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GRAMPA—An Automatic Technique for Exciting the Principal Modes of Vibration of Complex Structures

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Summary.

This Report describes a new technique for finding automatically, when using multipoint excitation, the force distribution required to produce in-phase responses at the excitation points on a structure.

It is shown how this technique is used to find natural frequencies and principal mode shapes. The construction of a five-channel, servo-controlled system, circuit details and operation are explained in detail.

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1. *Introduction.*

One of the major problems in structural dynamics is to establish the modes of vibration that a complex structure would have if it contained no damping. These modes will be called 'Principal modes', although the term 'Normal modes' is also commonly used.

During the last two decades the problem of obtaining satisfactory principal-mode data from aircraft or other complex structures, using experimental techniques, has received a good deal of attention. Several methods have been proposed for dealing with the problem but the main disadvantages, with even the few techniques which are known to produce satisfactory results, is that the information obtained from tests requires considerable processing to extract the principal-mode information.

Multipoint excitation is often employed in attempts to force complex, damped structures to vibrate in principal modes, but finding the necessary distribution of force amplitudes can be a long and tedious procedure. There is therefore, need for a technique which will automatically find the required force distribution in a very short time. A further need is for an exposure of the many pitfalls which should be, but often are not, avoided in using multiple exciter systems; pitfalls which can make nonsense of apparently satisfactory results.

The Report discusses briefly the theoretical background leading up to and justifying the principles involved in a new technique which has been developed at the Royal Aircraft Establishment for resonance testing aircraft and other complex structures. The limitations of current and recently proposed techniques are considered and it is shown how many of these limitations are overcome in the new technique.

The main body of the Report concerns the principle of operation of a multi-channel, servo-controlled, excitation system which automatically applies the necessary force distribution to a structure to make the responses at all the excitation points vibrate in phase with one another. The two significant variables (force distribution and excitation frequency) are controlled automatically so that the system 'homes' onto the frequency and force distribution that will produce in-phase responses.

A five-channel system has been constructed and used to resonance test a light aircraft; the system provides a rapid convergence on to the required response conditions, thus eliminating the tedious iterative procedure usually associated with multipoint excitation.

In the past, difficulties encountered in using multipoint excitation have led, sometimes deliberately and sometimes unwittingly, to the use of unjustifiable methods of achieving in-phase response. Some of these methods are discussed and the implications of the results are presented.

2. *Limitations of Current and Proposed Techniques for Finding Natural Frequencies and Mode Shapes.*

It is reasonable to have to justify the introduction of yet another technique to deal with the problem of resonance testing, so the first part of this Paper is directed towards explaining the state of the art as it exists and discussing generally and the background against which this new technique was conceived. The vibratory motion of an undamped, multi-degree-of-freedom structure may be considered as being composed of combinations of its principal modes of vibration, but the situation is considerably more complicated when the structure contains damping in an unknown form. It becomes necessary to make certain assumptions about the form of the damping, and then to find a satisfactory way either of interpreting the response of the structure to limited local excitation or of making the structure vibrate in each of its principal modes by using several exciters deployed around the structure. Both of these methods have achieved a degree of success, but both involve the user in a tedious process of interpretation or iteration.

2.1. *Natural Frequencies—Current Techniques.*

Of the methods in use at the present time, that due to Kennedy and Pancu[1] appears to hold most favour. This method involves excitation at one or more points on the structure with forces which are all in-phase (or in anti-phase) with one another, whilst recording the phase lag between the force and response vectors and the amplitude of the response over the frequency range of interest. Although this method involves a lengthy procedure and the plotted results must be analysed to obtain the required information, it can, theoretically, provide all that is required; natural frequencies are defined by the point on the response curve at which the maximum frequency spacing occurs. Other methods (Fig. 1) of finding natural frequencies are used with less justification; peak amplitude, maximum quadrature component and the quadrature response conditions, used as resonance criteria, can all be shown to give wrong estimates of the natural frequencies.

2.2. *Natural Frequencies—Proposed Techniques.*

Bishop and Gladwell[2] have considered Asher's[3] proposed method in which the determinant of the in-phase components of the receptances is plotted against frequency, first for one and then for an increasing number of excitation and measuring points. The successive intercepts on the frequency axis are used as increasingly closer estimates of the natural frequencies as the number of points is increased. Bishop and Gladwell show this to be unsound except for the lowest and highest natural frequencies. The intercepts, however, do determine the exact natural frequencies when the number of points used is equal to the number of degrees of freedom. The difficulty here is to know when to stop introducing further measuring points and in this connection Bishop and Gladwell suggest a method which involves plotting the in-phase components of a single direct receptance against frequency for several points on the structure. The number of degrees of freedom (n) is estimated from the maximum number ($2r-1$) of intercepts that occur in any of these plots, on the assumption that $r = n$. An $r \times r$ determinant of the in-phase components of receptances is evaluated and plotted against frequency; its intercepts—if only r occur—provide what may be the (n) true natural frequencies. However, in order to check finally that enough excitation points have been used, it is necessary to choose a different set of n measuring points and repeat the operation. This is likely to be a lengthy procedure and it is quite possible that experimental errors will have a significant effect on the accuracy of the frequencies obtained.

2.3. *Mode Shapes—Current Techniques.*

Mode shapes may be obtained from the vector-response diagrams in the Kennedy and Pancu technique, by fitting the circles of curvature at the points of natural frequency. The diameters of the circles indicate the modal contributions associated with that particular natural frequency and that point on the structure. In practice, experimental errors in the plotted response make the determination of the exact radius of curvature difficult. Other single point methods (peak amplitude, maximum quadrature component and quadrature response) can be shown—as in the determination of natural frequencies—to be inexact.

During recent years attempts have been made, after finding the natural frequencies, to distribute forces around the structure in such a way that the responses at all points on the structure are in phase with one another. Only limited success has been achieved largely owing to the difficulty involved in using a tedious iterative procedure to find the required force distribution.

2.4. *Mode Shapes—Proposed Techniques.*

A method, proposed by Traill-Nash[4] which, according to recent report, has had some initial success on a simple two-degree of freedom system, involves measurement of structural response to excitation that is distributed over the structure. Computation techniques are used on these results to determine the force distribution required to excite a normal mode. Apart from the fact that a computer is required to deal with the results from a system containing a number of degrees of freedom, experimental errors could well interfere with satisfactory operation.

2.5. *Summary of Excitation Requirements.*

A method for investigating dynamic behaviour is required that will provide:

(a) The natural frequencies of the structure,

(b) the principal modes, and probably

(c) the damping coefficients associated with those modes.

It should eliminate the need for interpretation of responses from vector plots, which means that some form of multipoint excitation must be employed. In order to use multipoint excitation it is necessary to eliminate the tedious manual iteration used at present.

The use of GRAMPA (Ground Resonance Automatic Multipoint Apparatus) achieves this objective; it provides information on natural frequency and mode shape at the excitation points immediately without further interpretation. This condition is achieved rapidly with enormous reduction in the skill required from an operator; there is also a large saving in time required to achieve this condition.

3. *Justification for using Multipoint Excitation.*

Provided certain simplifying assumptions are made about the form of damping existing in a structure—and some simplification is necessary in order to produce an analysis—it can be shown by receptance theory that a structural response is made up of contributions from several modes of the structure.

For a structure with a finite number of degrees of freedom, Fraeijs de Veubeke[5,6] has shown, theoretically, that for certain types of damping distribution a structure may be excited at any frequency with a number of exciters which are all in phase (or in anti-phase) in such a way that the measured responses are all in phase and that the common phase lag (the characteristic phase lag) is unique for the force distribution being used at that particular excitation frequency. Also there are, at this frequency, as many characteristic phase lags each with its own linearly independent force distribution as there are degrees of freedom in the structure. The characteristic phase lags correspond to pure modes of vibration and each mode excited under these conditions behaves as a single degree of freedom.

In order to simplify the real elastic system with its infinite number of degrees of freedom, it may be assumed that at a particular frequency only a limited number of modes will contribute significantly to the response. Although this simplifying assumption (due to Traill-Nash[7] and termed 'effective number of degrees of freedom') is not justified mathematically, it is a useful and realistic practical simplification. Traill-Nash has shown that in order to excite a principal mode there must be at least as many excitation points as there are effective degrees of freedom in the structure and further, that if the excitation frequency is exactly an undamped natural frequency of the structure, a principal mode can be excited even though damping couples the modes. In other words, if we can excite the structure exactly on the natural frequency many of the problems (for example damping coupling which makes it impossible to excite a normal mode away from a natural frequency) are removed. The method proposed by Bishop and Gladwell, mentioned in Section 2.2, locates the natural frequencies, but it does not meet the requirements that were laid down earlier for a desirable system. The other point to be emphasised here is that the modes excited, with the correct excitation, at the exact natural frequencies, are the principal modes of the structure.

4. *Automatic Mode Excitation.*

This Section deals with the principle of operation of the automatic resonance testing system—GRAMPA.

The system incorporates a set of five independent servo control loops each one using phase response as a resonance criterion. One of the loops, which will henceforth be referred to as Channel One, controls the frequency of all the loops including its own; the other loops control force amplitude only.

A block diagram of the five-channel system is shown in Fig. 2. The principle of operation is that the phase of the structural response with respect to the force input at the exciter on Channel One is compared with the demanded phase; any deviation from the demanded phase causes a change in voltage at the output of the control integrator and results in a change in frequency at the output of the frequency generator. The phase demand supplied to all the other channels is the same as that supplied to Channel One, so the conditions which the loops are looking for on the other channels is determined by the condition applied to Channel One.

In Channels 2 to 5 (force control loops) the same principle of operation is used except that the output from the control integrator is fed to a multiplier in which the output from the control integrator is multiplied by the reference sinusoidal signal supplied from Channel One; the signal emerging from the multiplier is therefore in phase with the signal from Channel One but of a different amplitude; the force amplitude at Channel One remains constant.

4.1. *System Operation.*

For the purpose of this discussion the velocity response vector is considered and the phase-resonance condition occurs when velocity and force are in phase.

The structure to be examined is excited first of all using only the exciter on the frequency control loop (Channel One). All other exciters, though attached, have zero force input. Normally the approximate value of the resonance frequency to be investigated will be found by manual adjustment using phase as the resonance criterion. It is important that in setting this condition the phase should be increasing as frequency is increasing, otherwise the introduction of further exciters will lead to an unstable condition (i.e. Channel One must always have a positive control characteristic). The next step is to introduce the phase-frequency servo (Fig. 2) which varies the frequency to obtain the required phase lag between force and response. The force level, set manually at this excitation point, is subsequently maintained constant and is used only as a reference value.

The condition which now exists at this point is that the sum of the quadrature velocity components from the effective modes is zero and the sum of the in-phase components is the measured amplitude.

A second exciter, in phase (or in anti-phase) with the first, is brought into operation, its force amplitude being varied manually until the response is in phase (or in anti-phase) with the response at the first excitation point. Note must be taken at this stage of the sign of the force-phase characteristic, positive characteristic being indicated by an increasing phase lag as the force amplitude is increased positively. Once it has been verified that the in-phase condition can be achieved by manual operation, automatic control of this channel may be selected (*see* Figs. 3 and 4). The first exciter will automatically maintain the required phase by varying the excitation frequency through the action of its own servo loop, the frequency changes of the second exciter following those of the first. The second exciter will regulate its force amplitude to maintain the in-phase condition.

Further exciters, if necessary, are introduced using the same procedure as was used for the second exciter.

When sufficient exciters have been introduced in this way—and that condition is determined by inspecting the phase of the responses around the structure—the frequency of excitation is the undamped natural frequency; the measured amplitudes define the principal mode shape and the forces define the damping distribution. In this condition, the servo systems are tuned to the behaviour of the structure (Fig. 5) and any change in the structure will be followed by the servo system; for example changing the mass distribution by adding a mass at a point on the structure will cause a change in the natural frequency and the amplitude of the responses around the structure; these changed conditions will be found automatically and quickly.

