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Some Comments on Air Condensation Effects in a Hypersonic Helium Tunnel

By L. Davies and J. E. G. Townsend

Aerodynamics Division N.P.L.

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Summary.

Apart from some earlier experimental work by Henderson and Swalley, there does not appear to be available in the literature an analysis of the effects of small amounts of air contamination on the flow of helium in a hypersonic nozzle. The main difference between this and the more common problem of condensation of water vapour in air or of a condensing diatomic gas alone, is that the mass fraction of the contaminant cannot be ignored.

In this Report a brief analysis of the problem is outlined and the more important points discussed. The theoretical and experimental results are compared with those of Henderson and Swalley.

*Replaces NPL Aero Report 1316—A.R.C. 32 118.

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1. *Introduction.*

Condensation in supersonic and hypersonic wind-tunnel nozzles has been investigated in great detail (at one reckoning there were well over three hundred references available in the literature[1]), and of course this interest arose because of the adverse effects on the test flow which result from the energy released. By far the majority of papers deal with the condensation of water vapour in air, or with the condensation of a diatomic gas in a hypersonic nozzle. The present problem, i.e., the effects of air or nitrogen condensation on helium flow, differs from these because the mass fraction of the small percentage of contaminant (the air or nitrogen) cannot be ignored. For example, if the percentage by pressure of the contaminant is 5 per cent, the mass fraction is about 27 per cent.

From our literature survey only one paper on this particular aspect was found[2], and this paper gave empirical equations for the variation in flow properties as a function of the amount of contamination present. The aims of the present Report are therefore to extend the well-known analyses of condensation in nozzles to cover the case of a light gas with a heavy-gas contaminant, and to compare our theoretical and experimental results with those of Henderson and Swalley[2].

As regards the theoretical approach for an exact analysis of this problem it is necessary to consider in detail spontaneous nucleation and droplet growth rate in the nozzle. Into this analysis will come the effects of nozzle shape, i.e., contoured, or conical, to account for the difference in expansion rates (*see* Daum and Gyarmarthy, Ref. 3). In the present analysis however we make use of the concept of the condensation shock, where the droplet formation process is completed in a very short distance, and assume that the mixture of test gas and contaminant expands isentropically in thermal equilibrium up to the critical cross section, where the condensation shock occurs, and that after the shock the mixture of test gas and droplets expand isentropically in equilibrium into the working section. At the shock we also assume that *all* the contaminant condenses out.

Using this technique it is found, as has been found previously in the case of water vapour, etc., that reasonably good estimates of the variation in flow properties can be made.

In the following sections we shall describe the nature of the contamination in the NPL helium tunnel, and follow this with an outline of the theoretical analysis of the condensation effects. Finally we shall compare our theoretical and experimental results and discuss the comparison between these data and the experimental results obtained by Henderson and Swalley[2]. In the Appendices the derivation of the various formulae are given in detail.

2. *Nature of the Contaminant.*

In routine operation contamination of helium-tunnel test gas is unavoidable. The major component is air, but a small amount of lubricating oil is also found to be present. It is therefore normal practice to incorporate a purifying plant into the circuit and this, operating on the molecular sieve principle, removes most of the contaminant.

The components of the contaminant are identified by using a gas chromatograph, and it is found that the proportions change from run to run. Immediately after purification the small amount of residual contaminant contains about 90 per cent nitrogen and 10 per cent oxygen. (The predominance of nitrogen is probably related to the fact that the molecular sieve operates at liquid nitrogen temperature and therefore favours the removal of oxygen with its higher liquefaction temperature.) After several runs however the ratio of nitrogen to oxygen tends towards the air value.

In order to eliminate the possibility of contamination pure helium could be used for each run and then discarded, but for economic reasons this is of course unrealistic. Or the gas could be purified after each run, but this would increase the time between each operation of the tunnel. A reasonable alternative is to run several times before purifying. The exact number of runs is determined by the amount of contamination of each run and by the maximum allowable contamination level. The latter restriction depends on the effect of contamination on the flow properties, and this is discussed in the next section.

3. *Derivation of Formulae.*

Several assumptions are made concerning the test gas as it passes through the nozzle. The approach to the problem is as follows:

We start with the test gas in the reservoir at temperature T_0 and pressure p_0. In the test gas there is a small amount of unwanted gas which we refer to as the 'contaminant'. In order to compute the free stream conditions through the throat and into the nozzle we assume that the test gas plus contaminant behaves as one gas with mean values of the specific heats and molecular mass calculated as is usual for gas mixtures. The next assumption affords considerable simplification; the test mixture is assumed to expand isentropically in thermal equilibrium until the cross section in the nozzle is reached where the temperature and pressure are the critical ones for the onset of condensation of the contaminant (*see* Fig. 1). (As we have pointed out in the Introduction the flow processes are much more complex than this, but we shall show results obtained using this simplified model which justify this approach.) At the critical cross-section condensation is assumed to occur instantaneously with the formation of a condensation shock of effectively zero thickness, and with realease of the total latent heat of vaporization, (i.e., all the contaminant is turned into droplets). The variation in flow properties across this shock are computed using the conservation equations for mass, momentum and energy. The subsequent flow is then assumed to be in equilibrium and the working section conditions are obtained assuming isentropic expansion from the critical cross-section. The post-shock test gas is considered to be a mixture of helium plus a heavy gas whose molecules are the droplets (*see* Clark Ref. 4).

