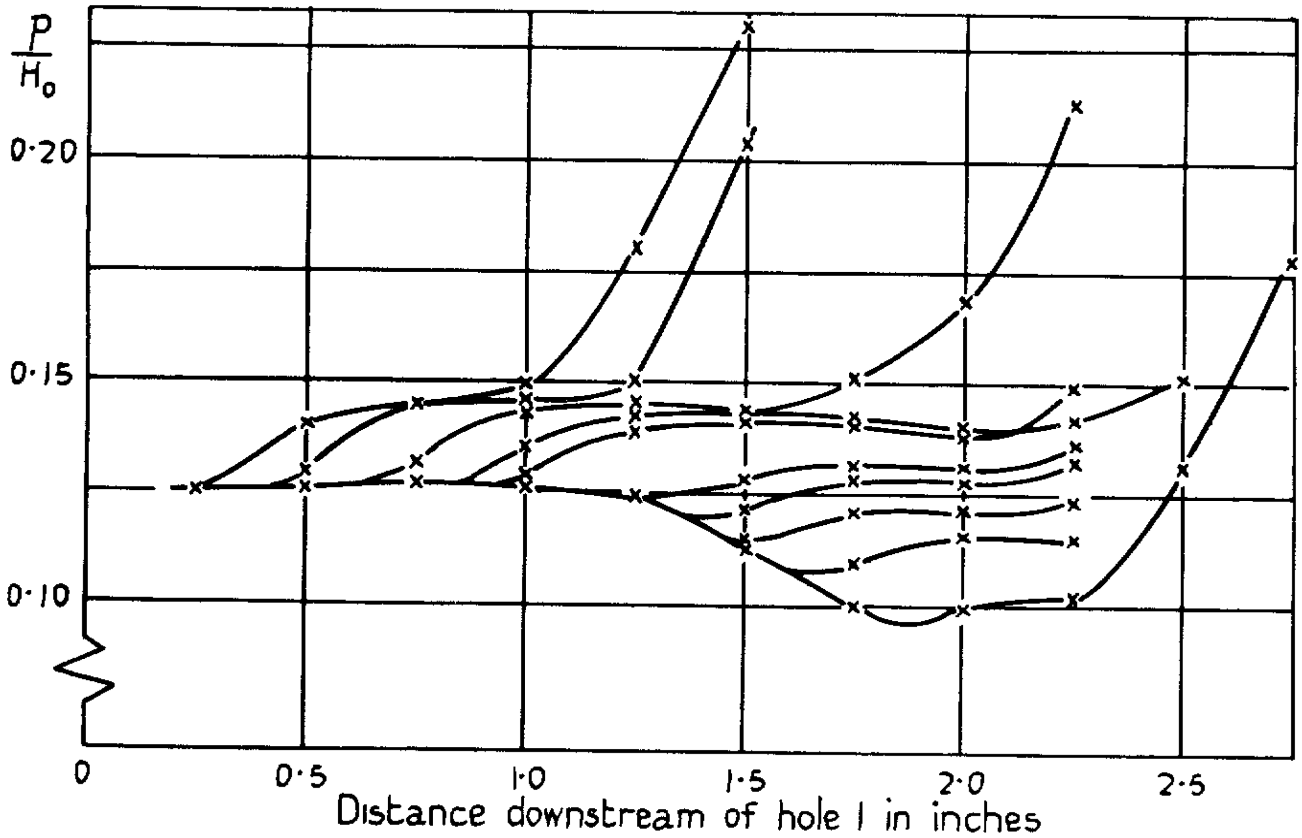
FIG 8.



Laminar foot pressure ratio for flat and curved plates at a Mach number of 3. The curved plate is curved downstream of J