Should it be found necessary to use a negative characteristic on the force control loops this is achieved by reversing the polarity of the exciting force at the exciter.

One of the major problems encountered in exciting structures such as aircraft is that it is difficult to determine exactly where to put the exciters. Using GRAMPA, with several exciters, the force levels required are considerably smaller than those required when using single point excitation. It would therefore be reasonable to deploy a much larger number of small exciters around the structure all permanently attached and switch control to the exciters which are most effective in particular modes of vibration.

4.2. *System Stability.*

The possibility of an instability arising in the system must be considered, as there will be interaction between the several servo loops. For example, the effect of introducing an exciter at the r^{th} point ($r < n$), where $n =$ the number of degrees of freedom) is to produce phase changes at the other $(r-1)$ points. At point number one a frequency change is initiated which is common to all exciters, whilst at the other $(r-2)$ points force changes are initiated.

In a structure with well-spaced natural frequencies, light damping and well chosen excitation points, the components from the off-resonant modes will be small. In these conditions the effect of introducing each successive exciter is to modify the force levels at the other points, but it should not change the sign of the force-phase characteristics which control the servo loops.

However, when two natural frequencies are close together and the damping is not light, the sign of the force-phase characteristic could feasibly change over a small frequency range if the receptance at a point has a larger component from an 'off-resonant' mode than from the mode being investigated. This condition may arise if exciters are badly placed (for example, near the node of the mode being excited).

Consider the response of the structure in more detail. If the principal modes are not coupled by damping, and displacements are measured at the excitation points, the response is defined by a receptance expression of the form

$$R|\alpha_{ij}| = \sum_{S=1}^{n} V_S \, K_i^{(s)} \, K_j^{(s)} \cos \theta_s$$

in which $R|\alpha_{ij}|$ is the in-phase component of the receptance connecting the response at j with a force at i or vice-versa

n = number of effective degrees of freedom

V_S is a positive expression involving damping, inertia, excitation frequency ω and stiffness terms

θ_s is the phase lag associated with the mode s (characteristic phase lag).

$K_i^{(s)}\,K_j^{(s)}$ defines the shape of the response in the mode s.

The $K_i^{(s)}$ may be positive or negative and take the form $\frac{dq_i}{dp_s}$ where q_i is the displacement at a point i and p_s defines the magnitude of the principal mode s.

When exciting near the first natural frequency (fundamental) with a single force and measuring the displacement response at the same point to obtain 90° phase lag (i.e. $R|\alpha_{ii}| = 0$), all the modes higher than the first provide a response in the first quadrant since the $\left(\frac{dq_i}{dp_s}\right)^2$ are all necessarily positive and the θ_s are all $< 90°$ for $S > 1$. The condition of 90° phase lag requires that $\cos\theta_1$ is negative i.e. $\theta_1 > 90°$ and hence $\omega > \omega_1$. It is assumed that $\frac{dq_i}{dp_s}$ is positive for all modes.

Now consider a cross receptance α_{ij}. In $R|\alpha_{ij}|$ under the same excitation conditions, $\frac{dq_i}{dp_s}$ is positive but $\frac{dp_j}{dp_s}$ can be either negative or positive depending on the choice of point j. A negative value of $\frac{dq_j}{dp_s}$ produces the same effect as adding 180° to θ_s and means that the total response is probably located in the third or fourth quadrants instead of in the first or second. The only way of predicting the sign of this term is from a knowledge of the behaviour of the structure in the appropriate mode.

A total response in the first or third quadrant indicates that the first mode response is small and that the point j is badly chosen (since $\omega > \omega_1$). Fig. 6 gives an indication of the form the response diagram might take under these conditions, with the contribution from different modes indicated by separate vectors.

Some of the measuring points j are likely to show response in the second or fourth quadrants not very far from 90° or 270° respectively. At points like these, little difficulty should be experienced in determining the correct sign for the force control characteristic.

However, in the range of the intermediate natural frequencies the behaviour of both the phase and amplitude of the measured response is complicated by the effects of contributions from lower and higher frequency modes. θ_s will be $>$ or $< 90°$ as ω is $>$ or $< \omega_s$ respectively so a condition of $R|\alpha_{ij}| = 0$ gives no indication of whether ω is $>$ or $< \omega_r$ (where ω_r is the undamped natural frequency of the mode being investigated).

$R|\alpha_{ij}|$ is complicated as before by the problem of allocating positive or negative signs to $\frac{dq_j}{dp_s}$ and though it may be possible, from a rough knowledge of the expected mode shape, to allocate the appropriate sign, the relative magnitude of the responses in the modes still remains unknown. However, from the practical point of view, the sign of the controlling characteristic should not be difficult to find unless the combined circumstances of very close natural frequencies and badly chosen excitation points occur.

4.3. *Force Amplitude as Phase Control—an Example.*

In order to make clear the principle of phase changing by varying the force amplitude a simple, two-degree-of-freedom system will be considered (*see* Fig. 7). Two equal in-phase forces operating at the same frequency (which is between the two natural frequencies on the system) are assumed to be acting at symmetrical points A and C. The total displacement vector response at A is made up of the vector responses due to the forces at both A and C (Fig. 8). Similarly the total vector response at C is made up from contributions from both forces. That is, each response is composed of two receptances multiplied by their respective force inputs; each receptance in turn contains two terms one from each of the modal contributions.

A total response vector may, therefore, be considered to be composed of four individual vectors (*see* Fig. 9). With these forces applied under these conditions, the total response vectors at points A and C are in phase with one another and a pure mode is excited. Now, if using these forces the frequency is changed so that the responses, whilst remaining in phase with one another, change phase to 90 degrees phase lag, the excitation frequency is the undamped natural frequency and the mode shape is the principal mode associated with that frequency.

Now consider the case in which the force at point A in the original conditions (Fig. 9) is maintained constant whilst the force at C is increased in amplitude (but not in frequency); inspection of the altered vectors shows what happens to the resultant phases. An increase in force at C produces an increase in the magnitude of all the vector components (in Fig. 9) with a second subscript C but those with a second subscript *A* remain unchanged. The effect of this is shown in Fig. 10; the total response at point A increases in both amplitude and phase lag whilst the total pressure at point C though increased in amplitude, is reduced in phase lag; i.e. responses are no longer in phase with one another. Further, it can be shown as the force at point C is decreased in magnitude through zero to a negative value (i.e. the force is now in anti-phase to the original force) the phase lag at point C is a continuous function of force amplitude. When the force amplitude is the same as it was originally, but of opposite sign, (the response also being in anti-phase at points C and A) the pure torsion mode is excited. The relevant point as far as the operation of GRAMPA is concerned is that the phase lag is normally a continuous monotonic function of force amplitude over the limited phase range required for adjustment.

5. *Equipment.*

In an attempt to make the unit as small and compact as possible transistor circuits have been used throughout, (Figs. 11, 12 and 13).

5.1. *Phase Measurement.*

Phase is measured using the well known mark-space ratio technique, presenting the output in the form of a square wave. The effect of the square wave (at the excitation frequency) is to superimpose a triangular waveform on the output of the control integrator (Fig. 14); this effect may be substantially reduced by supplying the phase demand signal also as a square wave. If motion is measured in the form of a velocity signal which is subsequently inverted, response and excitation have a phase difference of 180 degrees (i.e. they are anti-phase signals). A unity mark-space, square wave demand signal, in phase with the forcing signal and of opposite sign to the output from the phase measuring device will, when it is added to this output, produce zero summed voltage for the required operating condition; the integrator output will deviate from a constant value only over a very short time, during its controlling operation, when the two input signals are not exactly phased. This means that far less distortion occurs in the ensuing signals, (Fig. 15).

5.2. *Control Integrator.*

This is an integrator with a time constant which may be varied to alter the gain of the servo loop (Fig. 4).

5.3. *Frequency Control*[8,11]

This unit generates a fixed amplitude sinusoidal signal whose frequency is directly proportional to a dc voltage applied to the input.

5.4. *Force Control.*

Force level is controlled by feeding two signals to a multiplier, one of which is the output from the force-control integrator and the other is a reference, constant-amplitude, sinusoidal signal taken from the output of the frequency generator in Channel One, (Fig. 4). Its output, therefore, is a sine wave at the same frequency as, and in phase with, that of Channel One.

5.5. *D.C. Power Amplifier*[9]

This unit, a transistorised dc power amplifier, was developed especially for use with this equipment (Fig. 16 and 11). Its particular features are that it will operate down to dc and it has a high output impedance, (Fig. 17).

5.6. *Reliability.*

The problems involved in the reliability of the equipment can be divided into two categories: those concerned with the malfunction of individual components of the total equipment, and those concerned with the satisfactory operation of the system.

The component reliability has been found to be very good; very few failures have occurred during several months of continuous operation.

The main problem encountered with the system is that of zero drift in these components which influence the phase of the measured response or the excitation. These circuits have been improved so that less than 1 degree phase shift is caused by the inherent drift in the components.

5.7. *Circuits.*

5.7.1. *Phase detector.* The phase detector works on the well-known zero crossings principle. The two signals whose phase relationship it is desired to measure are each applied to separate limiters, which are used to detect the zero crossings of their input signals. The limiter outputs are used to control a dc potential, which is switched on at the zero crossing of one signal and off at the zero crossing of the other.

The input stage of the limiter (Fig. 18) has been arranged to have a high input impedance in order that:

(i) The signal source shall be only lightly loaded.

(ii) Phase shift caused by the ac coupling will be kept to a small value at low frequencies. The amplifier following the input stage has been used primarily to obtain amplification of the input signal; the balanced configuration makes possible symmetrical clipping of the signal.

The amplifier output is ac coupled to a Schmitt trigger circuit in order that a sharp pulse may be generated for triggering purposes. Because of the inherent backlash in the Schmitt trigger, it is adjusted to operate when the input goes to zero in one direction only.

The sensitivity of the circuit is such that the zero crossing is defined to better than 1 degree phase for an input signal with a dynamic range of 10 mV rms to 3 volts rms.

The switch (Fig. 19) consists of a conventional bi-stable multivibrator driving an inverted transistor *via* a buffer amplifier. The output of the inverted transistor is a dc potential which is switched between approximately zero volts and a fixed dc potential.

The limiters must have a low differential phase shift with respect to each other if errors in phase measurement at different frequencies are to be avoided.

The limiters in use at present were designed for a laboratory phase meter and to some extent suffer from drift with both temperature and supply voltage.

The design for a new limiter (Fig. 20) has just been completed and is presently being incorporated in the equipment. The new limiter is dc coupled thus avoiding phase shift at low frequencies, thereby making matching of units unnecessary.

Also, the backlash, zero crossing error and temperature stability have been considerably improved.

5.7.2. *Multiplier.* The multiplier[10] has been modified in order to eliminate the need for a 24 volt supply; this voltage is normally required for the inverting amplifier. However, if the output stage of the amplifier is 'current' rather than 'voltage' driven the 24 volt supply may be dispensed with, (Fig. 21).

5.7.3. *Characteristic Change—Amplifier A*3. At any given forcing station on the structure the phase change of the response signal for an increase of force may be positive or negative. When used in the automatic mode the sign of the force/phase characteristic must be taken into account. This may be done by making provision to invert the forcing signal at some place in the loop. The amplifier A3, Fig. 22, is used for this purpose.

6. *Some Pitfalls.*

The problem of resonance testing an aircraft is complicated by many factors which are not directly related to the actual resonance testing. Almost always the results are the best that can be achieved in the time allocated and there is little time available for a full investigation of any curious effects which may arise (and many of the effects which arise using multipoint excitation can have a profound effect on the quality of the results).

