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Measurement of Skin Friction at Low Subsonic Speeds by the Razor-Blade Technique

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Summary.
The technique consists of forming a surface pitot-tube by placing a small segment of razor blade on the surface with its tapered cutting edge above a static-pressure hole. The effects of limited changes in the razor-blade geometry on the measured pressure have been determined in a two-dimensional turbulent boundary layer and a calibration curve for a particular standardised geometry deduced. The variation of pressure due to yawing the blade segments in three-dimensional boundary layers, similar to those likely to occur on aerodynamic models, is found to be independent of the nature of the boundary layer. From this a method of using razor-blade segments to measure skin friction in three-dimensional boundary layers is proposed, which does not necessitate prior knowledge of the surface-flow direction.

LIST OF CONTENTS

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1. *Introduction.*

The direct method of measuring the skin friction as the force on a floating element of the surface has not been sufficiently developed to be suitable for use in normal wind-tunnel models. Consequently, the most promising approach to the problem of determining experimentally the skin-friction distribution on a model is to use an instrument which measures skin friction indirectly such as a Stanton or Preston tube.

In applying these devices to models two general requirements must be met. First, the flow over a model is in general three-dimensional and the direction of the surface streamlines cannot be estimated in advance. Therefore, the required device must either be insensitive to the direction of the skin-friction vector or be capable of being easily aligned with it. In both cases the direction would also have to be determined experimentally by some other means, such as oil flow-visualisation, and so ideally the device should be capable of measuring both the magnitude and the direction of the skin-friction vector without being accurately aligned. Secondly, to obtain useful information about the flow, it is usually desirable to determine the skin friction at a number of points and so the device must be both readily reproduceable and sufficiently small to cause a negligible obstruction to the flow.

An indirect measuring device which satisfies the above requirements to a large degree has been developed recently. This has been achieved by successive simplifications to the form of surface tube first used by Stanton, Marshall and Bryant[1]. In 1956 Hool[2] proposed a form of surface tube in which the 'tube' was formed on the one side by the surface of the model (as previously used by Stanton *et al.*) and on the other by the lower side of the tapered cutting edge of a segment of razor blade. The static pressure developed in this small enclosure was measured by a normal static hole in the surface of the model. With the razor blade removed, this same static-pressure hole could be used to measure the true static pressure and the difference between these two pressures calibrated against skin friction. Hool's calibration was obtained semi-empirically by placing the blade in a laminar flow and so, in common with Stanton *et al.*, can at best only be expected to apply to a turbulent boundary layer if the blade lies wholly within the laminar sub-layer. This restriction is not acceptable for model use where thin turbulent boundary layers can occur. However, the potential of this method was realised by Smith, Gaudet and Winter[3] who obtained a calibration in a turbulent supersonic boundary layer and, by cementing segments of razor blade over the static-pressure holes of a model, obtained the required skin-friction measurements. A similar method was used by Wyatt and Owen[4] and in a rather different form by Bradshaw and Gregory[5].

Two difficulties with the technique remained however. First, the solder or adhesive used to attach the blades had an unknown thickness and so the height of the blade edge above the model surface had to be measured accurately for each blade setting and, secondly, the arrangement could not be readily rotated into the local surface-streamline direction which, for instance, would in general change with model incidence. Both these shortcomings were overcome by the arrangement used by Wyatt and East[6] where the blades were attached magnetically. This was achieved by setting small magnets into the model surface, each containing a static-pressure hole. The razor-blade height was then taken as half the blade thickness and was assumed constant for all segments cut from the same type of razor blade. The blades were readily placed and removed by hand and so could be aligned with the local flow for each incidence in turn.

During the development of these techniques, each experimenter has used a different geometrical configuration. The present series of experiments was therefore undertaken to determine what effect limited changes in geometry had on the pressure recorded and, in consequence, to deduce what restrictions must be observed in order to avoid significant errors in the estimation of skin friction at low subsonic speeds in two-dimensional turbulent boundary layers. In addition some experimental information on the behaviour of razor blades in three-dimensional turbulent boundary layers has been obtained. From this information a technique has been deduced which enables the direction and magnitude of the skin-friction vector to be calculated without requiring prior knowledge of the direction.

The experiments were conducted in the R.A.E. Bedford 13 ft × 9 ft low-speed wind-tunnel and an auxiliary 4 ft × 3 ft tunnel between December 1964 and February 1966.

2. *The Characteristics of Razor Blades in a Two-dimensional Boundary Layer.*

2.1. *Dimensional Analysis.*

The basic experimental configuration is shown in Fig. 1. It is used in the same way as a Preston tube and this involves relating the skin friction (τ) to the difference (Δp) between the pressure recorded by the static hole with the blade in position and the true undisturbed static pressure. Assuming a universal inner law (or law of the wall), Δp can be expressed as:

$$\Delta p = \Delta p\,(\tau, \rho, \mu, h)\,,$$

where ρ and μ are the density and viscosity of the fluid and h is the razor-blade height (Fig. 1). Also, since the universal inner law is assumed to hold, the boundary layer thickness (δ) is not expected to be a relevant independent variable provided that $\delta/h >> 1$. The above relationship may be expected to be affected by changes in the proportions of the configuration, so that the general relationship becomes:

$$\Delta p = \Delta p\,(\tau, \rho, \mu, h, d, b, l, \Theta, \Delta x)\,,$$

which can be conveniently written as,

$$\frac{\tau h^2 \rho}{\mu^2} = F\left(\frac{\Delta p h^2 \rho}{\mu^2}, \frac{d}{h}, \frac{b}{h}, \frac{l}{b}, \Theta, \frac{\Delta x}{h}\right). \qquad (1)$$

The quantities d, b, l, Θ and Δx are shown in Fig. 1.

The intention of the first series of tests was to determine to what extent the basic calibration,

$$\frac{\tau h^2 \rho}{\mu^2} = F\left(\frac{\Delta p h^2 \rho}{\mu^2}\right) \qquad (2)$$

is affected by variations in the other non-dimensional quantities appearing in (1). The experiments were conducted in a closely two-dimensional flow with the blades carefully aligned and the effects of pressure gradients were not investigated.

2.2. *Experimental Arrangement.*

The experiments were conducted in the 13 ft × 9 ft low-speed wind-tunnel on an 8 ft × 4 ft flat plate (Fig. 2) which was mounted on the floor of the working section. The wooden plate was flat in the stream direction (8 ft dimension) and sagged transversely by no more than 1/16 in at any point. The leading edge of the plate consisted of a 3 : 1 semi-ellipse and the leading edge of the supports, which were 6 in. downstream, were semi-circular. Oil flow-visualisation tests showed that about 1 in. downstream of the leading edge a laminar separation bubble occurred which was not quite straight across the span. Accordingly a trip wire was fixed 1 in. from the leading edge so inducing a regular separation and reattachment across the span. The trip consisted of a single 26 s.w.g. steel wire stretched taut across the span and held against the surface by a continuous strip of $\frac{1}{4}$ in. wide adhesive tape.

Provision was made in the plate to locate two steel plates of dimensions 12 in. × 4 in. × $\frac{1}{4}$ in. One plate was plain and the other contained 8 static holes (0·030 in. diameter) and three static-hole/magnet combinations of different sizes as shown in Fig. 3. The plates were interchangeable and were located so that the row of static holes was, in turn, 13·5 in. (position A) and 79·5 in. (position B) from the leading edge of the wooden plate.

To investigate the effects of the non-dimensional quantities in equation (1), Δp was measured using configurations of different dimensions and was compared with the value of Δp recorded by one particular shape, which had been selected as a standard. Most of the non-standard shapes were selected so that only one non-dimensional quantity differed from the standard value and so the individual effect of each quantity could be determined. The standard ratios selected are $d/h = 6$, $b/h = 36$, $l/b = 1$ and $\Delta x/h = 0$, as these are the nominal values of the 0·005 in. thick blades which it had previously been found convenient to use. Two commercially available blades gave values of h of 0·0020 in. and 0·0050 in. and a third was manufactured with $h = 0{\cdot}0153$ in. Examination showed that the blade angle (Θ) was about 13 deg for the 0·0020 in. blade and about 11 deg for the 0·0050 in. blade. Since it was not intended that changes in Θ should be investigated the 0·0153 in. blade was made with $\Theta = 12$ deg and the slight differences in Θ have been subsequently ignored. The corresponding static-hole diameters for these blades ($d/h = 6$) are thus 0·012 in., 0·030 in. and 0·092 in. Blades were also made which gave in turn nominal values of b/h of 20 and 60 and of l/b of 0·6 and 1·8. Asymmetric blades, with the outer third removed as indicated in Fig. 1, were also tested as these were similar to those used by Smith *et al*[3]. The actual dimensions of the blades used are given in Table 1. No attempt was made to produce those blades which would have required nominal planform dimensions of less than 0·072 in. The blades were positioned by hand with the assistance of a microscope fitted with a micrometer eyepiece. The longitudinal position of the blade could be measured to $\pm 0{\cdot}0002$ in. and the allowable error in the positioning of the blades relative to the upstream edge of the static holes was limited to $\pm 0{\cdot}0004$ in. The pressures were measured on a multi-tube manometer containing alcohol, set at various angles between 5 and 10 deg to the horizontal chosen to suit the test conditions.

2.3. *Experimental Procedure.*

The experiment was conducted twice. In the first case speeds of 100 ft/s and 250 ft/s were used and consideration of the scatter of some of the results suggested that insufficient accuracy had been achieved in the measurement of the pressures. Accordingly, the experiment was repeated at 200 ft/s with additional precautions taken to increase the accuracy of the pressure measurements. Only a marginal decrease in the scatter was achieved and the repeatability of the results suggested that other factors, considered further in Section 2.4.5., were contributing significant errors. Apart from the construction of the razor-blade calibration, where the two experiments yielded complementary data due to the different air speeds used, the results given in this Report are taken from the second experiment.

Having established by oil flow visualisation that the surface flow appeared straight and parallel over the complete plate, tests were conducted with the measuring plate in each of the two positions (A and B). After measuring the static-pressure distribution, a single 2 mm (0·0825 in. actual diameter) Preston tube was traversed across the span of the steel plate. One inch intervals were used except that, over two separate one-inch sections of the traverse, steps of about $\frac{1}{8}$ in. were made. The three standard blades were then positioned over their respective static holes and, after taking pressure measurements in the standard position ($\Delta x = 0$), Δx was varied to find the dependence upon $\Delta x/h$. The standard blades were then positioned on different holes with $\Delta x = 0$ giving different values of d/h. Finally the rest of the blades were positioned over their respective static holes to give the dependence upon b/h and l/b. With the exception of the asymmetric blades the measurements were repeated with the blades inverted and re-positioned.

