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The Effect of Variations in Local Gravity and of Aircraft Speed on the Effective Weight of Aircraft in High Performance Cruise

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Summary

Numerical data are given of the changes in effective gravity experienced by aircraft in flight at various heights and speeds. In most flight conditions this results in a reduction in the effective weight experienced by aircraft which is shown to vary considerably with heading and latitude. For Concorde in cruise there is a reduction in effective weight of approximately 0·5 per cent in westbound flight and this increases to more than 2 per cent in eastbound flight.

CONTENTS

*Replaces RAE Tech. Report 69274—A.R.C. 32 167.

1. *Introduction*

When aircraft fly at speeds approaching a significant even though small proportion of orbital speed, they will experience an apparent reduction in gravity or weight. This effect assists the cruise performance of high speed aircraft. The present investigation is mainly concerned with Concorde, the first commercial aircraft, which can be considered as a partial orbiter. Although on this aircraft the contribution of gravity relief is no more than 2·3 per cent when flying at the most favourable course and 0·5 per cent in flight along the least favourable track, these effects are not without significance in a type of aircraft where performance margins are essentially small. When cruising speeds substantially higher than $M=2$ are considered, the centrifugal weight reduction will become progressively more important and will make a significant contribution to the overall economy.

The material presented here is, of course, in no way original and is concerned with an effect that is normally taken into account in studies of hypersonic flight, as for instance in Refs. 1–6, and is the basis of the mechanics of space flight. However, it appears that general aircraft designers, flight test analysts etc. are little aware of the fact, that these effects are already significant in flight at quite mundane supersonic speeds and not entirely negligible even in subsonic flight. The aim of this paper is therefore mainly to remind workers in this field of the relevance of celestial mechanics to aircraft performance in general and to present numerical data for the appropriate corrections to be readily made.

Two distinct contribution are considered here, firstly the variation of 'local' gravity with latitude and height as it would affect a stationary object at that location and secondly the additional effect of aircraft speed. Since the speed of the aircraft is added vectorially to the local circumferential speed of the earth, the latter effect varies with heading and latitude.

2. *Variations in Local Gravity with Latitude and Height*

The effective 'gravity' experienced by an object at rest in relation to earth is made up of two distinct contributions, gravitational attraction proper and centrifugal acceleration due to the rotation of the earth.

If one considers earth as a perfectly homogeneous and spherical body the gravitational acceleration is directed toward its centre. If we denote as $\bar{g}$ the associated gravitational acceleration appropriate to sea level, and R the radius of earth, then gravitational attraction F_g will vary with height H as

$$F_g = m\bar{g}\left(\frac{R}{R+H}\right)^2 = m\bar{g}\frac{1}{\left(1+\frac{H}{R}\right)^2}. \qquad (1)$$

With the assumptions made, this contribution is independent in magnitude of latitude. The angular velocity ω_E due to earth rotation generates a centrifugal force of magnitude

$$F_z = m(R+H)\cos\phi\,\omega_E^2. \qquad (2)$$

This contributions acts in a direction normal to the earth axis and varies with latitude ϕ.

The two components are added vectorially as indicated in Fig. 1. However since $F_z \ll F_g$ we can simplify the addition to give the total effective gravitational pull approximately as:

$$\frac{\Sigma F}{m} = \frac{F_g}{m} - \frac{F_z}{m}\cos\phi = \bar{g}\frac{1}{\left(1+\frac{H}{R}\right)^2} - (R+H)\cos^2\phi\,\omega_E^2. \qquad (3)$$

Equation (3) of course defines the local gravitational acceleration

$$g_L = \frac{\Sigma F}{m} = \bar{g}\frac{1}{\left(1+\frac{H}{R}\right)^2} - (R+H)\cos^2\phi\,\omega_E^2. \tag{4}$$

Taking

$$\begin{aligned} \omega_E &= 0{\cdot}728 \times 10^{-4}\ \text{rad/s}, \\ R &= 6360\ \text{km}. \\ \text{and } \bar{g} &= 9{\cdot}8307\ \text{m/s}^2 \end{aligned}$$

equation (4) has been evaluated and plotted in Fig. 2 to show the percentage variation of gravity with latitude and height in relation to the value at 45 degrees latitude and sea level which would be

$$g_L = 9{\cdot}80665\ \text{m/s}^2.$$

It should be noted that these idealized results differ from true geophysical data but this difference is at most 0·1 per cent and not a very significant amount in the present context.