The use of resonance testing techniques is justified assuming that the structure is linear, that the theory applies and that certain simplifying assumptions, explained earlier, are realistic. Experience has shown that many factors can interfere; there is some evidence to show that the assumption regarding linearity and the form of damping may both be suspect; coulomb damping and backlash are both likely to occur; temperature variations appear sometimes to have an effect on the damping coefficient and the derived natural frequency; the method of supporting large aircraft is notorious for introducing non-linearities into some modes of vibration and the attachment of large exciters with their additional mass and stiffness at different points of the aircraft to excite different modes, can, of course, upset the basic assumptions about the mass and stiffness distribution in the structure. The selection of suitable motion-measuring transducers is a compromise between speed and ease of attachment to the structure and convenience in use.

Grave doubts arise about the conclusions to be drawn from responses when using any of the existing techniques if there are non-linearities in the structure; though there is some consolation in using multipoint excitation to force in-phase responses, in that the structure may be considered to be linearised for the amplitudes being used when the required response conditions are achieved.

6.1. *Excitation.*

Most of the mistakes that occur when using multipoint excitation may be divided into two categories; those concerned with the excitation and those concerned with the measurement of the response.

It is all too easy in attempting to achieve the required in-phase response condition to neglect, or deliberately to distort, the required excitation conditions; for example there are known instances in which the relative phase of the force inputs has been deliberately changed to achieve in-phase responses. The implications of doing this are explained in the Appendix, but it can be stated here that the required condition is not produced.

Electromagnetic exciters are commonly used today as a means of forcing the structure and these have characteristics of their own which must be taken into account. They are commonly used in conjunction with low output impedance power amplifiers and in these circumstances the variation of the exciter impedance with both frequency and amplitude can have a significant effect on the phase difference between the applied voltage and the resulting current; a back emf proportional to the velocity of motion, is generated in an electromagnetic exciter and if the exciter current is supplied from a low impedance source two detrimental effects arise; firstly the back emf, which is in phase with the motion of the structure produces a phase shift, the magnitude of which depends on the phase between the motion of the structure and the exciter current. The second effect is that the back emf diminishes the amplitude of the current at the frequency of the structural vibration, but it does not diminish the harmonic content of the current; so an effective increase in the harmonic distortion of the current supplied to the exciter occurs. Both of these disadvantages disappear with the high output impedance amplifier, thus making it possible to use a single voltage source to supply several amplifiers and exciters and be sure of generating in-phase forces.

If supplied by low output impedance amplifiers, exciters generate forces at different phases with respect to one another if they are not operating at the same amplitude, and in an attempt to overcome this difficulty pairs of exciters are often fixed in series so that the current applied to both exciters is in-phase. Doing this involves the assumption that the exciters have the same force factor (which they rarely have) and involves also the assumption that the aircraft structure will behave exactly symmetrically; only slight variations in symmetry are sufficient to cause a substantial difference in phase at apparently symmetrical points on an aircraft.

Though it is reasonable to neglect the additional mass of the exciter coil and its attachments to the structure if the mass is small and the exciter remains permanently attached throughout the resonance tests, the stiffness of the attachment connecting the exciter coil to the exciter body may not so readily be ignored; this stiffness may introduce a phase shift of the effective force input if the exciter is not rigidly mounted.

These comments should not be taken to mean that the electromagnetic exciters cannot be used satisfactorily; rather it means that they should be used with caution.

6.2. *Response Measurement.*

Careful attention must also be paid to the response measuring equipment, so some comment should be made on the performance of transducers. The comments do not aim to cover the full range of equipment available but are intended to illustrate, generally, some of the problems that may arise.

For measuring mode shapes at a particular frequency it does not matter whether the quantity measured is acceleration, velocity or displacement as only the relative amplitudes are required; acceleration signals tend to increase and displacement signals decrease as frequency increases, whilst velocity signals strike a middle path between the two and limit the required dynamic range of any following equipment for examining the mode shapes over a frequency range of the structure.

Acceleration signals at low frequencies are small and require amplification; they also tend to be noisy. which can be an embarrassment if the signal is to be used in a phase-detecting circuit; combined integration and amplification will reduce the noise content and will usually introduce phase shifts in the signal. Seismic accelerometers such as the familiar piezo-electric types suffer the disadvantage of phase shift at low frequency unless used in conjunction with a good charge amplifier. They are also sensitive to changes in temperature.

Many transducers, designed to work below their natural frequencies, are damped so that they produce an approximately linear phase shift with frequency and phase correction is necessary as frequency is changed.

Velocity transducers can produce signals which require no amplification over a wide operating range and the signals are generally free from noise. The non-seismic type requires a rigid reference frame (as does the displacement transducer) to be constructed around the structure, whilst the seismic type which is usually damped, introduces a frequency-dependent phase shift.

Lightly damped transducers, operating above their natural frequencies, have a limited use in aircraft resonance testing as it is difficult to use them at the very low natural frequencies which occur on large aircraft, though they do minimise the problem of frequency-dependent phase shift.

Displacement transducers can provide a good working signal (though diminishing as the frequency increases) free from noise and phase shift but require a rigid reference frame to be constructed around the structure.

The choice of transducer will obviously be influenced by particular circumstances, but for multipoint excitation it is obviously best to avoid those systems which introduce phase shifts inherently. The non-seismic velocity transducer has much to commend it particularly for the force control points.

7. *Conclusions.*

It has been shown that there is considerable advantage to be achieved from using multipoint excitation to make a structure vibrate in a principal mode.

A technique has been devised for finding, automatically, using a set of servo systems, the distribution of forces required to produce in-phase responses at the excitation points on the structure.

This technique enables in-phase responses to be established at a number of points in a very short time. The level of expertise required of an operator is very much lower than that involved when manual iteration is employed to achieve the same condition.

A system, which comprises five channels of excitation controls, has been built and shown to work well. It enables a natural frequency, its associated principal mode shape and the required force distribution to be found. Modes other than (and including) the fundamental can be established and the system is being used to investigate the behaviour of a light aircraft.

The stability of the system is such that it is clearly possible to extend the system to include more channels of excitation control.

Acknowledgment.

The author wishes to thank Mr. G. A. Taylor for his keen and active interest in the development of suitable transistorised circuits required for the equipment; much of the credit for the successful demonstration of the feasibility of this technique is his.

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APPENDIX

The Interpretation of In-Phase Response when using Complex Excitation

A system of complex excitation forces $(F_1 + i\,F_2)\,e^{i\omega t}$ is assumed to be operating on a structure in such a way that the structural responses are all in phase with one another.

The behaviour is defined by the equation

$$A\,\ddot{q} + B\,\dot{q} + C\,q = (F_1 + i\,F_2)\,e^{i\omega t} \tag{A.1}$$

in which A, B and C are the square matrices defining the mass, damping and stiffness of the structure respectively and q is a set of displacements.

A solution of the form

$$q = \bar{q}\,e^{i(\omega t - \phi)} \text{ is sought,}$$

Substitution in equation (A.1) yields, after separation of real and imaginary parts

$$[(C - A\omega^2)\cos\phi + B\omega\sin\phi]\,\bar{q} = F_1\,, \tag{A.2}$$

$$[B\omega\cos\phi - (C - A\omega^2)\sin\phi]\,\bar{q} = F_2\,. \tag{A.3}$$

ϕ is a value common to all the measuring points q. Furthermore, the condition sought is that of quadrature in-phase response $\left(\phi = \dfrac{\pi}{2}\right)$ which reduces the equations A.2, A.3 to:

$$B\omega\bar{q} = F_1 \tag{A.4}$$

$$-(C - A\omega^2)\,\bar{q} = F_2\,. \tag{A.5}$$

At a particular frequency ω_1 for which this condition is achieved, the F_2 in equation (A.5), appear as modifications to the structure and the equation may be interpreted as

$$(C + C' - A\omega^2)\,\bar{q} = 0$$

where $C'\,\bar{q} = F_2$

or

$$[C - (A + A')\,\omega^2]\,\bar{q} = 0$$

where $A'\,\omega^2 = -F_2$.

The required condition is not achieved; the quadrature components (F2) of the forces appear as modifications to the structure and the measured mode is a principal mode of the modified structure.

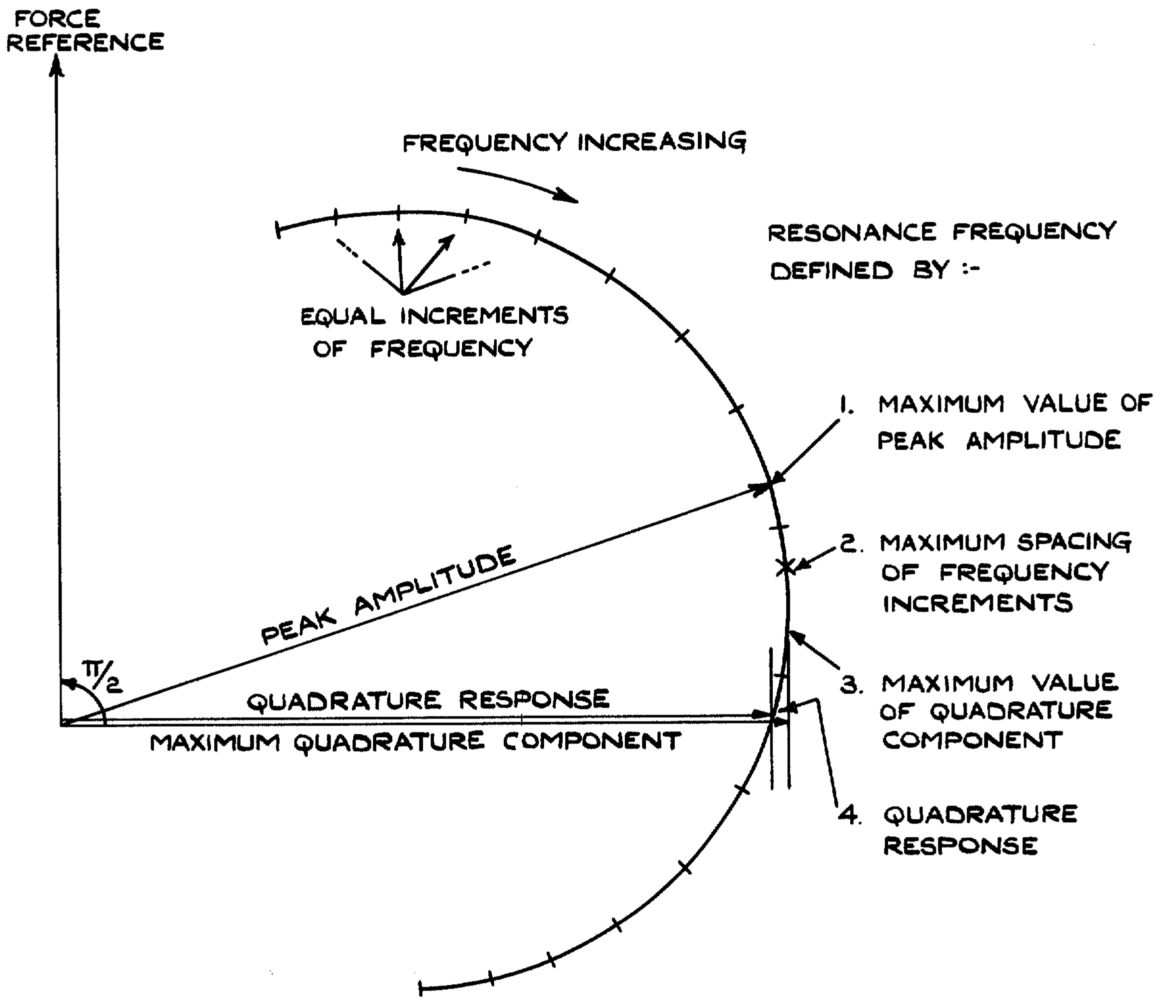


FIG. 1. Displacement vector-response plot showing various criteria for resonance frequency.