2.4. *Discussion of Results.*

2.4.1. *The static-pressure distribution.* The variation of static pressure across the steel plate in its two positions is shown in Fig. 4. The variation is very slight and the curves shown are taken to represent the true static pressure. The 0·090 in. hole ($z = 0$) measures a significantly higher pressure than the trend of the 0·030 in. holes. This error, which amounts to approximately $1{\cdot}4\tau$, agrees closely with the results of Shaw[7] for deep static holes when both are measured relative to the 0·030 in. holes. Also, according to Shaw, the reading of the 0·030 in. holes should be in error by about $0{\cdot}3\tau$ (about $0{\cdot}0008\ q_0$) but the 0·012 in. hole ($z = -3$ in.) does not show this trend and this small correction has not been applied*.

2.4.2. *The skin-friction distribution.* The variation of the pressure difference (Δp = Preston tube pressure – local static pressure) measured by a 2 mm Preston tube traversed across the steel plate is shown in Fig. 5. The general trend is towards a slight skin-friction minimum near the centreline. The variation about the mean curves shown are very small (about $\frac{1}{4}$ per cent of $\Delta p/q_0$) except at position A near the centreline where a distinct wave of magnitude about $\frac{1}{2}$ per cent is shown. However, these time-averaged spatial variations are more than an order less than those recorded by Bradshaw[8] and suggest that the boundary layer, though slightly three-dimensional on a large scale, is devoid of the small scale but large amplitude three-dimensional effects that have been found in some other turbulent boundary layers[8,9,10].

2.4.3. *Razor-blade calibration and the effect of blade asymmetry.* A calibration of the standard razor-blades can be constructed from the pressure difference recorded when they are positioned with $\Delta x = 0$ and the value of the skin friction estimated from the Preston tube measurements. The Preston tube calibration used is that of Patel[11]. In the present experiments the adverse pressure gradient is negligible,

$$\frac{\nu \dfrac{\partial p}{\partial x}}{\rho U_\tau^3} < 2 \times 10^{-5}$$

and the Preston tube Reynolds number limits of $300 < \dfrac{U_\tau h}{\nu} < 350$ are well within his experimental range. The resulting razor-blade calibration is given in Fig. 6a which includes the results of both sets of experiments. A close fit to the experimental points over the range $2 < x^* < 6$ is achieved with the equation,

$$y^* = -0{\cdot}23 + 0{\cdot}618\, x^* + 0{\cdot}0165\, x^{*2}, \tag{3}$$

where x^* and y^* are defined as,

$$x^* = \log_{10}\left(\frac{\Delta p h^2 \rho}{\mu^2}\right) \quad \text{and} \quad y^* = \log_{10}\left(\frac{\tau h^2 \rho}{\mu^2}\right).$$

Equation (3) is compared in Fig. 6b with the calibration curve of Bradshaw and Gregory[5] and unpublished points obtained at $M = 0{\cdot}2$ by the authors of Ref. 3. Neither is strictly comparable with the present curve owing to differences in the configurations used. Bradshaw and Gregory used a square hole and a piece of metal shim with its leading edge chamfered, while Smith *et al.* ground away most of the razor blade above the sharp leading edge. To investigate the effect of grinding away part of the blade, three blades were tested which were of the standard shape except that a third of the total thickness was removed from the top of each blade. The result of this modification can be seen in Fig. 6b by comparing the solid points with equation (3) (i.e. the change in x^* at constant y^*). The recorded pressure is reduced

*An error of $0{\cdot}3\tau$ in the value of Δp would cause an error of about $1\frac{1}{2}$ per cent in the derived value of τ when using the 0·002 in. blades and when using the larger blades the error would be appreciably less.

by between 5 and 35 per cent which brings the points into better agreement with Smith *et al.*, though the change is contrary to their observation[3] at supersonic speeds that the grinding-off process has no effect on the measured pressure.

2.4.4. *Razor-blade position.* The effect of varying the razor-blade position is shown in Fig. 7, where the measured Δp, which has been normalised with respect to $(\Delta p)_{\Delta x=0}$, is plotted against $\Delta x/h$. The behaviour of the 0·002 in. blade differs significantly from that of the 0·005 in. and 0·015 in. blades. The reason for this is not known but it may be associated with the probability that the 0·002 in. blade lay wholly within the laminar sublayer (h/δ was of order 4×10^{-3}). Since the three blade-configurations are similar, no significant difference will be made by plotting $\Delta p/\Delta p_0$ against $\Delta x/d$ in place of $\Delta x/h$, but for dissimilar configurations it has been found[6] that a better correlation is obtained with $\Delta x/h$. To obtain a 5 per cent accuracy in $\Delta p/\Delta p_0$ it is necessary to position the blade such that $|\Delta x/h| < 0{\cdot}5$, i.e. $|\Delta x/d| < 0{\cdot}08$ for the standard razor-blade configuration with $(d/h) = 6$.

Since $\Delta p/\Delta p_{(\Delta x=0)}$ is not a unique function of $\Delta x/h$, it follows that, for any particular chosen value of $\Delta x/h$, the calibration curve (Fig. 6a) should appear as a series of discontinuous curves with each curve fitting the points from one or more blade thicknesses. The standard value used in this Report ($\Delta x = 0$) was chosen for practical convenience and no attempt has been made to optimise $\Delta x/h$ to give the smoothest calibration curve. Fig. 6a shows that the points from the 0·002 in. and 0·005 in. blades join up without any apparent discontinuity although the different accuracies to which Fig. 6a and Fig. 7 are plotted should be noted. It is concluded that the use of $\Delta x/h = 0$ as a standard is justified though it is possible, in principle, that a different value might yield a smoother calibration curve.

2.4.5. *Razor-blade dimensions and static-hole size.* The effect on the recorded pressure of changes in the quantities d/h, b/h and l/b is shown in Fig. 8. In these figures the pressures recorded by the blades before and after being turned over are shown separately. Once again the 0·002 in. blade has shown peculiarities with some points improbably high. The fact that all the spurious points are high, regardless of whether the blades were inverted or not, suggests that the cause was imperfections in the blade segments such as burrs or lack of flatness. Such imperfections probably arose when the segments were cut out, since this process was done by hand. It was the appearance of these spurious points in the first experiment that was largely responsible for the repeat and in view of their likely cause they will be ignored. However, the results for the 0·0020 in. standard blade give a Δp minimum and a variation on being inverted consistent with the likely asymmetry of the blade cutting-edge. On another sample of 0·0020 in. blade it was found by optical measurement that $h = 0{\cdot}0019$ in. one way up and $h = 0{\cdot}0021$ in. when inverted. It is concluded that the standard blade was functioning correctly, which justifies using the results in Fig. 6a.

Fig. 8a shows that errors will result if the size of the static hole used is too small and suggests that providing $d/h > 3$ the standard calibration can be used. There is of course an upper limit to d/h, when using standard blades, at which the blades are too small to cover satisfactorily the static hole (in the present case the 0·072 in. × 0·072 in. blade could not be used on the 0·090 in. diameter hole). The effects of varying b/h and l/b independently are shown in Figs. 8b and c, together with the effect of keeping their product l/h constant and varying both b/h and l/b (Fig. 8d). Errors greater than ± 5 per cent do not result from these changes except for $h = 0{\cdot}005$ in. and $b/h = 20$. It is to be expected that below a certain value of b/h the effect of the open sides of the blades will start to reduce the pressure at the centre and also that, since b/d (which is proportional to b/h, h/d being constant) will be decreasing, the static hole itself will be increasingly influenced by the pressure towards the sides of the razor blades. Consequently, a value of b/h greater than 30 should be used. In the case of non-standard blades it is probably more important that $b/d > 5$.

It can also be deduced from Fig. 8 that the probable error in Δp resulting from the assumption that h is equal to half the razor-blade thickness will not be greater than ± 2 per cent for the particular brand of 0·005 in. blades used. A rather larger error may be expected with the 0·002 in. blades.

3. *The Application of Razor Blades to Three-dimensional Boundary Layers.*

3.1. *The effect of Razor-blade Yaw.*

The use of small razor-blade segments to measure skin friction is primarily intended for three-dimensional boundary layers. From an experimental point of view the determination of the surface-streamline direction is difficult and so, in addition to the influence of the quantities considered in Section 2, the effect of the misalignment of the blade with the local surface flow is of interest. It was not convenient to investigate this variable using the apparatus described in Section 2. Instead two magnets of diameter $1\frac{1}{2}$ in. were mounted flush with the surface in the roof of a 4 ft × 3 ft low-speed open-return tunnel. These magnets could be rotated and the angle of rotation measured to half a degree on a protractor. This method of rotating the magnet and razor-blade together from outside the tunnel was greatly superior to attempting to reposition the blade alone at different angles of yaw. Each magnet contained a concentric static hole, one of 0·030 in. diameter and the other of 0·090 in. diameter, so as to maintain standard dimensions with blades of height 0·005 in. and 0·015 in. respectively. Each pressure difference was measured with a Betz manometer having a range of 0 to 400 mm of water.

The effect on the measured Δp of varying β (the angle of yaw of the blade relative to the skin-friction direction) over a range of about ± 70 deg is shown in Fig. 9. These results were obtained with the tunnel empty and so correspond to a closely two-dimensional boundary layer. In addition to the two standard blades, two non-standard blades (with increased l/h) were tested and these results are also shown in Fig. 9. It will be seen that $\Delta p_\beta/\Delta p_0$ is closely symmetrical and the different configurations correlate well for $|\beta| < 50$ deg. The scatter of the results obtained using the 0·005 in. high blades is rather large due to the small Δp to be measured. At the top speed of the tunnel (about 90 ft/sec), at which these tests were performed, Δp_0 was approximately 2 mm of water. The curve $\Delta p_\beta/\Delta p_0 = \cos^2\beta$ which has been previously used[3,6] is also shown. A closer fit is obtained with $\Delta p_\beta/\Delta p_0 = \cos\sqrt{2}\,\beta$ which differs from the curve shown by less than $\pm 0{\cdot}02$ for $|\beta| < 50$ deg.