3. *Effect of Aircraft Speed*

If an object is moving at a steady speed in relation to earth this speed is superimposed on the circumferential speed of the earth and additional centrifugal acceleration is generated. We assume that the aircraft flies at constant height, i.e. it follows a circular path. We must take care properly to define the precise path of the aircraft. There is a distinct difference, e.g. between flight at constant heading and flight along a great circle. This is illustrated in Fig. 3 for two specific examples. In both cases the aircraft is at point 'A' at latitude ϕ on an easterly course with speed V_a.

In the first case it is flying constant heading. Hence it rotates about the earth axis with angular velocity $\omega_a = \frac{V_a}{R\cos\phi}$ at a radius ($R\cos\phi$). Its angular velocity is then linearly superimposed upon that of earth ω_E and the total centrifugal acceleration is

$$a_z = \frac{V_a^2}{R\cos\phi} + 2V_a\,\omega_E + R\cos\phi\,\omega_E^2$$

acting in a direction perpendicular to the earth axis. The component in the direction opposite to gravity proper is then

$$\Delta g = -a_z\cos\phi = -\frac{V_a^2}{R} - 2V_a\,\omega_E\cos\phi - R\cos^2\phi\,\omega_E^2.$$

In the second case (Fig. 3b) the aircraft is assumed to follow a great circle. Its own motion corresponds now to a rotation with angular velocity $\omega_a = \frac{V_a}{R}$ about the earth centre. This angular velocity has to be added vectorially to that of earth as shown in the illustration and the resulting angular motion of the aircraft with respect to inertial space is then a rotation with

$$\omega = \sqrt{\omega_a^2 + \omega_E^2 + 2\omega_a\,\omega_E\cos\phi}$$

about a radius arm

$$r = R\cos\varepsilon = R\sqrt{1-\left(\frac{\omega_E}{\omega}\sin\phi\right)^2}\,.$$

Hence centrifugal acceleration is

$$a_z = r\,\omega^2 = R\sqrt{1-\left(\frac{\omega_E}{\omega}\sin\phi\right)^2}\,(\omega_a^2+\omega_E^2+2\omega_a\,\omega_E\cos\phi)\,.$$

It is readily seen that this process involves elaborate geometric calculations which we shall not attempt here. Instead we shall derive strict answers for a few particularly simple cases and estimate intermediate conditions by interpolation.

The aircraft speed V_a relevant in this context is true speed in relation to a reference frame rotating with the earth, i.e. it is the sum of true airspeed plus the appropriate tail or headwind component. It should be noted that at high altitude the distance traversed by the path of an aircraft is greater than the corresponding projection on to the ground. We must therefore distinguish between what is commonly called ground speed and the speed with which we are concerned here. As in the following analysis aircraft speed is generally expressed in terms of Mach number it may be helpful to recall that 100 knots wind speed is equivalent to an increment in effective Mach number of approximately $M = 0{\cdot}172$. To obtain the correct result from the estimated effects (in Figs. 4 and 5) in the present report, we must interpret M as 'effective' Mach number and calculate it as the sum of aerodynamic Mach number plus the appropriate wind speed component.