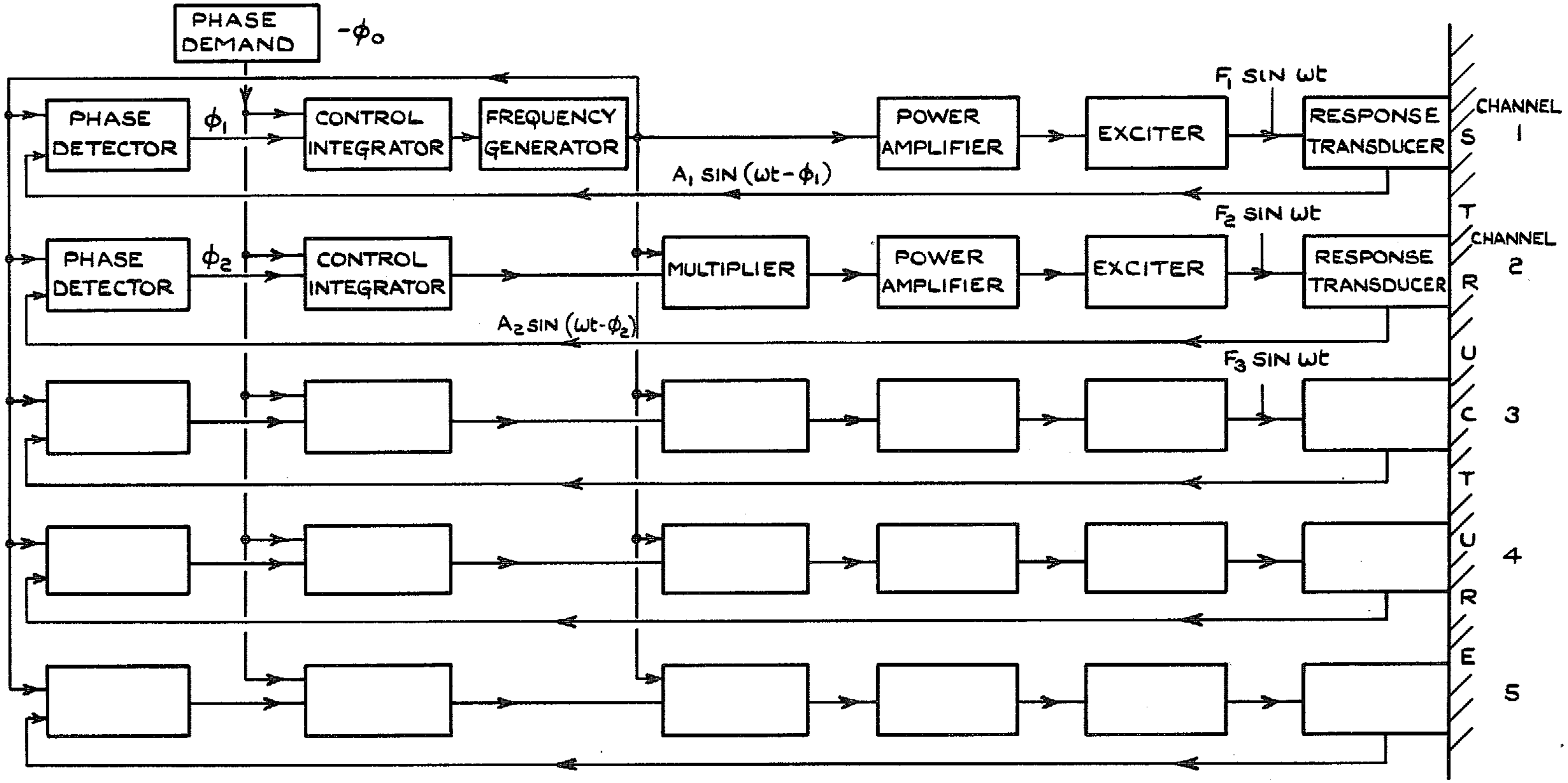


FIG. 2. Five channel block diagram of GRAMPA.

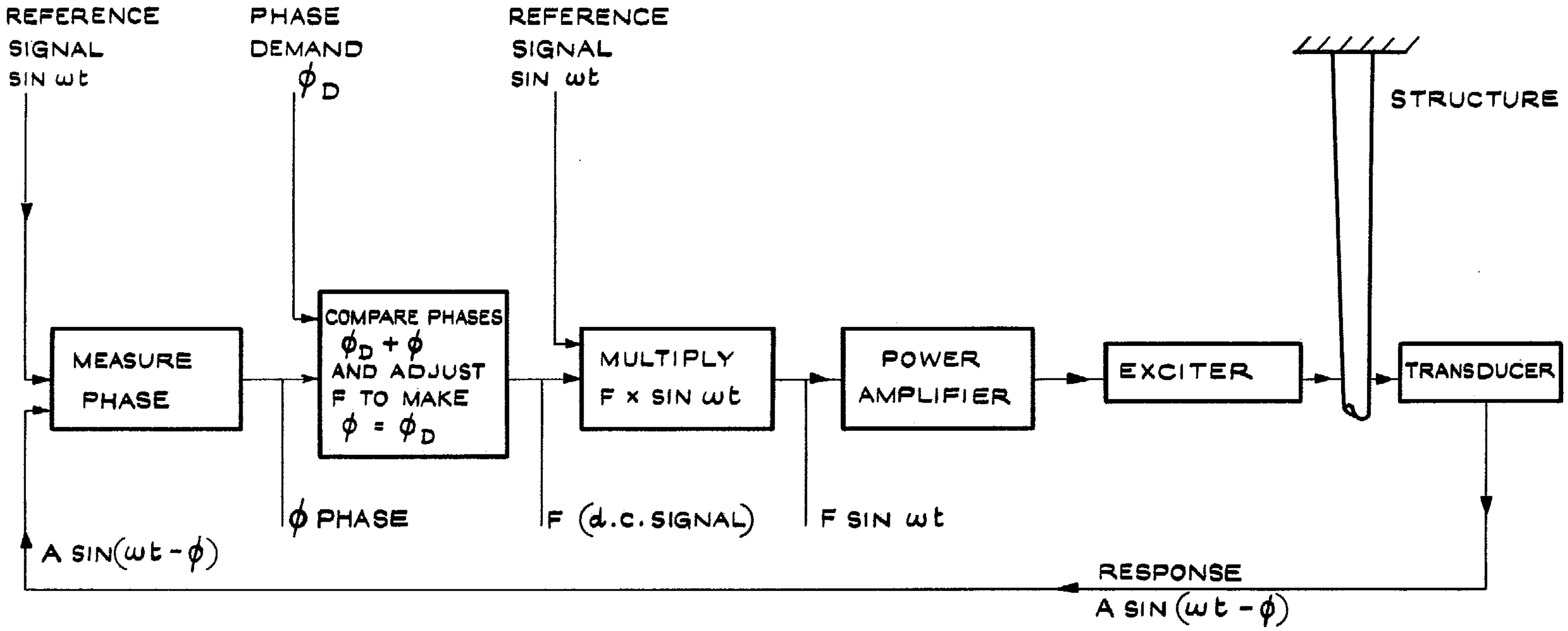


FIG. 3. Force level control—servo loop.

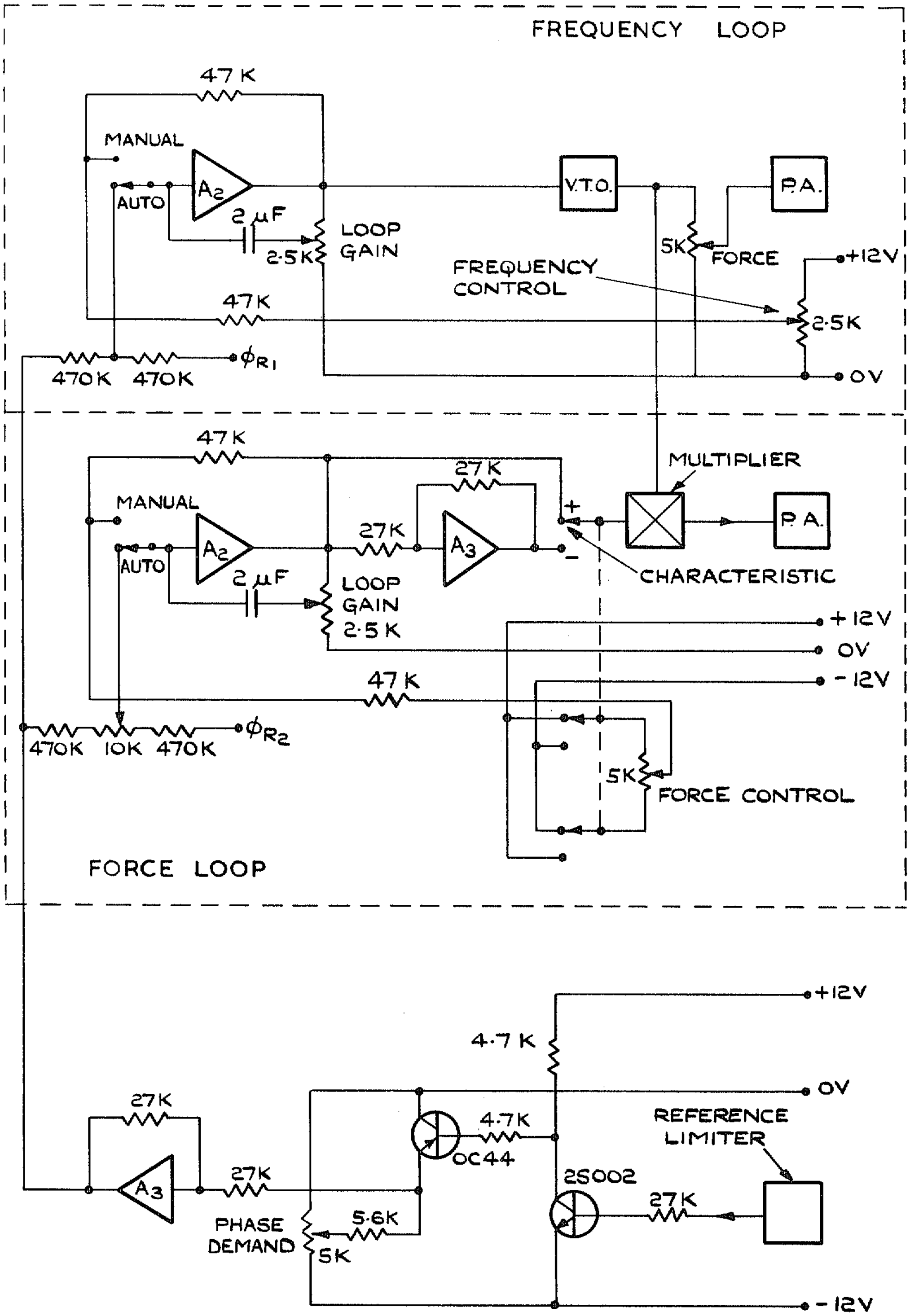


FIG. 4. Servo/manual control circuits.