A large obstruction was next put in the flow to produce a three-dimensional boundary layer in the vicinity of the magnets. The plan of this obstruction, which extended in height half way across the tunnel (i.e. 18 in.) is shown in Fig. 10a. In these tests only the magnet with the 0·030 in. diameter static hole was used and both the 0·005 in. and 0·015 in. high standard blades were tested on it in turn. This was done as it was necessary to obtain the same boundary-layer conditions for each blade and the two magnets were not interchangeable. Three positions were tested and the results are shown in Fig. 10a–c. The positions relative to the model are shown in the sketches and the cross flow in the boundary layer increased as the measuring point was moved closer to the model. The cross-flow angle, ϕ, (defined as the angle between the local freestream and the surface streamline) has been estimated approximately from previous unpublished measurements and is given in Fig. 10a to c as 15, 30 and 50 deg respectively. The approximate position of the three-dimensional separation line is also sketched and shows that the third position (Fig. 10c) is under the separated region of the flow. Fig. 10a to c show that symmetry is maintained in all cases. Further, the axes of symmetry ($\beta = 0$) lie, to within the experimental accuracy of the oil-flow technique (about ± 3 deg), along the surface-flow direction. This fact, together with the above mentioned symmetry, suggest that the change in crossflow angle with distance from the wall (the profile twist) has a negligible effect. This deduction is further supported by the fact that the values of the skin-friction coefficient, calculated from the measurements for the razor blade heights using equation (3), agree at each position to within 1 per cent. In calculating the skin friction from the Δp measured with the 0·015 in. high blade on the 0·030 in. static hole, which is not a standard configuration, a correction in accordance with Fig. 8a has been applied ($\Delta p/0{\cdot}925$). Referring again to Fig. 10a to c it will be seen that the deviation from the mean two-dimensional curve is small for $|\beta| < 30$ deg in Fig. 10a and b and does not exceed 7 per cent of Δp_β for $|\beta| < 45$ deg. The same is true for the 0·005 in. blade in Fig. 10c but the 0·015 in. blade shows a consistent error over the whole β range. Thus, for five of the six combinations of razor-blade sizes and three-dimensional boundary layers tested, the variation of $\Delta p_\beta/\Delta p_0$ with $|\beta| \leqslant 30$ deg is effectively the same as in a two-dimensional boundary layer. If experiments are to be undertaken under conditions for which this unique relationship holds, then a simple method of measuring the skin-friction vector can be deduced which does not require preliminary measurement of the vector direction, and this method is considered in detail in Section 3.2. A tentative estimate of the range of conditions for which the unique relationship holds can be made from the present data. It is assumed that the relationship changes significantly when the angle between the skin-friction vector and the velocity vector at $y = h$ reaches a certain value. This angle is very approximately proportional to the product of the boundary-layer crossflow angle, ϕ, and the proportion of the boundary-layer thickness (δ) spanned by the razor blade (i.e. h/δ). Using previous unpublished experimental results to estimate δ, it is tentatively suggested that significant deviations from the two-dimensional form of the curve of $\Delta p_\beta/\Delta p_0$ as a function of $|\beta|$ for $|\beta| < 30$ deg will not occur provided $h\phi/\delta < 0{\cdot}5$ deg. This limit is probably conservative since there is some doubt as to whether the results given in Fig. 10c for the 0·015 in. blade are entirely relevant in this connection as the configuration is not standard.

A probable limitation to the use of razor blades in three-dimensional boundary layers should be mentioned, which is inferred from the knowledge[11] that the calibration of Preston tubes in two-dimensional boundary layers is significantly affected by severe pressure gradients. It is probable that the razor-blade calibration will also be affected but as yet this has not been investigated. Since three-dimensional boundary layers are usually accompanied by pressure gradients, which can be very severe, it is likely that the skin friction deduced will be in error and will probably be an overestimation[11]. No estimation of the magnitude of this error can be deduced from the present work and indeed its experimental assessment would be very difficult.

3.2. *A Proposed Experimental Technique for Measuring Skin-friction Magnitude and Direction.*

3.2.1. *Description of the technique.* This technique rests solely on the assumption that the ratio $\Delta p_\beta/\Delta p_0$ is a unique function of $|\beta|$ up to a certain limit of $|\beta|$. Then if two measurements are made with the same blade positioned in two known directions, which are near to the skin-friction direction, the magnitude and direction of the skin-friction vector can be deduced. In particular if the angle between the two blade positions is kept constant at 30 deg then the direction of the skin-friction vector can be deduced from the ratio $\Delta p_\beta/\Delta p_{(\beta-30)}$ by the use of Fig. 11. A similar graph for any other blade spacing can easily be deduced from Fig. 9. If the skin-friction direction is not known to within 30 deg further readings can be taken at 30 deg intervals until the particular 30 deg arc in which the skin-friction vector lies is covered, and this will be evident from the nature of the successive values of Δp. Once the direction is known, Δp_0 can be deduced from Fig. 9. Experimentally this technique obviates the need to use the rather inconvenient oil-flow technique and enables the skin friction distribution on the model to be determined at any number of incidences from only two or three razor-blade settings at prescribed angles.

3.2.2. *Estimation of the accuracy of the technique.* Consideration has already been given in Section 2.4. to the errors that are likely to arise when using razor-blade segments in two-dimensional boundary layers. This showed that providing the configuration was standard, or at least came within the dimensional limits deduced, then the largest source of error in the measured value of $\Delta p_{(\Delta x=0)}$ was likely to result from insufficient accuracy in positioning the blade leading edge relative to the leading edge of the static hole. If the blades were placed accurately then the errors would be dependent upon the blade asymmetry and the measurement of Δp. The indirect method proposed in Section 3.2.1 for the measurement of Δp_0 and skin-friction direction in three-dimensional boundary layers may also be expected to contribute to the overall errors. This contribution is best estimated analytically using the empirical relationship:

$$\frac{\Delta p_\beta}{\Delta p_0} = \cos\sqrt{2}\,\beta \tag{4}$$

which differs from the empirical curve by less than 2 per cent of Δp_0 for $|\beta| < 50$ deg. From equation (4),

$$\frac{\Delta p_\beta}{\Delta p_{(\beta-30)}} = \frac{\cos\sqrt{2}\beta}{\cos\sqrt{2}(\beta-\pi/6)}.$$

Taking the logarithm of this equation and differentiating gives,

$$\frac{\delta(\Delta p_\beta/\Delta p_{(\beta-30)})}{(\Delta p_\beta/\Delta p_{(\beta-30)})} = -\sqrt{2}\,[\tan\sqrt{2}\beta - \tan\sqrt{2}(\beta-\pi/6)]\,\delta\beta \tag{5}$$

which can be written as

$$\delta\beta\,(\text{radians}) = -\left\{\frac{\cos(\pi/3\sqrt{2})+\cos\sqrt{2}\,(2\beta-\pi/6)}{2\sqrt{2}\sin(\pi/3\sqrt{2})}\right\}\frac{\delta(\Delta p_\beta/\Delta p_{(\beta-30)})}{\Delta p_\beta/\Delta p_{(\beta-30)}}$$

or

$$\delta\beta\,(\text{degrees}) = -\left\{22{\cdot}2+30{\cdot}0\cos\sqrt{2}\,(2\beta-\pi/6)\right\}\frac{\delta(\Delta p_\beta/\Delta p_{(\beta-30)})}{\Delta p_\beta/\Delta p_{(\beta-30)}}. \tag{6}$$

The error in the estimation of β in degrees is therefore, numerically, nearly equal to half the percentage error in $\Delta p_\beta/\Delta p_{(\beta-30)}$ for $\beta \simeq 15$ deg and is less for other values of β. This apparent improvement in the accuracy of β as β tends away from 15 deg is misleading for two reasons. First, the experimental data upon which the calibration curve is based show increasing scatter with β particularly for $\beta > 30$ deg and secondly Δp_β cannot in general be measured to a constant percentage accuracy since its value decreases as β increases. Since Δp_β is estimated in practice from the difference between two pressure differences measured at different times its accuracy should be sensibly independent of its magnitude. Thus, if a particular value of Δp_0 can be measured to an accuracy of $\varepsilon\Delta p_0$ then,

$$\delta\Delta p_\beta = \delta\Delta p_{(\beta-30)} = \varepsilon\Delta p_0$$

and therefore,

$$\frac{\delta\Delta p_\beta}{\Delta p_\beta} = \frac{\varepsilon\Delta p_0}{\Delta p_\beta} = \frac{\varepsilon}{\cos\sqrt{2}\beta}$$

and

$$\frac{\delta\Delta p_{(\beta-30)}}{\Delta p_{(\beta-30)}} = \frac{\varepsilon}{\cos\sqrt{2}\,(\beta-\pi/6)}.$$

The r.m.s. error is given by,

$$\frac{\delta(\Delta p_\beta/\Delta p_{(\beta-30)})}{\Delta p_\beta/\Delta p_{(\beta-30)}} = \varepsilon\left(\frac{1}{\cos^2\sqrt{2}\beta}+\frac{1}{\cos^2\sqrt{2}\,(\beta-\pi/6)}\right)^{\frac{1}{2}}. \tag{7}$$

Combining (5) and (7) gives after re-arrangement,

$$\delta\beta\,(\text{radians}) = -\varepsilon\frac{(\cos^2\sqrt{2}\beta+\cos^2\sqrt{2}\,(\beta-\pi/6))^{\frac{1}{2}}}{2\sin(\pi/3\sqrt{2})}. \tag{8}$$

Substituting values of β into this equation shows that the accuracy of β remains almost constant for $0<\beta<30$ deg and that $\delta\beta$ in degrees is, numerically, nearly equal to three quarters of the percentage accuracy to which Δp_0 can be measured.

Once β has been estimated from the measured values of Δp_β, $\Delta p_{(\beta-30)}$ etc. the value of Δp_0, from which the skin friction will be deduced, is found from one of the equations,

$$\Delta p_0 = \frac{\Delta p_\beta}{\cos\sqrt{2}\,\beta} = \frac{\Delta p_{(\beta-30)}}{\cos\sqrt{2}\,(\beta-\pi/6)} \text{ etc.}$$

Differentiation of the logarithm of the first of these gives,

$$\frac{\delta\Delta p_0}{\Delta p_0} = \frac{\delta\Delta p_\beta}{\Delta p_\beta} + \sqrt{2}\tan\sqrt{2}\,\beta\,\delta\beta$$

and so the r.m.s. error is given by,

$$\text{error} = \varepsilon\sqrt{\frac{1}{\cos^2\sqrt{2}\beta} + 2\left(\frac{\delta\beta}{\varepsilon}\right)^2 \tan^2\sqrt{2}\,\beta}\,. \tag{9}$$

This function rises rapidly with β and shows that for $\beta = 0$ the error is ε, for $\beta = 15$ deg the error is $1{\cdot}3\varepsilon$ and for $\beta = 30$ deg the error is $2{\cdot}2\varepsilon$. Thus, it is important to estimate Δp_0 from the measured value of Δp_β nearest to it and in this case the penalty resulting from this indirect method will not exceed 30 per cent of the basic experimental error that would have occurred in the direct measurement of Δp_0.