When the flow is brought to rest, for example at the nose of a blunt body, it is assumed that the heat of vaporization is then taken up in reversing the effect of the condensation shock and the stagnation temperature becomes T_0. Here we assume that no droplet of condensed contaminant escapes from the stagnation region before vaporising (as can occur for small diameter pitot probes, for example).

The derivation of the various formulae are given in the appendices, and only the main results are presented here.

Using the equation for the conservation of mass, momentum and energy, and assuming that the condensation region has zero thickness, it can readily be shown that the velocity ratio across the condensation shock as a function of contaminant mass fraction (x) and latent heat of vaporization (Q) is given by

$$V_{21} = \rho_{12} = \frac{\Delta}{(2\Delta-1)}\left(1+\frac{1}{\gamma M_1^2}\right) + \sqrt{\left[\frac{\Delta}{(2\Delta-1)}\left\{\left(1-\frac{1}{\gamma M_1^2}\right)-\frac{1}{\Delta}\right\}\right]^2 - \frac{2XQ}{C_{p_{He}} T_1 M_1^2(\gamma-1)(2\Delta-1)}} \quad (1)$$

Here Δ is a function of X and γ, and M_1 is the Mach number upstream of the condensation shock; γ is the ratio of specific heats for the mixture.

A further assumption made in deriving this equation is that the droplets of condensed contaminant move with the same velocity as the test gas (*see* Clark[4] and Thomann[1] for example). The density ratio across the condensation shock is reciprocally related to the velocity ratio and the pressure ratio is given by the equation

$$\frac{p_2}{p_1} = \gamma M_1^2(1-V_{21})+1 \quad (2)$$

Finally the Mach-number ratio is obtained using the relationship

$$\frac{M_2}{M_1} = \sqrt{\frac{V_{21}}{P_{21}}} \quad (3)$$

This modified flow expands isentropically into the working section. The test gas is brought to rest through a normal shock to a pressure related to the working section static pressure by the Rayleigh

supersonic pitot formula and initially at a higher temperature than the equilibrium value, T_0. Vaporization then occurs and the temperature falls to T_0 with a corresponding adjustment of pitot pressure.

The variation of working-section static pressure and temperature and the Mach number, together with the pitot pressure, as functions of the contamination ratio 'r' (p_{air}/p_0) are plotted in Figures 2 to 5. Also plotted for comparison are the experimentally observed variation in pitot and static pressure together with the variation observed by Henderson and Swalley[2]. These authors found from their experiments that the variation of static pressure with contamination ratio obeyed the relationship

$$\frac{p_\infty}{(p_\infty)_{r=0}} = e^{nr}, \tag{4}$$

and that the corresponding relation for pitot pressure was

$$\frac{p_t}{(p_t)_{r=0}} = e^{mr}, \tag{5}$$

where $n = 8{\cdot}5$ and $m = -2$. In our case we calculate n to be 7·62 and $m = -0{\cdot}61$ (*see* Section 4). As can be seen from Figures 2 and 3 there is only a small difference between the predicted variations using the two sets of values. For contamination ratios of 5 per cent or less the two sets of values agree within 4 per cent. Above this ratio the discrepancy increases considerably. Since even contaminations ratios of 5 per cent must be considered very large, the exact choice between using the theoretical and empirical values in actual cases is open.

Summarising at this stage it has been shown both experimentally and theoretically that the static and pitot pressures in contaminated flow are related to the uncontaminated values in terms of the contamination ratio. Conversely, it is therefore possible to obtain some simple expressions which, for example, allow the uncontaminated Mach number and the contamination ratio to be computed directly from the reservoir, static and pitot pressure measurements. These and other formulae are derived in some detail in the Appendix, and the more important equations are quoted here.

Using equations (4) and (5) for static pressure and pitot pressure in terms of their uncontaminated values, and the free-stream Mach numbers obtained from these expressions, we arrive at the following expression which relates the static, pitot and reservoir pressures (measured) with the contamination ratio (unknown).

$$\phi e^{ar}\left(\frac{p_t}{p_\infty}\right) - \left(\frac{p_0}{p_\infty}\right)^{0{\cdot}4} e^{br} + 1 = 0 \tag{6}$$

where $a = (m+n)/2$

$$b = (m/r)\left(\frac{\gamma_{He}-1}{\gamma_{He}}\right)$$

$$\phi = (0{\cdot}227).$$

From this equation a value for the contamination ratio can be computed, and using the expression, slightly modified from Henderson and Swalley,

$$(M_\infty)_{r=0} = M_\infty e^{ar} = 0{\cdot}825\left(\frac{p_t}{p_\infty}\right)^{\frac{1}{2}} e^{ar} \tag{7}$$

the true Mach number for the nozzle may be obtained.

If the true Mach number is known then the contamination ratio can be computed from the equation

$$r = \frac{1}{n} ln \left\{ \left(\frac{p_\infty}{p_0} \right) \left[(M_\infty)^2_{r=0} \left(\frac{\gamma_{He}-1}{2} \right) + 1 \right]^{\left(\frac{\gamma_{He}}{(\gamma_{He}-1)} \right)} \right\} \tag{8}$$

4. *Results and Discussion.*

In order to compute the variation of free-stream conditions with contamination ratio, it is necessary to determine the temperature and pressure in the nozzle at which condensation will occur. In this connection the paper by Daum and Gyamarthy[3] is most useful. The condensation temperature is of course dependent on the air partial pressure and therefore will vary with r, the contamination ratio. As a consequence the value of n or m which is computed will not be constant over the range, as is indicated below.