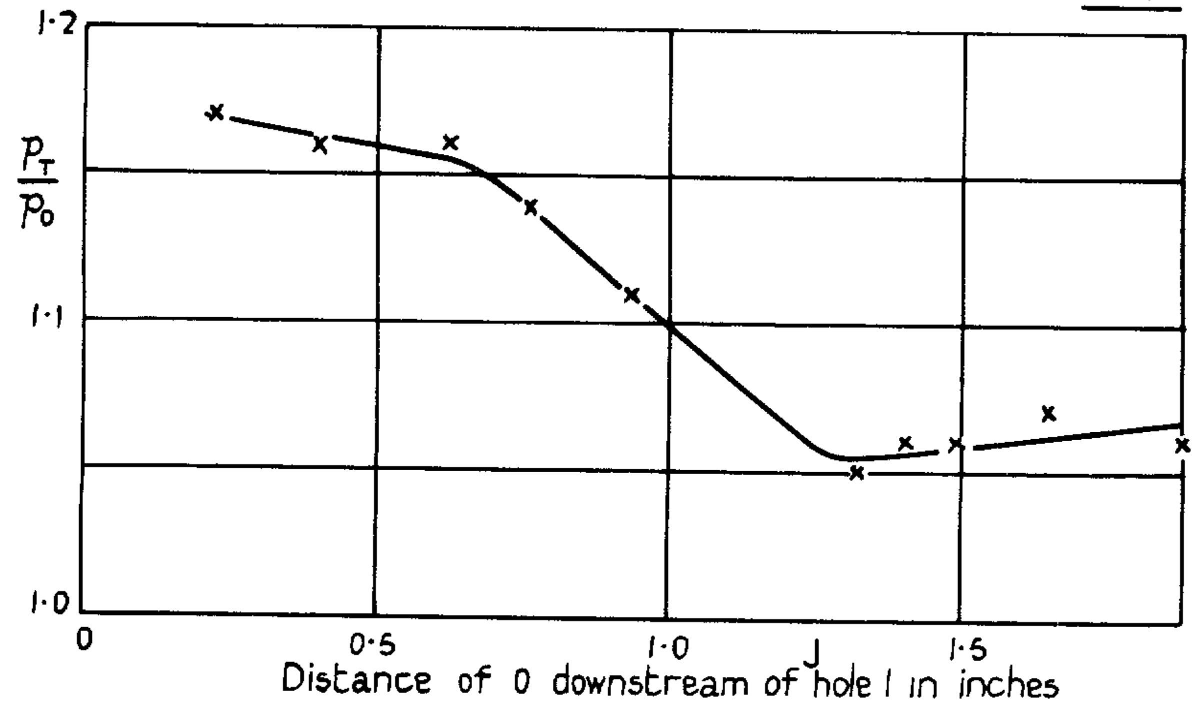
FIGS. 9 & 10.

FIG 9.



Pressure distributions on curved plate with a laminar layer at a free stream Mach number of about 2 Stagnation pressure H_0 = 0 67 atmospheres