3.1. *Zero Latitude (Equator).*

In flight round the equator in an easterly or westerly heading aircraft, speed is simply added to the circumferential speed of the earth

$$V_E = \omega_E R = 463{\cdot}5 \text{ m/s.}$$

Taking the + sign for easterly heading and − for westbound flight we get

$$\Delta g = -\frac{(V_E \pm V_a)^2}{R+H}\,. \tag{5}$$

Introducing Mach number, with a mean value for the speed of sound $a = 300$ m/s,

$$V_a = 300\text{M}\,,$$

we get

$$\Delta g = -\frac{(463{\cdot}5 \pm 300M)^2}{6360 \times 10^3+H}\,. \tag{6}$$

This expression defines the total reduction in effective gravity due to the combined effects of earth rotation and aircraft speed. Deducting from this the earth rotation contribution,

$$\Delta g^2 = -\frac{463{\cdot}5^2}{6360 \times 10^3+H}, \tag{7}$$

we can calculate the change in effective gravity due to aircraft motion alone. Corresponding results are presented as a percentage change in gravity, in Fig. 4 for 3 Mach numbers ($M = 2, 3, 4$). Within the height range appropriate to atmospheric flight this contribution is practically independent of height.

In flight at headings other than due east or west the two velocity components V_E and V_a have to be added vectorially but it can be shown that in this case the radius of rotation remains R and the answer is obtained quite simply. Results obtained by this process have been used to complete the curve given in Fig. 4 for zero latitude.

3.2. *Latitude* 90 *degrees* (*Pole*).

In flight over the pole, earth rotation does not contribute a centrifugal effect and aircraft speed only need be considered to give

$$\Delta g = -\frac{(300M)^2}{6360 \times 10^3 + H}. \tag{8}$$

Heading does not enter and is indeed meaningless at the pole so that the result as shown in Fig. 4 only depends on Mach number. The effect of height is negligible since for the range of heights of interest to flight $H \ll 6360 \times 10^3$.

Intermediate results for 45 degrees latitude have been obtained by graphical manipulation of the appropriate geometrical relations indicated in Fig. 3 and the results of this process are also given in Fig. 4. It was assumed that the aircraft follows a great circle in all cases.

In Fig. 5 these results have been crossplotted against Mach number for 90 and 270 degrees heading and various values of latitude. It should be noted that the answer for 90 degrees latitude is independent of heading. This particular curve can also be taken as an approximation for all latitudes for headings 0 and 180 degrees. Fig. 4 shows that at this heading, latitude has little influence.

4. *Application to Concorde.*

We apply these results now to the cruise condition of Concorde. The effective gravity relevant to cruising flight must take into account both of the contributions discussed above, namely the variation of local gravity with latitude and height and the additional effect of aircraft speed and heading. This total is obtained by simply adding the results given in Figs. 2 and 4 (or 5). The result is shown for 60,000 ft height and cruise at $M = 2$ in Fig. 6. This figure shows the reduction in the effective weight which is experienced by the aircraft over a range of latitudes and headings by comparisons with the weight it would register standing on the ground with the same loading at 45 degrees latitude. We see that the maximum relief is obtained in eastbound flight at the equator where it amounts to 2·3 per cent. At 45 degrees latitude it is still 1·85 per cent. In westbound flight the effect has a minimum of approximately 0·5 per cent. Unfortunately this is the direction in which prevailing winds make performance more marginal so that the gravitational relief, although not negligible and certainly welcome is perhaps disappointing. In eastbound flight, wind and gravity have both maximum favourable effect and the aircraft should be capable of substantially better performance as a result.

5. *Discussion and Conclusions.*

Two distinct influences are considered which effect the apparent gravity and hence the weight experienced by high performance aircraft. Although these effects are well known, being the basis of the mechanics of space flight and also normally considered in performance studies of hypersonic vehicles, they are not generally appreciated as relevant in ordinary aircraft work. The first is concerned with variations of 'local gravity' with latitude and height, the second with the additional centrifugal acceleration generated by the speed of the aircraft. The total of these effects results in a reduction in effective gravity and hence in a reduction in the aircraft weight one has to consider in performance calculations. For Concorde in its typical cruise condition, maximum gravitational relief is obtained in eastbound flight at low latitude, and this can reach 2·3 per cent. In westbound flight this reduces to approximately 0·5 per cent.

If speeds and cruising altitudes higher than those used by Concorde are considered these effects can become quite substantial. For instance flight at $M = 4{\cdot}0$ and $M = 80{,}000$ ft would result in an apparent loss of gravity of about 5·1 per cent in eastbound flight and 1·5 per cent in westbound flight. Such relief constitutes a quite substantial bonus on performance and should therefore not be ignored.