We arbitrarily chose the $r = 0{\cdot}05$ contamination ratio conditions for an initial calculation of n and m. For these conditions it is found by Daum and Gyarmarthy from experiment that the relevant nozzle temperature for the onset of condensation is that indicated by the nitrogen saturation value. In Figure 1 the intersection of the $r = 0{\cdot}05$ isentrope with the nitrogen and air saturation curves is shown. If we use these condensation data, then the value of n we obtain for these specific conditions is 7·62, and this is the value we have used in Figures 2 to 5 to represent the whole of the range of contamination ratios. However if we use the temperature and pressure appropriate to $r < 0{\cdot}01$, large degrees of supercooling occur because of the small air pressure and the value of n rises (typically to about 9·6 for $r = 0{\cdot}01$). The conclusion is therefore that n is not a constant for all r, as has been suggested by the data of Henderson and Swalley[2], but varies continuously with r.

The practical consequences are that the free-stream static pressures related to the low values of contamination ratio ($< 0{\cdot}01$ say) should lie above any mean line chosen to fit higher r values, and that the experimental curve should intersect the empirical one at some r. Subsequently the empirical equation should increasingly over-estimate the measured pressures with increasing r. Unfortunately this hypothesis is not readily tested, for two reasons. Firstly it is not possible to perform the experiments to the accuracy which would be required, and this is linked to the second reason, namely that over the range $0 < r < 0{\cdot}01$, any value of n from say 7·5 to 9·5 would fairly represent the experimental data with sufficient accuracy. As a rough guide a 10 per cent variation in m would represent only about a 4 per cent variation in static pressure. In order that the results be meaningful we should require an accuracy of about ± 1 per cent, and neither the data of Henderson and Swalley[2] nor our own results appear to be able to meet this requirement.

In Figures 2 and 3, where the static to reservoir and pitot to reservoir pressure ratios are plotted respectively, it is seen that our experimental data could be reasonably well represented by either Henderson and Swalley's curves or our own theoretical curves. In Figure 4 the 'actual' variation of working section Mach number is compared with the variation which might be inferred from the ratio of static or pitot pressure to reservoir pressure. In Figure 5 the continuous increase in free-stream static temperature with r is indicated.

As regards the maximum allowable contamination ratio, Henderson and Swalley have shown that to ensure no more than ε per cent error in free stream Mach number,

$$r \leqslant \frac{2}{(m+n)} \log_e \left[\frac{1}{1 - \frac{\varepsilon}{100}} \right]$$

When expanded in a series, for $\frac{\varepsilon}{100} << 1$,

$$r \leqslant \frac{0{\cdot}02\varepsilon}{(m+n)}.$$

5. *Conclusions.*

From this brief analysis of the effects of air contamination on the flow in a hypersonic helium tunnel we find that:

(i) The static and pitot pressure variations are simply related to the contamination ratio, as was found empirically by Henderson and Swalley[2].

(ii) Using a simplified theoretical model which assumes that the test gas flow is in thermal equilibrium before and after the condensation shock, and that all the contaminant condenses at the shock, it is possible to compute with a fair degree of accuracy the actual experimental variation in free stream conditions.

LIST OF SYMBOLS

A	Nozzle cross-sectional area
A_{CRIT}	Cross-sectional area at which condensation shock forms
A_*	Nozzle throat area
a, b	Constants in equation 6
C_p	Specific heat at constant pressure
C_v	Specific heat at constant volume
h	Specific enthalphy
m	Constant
$\bar{m}_0$	Mean molecular weight for initial reservoir gases
M	Mach number
n	Constant
p	Pressure
P_{ij}	p_i/p_j
Q	Latent heat of vapourisation
r	Contamination ratio p_{Air}/p_0
T	Temperature
V	Velocity
V_{ij}	Vi/V_j
X	Mass fraction of contaminant
γ	Ratio of specific heats $= C_p/C_v$
$\bar{\gamma}_0$	Mean value of γ for initial reservoir gases
Δ	$((1-X)+X/3{\cdot}125)\,\gamma/(\gamma-1)$
ϕ	Constant defined in equation 6
ρ	Density
ρ_{ij}	ρ_i/ρ_j

Subscripts

0	Reservoir conditions
1	Upstream of condensation shock
2	Downstream of condensation shock
∞	Working-section conditions
Air	Pure air values
gc	Gaseous contaminant

He	Pure Helium values
Lc	Condensed contaminant
t	Working-section stagnation conditions

REFERENCES

No.	Author(s)	Title, etc.
1	H. Thomann	Determination of the size of the crystals formed during condensation of water in wind tunnels and of their effect on boundary layers. FFA Report 101 (1964).
2	J. A. Henderson and F. E. Swalley	Effects of air contamination in a helium tunnel. NASA TN D-406 (1960).
3	F. L. Daum and G. Gyarmathy	Condensation of air and nitrogen in hypersonic wind tunnels. *AIAA Journal*, Vol. 6, No. 3, pp. 458–465 (1968).
4	D. R. Clark	On the flow in the nozzle of a condensing diatomic vapour. Coll. of Aeronautics, Cranfield, CoA Report Aero, No. 165 (1963).

