Boundary Layer Separation in Two-Dimensional Supersonic Flow

- By -
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1. Introduction

Whenever boundary layer separation occurs in two-dimensional supersonic flow there is a fairly sharp increase of pressure before separation. It is found that the ratio of the pressure at the separation point S to the pressure at the position O just upstream of the sharp pressure rise, as in Fig. 1, usually depends primarily on the external-flow Mach number and state of the boundary layer at O, and is not greatly affected by the nature of the particular agency provoking separation. This is in marked contrast to the state of affairs in subsonic flow, and in fact in this respect conditions in supersonic flow are much simpler than those at low speeds. The reason for this simplification is as follows:-

The pressure rise just before separation is usually sufficiently steep for the slope of the wall at S not to differ appreciably from that at O even though the wall may be curved. Hence unless compression waves generated by some upstream source strike the boundary layer between O and S, the compression between these two positions must arise from within the boundary layer, from the external flow being deflected due to the thickening of the boundary layer. This thickening, however, is in turn due to the compression. There must therefore be an equilibrium between the thickening and the compression, and this equilibrium condition ensures that the pressure distribution between O and S is primarily dependent only on conditions at O. Of course the position of O will depend on the particular agency used to provoke separation. Thus if separation is caused by a step in the wall, as in Fig. 2, the distance of separation upstream of the step will depend on the step height. But if steps of different heights are used and adjusted in position along the wall so that the position of O remains the same (conditions upstream of O being unaltered) then the pressure distribution between O and S will remain approximately unaltered. There may be small variations in it because the boundary conditions* imposed downstream by the particular step provoking separation probably affect not only the position of separation, but also to a subsidiary extent the shapes of the velocity profiles at separation. However these latter effects are small especially when separation occurs well upstream of the particular agency which causes it.

Examples/

Examples of possible causes of separation are (a) a step in the wall, as already mentioned, (b) a wedge on the wall, (c) an externally-generated oblique shock, or (d) a near-normal shock, as in Figs. 2-5. With types (b) and (c) the overall pressure increase imposed on the boundary layer must be greater than some minimum if separation is to occur at an upstream position where the equilibrium conditions discussed above apply. With near-normal shocks just strong enough to cause separation (at low upstream Mach numbers) the base of the shock becomes curved and is "softened" into a band of compression waves. Stronger normal shocks (at higher upstream Mach numbers) become bifurcated at the base, with an inclined front limb of compression waves arising from the separation region, as in Fig. 5. In both cases the compression waves in the region of separation conform to the boundary layer thickening and to the pressure gradient the boundary layer can stand. Probably the separation pressure is again dependent on the particular conditions downstream to a slight extent, but not very much so.

Hence in most cases in supersonic flow it is possible to treat the ratio of the pressure at separation to the "undisturbed" pressure just upstream as a function only of the upstream boundary layer condition and external-flow Mach number. The boundary layer condition depends primarily on whether the flow is laminar or turbulent, on Reynolds number, and on whether or not there is heat transfer between the wall and the boundary layer[1,2]. (The data discussed below refer to the zero heat transfer condition.) If favourable pressure gradients act on the boundary layer well upstream of separation, as is usually the case with an aerofoil, they will have some effect on the separation conditions, but probably not a very large one.

2. Results for Laminar Layers

When, say, an oblique shock strikes a boundary layer which in the absence of the shock is laminar, a flow pattern and pressure distribution as in Fig. 4 is often observed. Well ahead of the shock there is a "foot" on the pressure distribution curve. The pressure rises steeply just upstream of separation and downstream of separation the pressure gradients become much smaller. They increase again when transition occurs which, with fairly strong shocks, is often upstream of the shock. In these circumstances the equilibrium conditions discussed above apply to the whole of the laminar foot*. It is easy to determine from the pressure distribution (or, less accurately, from the angle of the flow at the edge of the separated laminar layer, as shown in a Schlieren photograph) the pressure at the "top" T of the foot. This position can be taken as the second point of inflexion in the pressure distribution, as shown in Fig. 4. The pressure at T is a function primarily of the boundary layer conditions at O, and is insensitive to shock strength because of the equilibrium conditions. Thus in Fig. 6 experimental results[3,4] obtained at a free-stream Mach number of 2 for the pressure coefficient C_{pT} (defined as $C_{pT} = \frac{2(p_T - p_o)}{\gamma M^2 p_o}$, where M and p_o are the free-stream Mach number and the pressure at O) are plotted against the Reynolds number R_o based on the distance from O to the leading edge of the plate on which the boundary-layer was formed. It is seen that the results for a wide range of shock strengths lie on a single curve.

Pressure/

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*However curvature of the wall may be a relevant factor since the slope of the wall may change appreciably over the whole length of the foot, which is much greater than the short length OS.

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Pressure distributions containing a laminar "foot" are also obtained when different agencies are used to provoke separation. In Fig. 7 experimental results from all available sources are plotted against R_o*. It can be seen that C_{pT} decreases with increasing Mach number and also with increasing Reynolds number, the latter variation being between $R_o^{-\frac{1}{4}}$ and $R_o^{-\frac{1}{2}}$. Most of the inconsistencies between the different experimental data are genuine effects due, probably, to differences in the free-stream turbulence conditions in different wind tunnels. As indicated above, the end of the laminar "foot" occurs at the transition position, and this occurs at a lower Reynolds number when the tunnel turbulence is high. This reduction in the length of the laminar "foot" is accompanied by a fall in the pressure p_T at the top of the "foot" so that the values of C_{pT} are low in tunnels with high turbulence levels.

