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JULIUS LUKASIEWICZ and J. K. ROYLE

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Summary.—The available theoretical and experimental information on condensation of water vapour in the supersonic flow of air is reviewed and the influence of condensation on operation of supersonic tunnels is considered.

The mechanism of condensation in supersonic flow is of molecular nature and does not depend on the presence of solid condensation nuclei in the air. As estimated by Oswatitsch and confirmed by experimental results, the condensation in supersonic flow of air is primarily a function of the adiabatic supercooling $\Delta T_{k \text{ to } ad}$ (defined in Fig. 1), which determines the conditions at which the condensation shock occurs. For medium-sized supersonic tunnels (say 1-ft square working section) the adiabatic supercooling is of the order of 50 deg C.

For most test purposes it is essential to eliminate the detrimental effects of condensation on flow distribution in the tunnel working section. The usual method is to use highly dried air, and the question of the required dryness is considered. Other methods, which do not rely on the dryness of air, are discussed. It is shown that by increasing stagnation temperature condensation can be avoided usually only at Mach numbers smaller than 1·5. Alternatively, condensation can be eliminated from the tunnel nozzle by pre-expansion in an auxiliary nozzle, as verified experimentally.

Introduction.—In the early days of experimental supersonic research the effects of air humidity on the flow in supersonic tunnels were largely neglected. At the 1935 High Speed, Volta Congress, Professor Prandtl[11] drew attention to the occurrence of a shock system a short distance downstream of the throat of supersonic nozzles, at that time a still unexplained phenomenon. Professor Wieselsberger commented that according to the observations made at Aachen[4], the position of a shock system of that type was a unique function of the air humidity. This proved in fact to be the correct explanation, the air humidity being the cause of the formation of disturbances known today as condensation shocks. Fig. 2 shows the appearance of a typical condensation shock while supersonic flow in a two-dimensional 1·86 Mach number nozzle is initiated.

With the development in Germany of supersonic research on a large scale during the Second World War, it was realized that in order to obtain experimental results with the required degree of accuracy the effects of air humidity must be eliminated. The obvious and radical method, which was universally adopted, was to use air dried down to a small water content, not exceeding 0·05 per cent of water in air by mass.

The early researches of Prandtl, Wieselsberger and others were followed by detailed investigations due to Oswatitsch[6, 7, 8, 9] who analysed the mechanism of condensation in supersonic flow and determined the factors governing the process. The results of his investigations, which indicate the importance of the time scale in condensation phenomena, are summarized in this paper and compared with the experimental evidence obtained from supersonic tunnel tests conducted in England.

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The two problems which are of major importance in estimating the effects of air humidity in supersonic tunnels, are, firstly, the determination of conditions in which a condensation shock occurs and, secondly, of its overall effect on flow in the working section. These problems are discussed in the first and second sections of the paper. In the third section the possibilities of avoiding condensation in the tunnel nozzle and working section either by using dry air or by other means, such as stagnation temperature control or pre-expansion in an auxiliary nozzle, are considered. The data relating to the humidity of air and saturation of water vapour in isentropic expansion of humid air, required in the calculations of humidity effects in tunnels, has been collected in the Appendix.

Condensation of water vapour in humid air, which is discussed in this paper in relation to the operation of supersonic tunnels, may also occur in subsonic tunnels and in actual flight.

Formation of mist over wings has been frequently observed in flight at high subsonic speeds. It seems, however, that in general the condensation would not produce appreciable effects on the aerodynamic forces in flight. The supersaturation reached in flight is many times smaller than that in a wind tunnel, the stagnation temperature and relative humidity being in flight much higher and smaller respectively as compared with conditions in a wind tunnel. Also, the very low water content of air at high altitudes* would limit the intensity of humidity effects to negligible proportions.

As regards the humidity effects in high-speed subsonic tunnels, it should be noted that a certain amount of work has been done in Great Britain, mainly in connection with the operation of atmospheric air, in the high-speed tunnels at the National Physical Laboratory (N.P.L.).

Two main types of condensation were observed,

(i) condensation upstream of and in the working section at high free-stream Mach number ($M \simeq 0 \cdot 75$ to $0 \cdot 85$), noticeable when a supersaturation of about 4 was reached, and

(ii) condensation near the model originating from local supersonic regions, *e.g.*, near surfaces of aerofoils.

In both cases working section conditions and test measurements are affected. The static pressure (in case (i)) and the true stagnation pressure are decreased when the condensation occurs, and it is not certain whether the pitot-tube reading gives the true stagnation pressure (*cf.* second footnote, section 1.2). Thus an accurate determination of the Mach number is not possible and a pitot-traverse method cannot be used for the determination of drag.

It seems that in view of the above difficulties and the uncertainty of any 'humidity corrections', condensation must be completely avoided in subsonic tunnels.

Notation.—Humidity and Supersaturation of Air.—Let Fig. 1 represent a section of the temperature-entropy diagram for water vapour and let the line *o-s-k* represent an adiabatic expansion of water vapour. Assuming the vapour to be supercooled between s and k and the condensation to occur at k, the ideal adiabatic expansion of the vapour between o and k, in absence of friction and other losses, is represented by an isentrope.

The relative humidity ϕ at the initial state o is defined as

$$\phi_o = P_o / P_{o\,\mathrm{to}\,s} ,$$

and the corresponding dew point temperature is $T_{d\,\mathrm{to}\,o}$.

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* The approximate values of absolute humidity at altitude are as follows.

Altitude	Absolute humidity
20,000 ft	$6 \cdot 5 \times 10^{-4}$
30,000 ft	$1 \cdot 0 \times 10^{-4}$
50,000 ft	$1 \cdot 5 \times 10^{-6}$

The expanding vapour becomes saturated at s ($T_{s\,\text{to}\,ad}$, $P_{s\,\text{to}\,ad}$) and super-saturated when the expansion is continued towards k. The supersaturation at k is defined as

$$\phi_{k\,\text{to}\,ad} = P_{k\,\text{to}\,ad}/P_{k\,\text{to}\,s}\,,$$

and the adiabatic supercooling as

$$\Delta T_{k\,\text{to}\,ad} = T_{s\,\text{to}\,ad} - T_{k\,\text{to}\,ad}\,.$$

Symbols

c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
h	Latent heat of evaporation (sublimation)
M	Mach number
P	Pressure
q	Heat input (positive when heat is added to the flowing gas)
Q	Dimensionless heat input $= \dfrac{q}{c_p T_o}$
T	Absolute temperature
ΔT_{ad}	Adiabatic supercooling (*see* Fig. 1)
w	Velocity of flow
x	Distance along the nozzle axis measured in the direction of flow
γ	Ratio of specific heats $= c_p/c_v = 1\cdot 4$
ρ	Specific density
ϕ	Relative humidity (supersaturation)
ψ	(auxiliary nozzle throat area)/(tunnel nozzle throat area)
Ω	Absolute humidity

Suffixes:

$_o$	Stagnation (initial) conditions
$_o{}^1$	Stagnation conditions downstream of a condensation shock or of an auxiliary nozzle
$_{ad}$	State reached in adiabatic (isentropic) expansion
$_k$	State at which condensation shock occurs
$_s$	Saturation state

Note: Where a double suffix is used both conditions implied by the suffixes o to s defined above apply.

1. *Occurrence of Condensation Shock in Supersonic Tunnels.*—1.1. *The Mechanism of Condensation in Slow and Rapid Processes.*—In slow processes, such as formation of atmospheric fog, the water vapour condenses on solid particles or condensation nuclei present in the air and practically no supersaturation is possible. The nature of the mechanism of condensation is different in rapid processes, in which large supersaturations are obtained.

An adiabatic expansion of humid air or steam, in which the vapour pressure decreases less quickly than the corresponding saturation pressure, leads in general to supersaturation and condensation. The above process is extremely rapid in high-speed or supersonic flow and results in condensation* of water vapour in an extremely small time and space interval, producing, at supersonic speeds, a flow discontinuity or so-called condensation shock.

* Condensation should be understood to include also sublimation, which, at low temperatures reached in supersonic tunnels, is likely to occur.

The fundamental difference between the mechanism of condensation in slow and rapid processes was demonstrated by Stodola[12], who compared rapid expansion of initially saturated steam in a de Laval nozzle with a slow expansion of the same steam and found that, whereas in the second case no supersaturation could be observed, in the nozzle a considerable supercooling occurred before the condensation, which was noticeable only downstream of the throat. This simple experiment proved that the condensation nuclei, present in the industrial steam used, which were mainly responsible for condensation in the slow expansion ($\simeq 0 \cdot 05$ sec. duration), did not influence the rapid expansion ($\simeq 4 \times 10^{-4}$ sec duration) in a supersonic nozzle.

This important result has since been confirmed by Oswatitsch[6, 7], who developed the theory of condensation in supersonic flow. Oswatitsch has shown that in the short time available in a supersonic nozzle, the amount of water which condenses on the available nuclei is negligibly small; the condensation originates in molecular collisions as soon as the vapour becomes supersaturated, and its subsequent development depends on the rate of formation of condensation nuclei (consisting of a small number of water molecules; such condensation nuclei will be called 'molecular condensation germs') and on their rate of growth. In this process water droplets or ice crystals are formed.

An expression for the rate of formation of molecular germs which persist has been given by Becker and Döring[14], and was used by Oswatitsch in his calculations of condensation in supersonic flow of steam and humid air.

According to these calculations, the rate of formation of molecular germs depends primarily on the adiabatic supercooling ΔT_{ad}, Fig. 1, and is an exponential function of ΔT_{ad}. Typical results obtained by Oswatitsch for adiabatic expansion of air from atmospheric conditions of normal temperature and humidity are given in Table 1; they show that, in the conditions assumed, practically no condensation is possible for adiabatic supercoolings smaller than 30 deg C. With 30 deg supercooling only about one condensation germ forms per cm^3 per cm, and the air would have to travel a distance of the order of 10 metres before any significant number of molecular germs were formed (10^3 cm^{-3}); with 40 deg supercooling, the same number of germs would be formed in a distance of $0 \cdot 01$ mm, whilst with 50 deg supercooling every $0 \cdot 01$ mm 100 million germs are being formed. This is a considerable number as compared with the 10^5 to 10^6 solid nuclei per cm^3 present in the ordinary air†.

TABLE 1

Effect of Adiabatic Supercooling on the Rate of Formation of Condensation Germs.

ΔT_{ad}, deg C	30	40	50	60	70
Number of molecular germs formed per cm along the direction of flow per cm^3 volume	<1	10^6–10^7	10^{11}	10^{13}	10^{14}–10^{15}

The high rate of formation of molecular condensation germs at large supercoolings, which usually occur in supersonic flow, provides a physical explanation of the appearance of condensation shocks.

The other factor which has to be considered in the analysis of the mechanism of condensation is the growth of molecular condensation germs. Oswatitsch has developed expressions for rate of growth of molecular condensation germs and of bigger droplets. On the basis of his calculations it appears that the effect of the growth of condensation germs on condensation is small as compared with that of the formation of condensation germs.

† According to Stodola[12] and Oswatitsch[6], the number of solid nuclei present in the air varies from 10^5 cm^{-3} in country to 3×10^6 cm^{-3} in towns and houses.

1.2. *Adiabatic Supercooling.*—In order to estimate the conditions in which condensation takes place in supersonic flow it is necessary to verify that, in fact, the adiabatic supercooling ΔT_{ad} is the governing factor and to determine the value of ΔT_{ad} applicable to flow in supersonic tunnels.

Two sets of experimental results, relating to condensation in small nozzles and in full-scale supersonic tunnels, are available

The first experiments in which humid air was used were described by Oswatitsch[6,7,8].

The humid air was expanded in a small two-dimensional supersonic nozzle (throat width of the order of 3 cm), and the position of the condensation shock was estimated from the static-pressure measurements along the nozzle walls. Three tests, with air of different humidities, were made and the same nozzle was used throughout, scale effects being thus eliminated.

The main results are summarized in Table 2, the suffix k denoting the state immediately upstream of the condensation shock.

TABLE 2

$\Delta T_{k\ \mathrm{to}\ ad}$ *in Flow of Humid Air in Small Nozzles*[7]

Test No.	ϕ_o	T_o, deg C	$T_{s\ \mathrm{to}\ ad}$, deg C	$\left(\frac{dT}{dx}\right)_{k\ \mathrm{to}\ ad}$, deg C/cm	$T_{k\ \mathrm{to}\ ad}$, deg C	$\Delta T_{k\ \mathrm{to}\ ad}$, deg C	$M_{k\ \mathrm{to}\ ad}$
1	0·90	15·0	13·0	15·6	−41	54·0	1·08
2	0·75	14·0	8·7	15·5	−48	56·7	1·17
3	0·40	19·7	2·7	16·4	−61	63·7	1·36

Although the initial conditions (ϕ_o) varied over a wide range, the supercooling $\Delta T_{k\ \mathrm{to}\ ad}$ changed only through 10 deg C and remained practically constant in tests 1 and 2. The corresponding supersaturation varied from about 80 to 220.

The results of Oswatitsch's tests with humid air have been compared[8] with the values of adiabatic supercooling $\Delta T_{k\ \mathrm{to}\ ad}$ obtained from pressure distribution along the nozzle walls, as observed by Yellot[13] and Binnie and Woods[1] in tests with steam. From Fig. 3, in which $\Delta T_{k\ \mathrm{to}\ ad}$ has been plotted in terms of the stagnation relative humidity ϕ_o, it is evident that with small nozzle sizes an adiabatic supercooling of about 55 deg C is obtained before a condensation shock occurs.

The results of small-scale experiments are compared in Fig. 3 with the values of adiabatic supercooling estimated from tests made in the 5·5-in. square supersonic tunnel at the Royal Aircraft Establishment (R.A.E.) and 11 × 11-in. N.P.L. supersonic tunnel.

In the R.A.E. tests the adiabatic supercooling $\Delta T_{k\ \mathrm{to}\ ad}$ has been estimated from schlieren observations of condensation shocks (Figs. 9 and 10) and from measurements of the axial static-pressure distribution in nozzles for 1·56 (Fig. 11) and 2·48 Mach number. The results (Table 3) indicate an adiabatic supercooling of about 52 deg C for the low Mach number and 60 deg C for the higher Mach number nozzle.

TABLE 3

$\Delta T_{k\ \mathrm{to}\ ad}$ *in Flow of Atmospheric Air in* $M = 1 \cdot 56$ *and* 2·48 *Nozzles*
(5·5 × 5·5-*in. R.A.E. Supersonic Tunnel*)

Nozzle Mach Number	ϕ_o	T_o, deg C	Ω_o	$T_{s\ \mathrm{to}\ ad}$ deg C	$\left(\frac{dT}{dx}\right)_{k\ \mathrm{to}\ ad}$ deg C/cm	$T_{k\ \mathrm{to}\ ad}$ deg C	$\Delta T_{k\ \mathrm{to}\ ad}$ deg C	$M_{k\ \mathrm{to}\ ad}$
1·56	0·66	19	0·0092	11	7	−40	51	1·126
1·56	0·58	21	0·0092	10·5	7	−44	54·5	1·196
2·48	0·57	17·6	0·007	7	14·5	−55	62	1·29

The results so far presented are based on static-pressure measurements, which in the regions of large pressure gradients present near the nozzle throat and in the vicinity of condensation shock, are subject to appreciable errors due to boundary-layer effects. Such errors and others, due to the two-dimensional character of the flow, were avoided in some measurements made in the N.P.L. 11-in. Closed-circuit Tunnel[2]. These measurements were made in about 1942, soon after the tunnel was built, to find if it were possible to develop quickly an operating technique that would make humidity effects negligible or, at worst, constant.

