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A Technique for the Automatic, Digital Analysis of Flight Dynamic Response Data

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Summary.

A technique is described for using a digital computer for the extraction of aerodynamic force and moment derivatives from measurement of the motion of aircraft or aircraft models, following a disturbance from the trimmed state. The analysis method is based on model matching by least squares using an automatic iterative method and has been found capable of handling coupled three-dimensional motions of much greater complexity than it has been possible to analyse previously. The particular problem of solution divergence which limits the usefulness of iterative methods has been overcome, giving a virtually fully-automatic system for response analysis that can be applied to a wide range of aircraft-like problems. A full description is given of the use of the techniques for the analysis of dynamic response data from free flight models. However, few modifications would be necessary to the computer programs for application to full-scale aircraft tests and even dynamic stability tests in windtunnels.

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1. Introduction

Methods of measuring aerodynamic force and moment derivatives by the study of the dynamic stability of aircraft and aircraft models have evolved over a number of years. The experimental technique depends on disturbing the model from the trimmed state in flight by the control surfaces or small pulse rockets and analysing the resulting response¹. Until recently, when new analysis methods rendered it unnecessary, care had to be taken to excite only either the short period pitching oscillation or the lateral Dutch roll oscillation. Analysis of the dynamic response to each disturbance was carried out almost entirely by hand using graphical methods² even though, at least for free flight, modern computer methods had made the actual data handling entirely automatic. On analysis the longitudinal derivatives, z_w , m_w and $m_{\dot{\theta}}$ were obtained by measurement of the frequency, damping and phase relationships of the pitch oscillation; and the Dutch roll was analysed by the time-vector method of Doetsch^{3,4}, which involved extracting phase and amplitude relationships to yield roll and yaw derivatives. Such analysis presupposed that the motion was linear; it was complicated and time consuming and therefore liable to error. However, its chief weakness was that only pure pitching or pure Dutch roll responses could be analysed. These responses were difficult to obtain experimentally and, when there was appreciable cross coupling, much of the data had to be discarded and with it much potentially valuable information. It was this problem of wishing to make full use of cross coupled responses and to a lesser extent handle non-linear motion which stimulated a search for a better and preferably automatic digital method of analysis.

The time-vector method, as well as limiting the type of motion that may be analysed, is not readily adapted for computer use and so a number of different approaches to the problem were considered. These fall into two broad categories: (1) methods which use the equations of motion in an unsolved form and thus make no use of the known relationships between different variables and the properties of the solution to the equations; (2) methods which use analytical or numerical solutions to the equations of motion to match characteristics of the measurements made of the motion. On a computer both approaches usually employ least squares fitting techniques either directly or implicitly. A useful comparison between various methods of both types is given in Ref. 5.

There are a number of methods of Type 1 designed for computer use. These include the derivative method (called least squares method in Ref. 5), the Laplace transform method, and the Fourier transform method. Shinbrot⁶ has shown that all these methods are equivalent and developed a generalised transform method which avoids some of the problems of the others. All these methods deal with equations of motion individually, not as a set, and require the measurement of most motion variables and their time derivatives. Thus the derivative method, which is the simplest and most commonly used, would take the normal force equation

$$Z(u, v, w, p, q, r) = m(\dot{w} - uq + pv - g(\theta, \phi))$$

and measure all variables u , v , w , \dot{w} etc. at n discrete times, so that n simultaneous equations in terms of the Z aerodynamic derivatives are obtained and solved redundantly by least squares. The procedure is repeated for each of the six equations in turn. This fairly obvious approach poses several problems in practice, some of which are avoided by the transform methods. In these the equation is multiplied through by a 'method function' and integrated over the duration of the record; the effect of this is to remove the need to know both w and \dot{w} , for example. In spite of the theoretical superiority of the transform methods, particularly Shinbrot's, their complexity has meant they have been used very little. Shinbrot's method has been used for the response analysis of full-scale aircraft⁷ but the straightforward derivative method is much more popular, particularly in Europe^{8,9}.

The great disadvantage of the equation of motion methods is that they demand the near-perfect measurement of most variables. Serious problems are caused by zero errors and noise in the data, and by the great difficulty of making pure measurement of some variables, e.g. \dot{v} and \dot{w} . Also because no advantage is taken of the known relationship between the variables, or the characteristics of aircraft motion, there is no easy way of checking for consistency in the data. It is particularly difficult to apply equation of motion methods for free flight response analysis because it is not normally possible to carry

sufficient instrumentation in a small model for the measurement of all variables. For these reasons no attempt has been made at RAE to use either Shinbrot's or any other equation-of-motion method for automatic aircraft response analysis.

The method finally adopted for analysing aircraft response comes into the second category mentioned earlier, in which the basic idea is to match the characteristics of a solution of the equations of motion to the characteristics of the actual measurements made. The measurements, therefore, are only used as a reference in a curve fitting process, and not used directly in calculations as in Type 1 methods. The practical advantages of the second type of method are that: (1) analysis is not necessarily crippled by noisy data; (2) instrument zero errors are easily included in the mathematical model; (3) measurement of all variables is unnecessary because theoretical relationships are imposed by the mathematical model. Moreover when several variables can be measured it is possible to gain the advantages of redundancy; (4) it is much easier to predict the readings of an instrument in terms of the variables than it is to unscramble the variables from instrument readings, as required by Type 1 methods.

The disadvantages of Type 2 methods lies in the problem of fitting complicated non-linear functions to data. This has to be accomplished by iterative techniques while Type 1 methods involve only the solution of simultaneous linear equations by well established methods. An obvious way of fitting the data to a mathematical model is by eye, with the assistance of an analogue computer to generate the functions and display on an oscilloscope. Judson has used such a method for analysing longitudinal short period response data¹⁰ but clearly an automatic method is much to be preferred. Automatic iterative curve fitting methods are usually based on a variant of the least squares technique known as the method of differential corrections. This is similar to manual analogue matching in that guesses must be made initially at the values of the parameters in the mathematical model such as initial conditions and aerodynamic derivatives. These initial guesses are then improved iteratively by the computer program to values that give a best fit to the data. The superiority of this method over all other methods of Types 1 and 2 is demonstrated in Ref. 5 where it is termed the Newton Raphson method.

A computer program based on the method of differential corrections was developed by the author in 1963 for the analysis of the angular motion of axially symmetric re-entry bodies¹¹ and many workers have since described similar schemes for a variety of applications^{12,13,14,15,16}. This original program has been adapted for the analysis of the much more complex motion of aircraft and of aircraft models. The technique is therefore not particularly new but, hitherto, it does not appear to have been applied successfully to problems of the complexity of the coupled motion of aircraft. This may be due to the lack of computers of sufficient size and speed but more likely it is due to that plague of iterative methods—solution divergence. The straight application of the method of differential corrections to mathematical models with more than about twelve unknown parameters is very prone to solution divergence problems. Now the model for the coupled non-linear motion of aircraft contains at least twenty parameters so the problem becomes very serious, and at one time it was feared that it would be unsurmountable. Fortunately, after several attempts, a computing procedure has been devised which makes the latest version of the program able to accept such large errors in initial guesses without risk of divergence that it is virtually a fully automatic system. In achieving this end, it is perhaps only fair to warn the reader, that it was found expedient to graft in a feature of Type 1 methods and make direct use of measured roll rate. Suffice it to say here that this particular sacrifice of a statistical ideal resulted in no disadvantages but rather conferred very considerable advantages over a pure curve fitting method.

The response analysis program described in this Report is tailored to the specific needs of free flight experiments using a particular instrumentation system. However, only trivial modifications involving perhaps a dozen program cards, would be needed to apply the program to analysis of the motion of full-scale aircraft, or of glide models; it could even be applied to dynamic stability testing in windtunnels. Therefore to help the reader with interest different from free flight, most of the main text of this Report is confined to describing the response analysis technique in general terms. Only in Section 5 is the technique set in a free flight context, while the Appendices contain material concerned with the general theory of the method of differential corrections, with the data handling problems of free flight experiments and with the smoothing of free flight trajectories.

2. Model Matching by the Method of Differential Corrections

The method of analysis, of matching a mathematical model to observations, is essentially a curve fitting technique akin to regression analysis, of which the simplest and best known form is fitting a curve to a set of measurements y_i made at different values of an independent variable x_i . If the curve has the form $y(x) = a + bx$, values of the parameters a and b can be found which minimise the sum of the squares of the errors $\Sigma(y_i - y(x))^2$; these being the values of a and b for which $\partial\Sigma/\partial a = \partial\Sigma/\partial b = 0$. The procedure leads to two equations, termed the NORMAL EQUATIONS, which may be solved explicitly for a and b . High order polynomials and complicated functions can be handled in an analogous way provided the functions are linear in the parameters to be measured, e.g. $y = ax + bx^2 + c \sin(x)$ is linear in the constants a , b and c , and leads to three linear simultaneous equations that may be solved for a , b and c . Regression analysis can be applied to multidimensional observations with no difficulty in principle, e.g. finding a and b so as to minimise $\Sigma[(y_i - y(x))^2 + (z_i - z(x))^2]$ where $y(x) = a + bx$ and $z(x) = ax + bx^2/2$ and y_i and z_i are observations.

Unfortunately there are many problems, such as that of response analysis, where the function to be fitted is not linear in the parameters to be deduced from the data, and an explicit solution for the parameters is not generally possible, e.g. $y(x) = a \sin(bx)$ on partial differentials does not lead to equations that may be solved explicitly for a and b . The technique for non-linear problems is best described as linearisation; the fundamental idea is, that we start by guessing the value of the parameters so that we can obtain a function that is linear in terms of the error in the parameters by means of a Taylor series expansion. Thus for the example cited: $y(x) = y(x)_0 + \delta a \sin(bx) + \delta b a \cos(bx)$, or more generally: $y(x) = y(x)_0 + \delta a \partial y/\partial a + \delta b \partial y/\partial b +$ second order terms, where $\partial y/\partial a$ and $\partial y/\partial b$ are the partial derivatives of $y(x)_0$ with respect to a and b . This function is now linear in terms of the errors in the parameters δa and δb provided these are small enough for second order terms to be neglected; if this is not so a and b can be corrected by δa and δb and the process repeated iteratively. The method therefore consists of starting with a approximate fit to the observations and then repeatedly correcting the parameters until a best fit is achieved, hence the name—the method of differential corrections.

The formal proof of the method for multiple dimensions and parameters is best put in terms of matrix algebra; this is given in Appendix I but it will be helpful to illustrate the main features of the technique with a simple smoothing problem.

2.1. Application to Simple Problems

Let us suppose we have made a series of measurements of y and z in two dimensions at n discrete times t and that we believe y and z can be calculated from the functions $y(t) = Y(U, A, t)$ and $z(t) = Z(U, A, t)$. We wish to find the values of U_0 and the constant A so as to minimise the function $\Sigma[w_1(y_i - y(t))^2 + w_2(z_i - z(t))^2]$, where w_1 and w_2 are the 'weights' of the observations or scaling factors chosen so that the accuracies of y and z are made numerically equal. The functions Y and Z are non-linear in U and A and may be assumed to include numerical solutions to differential equations.

The first step is to guess the values of U_0 and A to be approximately U'_0 and A' and then to use $Y(U'_0, A', t)$ and $Z(U'_0, A', t)$ to calculate $y'(t)$ and $z'(t)$ at every observation time t_i . At the same time we calculate the residuals $(y_i - y'(t))$ and $(z_i - z'(t))$ and compute the partial derivatives: $f_1 = \partial y'/\partial U_0$, $f_2 = \partial z'/\partial U_0$, $f_3 = \partial y'/\partial A$ and $f_4 = \partial z'/\partial A$ (see Section 2.2). The normal equations, expressed by equation (I.5) may then be calculated as follows:

$$\delta U_0 \Sigma(w_1 f_1^2 + w_2 f_2^2) + \delta A \Sigma(w_1 f_1 f_3 + w_2 f_2 f_4) = \Sigma(w_1 f_1 (y_i - y') + w_2 f_2 (z_i - z'))$$

and

$$\delta U_0 \Sigma(w_1 f_1 f_3 + w_2 f_2 f_4) + \delta A \Sigma(w_1 f_3^2 - w_2 f_4^2) = \Sigma(w_1 f_3 (y_i - y') + w_2 f_4 (z_i - z')).$$

These are then solved to give the corrections δU_0 and δA to be made in the guessed values of U_0 and A' to obtain U_0 and A . As explained in Appendix I, estimates of probable errors in U_0 and A and the goodness of fit can also be made. Usually the above procedure would be repeated at least once to ensure that the best values had been obtained.

2.2. The Problems of Obtaining the Partial Derivatives

A major problem of the method of differential correction is meeting the need for accurate partial derivatives which represent the sensitivity of the model to changes in parameter values. The necessary calculations can absorb over 90 per cent of the computing time and inaccuracy here is a major cause of solution divergence and the consequent wastage of computer time. The earlier versions of the response analysis program used the simple perturbation technique for finding partial derivatives, favoured by many workers¹¹⁻¹⁵. The success of the later programs, however, must in part be ascribed to the decision to use the parametric differentiation technique used by Chapman and Kirk¹⁶, which is so much more accurate and considerably faster than the perturbation method.