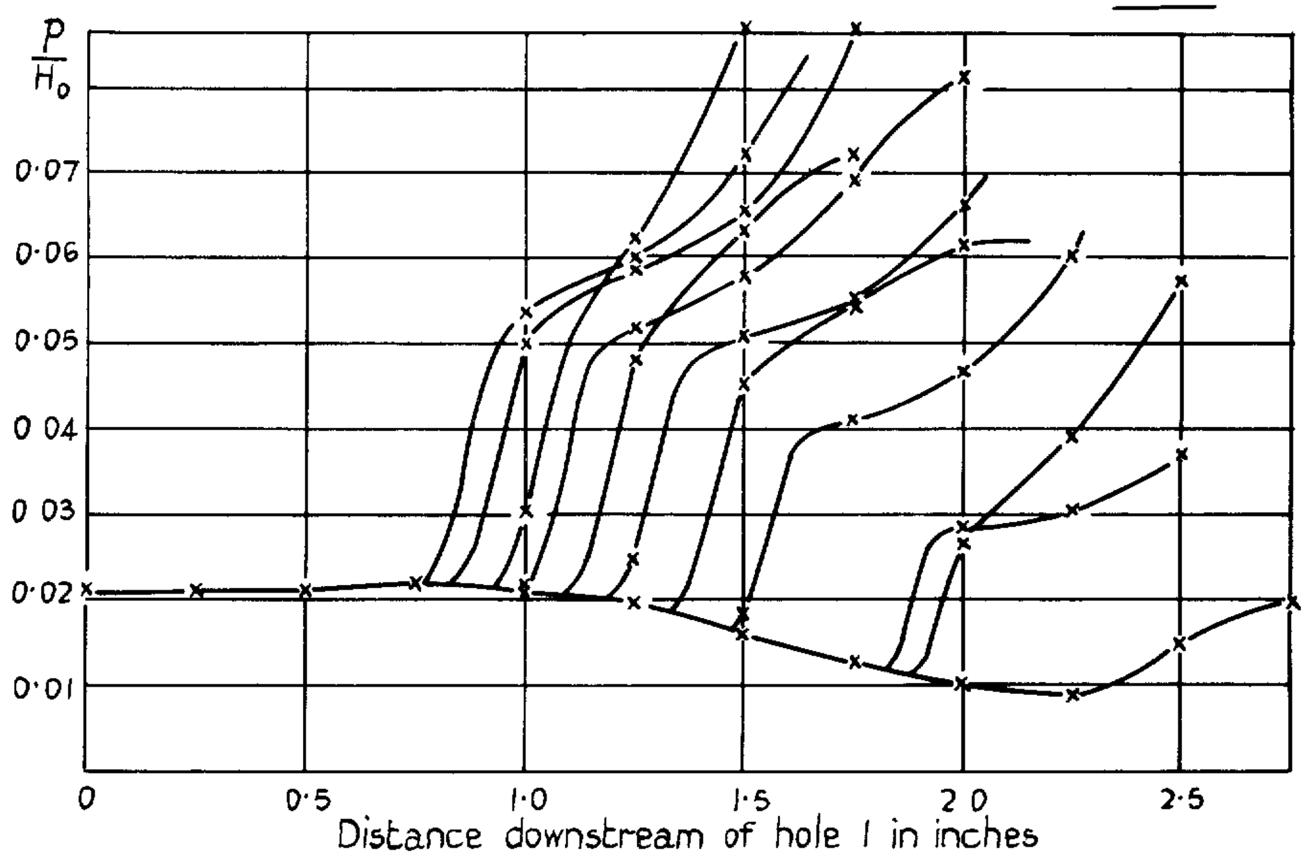
FIG 10



Laminar foot pressure ratio for curved plate at a Mach number of 2. The curved plate is curved downstream of J