APPENDIX I

In this Appendix we shall derive, or quote where well known formulae are used, the equations which have been used in analysis of the contaminated flow in the helium tunnel. Firstly the properties of the mixture of test gas plus contaminant will be obtained, then the equations for the flow in the nozzle are given and the critical cross-section at which the condensation shock forms is determined. The conservation equations for mass and momentum will be used together with the appropriate form of the energy equation, to compute the change in flow conditions across the condensation shock. Lastly the equations for the isentropic expansion of the flow into the working section following the critical cross-section are quoted.

(i) *Reservoir conditions.*

In the reservoir we have helium at a partial pressure p_{He}, and contaminant at partial pressure p_C, the total pressure of the mixture has value p_0 say. The contaminant pressure fraction r, which we shall term the 'contamination ratio', is equal to p_C/p_0. If, as in the present case, the contaminant is air, then its molecular weight is 28·96 and using the molecular weight of helium (4) the mean molecular weight ($\bar{m}_0$) for the mixture is obtained using the equation

$$4(1-r)+28{\cdot}96r = \bar{m}_0 \tag{I.1}$$

The mass fraction of the contaminant is $X = 28.96.r/\bar{m}_0$.

The mean specific heat at constant pressure ($\bar{C}_{p0}$), and at constant volume ($\bar{C}_{v0}$), can be obtained from the following equations:

$$\bar{C}_{p0} = (1-X)\,C_{p_{He}}+XC_{p_{\mathrm{Air}}} \tag{I.2}$$

and

$$\bar{C}_{v_0} = (1-x)\,C_{v_{He}}+XC_{V_{\mathrm{Air}}} \tag{I.3}$$

The mean value of their ratio $\bar{\gamma}o$, is given by $\bar{\gamma}o = \bar{C}_{p0}/\bar{C}_{v0}$.

The flow properties in the nozzle, in particular the variation of static pressure and temperature and free stream Mach number are then computed from the following equations:

$$\frac{T}{T_0} = \left(1+\frac{(\bar{\gamma}_0-1)\,M_1^2}{2}\right)^{-1}, \quad \frac{p}{p_0} = \left(1+\frac{(\bar{\gamma}_0-1)\,M_1^2}{2}\right)^{\frac{\bar{\gamma}_0}{1-\bar{\gamma}_0}} \tag{I.4}$$

where M_i is obtained in terms of the area ratio A/A_* from

$$\frac{A}{A_*} = \frac{1}{M_1}\left\{\frac{2}{(\bar{\gamma}_0+1)}\left(1+\frac{(\bar{\gamma}_0-1)\,M_1^2}{2}\right)\right\}^{\frac{\bar{\gamma}_0+1}{2(\bar{\gamma}_0-1)}} \tag{I.5}$$

(ii) *Conditions across the condensation shock.*

In deriving the equations for the variation in flow properties resulting from condensation of the small amount of contaminant in the helium we make the following assumptions: The droplets of condensed contaminant may be considered as the molecules of a heavy gas (*see* Clark), diffusion and heat transfer are neglected, the droplets are assumed to move with the same velocity as the helium, and all gases are thermally perfect.

The energy equation is as follows:

$$(1-X)dh_{He}+X\,\{(1-\mu)dh_{gc}+\mu dh_{Lc}+\mu dQ\}+VdV = 0 \tag{I.6}$$

where X is the mass fraction of contaminant, μ is the amount of contaminant condensed measured as a fraction of the uncondensed contaminant gas, dh_{He} is the helium enthalpy, dh_{gc} is the enthalpy of the gaseous contaminant dh_{Lc} is the enthalpy of the condensed contaminant and Q is the specific latent heat of vaporisation. If we assume that all the contaminant is condensed at the shock then $\mu = 1$ and equation I.6 can be integrated to give the following expression:

$$(1-X)h_{He}+Xh_{Lc}+XQ+\frac{V^2}{2} = \text{Constant} \cdot \tag{I.7}$$

Using the concept of the heavy gas molecules described above, then we have

$$(1-X)h_{He}+Xh_{Lc} = h_m = \text{mixture enthalpy} \tag{I.8}$$

we now solve equation (I.7) for V_2/V_1 $(= V_{21})$ using the appropriate equations for conservation of mass and momentum,

Mass...
$$\rho_1 V_1^2 = \rho_2 V_2^2 \tag{I.9}$$

Momentum...
$$\rho_1 V_1^2+p_1 = \rho_2 V_2^2+p_2 \tag{I.10}$$

Here conditions (1) are upstream and (2) downstream of the shock respectively. The two roots obtained are:

$$V_{21} = \frac{\Delta}{(2\Delta-1)}\left(1+\frac{1}{\gamma M_1^2}\right) \pm \left(\left[\left(\frac{\Delta}{2\Delta-1}\right)\left\{\left(1-\frac{1}{\gamma M_1^2}\right)-\frac{1}{\Delta}\right\}\right]^2 - \frac{2XQ}{C_{p_{He}}T_1 M_1^2(\gamma-1)(2\Delta-1)}\right)^{\frac{1}{2}}$$

where
$$\Delta = ((1-X)+X/3.12)\gamma/(\gamma-1) \tag{I.11}$$

In order to decide which root is the appropriate one in the present case we put $X = 0$, and obtain the following two values for V_{21}

$$\left.\begin{aligned} +ve \text{ sign } V_{21} &= 1 \\ -ve \text{ sign } V_{21} &= \frac{(\gamma-1)M_1^2+2}{(\gamma+1)M_1^2} \end{aligned}\right\} \tag{I.12}$$