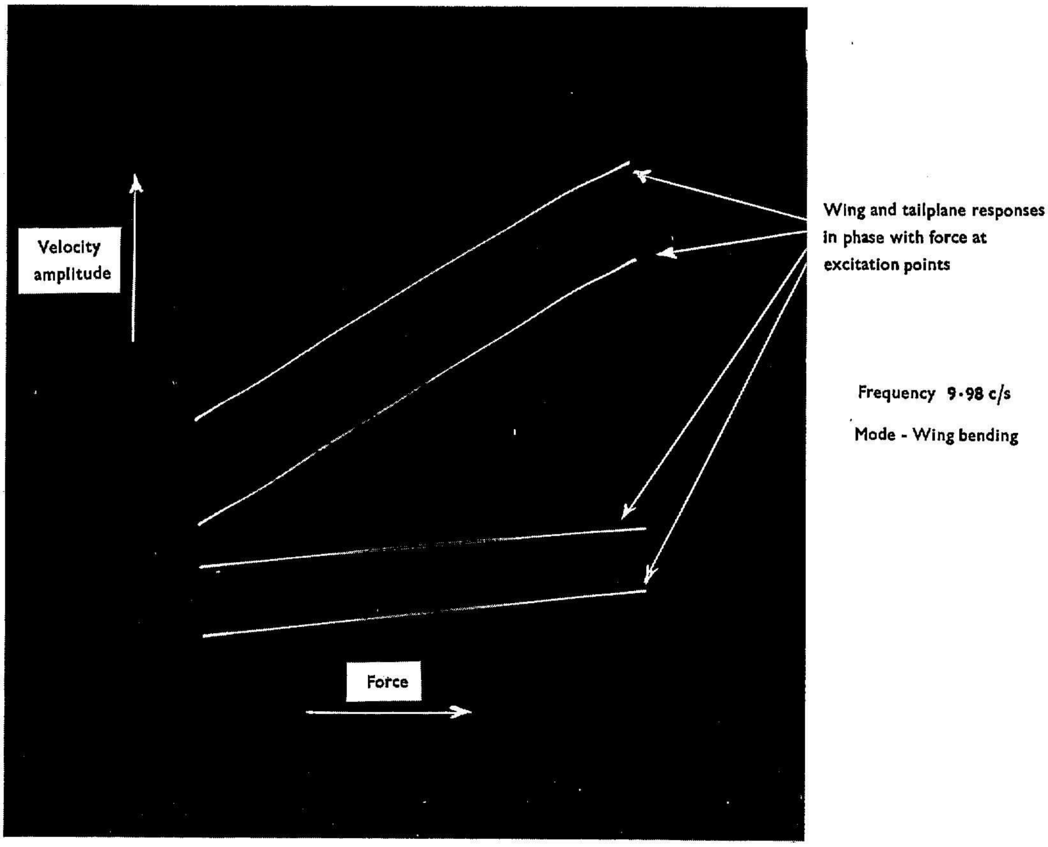


FIG. 5. In-phase responses at excitation points when using automatic system.

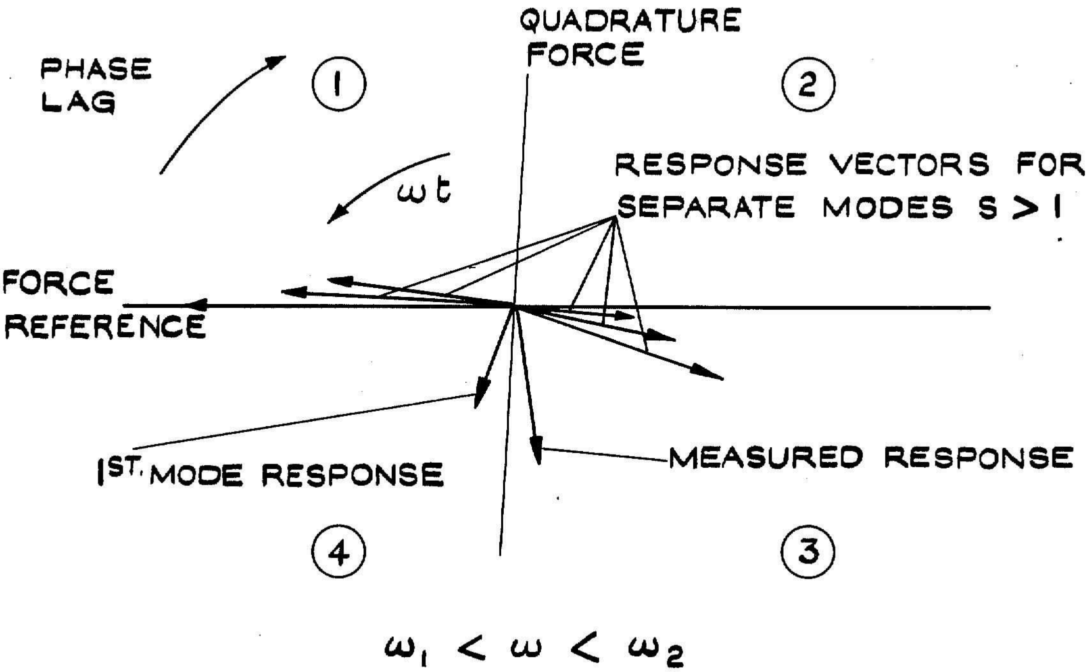


FIG. 6. Vector response showing contributions from six modes.

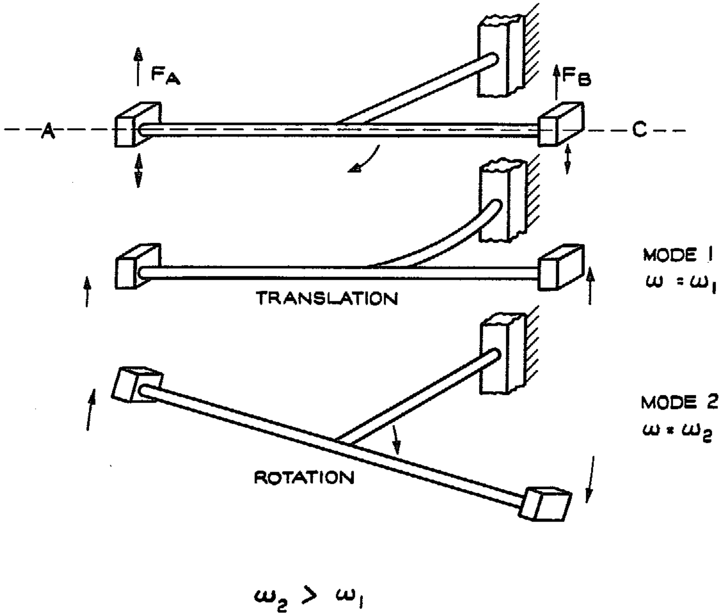
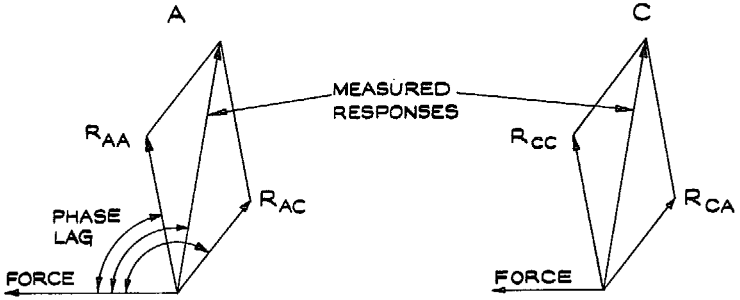


FIG. 7. Two degree of freedom system showing principal modes of vibration.



R_{AA} IS RESPONSE AT A DUE TO FORCE AT A.
R_{AC} IS RESPONSE AT A DUE TO FORCE AT C.

FIG. 8. Vector responses showing components from two forces.

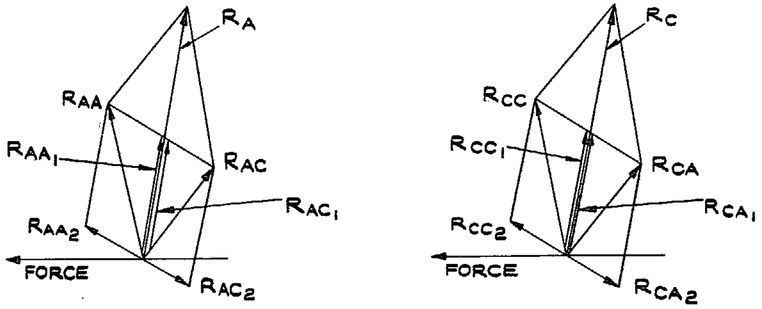


FIG. 9. Vector diagrams showing addition of modal contributions to produce in-phase responses.

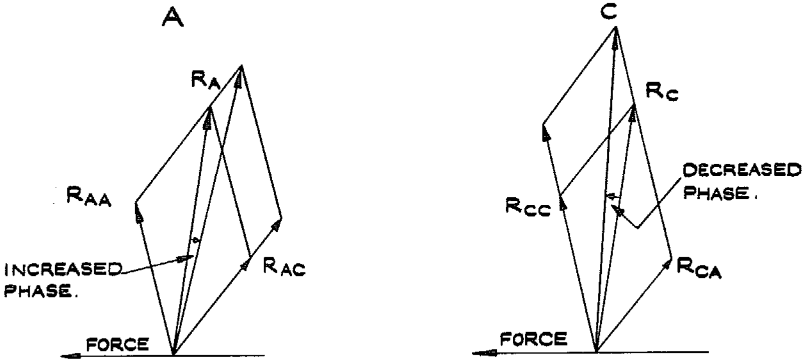


FIG. 10. Vector diagrams showing the effect on phase lag of increasing force level at C.

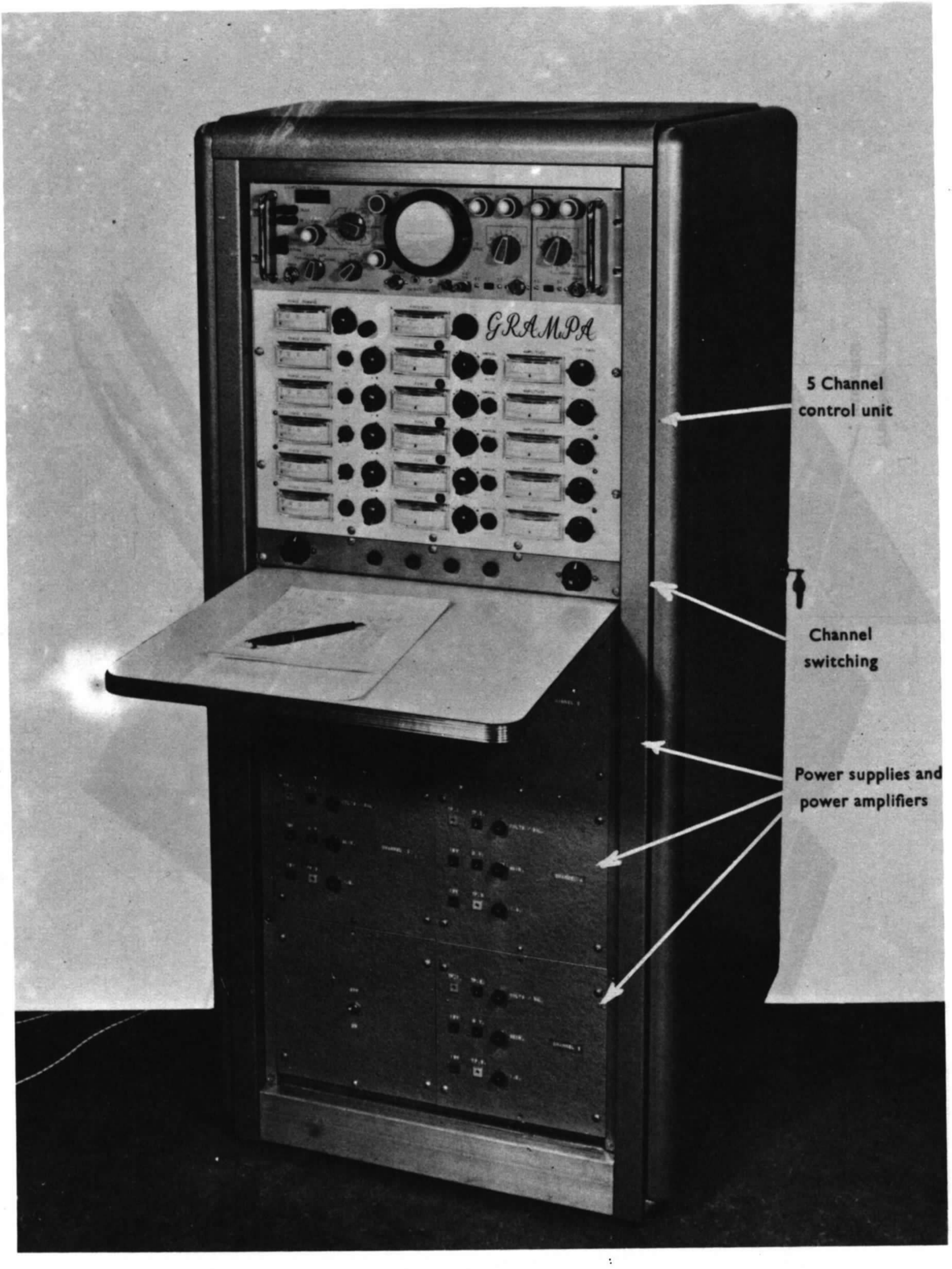


FIG. 11. GRAMPA—5 Channel automatic force and frequency control unit.