It is perhaps worth mentioning that the effects discussed in this Report affect also flight testing. If they are not properly accounted for, C_L for instance would be overestimated by the percentage gravity relief pertaining in these conditions. One must, however, note that an accelerometer carried by an aircraft gives in all conditions a rue mearure uf lift in relation to its mass rather than weight and that in level flight it will generally give a reading differing from 1 g by the factor enumerated in this Report.

LIST OF SYMBOLS

a (m/s^2)	Centrifugal acceleration
F (N)	Gravitational force
F_g (N)	Force due to gravity proper
F_z (N)	Centrifugal force
g (m/s^2)	Gravitational acceleration
$\bar{g}$ (m/s^2)	Nominal 'pure' gravitational acceleration at sea level
g_L (m/s^2)	Local gravity
H (m)	Height
m (kg)	Mass of aircraft
R (m)	Radius of earth
r (m)	Radius of rotation
V_a (m/s)	Aircraft speed
V_E (m/s)	Circumferential speed of earth
ϕ	Latitude
ω (rad/s)	Angular velocity
ω_E	Angular velocity of earth
ω_a	Angular velocity of aircraft in relation to earth

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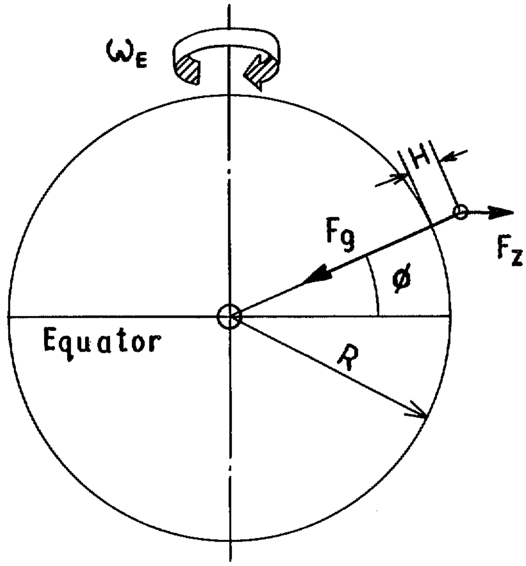


FIG. 1. The two connnnntributions to the apparent gravity experienced by a stationery object on or above the earth surface.

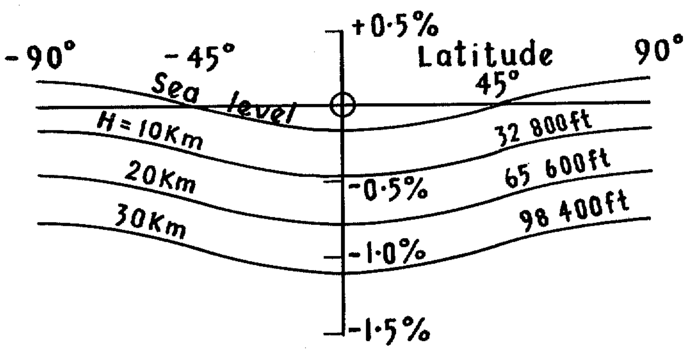
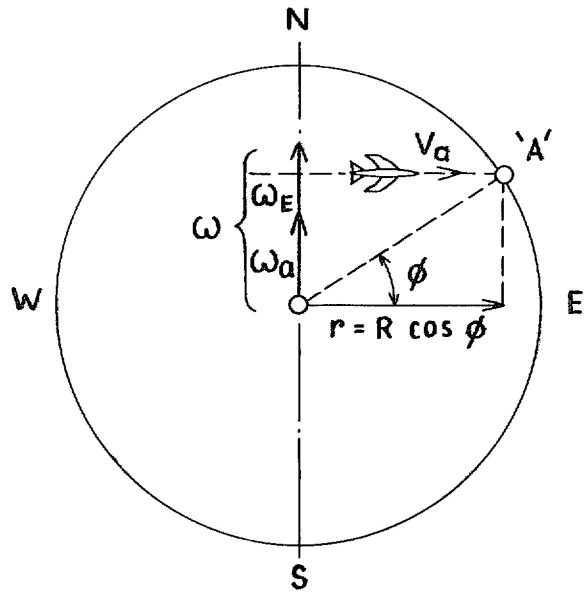
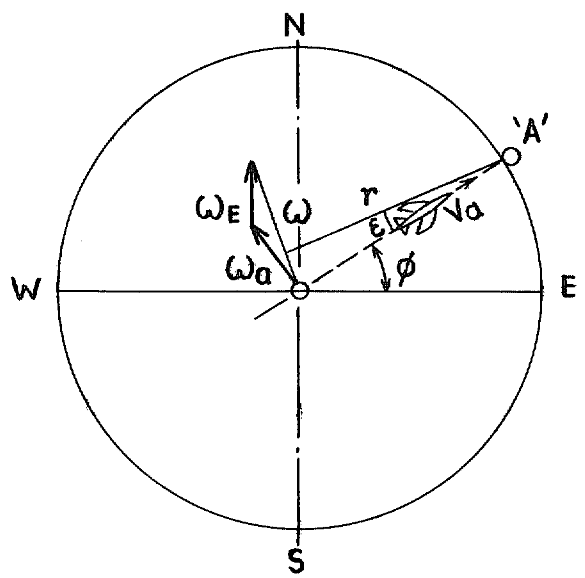


