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Some Tests on the Spread of Velocity in a Cold Jet Discharging with Excess Pressure from a Sonic Exit into Still Air

By

J. Seddon, Ph.D. and L. Haverty

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Some tests on the spread of velocity in a cold jet discharging with excess pressure from a sonic exit into still air

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SUMMARY

The spread of the half-velocity circle in a jet discharging with excess pressure from a length of parallel pipe into still air has been examined over the range 40 to 200 diameters from the exit. Jet pressure ratios (outside static pressure ÷ jet total pressure) were from 0.479 to 0.042, corresponding to isentropic Mach numbers from 1.08 to 2.72.

It is found that rates of spread are not greatly different from that for a subsonic jet. The effective exit of an under-expanded jet is, however, displaced some distance downstream of the actual exit. For a jet of isentropic Mach number 1.51, the effective exit position is about 8 diameters downstream and the half-velocity circles lie on a cone of semi-angle 5.7° from the centre of the effective exit.

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1 Introduction

It is known that the rate of spread of a subsonic jet blowing into still air is such that, downstream of about 10 diameters from the jet exit, the "half-velocity circles"* lie close to a cone of semi-angle 5°, having its apex at the centre of the exit. Further, the presence of an external stream flowing in the same direction as the jet restricts the rate of spread to smaller angles. These general results were given by Squire[1] in a review and analysis of existing data.

If the jet is sonic or supersonic at exit, the exit pressure may be different from that in the surrounding atmosphere. In such a case the development of the jet, at least in the early stages, could be expected to differ appreciably from that of a subsonic jet. It was thought desirable to obtain some experimental information on this point and in particular to check whether marked variations could occur in the size of the jet at downstream distances of the order of 100 diameters, when the jet is discharging into a rarefied atmosphere representative of conditions at high altitudes.

2 First experiment

A jet of cold compressed air was blown from a straight pipe into a chamber containing still air. The inside diameter of the pipe was 0.08 inches; the chamber was the working section of a $5\frac{1}{2}" \times 5\frac{1}{2}"$ supersonic tunnel. The total pressure of the jet was varied between 20 and 75 lb/sq in. absolute. The pressure in the chamber was maintained either at ground level atmospheric value (14.7 lb/sq in.) or, by bringing the tunnel suction plant into operation, at an absolute value of 2.5 lb/sq in., which corresponds to atmospheric pressure at 48,000 ft.

A survey of dynamic head in the jet was made at two stations, 62.5 and 100 jet diameters from the exit of the pipe, by means of a pitot-static rake connected to a water manometer. To obtain the jet velocity profiles from these measurements, it was assumed that at the measuring stations the temperature in the jet, and hence the density also, were the same as in the surrounding medium.

Velocity profiles at the two stations, obtained for various total pressures in the jet with atmospheric pressure in the tunnel, are given in Fig.1. Corresponding results obtained with the tunnel evacuated to 2.5 lb/sq in. pressure are given in Fig.2. It is seen that, over the range of pressure ratio covered, there are no large changes in the amount of spreading of the jet at a given position downstream. In Fig.3 the ratio r/x, where r is the radius of the half-velocity circle at distance x from the exit, is plotted against jet pressure ratio and the corresponding isentropic Mach number. The latter is given merely as an indication of the Mach number attainable in the jet by expansion to ambient pressure. Results for the two stations are indicated by different symbols. At all pressure ratios, Squire's rule for subsonic jets is seen to fit well as a mean result for the two stations. There is however, some indication of curving outwards, that is to say the equivalent cone angle is consistently the higher for the station further downstream, and this effect becomes more pronounced as the jet Mach number increases.

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* The "half-velocity circle" is that circle, in a section normal to the axis of the jet, on which the value of velocity is a mean of the values on the axis and in the surrounding medium.

3 Second experiment

In order to resolve the problem further, a second test was made in which the apparatus was rigged on a laboratory bench, away from possible constraint effects of the tunnel walls, and a more complete survey of the jet was made for one pressure ratio. Profiles were obtained at six stations ranging from 37.5 to 200 diameters from the exit. The pressure ratio for this test was necessarily restricted, the isentropic jet Mach number being only 1.51.

The velocity profiles are given in Fig.4, and Fig 5 shows the radius of the half-velocity circle plotted against distance downstream. A comparison of values of this radius at 62.5 and 100 diameters again shows the effect observed in the first results and it is seen that a better "fit" than the 5° cone from the centre of the exit is a cone of half-angle 5.7° from a point on the axis about 8 diameters downstream of the exit. From this, two general results are inferred.

(1) The effective exit position of a jet emerging with an excess pressure ratio from a sonic nozzle is some distance downstream of the actual exit. This distance appears to be between 5 and 10 exit diamaters when the isentropic Mach number of the jet is 1.5.

(2) The angle of spread of the half-velocity circle from this effective exit position is somewhat greater than the 5° given by Squire for a wholly subsonic jet. For a jet of isentropic Mach number 1.5, an angle of 5.7° holds well up to 200 diameters downstream.

The first of these results is as could be expected from the fact that the jet is under-expanded at the sonic nozzle. The point can be followed up through an examination of schlieren pictures of the jet discharging into atmosphere at four values of internal pressure (Fig.6). At the first pressure the jet is only just sonic, hence the core is not visible, but at the other three pressures the length of the core can be judged approximately by the distance over which the chain of supersonic expansion and compression is visible. These distances are plotted in Fig.7, where it is seen that they form a plausible extension of Squire's result that the core of the subsonic jet extends for about 5 diameters.