The case usually considered trivial for conventional compression shocks, *viz* $V_{21} = 1$, is the one we must adopt because when there is no contamination no velocity change occurs. The density ratio across the shock is the reciprocal of the velocity ratio, and the pressure ratio is obtained from the equation:

$$P_{21} = \frac{p_2}{p_1} = \gamma M_1^2(1-V_{21})+1 \tag{I.13}$$

Finally the Mach number ratio across the condensation shock is given by

$$\frac{M_2}{M_1} = \sqrt{\frac{V_{21}}{P_{21}}} \tag{14}$$

(iii) *Working-section conditions.*

The conditions in the working section are obtained by assuming that the test gas after condensation

expands isentropically. The appropriate equations are

$$\frac{A_\infty}{A_{\text{CRIT}}} = \frac{M_2}{M_\infty}\psi^{\frac{\gamma+1}{2(\gamma-1)}}, \quad \frac{T_\infty}{T_2} = \psi^{-1}, \quad \frac{p_\infty}{p_2} = \psi^{\frac{\gamma}{1-\gamma}} \tag{I.14}$$

where

$$\psi = \left(\frac{2+(\gamma-1)M_\infty^2}{2+(\gamma-1)M_2^2}\right)$$

(A_∞ = nozzle exit cross-sectional area.

A_{CRIT} = cross-sectional area in nozzle where condensation shock occurs, and condition '∞' are those in the working section.)

The values computed using the above theory are presented in Figures 2, 3, 4, 5 and are discussed fully in the main body of the text in the section headed 'Results and discussion'.

APPENDIX II

In this Appendix we shall examine the equations relating to the flow properties in the working section, which have been obtained empirically, and compare them with the analytical values. Initially we shall derive some simple expressions which allow the contamination ratio and 'true' free-stream Mach number to be calculated using measured pressures.

(i) *Determination of r and* $(M_\infty)_{r=0}$ *from pressure measurements.*

The 'pure' and 'contaminated' values of working section pressures and Mach numbers are related to the contamination ratio by the following equations.

$$\frac{(p_\infty)}{(p_\infty)_{r=0}} = e^{nr} \tag{II.1}$$

$$\frac{(p_t)}{(p_t)_{r=0}} = e^{mr} \tag{II.2}$$

$$(M_\infty)_{r=0} = e^{\left(\frac{n-m}{2}\right)}, \quad M_\infty = e^{\left(\frac{n-m}{2}\right)r}\, 0{\cdot}825 \left(\frac{p_t}{p_\infty}\right)^{\frac{1}{2}} \tag{II.3}$$

The values of m and n are as follows:

Empirical (Henderson and Swalley) ..... $n = 8{\cdot}5$

$m = -2$

Analytical (Present report)..... $n = 7{\cdot}6$

for $r = 0{\cdot}05$ $\qquad m = -0{\cdot}61$

In terms of the ratio of static to reservoir pressure, and the contamination ratio the uncontaminated free-stream Mach number is given by the equation

$$(M_\infty)_{r=0} = \left(\frac{2}{(\gamma-1)}\left\{\left(\frac{p_\infty}{p_0}\cdot e^{-nr}\right)^{\frac{1-\gamma}{\gamma}} - 1\right\}\right)^{\frac{1}{2}} \tag{II.4}$$

This equivalent free-stream Mach number is also related to the ratio of pitot to static pressure and the contamination ratio by equation (II.3).
Therefore from equations II.3 and II.4 we have

$$\frac{2}{(\gamma_{He}-1)}\left\{\left(\frac{p_\infty}{p_0}e^{-nr}\right)^{\frac{1-\gamma_{He}}{\gamma_{He}}}\right\} = (0{\cdot}825)^2\left(\frac{p_r}{p_\infty}\right)e^{(n-m)r} \tag{II.5}$$

and with some simple manipulation we arrive at the following equation

$$\frac{(\gamma_{He}-1)}{2}(0{\cdot}825)^2\left(\frac{p_t}{p_\infty}\right)e^{(n-m)r} - \left(\frac{p_0}{p_\infty}e^{nr}\right)^{\frac{\gamma_{He}-1}{\gamma_{He}}} + 1 = 0\,. \tag{II.6}$$

This equation is quoted in the text (equation 6) and allows the contamination ratio to be computed from the measured pitot, static and reservoir pressures, (without a prior knowledge of the uncontaminated

value of the free-stream Mach number), and thereby the 'pure helium' Mach number is obtained using equation II.3.

If the true Mach number is known, then the contamination ratio can be computed readily from the ratio of the static to reservoir pressure as follows:

Solving for r in equation II.4 we have:

$$r = \frac{1}{n} l_n \left\{ \left(\frac{p_\infty}{p_0} \right) (M_\infty)^2_{r=0} \left(\frac{\gamma_{He}-1}{2} \right) + 1 \right]^{\frac{\gamma_{He}}{(\gamma_{He}-1)}} \right\} \tag{II.7}$$

(ii) *Choosing values for m and n.*

In order to relate the free-stream conditions to their uncontaminated levels it is necessary to decide which values of m and n to use. Both pairs of values have been used in computing the pressure variations with contamination ratio, and it is found that for contamination ratios less than 5 per cent the difference in static pressure computed using 8·5 or 7·62 is less than 4 per cent, and this difference falls to about 0·82 per cent for a contamination ratio of 1 per cent. The percentage difference is roughly 0·8 times the contamination ratio. Similar results are obtained for the pitot pressure, which is much less affected by contamination as a result of vaporisation in the stagnation region and where the percentage difference is roughly 1·4 times the contamination ratio.