Since separation occurs in the region of relatively steep pressure gradient at the upstream end of the laminar "foot", the pressure coefficient at separation is lower than the coefficient at the top of the foot. Moreover, for the reasons described above, the pressure coefficient at separation should be independent of the position of transition downstream so that good correlation would be expected between the results obtained in different wind tunnels. However, the position of separation is very difficult to determine experimentally and this leads to inconsistencies. The measurements reported in Ref. 4 gave values of p_s/p_o of 1.14, 1.14, and 1.33 at $M = 2$, 3 and 4 respectively (the corresponding values of C_{ps} are 0.050, 0.022 and 0.029). The Reynolds number R_o varied in the experiments between 2×10^5 to 4×10^5 and over this range appeared to have no effect on p_s/p_o. This may appear to be inconsistent with the variation of p_T/p_o with R_o (see Fig. 7), but this is not so since p_T/p_o would be expected to decrease with increasing R_o even for constant p_s/p_o because the laminar "foot" becomes shorter as R_o is raised. However, in view of the difficulties associated with the accurate measurement of p_s it is possible that there is some variation with Reynolds number which has passed undetected. The theories of Ritter and Kuo[9] and Gadd[10] predict that the pressure increase at separation $p_s - p_o$ should vary as $R_o^{-\frac{1}{4}}$. Donaldson and Lange[11], using a simple dimensional argument, suggested that $p_s - p_o$ is proportional to $R_o^{-\frac{1}{2}}$. Stewartson[12] who assumed (in the authors' view incorrectly) that the pressure distribution acting on the boundary layer upstream of separation is the same as the theoretical pressure distribution through a shock wave with the same pressure increase, predicted a variation as $R_o^{-2/5}$.

As regards the magnitude of the pressure rise at separation Gadd predicted[10] that p_s/p_o for $M = 2$, 3 and 4 and $R_o = 2.5 \times 10^5$ should be 1.10, 1.18 and 1.27 respectively, whilst Ritter and Kuo found[9] that $p_s/p_o = 1.10$ for $M = 2$ and $R_o = 5 \times 10^5$. It is seen that these values are of the same order as the experimental values given above.

It is worth noting that according to Gadd's theory the pressure ratio at separation is roughly equal at all Mach numbers in the range 2-4 to the maximum pressure ratio produced by the reflexion of a shock of 1° deflexion angle, or by a 2° wedge on the wall.

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*When the boundary-layer was not formed on a flat plate in the experiments, R_o is the effective flat plate Reynolds number defined as follows. Consider a flat plate in a stream with the same external flow velocity, kinematic viscosity and Mach number as at O. At some position P on the plate the boundary-layer thickness will be the same as the boundary-layer thickness at O. Then R_o is defined as the Reynolds number based on the distance from P to the leading edge of the flat plate.

3. Results for Turbulent Layers

When, say, a strong oblique shock strikes a turbulent boundary layer, a flow pattern and pressure distribution as in Fig. 8 is often observed. In these circumstances, separation occurs well ahead of the shock. There is a corresponding kink in the pressure distribution at the wall because the pressure gradient is very steep upstream of separation but becomes much less steep where the boundary layer is well separated. It is possible to define a kink pressure p_K as the intersection of the maximum and minimum slope tangents, as in Fig. 8. A kink pressure can also be determined in interactions with normal shocks and wedges on the wall if these are such as to cause a considerable extent of separated flow. The equilibrium conditions discussed above then apply to the flow in the kink region, so that p_K/p_o and the corresponding pressure coefficient C_{pK} are mainly dependent only on conditions at O. The pressure p_s at the separation position, which can be determined by surface tube measurements, is near to p_K.

In interactions between a step and a turbulent boundary layer it is less easy to define a kink pressure because the pressure distribution is then rounded over, as in Fig. 2. However if the step is sufficiently high there is a well defined peak P in the pressure distribution ahead of the step. The pressure p_P at the peak can easily be determined from the pressure distributions or less accurately from the angle of the flow at the edge of the boundary layer in a schlieren photograph. The ratio p_P/p_o and the corresponding pressure coefficient C_{pP} are again mainly dependent on conditions at O, and are insensitive to step height for sufficiently large step heights. The peak pressure is, as indicated in Fig. 2, appreciably higher than the pressure at separation.

In Fig. 9 all the available data on the kink pressure and the separation pressure are presented. The kink pressure coefficient C_{pK} and the separation pressure coefficient C_{ps} are plotted against Reynolds number R_t based on the distance from O to the effective leading edge of the turbulent boundary-layer. Although there is a tendency for C_{pK} and C_{ps} to decrease with increasing R_t, the results are too few and scattered for the magnitude of the decrease to be reliably estimated. The decrease is, however, evidently quite small and in Fig. 10, where the pressure coefficients are plotted against Mach number, mean curves are drawn which take no account of the variation with Reynolds number. Most of the experimental values of C_{ps} in Figs. 9 and 10 were obtained from surface-tube measurements. However, the data in Fig. 10 at the lowest Mach number were obtained from tests on aerofoils at transonic speeds, and here the onset of separation was deduced from Schlieren photographs and from the way in which the kink pressure p_K varies with p_o when the free-stream Mach number is gradually increased through the region in which separation first occurs[16].

It is useful to note that, according to the curve in Fig. 10, the pressure ratio at separation is roughly equal at all Mach numbers greater than 1.5 to the maximum pressure ratio produced by the regular reflection of a shock of 5 deg. deflection angle or by a 10 deg. wedge attached to the wall.

Comparison is possible between the experimental results described above and the predictions of several theories. Gadd[10] considers the pressure at separation, and finds that it is independent of Reynolds number and related to the Mach number as shown in Fig. 10. The curve shown corresponds to a different choice of empirical constant to that made in Ref. 10. The theory of Tyler and Shapiro[20] is concerned with the peak pressure, and it predicts a small effect of Reynolds number in the opposite sense to that found by experiment. However, as can be seen from Fig. 12 the predicted values of C_{pP} are of the same order as those measured, and the trend of variation with Mach number is of the right sign. Crocco and Probstein[21] are also concerned with the peak pressure coefficient C_{pP}. Their results are independent of Reynolds number and as can be seen in Fig. 12 agree quite well with experiment if a suitable choice of empirical constant is made.