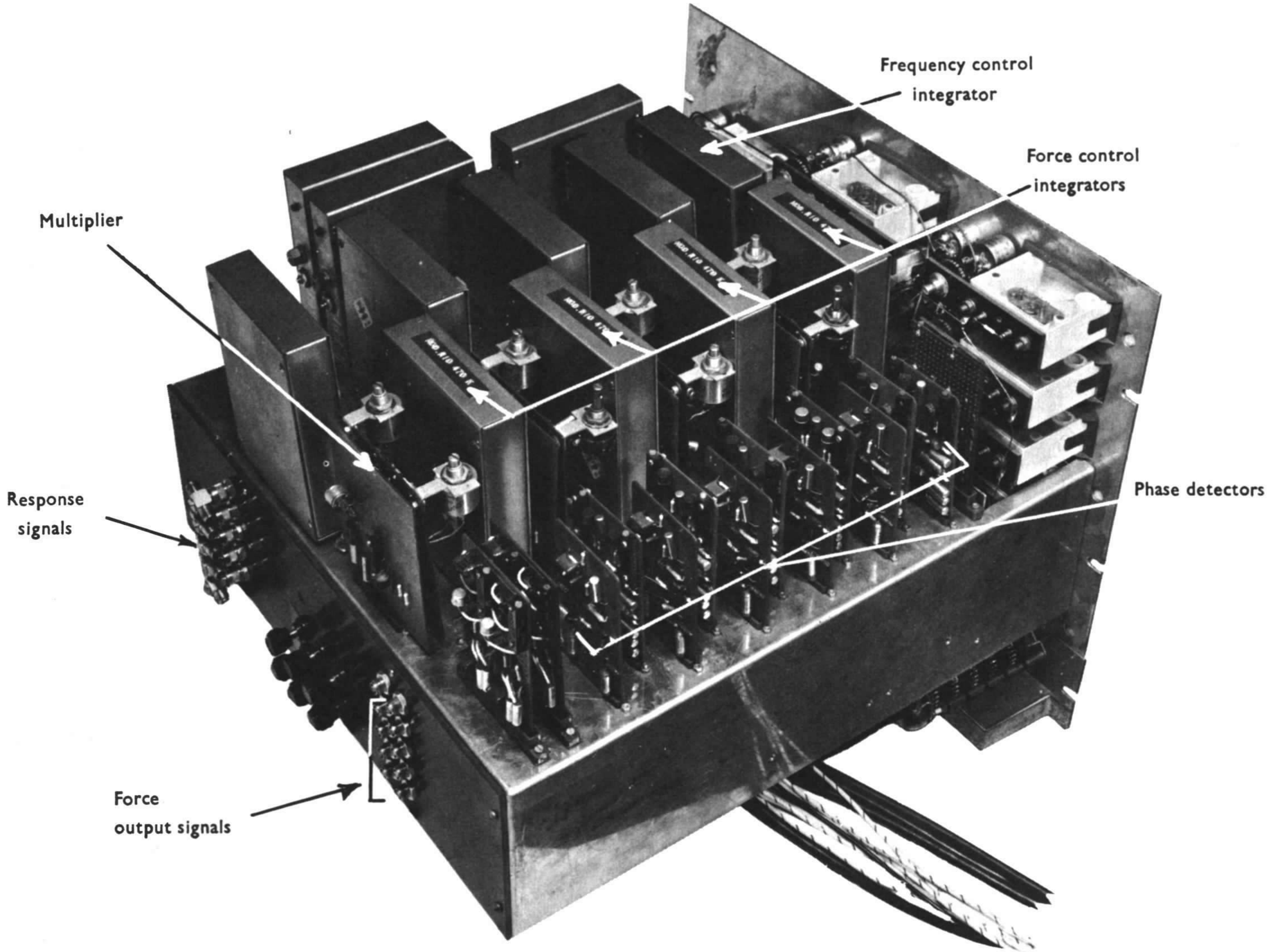


FIG. 12. 5-channel control unit.

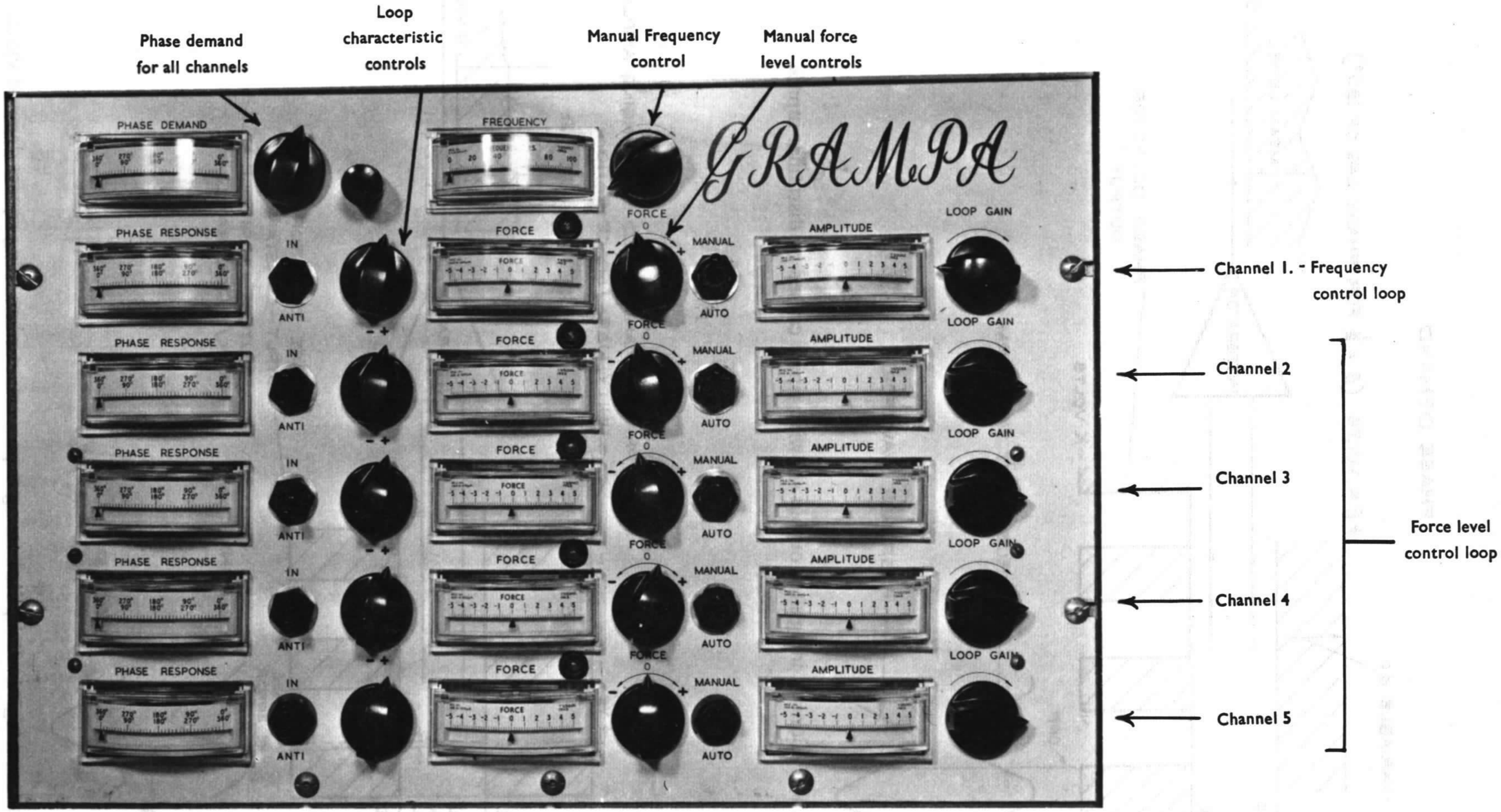


FIG. 13. View of 5-channel control panel.

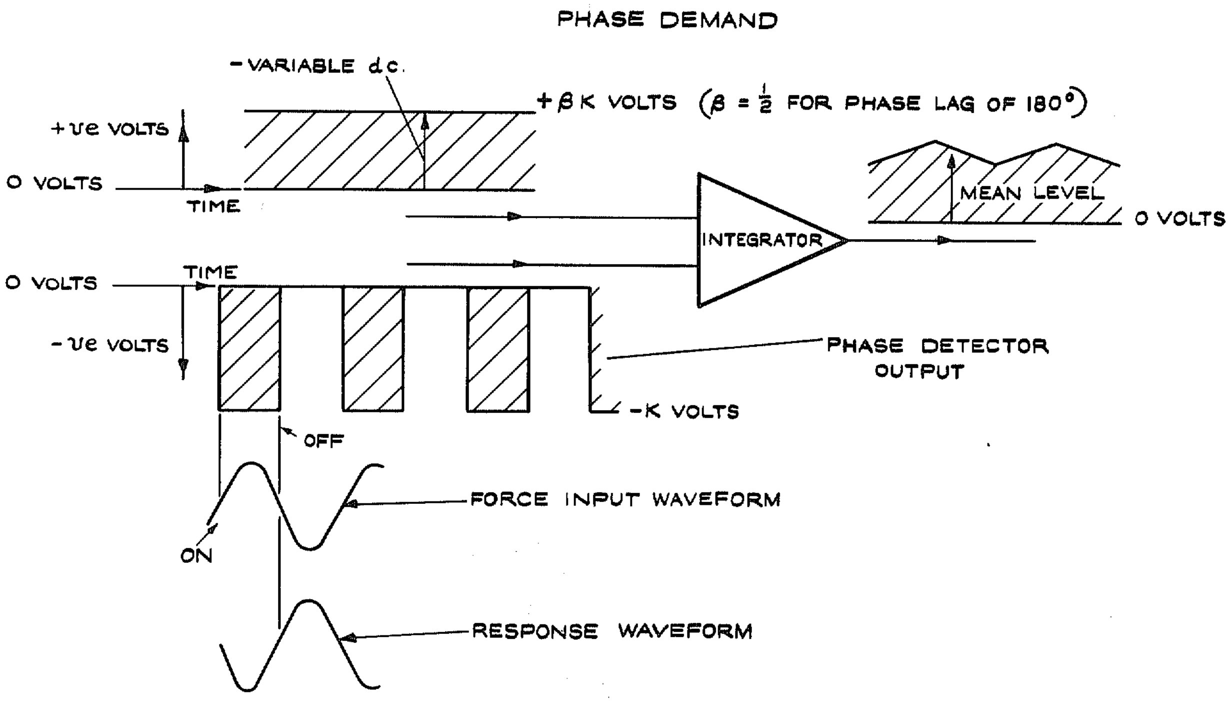


FIG. 14. Control integrator output showing ripple caused by dissimilar inputs.