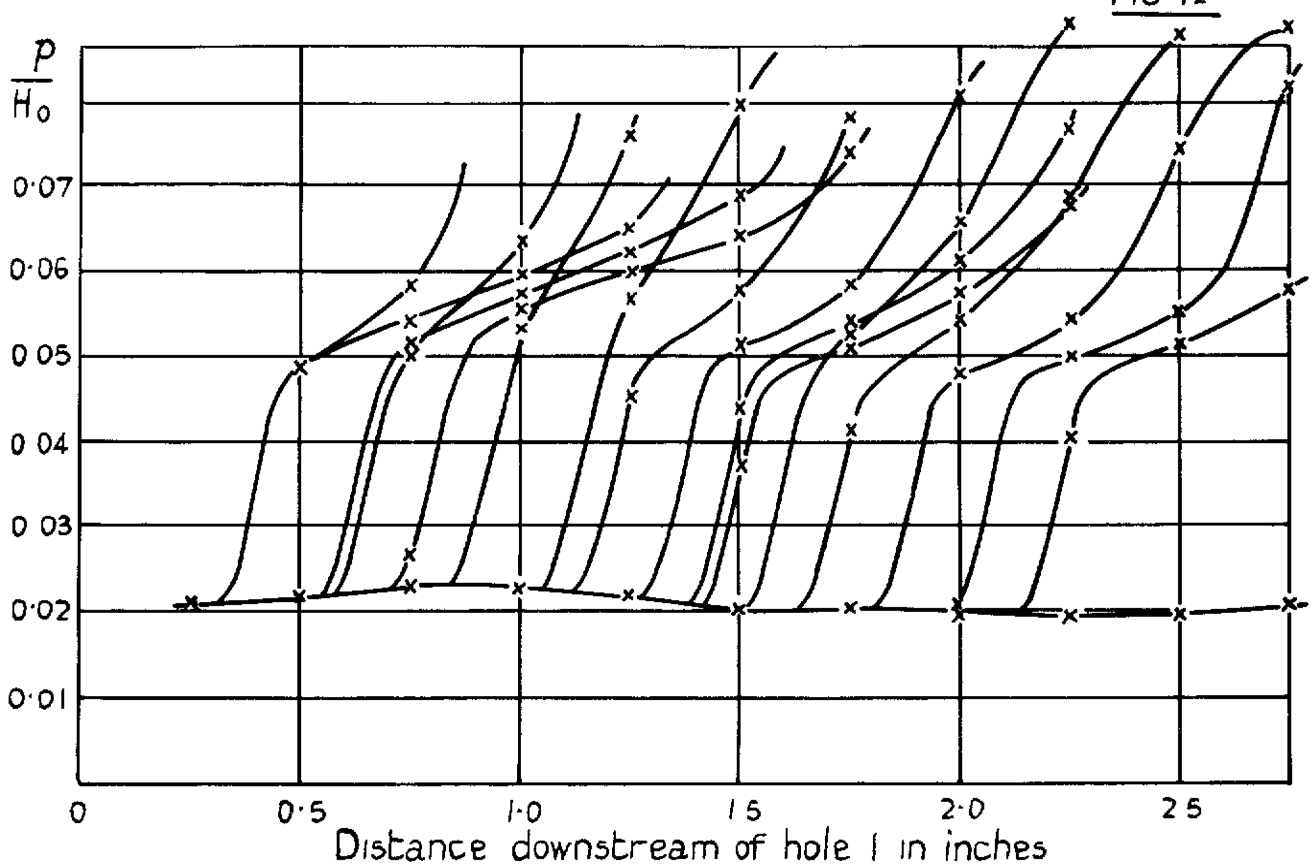
Figs 11 & 12

Fig. 11.



Pressure distributions on curved plate with a turbulent layer at a free stream Mach number of about 3 Stagnation pressure H_o = 7·8 atmospheres, transition-promoting strips attached near hole 1.

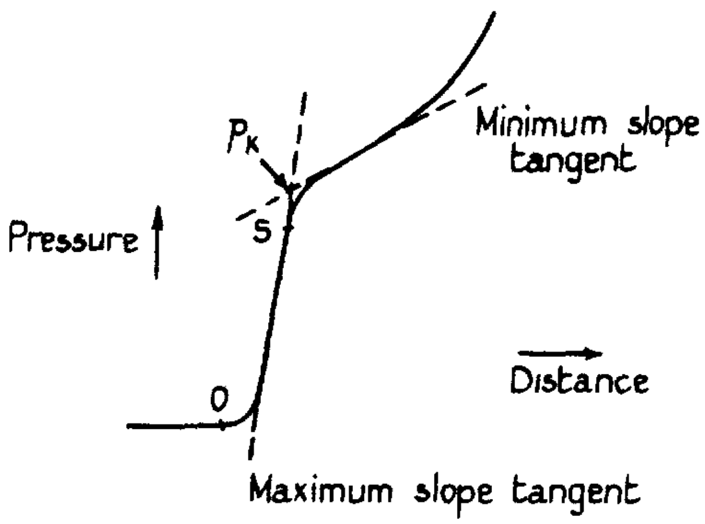
Fig 12



As Fig. 11, but for flat plate.