In the measurements, the results of which are shown in Fig. 4 and analysed in Table 4, the stagnation temperature in the nozzle, T_o, was altered while stagnation pressure P_o was kept constant; the variation with the temperature T_o of the pitot and static pressure, measured at one point of the working section, was observed and the corresponding behaviour of the Mach number computed. A nozzle giving a nominal Mach number of $1 \cdot 4$ was used* and tests were made at two different stagnation pressures P_o.

In the first series (see Fig. 4) with $P_o = 230 \cdot 6$ mm of mercury, the pitot and static pressures in the working section were observed and Mach number computed; also, the base pressure of a streamlined projectile has been measured.

In the second series, a stagnation pressure $P_o = 433 \cdot 8$ mm of mercury was maintained and only the static pressure in the working section was observed.

The tunnel air was wholly derived from the air in the room, and was, therefore, assumed to have the same constant absolute humidity (water content) Ω of $0 \cdot 005$. The stagnation relative humidities ϕ_o corresponding to various stagnation temperatures T_o, for the above Ω and the two values of P_o used, are marked in Fig. 4.

In all tests, for stagnation temperatures exceeding a certain minimum value, no changes in the static pressure and Mach number were recorded and the mist disappeared from the tunnel. The pitot pressure remained substantially constant†.

The conditions at the minimum stagnation temperature in which the effects of condensation disappear are analysed in Table 4. These conditions correspond to the state $(\ )_{k \text{ to } ad}$ immediately upstream of the condensation shock; for lower T_o (*i.e.*, higher ϕ_o) the condensation occurs inside the nozzle and the flow in the working section is affected.

Taking the variation of the static pressure as the criterion, Table 4 shows that in all cases‡ the condensation occurs when an adiabatic supercooling $\Delta T_{k \text{ to } ad} \simeq 48$ deg C is reached. In the test (iii) a slightly smaller $\Delta T_{k \text{ to } ad}$ is obtained if based on the disappearance of mist.§

It is interesting to note that a good agreement exists between the disappearance of condensation in the tunnel nozzle and in the local expansion near a model. It follows that the maximum humidity at which no condensation effects appear must be based, for a given $\Delta T_{k \text{ to } ad}$, on the maximum local expansion and not merely on the free-stream tunnel Mach number.

On the basis of the above R.A.E. and N.P.L. tests a value of $\Delta T_{k \text{ to } ad} \simeq 50$ deg C can be considered as applicable to medium-sized supersonic tunnels (say up to 1 ft square working section).

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* Similar results were observed with a nozzle for $M = 1 \cdot 5$.

† The value of the pitot pressure is influenced by: (i) the loss of the stagnation pressure due to condensation upstream, which is counteracted by (ii) the decrease of Mach number due to condensation upstream and hence of the normal-shock losses at the pitot-tube entry and (iii) the increase of stagnation pressure due to evaporation of condensed water in the subsonic compression in the pitot-tube. The combined effect of these factors may be such as to render the pitot pressure relatively insensitive to condensation in supersonic flow.

‡ In the case of the streamlined projectile, $\Delta T_{k \text{ to } ad}$ and other values were determined on the assumption of an isentropic expansion from the stagnation pressure to the observed base pressure.

§ The visibility of fog depends on number and size of drops, thickness of the fog layer and illumination, so that in general no conclusions as to the occurrence of condensation should be based on visual observations of fog. In supersonic tunnels mist can usually be seen only at lower Mach numbers.

1.3. *Secondary Effects.—Saturation Temperature and Temperature Gradient.*—The variations in the value of $T_{k\ \mathrm{to}\ ad}$, such as observed in the small-scale nozzle tests and in the R.A.E. tests of 1·56 and 2·48 Mach number nozzles, were ascribed by Oswatitsch to the influence of saturation temperature $T_{s\ \mathrm{to}\ ad}$ and temperature gradient in the neighbourhood of the condensation shock, $(dT/dx)_{k\ \mathrm{to}\ ad}$. On the basis of experiments with humid air, Oswatitsch[6] arrived at the functional relationship between $T_{s\ \mathrm{to}\ ad}$, $\Delta T_{k\ \mathrm{to}\ ad}$ and $(dT/dx)_k$ which is represented in Fig. 5; larger supercoolings are possible with lower values of $T_{s\ \mathrm{to}\ ad}$ (*i.e.*, for a given T_o, lower initial humidities) and larger gradients $(dT/dx)_k$. The results of R.A.E. tests are in qualitative agreement with these estimates, but the N.P.L. experimental points, which correspond to vanishingly small temperature gradients, cannot be correlated to curves of Fig. 5.

For a given value of $T_{s\ \mathrm{to}\ ad}$ or with constant stagnation conditions, the effect of $(dT/dx)_k$ is equivalent to the scale effect; with smaller nozzles, higher values of $\Delta T_{k\ \mathrm{to}\ ad}$ are obtained, the condensation shock occurring at a relatively larger distance from the throat. This behaviour of condensation was observed in Aachen investigations[4], in which geometrically similar nozzles of different sizes were used.

2. *Effects of Condensation on Flow in the Working Section.*—The conditions in which a condensation shock occurs can be estimated with a sufficient degree of accuracy; the next problem to be considered is the influence of condensation on flow in the tunnel working section.

TABLE 4

Conditions in which Condensation First Disappears

(Computed from tests of the 11×11-in. N.P.L. Supersonic Tunnel, Fig. 4)

Test	Absolute humidity Ω_o	Initial stagnation pressure P_o, mm mercury	Initial stagnation temperature T_o, deg C (lowest initial stagnation temperature at which humidity effects disappear)	Initial stagnation relative humidity ϕ_o	$T_{s\ \mathrm{to}\ ad}$ deg C $M_{s\ \mathrm{to}\ ad}$	$\Delta T_{k\ \mathrm{to}\ ad}$ deg C	$M_{k\ \mathrm{to}\ ad}$	Static pressure $P_{k\ \mathrm{to}\ ad}$ mm mercury, in. mercury	Super-saturation $\phi_{k\ \mathrm{to}\ ad}$	$T_{k\ \mathrm{to}\ ad}$ deg C
						In the working section				
(i)	0·005	230·6	20	0·105	−15·5 0·83	49	1·42	71·1 2·8	129	−64·5
						At the base of streamlined projectile				
(ii)	0·005	230·6	30	0·058	−17 0·96	48	1·52	61 2·4	121·75	−65
						In the working section				
(iii)	0·005	433·8	32 Static pressure unaffected but mist still noticeable	0·097	−9 0·88	47	1·42	132·1 5·2	75·5	−56
						In the working section				
(iv)	0·005	433·8	34 No mist observed	0·087	−9 0·903	45·5	1·42	132·1 5·2	62·8	−54·5

2.1. *Addition of Heat in Compressible Flow.*—By applying the theoretical laws of the rate of molecular germ formation and their growth, together with suitably modified one-dimensional equations of flow, and by making a number of assumptions, Oswatitsch[7] was able to obtain theoretical pressure distributions for flow of steam in supersonic nozzles which agreed well with the observed position and intensity of the condensation shock. This method involves laborious step-by-step calculations and, moreover, in the case of flow of humid air, the validity of its results is uncertain due to theoretical difficulties connected with the condensation or sublimation at low temperatures. Alternatively, one can estimate the condensation effects by looking at the phenomenon from a purely thermodynamic angle, which will be here considered.

The overall effect of condensation in compressible flow may be regarded as equivalent to the addition of the latent heat. For flow in a duct of constant cross-section the effects of heat addition can be easily investigated provided the friction is neglected between the cross-sections of the duct considered between which the heat is added, and provided the flow is assumed to be truly one-dimensional at these sections. If the quantity of the heat added is accurately known, the above simple treatment gives an accurate estimate of the phenomenon. This simple treatment ignores the changes in specific heat, γ, and gaseous mass flow, the effects of which are all negligible.

In the case of ducts of variable cross-section, *e.g.*, supersonic nozzles, the results obtained for constant cross-section ducts would apply if the heat were added instantaneously (at a constant cross-section), thus producing a discontinuity of flow, and if the flow on the two sides of the discontinuity remained one-dimensional. One would then speak of one-dimensional diabatic shocks, analogous to normal adiabatic shocks.

As regards condensation effects in supersonic flow, which approximate to flow discontinuities, the latter interpretation of the theory is useful, although the condensation shocks as observed in supersonic nozzles are not, in general, one-dimensional.

The effects of heat addition in compressible flow have been investigated in a number of papers[3, 4, 5, 9]. The theoretical treatment is identical with that of an adiabatic normal shock, except for an additional term in the energy equation. The following dimensionless solutions in a convenient form for the investigation of condensation effects have been obtained, in terms of Mach number M_1 (before the addition of heat) and $Q = q/(c_p T_{o1}) = (T_{o2}/T_{o1}) - 1$, the ratio of the heat input q to the initial stagnation enthalpy.

$$\frac{P_2}{P_1} = \frac{\gamma}{\gamma+1}(M_1^2 - 1)[1 \pm \sqrt{(1-Z)}] + 1 , \quad \ldots \quad (1)$$

$$\frac{\rho_1}{\rho_2} = \frac{w_2}{w_1} = 1 - \frac{1}{\gamma+1}\frac{M_1^2 - 1}{M_1^2}[1 \pm \sqrt{(1-Z)}] , \quad \ldots \quad (2)$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = \frac{P_2}{P_1}\frac{w_2}{w_1} , \quad \ldots \quad (3)$$

$$\frac{M_2}{M_1} = \left\{\frac{1 - \dfrac{1}{\gamma+1}\dfrac{(M_1^2-1)}{M_1^2}[1 \pm \sqrt{(1-Z)}]}{1 + \dfrac{\gamma}{\gamma+1}(M_1^2-1)[1 \pm \sqrt{(1-Z)}]}\right\}^{\frac{1}{2}} , \quad \ldots \quad (4)$$

where

$$Z = \frac{M_1^2}{(M_1^2-1)^2}\left(M_1^2 + \frac{2}{\gamma-1}\right)(\gamma^2 - 1)\,Q , \quad \ldots \quad (5)$$

and suffixes 1 and 2 denote respectively the conditions before and after the heat addition.

The above solutions can be interpreted[3] with the aid of Fig. 6 in terms of M_1 and M_2 for the whole range of Q values.

The $M_1 - M_2$ plane is divided by the $Q = 0$ lines (one of which represents the identity $M_1 \equiv M_2$, the other solutions for adiabatic shock) into four regions which meet at $M_1 \equiv M_2 = 1 \cdot 0$. Each pair corresponds to the endothermic ($Q < 0$) and exothermic ($Q > 0$) processes.

The Q = constant lines, for equal absolute values of Q (drawn in Fig. 6 for $Q = \pm 0 \cdot 17$) are symmetrical about the $M_1 \equiv M_2$ line. This is due to the fact that a change in the sign of Q (or q) is equivalent to interchanging M_1 with M_2.

In the shaded area ($0 < M_1 < 1 \cdot 0$, $M_2 > 1 \cdot 0$) no solutions physically possible exist, since a decrease in entropy would be involved. Thus when $M_1 < 1 \cdot 0$ only one physically real solution exists depending on the value of Q, with $M_2 < 1 \cdot 0$ always. For $M_1 > 1 \cdot 0$ two solutions are possible for endothermic processes ($Q < 0$); for exothermic processes ($Q > 0$) there may be two, one or none, according to the values of Q and M_1.

Confining our attention to the region into which the condensation effects fall, *i.e.*, $Q > 0$, we have $Z > 0$, and in order to obtain algebraically real solutions there must be $Z \leqslant 1 \cdot 0$ (*cf.* equation (1)).

This gives, for any Mach number M_1, the maximum amount of heat Q which can be added as

$$Q_{\max} = \frac{(M_1^2 - 1)^2}{M_1^2 \left(M_1^2 + \frac{2}{\gamma - 1}\right)(\gamma^2 - 1)}, \quad \ldots \quad (6)$$

and in the limit, for $M_1 \to \infty$,

$$Q_{\max} = \frac{1}{\gamma^2 - 1} = 1 \cdot 041 \text{ for } \gamma = 1 \cdot 4.$$

Function (6) is tabulated for $\gamma = 1 \cdot 4$ in Table 5 and drawn in Fig. 7*.

TABLE 5

$Q_{\max}$: M_1 *Function* ($\gamma = 1 \cdot 4$)

M_1	$Q_{\max}$	M_1	$Q_{\max}$	M_1	$Q_{\max}$	M_1	$Q_{\max}$
0·0	∞	0·7	0·101	1·4	0·070	2·5	0·408
0·1	20·376	0·8	0·037	1·5	0·100	3·0	0·529
0·2	5·064	0·9	0·008	1·6	0·131	3·5	0·624
0·3	1·883	1·0	0·000	1·7	0·163	4·0	0·698
0·4	0·890	1·1	0·006	1·8	0·196	4·5	0·755
0·5	0·446	1·2	0·022	1·9	0·228	5·0	0·800
0·6	0·221	1·3	0·044	2·0	0·260	∞	1·041

Putting the value of $Q_{\max}$ in the equations (1) to (5), one obtains only one solution which corresponds, as is evident from (4), to sonic velocity after the heat addition ($M_2 = 1 \cdot 0$). For $Q < Q_{\max}$ there are two solutions, one subsonic and the other supersonic, depending on whether one takes the positive or the negative sign respectively before the root, and related by the usual functions of an adiabatic shock. With $Q = 0$ one obtains, in the supersonic region, the usual solutions for adiabatic shock.

In the subsonic case ($M_1 < 1 \cdot 0$) the addition of heat causes the Mach number to increase (to the sonic value in the limit), with a corresponding decrease in the static and stagnation pressure.

* Table 5 and Fig. 7 are also useful to obtain the value of Z. By (5) we have

$$Z = Q/Q_{\max},$$

where $Q_{\max}$ is taken for the appropriate M_1.

In the supersonic region ($M_1 > 1 \cdot 0, Q > 0$), which we shall here consider in detail, the variation of the static pressure ratio P_2/P_1 is shown, as a function of Q and M_1 ($\gamma = 1 \cdot 4$) in Fig. 8. It is seen that, for a given M_1, P_2/P_1 attains its maximum value for adiabatic shock ($Q = 0$); when $M_2 < 1 \cdot 0$, P_2/P_1 increases with decreasing Q, but when $M_2 > 1 \cdot 0$, P_2/P_1 increases with increasing Q. For a given value of Q = constant, the pressure ratio attains a minimum with $M_2 > 1 \cdot 0$.

In all cases in the range considered the heat addition is associated with a stagnation pressure loss which can be calculated, using (1) and (4), from the expression

$$\frac{P_{o2}}{P_{o1}} = \frac{P_2}{P_1}\left[\frac{2 + (\gamma - 1) M_2^2}{2 + (\gamma - 1) M_1^2}\right]^{\gamma/(\gamma-1)}, \qquad (7)$$

where P_{o1} and P_{o2} are the stagnation pressures before and after the heat addition.

Expression (7) applies to compressible flows in general.

The solutions obtained above show that two kinds of one-dimensional discontinuities are possible, leading to supersonic or subsonic velocities. Provided the pressure ratio is sufficient to ensure supersonic flow in the tunnel working section, the supersonic solution will be obtained and this solution corresponds to the condensation shock. The subsonic solution is obtained by a pitot tube downstream of a condensation shock (if evaporation is neglected) or may occur when flow is not truly one-dimensional.

2.2. *Changes of State in the Working Section due to Condensation.*—Using equations (1) to (4) one can estimate changes in the state of air due to a condensation shock assumed to form at a given $\Delta T_{k\,\mathrm{to}\,ad}$ and its maximum intensity (corresponding to $M_2 = 1 \cdot 0$). The one-dimensional calculations can only indicate the order of condensation effects and mean deviations in the working section from isentropic (dry) flow. In practice usually two-dimensional condensation shocks form and have appreciable effects on distribution of flow in the working section.