If the contamination ratio is low, say less than 1 per cent, then the choice is open. Henderson and Swalley's experimental data in this region could be fitted with a range of values of m and so 7·62 is as likely to be correct as 8·5. It is also observed that for high values of r, (about 0·65), three out of four of the values of static to reservoir pressure ratio would favour the lower value of m, i.e., 7·62.

The conclusion is therefore that under practical conditions where the value of r would be less than 0·01, the choice of m between the analytical and empirical values, is arbitrary. Similar observations apply to the pitot pressure.

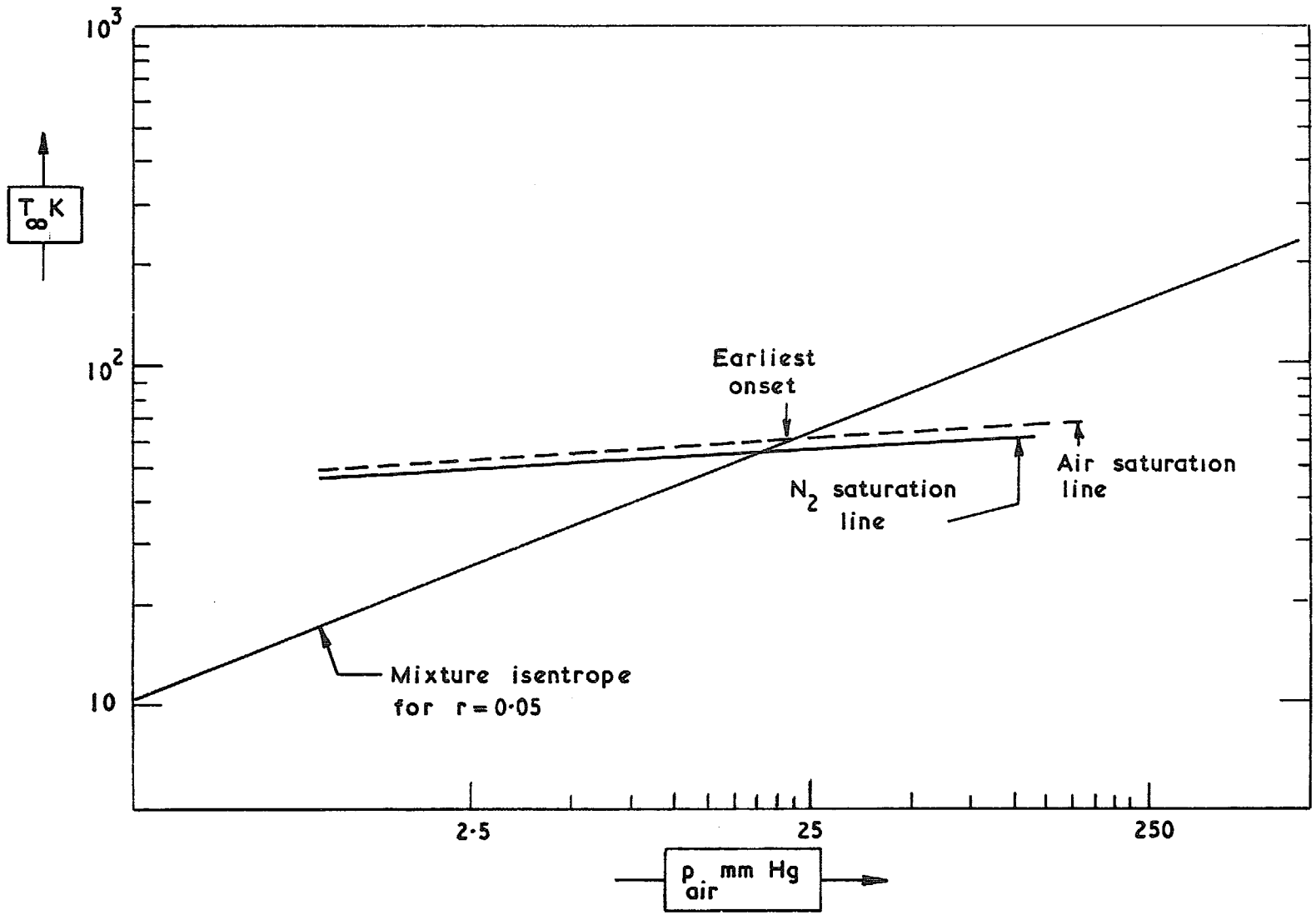

FIG. 1. Determination of condensation onset points for air.