The perturbation method would find $\partial y_i / \partial U_0$ in the above example by calculating $Y((U_0 + \epsilon), A, t_i)$ in addition to $Y(U_0, A, t_i)$ so that $\partial y_i / \partial U_0 = [Y((U_0 + \epsilon), A, t_i) - Y(U_0, A, t_i)] / \epsilon$. The merit of this technique is that it permits the writing of a general purpose program that requires only a different routine for different mathematical models. Unfortunately it is difficult to choose a perturbation ϵ which gives sufficient accuracy at the end as well as at the beginning of a response. Successful analysis, therefore, is likely to depend very much on the experience of the program user which is hardly desirable in a fully automatic system. Generally speaking, although programs using the perturbation technique are easy to write, they can be very difficult to use and are liable to be very wasteful of computer time.

The parametric differentiation technique, as the name implies, involves differentiating the mathematical model partially with respect to the parameters. There is nothing new about the method in principle and it has always been preferred to the perturbation method for simple curve fitting problems because it is so much quicker and more accurate to obtain the derivative of, say, x^2 from $2x$ than $[(x + \epsilon)^2 - x^2] / \epsilon$. The only objection in principle to parametric differentiation arises when the function includes integrals with respect to time such as occur in the mathematical model of aircraft response, because then it is necessary to assume that the order of differentiation can be reversed. Thus, for example, if the function involves the integration of dy/dt and the partial derivative $\partial y / \partial A$ is required, we assume that $d/dt(\partial y / \partial A) = \partial / \partial A(dy/dt)$ in order that the partial derivative can be obtained by an integration with respect to time. Fortunately, this assumption is justified in almost all the functions which arise in response analysis, since a measured response is inevitably continuous.

Parametric differentiation is not as difficult to program as might be imagined since partial differentiation often yields quite simple expressions from the most complex of functions. The extra accuracy and higher speed of the method are well worth the extra programming effort. Certainly the programs using the technique have been difficult to write but they are very easy to use and save considerable proportions of computing time.

2. The Application of Model Matching to the Analysis of the Short Period Motion in Aircraft

The application of the method of differential corrections to the analysis of measurements of any dynamical system depends on making a sufficiently accurate mathematical model. The number of parameters to be determined by the analysis must be as few as possible because of the problems of solution divergence common to many iterative methods. Our difficulty is that a general mathematical model for the motion of an aircraft is exceedingly complex, largely because most aerodynamic forces and moments are non-linear functions of several variables and have to be represented by polynomials or similar functions. In order to allow simplification it is necessary to impose restrictions and confine analysis to particular problems. It will be assumed therefore, that the analysis of the short period motion of an aircraft involves (a) measurements covering only a few seconds of flight and (b) small angular perturbations from some datum, so that the aerodynamic forces and moments can be reasonably described in terms of aerodynamic derivatives. The responses to be analysed could either be damped oscillations

following an initial disturbance such as is often used in free flight experiment or be forced oscillations as often used in wind-tunnel experiments and sometimes in full-scale aircraft tests.

In the following sub-sections we shall first consider the basis of the mathematical model—the equations of motion of a rigid body under aerodynamic and gravitational forces, with approximations validated by the restrictions imposed. We then discuss the measurements required to define the motion, taking into consideration what is practicable. The mathematical model of the actual measurement made can then be written down in terms of the variables calculated from the equation of motion. Finally, some consideration is given to the formidable problem of calculating the partial derivatives expressing the sensitivity of model predictions to parameter changes.

3.1. Equations of Motion

Following Hopkin¹⁷ the general equations of motion of a rigid object in still air in terms of body axes $Oxyz$ with origin at the CG, are expressed as follows:

$$\text{and } \left. \begin{aligned} m(\dot{u} + qw - rv) &= X + mg_x, \\ m(\dot{v} + ru - pw) &= Y + mg_y, \\ m(\dot{w} + pv - qu) &= Z + mg_z; \end{aligned} \right\} \quad (1)$$

also

$$\text{and } \left. \begin{aligned} I_x \dot{p} - I_{zx}(\dot{r} + pq) - (I_y - I_z)qr &= \mathcal{L}, \\ I_y \dot{q} - I_{zx}(r^2 - p^2) - (I_z - I_x)rp &= \mathcal{M} \\ I_z \dot{r} - I_{zx}(\dot{p} - qr) - (I_x - I_y)pq &= \mathcal{N}. \end{aligned} \right\} \quad (2)$$

The products of inertia I_{xy} and I_{yz} have been omitted from the above equations because Ox is usually defined as forward and Oy through the wing of an aircraft whether model or full-scale, so that all but I_{zx} is usually negligible. In general, when calculating the forces, moments and gravity terms, it will be necessary to also know the position and orientation of the body relative to the earth, and this is considered later. Since we intend only to analyse small perturbations relative to data levels, u_e, v_e, w_e, p_e, q_e and r_e , it is convenient to replace u by $(u' + u_e)$, v by $(v' + v_e)$, etc. in the equations. In doing this we shall drop the prime as all subsequent development will be in terms of perturbation quantities. Assuming that oscillations of u are of negligible importance compared with steady changes, the equations of motion become:

$$\dot{u} = X_e/m + g_x, \quad (3)$$

$$\dot{v} + r(u + u_e) + ur_e - p_e w - w_e p - pw = Y/m + g_y + (Y_e/m - u_e r_e + p_e w_e), \quad (4)$$

$$\dot{w} + p_e v + v_e p + pv - q(u + u_e) - q_e u = Z/m + g_z + (Z_e/m - p_e v_e + q_e u_e), \quad (5)$$

$$I_x \dot{p} - I_{zx}(\dot{r} + pq + p_e q + q_e p) - (I_y - I_z)(qr + q_e r + r_e q) = \mathcal{L} + (0), \quad (6)$$

$$\text{and } I_y \dot{q} - I_{zx}(r^2 - p^2 + 2r_e r - 2p_e p) - (I_z - I_x)(rp + r_e p + p_e r) = \mathcal{M} + (0) \quad (7)$$

$$I_z \dot{r} - I_{zx}(\dot{p} - qr - q_e r - r_e q) - (I_x - I_y)(pq + p_e q + q_e p) = \mathcal{N} + (0). \quad (8)$$

The constant terms involving products of datum values have been placed on the right hand side and equated to zero in the case of the moment equations with trim moments. In the case of the force equations,

it is usual to equate constant terms, gravity and trim forces to zero, but this is not convenient in the general response analysis problem. It would be satisfactory for full-scale aircraft with a pilot in control or in windtunnel dynamic stability tests but in free flight tests with models with fixed controls, a steady roll rate is necessary to ensure a ballistic flight path is followed, which means the gravity terms oscillate. The variation in datum caused by the gravity terms is much smaller than the usual magnitude of perturbations so there is little loss in generality if the gravity terms are not included in the definition of trim. The response analysis programs therefore impose the conditions

$$Y_e/m - u_e r_e + p_e w_e = 0 \quad (9)$$

and

$$Z_e/m = p_e v_e + q_e u_e = 0. \quad (10)$$

The perturbation forces and moment functions may now be expanded in terms of aerodynamic derivatives as follows:

$$Y = [y_v v + y_p p s + y_r r s + \dots] \rho V S,$$

$$Z = [z_w w + z_q q \bar{c} + z_\eta \eta + \dots] \rho V S,$$

$$\mathcal{L} = [l_v v + l_{vw} vw/V + l_p p s + l_r r s + l_t V] \rho V S,$$

$$\mathcal{M} = [m_w w + (m_q + m_{\dot{w}}) q \bar{c} + m_\eta \eta + m_t V + \dots] \rho V S \bar{c}$$

and

$$\mathcal{N} = [n_v v + (n_r - n_{\dot{v}}) r s + n_p p s + n_t V + \dots] \rho V S s.$$

Only the aerodynamic derivatives that have been found to be significant in recent free flight analysis have been included but others could easily be added. The terms l_t , m_t , n_t are small trim adjustments, needed because of the inadequacies of equations (9) and (10). The elevator force and moment coefficients z_η and m_η have been included to cover the case of forced oscillations; they are not included in the analysis program for free flight analysis. It has been assumed that q and \dot{w} , and r and \dot{v} are too close in phase for m_q and $m_{\dot{v}}$, and n_r and $n_{\dot{v}}$ to be separated. This is certainly true of free oscillation experiments but may not be true for forced oscillations. Similarly l_p and l_r , and n_p and n_r are difficult to separate, so that l_r and n_p are treated as constant in the present analysis programs.

We must now consider the subsidiary problem of calculating the gravity terms. The direction of the gravity vector in body axes can be defined by two angles, θ the inclination of the $0x$ axis to the local horizontal and Φ the bank angle of the $0y$ axis. Therefore we have:

$$g_x = -g \sin \theta, \quad g_y = g \cos \theta \sin \Phi \quad \text{and} \quad g_z = g \cos \theta \cos \Phi. \quad (11)$$

The effect of these terms on short period responses is often small; even the changes of datum caused by gravity are barely significant so that oscillations in θ during a pitching response are certainly negligible. If γ , the elevation of the velocity vector is known, then θ is approximately $(\gamma + w_e \sin \Phi/V)$, but assuming that $\theta = \gamma$ is usually quite adequate.

For windtunnel and full-scale aircraft experiment it is usual for these gravity terms to have constant average values and for the velocity V and air density ρ to be sensibly constant. In free flight experiments, such steady conditions can be achieved only rarely and although small, the changes are worth taking into consideration. The information is derived by measuring the trajectory of the model with ground based instrumentation. It is then smoothed by a computer program making use of additional informa-

tion such as on-board acceleration a_x and roll angle Φ , as described in Appendix IV. This trajectory analysis program provides not only smoothed values of velocity V , climb angle γ and altitude \bar{Z} , but estimates of the mean drag coefficient C_D , lift coefficient C_L and trimmed incidence during a response. The response analysis program then uses the initial conditions $V_0, \gamma_0, h_0, \Phi_0$ at the start of a response to calculate the values of u, V, θ, \bar{Z} and Φ during a response by assuming $u_e = V_0$ and $\theta = \gamma$ and using the following equations:

$$\dot{u} = \dot{V} = -\frac{1}{2}\rho V^2 S C_D/m - g \sin \theta, \quad (12)$$

$$V\dot{\theta} = (Z_e/m) \cos \Phi - g \cos \theta, \quad (13)$$

$$\dot{\bar{Z}} = V \sin \theta \quad (14)$$

and

$$\dot{\Phi} = p_e. \quad (15)$$

Where Z_e/m is the trimmed normal acceleration (a_{z_e}) and air density ρ is obtained from a table of ρ as a function of \bar{Z} , as measured on the day of the free flight trial.

3.2. Measurement of Response Parameters

A basic instrumentation system might be designed to measure the six perturbation quantities u, v, w, p, q, r . Aerodynamic derivatives would then be extracted by fitting these measurements to values obtained by integration of the equations of motion. In practice, even if it were possible to measure these quantities directly, it would mean that estimates of force and moment derivatives such as z_w would depend on the phase relationship between linear and angular velocities. Better accuracy would be obtained by measuring the components, a_y and a_z , of the translational accelerations (at the CG) so that force derivative estimates would depend on both amplitude and phase relationships. A more practical minimal system would therefore have to provide $u, v, w, p, q, r, a_y, a_z$. In addition it would have to provide measures of the datum levels but generally no extra instruments would be needed. Obviously, eight instruments measuring combinations of variables could be used provided it is possible in principle to extract all eight quantities from the data.

In practice the angular velocity resolutes p, q, r pose the problem that the rate gyros required are not easily accommodated in the average free flight or windtunnel model. It is much easier to measure angular accelerations \dot{p}, \dot{q} and \dot{r} by angular accelerometers or by pairs of separated linear accelerometers and although the datum levels of p_e, q_e and r_e then cannot be measured directly, they are available from the assumed relationship between the datum levels, equations (9) and (10). In the same way u is obtained by integration of \dot{u} measured as a_x . Therefore the analysis program in use for free flight analyses uses the basic measurements $a_x, v, w, \dot{p}, \dot{q}, \dot{r}, a_y$ and a_z .

A particular virtue of the method of analysis is that because the mathematical model imposes certain relationships between the eight variables, the measurements of v and w are redundant. This is because frequency, damping, and amplitude and phase relationships are implicit in the data so that the amplitude of v and w is implicit in \dot{q} and \dot{r} , while a_y and a_z determine the phase. Thus v and w need not be measured except to find the trim values v_e and w_e , and these are sometimes available from other sources, e.g. it is always obvious from measures of acceleration at the CG when trim is zero. Of course u and u_e can nearly always be measured by external means, as well as by instruments on board. So that very often analysis can be based on measurements of $\dot{p}, \dot{q}, \dot{r}, a_y, a_z$ only.

The general scheme for instrumentation of free flight models uses the following eight instruments:

- 2 differential pressure probes measuring CP_v and CP_w for v and w ,
- 2 linear accelerometers measuring a_y and a_z near the CG,
- 2 linear accelerometers near the model tail for \dot{q} and \dot{r} ,
- 1 linear accelerometer near the CG for a_x and thus \dot{u} and
- 1 angular accelerometer for measuring \dot{p} .

In addition a magnetometer with the sensitive axis parallel to Oz is used to measure roll angle Φ and thus trim roll rate p_e .