The N.P.L. tests show large effects of condensation on conditions in the working section: in test (i), Fig. 4, a change of 1 deg C in the stagnation temperature causes average changes of 1·7 per cent in the Mach number and 3·4 per cent in the static pressure. An attempt was made to estimate such deviations using the one-dimensional theory. The procedure was first to determine, by taking a suitable value of $\Delta T_{k\,\mathrm{to}\,ad}$, the state at which the condensation shock occurs, and then to calculate, using expression (1) to (5), the changes across the condensation shock. These depend on the amount of water vapour assumed to condense, which cannot, however, exceed the value corresponding to $Q_{\max}$, equation (6)*. Good agreement was obtained if the amount of water vapour condensed in the shock was so chosen (by trial and error calculation) that the remaining water vapour was saturated after the shock. After the condensation shock the flow was again assumed to be isentropic and the state in the working section was obtained from the known expansion of the nozzle (on the basis of test (i), no-condensation conditions, the nozzle was assumed to have an area expansion of 1·126 corresponding to $M = 1 \cdot 42$).

Calculations of this type were made for three test points. In all cases an adiabatic supercooling $\Delta T_{k\,\mathrm{to}\,ad}$ of 48 deg C was taken (*cf.* Table 4) and the state of the air in the working section has been calculated, assuming

(a) all present water vapour to condense, and

(b) the water vapour to be saturated after condensation.

For the case T_o = 12 deg C in test (iii), it has been found that if all the water vapour condensed the theoretical value $Q_{\max}$ would be exceeded, so that no result for the assumption (a) could be obtained. Instead, when the theoretical $Q_{\max}$ value was taken, it was found to correspond exactly to the assumption (b).

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* In computing the value of Q, 690 C.H.U./lb was taken for the latent heat h and 0·24 C.H.U./lb deg C for c_p of air. Values of h at low temperatures are given in the Appendix, Table 7.

and corresponding values of $M_{k\,to\,ad}$ (for $\Delta T_{k\,to\,ad} = 50$ deg C) and $M_{s\,to\,ad}$ are shown as functions of T_o. Although the relative humidity is higher in winter, the lower atmospheric temperature results in a lower relative humidity after heating to a given T_o, so that slightly higher $M_{k\,to\,ad}$ are obtained for the winter conditions.

In the above cases the air was assumed to be heated in such a way that no additional moisture was introduced (*e.g.*, electrically or in heat exchangers); slightly lower Mach number values would have been obtained were the air heated by direct fuel combustion.

From Fig. 13 it is evident that the scope of the stagnation temperature* control of humidity effects is limited to low Mach numbers. With a moderate increase in the stagnation temperature, say up to 50 deg C, the humidity effects could be eliminated for Mach numbers not exceeding 1·5. The use of higher stagnation temperature, of the order of 100 to 200 deg C, would increase the Mach number up to 2·5, but presents in itself a serious engineering problem, so that the use of driers would be preferred.

3.3. *Condensation in an Auxiliary Nozzle.*—Instead of controlling the initial state of air (by drying or heating) so that no significant condensation can occur in the tunnel nozzle, it should be possible to obtain the same result by expanding the humid air to a slightly supersonic velocity in a first auxiliary nozzle, the flow then reverting to subsonic through a system of shocks and expanding adiabatically in the main tunnel nozzle. In order to obtain the required expansion, the throat of the first nozzle must be adjustable, the exact nozzle shape being, however, immaterial. With atmospheric air, the first expansion need not exceed $M = 1 \cdot 5$.

The success of this method, originally suggested by R. S. Rae, depends mainly on

(i) whether the occurrence of a second condensation shock in the main tunnel nozzle is possible;

(ii) the magnitude of the losses due to expansion in the auxiliary nozzle;

(iii) the extent to which the subsonic flow distribution before the main nozzle affects the flow distribution in the working section;

(iv) effects of interference of water droplets or ice crystals with model;

(v) whether there is appreciable re-evaporation in the subsonic portion of the auxiliary nozzle.

Preliminary tests which were carried out at the R.A.E. throw some light on factors (i), (ii) and (iv).

A 4 × 4-in. open-circuit supersonic tunnel (Fig. 14), incorporating between the atmospheric inlet and the 1·65 Mach number nozzle an auxiliary variable-area condensation nozzle and a honeycomb, was used. The flow in the auxiliary throat could not be observed visually, but static-pressure distribution was measured on the tunnel wall opposite the central variable-thickness aerofoil-shaped section, and losses were estimated from pitot-static pressure readings before the main nozzle. Flow in the main nozzle was observed by schlieren apparatus.

The tunnel dimensions were such that, with the 1·65 Mach number nozzle, the minimum Mach number in the long entry duct (about 4 ft) equalled about 0·36, corresponding to a maximum duration of flow of $\simeq 0 \cdot 01$ sec before the main nozzle.

The tests were carried out over a range of atmospheric conditions ($7 \leqslant T_o \leqslant 17$ deg C, $0 \cdot 59 \leqslant \phi_o \leqslant 0 \cdot 93$) and observations were made while the auxiliary throat area was varied. Two distinct types of phenomena were observed.

At stagnation relative humidities $\phi_0 > 0 \cdot 85$ a condensation shock did not form in the nozzle even with first throat fully open, but mist was visible in and upstream of the nozzle and severe icing of the nozzle liners and pitot tubes in the working section occured (with $T_0 \simeq 7$ deg C).

* The effect of the stagnation temperature on Reynolds number is not serious. With increasing stagnation temperature, for constant stagnation pressure and for a given working-section Mach number, the Reynolds number decreases and is proportional to $T_o^{-1 \cdot 27}$ (on the assumption of viscosity $\propto T^{0 \cdot 77}$). For the extreme case of an increase in T_o from 0 deg C to 200 deg C, the Reynolds number would be halved.

2.5. *Mechanical Interference Effects of Condensation.*—Water droplets or ice crystals formed during condensation may interfere with models and cause additional impulse, changes in the boundary-layer flow and in the model shape (due to icing). In general these effects increase with the size of drops or ice crystals. With small sizes of drops (or crystals) formed in condensation shocks usually no interference effects are observed, but when condensation occurs in subsonic flow upstream of the tunnel nozzle they can be appreciable (*cf.* section 3.3), particularly at low stagnation temperatures, when icing is severe.

3. *Elimination of Condensation Effects from the Working Section.*—For most test purposes it is essential to eliminate the effects of condensation from the tunnel working section. The usual method is to use highly dried air, and the question of the required dryness then arises. Alternative methods, which do not rely on the dryness of air, will also be considered.

3.1. *Required Dryness of Air.*—In order to avoid humidity troubles in supersonic tunnels, the standard practice in Germany during the Second World War was to keep the water content down to at least 0·5 grams per Kg of air ($\Omega = 0{\cdot}0005$, corresponding to a dew point of $-22{\cdot}5$ deg C at 760 mm mercury pressure, Fig. 19); in some American tunnels the maximum permissible absolute humidity is specified as low as 0·0001 (dew point, $-37{\cdot}6$ deg C).

Assuming the air to be at normal pressure and temperature (760 mm mercury, 15 deg C) after drying, an absolute humidity of 0·0005 corresponds to a relative humidity $\phi_0 = 0{\cdot}048$ and a dew point of about $-22{\cdot}5$ deg C. For these stagnation conditions the water vapour becomes saturated in an isentropic expansion at $T_{s\,\mathrm{to}\,ad} = -28$ deg C (*cf.* Fig. 21) corresponding to a Mach number $M_{s\,\mathrm{to}\,ad} = 0{\cdot}04$. With an adiabatic supercooling of, say, 50 deg C, a maximum Mach number $M_{k\,\mathrm{to}\,ad} = 1{\cdot}54$ would be reached in a condensation-shock free flow.

It thus appears that, except at low Mach numbers ($M < 1{\cdot}5$), the condensation effects would not completely disappear even with the air dried down to $\Omega = 0{\cdot}0005$, but it can be shown that their intensity would be very small.

Using the method already described, the maximum intensity of a condensation shock can be estimated for the above conditions. Assuming the whole of the water vapour present to condense, the pressure across the condensation shock is found to increase by 1·3 per cent and the Mach number to decrease by less than 1 per cent. Such changes are of the order of deviations usually present in supersonic nozzles and can, in most cases, be ignored. So far little evidence is available on the effects of condensation at these low humidities, and a highly accurate experimental technique combined with a refined nozzle design must be used in order to detect such effects.

3.2. *Stagnation Temperature Control.*—In some cases the effects of humidity can be avoided by increasing the tunnel stagnation temperature, keeping the absolute humidity constant and so reducing the relative humidity. This method, for not too high a stagnation temperature, is easily applicable to closed-circuit tunnels and, in fact, has been successfully used in the 11 × 11-in. N.P.L. supersonic tunnel.

Fig. 12 gives the maximum Mach number $M_{k\,\mathrm{to}\,ad}$ for condensation-shock free flow in terms of stagnation relative humidity ϕ_o, for various stagnation temperatures T_o (= -10, 20, 50 deg C) and adiabatic supercooling $\Delta T_{k\,\mathrm{to}\,ad}$ (= 40, 50, 60 deg C). Over the range of ϕ_o considered, the maximum Mach number never exceeds 2 and, for a given ϕ_o, the effect of T_o is small. It will be noticed that at large ϕ_o values and for a given $\Delta T_{k\,\mathrm{to}\,ad}$, since $M_{k\,\mathrm{to}\,ad}$ is a function of $T_{k\,\mathrm{to}\,ad}/T_o$, a higher $M_{k\,\mathrm{to}\,ad}$ can be reached for low T_o values.

A better appreciation of the practical possibilities of the stagnation-temperature control can be gained from Fig. 13. The two cases examined correspond to typical atmospheric conditions in the British Isles at ground level, in daytime*, in winter and in summer. The air is assumed to be heated from these standard conditions to stagnation temperatures up to $T_o = 200$ deg C

* At night the relative humidity is seldom smaller than 90 to 100 per cent.

The assumption (a) gives usually too large values of the static pressure and too small Mach number, as would be expected. The results obtained with the assumption (b) are indicated in Fig. 4; a particularly good agreement with the observed values has been obtained in test (i), while in test (iii) the difference amounts to $-3 \cdot 7$ and $+5 \cdot 9$ per cent of the corresponding observed static pressure, at the two points considered.

The conditions in the working section are most sensitive to the condensation when it is about to disappear, and may remain practically constant with high humidities and Mach numbers. This is well illustrated by test (iii), Fig. 4, in which the static pressure remained practically constant for $\phi_0 \geqslant 0 \cdot 18$ (with constant absolute humidity). Moreover, at $2 \cdot 5$ Mach number[2], when the stagnation temperature was varied over a 25 deg C range, no changes in the Mach number were detected. Similar results were obtained in Aachen[4] investigations in $2 \cdot 29$ Mach number nozzle over a range of stagnation relative humidities ($0 \cdot 30$ to $0 \cdot 54$). These observations are in agreement with calculations (of the type described above) made for $2 \cdot 5$ Mach number nozzle over the range involved.

When the humidity range is extended to sufficiently low values, the variation of state in the working section can be appreciable at high Mach numbers[15].

2.3. *Effects of Condensation on Flow Distribution in the Working Section.*—When a condensation shock occurs in a supersonic tunnel nozzle designed to produce, with isentropic flow, a uniform distribution in the working section, the nozzle profile is no longer correct, and not only the mean parameters but also the uniformity of flow in the working section is affected. The deterioration of the flow distribution increases with the intensity of the condensation shock, and additional disturbances are introduced when the condensation shock is not one-dimensional.

Typical schlieren photographs of condensation shocks in two-dimensional nozzles for various Mach numbers, from $M = 1 \cdot 56$ to $M = 3 \cdot 24$, are shown in Figs. 9, 10 and 15. When the curvature of the nozzle profile is small in the vicinity of throat, the flow is approximately one-dimensional and the condensation shock assumes the form shown in Fig. 15. A different configuration results in two-dimensional flow (Figs. 9 and 10) giving rise to the so-called X-shock. The nozzles for $M = 1 \cdot 56$, $1 \cdot 86$, $2 \cdot 48$ and $3 \cdot 24$ were designed with circular throat profiles, the radius being equal to the throat gap. In all these cases the condensation shock is not normal to the flow direction, so that its reflection at the plane of symmetry produces adiabatic disturbances which are propagated downstream. The resulting shock system is a function of condensation and of boundary conditions. In some cases reflection of the Mach type occurs, as shown in Fig. 9 (atmospheric air). This shock system has been investigated by means of axial pitot and static-pressure traverses, and the result, in terms of Mach number distribution, is given in Fig. 11; for comparison a dry-air traverse is included. The effect of condensation on Mach number distribution is very marked. The central Mach shock leads to subsonic velocity and is almost of the intensity of an adiabatic normal shock; second and third reflections of appreciable intensity occur before the end of the nozzle.

In general, the intensity of reflections decreases as they are propagated downstream. In high Mach number nozzles (Fig. 10) the disturbances tend to coincide with the Mach waves defining the working section rhombus. In no case was a breakaway of flow from the walls near the points of reflection observed.

2.4. *Condensation in Multi-nozzles.*—The above effects of condensation on flow distribution can be largely avoided in multi-throat nozzles. If, in spite of the disturbances inherent in the flow obtained with multi-nozzles (shocks and wakes), a sufficiently good flow distribution could be obtained, it would not be affected by relatively weak disturbances due to condensation. The effects of condensation on the mean flow parameters would, however, still be present. But on the basis of the R.A.E. tests[15], it appears that frictional losses in multi-nozzles are in general too large to warrant their use in supersonic tunnels.

When the first nozzle throat was sufficiently reduced the heavy mist and icing disappeared.

With $\phi_o < 0 \cdot 85$, a condensation shock was visible in the nozzle and disappeared when the first throat area was sufficiently decreased. A typical sequence of events as the humidity throat was reduced is shown in Figs. 15 and 16, where maximum Mach number in the first throat (based on static pressure) and loss of stagnation pressure in the first nozzle and honeycomb are indicated. With $\phi_o = 0 \cdot 80$ and $T_o = 7 \cdot 6$ deg C, the condensation shock, Fig. 15a, moved downstream, Figs. 15b, 15c, and was on the point of disappearing when the maximum Mach number measured at the wall in the first throat was $0 \cdot 68$, Fig. 15d. From this point onwards mist was observed throughout, upstream and downstream of the nozzle throat, and icing of pitot tubes occurred. At $M_{max} \simeq 1 \cdot 1$ in the first throat the mist disappeared and was observed only as occasional gusts; the icing disappeared simultaneously. No further changes occurred when the first throat area was reduced to its minimum value, giving $M_{max} \simeq 1 \cdot 5$.

It is apparent from the above results that the condensation shock can be eliminated from the tunnel nozzle by condensation either in purely subsonic flow, provided the relative humidity is sufficiently large, or in local supersonic flow. The first method is undesirable in that it results in formation of heavy mist and large drops or ice crystals, which can interfere with models, and at low stagnation temperatures may cause icing. It appears that in this case the mechanism of condensation is largely affected by presence of mist formed in the slow condensation on dust particles available in the air. The conditions of $\phi_o = 0 \cdot 85$ and $T_o = 7$ deg C at which the condensation shock disappears with fully open first throat correspond to an adiabatic supercooling of 4 deg C ($M = 0 \cdot 36$) at entry to the main tunnel nozzle, so that, according to Table 1, no appreciable condensation of the molecular type can take place. On the other hand, the duration ($\simeq 0 \cdot 01$ sec) of subsonic flow is sufficient to allow appreciable condensation of the slow type, its degree depending on the number of condensation nuclei present in the air. Assuming the maximum possible condensation, *i.e.*, air to be saturated at entry to the nozzle (only 25 per cent of the water vapour having condensed), sonic velocity corresponds to a maximum supercooling of 40 deg C, so that, on the basis of previously described results, one should expect a condensation shock to form in the nozzle. Its absence indicates that most of the vapour condenses in subsonic flow between the nozzle entry and throat and that this process, in which condensation of the molecular type presumably occurs, depends on the prior slow condensation.