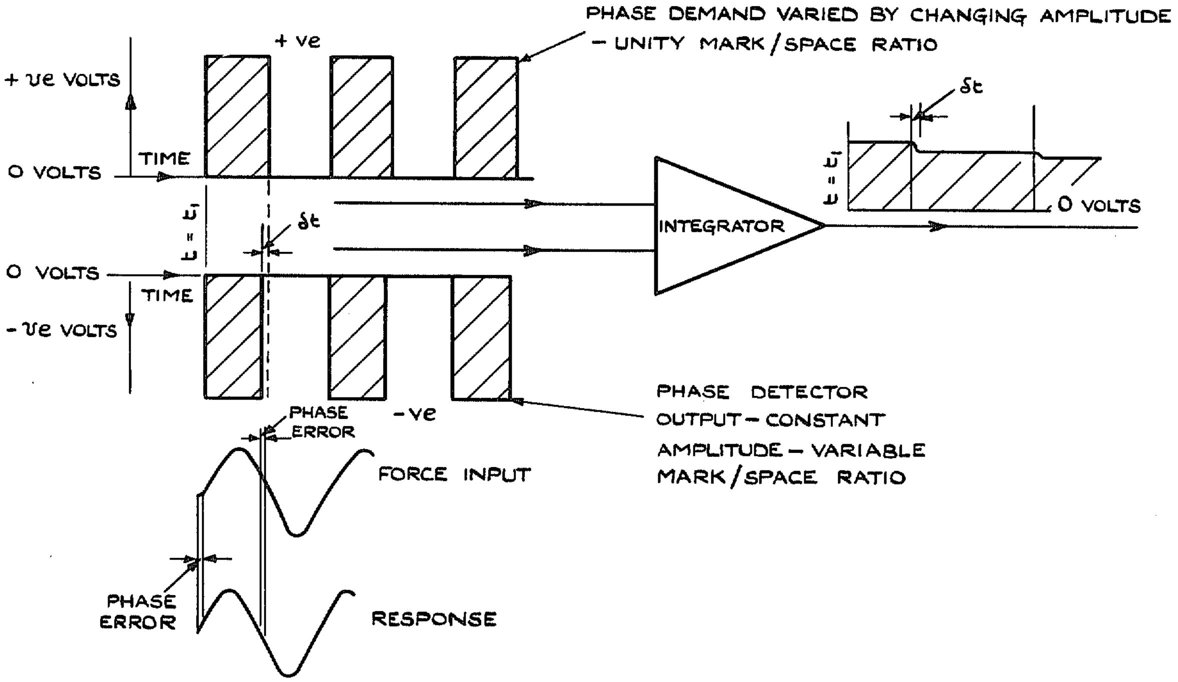


FIG. 15. Control integrator output showing improvement when using square demand signal.

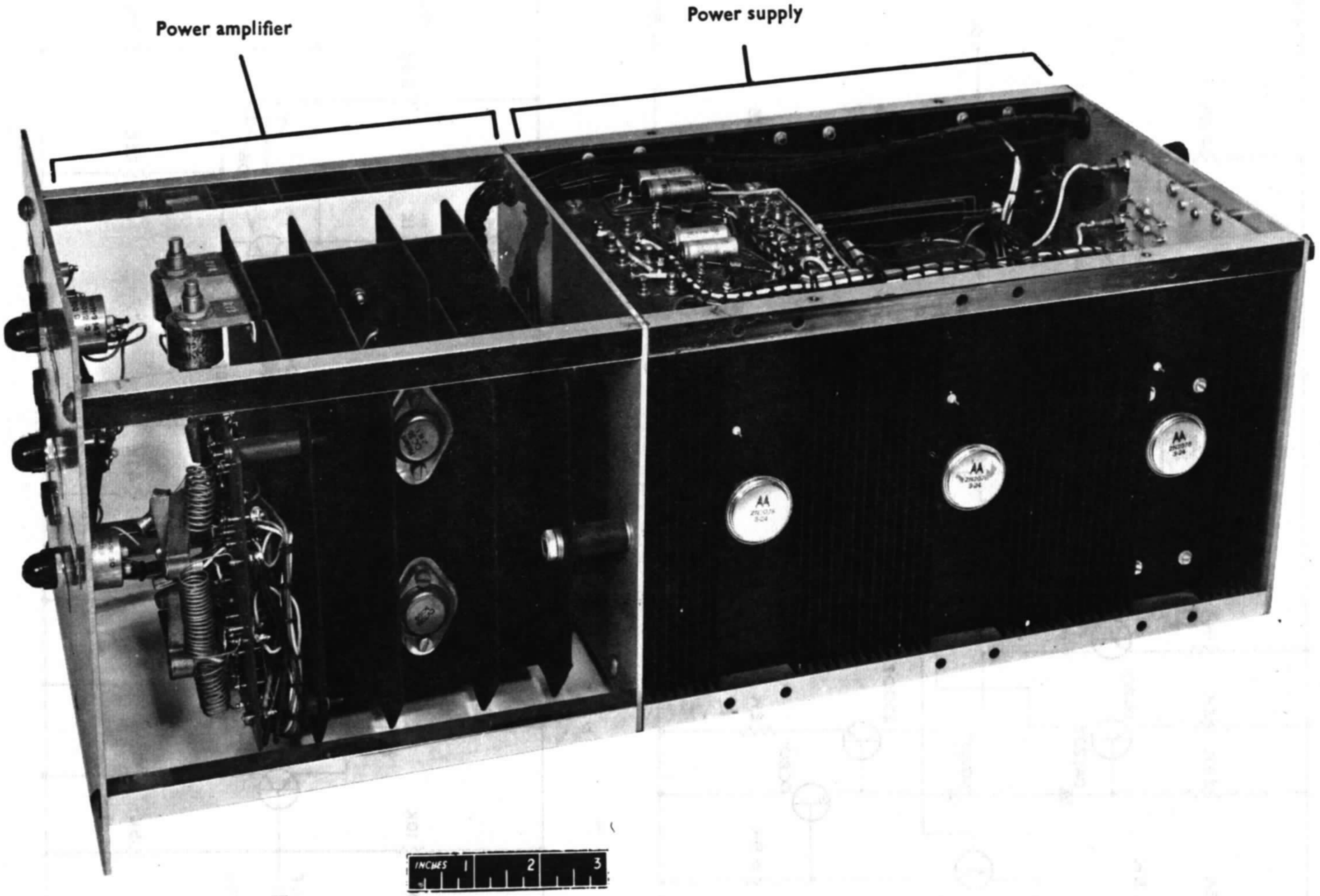


FIG. 16. Power amplifier.

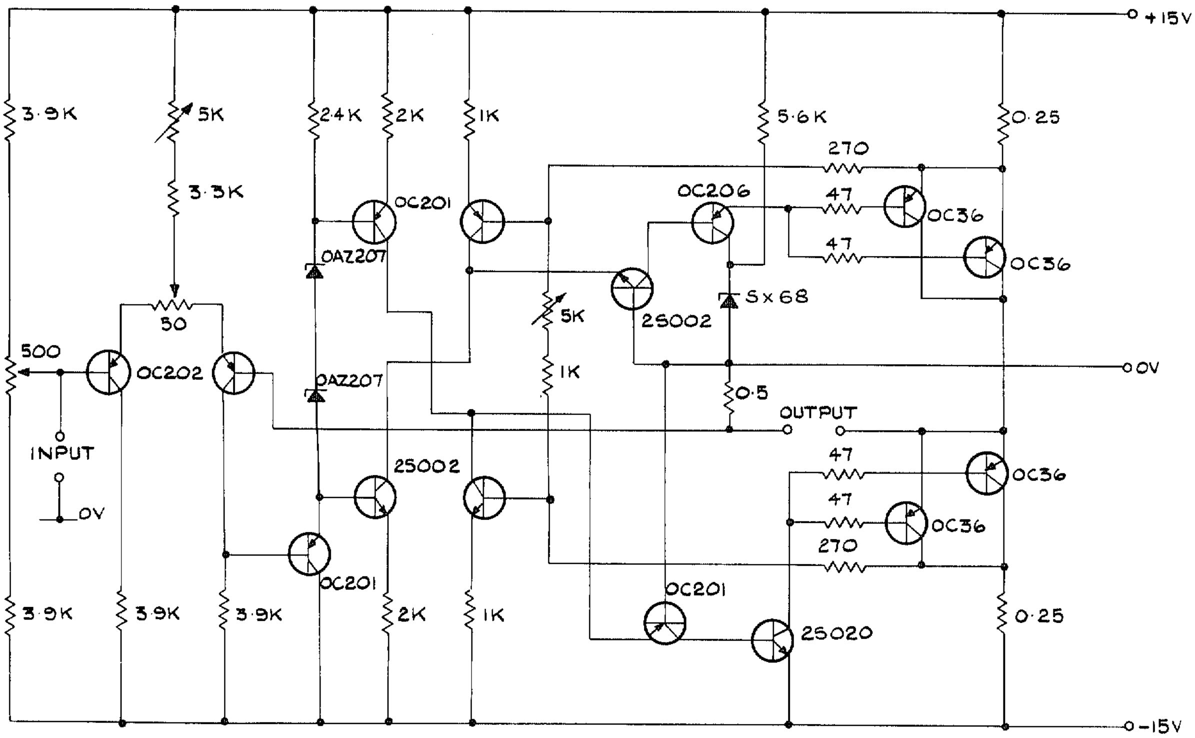


FIG. 17. Power amplifier.

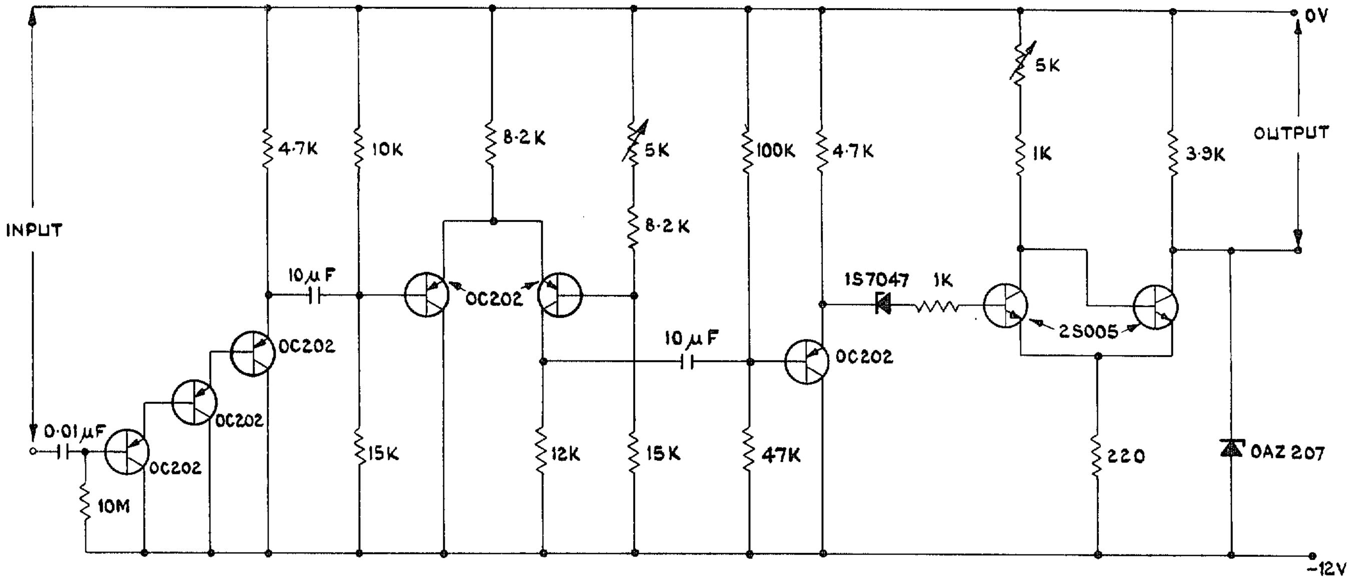


FIG. 18. Phase detector—limiter circuit.