We may interpret the results in Fig.7 as an effective downstream shift of the nozzle exit when the jet is supersonic, compared with the subsonic jet. For the jet at 55 lb/sq in. pressure ($P_{\infty}/p_{oj} = 0.268$) this downstream shift would be about 8 diameters (a core length, supersonic of 13 diameters, compared with 5 diameters subsonic): this agrees well with the previous suggestion that the effective exit position is 8 diameters downstream.

Using the subsonic core length of 5 diameters as a datum, the position of the effective exit, supersonically is as shown by the dotted curve in Fig.7. Although it is not possible to extrapolate this curve to really low pressure ratios (high Mach number jets) it seems probable that in such cases the downstream displacement of effective exit would be very considerable. This result is of interest because some recent results of measurements in the wake of a rocket jet,* tested at ground level, suggested that at a downstream distance of the order of 100 diameters the radius of the half-velocity circle corresponded to a cone semi-angle of only about 4° from the jet exit. It is seen that the result can probably be explained in terms of downstream displacement of the effective exit of an under-expanded jet.

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* Tests by R.P.D. Westcott, not yet published.

As regards the mean angle of spread downstream of an effective exit, the reason for the discrepancy between Squire's rule and the 5.7° of the present tests is not clear, particularly as the supersonic jet, outside and beyond the core rapidly becomes subsonic. It is suggested that the second conclusion above should be regarded as provisional only, until further experimental evidence for supersonic jets, particularly at higher Mach numbers than apply to the present tests, becomes available.

4 Additional features of the results

4.1 *Jet momentum*

In the development of a jet by mixing it is normally assumed that the total momentum of the jet is constant. At a downstream station the total momentum is

$$\int \rho V^2 dA$$

At the jet exit the total momentum (assuming uniform conditions) is:

$$(\rho_j V_j^2 + p_j - p_\infty)\, A_j$$

which, for a sonic exit, may be written

$$[(\gamma + 1)\, p_j - p_\infty]\, A_j$$

Thus, with $\gamma = 1.4$ and since $p_j = 0.528\, p_{oj}$

$$\frac{\text{Momentum}}{A_j\, p_{oj}} = 1.269 - \frac{p_\infty}{p_{oj}} \qquad (1)$$

Using results from the first experiment, the jet momentum at the two downstream stations was determined by graphical integration of the measured profiles. This was done for one jet total pressure at each of the two external pressures. The results are shown in Fig.8 where they are also compared with values calculated for the jet exit by means of equation (1). It is seen that good agreement between the two stations is obtained and that, at both pressure ratios, the downstream values agree with the exit value provided that the effective exit area of the jet pipe is taken to be 93% of the actual area. This is thought to be a plausibly correct value for the particular jet pipe used, which was a long parallel pipe with no final contracting nozzle.

4.2 Axial velocity

From the result of the second experiment, the variation of velocity on the axis of the jet is as shown in Fig.9. The curve is approximately hyperbolic in form. Squire shows that for a subsonic jet $V_a x/V_j d$ is constant, where V_a is the velocity on the axis at distance x/d, and V_j is the jet exit velocity. For a supersonic jet we substitute V_m, the maximum velocity corresponding to the jet pressure ratio p_∞/p_{oj}, for V_j and allow for the fact that with a sonic exit followed by free expansion, the effective exit position is, as discussed in section 3 above, some distance downstream of the actual exit - about 8 diameters in the present case. Fig.9 shows that the value of the expression:

$$\frac{V_a}{V_m}\left(\frac{x}{d} - 8\right)$$

is constant over the whole range of the test. If the value of V_m is calculated on the assumption that the stagnation temperature of the jet is equal to laboratory temperature, the constant value of this expression is 8.2 which is in agreement with Rolls Royce and previous R.A.E. data quoted by Squire.

LIST OF SYMBOLS

ρ = stream density

V = stream velocity

M = Mach number

p = pressure

A = cross sectional area of jet at relevant station

d = internal diameter of jet pipe

x = distance along jet from pipe exit

y = distance out from jet axis

r = radius of half-velocity circle

∞ refers to outside atmosphere

j refers to jet exit

o refers to stagnation condition

a refers to axis of jet

m maximum velocity on jet axis

REFERENCES

No.	Author	Title, etc.
1	H. B. Squire	Jet flow and its effects on aircraft. R.A.E. Report No. Aero.2171, September, 1946. A.R.C.10189