A further evidence of the influence of slow condensation was obtained from a test in which the throat of the main tunnel nozzle was reduced (by tilting the nozzle liners) to $1 \cdot 57$ in. in order to decrease the subsonic approach velocity to $M = 0 \cdot 17$. With the small throat a condensation shock was clearly visible (Fig. 17a), even with high stagnation relative humidity, but when no supercooling occurred before the nozzle entry; in the case shown ($\phi_o = 0 \cdot 92$, $T_o = 5$ deg C) the air became just saturated at entry. When, in the same atmospheric conditions, the main nozzle throat width was increased to the original value of $3 \cdot 05$ in. (resulting in an adiabatic supercooling of $5 \cdot 5$ deg C before the nozzle entry) no condensation shock was observed (Fig. 17b).

The same result was obtained in $5 \cdot 5 \times 5 \cdot 5$-in. R.A.E. supersonic tunnel, in which the condensation shock was eliminated from $2 \cdot 48$ Mach number nozzle by an auxiliary nozzle, as shown in Fig. 18. The flow in the main nozzle was observed while the first nozzle throat area was decreased. As shown in Fig. 18a, a condensation shock in the tunnel nozzle was visible when the auxiliary nozzle was not mounted in the tunnel. Fig. 18b, 18c and 18d correspond to different auxiliary throat settings, the last one showing the flow in both nozzles, the condensation shock being visible in the first one. The ratio of the stagnation pressures before the nozzles is indicated in each case; considering the shape of the first nozzle, the losses are surprisingly small. In the set-up tested the flow between the two nozzle throats was not steady, and the shock system, through which the flow reverted to subsonic velocity, is not visible in the long-exposure schlieren pictures. In Fig. 18d a sufficiently large region of steady supersonic flow was obtained to observe the condensation shock.

On the basis of the above tests, the elimination of condensation shock from the tunnel nozzle by pre-expansion appears to be a practical proposition. In order to avoid deposition of condensate on the model, an expansion to supersonic velocity is necessary; the resulting losses amount to about 20 per cent of stagnation pressure and could presumably be decreased by a suitable design of the condensation section. On a tunnel designed to give low velocities before the main nozzle, the success of the scheme may be limited by re-evaporation between the two throats.

4. *Conclusions*.—Both theoretical and available experimental results show that the condensation in supersonic flow of air depends primarily on the adiabatic supercooling $\Delta T_{k\,\mathrm{to}\,ad}$ (defined in Fig. 1) which determines the conditions at which a condensation shock occurs for a given stagnation state of air. For medium-sized supersonic tunnels (say 1 ft square working section) the value of $\Delta T_{k\,\mathrm{to}\,ad}$ is of the order of 50 deg C (Fig. 3).

For a given constant supercooling $\Delta T_{k\,\mathrm{to}\,ad}$, the Mach number $M_{k\,\mathrm{to}\,ad}$ at which the condensation shock occurs depends mainly on the stagnation relative humidity ϕ_o (Fig. 12). The effect of stagnation temperature T_o, *i.e.*, for a given ϕ_o of the absolute humidity, on the position of the shock is small.

On the other hand, the intensity of the condensation shock depends primarily on the amount of the heat liberated (Fig. 8) or absolute humidity, and increases, for a given position ($M_{k\,\mathrm{to}\,ad}$) of the shock, with the quantity of heat.

For most test purposes it is essential to eliminate the detrimental effects of condensation on flow distribution in the tunnel working section. The usual method is to use highly dried air; in this way, although usually the condensation is not eliminated from the tunnel nozzle, the intensity of the condensation shock is greatly reduced. It is shown that with an absolute humidity of 0·0005 the disturbances due to condensation are of the order of flow deviations usually present in supersonic nozzles. So far little evidence is available on the effects of condensation at such small absolute humidities, and a highly accurate experimental technique combined with a refined nozzle design must be used in order to detect such effects.

Two other methods, which do not rely on the dryness of air, can be used to eliminate condensation from the tunnel nozzle.

By increasing stagnation temperature condensation can usually be avoided only at Mach numbers smaller than 1·5 (Fig. 13).

Alternatively, it can be eliminated by pre-expansion in an auxiliary nozzle. Preliminary tests of this method, carried out in small R.A.E. supersonic tunnels (about 6 in. square working section) have shown that no appreciable evaporation takes place between the two nozzles and the condensation shock can be removed from the main nozzle (Figs. 15, 16 and 18). Provided supersonic velocity is reached in the auxiliary nozzle no deposition of drops or ice crystals on the model occurs. The resulting stagnation-pressure loss amounts to about 20 per cent.

The principle of the method has been thus verified experimentally. Before its practical application the question of the influence of flow distribution before the main nozzle throat on flow in the working section will have to be investigated.

Acknowledgement.—The authors express their indebtedness to Messrs. W. F. Cope and N. D. G. Vincent for permission to quote and discuss the results of their investigations.

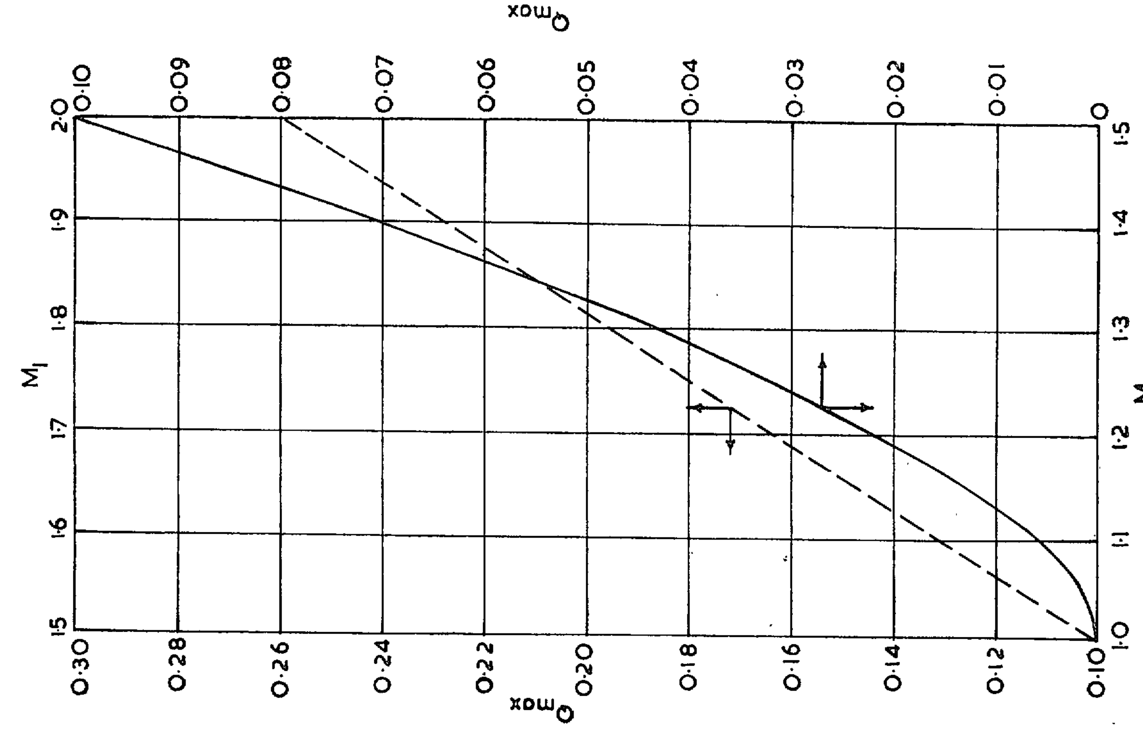

FIG. 7. $Q_{max} : M_1$ function, $1 \leqslant M_1 \leqslant 2$.

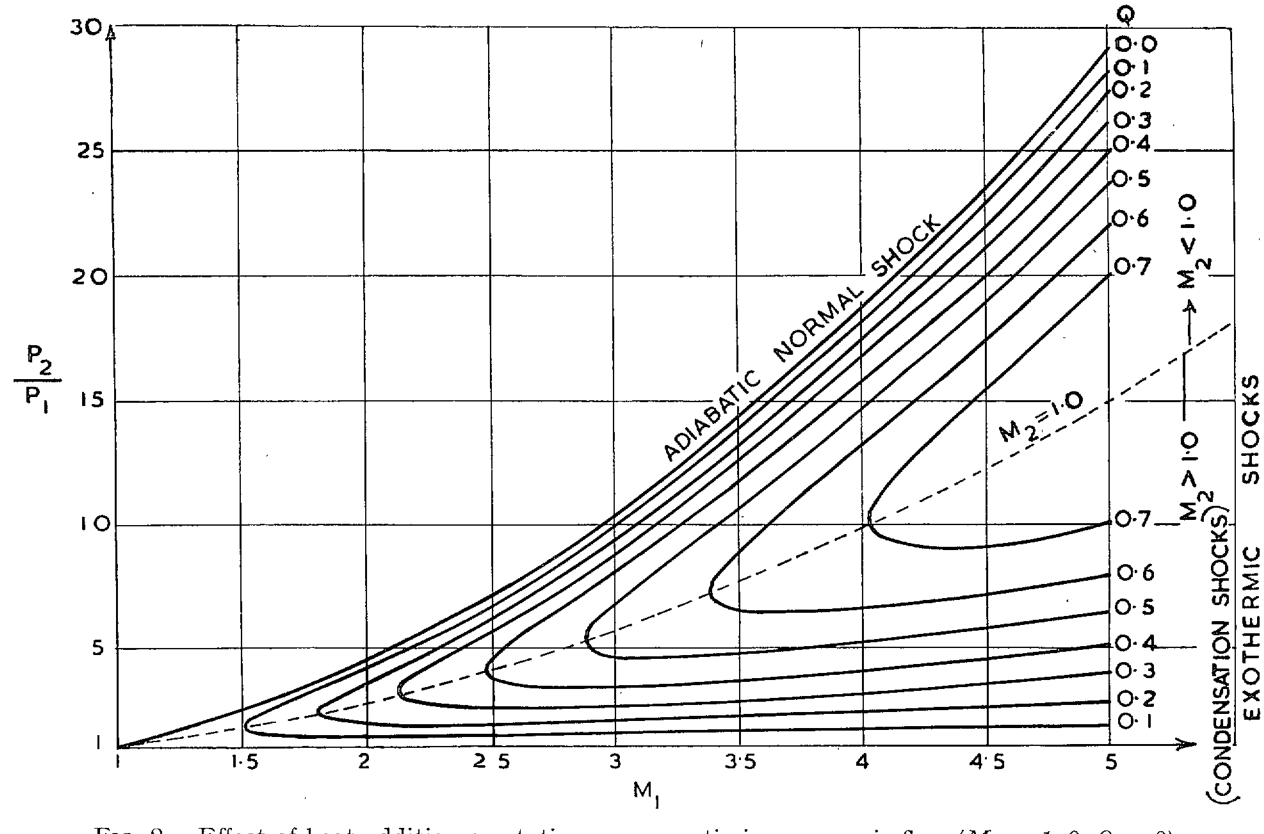

FIG. 8. Effect of heat addition on static pressure ratio in supersonic flow ($M_1 > 1 \cdot 0$, $Q > 0$).

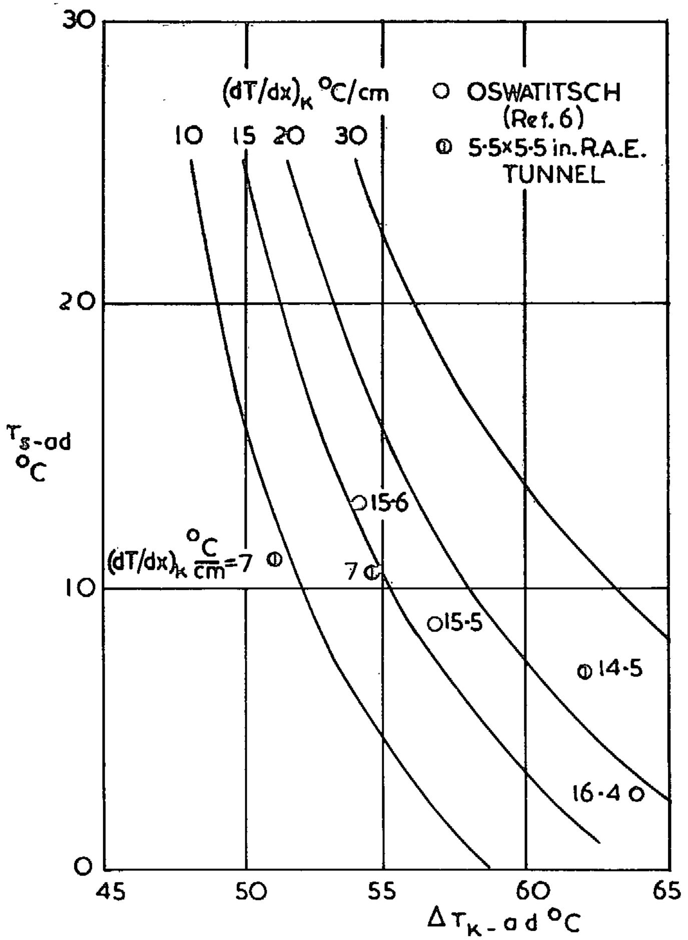

FIG. 5. Effect of saturation temperature $T_{s\ \mathrm{to}\ ad}$ and temperature gradient $(dT/dx)_k$ on adiabatic supercooling $\Delta T_{k\ \mathrm{to}\ ad}$ (Ref. 6).