4. The Conditions Leading to Boundary-Layer Separation

Separation does not necessarily occur whenever the boundary layer is subjected to a pressure increase greater than that corresponding to the separation pressure ratio discussed in the preceeding sections. This is because in some types of interaction the equilibrium conditions apply only after extensive separation has occurred, and the separation point has moved well upstream of the agency used to impose the pressure rise. The conditions leading to the first occurrence of separation then depend not only on the free-stream Mach number and the characteristics of the undisturbed boundary-layer, but also on the nature of the agency used to produce the pressure rise. For example; in interactions with oblique shock waves generated by an external wedge or a wedge attached to the wall, whether separation is present or not, present a considerable part of the pressure rise occurs downstream of the point where the shock strikes the boundary-layer or the wedge apex position. Separation does not then occur immediately the total pressure rise exceeds the pressure rise to separation considered above, because the appropriate equilibrium conditions do not apply. Especially at high Mach numbers a considerably greater pressure rise is in fact required before any appreciable extent of separated flow is formed. 2

In other cases it is possible that separation will occur when the overall pressure rise is less than the pressure rise to separation considered above. Thus in the flow up a step, separation must occur however small the step height, but the overall pressure increase produced by the step will presumably fall smoothly to zero as the step height is reduced to zero. For small step heights the equilibrium conditions discussed above will not apply since the local effects of the step will become confused with the effects due to boundary-layer thickening. This is only true for very small step heights, however, and for turbulent boundary-layers the equilibrium conditions appear to apply at separation, provided that the step height exceeds about half a boundary-layer thickness.

For normal shock waves, the pressure increase becomes very large at high upstream Mach numbers, and separation always occurs. On two-dimensional aerofoils in transonic flow with turbulent boundary-layers, separation is found to occur[16,22] when the local Mach number immediately upstream of the shock exceeds approximately 1.23. The corresponding pressure coefficient across the shock is found to be about 0.38 which is in good agreement (see Fig. 10) with the pressure coefficient for separation under equilibrium conditions. It should be noted, however, that the pressure ratio across the shock is considerably less than that calculated for the upstream Mach number by means of the normal-shock equations. This means that the shock must be followed closely by an expansion although, at the wall, a continuous increase of pressure occurs because the boundary-layer smooths out the local pressure peak. This expansion arises partly because the thickening of the boundary-layer at the shock reduces the stream-tube areas in the subsonic flow downstream, and partly[7] because of the pressure gradient in the limited region of supersonic flow. Thus, the Mach number just ahead of the shock varies from a maximum near the wall to unity at the outer edge of the supersonic region so that the pressure immediately behind the shock must be greater near the wall than further out in the flow. To enable the flow to follow the convex surface of the aerofoil, the pressure must fall as the wall is approached and suitable expansions must occur to make this possible.

The conclusions which emerge from the foregoing evidence are that whenever any appreciable extent of separated flow occurs the pressure ratio at separation is approximately as discussed in sections 2 and 3, but it is seldom possible to formulate simple rules as to what conditions are necessary for separation to occur with any particular configuration. However it is certainly safe to say that if the usual separation pressure ratio is not exceeded, separation, if it occurs at all, will be very limited in extent.

List of Symbols

x_o distance from leading-edge to 0.

R_o Reynolds number based on length x_o and free-stream values of velocity and kinematic viscosity.

x_t distance from effective leading edge of turbulent boundary-layer to 0.

R_t Reynolds number based on length x_t and free-stream values of velocity and kinematic viscosity.

M free-stream Mach number (just outside boundary layer at 0).

p static pressure

p_o p in free-stream or at point 0 just upstream of region of interaction

p_k p at "kink" in pressure distribution at wall with separated turbulent boundary-layers (see Fig. 8).

p_{max} peak value of p attained at wall in region of interaction.

p_s p at wall at separation point.

p_T p at "top" (point of inflexion) of laminar foot (see Fig. 4).

p_P p at the peak in the pressure distribution ahead of a step (see Fig. 2).

C_p pressure coefficient $\frac{2}{\gamma M^2}\left(\frac{p}{p_o} - 1\right)$, where $\gamma = 1.4$: suffices corresponding to p.

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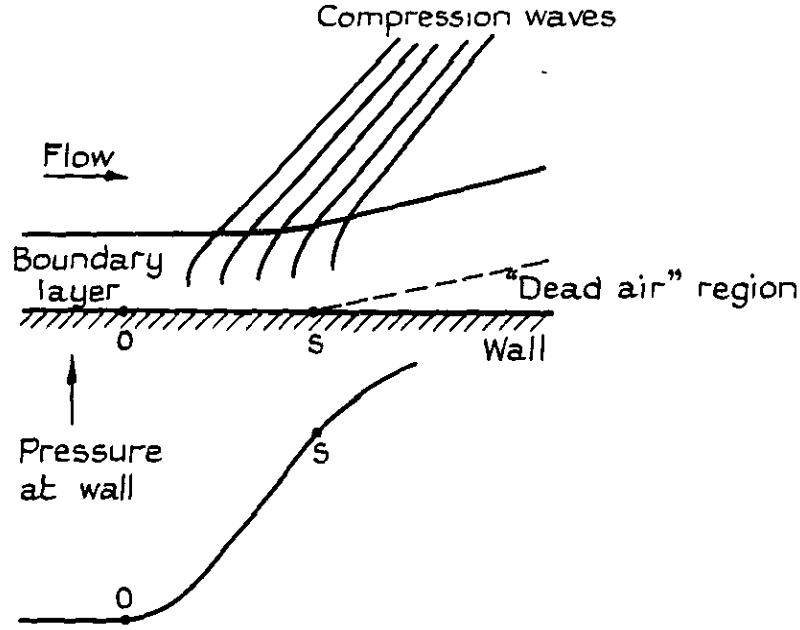
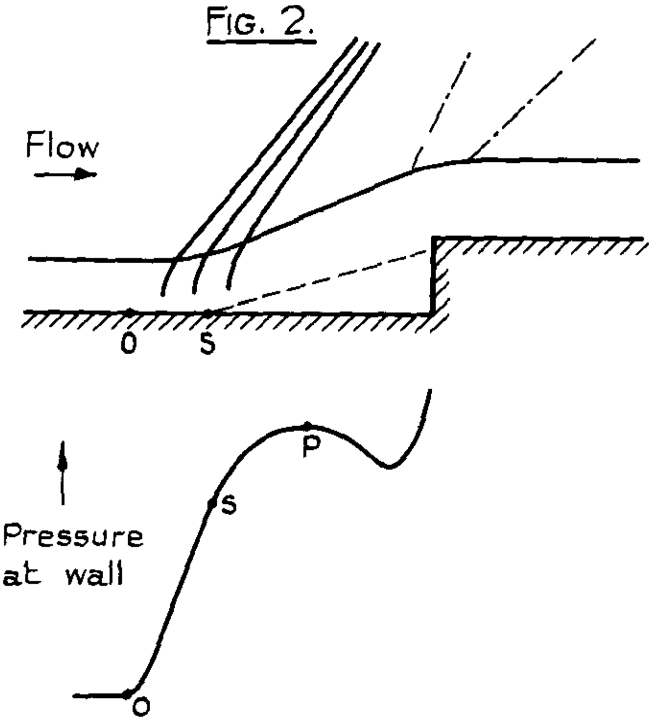
FIGS. 1 – 3