A similar scheme could probably be developed for windtunnel dynamic stability testing. For full-scale aircraft it would almost certainly be advantageous to include rate gyros for the direct measurement of p, q, r . This would introduce even more redundancy which is no bad thing but it would perhaps be best to measure \dot{q} and \dot{r} , if required, by angular accelerometers rather than spaced linear accelerometers.

3.3. Mathematical Model for on-board Measurement

The analysis program uses a mathematical model to predict the actual readings of each instrument rather than attempting the bad statistical practice of unscrambling v, w, p, q, r , etc. from the measurements. Instrument response characteristics are not included in the mathematical model because of the computational problems this would introduce. Instead corrections for instrument response characteristics are made when the basic data is processed in preparation for analysis as described in Appendix III.

Measurements from longitudinal acceleration a_x and the magnetometer are not given directly to the response analysis program, only implicitly in the form of equations (12)–(16). The seven remaining instruments are given names in capital letters for the purpose of the computer program, viz:

NC for acceleration a_z measured near the CG at coordinates x_1, y_1, z_1 ,
NA for acceleration a_z measured near the tail at coordinates x_2, y_2, z_2 ,
LC for acceleration a_y measured near the CG at coordinates x_3, y_3, z_3 ,
LA for acceleration a_y measured near the tail at coordinates x_4, y_4, z_4 ,
 \dot{P} for \dot{p} and *CPV* and *CPW* for probe differential pressures CP_v and CP_w .

It should be noted that *NC* and *NA*, in spite of the name, have the opposite sign to the normal acceleration because Oz and therefore a_z is positive downwards. The mathematical model in terms of $u, v, w, p, q, r, \dot{p}, \dot{q}, \dot{r}$, derived from the equations of motion is as follows:

$$NC = a_{z_e} + Z/m + [(p + p_e)(r + r_e) - \dot{q}]x_1 + [(q + q_e)(r + r_e) + \dot{p}]y_1 - [(p + p_e)^2 + (q + q_e)^2]z_1 + ENC, \quad (16)$$

$$NA = a_{z_e} + Z/m + [(p + p_e)(r + r_e) - \dot{q}]x_2 + [(q + q_e)(r + r_e) + \dot{p}]y_2 - [(p + p_e)^2 + (q + q_e)^2]z_2 + ENA, \quad (17)$$

$$LC = a_{y_e} + Y/m + [(p + p_e)(q + q_e) + \dot{r}]x_3 - [(p + p_e)^2 + (r + r_e)^2]y_3 + [(q + q_e)(r + r_e) - \dot{p}]z_3 + ELC, \quad (18)$$

$$LA = a_{y_e} + Y/m + [(p + p_e)(q + q_e) + \dot{r}]x_4 - [(p + p_e)^2 + (r + r_e)^2]y_4 + [(q + q_e)(r + r_e) - \dot{p}]z_4 + ELA, \quad (19)$$

$$\dot{P} = \dot{p}, \quad (20)$$

$$CPW = KK_w \rho V(w - qx_p) + ECPW \quad (21)$$

and

$$CPV = KK_v \rho V(v - rx_p) + ECPV. \quad (22)$$

The prefix *E* is used to denote zero errors in all instruments, e.g. *ENC*. If possible such errors are removed before analysis but some are liable to drift and may have to be estimated during analysis. The

factor K is a correction factor for probe sensitivity, x_p is the distance of the probe from the CG. The trim accelerations $a_{y_e} = Y_e/m$ and $a_{z_e} = Z_e/m$ are also known as *LC* (trim) and *NC* (trim) in the computer program.

These equations illustrate the chief problem of using linear accelerometers, viz. that because they respond to linear, angular and gyroscopic accelerations, their positions relative to the CG must be accurately known and the computing problem is very complex. The great advantage of rate gyros and angular accelerometers is that position does not matter, only alignment, so that generally better accuracy can be expected, as was found in re-entry dynamics work¹¹.

3.4. The calculation of the Partial Derivatives

The partial derivatives representing the sensitivity of the mathematical model to changes in parameters are found by the method of parametric differentiation which simply means that each measurement prediction is differentiated analytically with respect to each parameter. For response analysis with twenty or more parameters this presents a formidable computing problem and has strongly influenced the design of the computer program. For example, consider the problem of finding the partial derivatives $\partial NC/\partial x$ of *NC* with respect to every parameter x . Differentiating equation (16) gives

$$\begin{aligned} \partial NC/\partial x = & \left[z_w \partial w/\partial x + w \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{z_w} + \bar{c} z_q \partial q/\partial x + \bar{c} q \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{z_q} \right] \rho VS/m \\ & + x_1 \left[(r + r_e) \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_e} + (p + p_e) \partial r/\partial x - \partial \dot{q}/\partial x \right] \\ & + y_1 [(r + r_e) \partial q/\partial x + (q + q_e) \partial r/\partial x] + 2z_1 \left[(p + p_e) \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_e} + (q + q_e) \partial q/\partial x \right] + \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{ENC} \end{aligned}$$

where terms of the form $\begin{pmatrix} 0 \\ 1 \end{pmatrix}_{a}$ have a zero value except when $x = a$ when the value of 1 is taken; the reason $\partial p/\partial x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0}$ will be explained below. Although some of the terms of this equation and in those of the other instruments are trivial, the general complexity provides a powerful motivation towards either simplification of the mathematical model or at least to modifications which simplify the computing problem. This is particularly complicated because the partial derivatives of the state variables $\partial v/\partial x$, $\partial q/\partial x$ etc. can only be obtained by integration of sets of equations derived by partial differentiation of the equations of motion, Figs. 4–8. This means that five equations have to be integrated for every parameter except those like *ENC*.

To make any more simplification in the mathematical model would mean loss of accuracy which is unacceptable. We must consider, therefore, whether there is any modification to the curve fitting process that would greatly simplify the computing problem. In particular a modification is needed which reduces the number of partial derivatives whose derivation depends on the integration of differential equations. Unfortunately the only useful idea involves sacrificing the statistical ideal, discussed in the introduction, that 'measured data should not be used directly in calculations but only as references in a curve fitting process'. The objection to using measurements directly is that serious problems arise when there are zero errors and high noise levels in the data, but it would be foolish to fail to make direct use of some quantities that can be measured accurately. Now it is not fortuitous that the only quantity, in the instrumentation system discussed above, that is measured accurately and directly is \dot{p} , the roll angular acceleration, because it is p and \dot{p} which occur most frequently in the mathematical model and contribute most to cross coupling effects. Integrating the measured \dot{p} to yield a p , which is then used instead of values of p computed from the mathematical model, has the following considerable advantages:

(a) All the partial derivatives of p are zero with respect to all parameters except the initial level p_0 , i.e. $\partial p/\partial x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0}$. Consequently integration of equations of the form $d(\partial p/\partial x)/dt$ is rendered unnecessary so that only four differential equations must be integrated for every aerodynamic derivative treated as a parameter except the roll derivatives. In practice this means a reduction of 40 per cent in the number of differential equations requiring numerical integration.

(b) Because most of the partial derivatives of p are zero, many terms of the other functions are eliminated on partial differentiation. So that not only are the equations to be integrated much fewer, they are also much simpler.

(c) Several terms in p are linearised in the mathematical model in the same way that terms are linearised in Shinbrot's or other equations-of-motion methods. Consequently more rapid convergence of the iterative analysis procedure might be expected.

All these advantages of using \dot{p} directly taken together, have resulted in practice in a 50 per cent saving in computing time per iteration and a 30 per cent reduction in the number of iterations required for a solution. Also, there is no significant loss in accuracy provided one of the modern high performance angular accelerometers is used for the measurement of \dot{p} . The problem of calculating the partial derivatives, therefore, has encouraged us to produce a hybrid of the Type 1 and Type 2 methods, discussed in the introduction, but we have gained the best features of the Type 1 without many of the disadvantages of using measurements directly. It should be made clear that we still do a least squares fit to measured \dot{p} ; all that is not done is the time-wise integration of the computed \dot{p} . If at the end the measured \dot{p} is not fitted accurately by the computed \dot{p} , then the mathematical model is inadequate and the direct integration of \dot{p} for p will not have been justified but it was still the best that could be done. If roll rate p was measured directly this could no doubt be used directly for the same purpose and further simplification of the computing problem could be effected by using direct measurement of \dot{q} and \dot{r} (or q and r) but caution would no doubt be necessary.

For the present response analysis scheme the four differential equations required for partial derivatives have the following form:

$$d(\partial v/\partial x)/dt = \left[v \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{y_v} + y_v \partial v/\partial x + s p \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{y_p} + s y_p \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0} \right] \rho V S/m$$

$$+ (p_e + p) \partial w/\partial x - (u + u_e) \partial r/\partial x + w \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0},$$

$$d(\partial w/\partial x)/dt = \left[w \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{z_w} + z_w \partial w/\partial x + \bar{c} q \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{z_q} + \bar{c} z_q \partial q/\partial x \right] \rho V S/m$$

$$- (p_e + p) \partial v/\partial x + (u + u_e) \partial q/\partial x - v \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0},$$

$$I_y d(\partial q/\partial x)/dt = \left[V \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{m_t} + q \bar{c} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{m_q} + \bar{c} (m_q + m_w) \partial q/\partial x + w \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{m_w} + m_w \partial w/\partial x \right] \rho V S \bar{c}$$

$$+ (I_z - I_x) \left[(p_e + p) \partial r/\partial x + r_e \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0} \right] + I_{zx} \left[2(r + r_e) \partial r/\partial x - 2(p + p_e) \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0} \right]$$

and

$$I_x d(\partial r/\partial x)/dt = \left[V \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{n_t} + r s \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{n_r} + s (n_r - n_v) \partial r/\partial x + p s \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{n_p} + s n_p \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0} + v \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{n_v} + n_v \partial v/\partial x \right] \rho V S s$$

$$+ (I_x - I_y) \left[(p_e + p) \partial q/\partial x + q_e \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{p_0} \right] - I_{zx} [(q + q_e) \partial r/\partial x + r_e \partial q/\partial x].$$

Here again terms of the form $\begin{pmatrix} 0 \\ 1 \end{pmatrix}_a$ have the value of zero except when $x = a$, when the value of unity is taken. It will be noted that eighteen out of the thirty four terms are of this type so that the equations are much simpler than they appear.

The sets of differential equations are integrated numerically in parallel with the equations of motion. The initial conditions are trivial at $t = t_0$, all are zero except for $\partial v/\partial v_0$, $\partial w/\partial w_0$, $\partial q/\partial q_0$, and $\partial r/\partial r_0$ which are unity. The minimum number of aerodynamic derivatives an analysis program must try to extract is twelve, viz. $l_v, l_{vw}, l_p, l_t, m_w, m_q, m_t, n_v, n_r, n_t, z_w, y_v$, but, of these, the four roll derivatives do not need the integration of equations which means only eight sets of equations are required. The five initial conditions p_0, q_0, r_0, v_0, w_0 each need a set of equations bringing the total number of sets to thirteen, i.e. fifty two differential equations. There are also nine equations of motion including the trajectory, so that sixty one equations must be integrated in the simplest analysis scheme. This compares with ninety four rather more complex equations if the \dot{p} simplification is not used and ninety four very complicated equations if the inaccurate perturbation method was used. An extra four equations are needed for every extra aerodynamic derivative (other than rolling moment derivatives) treated as an unknown parameter.

4. The Response Analysis Computer Program

4.1. Early Versions and Development

As mentioned in the Introduction, the first programs for the analysis of response data were adaptations of those used for the analysis of re-entry vehicle motion^{11,18}. Separate programs were written for the analysis of longitudinal motion (based on the \dot{w} and \dot{q} equations of motion) and for lateral motion (based on the \dot{v} , \dot{r} and \dot{p} equations of motion) but some approximate allowance for cross coupling effects was included by either using some measurements directly, or by using results from the analysis of one made in the analysis of the other. In its final form the two part analysis scheme could give satisfactory results¹⁹ but only after many computer runs of which 50 per cent could be abortive due to solution divergence. So much time had to be spent on initial graphical analysis that the method could hardly be described as automatic. It was more of an aid for the analysis of response data by the expert than a method in its own right.

Divergence is a common problem with this type of iterative analysis scheme. It is caused in part by ill conditioned normal equations and partly by large second order terms ignored in the Taylor series expansion. The risk of divergence increases with the number of parameters and at one time it seemed hardly possible that a single program using all five equations of motion and analysing data from seven instruments simultaneously could ever be made to work with the necessary twenty or more variable parameters. It was feared that parameters would have to be estimated by graphical methods so accurately that the computer method would be virtually useless. Various methods of reducing the risk of divergence were considered such as steepest descent techniques with limited step size but all bought stability at the price of greatly increased computing time and program complexity. A simpler idea was to do a preliminary analysis with a program that determined only the most important parameters and it was found experimentally that if the force and rate damping derivatives such as z_w and m_q were treated as constants in the mathematical model, it was almost impossible to persuade the preliminary fitting process to diverge. This preliminary, followed by full analysis idea is perhaps the most important feature of the present highly stable analysis programs, but parameter differentiations to obtain partial derivatives, the integration of measured \dot{p} to obtain p and the redundancy inherent in the simultaneous analysis of seven instruments, are all later developments which have improved stability and reduced computing time.