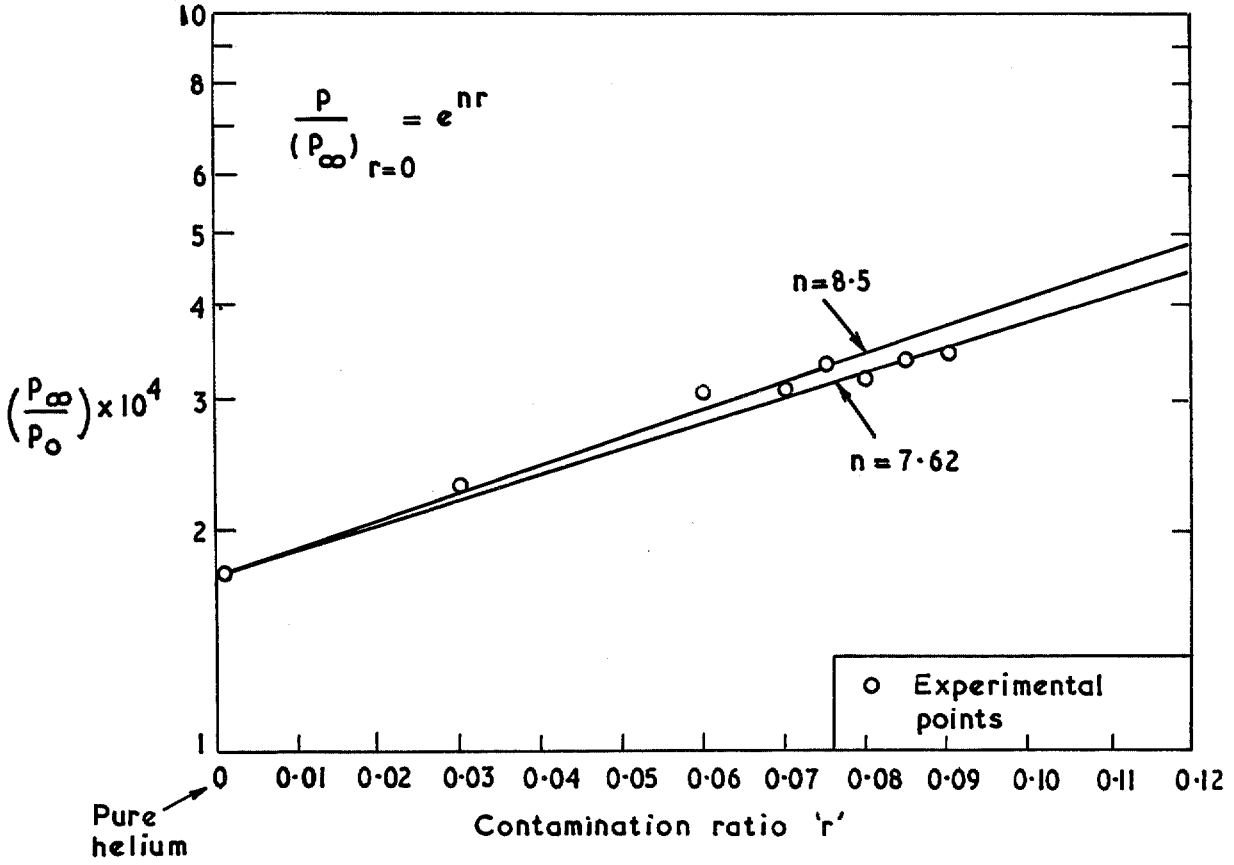

FIG. 2. Static pressure divided by reservoir pressure (p_∞/p_0), plotted as a function of the contamination ratio 'r' (M_∞ = 10·22).

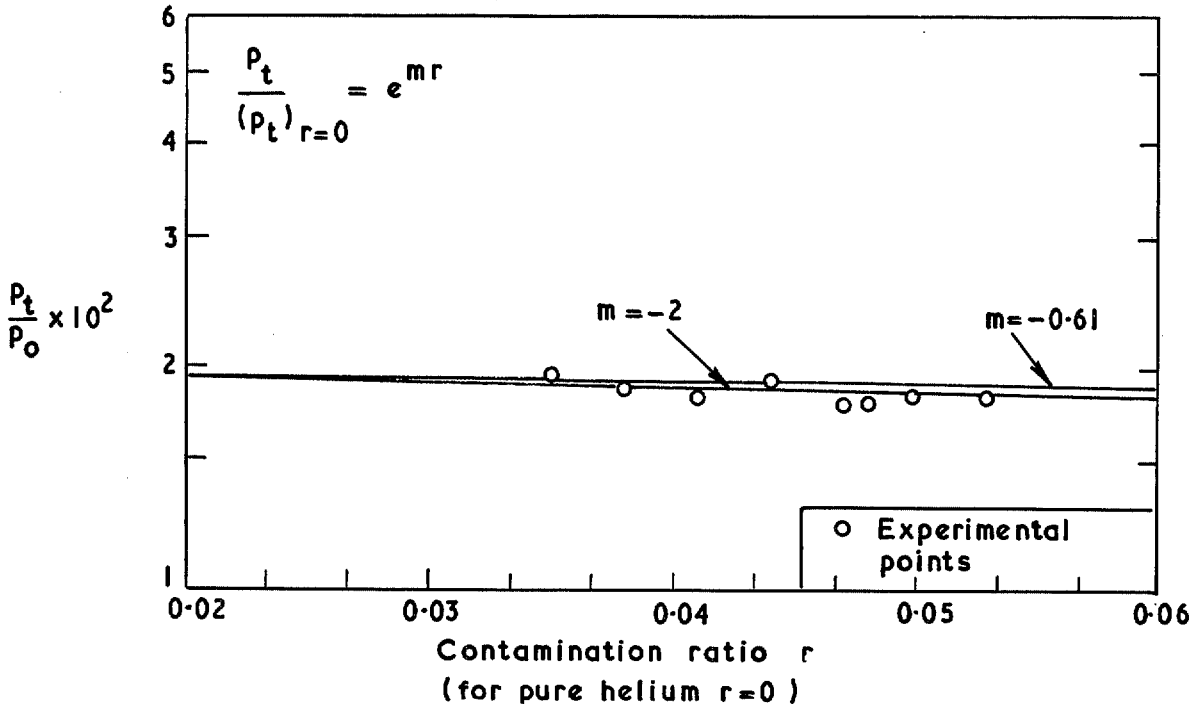

FIG. 3. Working section pitot pressure divided by reservoir pressure plotted as a function of contamination ratio, r (M_∞ = 10·22).