It is concluded that the additional errors due to the use of this technique are small and that the overall accuracy is in practice still mainly dependent upon the accuracy with which the razor blade can be positioned relative to the static hole.

3.2.3. *Trial application of the technique.* As a test of the technique proposed in Section 3.2.1, the skin-friction distribution under a leading-edge vortex was measured in the 13 ft × 9 ft wind tunnel. The model used was the 30 deg rhombic cone previously used by Wyatt and East[6], and its plan and section are shown in Fig. 12, together with the characteristic features of the flow. Measurements were made along a spanwise line at $x/c = 0{\cdot}777$. For two speeds (150 and 250 ft/s) and three incidences ($\alpha = 6$, 10 and 14 deg) measurements were made, both with the razor blades aligned with the local surface-flow, as given by oil flow, and at angles of 0, 30 and 60 deg to the model centreline. An attempt was also made to measure the local surface-flow direction from the oil flow patterns. The complete procedure was then repeated with a different person setting up the razor blades. It is estimated that the maximum value of $h\phi/\delta$ encountered during this test was approximately unity and so was above the limit deduced in Section 3.1.

The results are presented in Figs. 13 and 14. The surface cross-flow angles relative to the model centreline have been computed from the razor blade measurements at $\theta = 0$, 30 and 60 deg using Fig. 11. These are compared in Fig. 13 with the values obtained by direct measurement of the oil-flow pattern. Reasonable agreement has been achieved and it is not thought that there is sufficient disagreement or inconsistency in the results to suggest a deficiency in the method.

The values of Δp measured with the razor blades aligned with the oil-flow pattern are compared in Fig. 14 with those deduced from the measurements taken at angles of 0, 30 and 60 deg. Since the skin friction would in both cases be deduced from Δp by the use of the two-dimensional calibration (3), this comparison of the values of Δp is sufficient. Once again, the results do not suggest a deficiency in the method and indeed it appears that the indirect method has produced marginally greater consistency between the two runs than the direct method. This is probably due to the greater accuracy that can be achieved when setting the blades to a prescribed angle than to an oil flow pattern. The latter process was done entirely by eye and consisted of pushing the blade forward over the static hole until the hole was 'just not visible'. In the former case a small rectangular metal block, which adhered to the magnet, was positioned so that one of its faces normal to the model surface was tangential to the static hole. This could be done fairly accurately as the static hole was not covered and its reflection could be seen in the face of the block. Adjusting the block so that the static hole and its image appeared to touch ensured that it was in the required position. The angular position of the block was set up by either aligning the opposite face with a fixed straight edge (for 0 deg) or *via* two edges of a 30–60 deg square to the straight edge (for 30 deg and 60 deg). Once the block was in position the razor blade was pushed forward over the static hole until it touched the face of the block, which was then removed.

This short test has shown that the technique can be applied satisfactorily and easily to models and that it will give results which in practice are at least as accurate as the direct method. A proviso must however be placed on this conclusion concerning the rather ill-defined limit at which the technique may be expected to break down on account of the rapidity of rotation of the velocity vector with height from the surface.

4. *Conclusions.*

The conclusions are given below in terms of the List of Symbols. The razor blade configuration is shown in Fig. 1.

4.1. *Two-dimensional Turbulent Boundary Layers at Low Subsonic Speeds.*

(1) The calibration in zero pressure gradient of surface-pitot tubes formed by razor blades is not significantly affected by changes in the proportions of the dimensions of the configuration, providing the proportions lie within certain limits. These limits are that b/h should be greater than 30 and d/h greater than 3, with the probable additional condition that b/d should be greater than 5.

(2) A razor blade shape which has been found convenient in use and which complies with the limits given in (1) has the proportions $d/h = 6$, $b/h = 36$ and $l/b = 1$ and the blade is positioned so that $\Delta x = 0$. This configuration has been taken as a standard and a calibration curve has been produced.

(3) The greatest source of error arises from Δx not being zero. This error does not exceed 5 per cent of Δp providing $|\Delta x/h| < 0{\cdot}5$, which is equivalent to $|\Delta x/d| < 0{\cdot}08$ when using the suggested value of $d/h = 6$.

(4) Removing a portion of the thickness of the blade asymmetrically above the leading edge (Fig. 1), so as to reduce its effect upon the external flow, will change the calibration.

4.2. *Three-dimensional Turbulent Boundary Layers at Low Subsonic Speeds.*

(5) Using razor blade segments of different thickness, but of symmetrical section, aligned with the surface-flow direction, the same value of skin friction was obtained when the two-dimensional calibration curve was used. This suggests that the true skin friction was being measured, although there may be an error due to the effect on the calibration of static-pressure gradients, which cannot be assessed.

(6) The effect on the measured Δp of yawing a razor blade relative to the local skin-friction vector direction is symmetrical about that direction to within the accuracy of determining the direction by the oil-flow technique (± 3 deg).

(7) $\Delta p_\beta/\Delta p_0$ is found to be a unique function of $|\beta|$ for $|\beta| \leqslant 30$ deg for both two-dimensional and some three-dimensional boundary layers. The limits at which the function ceases to be unique is uncertain, though the present results suggest that providing $h\phi/\delta < 0{\cdot}5$ deg the function will be unique and it is possible that this limit is conservative.

(8) A technique based on (7) has been proposed whereby both the magnitude and direction of the skin-friction vector can be measured. The technique has been tested on a model and found to be convenient and accurate. It is particularly appropriate when measurements are required for a large number of model incidences and tunnel speeds and it might also be possible to use it to obtain full-scale flight measurements, though the difficulties arising from the lack of repeatability of flight conditions would have to be overcome.

LIST OF SYMBOLS

b	Breadth of razor-blade segment (Fig. 1)
c_0	Root chord of rhombic-cone model (Fig. 12)
C_P	Static-pressure coefficient measured relative to working-section reference pressure
d	Diameter of static-pressure hole (Fig. 1)
h	Height of razor blade from model surface to cutting edge (Fig. 1): outside diameter of Preston tube
l	Length of razor-blade segment (Fig. 1)
M	Mach number
Δp	Pressure measured by razor blade or Preston tube minus local static pressure (blade or tube removed)
Δp_β	Value of Δp measured with blade at an angle of β to skin-friction vector (similarly Δp_0, $\Delta p_{\beta-30}$, etc.)
Δp_s	Mean value of Δp measured by standard razor blade both ways up
q_0	Reference freestream kinetic pressure
s_x	Local semi-span of rhombic cone
U	Freestream velocity
U_τ	Friction velocity. $U_\tau = (\tau/\rho)^{\frac{1}{2}}$
x	Streamwise distance measured from the leading edge of the plate (Fig. 2) or from the apex of the rhombic cone (Fig. 12)
Δx	Position of razor blade relative to the upstream edge of the static hole measured normal to the razor-blade cutting edge (Fig. 1)
x^*	Defined as $\log_{10}\left\{\frac{\Delta p h^2 \rho}{\mu^2}\right\}$
y	Transverse distance across rhombic cone from centreline (Fig. 12)
y^*	Defined as $\log_{10}\left\{\frac{\tau h^2 \rho}{\mu^2}\right\}$
z	Transverse distance across plate measured from centreline (Fig. 3)
α	Angle of incidence of rhombic cone
β	Angle of yaw relative to the direction of the skin-friction vector measured in the plane of the model surface
δ	Boundary-layer thickness
Θ	Blade cutting angle (Fig. 1)
θ	Angle of yaw relative to the undisturbed freestream measured in the plane of the model surface
μ	Fluid viscosity
ν	Kinematic viscosity of fluid

LIST OF SYMBOLS—*continued*

ρ	Fluid density
τ	Skin friction
ϕ	The boundary-layer crossflow angle; the angle between the local freestream and the surface-flow direction

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TABLE 1

Dimensions of Razor-blade Segments (in inches)

h = 0·0020 in.		h = 0·0050 in.		h = 0·0153 in.	
l	b	l	b	l	b
0·072	0·071	0·183	0·180	0·541	0·541
		0·100	0·100	0·301	0·301
0·122	0·116	0·302	0·297	0·902	0·902
		0·105	0·181	0·304	0·542
0·120	0·072	0·297	0·178	0·900	0·541
		0·180	0·100	0·541	0·301
0·072	0·121	0·183	0·301	0·542	0·900

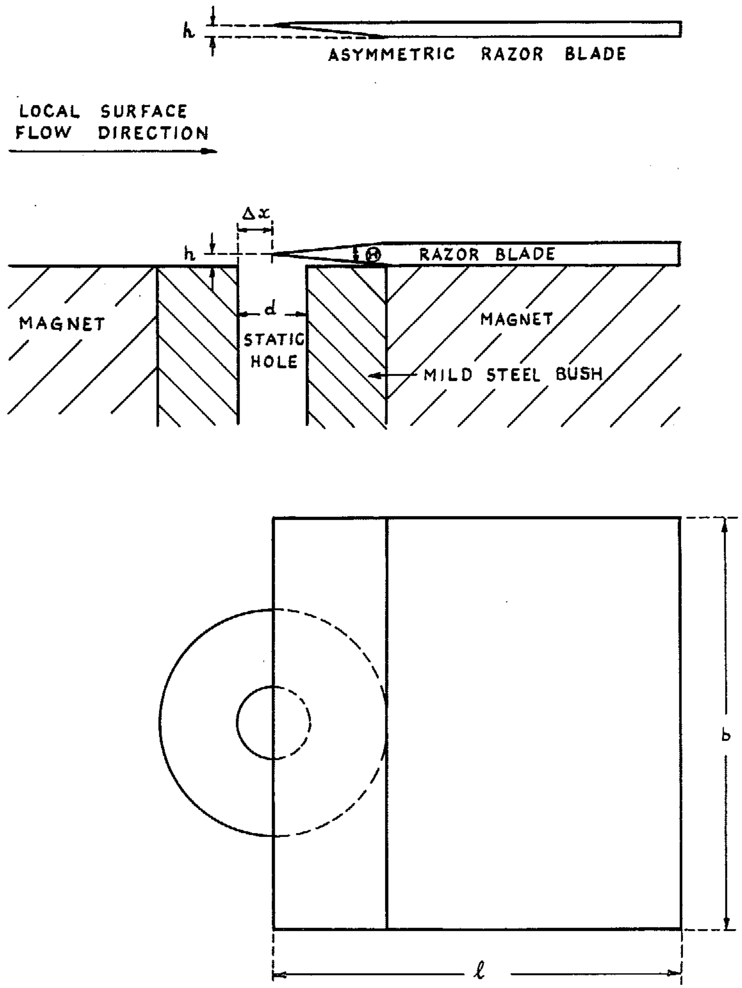


FIG. 1. Surface pitot tube.