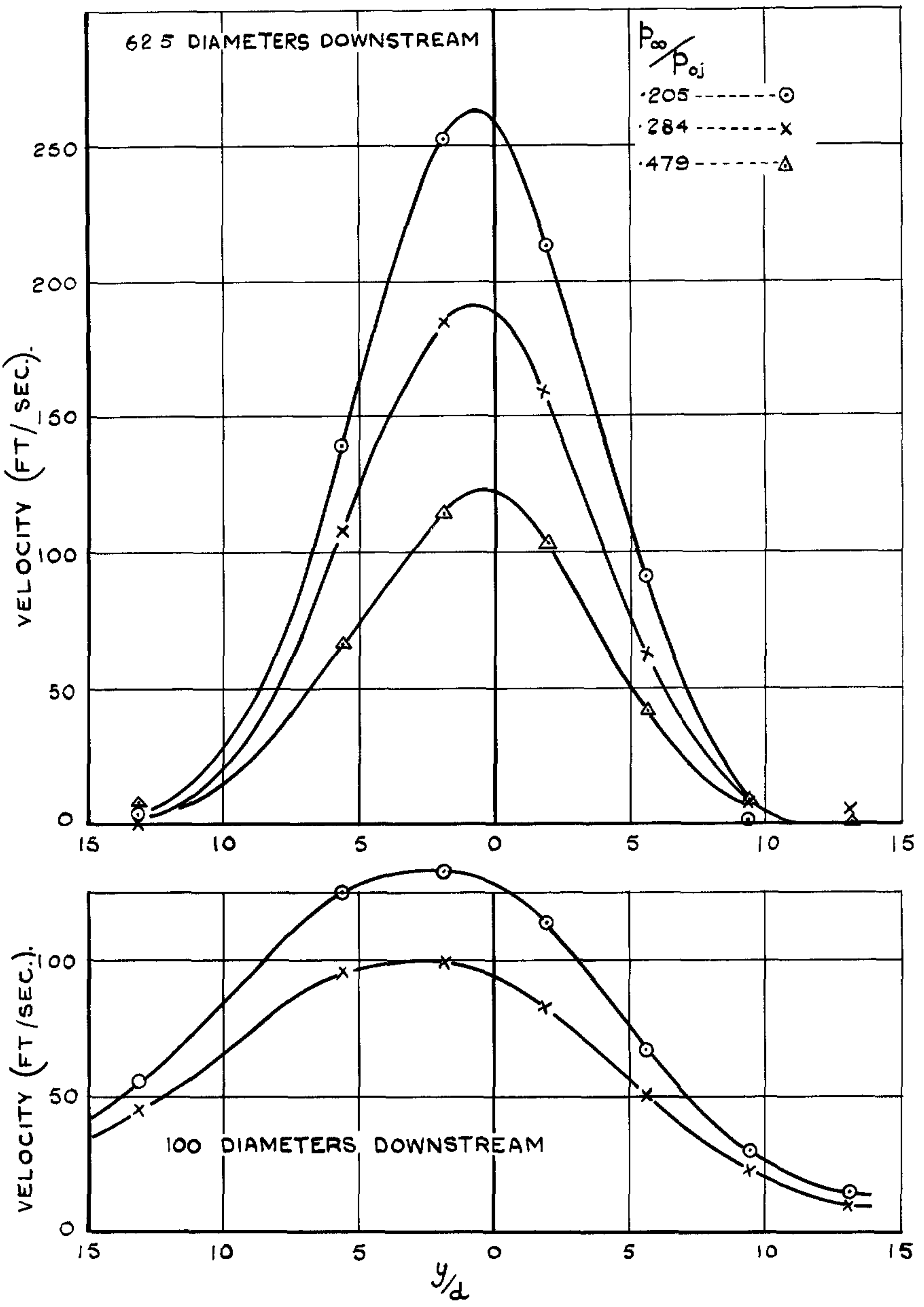


FIG. I. VELOCITY PROFILES WITH ATMOSPHERIC PRESSURE IN THE TUNNEL.

FIG. 2.