4.2. Finalised Versions

The latest program uses the mathematical model specified by equations (3)–(15) for the analysis of data from up to seven instruments, to extract up to twenty three model parameters including eleven

aerodynamic derivatives. The specification of this program A51B is given in full in Appendix V. The program can be instructed to extract only the more important parameters and subsequently to extract the full twenty three, only if analysis proceeds satisfactorily. To facilitate this feature the variable parameters are listed in order of importance as follows: $ENA, ELA, l_t, l_v, l_{vw}, l_p, p_0, w_0, q_0, v_0, r_0, m_r, n_r, m_w, n_w, (m_q + m_w), (n_r - n_w), z_w, y_w, (z_q + z_w), y_p, ELC$ and K . If for example the number of parameters is specified as fifteen, the parameters in the list from m_q onwards would be treated as constants. All other parameters are permanently treated as model constants including trim levels and the aerodynamic derivatives l_p and n_p . The standard version of the program can also be instructed to omit data from one or both probe measurements. If less than five data channels are to be used for analysis or different variable parameters are to be extracted, non-standard versions of the program would be necessary. In use the program iteratively improves the value of each variable parameter until each parameter is changing by less than a specified Δ . Thus the accuracy to which the program will attempt to work is governed by the constants $\Delta_1 - \Delta_{23}$ chosen by the user. When data is being analysed for the first time the number of parameters is usually set initially to fifteen or less and the program automatically uses only half the available data. If parameters have not converged to within their value of Δ after a given number of iterations (usually five), the computation stops automatically and the cause of non convergence is investigated off the machine. If the case is convergent, the number of parameters is then automatically increased to the final value, the total amount of data is used, and analysis proceeds until the specified Δ limits for all parameters is achieved or the maximum number of iterations allowed has been exceeded. By this technique the maximum amount of progress is made on one computer run and the analysis usually completed. If divergence does occur sufficient information is printed out for the analysis to be completed at the second attempt. Very large errors can be tolerated in the initial estimates of parameters, so that the analysis can fairly be described as fully automatic. Failure to converge is usually the result of card punching errors such as giving l_v, m_w or n_w the wrong sign.

4.3. Accuracy of the Analysis Programs

One advantage of this type of analysis method over analysis by graphical methods is that errors are treated in a consistent and proper statistical manner. As discussed under Section 2.1 and Appendix I the so called random errors can be estimated by the computer programs. Unfortunately, because these error estimates are printed out by the computer, there is a temptation to believe they constitute the whole of probable errors in parameters. It should not be forgotten that instrument errors, approximation in the mathematical model and errors in meteorological measurements can greatly exceed the random error computed by the program.

The trajectory analysis scheme is usually able to fit the trajectory data to within 5 metres and velocity to better than 0.4 m/sec, these errors arising mainly from the mathematical model. However, actual velocity may be in error by as much as 2 m/sec because of wind variations. The probable zero error in the longitudinal accelerometer is removed on analysis so that total drag is probably measured to better than 1 per cent. However, the estimated C_D and Mach number depend on meteorological soundings which at Aberporth are made every three hours so that errors of 2 per cent or more are possible.

The errors in response analysis are contributed by many sources including trajectory errors, instrument position and calibration errors, and by approximations in the mathematical model being used for analysis. Most of these can be assessed only by referring to results from particular experiments but some consideration of errors inherent in the mathematical model will be appropriate in this Report.

It will be recalled that approximations were made in the mathematical model described in Section 3 largely for the sake of keeping the computing problem to manageable proportions. The more important approximations are that u is assumed equal to the average value of V obtained by trajectory analysis, that $\theta = \gamma$ obtained by trajectory analysis, and that the oscillatory component in θ and Φ arising from q and r oscillations can be ignored. To check that these assumptions are reasonable the equations of motion were programmed in full with no such approximations and used to predict the standard seven instrument readings. The simulated data thus produced was then analysed with the standard response analysis program in order to determine how far the approximations affect results. As far as possible the

simulated responses were made to be typical of those from free flight data being analysed both from lifting and non-lifting aircraft models. It was shown that the approximations have an effect on the extracted stability derivatives that can usually be ignored.

The simulated data was also used to check the importance of scaling factors, the effect of small errors in accelerometer positions, the effect of errors in trim terms, and errors in mass and moments of inertia on the stability derivative values. Most of these had effects that were of course, predictable and it became evident that approximations in the mathematical model used to analyse the data, had effects that were insignificant compared with likely instrumentation errors. As a result of this exercise with simulated data it was concluded that the analysis scheme now in use is more than adequate for handling the data from instruments that are likely to be available in the foreseeable future. Time has not permitted a more comprehensive study of the effects of noise in the data and the influence of telemetry multiplexing and smoothing on the noise. However, the very extensive study of errors by Hill *et al.*²⁰ for a similar type of analysis scheme for aircraft response data gave encouraging results as well as vividly illustrating how the effects of less important derivatives can often be obscured by noise.

5. Analysis of Free Flight Data in Practice

The flow diagram in Fig. 2 summarises the procedure followed from construction of a model to extraction of the aerodynamic characteristics. The first steps involve careful calibration of the instruments including response characteristics, the positions of instruments within the model and the measurement of the mass, centre of gravity and moment of inertia of the model. The firing of the model and some data processing, takes place at the range (usually Aberporth) as indicated by the dashed line box in Fig. 2.

The telemetry signal is recorded in analogue form on magnetic tape and is also used directly to produce a paper trace record of each instrument reading. The latter is used for a quick assessment of the trial and later for estimating some of the parameters prior to response analysis on the computer. A replay in the Aberporth BRAMBLE equipment of the analogue tape is used to convert the data to digital form which is then recorded on magnetic tape in the form specified in Appendix II. The radiosonde measurements of the atmosphere made at the Range at three hourly intervals are used for the estimation of air density and sonic speed, and for making wind speed corrections to the trajectory observations.

The trajectory data is gathered from kinetheodolites, radar and the radio Doppler trebler system, although not all these systems are necessarily used in every case. The processing at the range includes some smoothing, and correction is also made for the effect of the mean steady winds by a polynomial fitting method. No elaborate analysis of the type described in Appendix IV is attempted at this stage.

On receiving data from the range the first tasks are to apply calibrations to the digital telemetry readings, to make the corrections for instrument response characteristics and multiplexing effects, and to produce the calibrated data in a form suitable for use by the analysis programs. This is achieved with a Fortran program A51A written specifically for reading the Bramble-produced magnetic tape on the ICT 1907 computer. A full description is given in Appendix III. All data channels are reproduced on a line printer but only eight channels on punched cards. Usually two computer runs are necessary, the first to check readings of the instrument and the calibrations just before launch, the second to produce punched cards suitable for stability analysis and other relevant data on the line printer, such as magnetometer readings. The standard data on a card is as follows,

$T(\text{sec})$ AX NC NA CPW LC LA \dot{P} CPV.

The data is punched in free format with two spaces between each number, the time is given to three decimal places, the data to two places. These channels are line printed in the same order, usually followed by the magnetometer reading and then standard calibration channels. There is no easy way to turn single axis magnetometer readings into roll angle on the computer; the output is approximately sinusoidal with maxima at zero roll angle but the amplitude is dependent on climb angle and direction of flight. The present method is to find the times of maxima by inspection and to plot against peak number, thus

obtaining the roll angle at intermediate times by interpolation. When magnetometer data are not available, roll angle can also be deduced approximately by measuring the direction of the plane of polarisation of the telemetry signal.

The telemetry data are produced at 10 msec intervals, and this is unnecessarily frequent for trajectory analysis, for which a 1 sec interval is often adequate; cards direct from Program A51A are therefore not used. The specification for the trajectory analysis program A51T is given in Appendix IV; of the two versions in use Version 2 is preferred, Version 3 is used only when an incidence probe is not fitted and incidence has to be calculated using an assumed value of $C_{N\alpha}$. The trajectory information to be analysed is listed as Table IV3 in Appendix IV. The average or trim values of AX , NC , CPW and ϕ at each time should be used if the time interval is 0.1 sec or more. The parameters determined by the least squares fitting process are the initial conditions V_0 , γ_0 , χ_0 , X_0 , Y_0 , Z_0 , and the zero error $\Delta(Ax)$ in the longitudinal acceleration. Zero shifts often arise as a result of the high overload caused by the rocket boost and must be removed or large position errors would accumulate on integration. Having obtained a best fit, the smoothed trajectory is printed out along with Mach number, incidence α , $Q(\frac{1}{2}\rho V^2)$ and C_D and C_L . An example is reproduced in Fig. 3, it will be noted that most of the input information is also printed out. Further details are given in Appendix IV.

Having obtained a smoothed trajectory, the analysis of the response to each disturbance may proceed with Program A51B, and the specification for the input data required is listed in Appendix V. The initial time of the response T_0 must be later than the all-burnt time of the pulse rocket, and preferably near the peak of at least NC and NA so that q_0 and perhaps r_0 can be taken as zero. The values of the trajectory parameters V_0 , γ_0 , H_0 and ϕ_0 are all taken from the smoothed trajectory along with the average value of C_D during the response. Only C_D is needed because the lift component is obtained from the value of the trimmed normal force $NC(\text{trim})$. The values of $NC(\text{trim})$ and $LC(\text{trim})$ are average values of NC and LC before and after the response less any zero errors believed to be present. The value of $w(\text{trim})$ is taken directly from Vx deduced by trajectory analysis, a value of $v(\text{trim})$ can also be calculated although it is often easier and reasonable to assume $LC(\text{trim})$ and $v(\text{trim})$ are zero. The average value of roll rate $p(\text{trim})$ is deduced from the magnetometer data. Of the variable parameters ENA , ELA , l_t , l_{vw} , m_r , n_t , z_q , y_p and ELC can be taken initially as zero, K is given the nominal value of 1, theoretical or windtunnel values are assigned to l_p , m_q , n_r , z_w and y_v while m_w and n_v are calculated from the observed frequencies, e.g.:

$$m_w = I_y \omega^2 / \rho V^2 S \bar{c},$$

where ω is the frequency in pitch. The amplitude of \dot{q} and \dot{r} may be estimated from NA and LA , and hence the amplitude of w , v , q and r using m_w and n_v , and thus to estimates of w_0 , v_0 , q_0 and r_0 . From the amplitude of \dot{p} and v , the value of l_v can be estimated. Very little time or effort is used in making these estimates since the analysis program can tolerate very large errors.

When a batch of data is to be put on the computer for the first time, the number of parameters allowed to vary initially, P_0 , is set to 15, the final number is set to 23 for a straightforward-problem. This means the computer handles only half the data for the first few iterations and as a general rule this data must include at least two cycles of the response oscillation, so at least four cycles must be covered by all the data. The initial value of the iteration counter is set to 1 unless a print out of the first guess at the response is needed when it should be set to 0. A maximum iteration counter setting of 5 is usually satisfactory unless unrealistic terminal accuracies have been listed in the $\Delta_1 \rightarrow \Delta_{23}$ table.

The final problem is estimating the instrument reading scaling factors or weights, whose purpose is to ensure that analysis is not dominated by the large numerical value of readings from an instrument which may also be making correspondingly large errors. In general these scaling factors should be inversely proportional to the full-scale ranges of the instruments if the percentage accuracy of the instruments can be taken to be the same—usually 1 per cent. Less accurate instruments can then be given even smaller scaling factors. It is always advisable to do a repeat analysis with different scaling to check that results are not unduly sensitive to scaling value.

An example of the print out of a typical problem up to the end of the first iteration is shown in Fig. 4. It should be noted that the atmosphere table and some of the program control parameters are not listed. The DELTA printed under the first parameter estimates are the accuracy demands $\Delta_1 \rightarrow \Delta_{23}$, the DELTA printed subsequently are estimates of the actual accuracy. SIGMA is the rms of the weighted (or scaled) residuals. This particular example is varying all twenty three parameters, and is actually a repeat computer run. The actual fit finally achieved for this case is plotted in Figs. 5 and 6 of which the fit to the longitudinal response can scarcely be faulted. The cross coupling to the lateral response is very marked, but the fit is very good taking into consideration the low amplitude of only one tenth full-scale of the lateral accelerometers. This quality of performance is typical, although for many responses there is insufficient amplitude in either the lateral or the longitudinal mode for all deduced derivatives to be reliable. Often, too, the data contains too much noise for the estimates of the force derivatives z_q and y_p to be reliable, so that like n_p and l_r it is probably best in many cases to hold these derivatives at constant values based on theoretical estimates.

6. Conclusions

A system has been successfully developed using a digital computer for the extraction of aerodynamic derivatives from dynamic response measurements from aircraft and aircraft models. A multidimensional curve fitting technique is employed, using an automatic iterative method based on the principle of least squares, known as the method of differential corrections. The technique, which demands the use of a large high speed computer, has become widely used in recent years for many applications including aircraft response analysis, but hitherto it does not appear to have been successfully applied to problems of the complexity of the coupled motion of aircraft. The reason for this may be the problem of solution divergence common to iterative techniques. By overcoming the problem a virtually fully automatic analysis scheme has been produced with wide application in the field of aircraft response analysis.