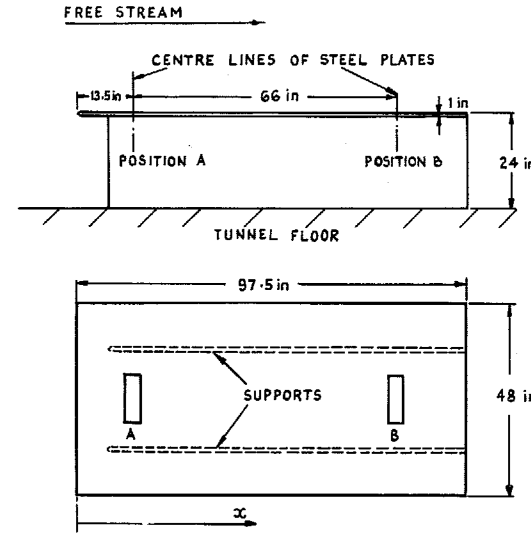


FIG. 2. General arrangement of flat plate.

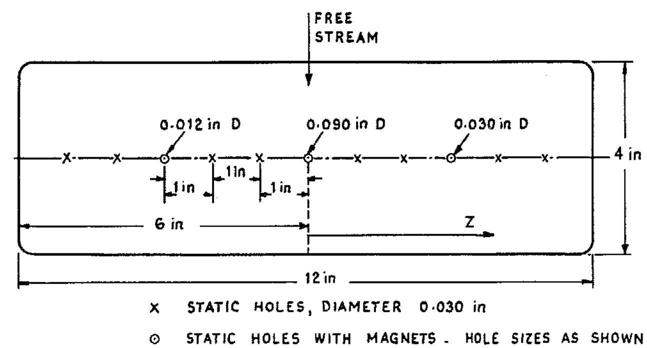


FIG. 3. Details of instrumented steel plate.

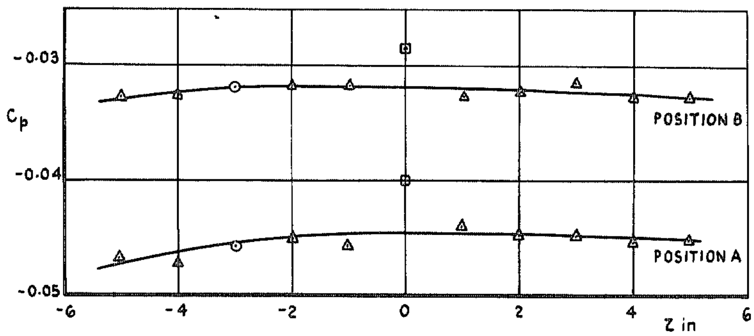


FIG. 4. Static-pressure distribution across steel plates. Diameter of static tappings; ⊙ 0·012 in, Δ 0·030 in, ⊡ 0·090 in, free-stream velocity = 200 ft/s.

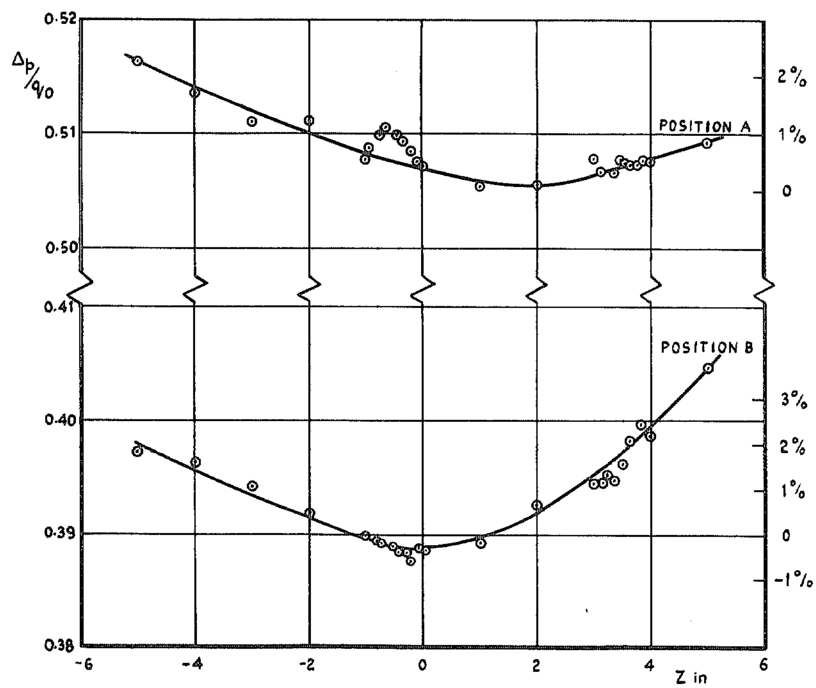
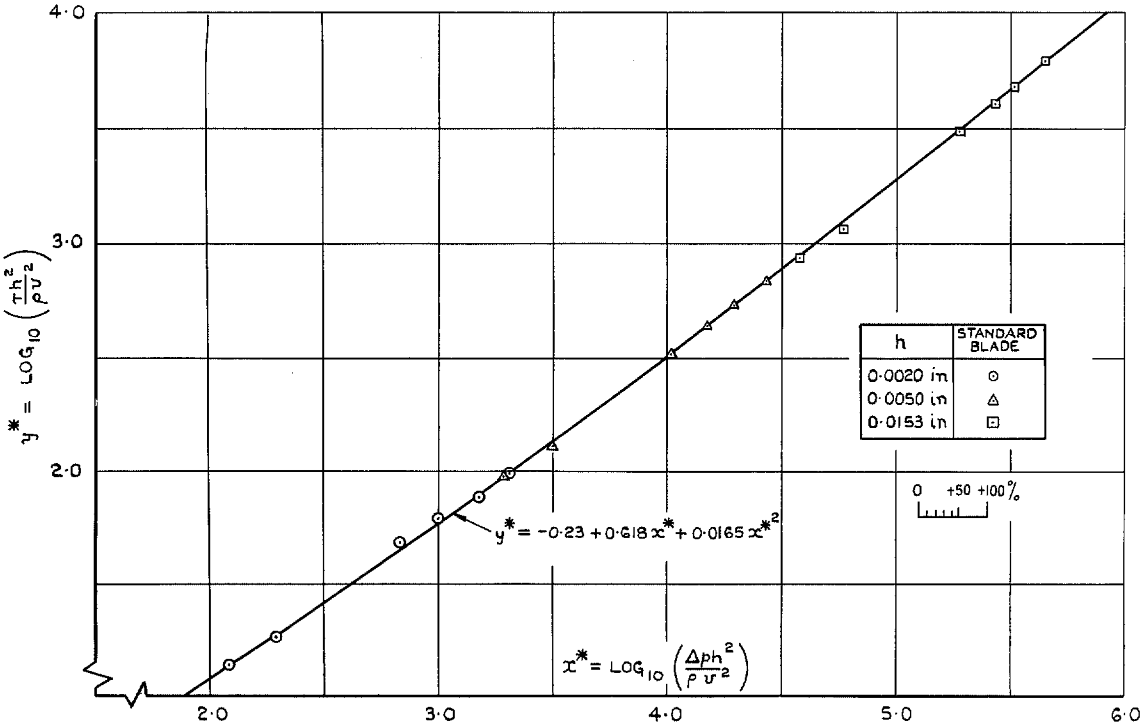
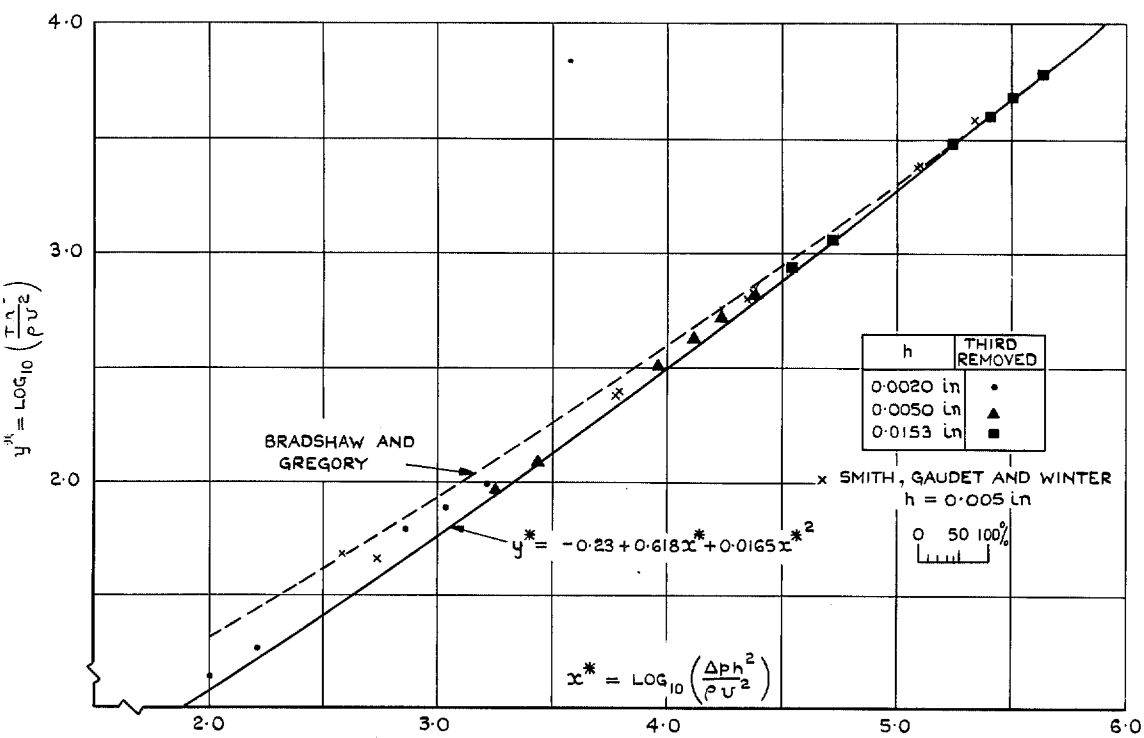


FIG. 5. Preston-tube pressure distribution across steel plates, free-stream velocity = 200 ft/s.



(a) Calibration of standard razor-blade surface-pitot tubes.



(b) Comparison of several surface-pitot calibrations. For details of configurations used *see* Section 2.4.3.

FIGS 6 a & b. Surface-pitot calibrations in low-speed turbulent flow.