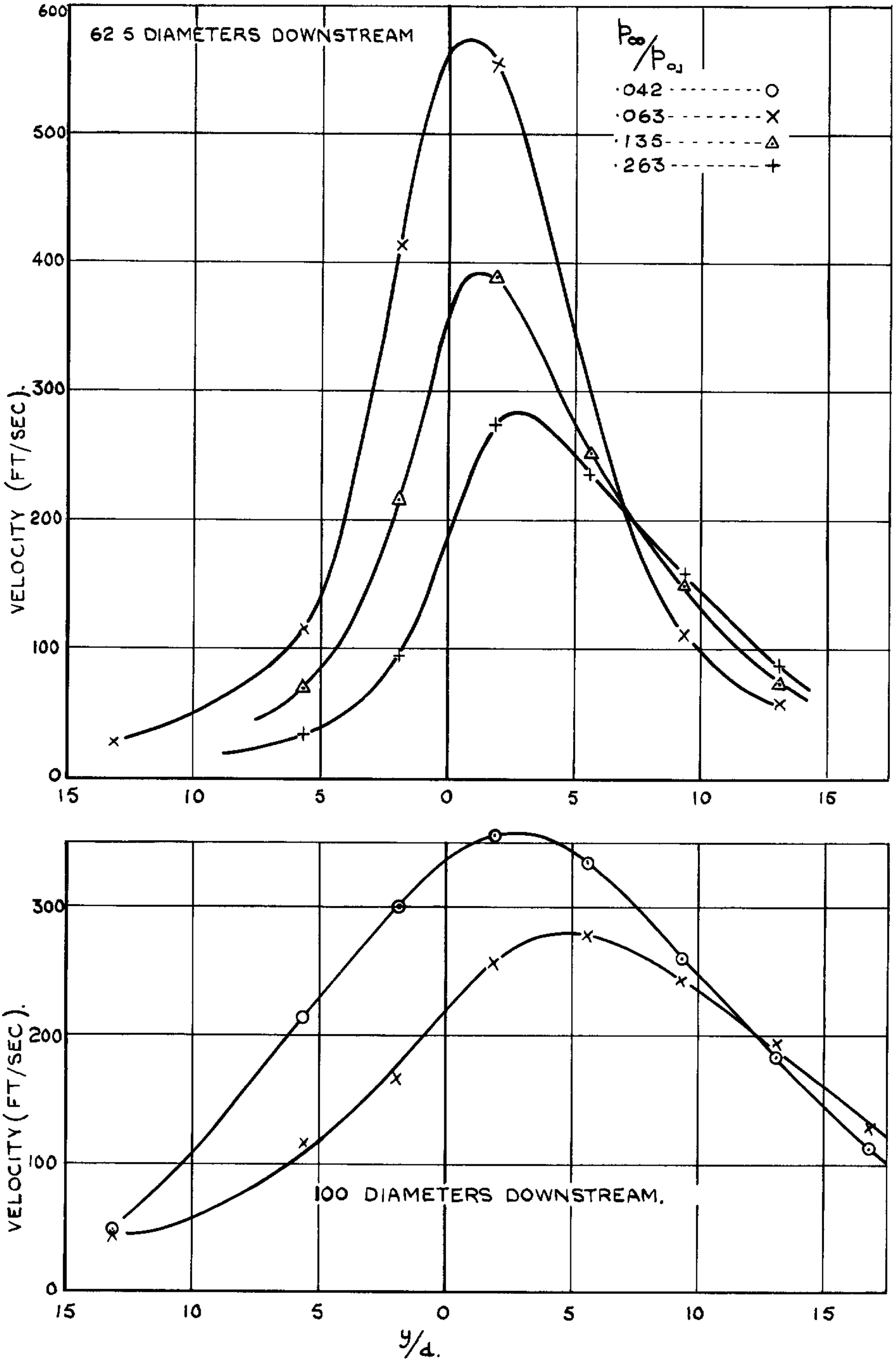


FIG.2 VELOCITY PROFILES WITH 2·5 LBS/SQ. IN. (ABS.) PRESSURE IN THE TUNNEL.