Compared with other methods in current use, this analysis scheme offers the following advantages:

- (a) A mathematical model of considerable complexity in the form of the basic equations of motion may be used, including inertial and aerodynamic cross coupling, and non-linear aerodynamics.
- (b) As a consequence of (a) coupled responses may be analysed that are difficult to avoid particularly in free flight so that a much greater proportion of expensive data can be made to yield valuable results.
- (c) Because of the relationships imposed by the mathematical model, incomplete sets of measurements can be analysed. As a corollary, when most state variables and their derivatives can be measured, valuable cross checks on consistency are afforded by the inherent redundancy.
- (d) It is unnecessary to go through the bad statistical practice of unscrambling the state variables from instruments like accelerometers which cannot make pure measurements. This is because the mathematical model predicts the actual instrument readings.
- (e) As a consequence of (d) errors and noise in data are handled in a consistent statistical manner which is invaluable when results from several experiments are being compared. The method is also able to extract results in the presence of relatively high noise levels.

In short, this analysis system is able to make the best possible use of measurements of response. Its use has greatly extended the accuracy and usefulness of the free flight technique of measuring aerodynamic derivatives by the dynamic method, and it could be equally useful for full-scale aircraft and for dynamic windtunnel tests. However, no analysis method can overcome the basic limitations of the experimental technique, which is that only aerodynamic parameters which significantly affect response can be extracted. Generally, parameters which have negligible effects are of negligible importance but there are some examples of parameters which cannot be measured because they have an effect which is indistinguishable from that of another, e.g. $m_{\dot{\delta}}$ cannot usually be separated into its components of m_w and m_q . This can be important in extrapolating results to some different CG position or in control system design. This limitation can only be overcome by changes in experimental technique such as using forced instead of free oscillation methods in flight. The analysis system can be very easily modified to obtain the best results from a different experimental technique.

LIST OF SYMBOLS

a_x, a_y, a_z	Components of acceleration at (x, y, z) in body axes
C	Matrix of partial derivatives
CPV, CPW	Differential pressures in yaw and pitch from probe
C_D	Drag coefficient
C_L	Lift force coefficient
C_Y	Side force coefficient
$C_{L\alpha}$	$\partial C_L / \partial \alpha$
\bar{c}	Geometric mean chord
D	Drag force. Also column vector of residuals
e	As subscript to denote trim or datum value
E	Column vector of parameter errors or corrections
$ENC, ENA, ECPW$ etc.	Zero errors in NC, NA , etc.
f	Partial derivative of instrument reading with respect to a parameter
g	Acceleration due to gravity
h	Instrument damping factor, or altitude
I_x, I_y, I_z	Moments of inertia in roll, pitch and yaw
K	Probe scale factor
KV, KW	Probe sensitivities in yaw and pitch
L	Lift force or rolling moment
L_p, L_r, L_v	Rolling moment derivatives, e.g. $L_p = \partial L / \partial p$
L_t	Rolling moment associated with trim
l_p	$L_p / \rho V S s^2$
l_r	$L_r / \rho V S s^2$
l_v	$L_v / \rho V S s$
l_t	$L_t / \rho V^2 S s$
LC, LA	Lateral acceleration measurements
M	Mach number or pitching moment
$M_q, M_w, M_{\dot{w}}$	Pitching moment derivatives, e.g. $M_q = \partial M / \partial q$
M_t	Pitch moment associated with trim
m	Mass of aircraft
m_q	$M_q / \rho V S c^2$
m_t	$M_t / \rho V^2 c S$
m_w	$M_w / \rho V S c$
$m_{\dot{w}}$	$M_{\dot{w}} / \rho S c^2$

NC, NA	Normal acceleration measurement (but <i>-ve</i> sign)
N or n	Number of data points
N_p, N_r, N_v, N_i	Yawing moment derivatives, e.g. $N_p = \partial N / \partial p$
N_v	Yawing moment associated with trim
n_p	$N_p / \rho V S s^2$
n_r	$N_r / \rho V S s^2$
n_i	$N_i / \rho V^2 S s$
n_v	$N_v / \rho V S s$
n_i	$N_i / \rho S s^2$
p, q, r	Angular velocity resolutes in roll, pitch and yaw
P or p	Number of parameters
\dot{p}	Roll angular acceleration
R	Residual—difference between measured and calculated value
R_e	Reynolds number
S	Standard deviation of parameter
S	Wing area
s	Semi-span
t	Time
U	Sum of the squares of the weighted residuals
u	Velocity along x body axis
V	Velocity along flight path
v, w	Lateral and normal perturbation velocity
W	Matrix of weights (importance of) of observations
w_{π_i}	Matrix of weight of i th observation π_i
X_K	The true value of the k th parameter
x_K	Estimate of k th parameter
x, y, z	Coordinates relative to CG, z vertically down
$\bar{X}, \bar{Y}, \bar{Z}$	Earth fixed coordinates, \bar{Z} vertical upwards
Y	Side force
Y_r, Y_p, Y_v	Side force derivatives
y_p	$Y_p / \rho S V s$
y_r	$Y_r / \rho S V s$
y_v	$Y_v / \rho S V$
Z_q, Z_w	Normal force derivatives

z_q	$Z_q/\rho SV\bar{c}$
z_w	$Z_w/\rho VS$
α	Angle of attack
β	Side slip
γ	Flight path elevation
Γ	Instrument reading
σ	Standard deviation of residuals
θ	Body attitude in pitch
Ψ	Matrix product C^*WC
χ	Body attitude in yaw and flight path bearing
π	Observation vector
ρ	Air density
Φ	Body roll angle
$\begin{pmatrix} 0 \\ 1 \end{pmatrix}_a$	Partial derivative with respect to parameter x_k having a value of zero except when $x_k = a$

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APPENDIX I

Model Matching by the Method of Differential Corrections

The mathematical model consists in its basic form of the assumed equations of motion of the aircraft. Provided certain parameters, including the initial conditions are known, the equations may be integrated to predict the state of the aircraft at any time, t . The state variables at this time, t , can then be used to predict the readings of instruments, such as accelerometers, carried by the aircraft. If we represent the m instrument readings at the i th instant, t_i , by an m -dimensional column vector, π_i , and if there are p undetermined parameters X_k ($k = 1, 2, \dots, p$) in the mathematical model, and there are no errors in the observations or the model, then the observations would be related to the parameters by equations of the form:

$$\pi_i = \pi(X_1, X_2, \dots, X_p, t_i).$$

If there are n observations and $mn > p$, the parameters may be obtained by solving the equations redundantly, using a least squares procedure to determine the best values.

Let π_{ic} be computed values close to the observed values based on estimates x_k of the true parameters X_k (capital X denotes true value), such that

$$\pi_{ic} = \pi(x_1, x_2, \dots, x_p, t_i)$$

and let the residuals be of the form $R_{\pi i} = \pi_i - \pi_{ic}$. Then the procedure is to determine the values of x_k so as to minimise the quantity $U = \sum_{i=1}^n w_{\pi i} R_{\pi i}^2$, where $w_{\pi i}$ is an $m \times m$ diagonal matrix with the weight of the observations along the diagonal. Strictly speaking, since $R_{\pi i}$ is a vector, this expression should be written:

$$U = \sum_{i=1}^n R_{\pi i}^* w_{\pi i} R_{\pi i} \quad (I.1)$$

where the asterisk * denotes the transpose. (In this case column to row.)

Suppose x'_k are approximations to x_k so that

$$x'_k = x_k + \delta x_k$$

and

$$\pi'_{ic} = \pi(x'_k, t_i). \quad (I.2)$$

Linear equations may be obtained by expanding equation (I.2) as a Taylor series:

$$\pi'_{ic} = \pi_{ic} + \sum_{k=1}^P f_{ki} \delta x_k + O((\delta x_k)^2, t_i)$$

where f_{ki} are m -dimensional column vectors of partial derivatives at time t_i of the form $\partial \pi(x_k, t_i) / \partial x_k$. The correction procedure is based on the assumption that $(\delta x_k)^2$ is negligible. In practice, this is not generally true, and the minimization of U is obtained by repeated application of the procedure. Let

$$R'_{\pi i} = \pi_i - \pi'_{ic}$$

so that

$$R'_{\pi i} = R_{\pi i} - \sum_{k=1}^P f_{ki} \delta x_k \quad (I.3)$$

and

$$U' = \sum_{i=1}^n R'_{\pi i} w_{\pi i} R_{\pi i}. \quad (\text{I.4})$$

To minimise U' we must find values of δx_k for which $\partial U'/\partial(\delta x_k)$ is zero. Taking all n observations define D as a column vector of element $R_{\pi i}$, E as a column vector of δx_k , W as an $mn \times mn$ diagonal matrix of the weights $w_{\pi i}$ and let

$$C = \begin{vmatrix} f_{11}, & f_{21}, & \dots & f_{p1} \\ f_{12}, & f_{22}, & \dots & f_{p2} \\ \dots & \dots & \dots & \dots \\ f_{1n}, & f_{2n}, & \dots & f_{pn} \end{vmatrix}.$$

Then equation (I.4) may be written as:

$$U' = D' * W D'$$

and from equation (I.3) we have

$$D' = D - CE.$$

Therefore

$$U' = (D' - E * C') W (D - CE)$$

and

$$\frac{\partial U'}{\partial(\delta x_k)} = -2C' * W D + 2C' * W C E = 0$$

or

$$C' * W C E = C' * W D \quad (\text{I.5})$$

which is the matrix form of what are commonly called the 'normal equations'. Defining $\Psi = C' * W C$ then:

$$\Psi E = C' * W D$$

and

$$E = \Psi^{-1} C' * W D, \quad (\text{I.6})$$

which is the desired solution and gives the increments to be added to the parameters. The procedure is repeated using the new parameters as starting values, until E is negligible when $(\delta x_k)^2$ must be negligible, and the best estimate x_k of the parameters X_k obtained. Equation (I.6) represents the most general form of the result obtained in Ref. 18 which was concerned with the three-dimensional case (i.e. $m = 3$). Estimates of the accuracy of the observations and the parameters were also derived in Ref. 18 and will be quoted without proof.

The best estimate of the accuracy of the observations σ^2 or the minimum value of U' is given by

$$\sigma^2 = \sigma_0^2 - \frac{E^*C^*WD}{3n - p} \quad (I.7)$$

where σ_0^2 is the rms of the residuals of the observations using the uncorrected parameters.

The covariance matrix of the parameters is given by

$$\text{cov}(E) = \sigma^2\Psi^{-1}.$$

If the parameters are uncorrelated all the terms in $\text{cov}(E)$ will vanish except for the diagonals. Therefore the variance S_k^2 of X_k is given by

$$S_k^2 = \sigma^2\Psi_k^{-1}$$

where Ψ_k^{-1} is the k th diagonal element of Ψ^{-1} . At the 95 per cent probability level, the probable error in X_k is given by

$$\Delta X_k = 2S_k = 2\sqrt{\sigma^2\Psi_k^{-1}}. \quad (I.8)$$

This estimate is only true when $(\delta x_k)^2$ and the non-diagonal terms of Ψ^{-1} are all negligible.

The computer program is designed to print out σ , $(n - p)$ and the ΔX_k after every iteration and the numbers provide a useful way of judging the confidence that may be placed in particular results.

APPENDIX II

Specification for the Format of Industry Compatible Magnetic Tape from BRAMBLE for Use on the ICT 1907 Computer at RAE, Farnborough

II.1. Introduction

The BRAMBLE system is used to digitise the 24/48 channel 465 MHz telemetry data from Aberporth and Woomera trials. The digitised data is to be recorded on the now standard Industry Compatible Magnetic Tape. This specification relates to the format in which the data is to be recorded so that it is compatible with the ICT 1907 computer.

II.2. Number of Tracks

There will be seven tracks, numbered 1 to 7. Track No. 1 will be used for parity checking. Tracks 1 to 6 for the numerical information in BINARY code. Parity is odd. And it is emphasised that no alpha numeric code is to be used, all characters are pure BINARY numbers.

II.3. Packing Density

The recording density shall be a 200 bit/inch. The beginning of tape marker must be at least 14 ft from the beginning of the tape and the end of tape marker at least 25 ft from the end. The *interblock gap* shall be the standard 0.75 inch.

II.4. Tape Marks

The tape marks shall be special one character blocks of the form X0X1111, where X may be either 0 or 1. The 1907 ignores both bit 5 and the parity bit on reading a tape mark.

II.5. The Format

The first block on the tape shall be a HEADER followed by a tape mark. This will be followed by five groups or runs of blocks each followed by a tape mark. These runs will be (1) BRAMBLE CALS, (2) FLIGHT DATA, (3) BRAMBLE CALS, (4) AERO CALS, (5) ABERPORTH CALS, (or WOOMERA). Before going into details it should be noted that the 1907 has been programmed to construct words of 12 bits out of pairs of tape characters. This applies whether a header or ordinary block is being read, so that alpha numeric characters in the header are not acceptable.

II.5.1. The Header

The header shall consist of 20 characters and be used to store a serial number, the trial number and the firing date. The trials numbers used by Aberporth and Woomera usually contain letters, these will have to be omitted or be replaced by numerals for BRAMBLE purposes. All numbers must be translated into BINARY form, one decimal digit per character. Characters will be allocated as follows:

Characters 1 → 4	Tape Serial No.
Characters 5 → 6	Trial No.* (numeral substitute of letters)
Characters 7 → 12	Trial No.
Characters 13 + 14	Day of firing
Characters 15 + 16	Month of firing
Characters 17 → 20	Year of firing.