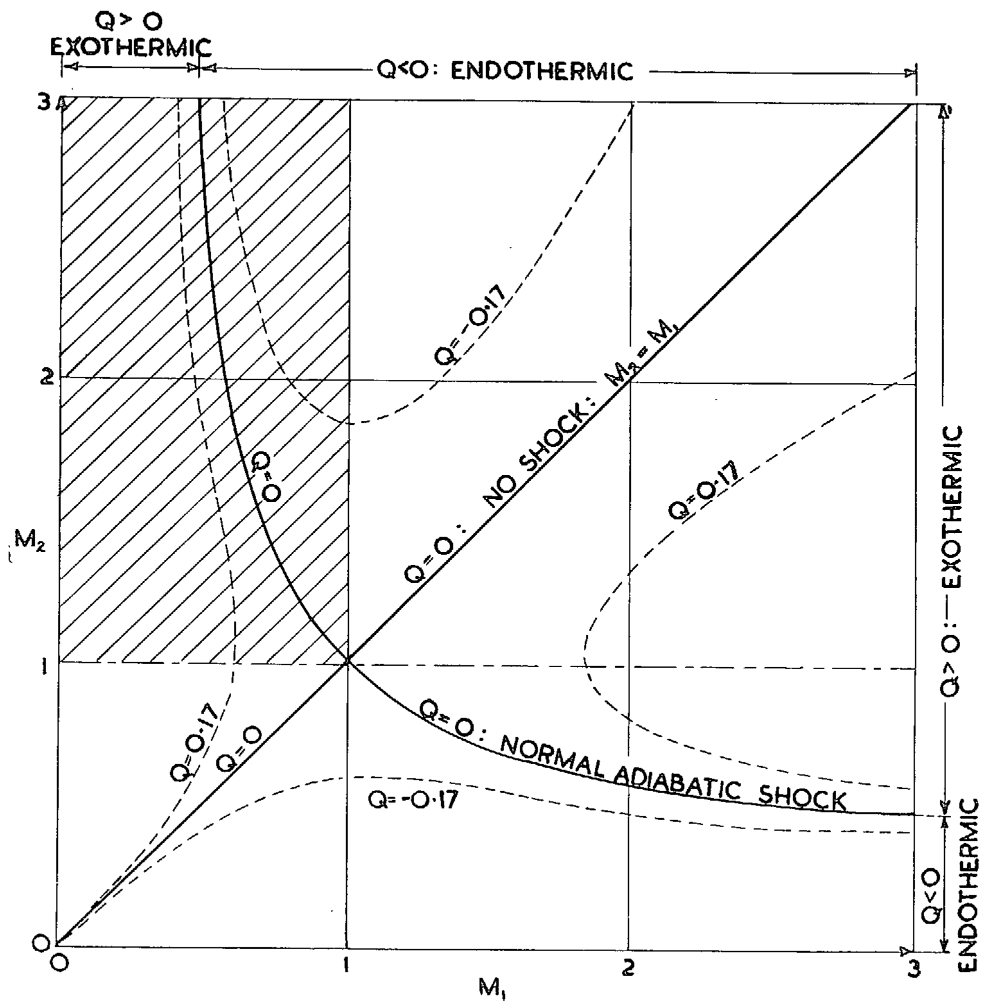

FIG. 6. Effects of heat addition (subtraction) in compressible flow[3].

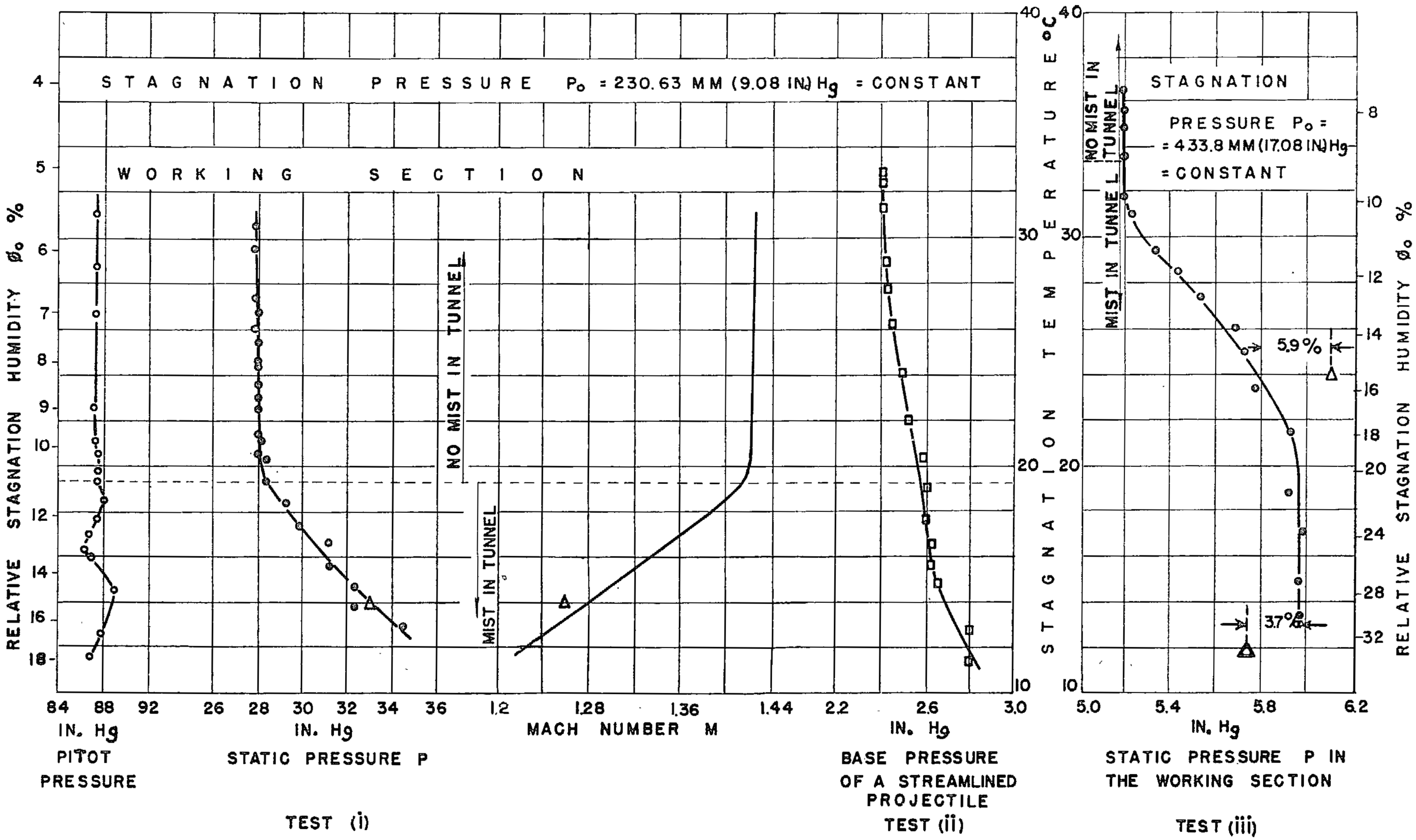

FIG. 4. Investigation of humidity effects in the N.P.L. 11 in. Supersonic Tunnel[2]. Nozzle nominal $M = 1{\cdot}4$. Water content of air $\Omega = 0{\cdot}005$ = constant. Δ denotes calculated values for water vapour saturated after condensation.

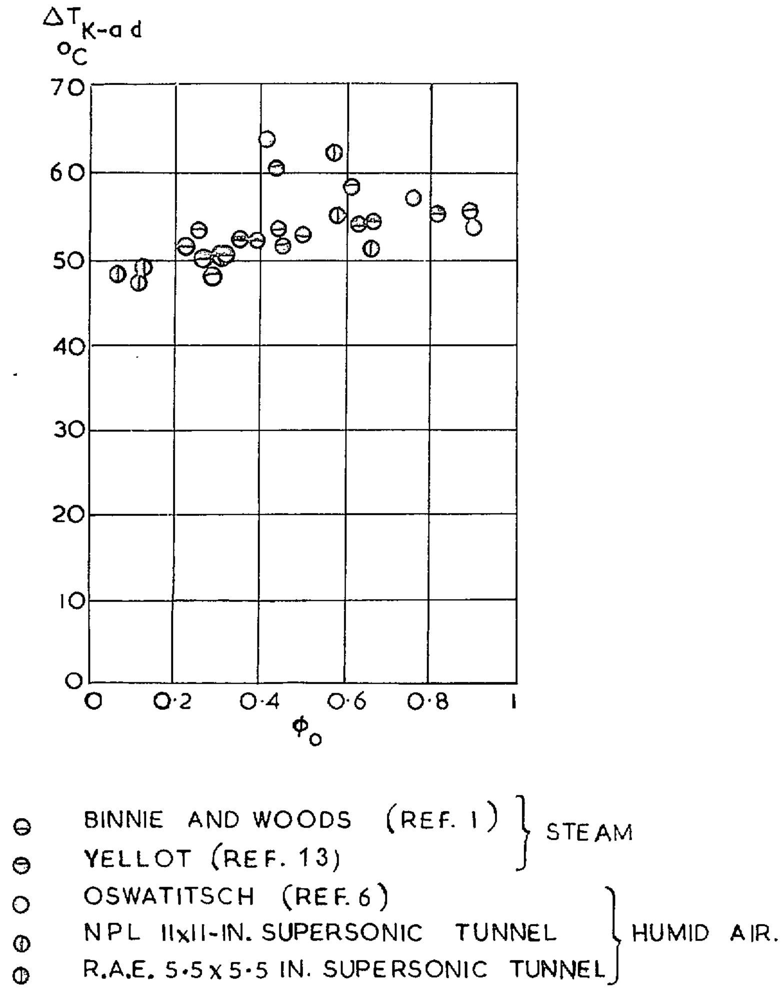

FIG. 3. Adiabatic supercooling $\Delta T_{k\ \mathrm{to}\ ad}$ in supersonic flow of steam and humid air.

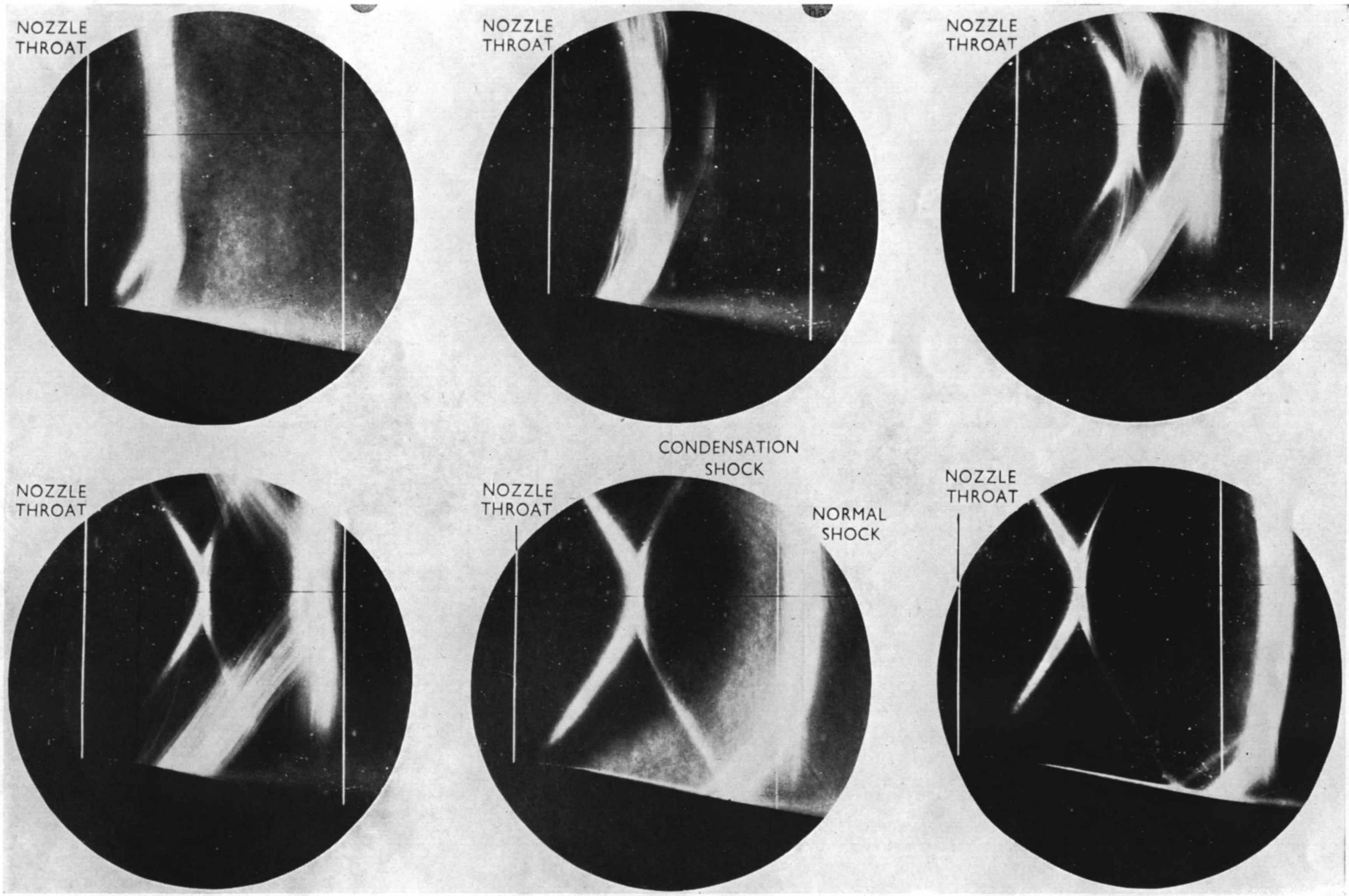

FIG. 2. Schlieren photographs showing separation of a condensation and normal shock when starting supersonic flow in a two-dimensional nozzle for $M = 1 \cdot 86$. Atmospheric air, $T_o = 20$ deg C, $\phi_o = 0 \cdot 62$, $\Omega_o = 0 \cdot 009$.

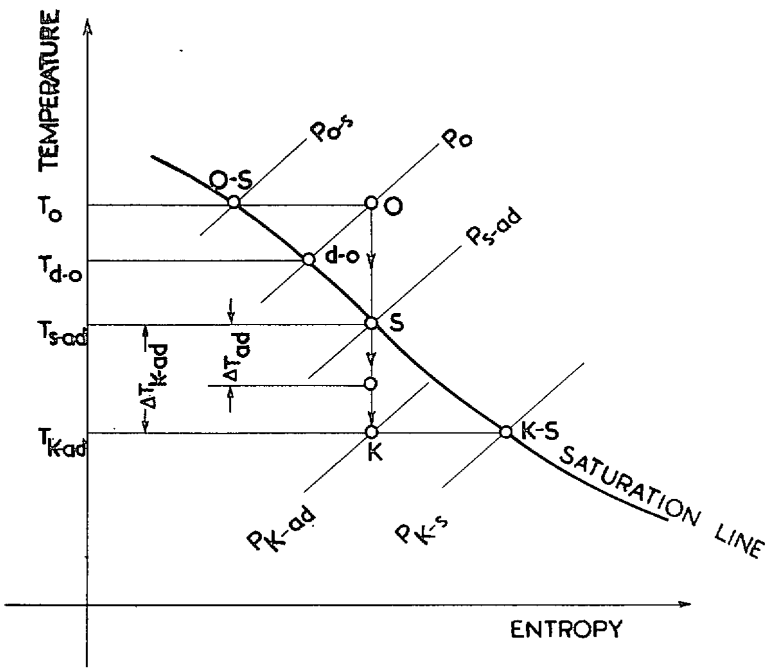

FIG. 1. Adiabatic expansion of water vapour. Notation.

determined from Fig. 21. Since only partial vapour pressures are here involved, the results hold for any air pressure.

The corresponding Mach number at saturation can be obtained from the temperature ratio as

$$M_{s\,to\,ad} = \sqrt{\left\{\frac{2}{\gamma - 1}\,(T_o/T_{s\,to\,ad} - 1)\right\}}.$$

TABLE 6

Saturation Pressure and Temperature of Water Vapour

(From International Critical Tables, Vol. 3, p. 210 and 233, McGraw Hill Book Co., 1928)

T_s deg C	P_s mm mercury	T_s deg C	P_s mm mercury	T_s deg C	P_s mm mercury	T_s deg C	P_s mm mercury
Over ice:							
−90	0·00007	−65	0·0040	−40	0·0966	−15	1·241
−85	0·00017	−60	0·0080	−35	0·1681	−10	1·950
−80	0·00040	−55	0·0157	−30	0·2859	− 5	3·013
−75	0·00090	−50	0·0295	−25	0·476	0	4·579
−70	0·00194	−45	0·0541	−20	0·776		
Over water:							
−15	1·436	30	31·824	75	289·10	140	2710·92
−10	2·149	35	42·175	80	355·10	150	3570·48
− 5	3·163	40	55·324	85	433·60	160	4636·00
0	4·579	45	71·88	90	525·76	170	5940·92
5	6·543	50	92·51	95	633·90	180	7520·20
10	9·209	55	118·04	100	760·00	190	9413·36
15	12·788	60	149·38	110	1074·56	200	11659·16
20	17·535	65	187·54	120	1489·14		
25	23·756	70	233·70	130	2026·16		

TABLE 7

Latent Heat of Evaporation (Sublimation) of Water (Ice) (From Ref. 9)

T deg C	h C.H.U./lb	
	water	ice
30	580	
20	586	
10	592	
0	597	677
−10	602	679
−20		681
−30		683
−40		685

APPENDIX

Data on Humid Air

Absolute and Relative Humidity, Dew Point, Latent Heat.—For a mixture of perfect gases we have, at any temperature T,

$$\frac{P_a}{P_v} = \frac{W_a R_a}{W_v R_v},$$

with P = pressure (partial),

W = mass of component gas,

R = characteristic gas constant,

and indices a and v denoting air (dry) and water vapour respectively.