FIG. 1.



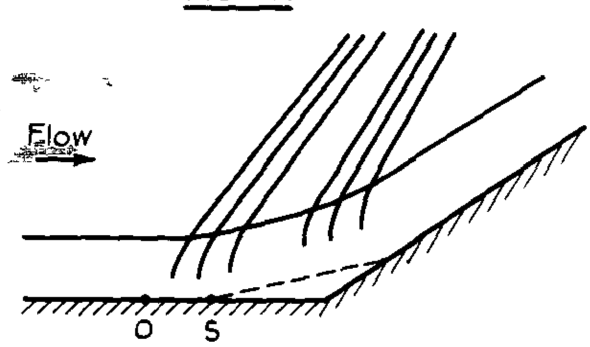
The flow and pressure distribution in the vicinity of separation

FIG. 2.



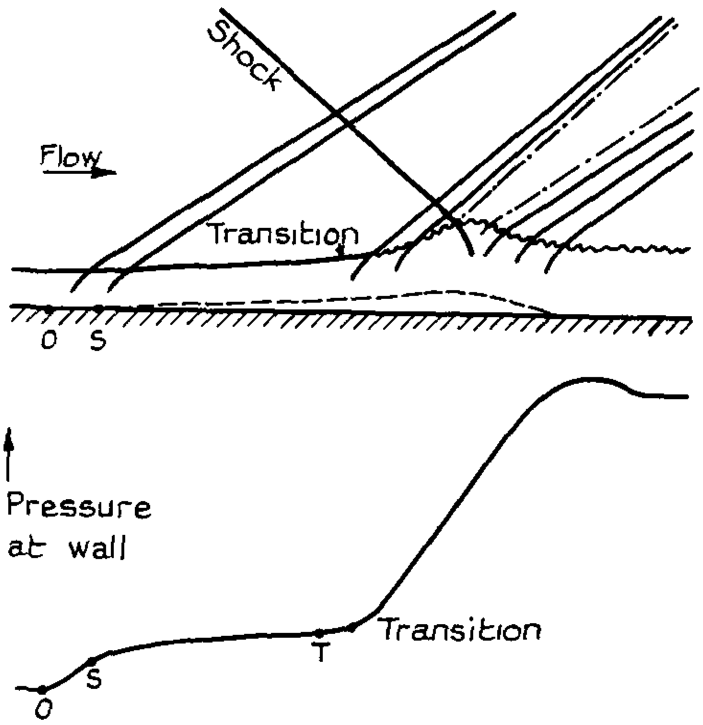
The flow up a step

FIG. 3.



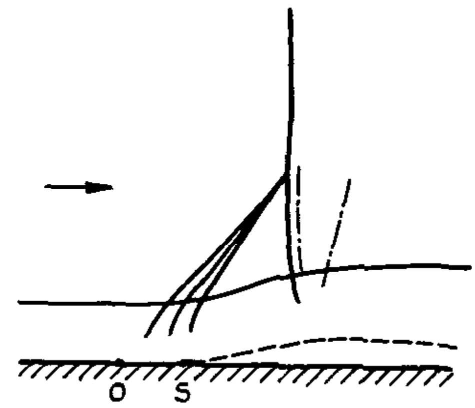
The flow up a wedge

FIG. 4.