* For example the letter X could be replaced by 24, XY by 2425, but little importance is attached to this matter as long as tapes cannot be confused.

II.5.2. Cals and Flight Data Blocks

Each block shall consist of 1004 characters. The first 4 characters of each block shall each contain a beginning of block marker (BBM) of decimal values 26, 28, 29, 30 or 31 depending on the type of run, as listed in Table II.1. For example the BBM for flight data will always be 28. The remaining 1000 characters will record either 20 cycles of 24 channel telemetry or 10 cycles of 48 channel telemetry. In the first case time will be followed by 23 data channels, in the case of 48 channel telemetry, time will be followed by 46 channels but 4 extra BBM characters will be inserted before the time.

The time will use 4 characters and each channel 2 characters. So that each cycle of 24 channel telemetry will use 50 characters. Each cycle of 48 channel telemetry will use 100 characters (including 2 blank characters at the end).

Where time is not known as in the case of calibrations, a standard time starting at 400 sec will be used as listed in Table II.1.

II.6. Requests for BRAMBLE Processing

The only difference between trials will be the type of telemetry sender and the times of the firing data required. Dummy times of calibration and the values of the BBM will be standard. This simplifies the request form.

TABLE II.1

Run	Data	Times (sec)		Beginning of block marker
		Start	Stop	
1	BRAMBLE Cals	400	403	26
2	Firing Data			28
3	BRAMBLE Cals	420	423	29
4	Aero Cals	440	443	30
5	Aberporth Cals	460	463	31

APPENDIX III

BRAMBLE Data Processing Program A51A

The BRAMBLE system at RAE Aberporth is used to provide digitised telemetry data on magnetic tape to the specification listed under Appendix II. The purpose of program A51A is to process this raw digital data into forms suitable for subsequent analysis. It performs the following basic tasks:

- (1) Sorts any number of channels into any prescribed order and provides output in this order on line printer and/or punched cards.
- (2) Corrects for the time delays between channels caused by the sequential multiplexing system of the telemetry. This is done by correcting each reading to the value it would have had if sampled at the time of the preceding synchronisation pulse.
- (3) Corrects for instrument response characteristics so that all readings take the most probable value of the quantity being measured.
- (4) Combines the values of quantities sampled twice during a multiplexing cycle.
- (5) Applies instrument calibrations.
- (6) Corrects for drift in instrument rest levels.

Operations (2), (3) and (4) are achieved by fitting a quadratic of the form $\Gamma_0 = at^2 + bt + c$ to 5 sequential samples of a channel, where Γ_0 is the reading and t is measured relative to the central data point. Having found a , b and c the smoothed reading at any time t can be computed. The highest frequency component in the data is usually much less than 10 cycles per second so that with a sampling interval of only 6 msec the distortion caused by smoothing is negligible. If only very low frequency components are known to be present, 11 point smoothing is available in the program for use with very noisy data. Instrument response characteristics are corrected by assuming that the true reading Γ is related to the observed reading Γ_0 by $\Gamma = \Gamma_0 + \dot{\Gamma}_0(2h/\mu) + \ddot{\Gamma}_0(1/\mu^2)$, where h is the damping factor of the instrument and μ the natural frequency. Since the smoothing process has determined a , b and c then $\dot{\Gamma}_0$ and $\ddot{\Gamma}_0$ are given by $\dot{\Gamma}_0 = 2at + b$ and $\ddot{\Gamma}_0 = 2a$. The reading corrected for time t and instrument response is then given by:

$$\Gamma = at^2 + (2a(2h/\mu) + b)t + 2hb/\mu + 2a/\mu^2 + c.$$

Quadratic representation of the instrument calibrations are also used for operation (5). Calibration values are given to the program at 5 equally spaced intervals and converted to quadratic form in the computer. The program must not be used if calibrations cannot be accurately represented as quadratics.

Input Parameter List for Program A51A—BRAMBLE Data Processing

No. of cards	Fortran symbol	Units	Comments
1	HLD	—	This is the value of the beginning of block marker for the data to be processed, <i>see</i> Mag Tape Specification.
	BZ	msec	This is the time of the BRAMBLE zero in range or trajectory time, usually ~ -1000 msec.
	H	msec	The expected time interval between sync pulses.
	KA	—	Number of channels (or pairs of channels) to be processed up to a maximum of 23.
	NC	—	Number of cases (max. 25), i.e. number of separate periods for which data is to be processed. NC would be 1 if data is to be extracted in one continuous run.
			NB All numbers on this CARD are INTEGERS in FREE FORMAT.

No. of cards	Fortran symbol	Units	Comments
NC	IT LT	msec msec	Initial time for which data is required. Final time for which data is required. NOTE IT and LT are punched as INTEGERS in FREE FORMAT on one card. There must be NC cards, one for each time period defined.
KA	C(4) C(5) C(6) C(7) C(8) X(1) X(2) X(3) X(4) X(5)	See comments See comments See comments	Channel data of 10 <u>real</u> numbers on one card in free format for each of the KA channels. Channel No. of telemetry. (NB smallest values = 2). Pairs indicator. If 1 then program assumes that channel C(4) is paired with channel (C(4) + 12), if -1 then no pairs are assumed. Zero error correction for calibration curves, i.e. this number is added to result after calibration has been applied to a data point. Instrument damping term = $2000 h/\mu$ where h is the damping factor (usually about 0.5) and μ is the natural frequency in rad/sec. Instrument natural frequency term = $10^6/\mu^2$. Calibration for 150 BRAMBLE units. Calibration for 250 BRAMBLE units. Calibration for 350 BRAMBLE units. Calibration for 450 BRAMBLE units. Calibration for 550 BRAMBLE units. NOTE The program takes these 5 calibrations and fits a quadratic of the form $y = ax^2 + bx + c$, and stores the values of a , b and c in III(1), III(2) and III(3) respectively.

The data cards (1 + NC + KA in number) are placed at the end of the program deck. On the ICT 1907 computer the following operator instructions are given with the program and magnetic tape.

Output Formats for Program A51A

Peripheral instructions	Console instructions
Load Program	FI X FAE Gθ X FAE 21
Load Private File on MT Deck nn	GI A51A ∇ nn ∇ 0 ∇ * 4 Gθ A51A 20

The analysed data is punched on cards and printed on the Line Printer. Supplementary information including calibration checks and the magnetic tape header are printed on the Line Printer only.

III.1. Line Printer Output

III.1.1. The magnetic tape header is printed giving the Tape No., the Trial No. and the firing data as in the following example:

TN 0123 TRIAL NO 2425931 DATE 20 6 1969

A space is often inserted instead of zero, e.g. 02 is printed 2.

III.1.2. The calibration being used and the channel numbers are printed next. A quadratic is fitted to the calibration on the data cards and the calibrated values corresponding to 100, 350 and 600 BRAMBLE units are calculated from the quadratic and printed. The calibration zero shift III(6) is not added. The format for three channels is as follows:

Time	Channel No.		
	5	12	2
CAL 100	-5.00	20.00	-1.00
CAL 350	0.00	0.50	-0.50
CAL 600	5.00	-19.50	0.00

Note that the channels are printed in the order of the calibration cards.

III.1.3. When the program encounters a tape mark it prints TAPEMARK, there could be several of these before data is printed.

III.1.4. The calibrated and smoothed data is then printed under the same columns as the calibrations except that the time is printed in this first column, thus:

4.620	-7.10	18.36	0.01
4.635	-7.00	15.41	0.01

There may be up to 23 data channels.

III.2. Card Punch Output

The information printed on the cards is identical to that printed by the line printer under the heading III.1.4, except that not more than 8 channels can be accommodated on one card. The format of the card output is easily changed to suit particular needs, for example to give data suitable for Program A51B on one card and A51T on another. The only problem is that cards must then be sorted by hand. It is also possible to arrange that only some channels are reproduced on punched cards.

APPENDIX IV

Trajectory Smoothing and Drag Analysis Programs A51T

These programs are simplified versions of the trajectory analysis programs described in Ref. 18 for the analysis of ballistic trajectories in three dimensions. The simplifications are based on the assumption that the trajectories cover ranges of a few miles only and are dominated by aerodynamic forces. The position of the free flight model is defined relative to earth fixed axes $0\bar{X}\bar{Y}\bar{Z}$, while forces on the model are resolved tangentially and normal to the flight path whose direction is defined by the elevation γ and bearing χ with respect to the horizontal \bar{X} , \bar{Y} plane as shown in Fig. 1. The lift force L and the side force Y roll with the model and their direction, relative to the flight path, is defined by the bank angle Φ . The equations of motion are

$$m\dot{V} = -D - mg \sin \gamma,$$

$$mV\dot{\gamma} = L \cos \Phi - Y \sin \Phi - mg \cos \gamma,$$

$$mV\dot{\chi} \cos \gamma = L \sin \Phi + Y \cos \Phi,$$

$$\dot{\bar{X}} = V \cos \gamma \sin \chi,$$

$$\dot{\bar{Y}} = V \cos \gamma \cos \chi$$

and

$$\dot{\bar{Z}} = V \sin \gamma$$

where Φ is assumed to be a known function of time, $D = \frac{1}{2}\rho V^2 SC_D$, $L = \frac{1}{2}\rho V^2 SC_L$, $Y = \frac{1}{2}\rho V^2 SC_Y$ and air density ρ is given in tabulated form as a function of altitude \bar{Z} .

The purpose of the analysis programs is to find the values of the initial conditions, V_0 , γ_0 , χ_0 , \bar{X}_0 , \bar{Y}_0 , \bar{Z}_0 at $t = t_0$, and in most versions estimates of C_D and C_L , such that values of \bar{X} , \bar{Y} , \bar{Z} at n values of t obtained by integration of the equations of motion best fit the observed \bar{X} , \bar{Y} , \bar{Z} . In the process smoothed values of V , γ and χ are obtained at each value of t .

Program A51T Version 1 assumes that L and Y are zero and determines the constant value of C_D to give a best fit trajectory. Version 2 assumes Y is zero but obtains L and D from on-board acceleration measurements a_x and a_z , plus measurements of α from a differential pressure yaw meter. The following relationships are assumed:

$$-D/m = a_x + a_z \alpha,$$

$$L/m = a_x \alpha - a_z$$

and

$$\alpha = CPW / \frac{1}{2}\rho V^2 KW$$

where CPW is the differential pressure and KW the probe sensitivity.

Versions 3 and 4 are intended for use when some of the supplementary data is not available. Version 3 is used when both Φ and α are unknown, and assumes that Φ is changing rapidly enough for the net effect of L to be zero and that α can be obtained from a_z and an assumed value of $C_{N\alpha}$. Version 4 obtains α in the same way but assumes measurement of bank angle Φ are available.

Versions 2-4 are all designed to detect and remove a possible zero error in a_x , the longitudinal acceleration.

*Input Parameter List for Programs A51T Versions 2 to 4
for Trajectory Smoothing and Drag Analysis*

Entry label number	Symbol	Units	Comments
1	DATA TITLE XXXXXXXXXX	—	On a line by itself or a card write DATA TITLE. Any wording or numbers on following line or card will be copied to output document. The program will SEARCH for these two cards or lines which must be present or no data will be input.
2	S		This is the LABEL No. to be entered next, enabling parts of the input routine to be by-passed so that standard data once in the computer need not be entered for subsequent cases.
3	Table 1 N' A P T Table 2 N' M KW D' C' $\Delta_1 \rightarrow \Delta_7$	— ft (m) mbs °C — rad ⁻¹	N' is the number of rows in the atmosphere table each row being A, P, T : the altitude, pressure and temperature. A single card is recommended for each row. Maximum number of rows is 26. N' is the number of rows of M, KW in the probe sensitivity table. Mach number. Incidence probe sensitivity ($\Delta P / \frac{1}{2} \rho V^2$) per rad. Table 2 is omitted in Version 3. CONSTANT PARAMETERS S/m area mass ratio. $C_{N\alpha}$ normal force derivative. (Versions 3 and 4 only.) List of accuracies required for all variable parameters, in the same order as the variable list. There must always be 7 numbers in this list.
4	T_0 V_0 GAMMA ₀ KI_0 $\Delta(Ax)$ X_0 Y_0 Z_0	sec ft/sec (m/sec) rad rad g ft (m) ft (m) ft (m)	PARAMETERS LIST (of 5 numbers) Initial time. Initial velocity Initial climb angle. Initial azimuth angle. Zero error in longitudinal acceleration. } Initial plan position. Initial height. NOTE All these parameters, except T_0 , are varied to obtain a best fit to the observed trajectory.
5	E max ACCURACY		PROGRAM CONTROL PARAMETERS Initial data rejection level, should be set very large. Numbers in the parameter accuracy list are multiplied by this factor.

Entry label number	Symbol	Units	Comments
5	DELTA T N' Q'		Maximum integration step length. Maximum number of iterations. Final case indicator, if 2 program halts at end of case.
8	Table 3 M T X Y Z AX AZ or NC CPW Φ	 sec ft (m) ft (m) ft (m) g g psi rad	M is the number of rows in the data table, each row being the seven numbers in the list and punched on 1 card. Height. Longitudinal acceleration. Acceleration along Z axis of body (= minus normal acceleration). Differential pressure of incidence probe. (Omit for Versions 3 and 4.) Roll angle measured from horizontal. (Omit for Version 3.) NB There must be at least M data rows. If there are more than M rows the remainder are rejected when the program searches for the DATA TITLE of the next case.