The total pressure of the mixture is equal to

$$P = P_a + P_v ,$$

and defining the absolute humidity Ω as

$$\Omega = W_v/W_a,$$

we obtain

$$\Omega = \frac{R_a}{R_v}\frac{P_v}{P_a} = \frac{R_a}{R_v}\frac{P_v}{P - P_v}.$$

The above gives for air and water vapour, with $R_a/R_v = 18 \cdot 02/28 \cdot 95 = 0 \cdot 622$,

$$\Omega = 0 \cdot 622 \frac{P_v}{P_a} = 0 \cdot 622 \frac{P_v}{P - P_v}, \qquad \text{(A1)}$$

or approximately, since P_v is usually small as compared with P,

$$\Omega \simeq 0 \cdot 622\, P_v/P .$$

From the above expressions the water-vapour pressure, for any air pressure and absolute humidity, can be found and the corresponding relative humidity determined.

The saturation pressure and temperature over water and ice are given in Table 6, below.

With the aid of Table 6 and using equation (A1) for the absolute humidity, Fig. 19 has been prepared giving the absolute humidity Ω as a function of the dew-point temperature, at pressures of 760 and 700 mm of mercury.

The relative humidity is a function of the dew point and actual air temperature only. Curves of constant dew-point temperature are drawn in terms of ϕ and T in Fig. 20.

With the aid of the above diagrams the equivalent definitions of humidity (Ω, ϕ and dew point) for given conditions can be conveniently found.

The latent heat of evaporation (sublimation) of water (ice) is given in Table 7.

Isentropic Expansion of Humid Air.—In the case of expansion of humid air in high-speed flow it is of interest to determine, for given initial humidity, at what point of the expansion the air becomes saturated.

When the initial relative humidity ϕ_o and air temperature T_o are known, the temperature $T_{s\,\text{to}\,ad}$ at which saturation is reached in an isentropic expansion of humid air can be accurately

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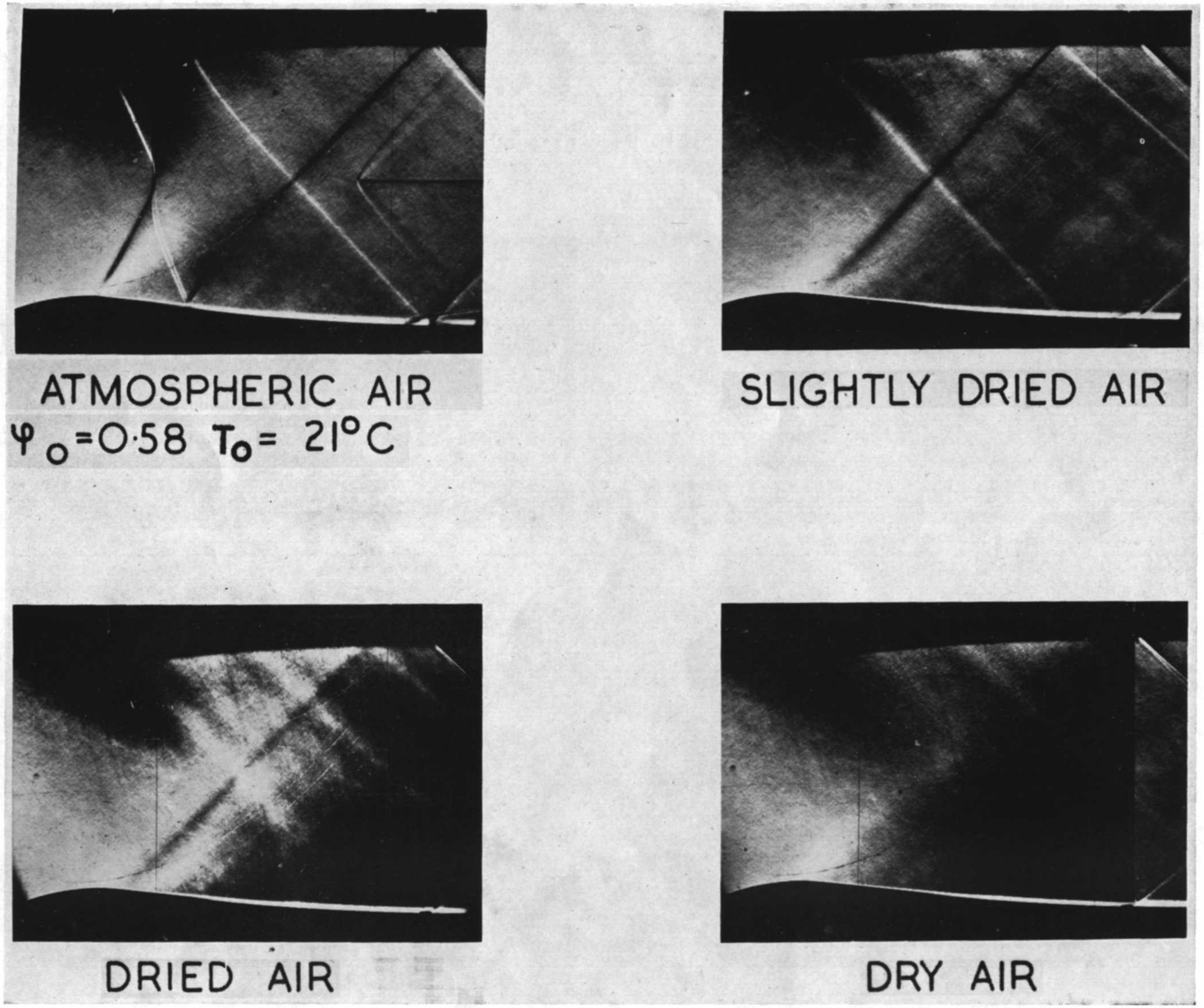
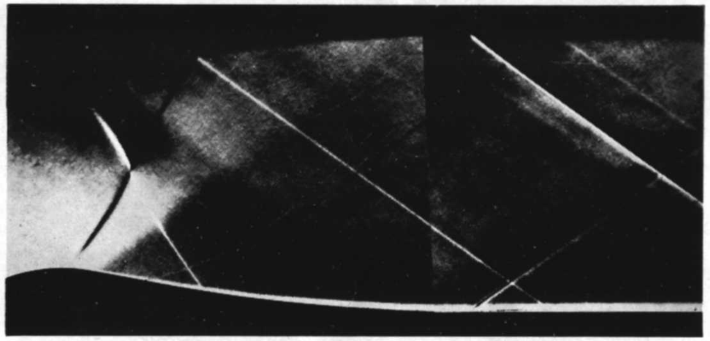
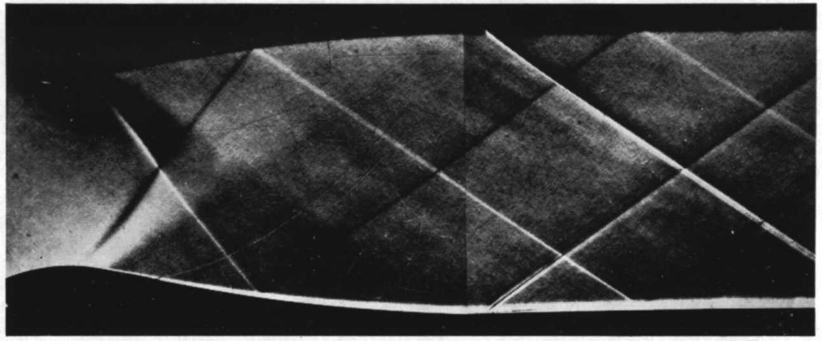


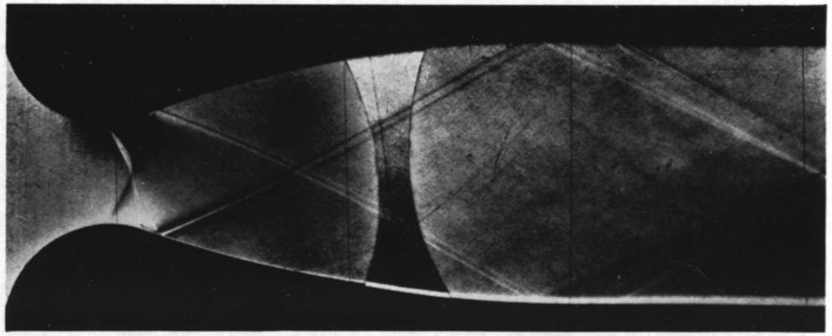
FIG. 9. Condensation shock in 1·56 Mach number nozzle. Atmospheric and dried air.



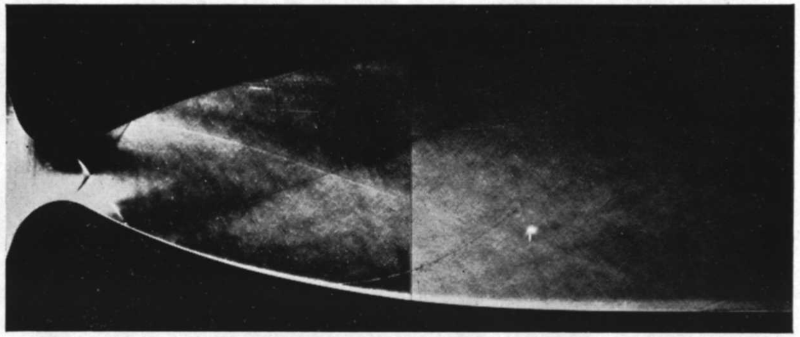
$M = 1 \cdot 86$



$M = 1 \cdot 86$



$M = 2 \cdot 48$



$M = 3 \cdot 24$

FIG. 10. Condensation shocks in 1·86, 2·48 and 3·24 Mach number nozzles. Atmospheric air.

Wt. 14/655 K.9 4/53 G.G. 34-262

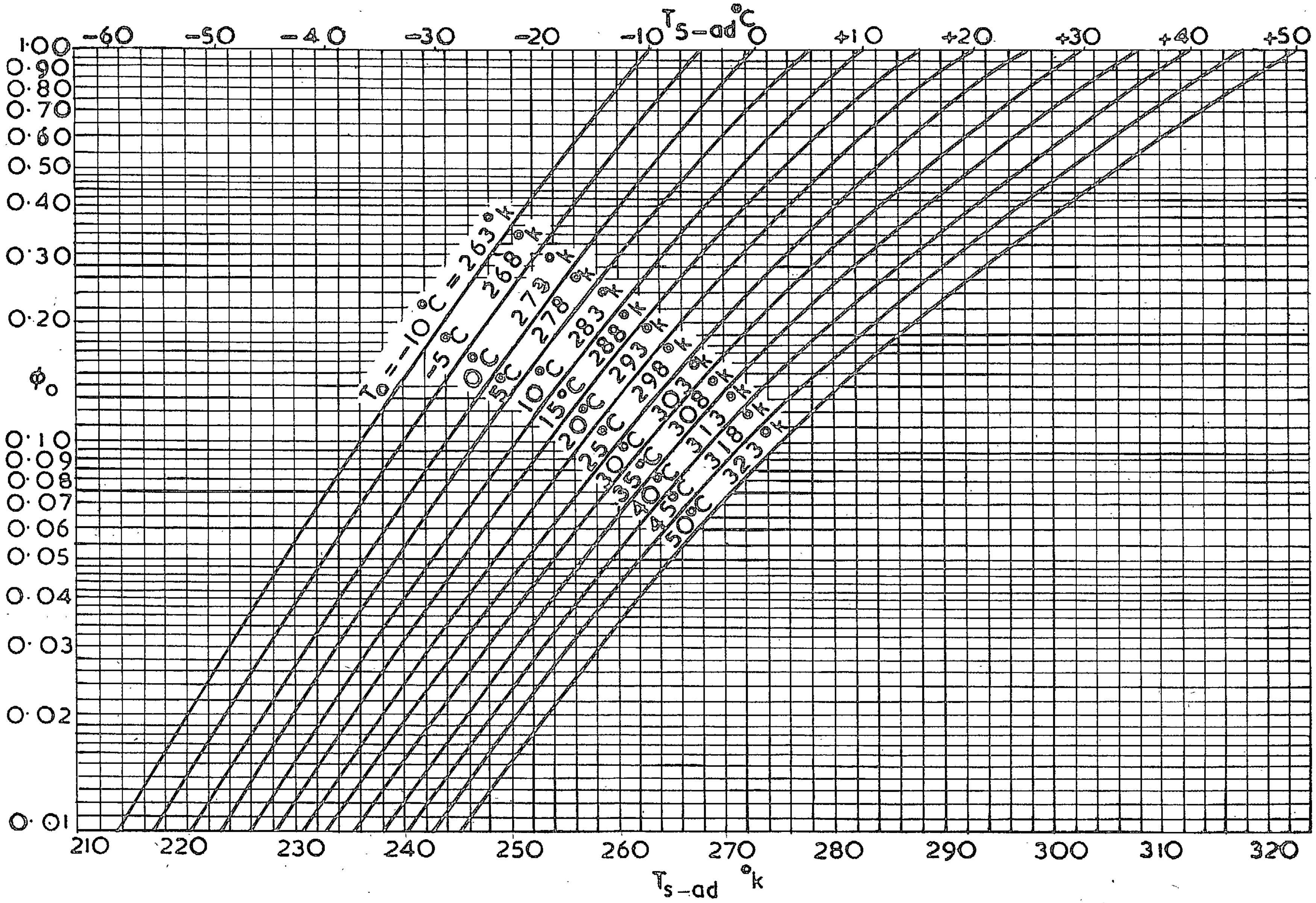


FIG. 21. Temperature $T_{s\ to\ ad}$ at which saturation is reached in isentropic expansion of humid air, in terms of initial stagnation, relative humidity ϕ_o and stagnation temperature T_o.