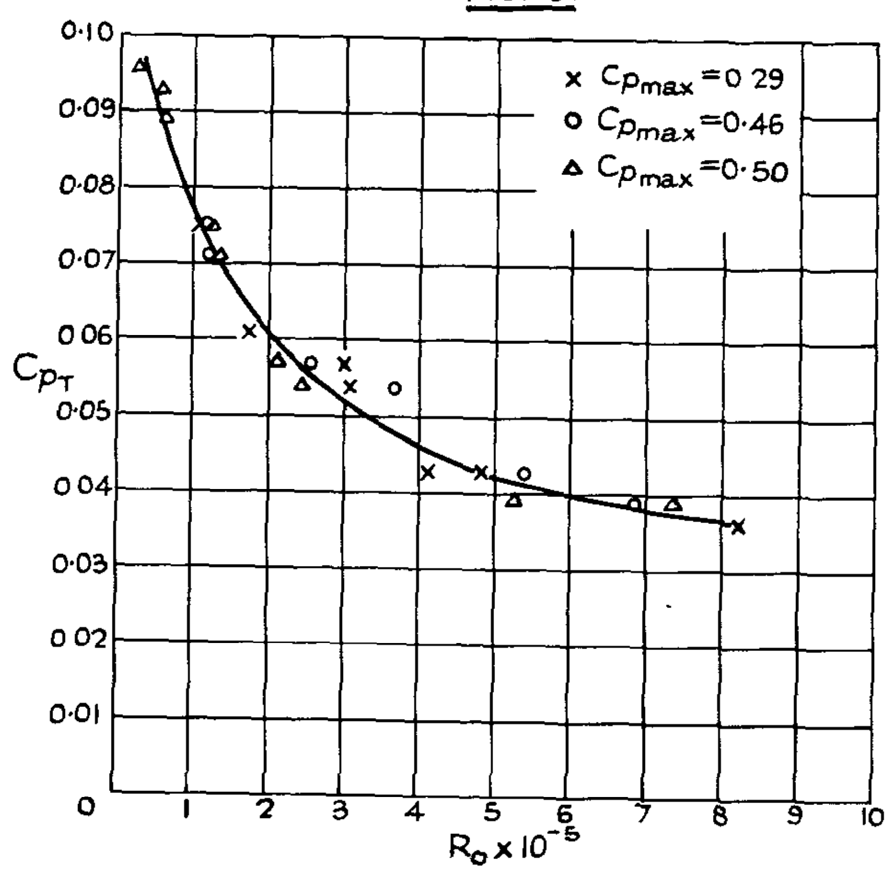
A typical flow pattern and pressure distribution at the wall for an externally-generated oblique shock and an initially laminar boundary layer.

FIG. 5.

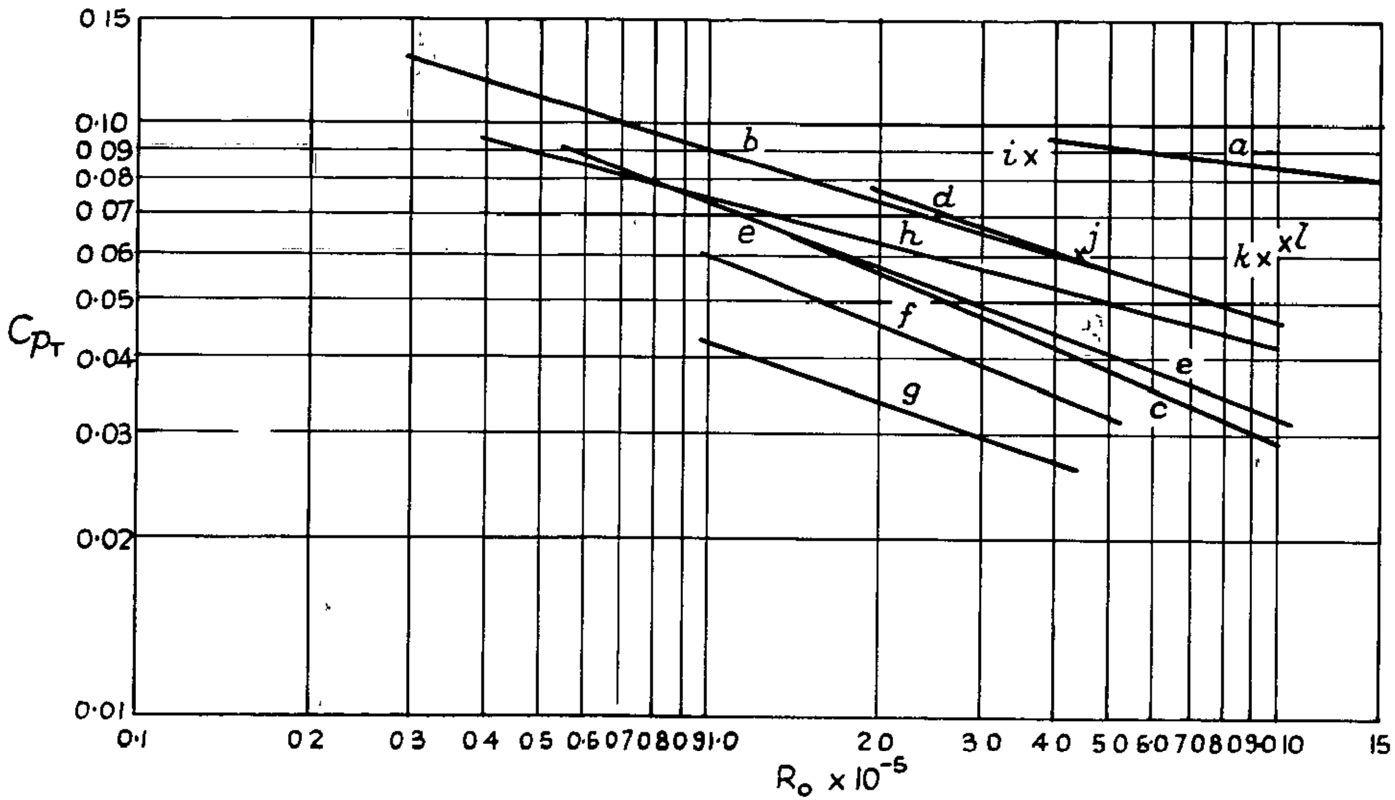


Flow separation caused by a normal shock

FIG. 6.