FIG. 2. Variation of apparent gravity with latitude and height for a stationary object.



a Flight at constant heading



b Flight along great circle

FIG. 3 a & b. Superimposition of aircraft motion and earth rotation illustrated for two specific examples.

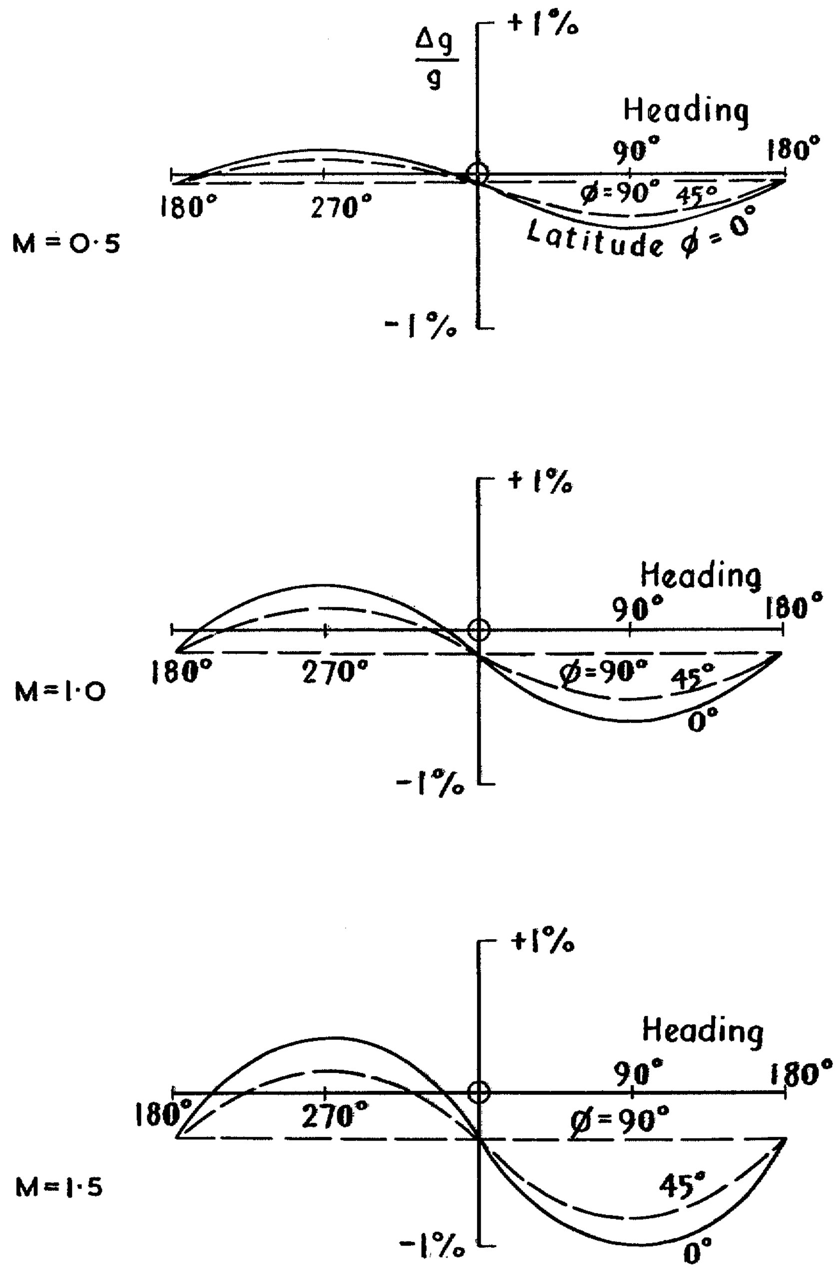
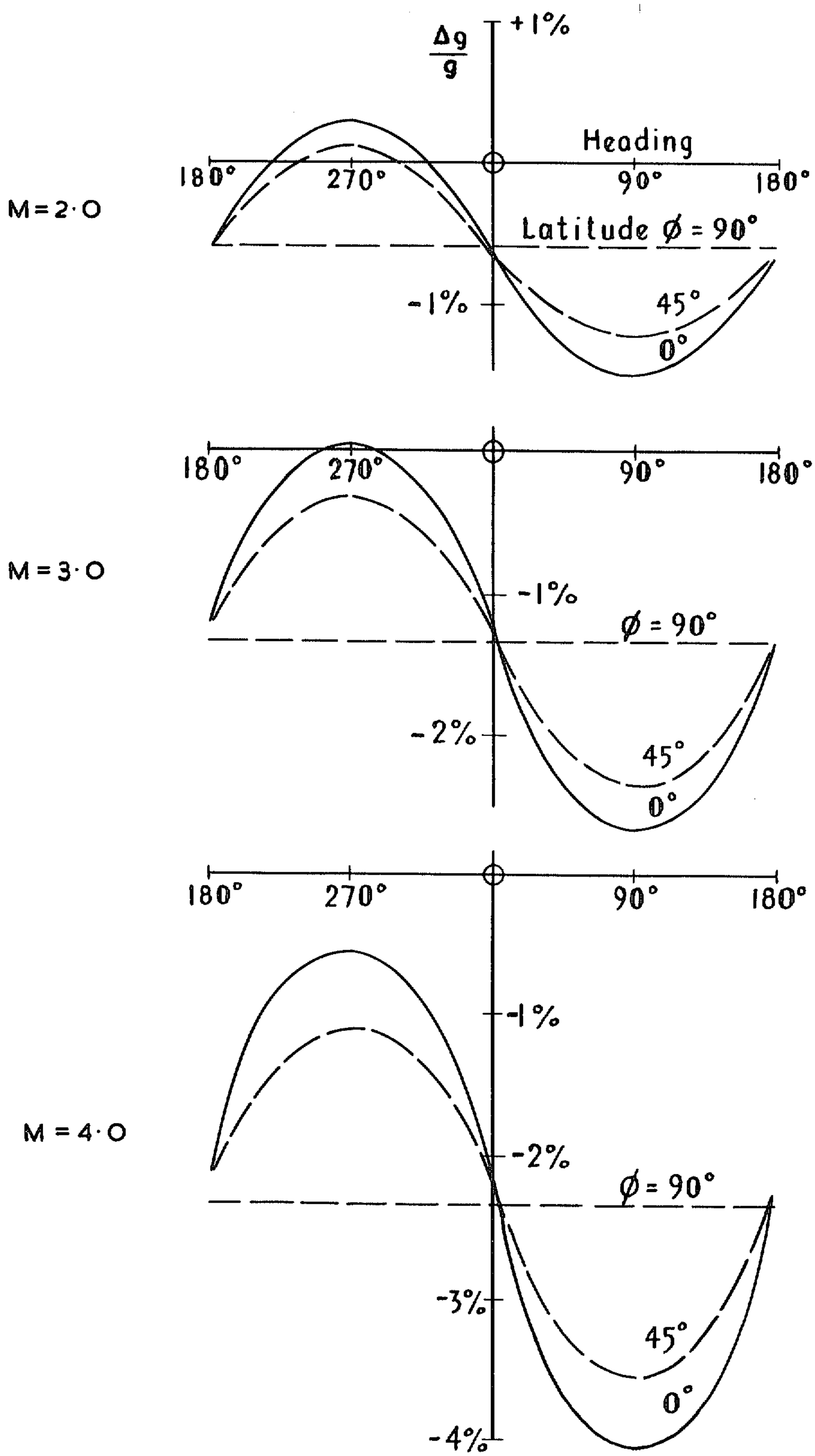