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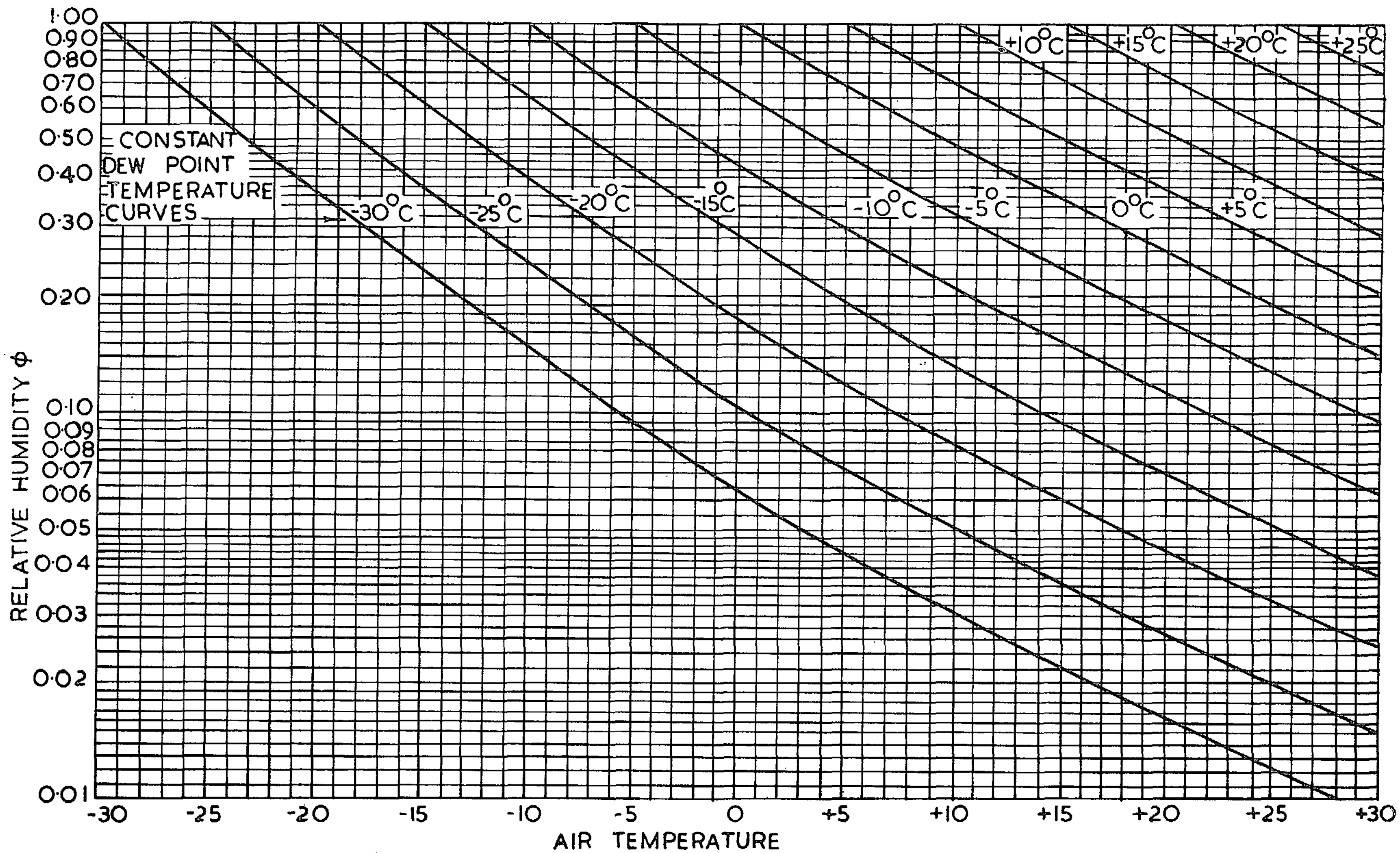


Fig. 20. Relative humidity ϕ and dew-point temperature as a function of air temperature.

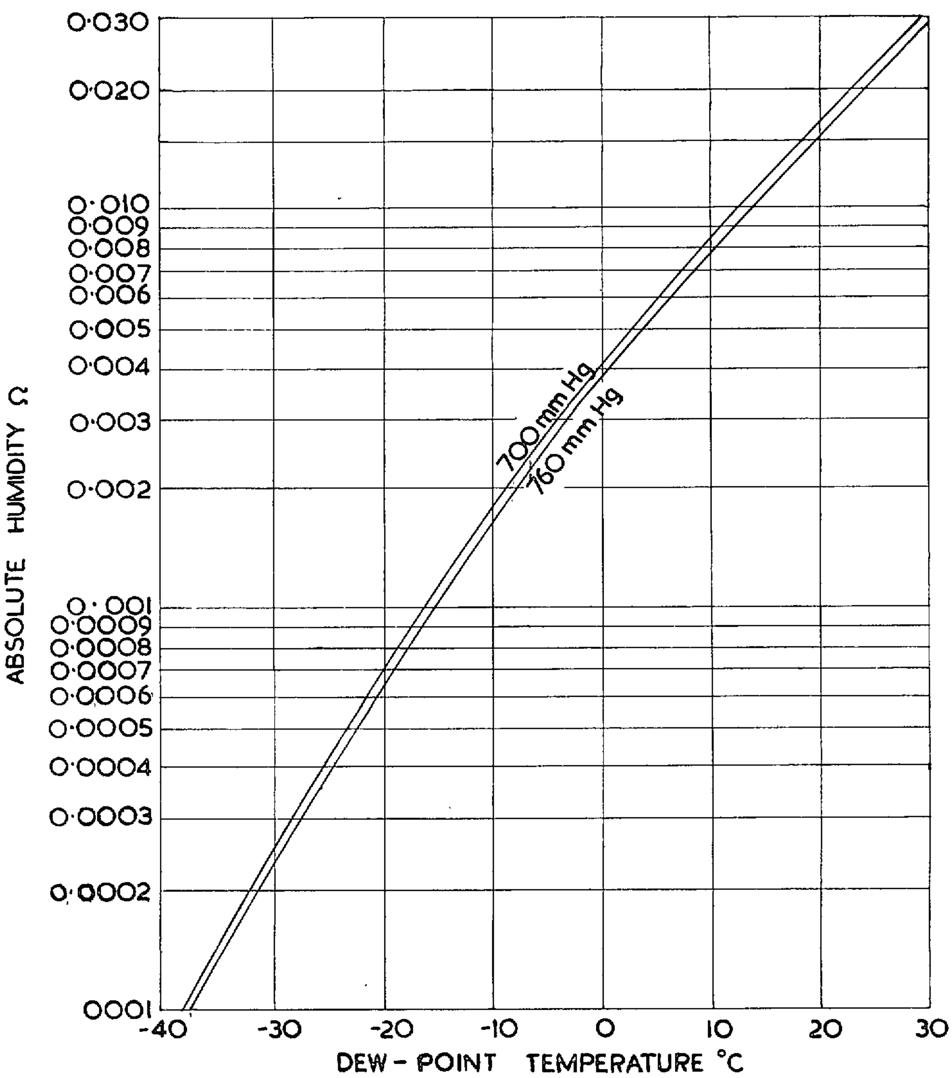


FIG. 19. Dew-point temperature and absolute humidity Ω of air at 760 and 700 mm of mercury pressure.

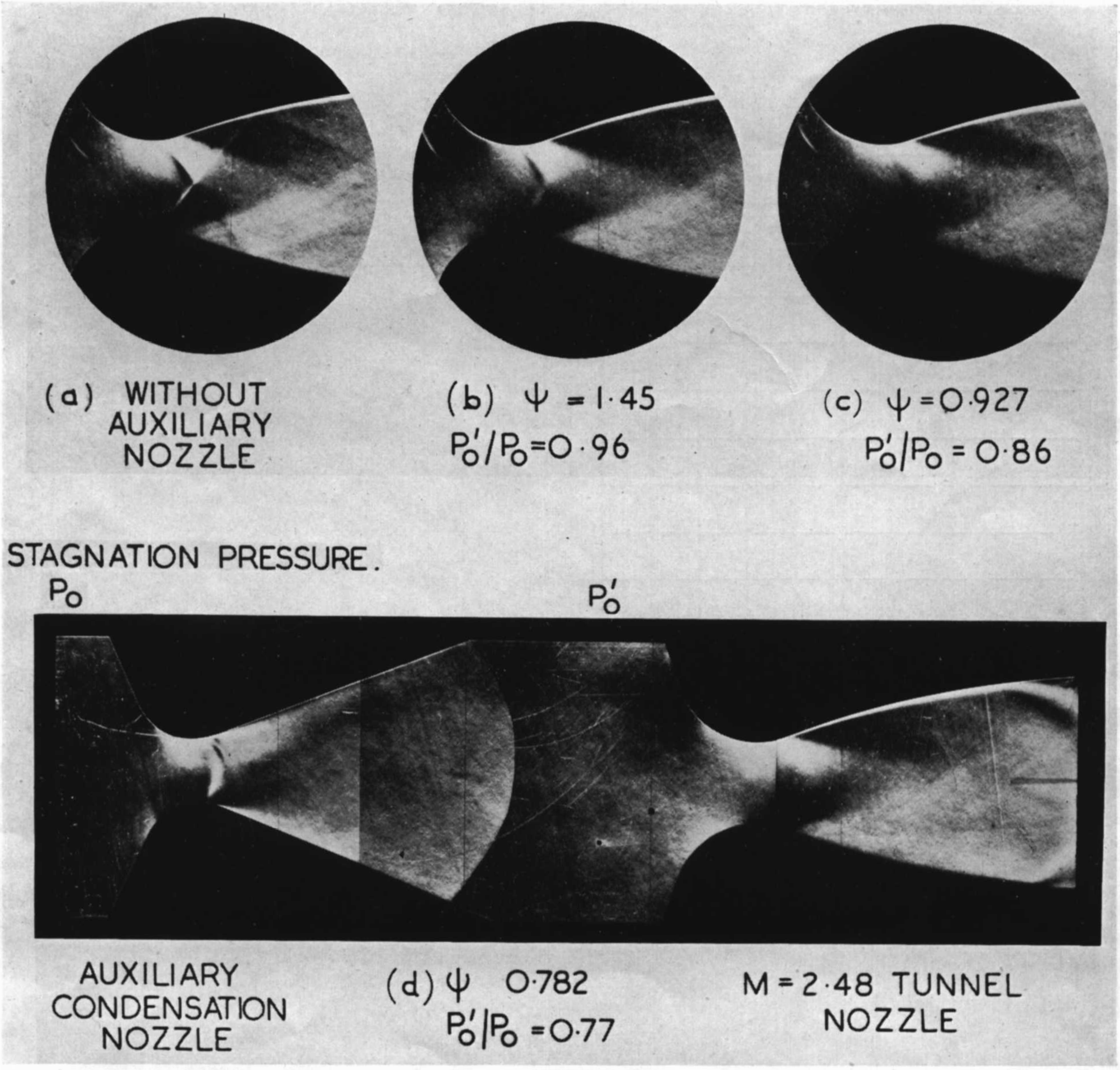


FIG. 18. Elimination of condensation shock from 2·48 Mach number nozzle by pre-expansion (5·5 × 5·5 in. supersonic tunnel). $\phi_o = 0 \cdot 57$, $T_o = 17 \cdot 6$ deg C.

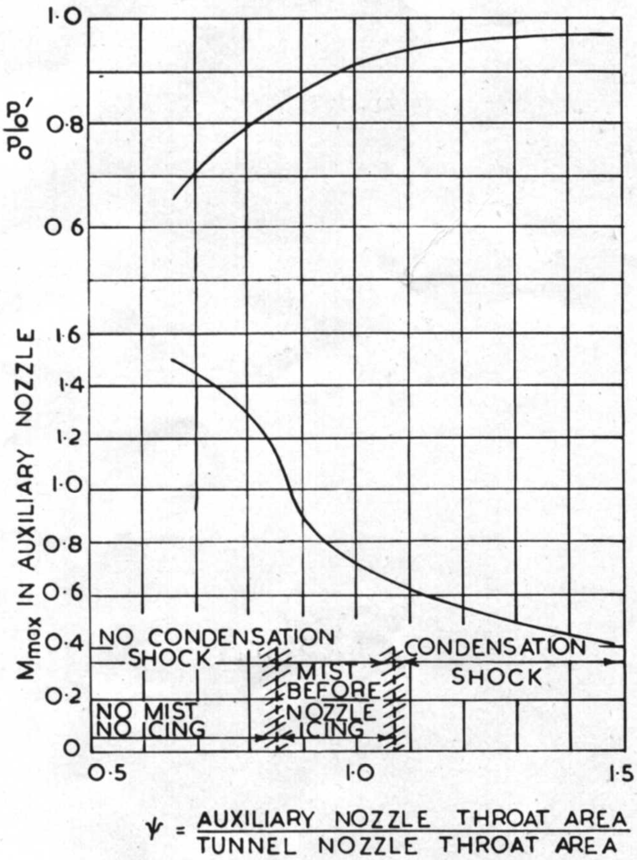
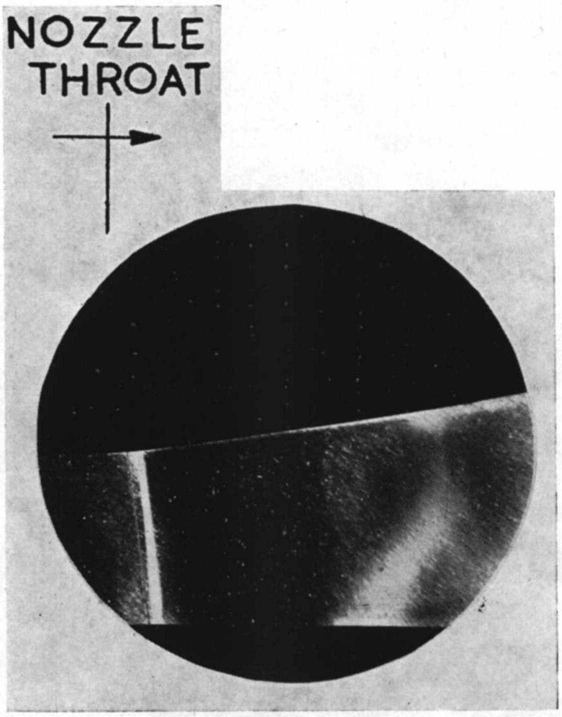
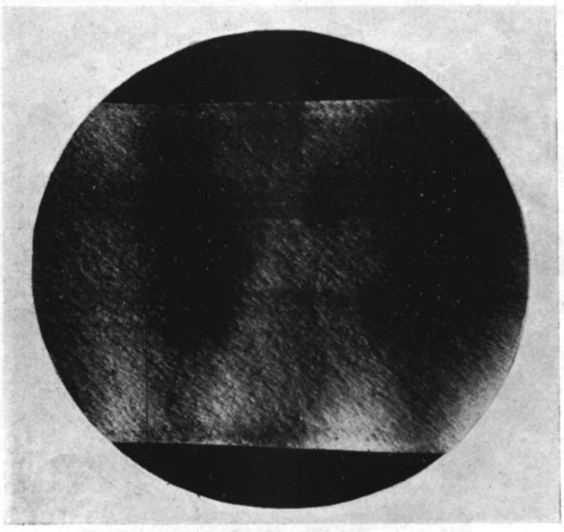


FIG. 16. Elimination of condensation shock from 1·65 Mach number nozzle by pre-expansion (4 × 4 in. tunnel). M_{max} in auxiliary nozzle and stagnation pressure ratio.



(*a*) M approach $= 0{\cdot}17$



(*b*) M approach $= 0{\cdot}36$

FIG. 17. Flow in the tunnel nozzle with low and high approach velocity. $\phi_o = 0{\cdot}92$, $T_o = 5$ deg C.