M = 2 Pressure coefficient C_{p_T} at top of laminar foot for various shock strengths

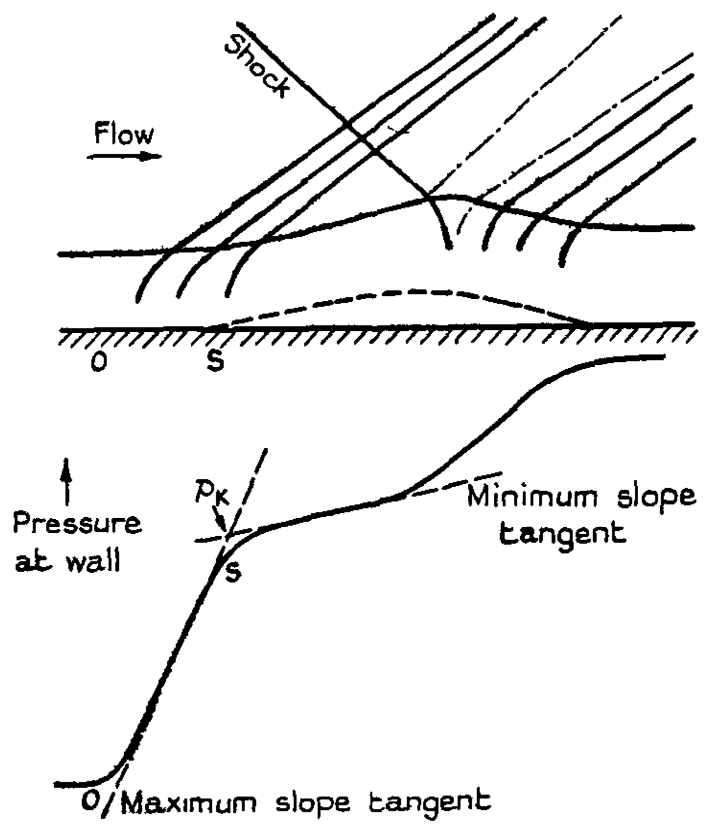


a	Lee[5]	M = 1·56
b	Lee	M = 2·48
c	Lee	M = 3·0
d	Gadd et al.[4]	M = 1·5
e	Gadd et al	M = 2·0
f	Gadd et al	M = 3·0
g	Gadd et al	M = 4·0
h	Barry et al[6]	M = 2·05
i	Ackeret et al.[7]	M = 1·14
j	Ackeret et al	M = 1·22
k	Liepmann et al[8]	M = 1·44
l	Liepmann et al	M = 1·40

FIG. 7.

C_{p_T} for laminar separations as determined by various workers at various Mach numbers.

FIG. 8



A typical flow pattern and pressure distribution at the wall for a strong externally generated oblique shock and a turbulent boundary layer.

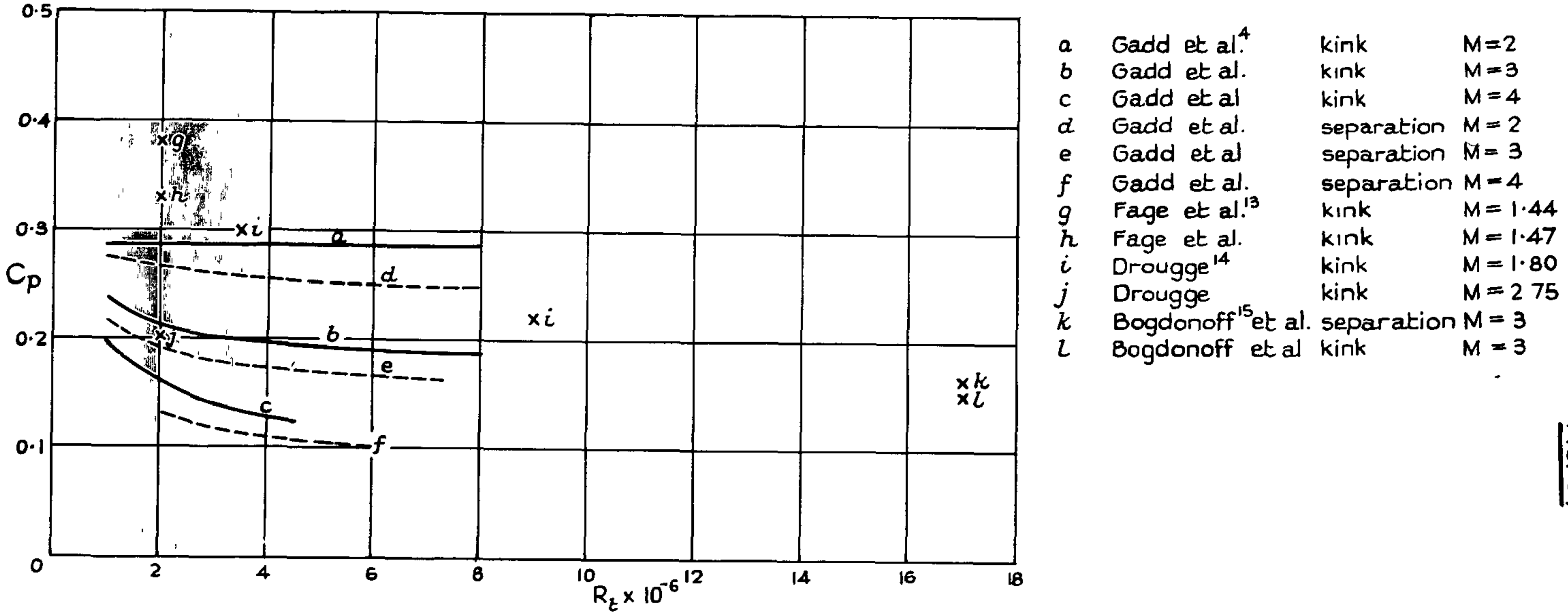
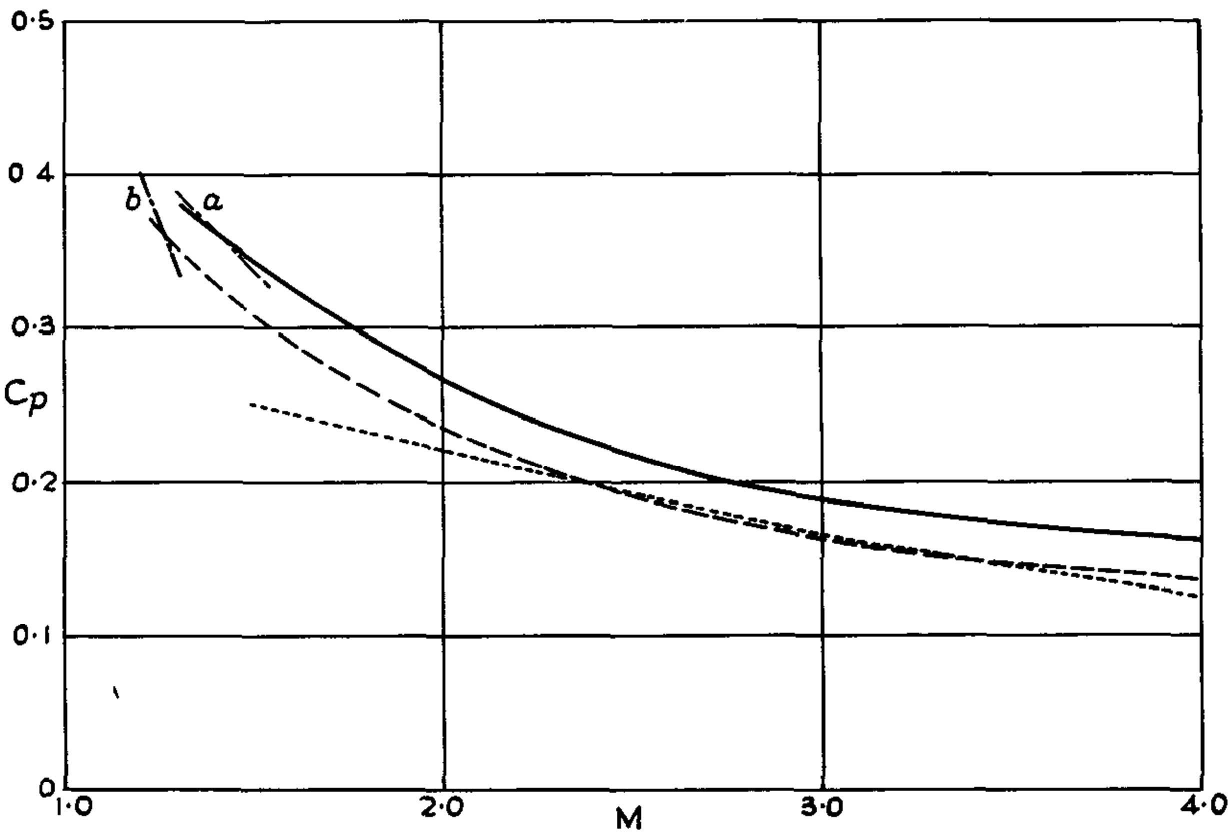