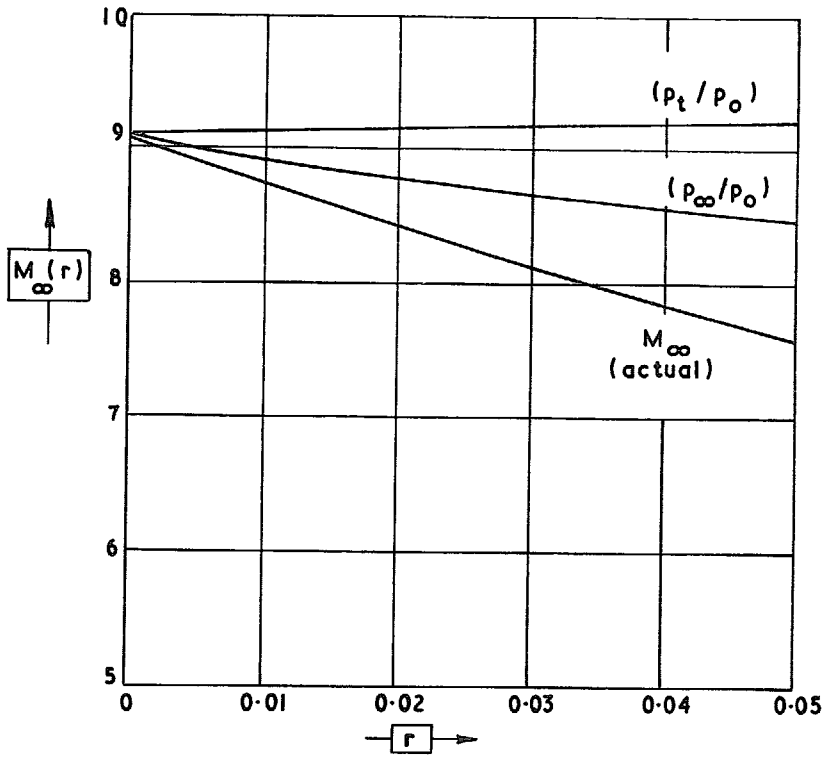

FIG. 4. Working section Mach-number variation with contamination ratio as computed, and as would be measured from the pitot/reservoir (p_t/p_0) and static/reservoir pressure (p_∞/p_0) ratios.

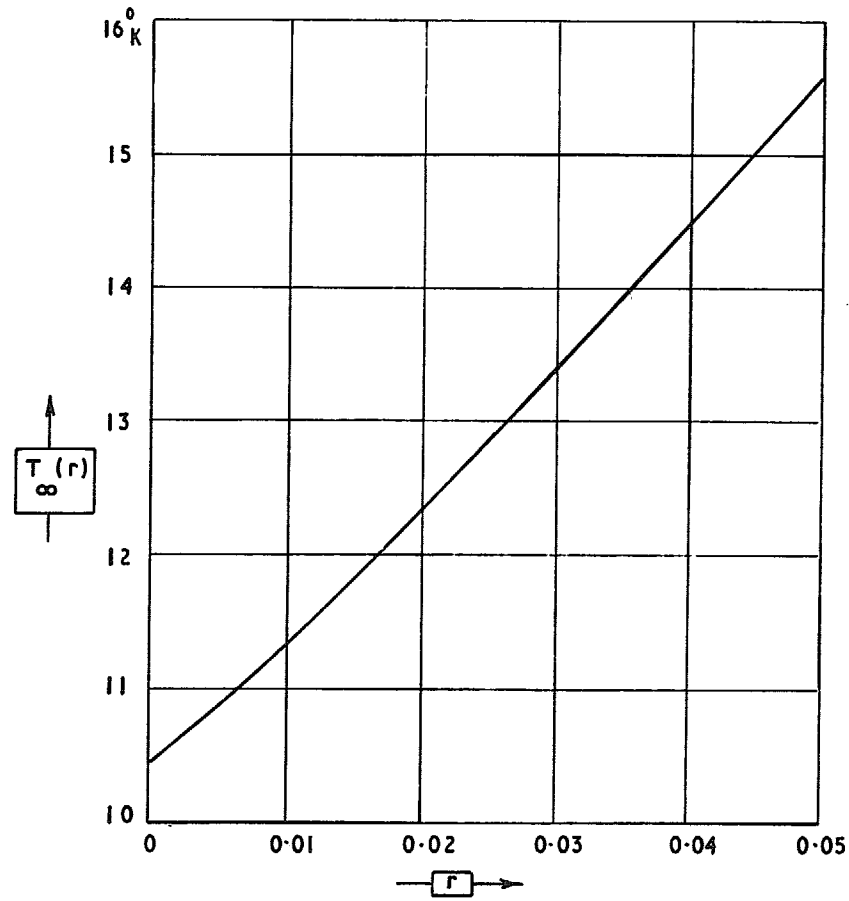

FIG. 5. Increase in working section static temperature with contamination ratio.

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