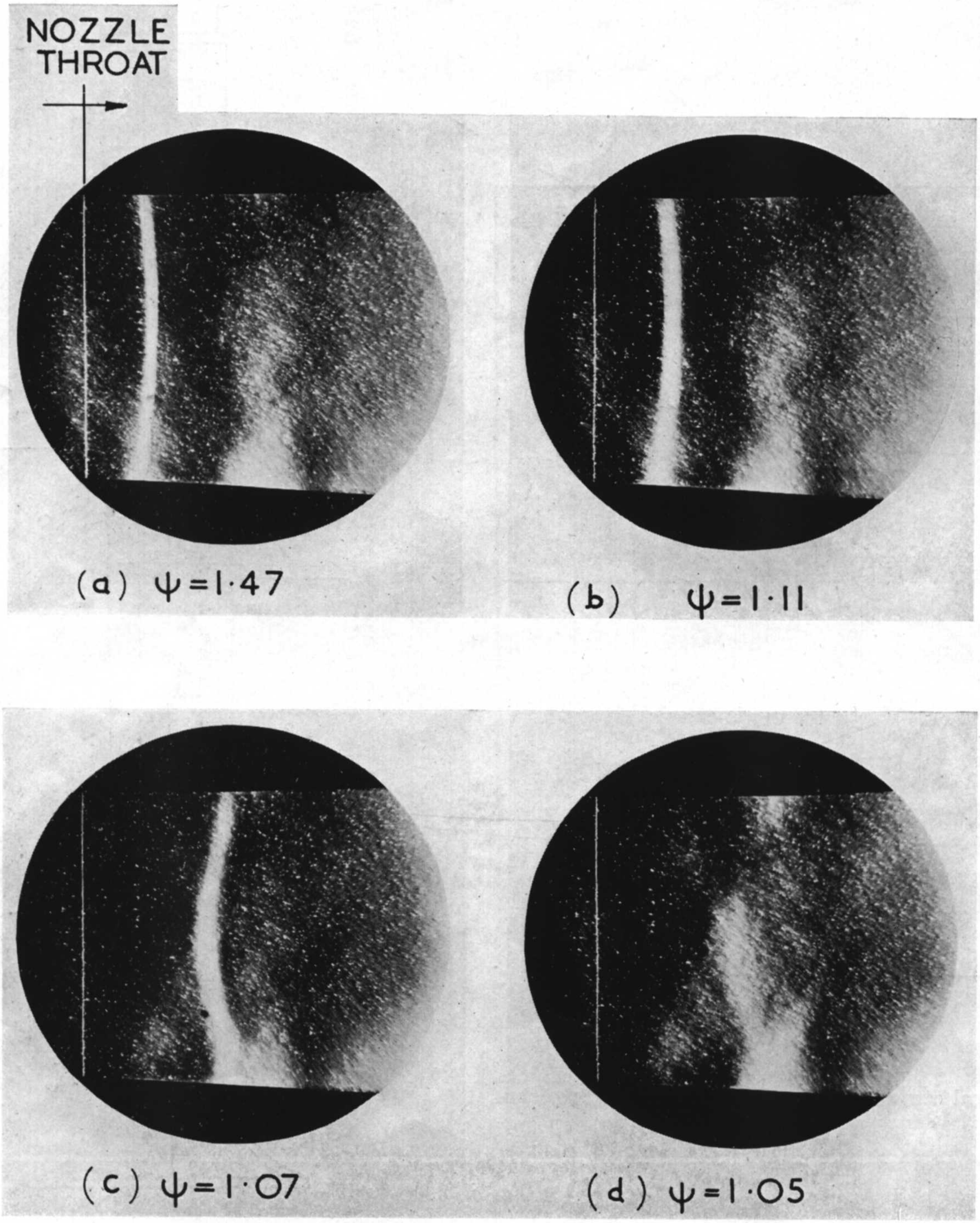


FIG. 15. Elimination of condensation shock from 1·65 Mach number nozzle by pre-expansion (4 × 4 in. Tunnel). $\phi_o = 0{\cdot}80$, $T_o = 7{\cdot}6$ deg C.

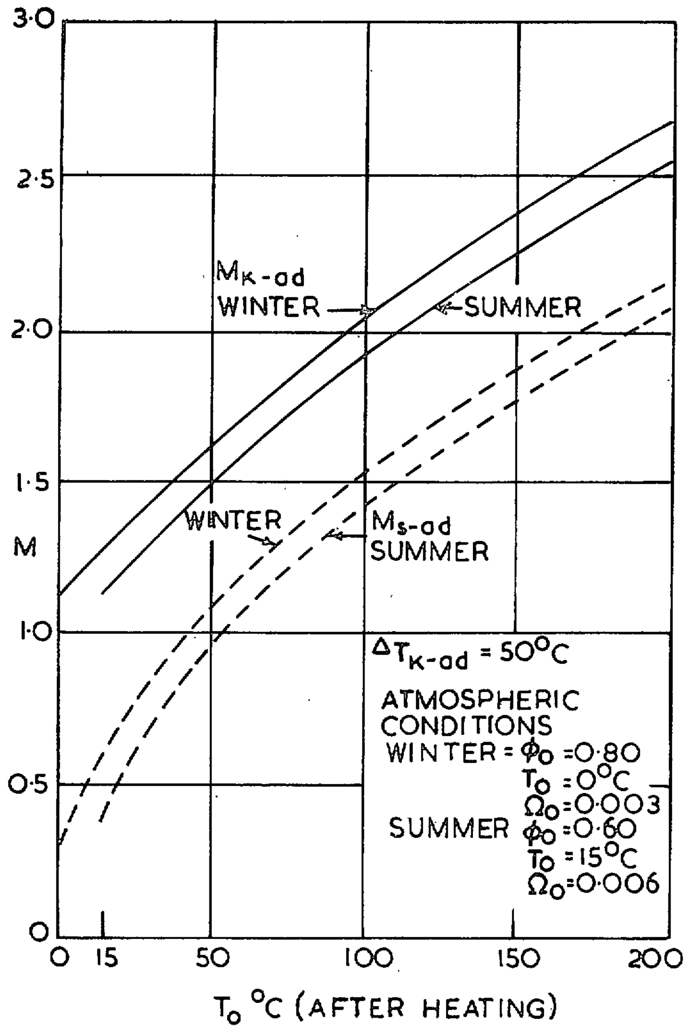


FIG. 13. Effect of stagnation temperature T_o on $M_{k\ to\ ad}$ for typical atmospheric conditions in the British Isles.

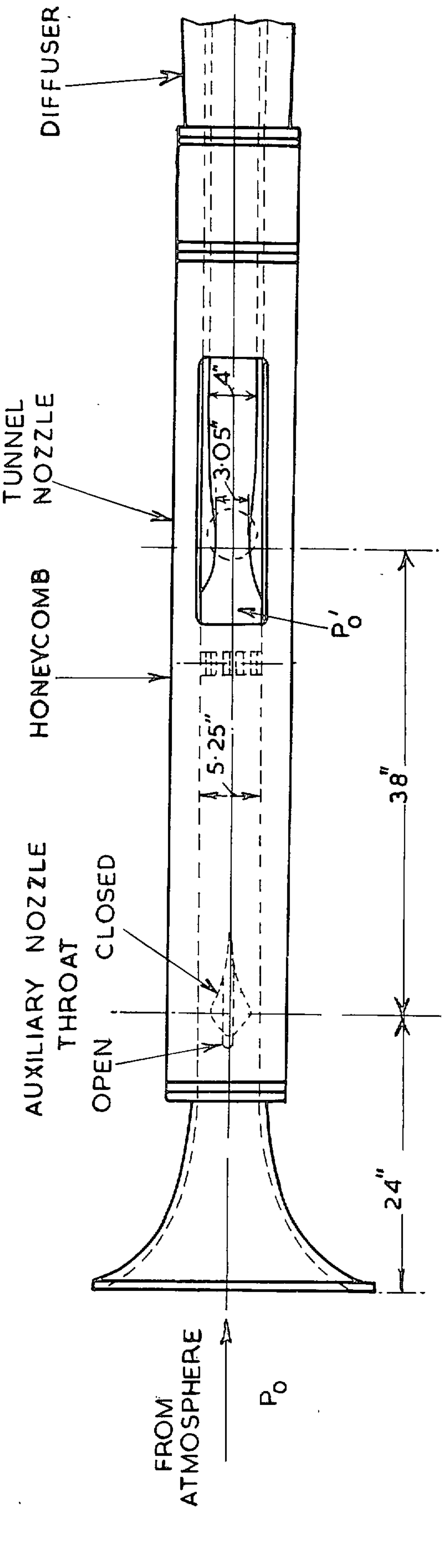


FIG. 14. 4 × 4 in. Supersonic Tunnel with auxiliary condensation nozzle.

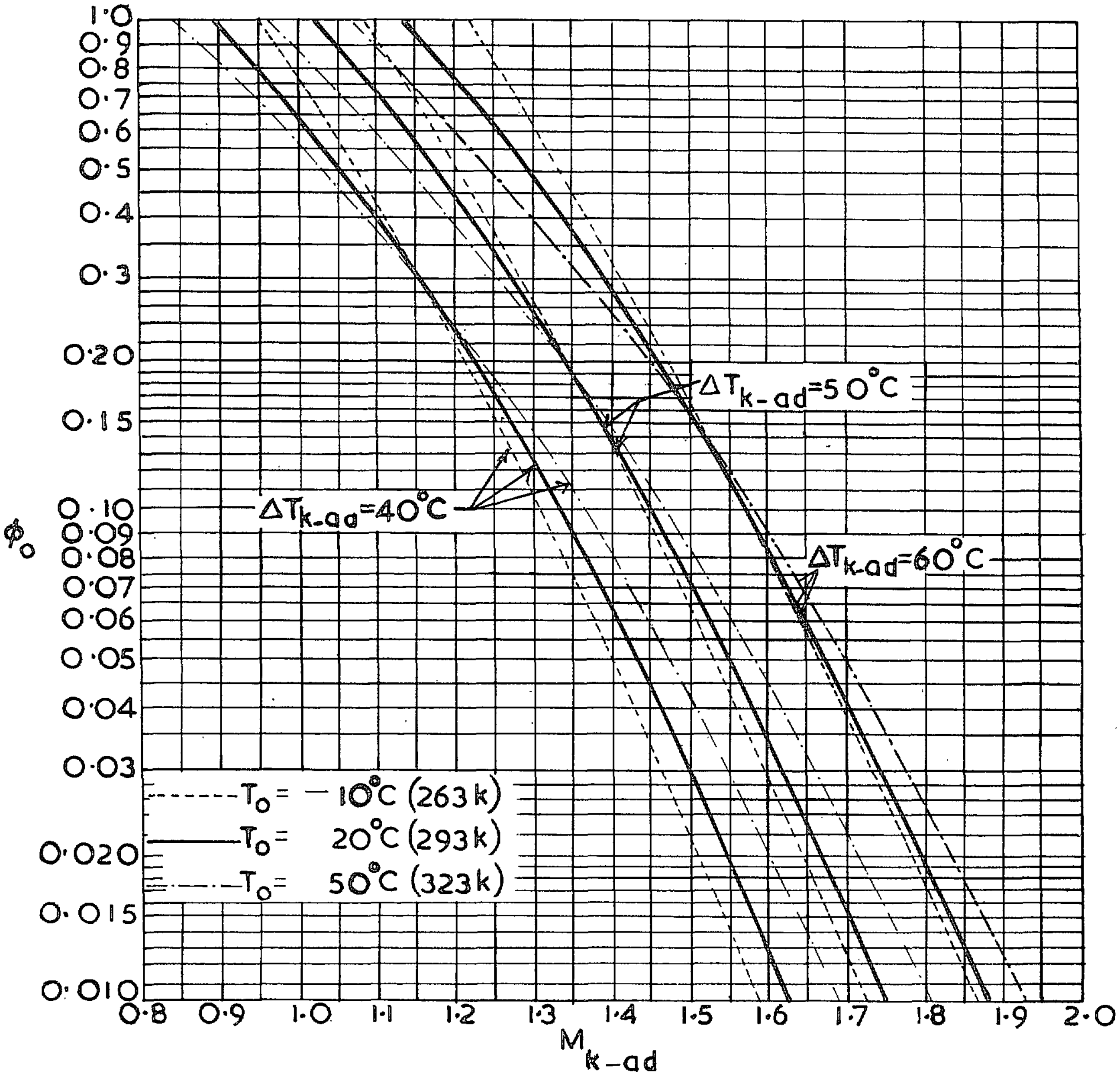


FIG. 12. Mach number $M_{k\ \text{to}\ ad}$ which is reached with adiabatic supercoolings $\Delta T_{k\ \text{to}\ ad} = 40$, 50 and 60 deg C for various stagnation temperatures T_o and initial stagnation relative humidity ϕ_o.

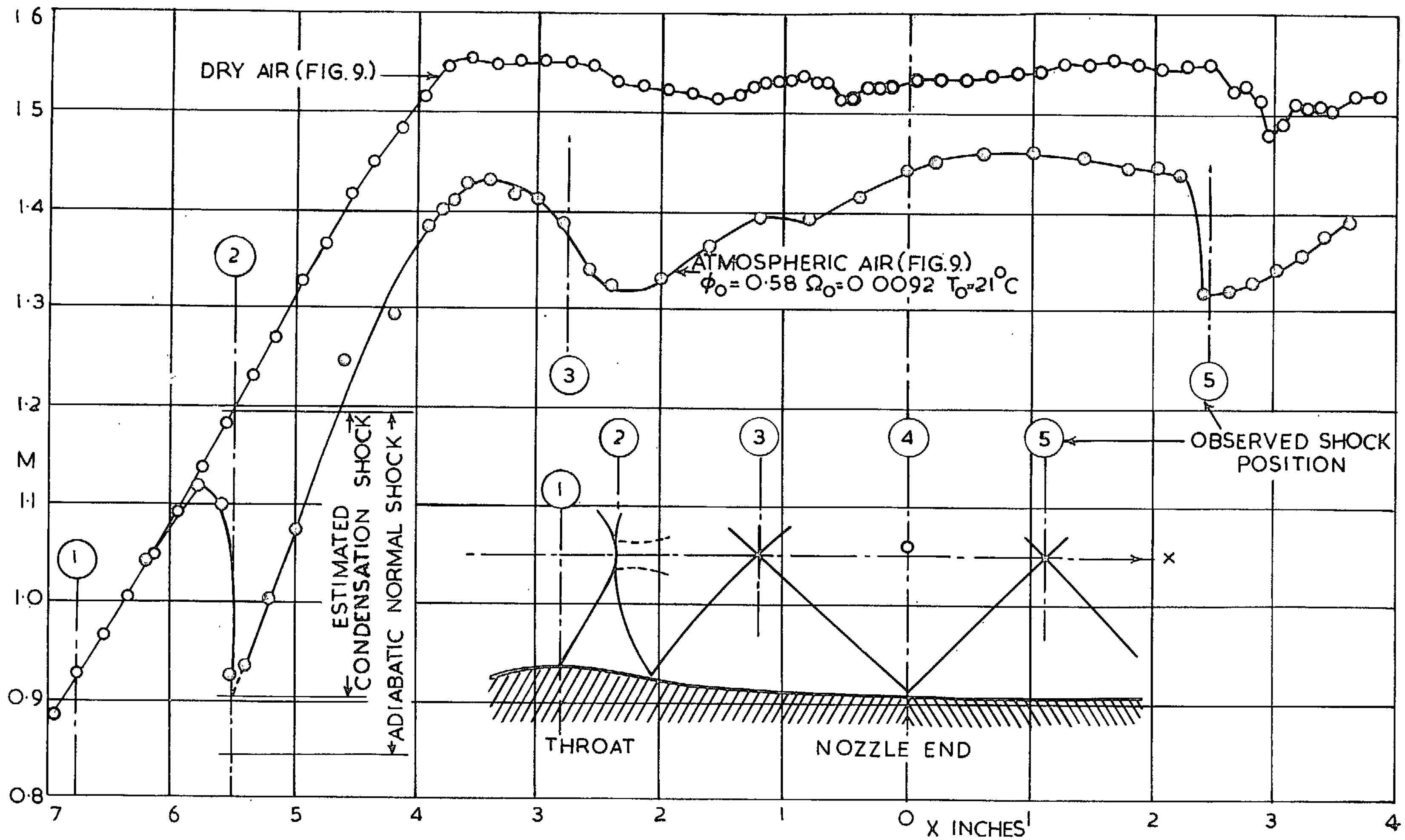


FIG. 11. Mach number distribution in 1·56 Mach number nozzle. Atmospheric and dry air.

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