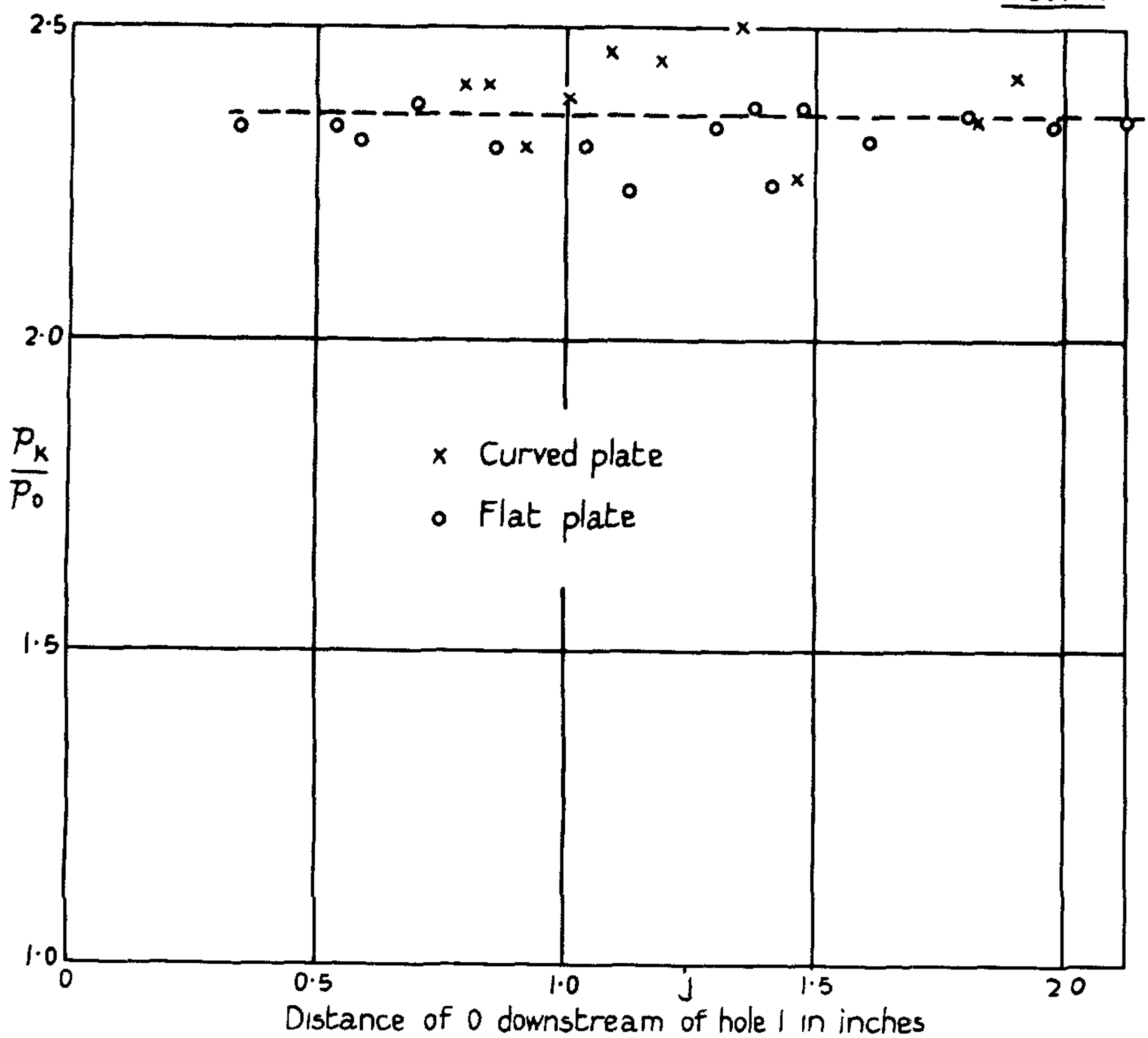
Figs. 13 & 14.

Fig 13.



Definition of kink pressure p_K