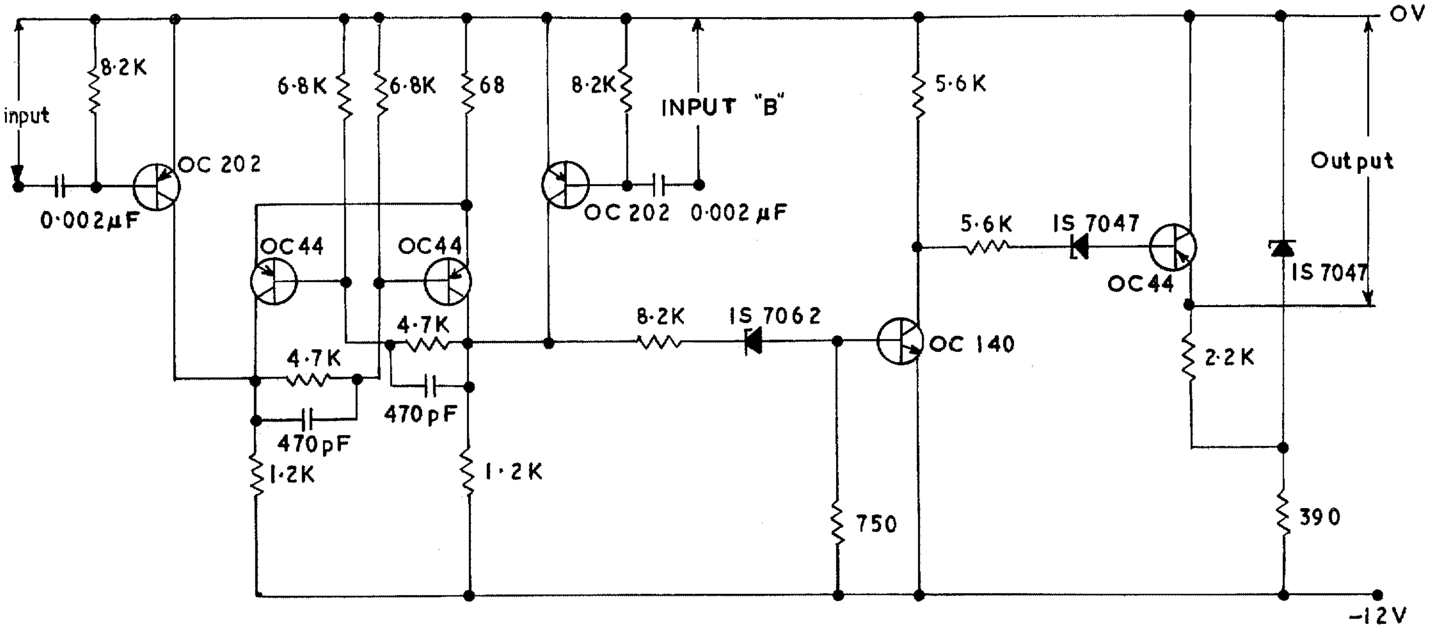


FIG. 19. Phase detector—switch circuit.

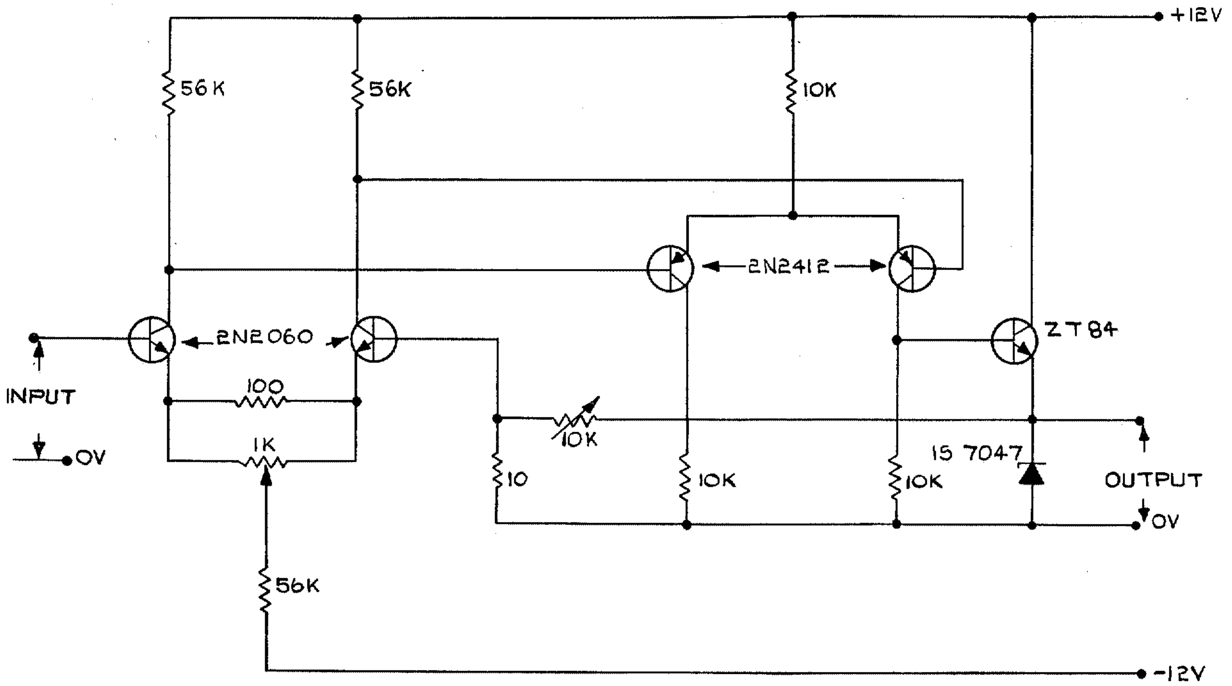


FIG. 20. Phase detector—improved limiter circuit.

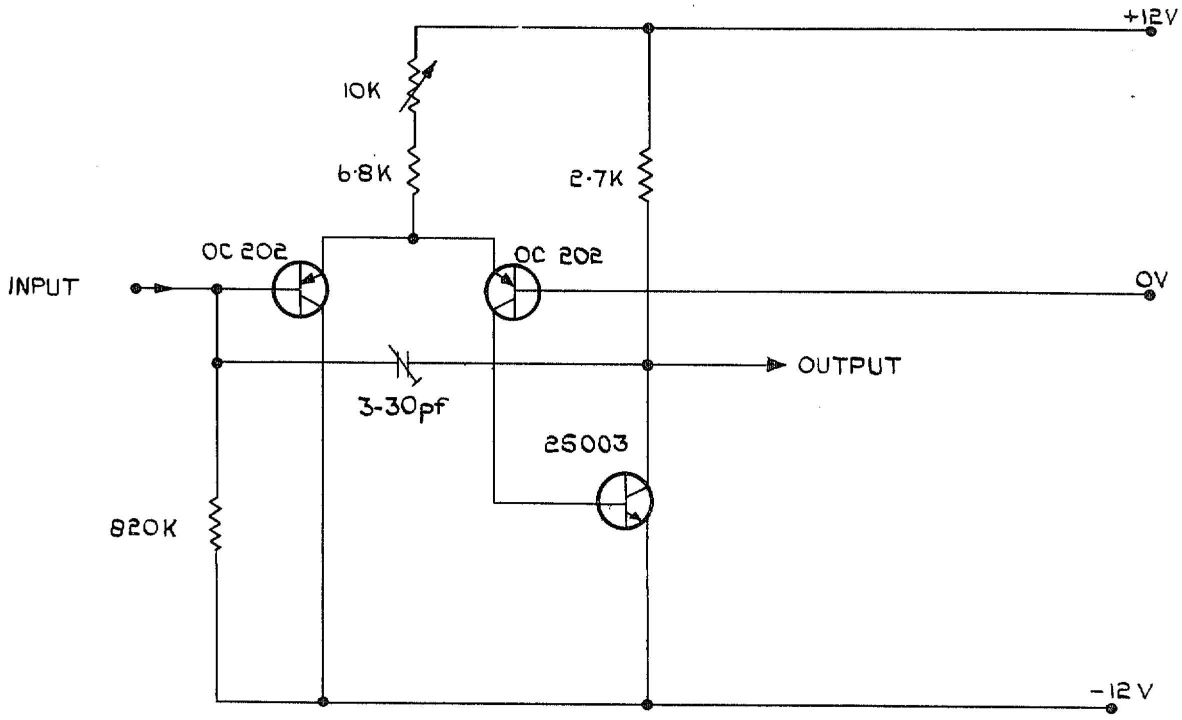


FIG. 21. Modified amplifier circuit for multiplier.

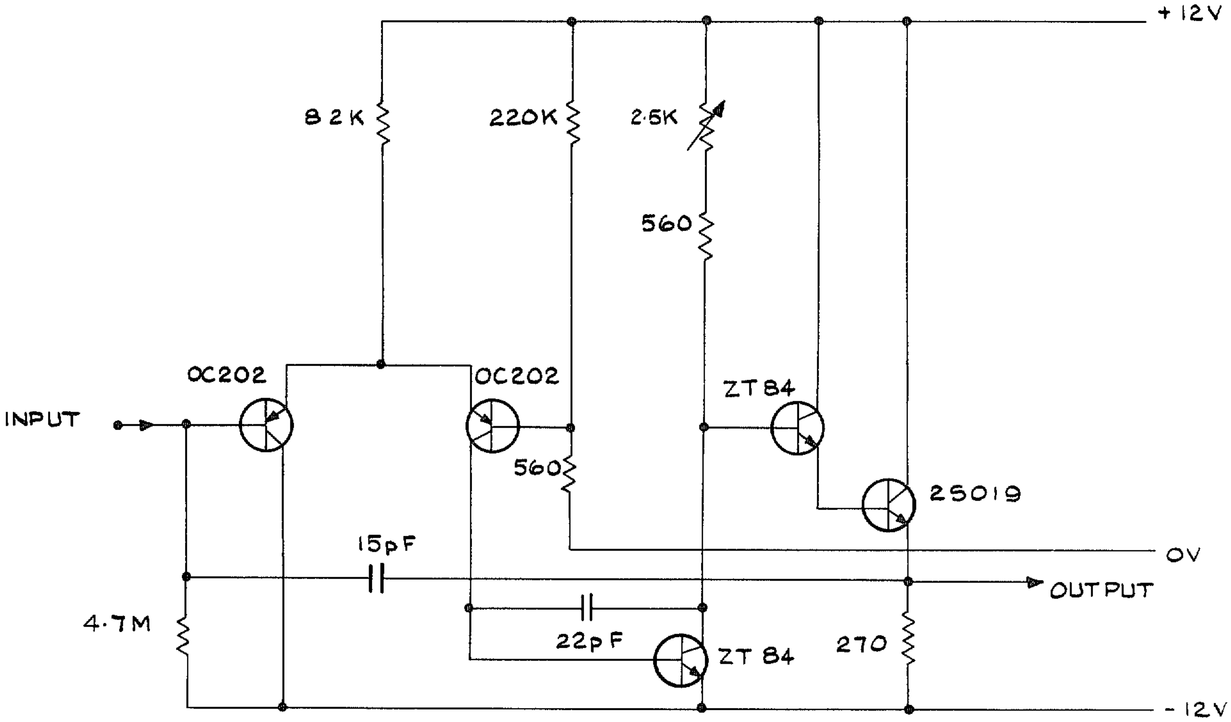


FIG. 22. Amplifier 'A3'.

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