Output Format of Program A51T

This will be readily understood from the example in Fig. 3 and it will be noted that the important input parameters are printed out as well as the best fit to the data. However Table 1, Q' , N' and Table 3 are not printed. The required accuracies $\Delta_1 \rightarrow \Delta_7$ are printed in the row beneath the first guesses of the variable parameters. Thereafter DELTA denotes the estimated probable errors of the variable parameters. SIGMA is the predicted rms of the residuals, N-P the number of degrees of freedom, while R_X , R_Y and R_Z are residuals.

APPENDIX V

Response Analysis Program A51B

Input Parameter List

Entry label number	Symbol	Units	Comments
1	DATA TITLE XXXXXXXXXX	— —	On a line by itself or a card write DATA TITLE. Any wording or numbers on following line or card will be copied to output document. The program will search for these two cards which MUST BE PRESENT or no data will be input.
2	S		This is the LABEL No. to be entered next, enabling parts of the input routine to be by-passed so that the standard data once in the computer need not be entered for subsequent cases.
3	Table 1 <i>N'</i> <i>A</i> <i>P</i> <i>T</i> <i>X NC, X NC,</i> <i>Y NC; X NA,</i> <i>Y NA, Z NA;</i> <i>X LC, Y LC,</i> <i>Z LC; X LA,</i> <i>Y LA, Z LA;</i> <i>IX, IY, IZ</i> <i>IXZ</i> <i>M</i> <i>S</i> <i>L1</i> <i>L2</i> $\Delta_1 \rightarrow \Delta_{23}$	— ft (m) mbs °C ft (m) slug ft ² (kg m ²) slug ft ² (kg m ²) slugs (kg) ft ² (m ²) ft (m) ft (m)	<i>N'</i> is the number of rows in the atmosphere table, each row being <i>A, P, T</i> ; the altitude, pressure and temperature. A single card is recommended for each row. Maximum number of rows is 26. Accelerometer coordinates. The convention is that <i>NC</i> and <i>LC</i> are normally at the CG while <i>NA</i> and <i>LA</i> are towards the tail of the aircraft. Moment of inertia. Cross product of inertia. Mass. Reference area. Reference lengths for longitudinal derivatives. Reference length for lateral derivatives. List of accuracies required for all variable parameters in the same order as the variable parameter list. There must always be 23 numbers in this list. Two or more lines or cards may be used.
4	<i>I</i> <i>KW, KV</i> <i>XP</i> <i>ECPW, ECPV</i>	ft (m)	NUMBER OF INSTRUMENTS. This would normally be 5 for accelerometers only and 7 when probes are included. Probe sensitivity in pitch and yaw. Probe position ahead of CG. Probe zero errors. } OMIT if <i>I</i> is 5 or less.

Entry label number	Symbol	Units	Comments
6	N_1 Q' $SNC, SNA, SP,$ $SLC, SLA,$ $SCPW, SCPV$ S		Maximum value of iteration counter. NB Number is reset to 1 when changing from P_0 to P_1 . Final case indicator, if set to 2 program is halted at end of case. These scaling factors are the weights to be applied to each of the 7 instruments, and are inversely proportional to the probable errors of the instrument. Usually the scale factors are made inversely proportional to the full scale sensitivity of each instrument, which normally have the same 1 per cent accuracy. LABEL No. to be entered next, as with 2.
8	Table 2 M T A NC NA CPW LC LA P CPV	 sec g g g g g g rad/sec ²	M is the number of rows or cards in the DATA table, each row being the 9 numbers in the list. NOTE (1) that the second number A, the longitudinal acceleration is discarded by the program after reading, (2) that only $M/2$ rows are initially used when $P_0 \neq P_1$ so that M should be at least 80, or cover about 1 sec or at least 3 cycles of oscillation, (3) units for CPW and CPV depend on units chosen for KW and KV .
11			Entry point for re-run with same Table 2 data.

Output Format of Program A51B

This is of the same form as that from Program A51T as shown in Fig. 3 but is very much more lengthy, so only the first part is reproduced in Fig. 4. All the input parameters (except the control parameters P_0, P_1, N_0, N_1 and Q' and Tables 1 and 2) are printed first. The accuracies $\Delta_1 \rightarrow \Delta_{2,3}$ are printed under the first guesses at the variable parameters. After each iteration an estimate of the expected variance SIGMA and number of degrees of freedom are printed above the new list of variable parameters. The DELTA printed under the VARIABLES are estimates of the probable errors, except when the VARIABLE is frozen, in which case the accuracy required continues to be printed. At each data point the values of u, v, w and p, q and r are printed in addition to the predicted instrument readings and the residuals. If no residuals are printed then either there were no observations at that time or at least one of the residuals is greater than E max and the data point has been rejected. Note that the weighted value of the residuals are printed (residual) \times (scaling factor) and SIGMA is the rms of the weighted residuals.

Other Versions of Program A51B

The above description refers to the program as originally written in the EMA language. However, the program has since been translated into FORTRAN and several versions have been developed.

One version is almost identical to the EMA program, another was specially developed for analysis of Dutch Roll Motion of free flight models. Both these versions are described, with full program listing in RAE Tech. Memo. Aero 1335.

Another version, A51C, has been successfully developed in cooperation with BAC for the analysis of the Dutch Roll Motion of full scale aircraft. Details may be obtained from Aerodynamics Department RAE.

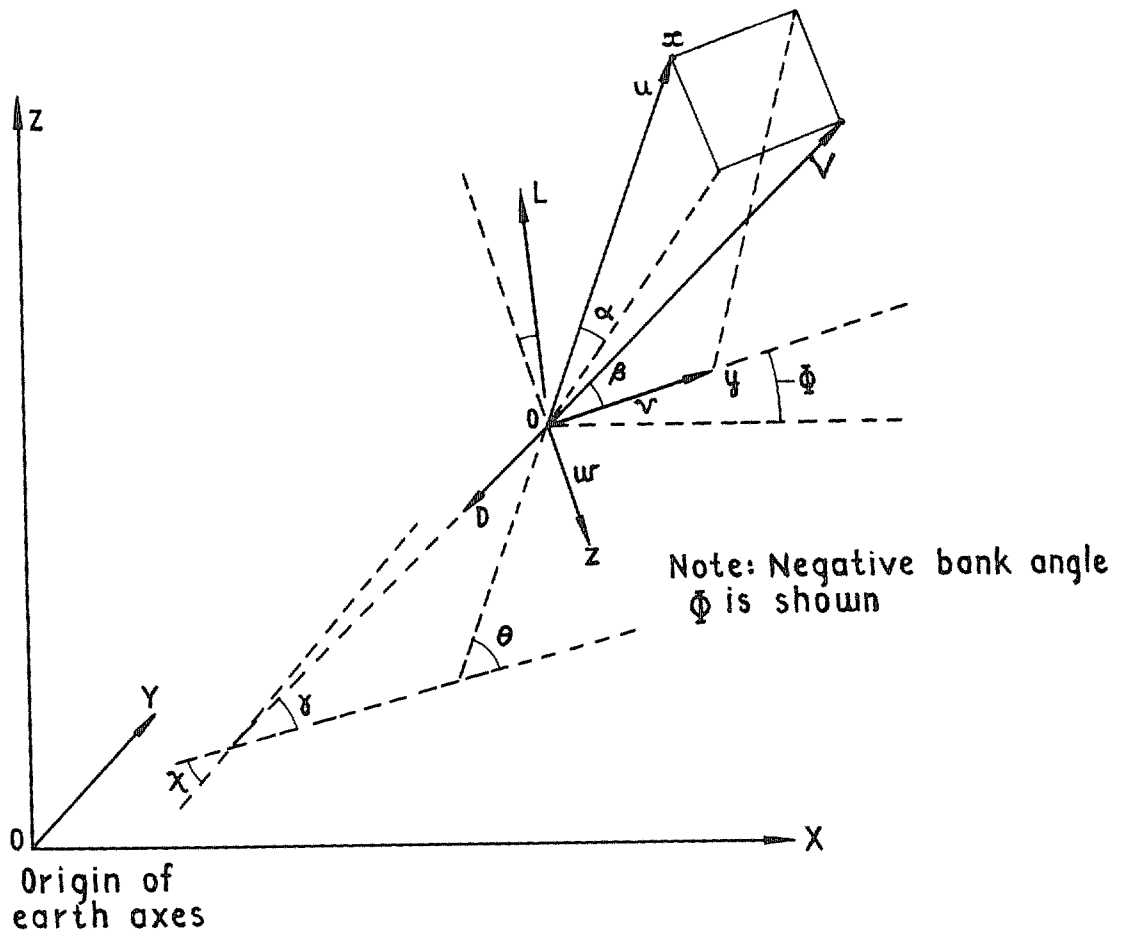


FIG. 1. Definition of notation and of forces on aircraft in body and earth axes.

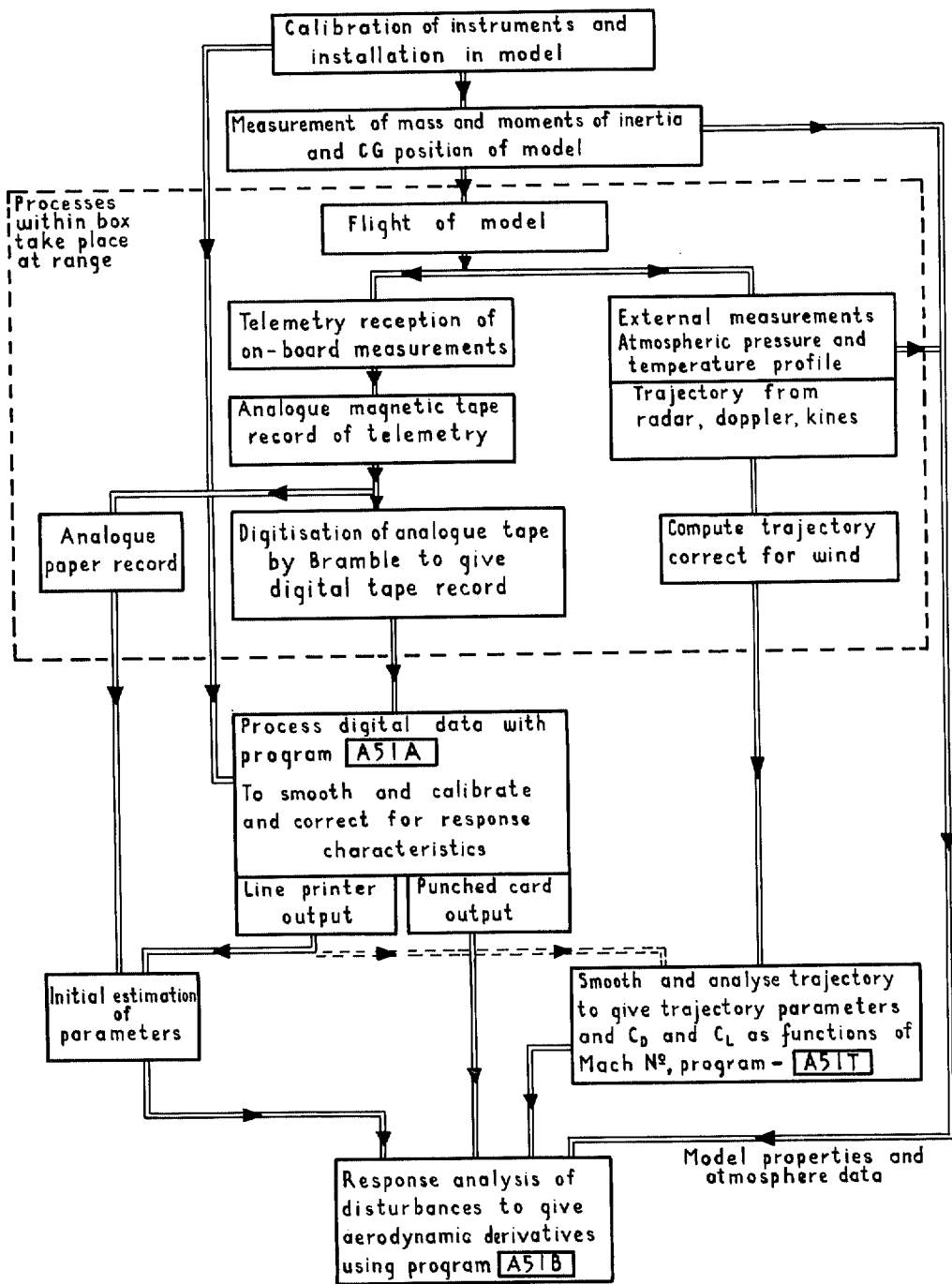


FIG. 2. Flow diagram of free flight dynamic stability measurement and analysis procedure.