FIG. 4. Percentage change in apparent gravity due to aircraft velocity, heading and latitude.



Δg/g
+1%
Heading
180°
270°
90°
180°
M = 2·0
Latitude ø = 90°
-1%
45°
0°
M = 3·0
-1%
ø = 90°
-2%
45°
0°
M = 4·0
-1%
-2%
ø = 90°
-3%
45°
0°
-4%

FIG. 4. (contd.)

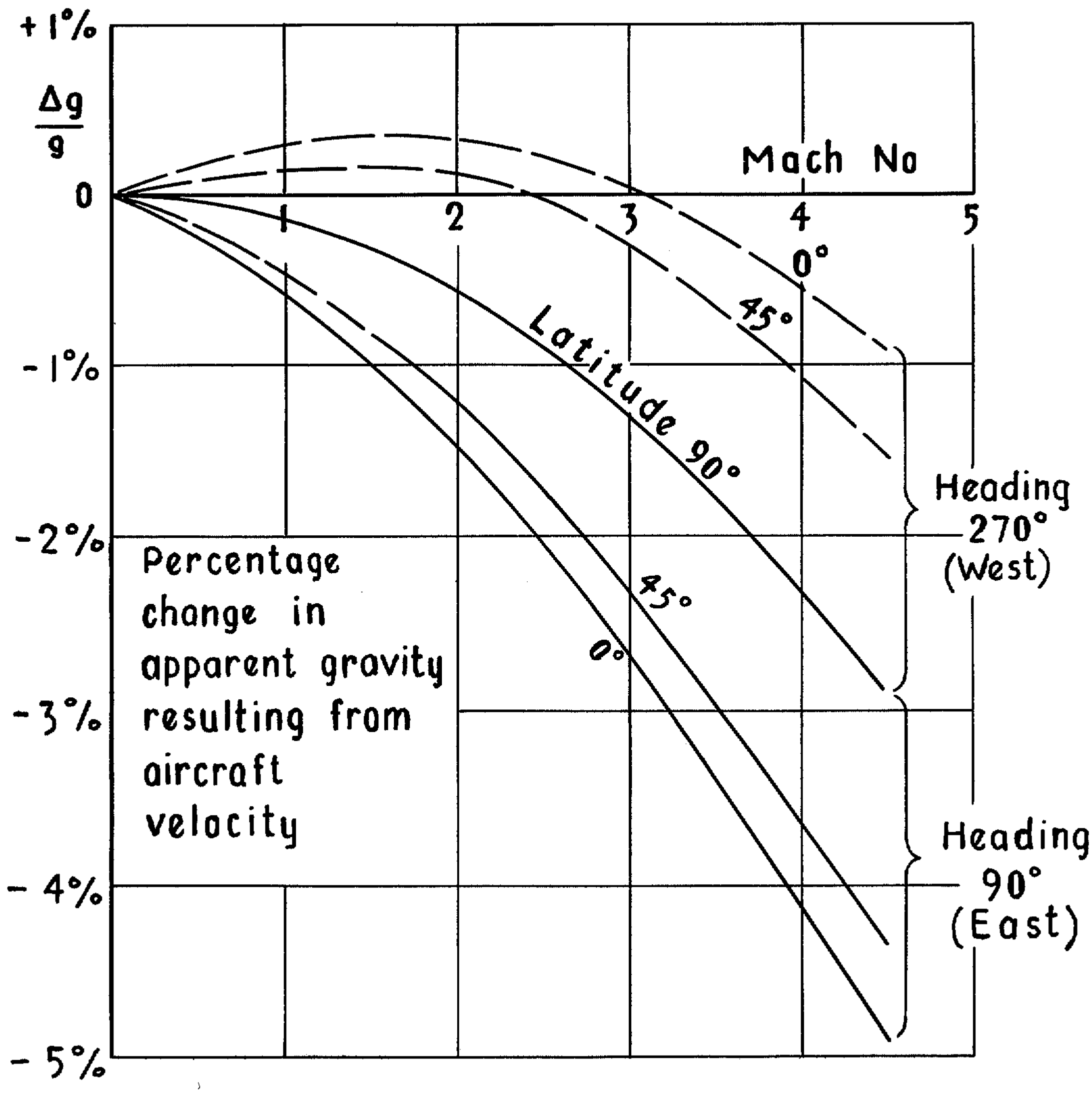