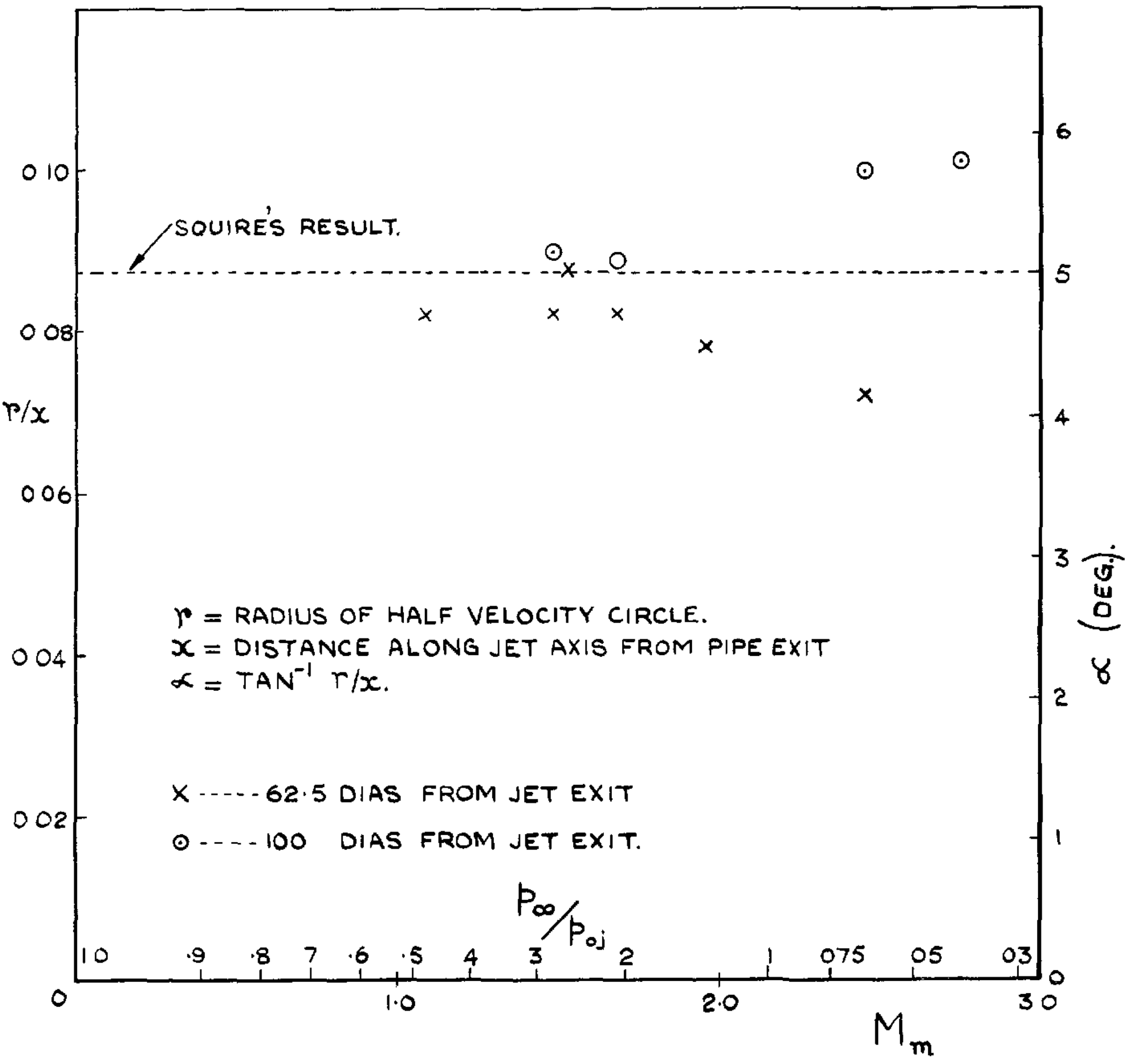
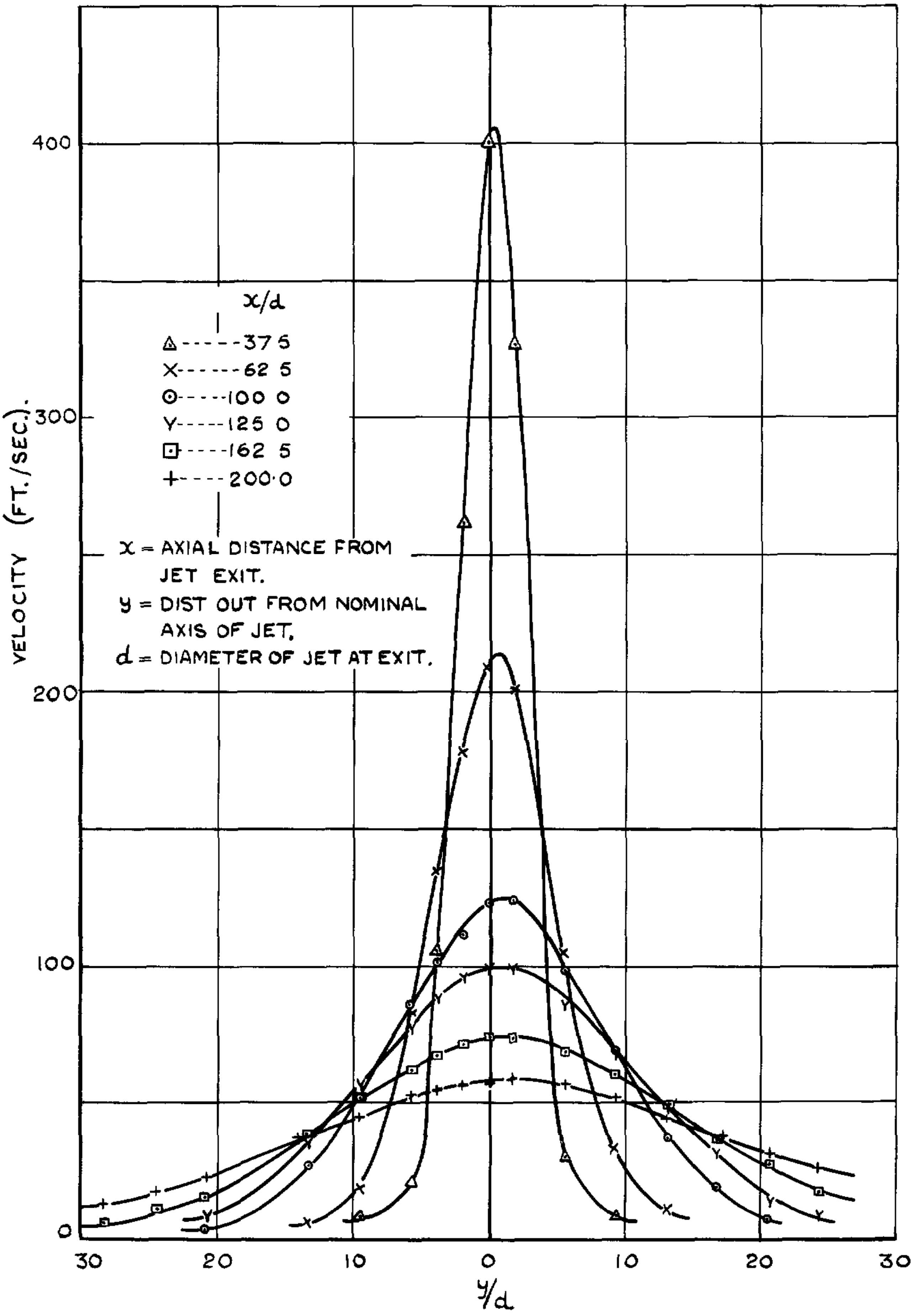


FIG. 3. r/x v. JET PRESSURE RATIO AND CORRESPONDING ISENTROPIC MACH NUMBER.

FIG. 4.



VELOCITY (FT./SEC.).
400
300
200
100
0
30
20
10
0
10
20
30
y/d
x/d
△-----37·5
×------62·5
⊙-----100·0
Y-----125·0
⊡----162·5
+----200·0
x = AXIAL DISTANCE FROM JET EXIT.
y = DIST OUT FROM NOMINAL AXIS OF JET.
d = DIAMETER OF JET AT EXIT.