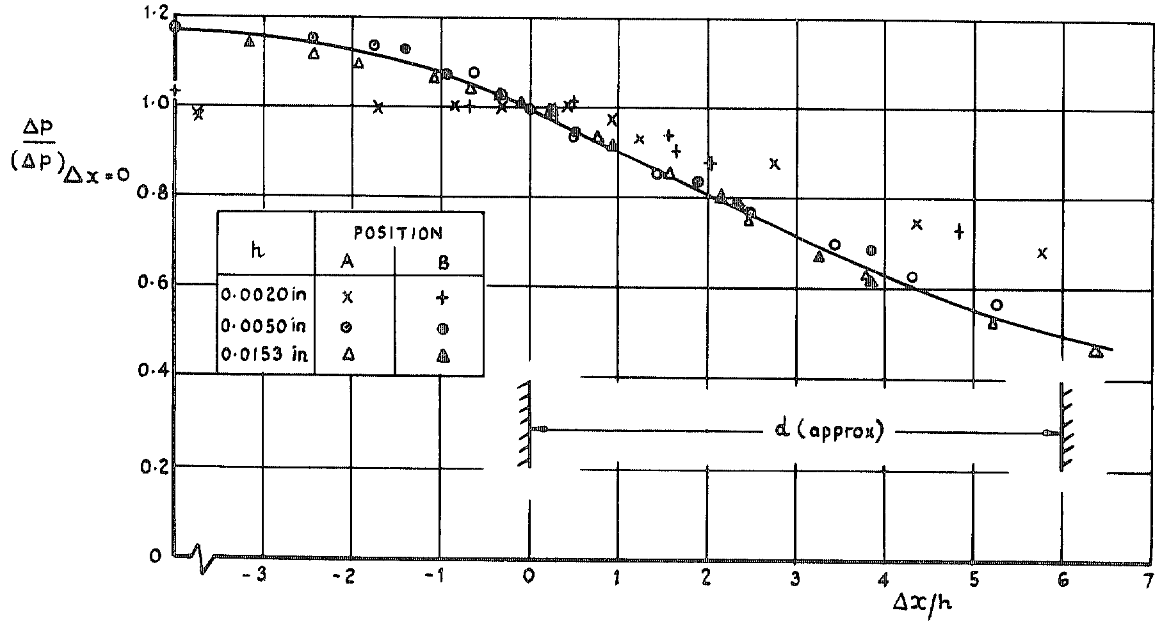
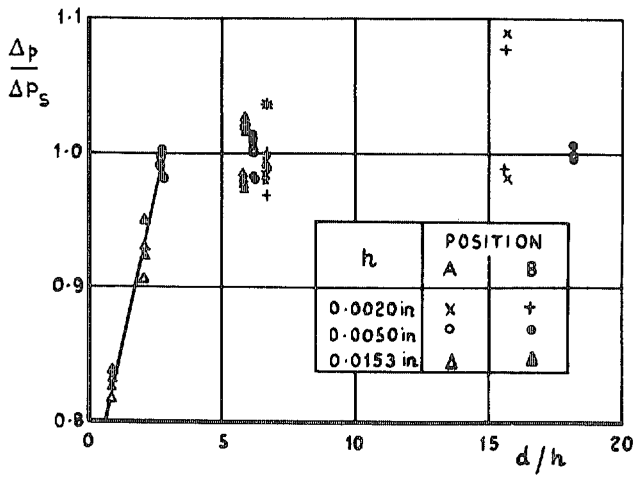
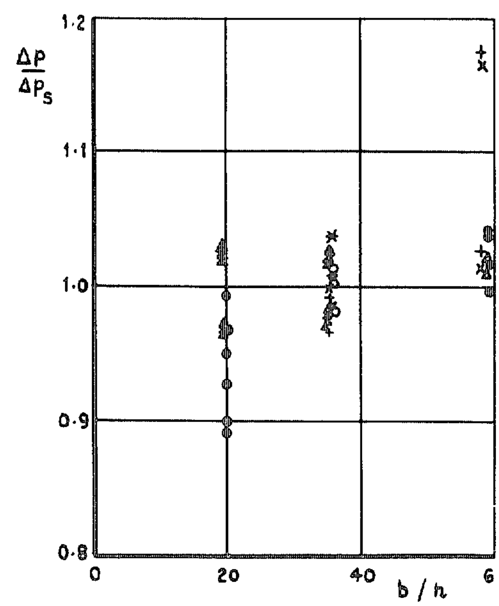


FIG. 7. The effect on Δp of varying the razor-blade longitudinal position relative to the static hole.

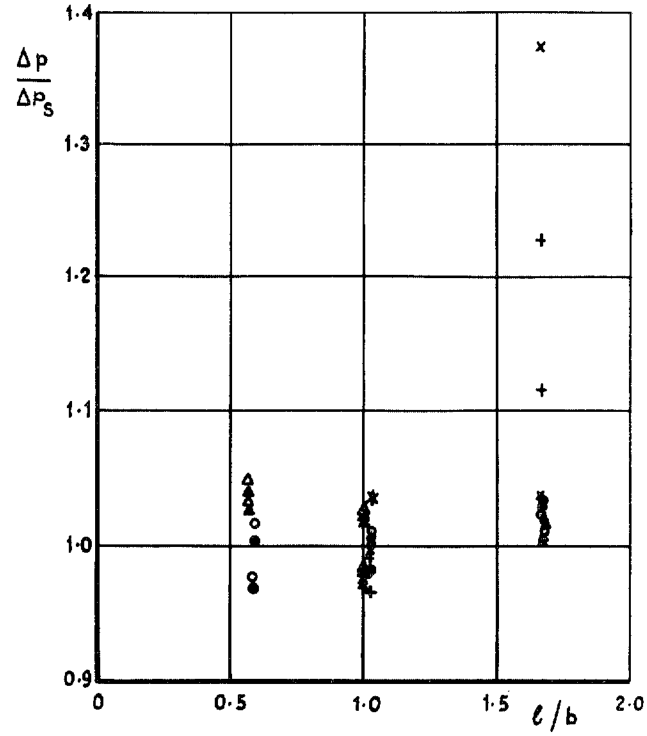


(a) $b/h = 36, \quad l/b = 1$

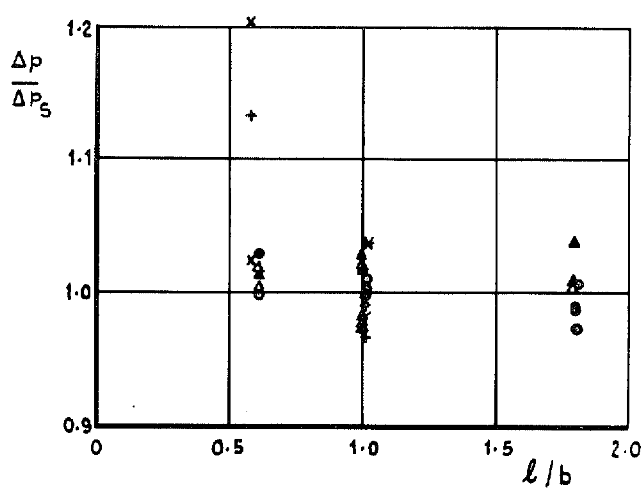


(b) $d/h = 6, \quad l/b = 1$

FIGS. 8a & b. The effect on Δp of varying surface pitot-tube dimensions.



(c) $d/h = 6, \quad b/h = 36$



(d) $d/h = 6, \quad l/h = 36$

FIGS. 8c & d. The effect on Δp of varying surface pitot-tube dimensions.

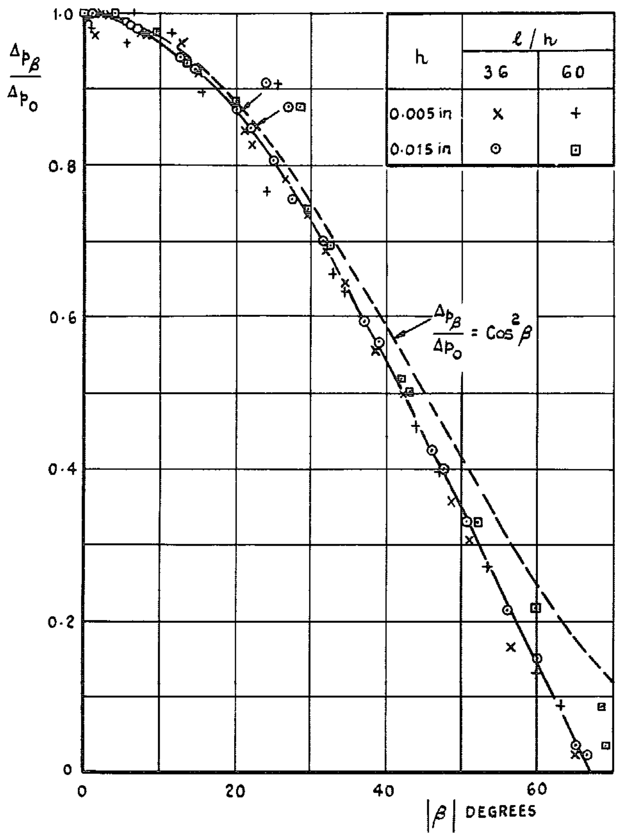
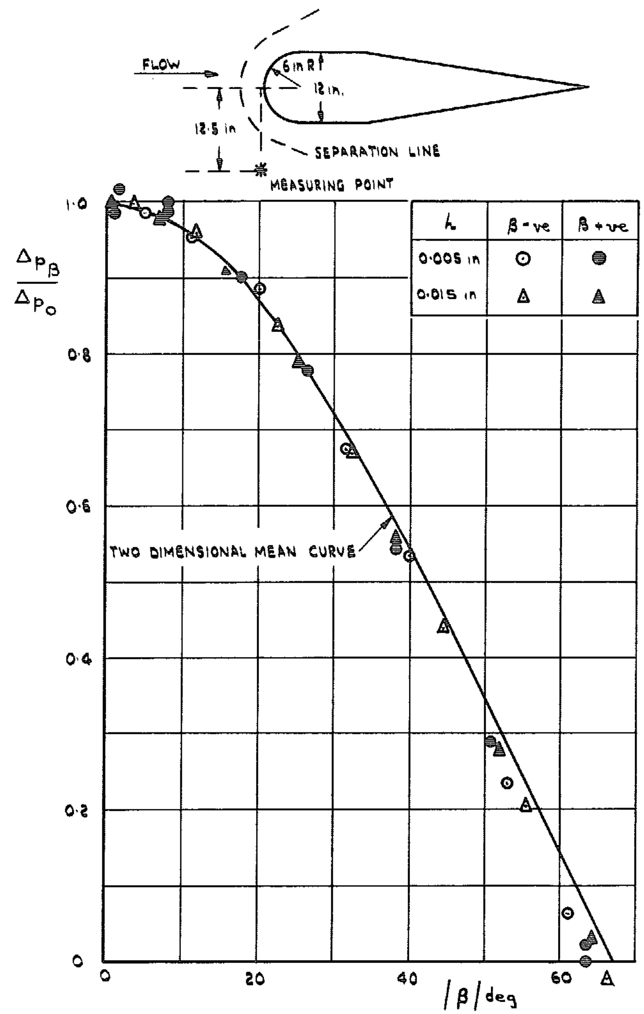
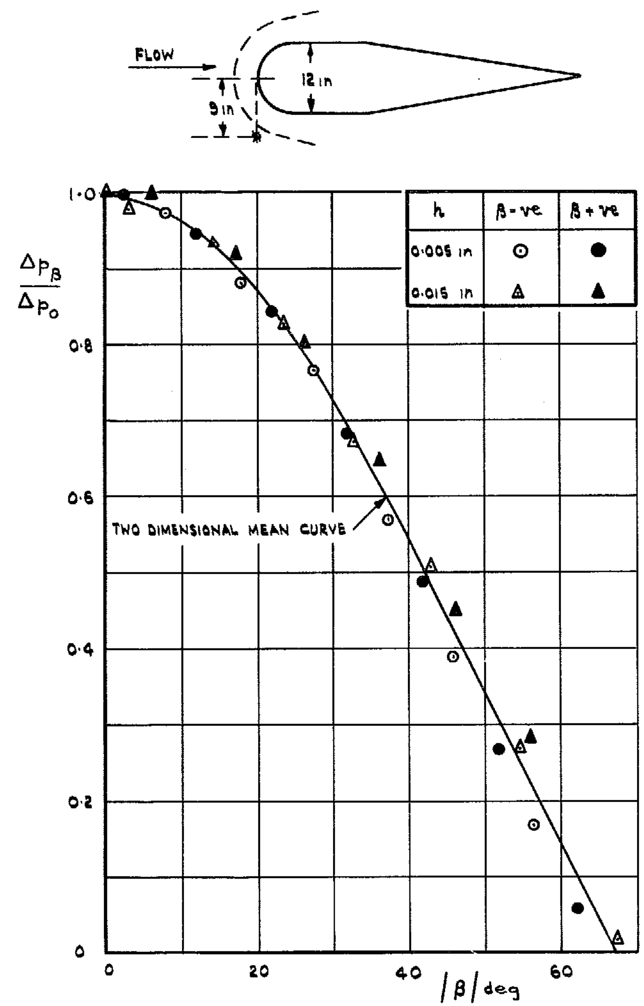