FIG. 5. Results of Fig. 4 crossplotted against Mach number.

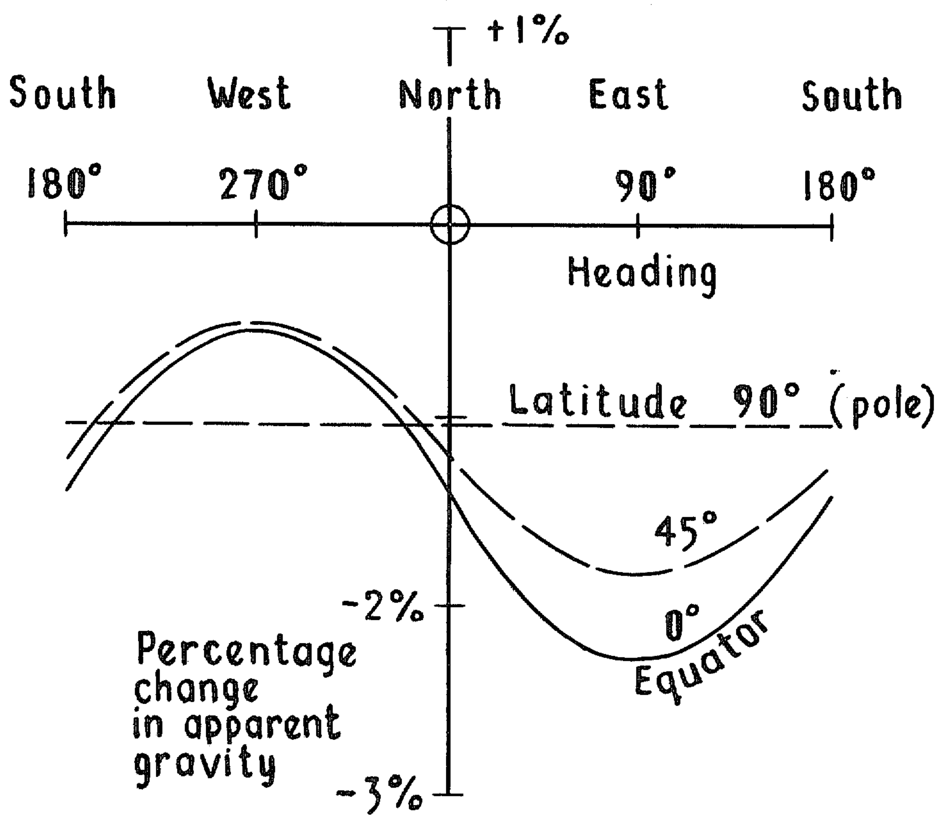


FIG. 6. Combined effects of variations in local gravity and aircraft speed on apparent gravity experienced by Concorde in typical cruise condition M = 2·0 H = 65,000 ft.

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