FIG. 9.

Pressure coefficients at kink and separation as functions of Reynolds number for turbulent boundary layers

FIG. 10.



——————— Curve for kink pressure at "average" Reynolds No., from Fig. 9.

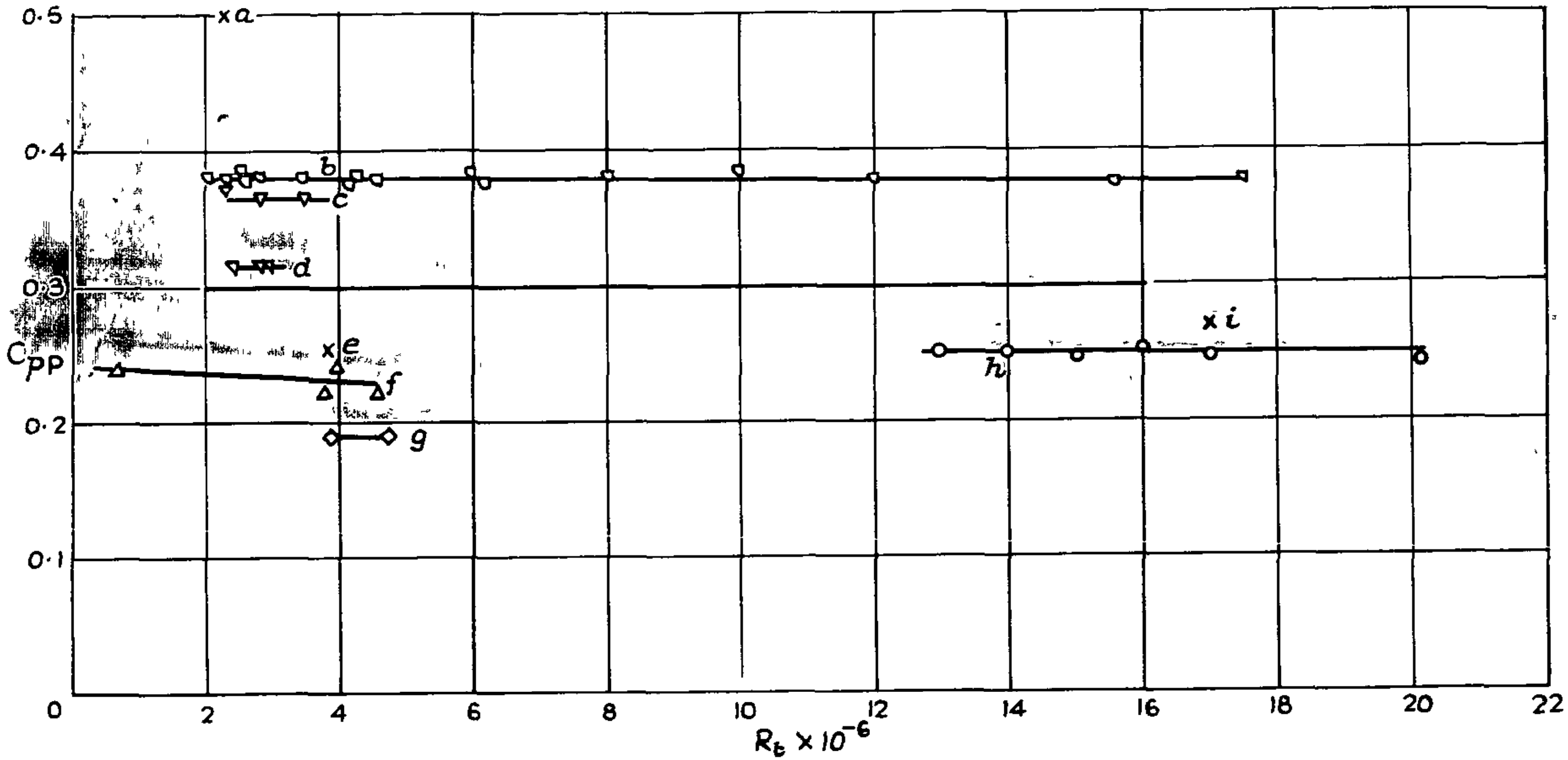
– – – – – Curve for separation pressure at "average" Reynolds No., from Fig 9.