for turbulent separations
s = separation point

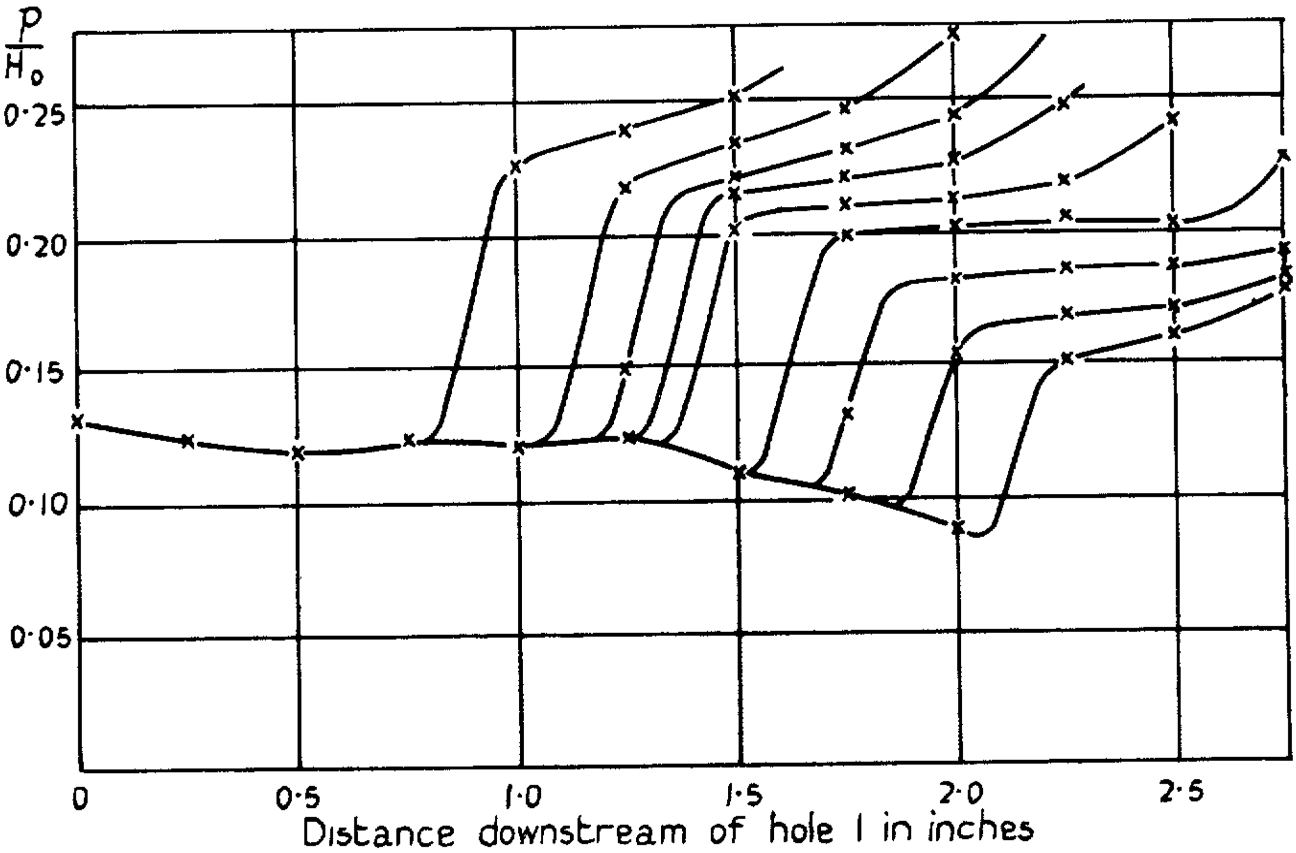
Fig. 14.