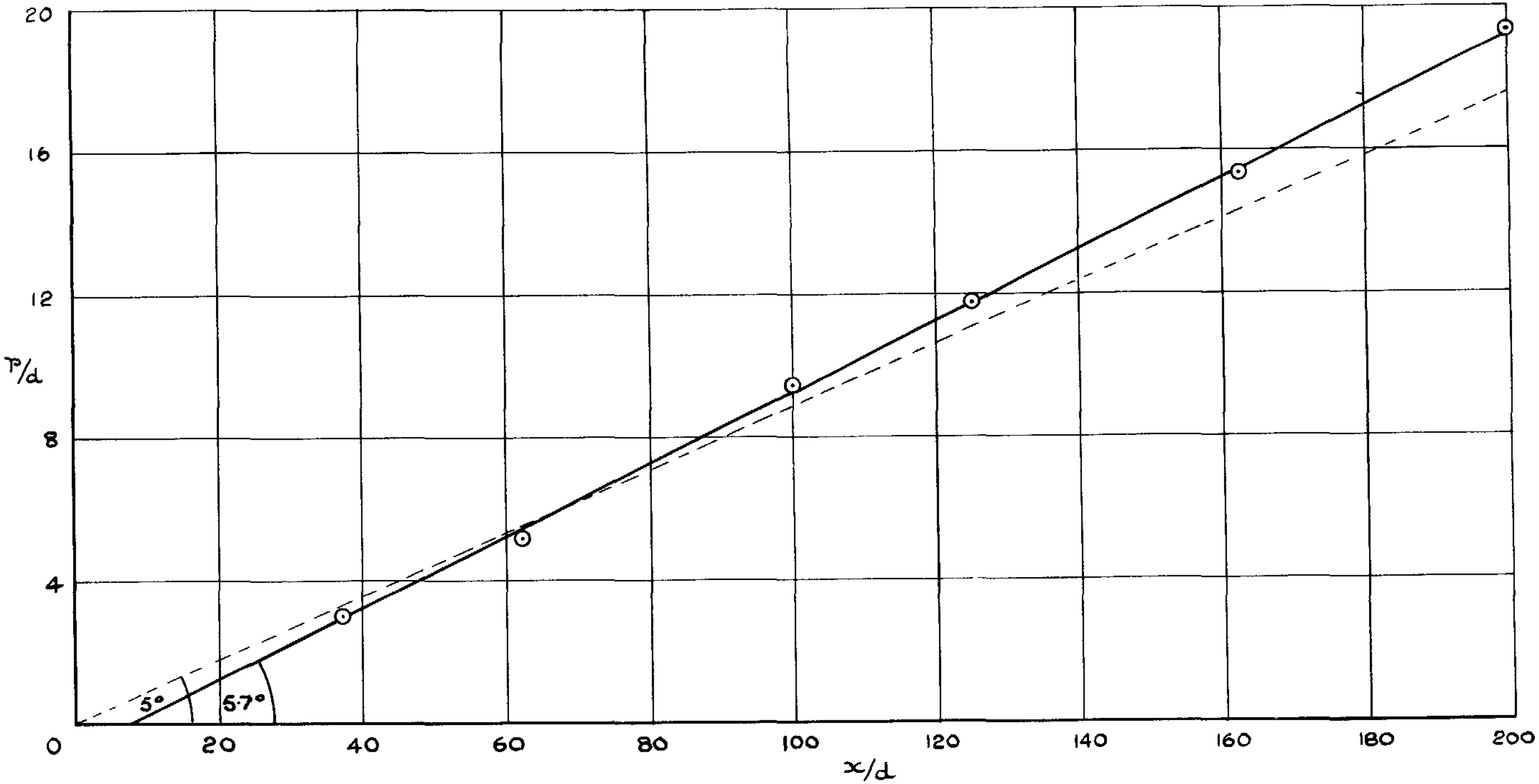
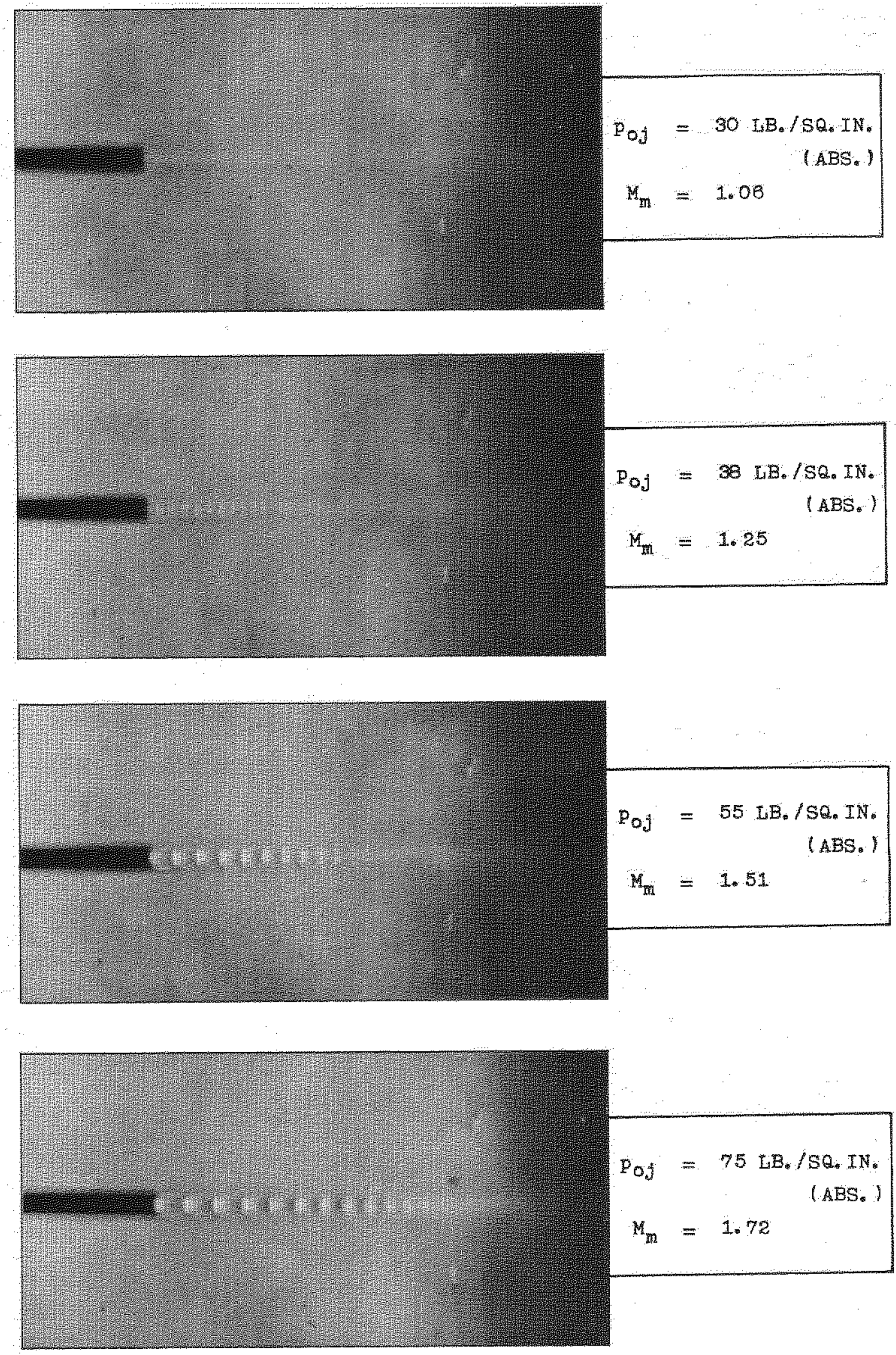