PROGRAM A51T VERSION 2A 3D TRAJ. ANAL. USING AX,AZ,CPW & PHI. 4.11.70

ORION 25

M.NO. KW
 0.900 3.930
 1.000 3.670
 1.100 3.450
 1.200 3.360
 2.500 3.350

B/M= 1.360

E MAX ACCURACY DELTA T
 10000.000 1.000 1.010

VARI.	V(FT/SEC)	GAMMA(RAD)	KI	AX ERROR	X0(FT)	Y0(FT)	Z0(FT)
DELTA	1.00	0.01	0.01	0.01	10.00	10.00	10.00

SIGMA= 3.36246 N-P = 26

VARI.	V(FT/SEC)	GAMMA(RAD)	KI	AX ERROR	X0(FT)	Y0(FT)	Z0(FT)
DELTA	1.40	0.00	0.00	0.23	2.24	3.67	2.27

T	v	M.NO	GAMMA	Z(FT)	PHI	ALPHA	CD	CL	Q	RX	RY	RZ
22.000	1294.0	1.217	12.76	15760	-1399.84	1.88	0.0135	0.0357	1212.7			
23.000	1264.9	1.191	13.82	16053	-1473.18	2.01	0.0137	0.0379	1147.7	-7	-3	1
24.000	1236.2	1.165	13.32	16352	-1544.24	2.15	0.0139	0.0403	1085.0	1	2	4
24.160	1231.7	1.161	12.92	16397	-1559.71	2.17	0.0141	0.0406	1075.5			
25.000	1209.5	1.141	9.69	16600	-1627.89	2.28	0.0141	0.0427	1029.7	1	8	0
26.000	1185.6	1.119	6.43	16763	-1709.26	2.39	0.0144	0.0449	983.8	5	-4	-4
27.000	1163.0	1.098	6.74	16892	-1799.79	2.51	0.0147	0.0478	942.5	4	-1	-0
28.000	1140.3	1.077	7.03	17036	-1893.77	2.61	0.0152	0.0508	901.7	0	1	-1
29.000	1118.9	1.058	3.14	17141	-1995.19	2.69	0.0154	0.0537	865.3	-0	2	-3
30.000	1100.3	1.040	0.34	17167	-2097.75	2.75	0.0155	0.0565	836.3	0	2	1
31.000	1082.7	1.024	1.52	17184	-2211.78	2.82	0.0152	0.0596	809.2	-2	3	0
31.790	1069.2	1.011	-0.04	17200	-2297.73	2.87	0.0151	0.0620	788.8			
32.000	1065.9	1.008	-1.01	17198	-2309.76	2.88	0.0149	0.0627	784.0	-0	4	-4
32.500	1058.7	1.001	-3.64	17176	-2356.18	2.96	0.0144	0.0651	774.0			

FIG. 3. Example of line printer output from program A51T.

PROGRAMME B VERSION 2 19.5.70 AIRCRAFT RESPONSE ANALYSIS.

ORION 24 OSC.1

ACCELEROMETER POSITIONS

X.NC	Y.NC	Z.NC	X.NA	Y.NA	Z.NA	X.LC	Y.LC	Z.LC	X.LA	Y.LA	Z.LA
0.003	0.017	-0.110	-2.106	-0.058	0.004	0.086	0.017	-0.081	-2.552	0.232	0.001
IX	IY	IZ	IZX	M	S	L1	L2				
1.925	21.100	22.120	-0.120	9.440	12.813	3.843	1.667				

NUMBER OF INSTRUMENTS = 7

PROBE CHARACTERISTICS

KW	KV	XP	ECPW	ECPV						
0.011	0.011	3.360	1.060	-0.620						
TO	VO	THETA 0	HO	PHI 0	CD	XE	LR	NP		
5.616	1451.000	0.458	13410.000	-2.510	0.013	2.500	0.050	-0.002		
NC(TRIM)	LC(TRIM)	W(TRIM)	V(TRIM)	P(TRIM)	Q(TRIM)	R(TRIM)				
-1.800	0.005	40.000	-0.500	-0.240	0.040	-0.007				
E MAX	ACCURACY	DELTA T								
10.000	3.000	0.015								

SCALING OF

NC	NA	P.	LC	LA	CPW	CPV
0.200	0.200	0.050	1.000	1.000	0.200	0.200

VARI	ENA	ELA	LT	LV	LVW	LP	PO	WO	QO	VO	RO	
DELTA	0.057	-0.013	0.020	-0.022	-1.000	-0.070	-0.140	-62.000	0.062	-3.100	0.053	
	0.050	0.050	0.100	0.003	0.200	0.010	0.500	1.000	0.050	1.000	0.010	
VARI	MT	NT	MW	NV	MQ	NR	ZW	YV	ZQ	YP	ELC	K
DELTA	0.072	0.010	-0.205	0.133	-0.330	-0.230	-0.810	-0.120	0.100	-0.077	0.000	1.070
	0.500	0.500	0.002	0.002	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050

SIGMA= 0.01360 N-P = 530

VARI	ENA	ELA	LT	LV	LVW	LP	PO	WO	QO	VO	RO	
DELTA	0.043	-0.014	-0.013	-0.024	-0.786	-0.035	-0.237	-62.374	0.039	-4.186	0.057	
	0.009	0.000	0.003	0.000	0.035	0.004	0.007	0.136	0.007	0.260	0.000	
VARI	MT	NT	MW	NV	MQ	NR	ZW	YV	ZQ	YP	ELC	K
DELTA	0.042	0.122	-0.205	0.135	-0.334	-0.310	-0.837	-0.130	-0.181	-0.035	-0.025	1.079
	0.010	0.007	0.000	0.000	0.002	0.010	0.006	0.003	0.071	0.016	0.003	0.029

FIG. 4. Example of output of A51B up to the end of the first iteration.

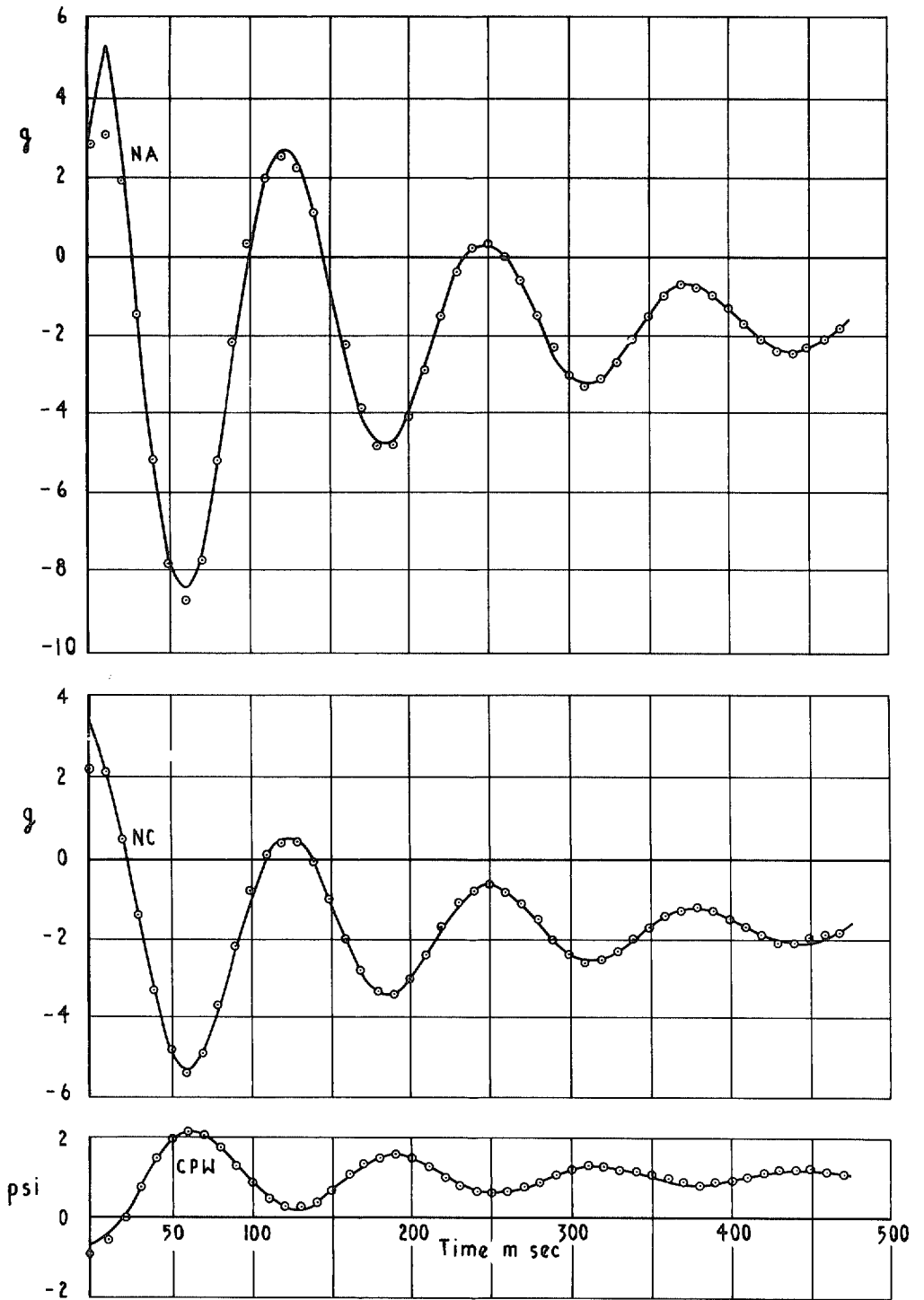


FIG. 5. Fitted curve to longitudinal response from Orion 24.

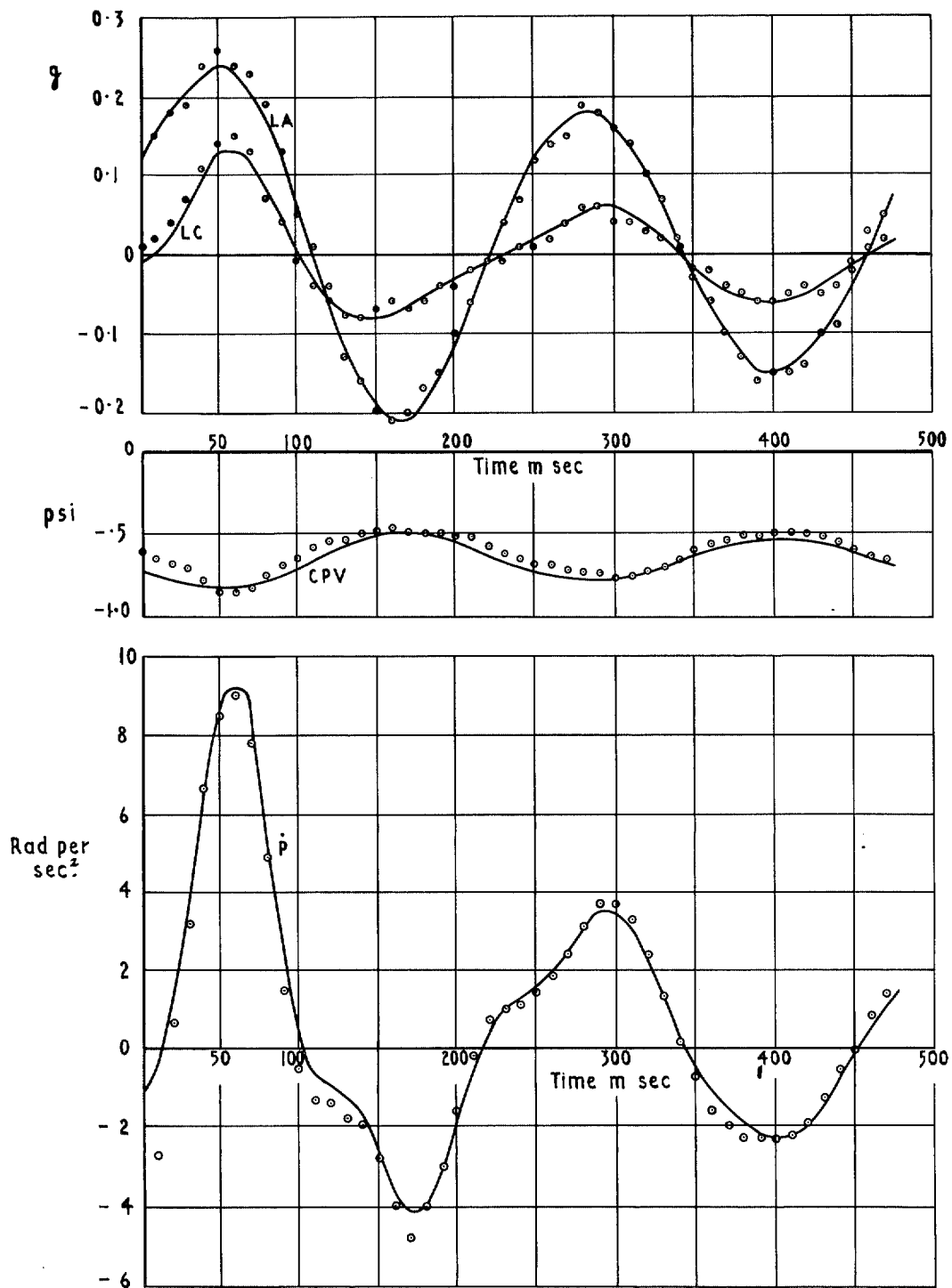


FIG. 6. Fitted curve to lateral response from Orion 24.

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