FIG. 9. The effect of razor-blade yaw in a two-dimensional boundary layer.

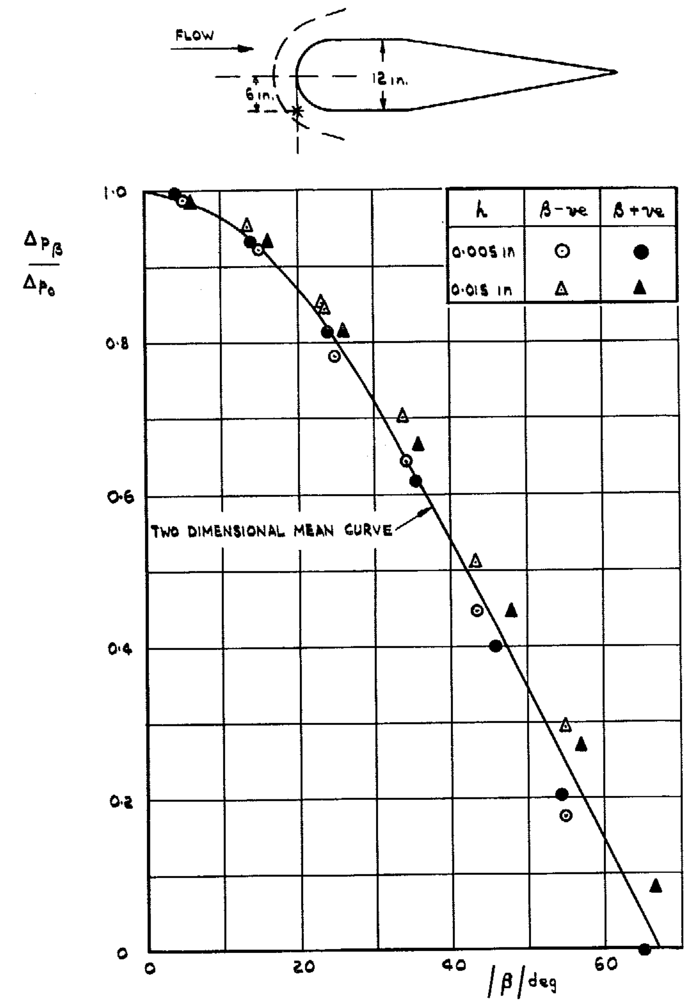


(a) $\phi \approx 15°$

FIG. 10a. The effect of razor-blade yaw in a three-dimensional boundary layer.



(b) $\phi \approx 30°$



(c) $\phi \approx 50°$

FIGS. 10b & c. The effect of razor-blade yaw in a three-dimensional boundary layer.

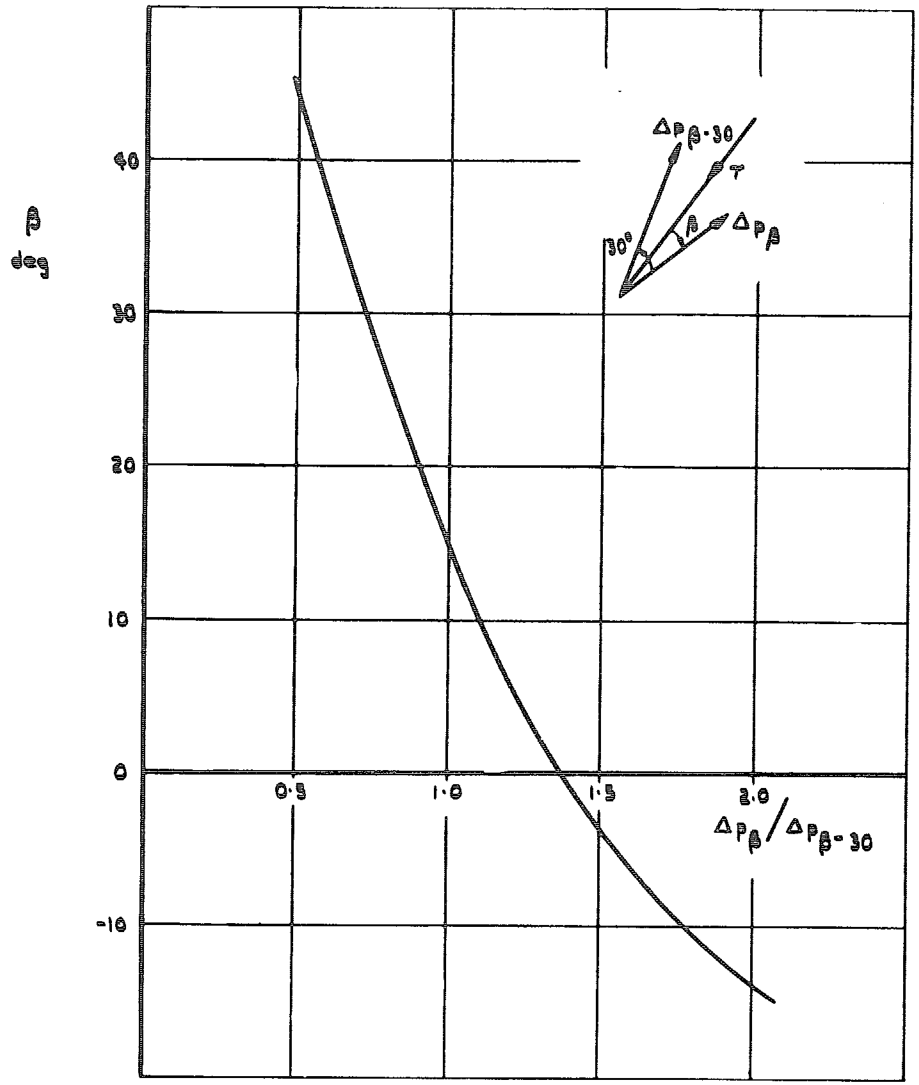
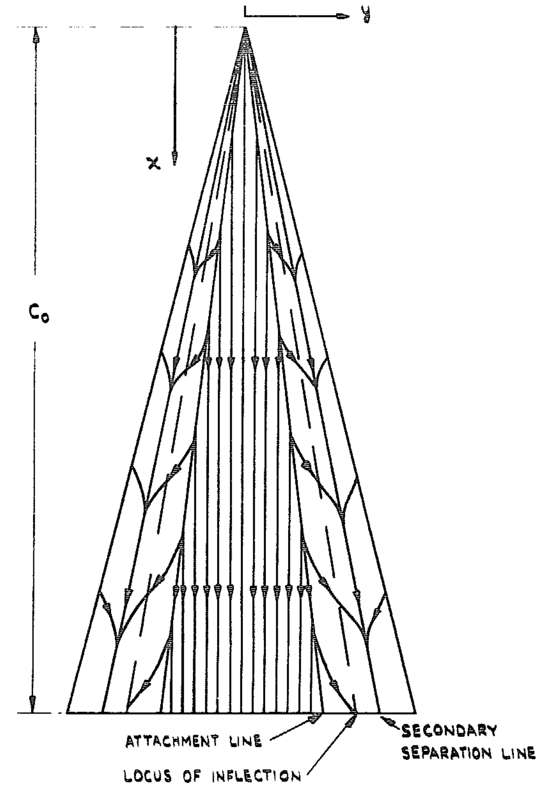
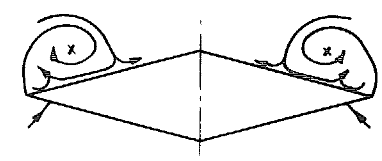


FIG. 11. The relationship between β and $\Delta p_\beta / \Delta_{P(\beta-30)}$ deduced from empirical curve of Fig. 9.



(a) Surface-flow pattern.



(b) Conic section of flow.

FIGS. 12a & b. Flow over a rhombic cone at incidence.