—·—·—·— Curves from N.P.L. aerofoil tests, uncertain Reynolds No., "artificial" transition (a) kink, (b) separation

------------ Theoretical curve for separation deduced as in Ref. 10 but with empirical constant taken as 0·54 instead of 0·60

Pressure coefficients at kink and separation as functions of Mach numbers for turbulent boundary layers.

E.E.M.



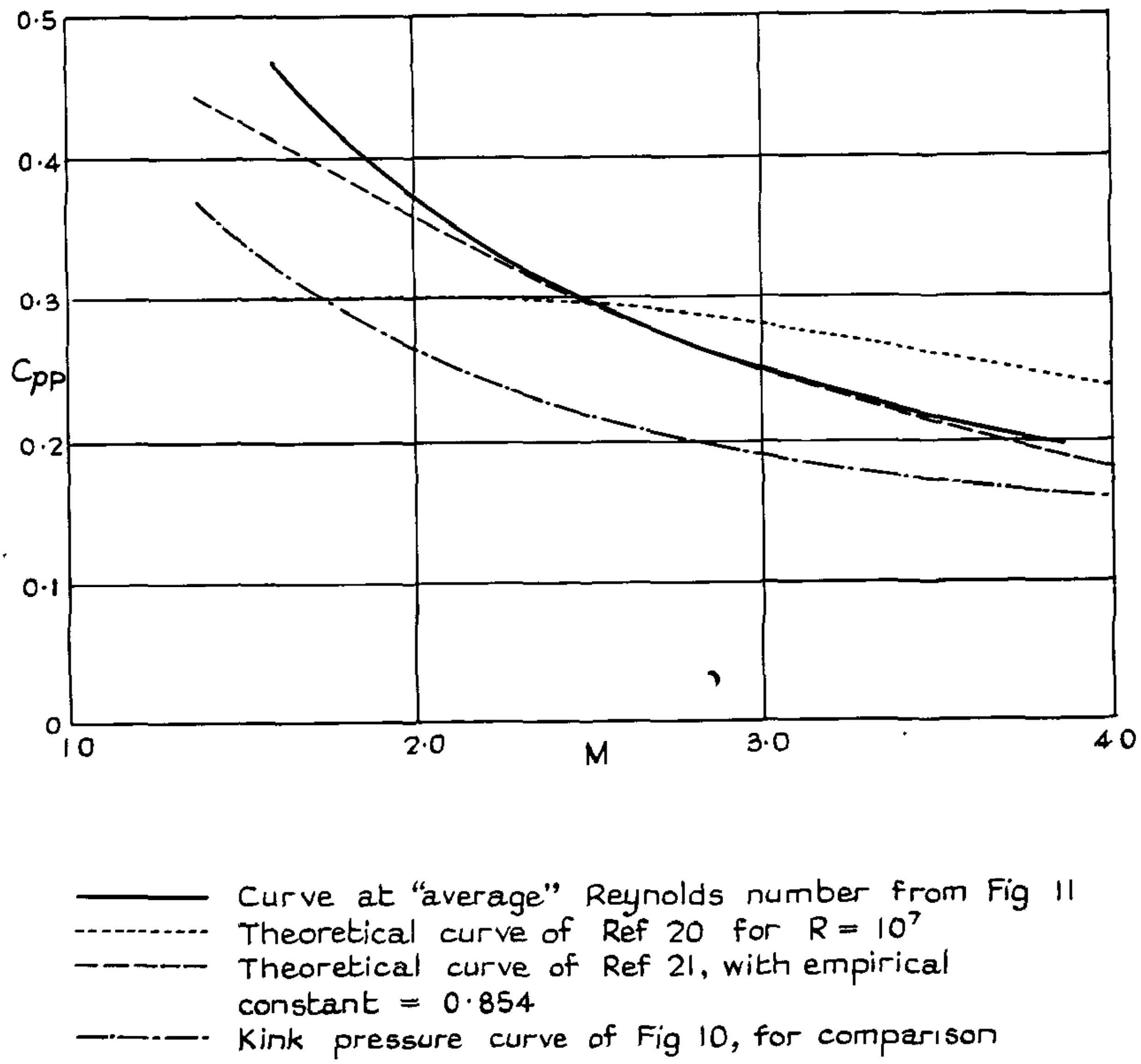
	a	Moeckel[17]	M = 1·84	From photographs	
▽	b	Data quoted by Lange[18]	M = 1·55 and 1 62		
▽	c	Data quoted by Lange	M = 1 93		
▽	d	Data quoted by Lange	M = 2·41		
	e	Data quoted by Lange	M = 2·45		From pressure distributions
△	f	Data quoted by Lange	M = 2 80		
◇	g	Data quoted by Lange	M = 3·65		
	h	Data quoted by Lange	M = 3·03		
	i	Kepler et al[19]	M = 3		
	j	Donaldson et al[11]	M = 3·03	From photographs	

Pressure coefficients at peak for flow up steps as functions of Reynolds number for turbulent boundary layers.

FIG. 11.

EEM

FIG 12.



Pressure coefficients at peak for flow up steps as function of Mach number for turbulent boundary layers.

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