Turbulent kink pressure ratio for flat and curved plates at a Mach number of 3 The curved plate is curved downstream of J

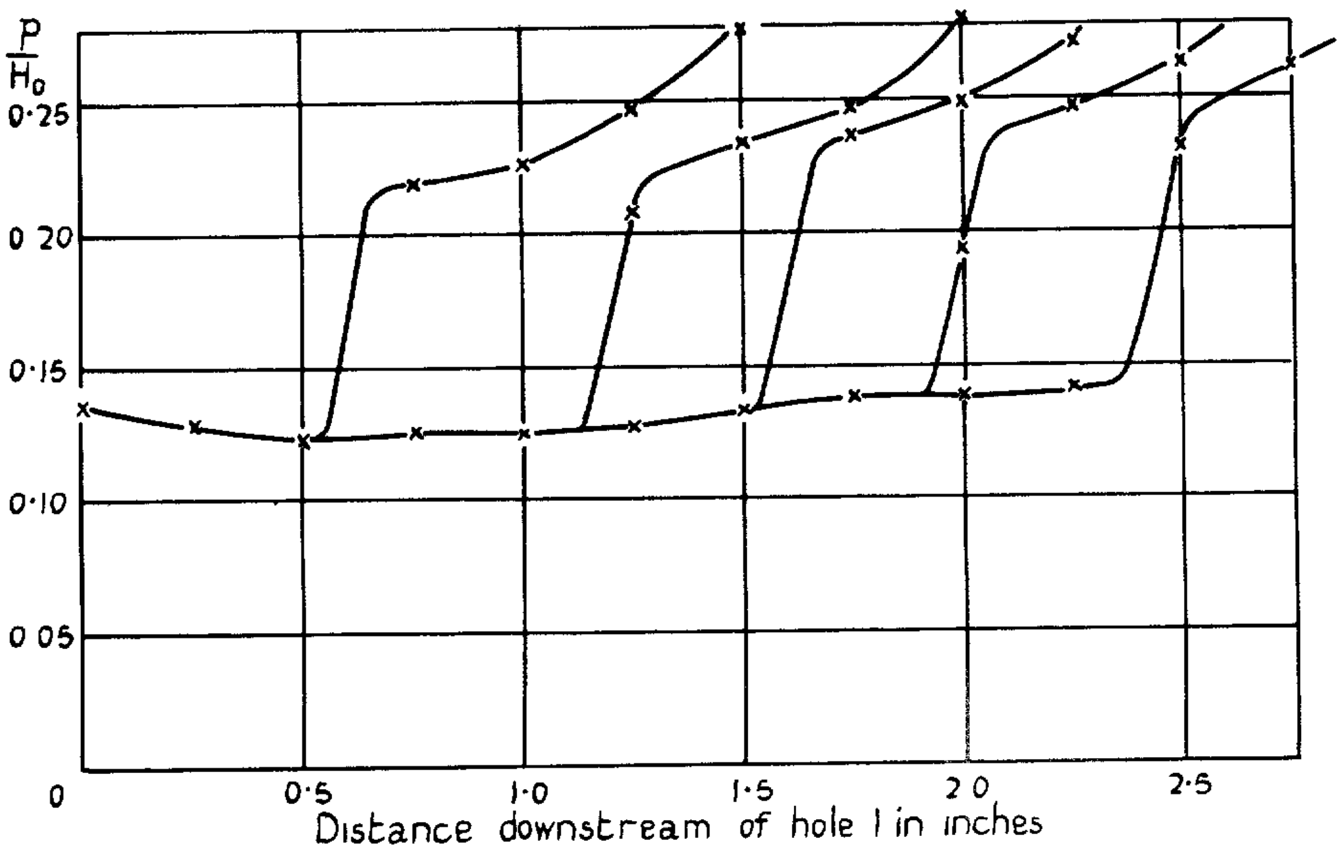
FIGS. 15 & 16

FIG 15.



Pressure distributions on curved plate with a turbulent layer at a free stream M of about 2. Stagnation pressure H_o = 6 atmospheres transition free.