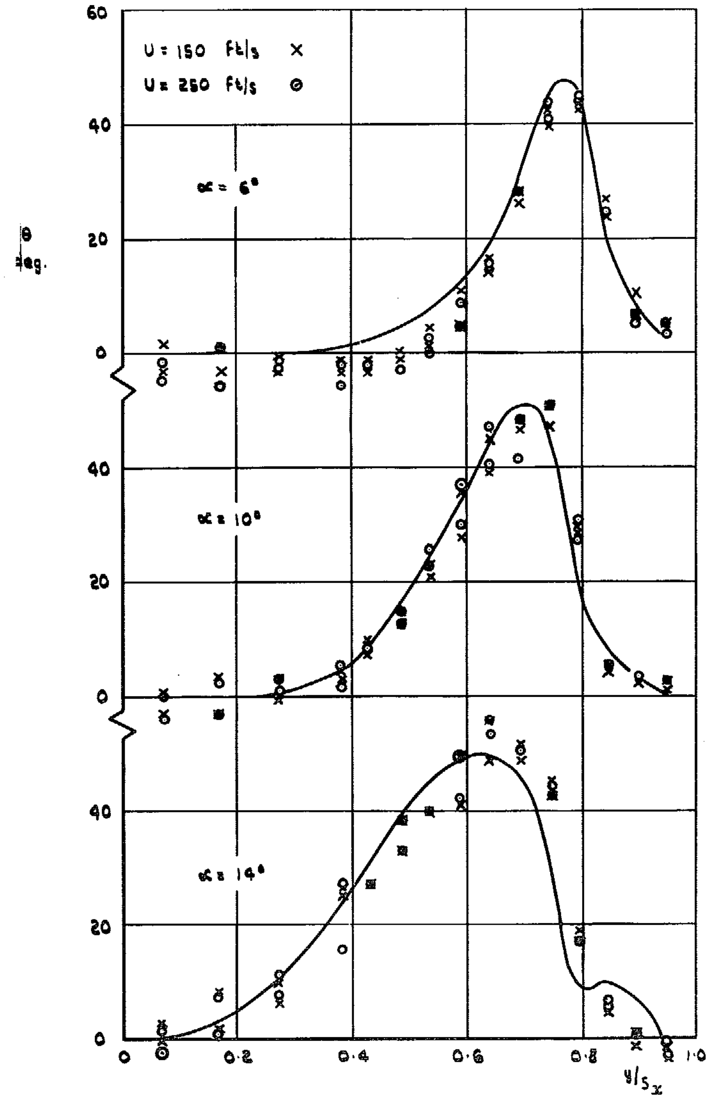


FIG. 13. Surface-flow direction on the upper surface of a rhombic cone at $x/c_0 = 0{\cdot}777$. Curves are from oil-flow measurements.

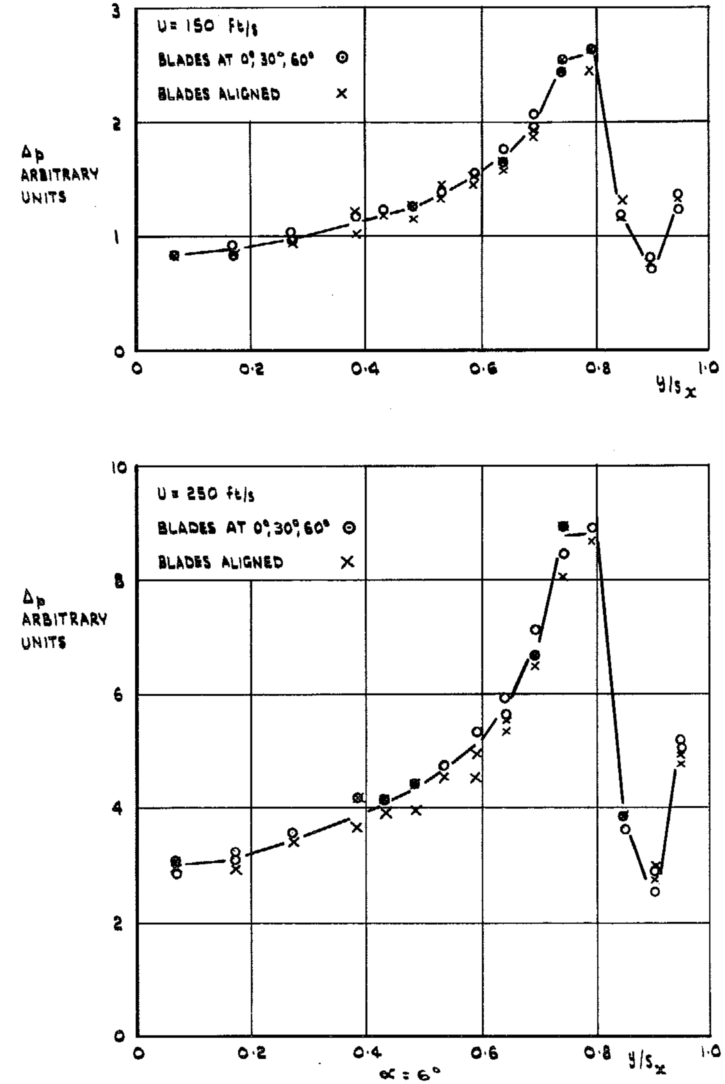
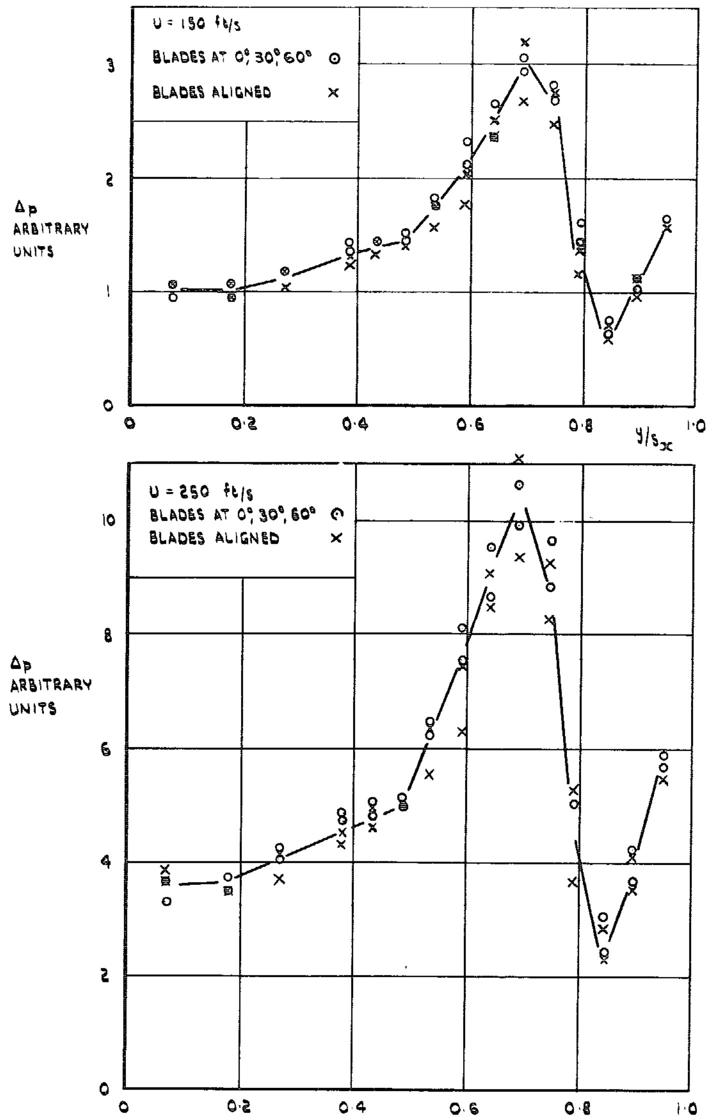
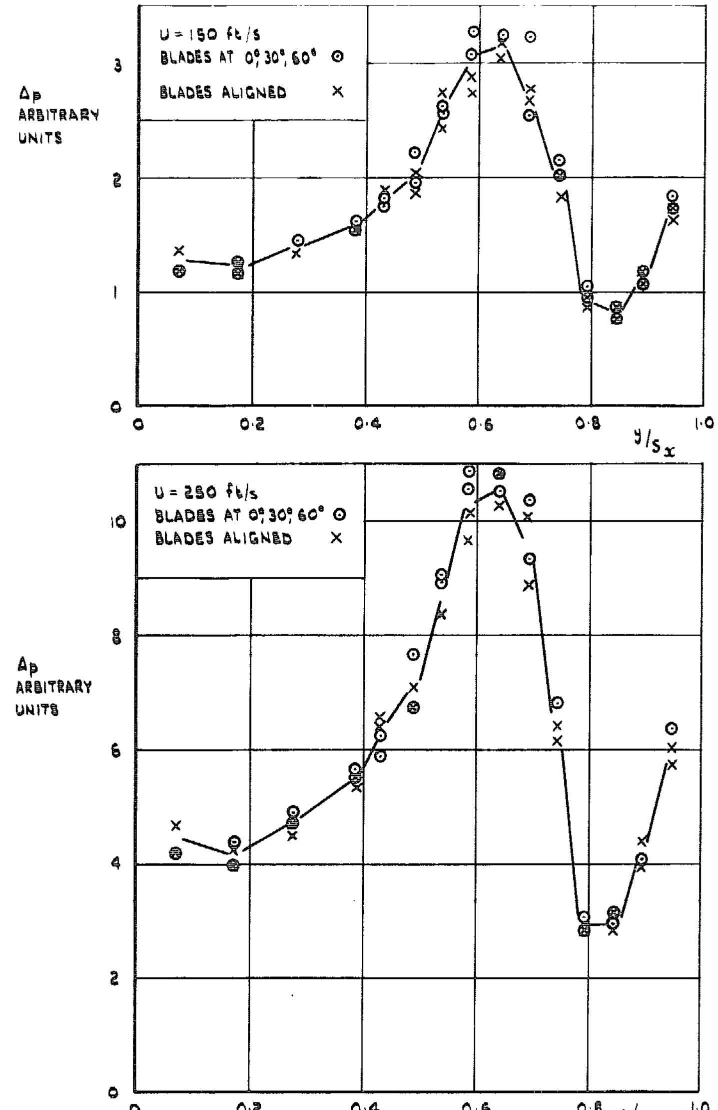


FIG. 14. Comparison between Δp measured with the blades aligned with the surface oil flow and Δp deduced from blades set at $\theta = 0, 30$ and 60 deg.



$\alpha = 10°$

FIG. 14. (cont'd). Comparison between Δp measured with the blades aligned with the surface oil flow and Δp deduced from blades set at $\theta = 0$, 30 and 60 deg.



$\alpha = 14°$

FIG. 14. (concl'd). Comparison between Δp measured with the blades aligned with the surface oil flow and Δp deduced from blades set at $\theta = 0$, 30 and 60 deg.

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