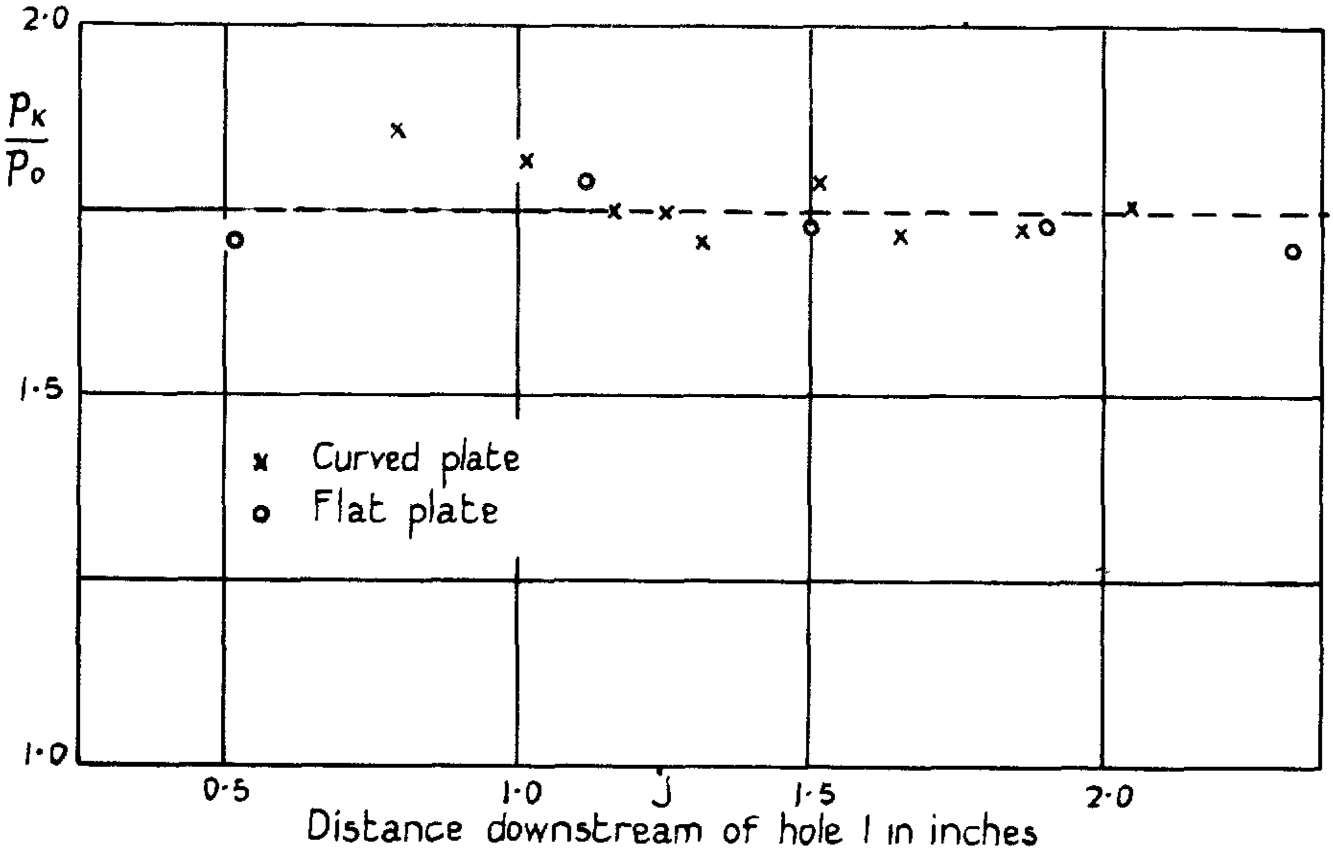
FIG 16



As Fig. 15, but for flat plate

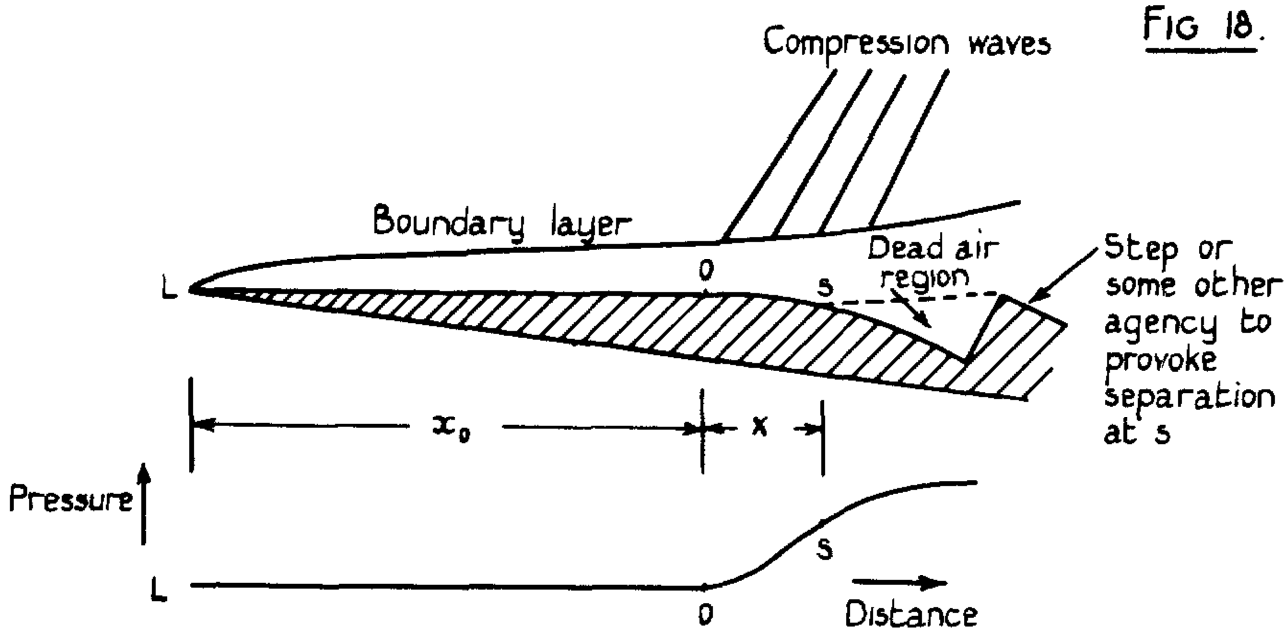
FIGS. 17 & 18.

FIG 17.



Turbulent kink pressure ratio for flat and curved plates at a Mach number of 2. The curved plate is curved downstream of J

FIG 18.



LO flat; constant curvature downstream of O.
Effect of growth of boundary layer just behind leading edge on pressure distribution ignored: i.e. pressure assumed constant between leading edge L and point O where it begins to rise.

Shape of wall and of pressure distributions assumed in theoretical analysis for laminar flow.

DS 48112/1/WT8 K3 10/56 CL

C.P. No. 289
(18,038)
A.R.C. Technical Report

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