FIG. 5. DISTANCE DOWNSTREAM v. RADIUS OF HALF-VELOCITY CIRCLE.
(FOR $p_\infty/p_{oj} = \cdot 268$, $M_m = 1\cdot 51$)

FIG.6



P_{oj} = 30 LB./SQ. IN. (ABS.)
M_m = 1.06

P_{oj} = 38 LB./SQ. IN. (ABS.)
M_m = 1.25

P_{oj} = 55 LB./SQ. IN. (ABS.)
M_m = 1.51

P_{oj} = 75 LB./SQ. IN. (ABS.)
M_m = 1.72

FIG.6. SCHLIEREN PICTURES FOR VARIOUS VALUES OF p_{oj} AT ATMOSPHERIC p_∞

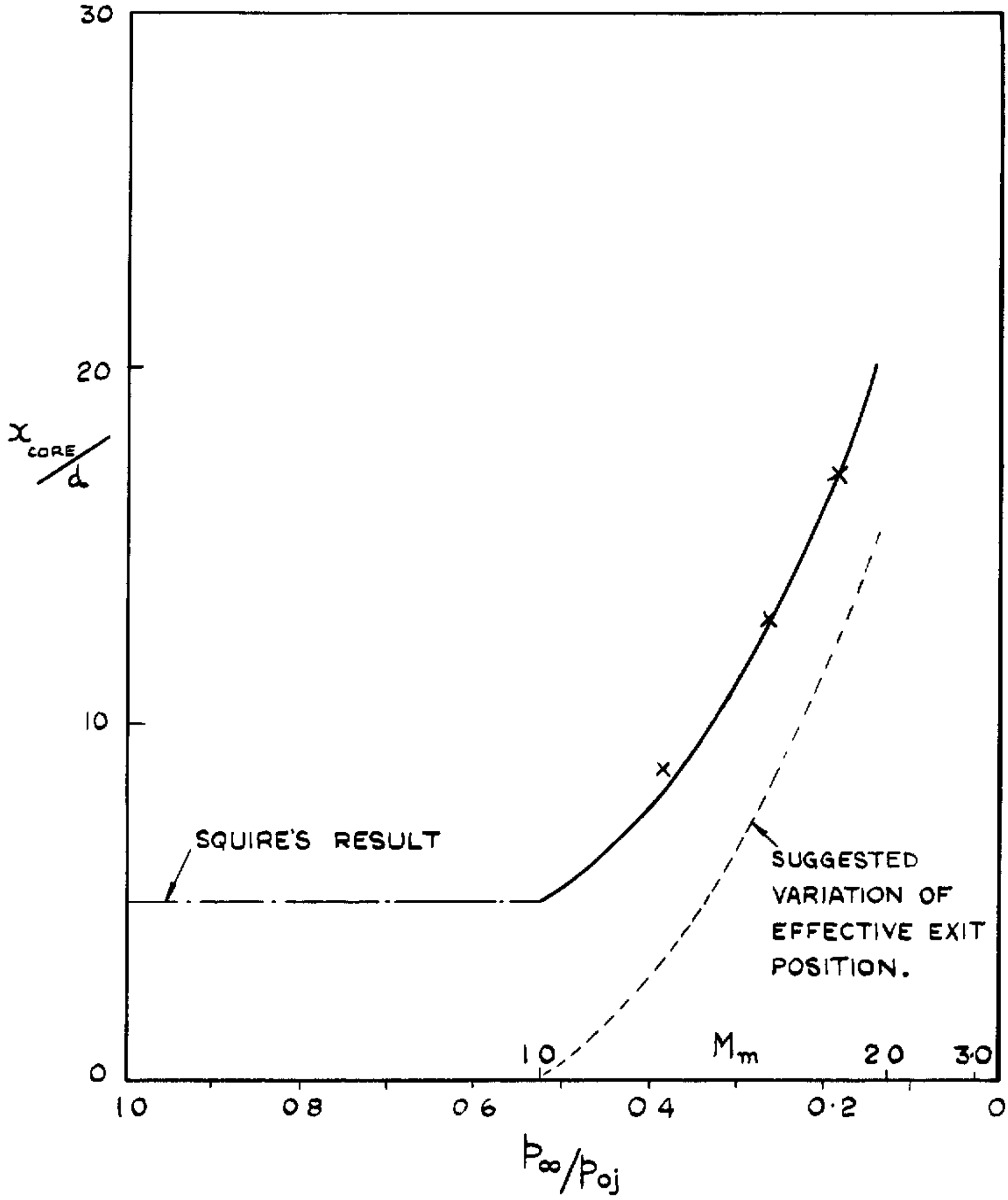


FIG. 7. CORE LENGTH FOR SUPERSONIC JETS

FIG. 8.

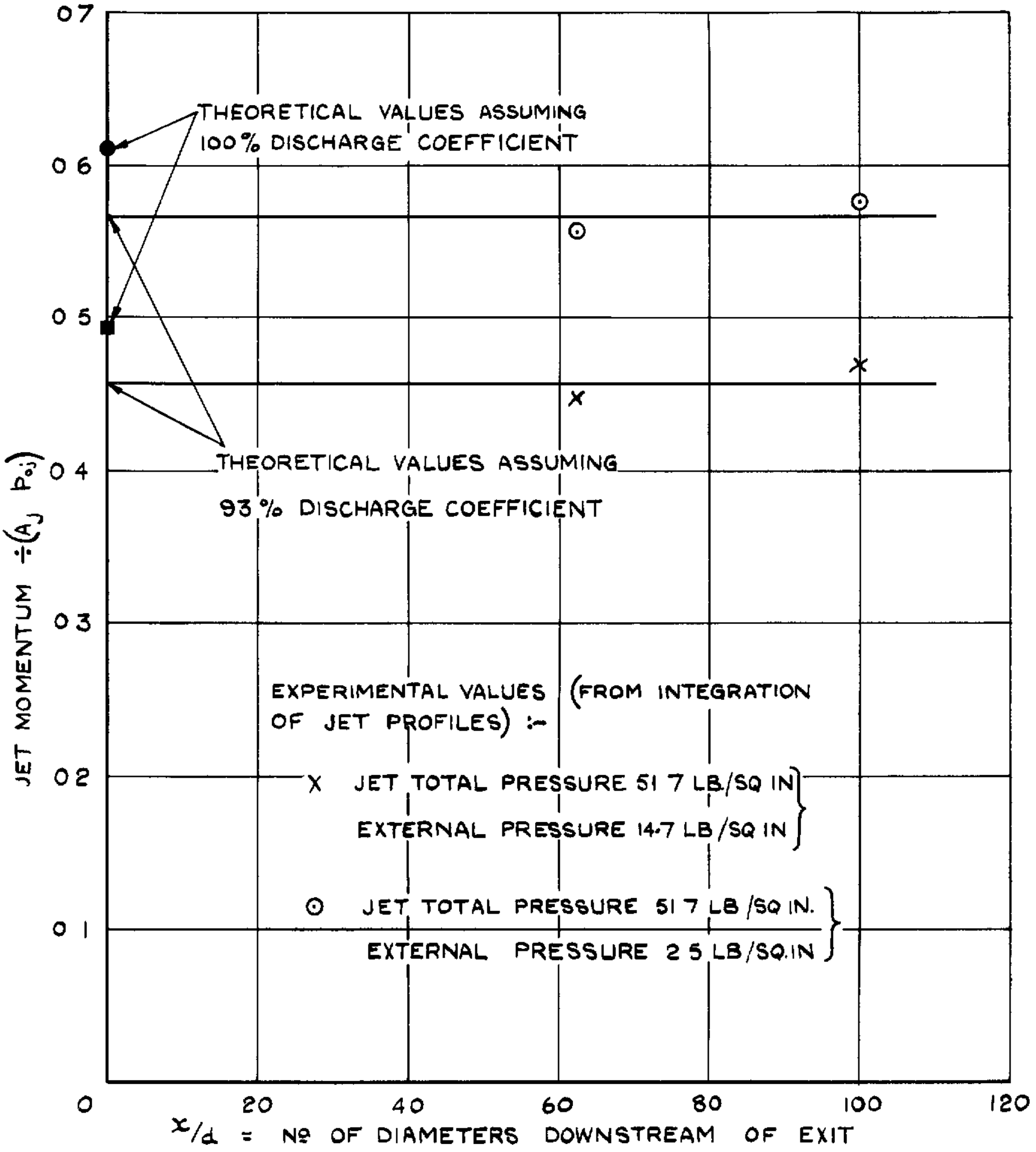


FIG. 8. CONSTANCY OF JET MOMENTUM.

FIG. 9.

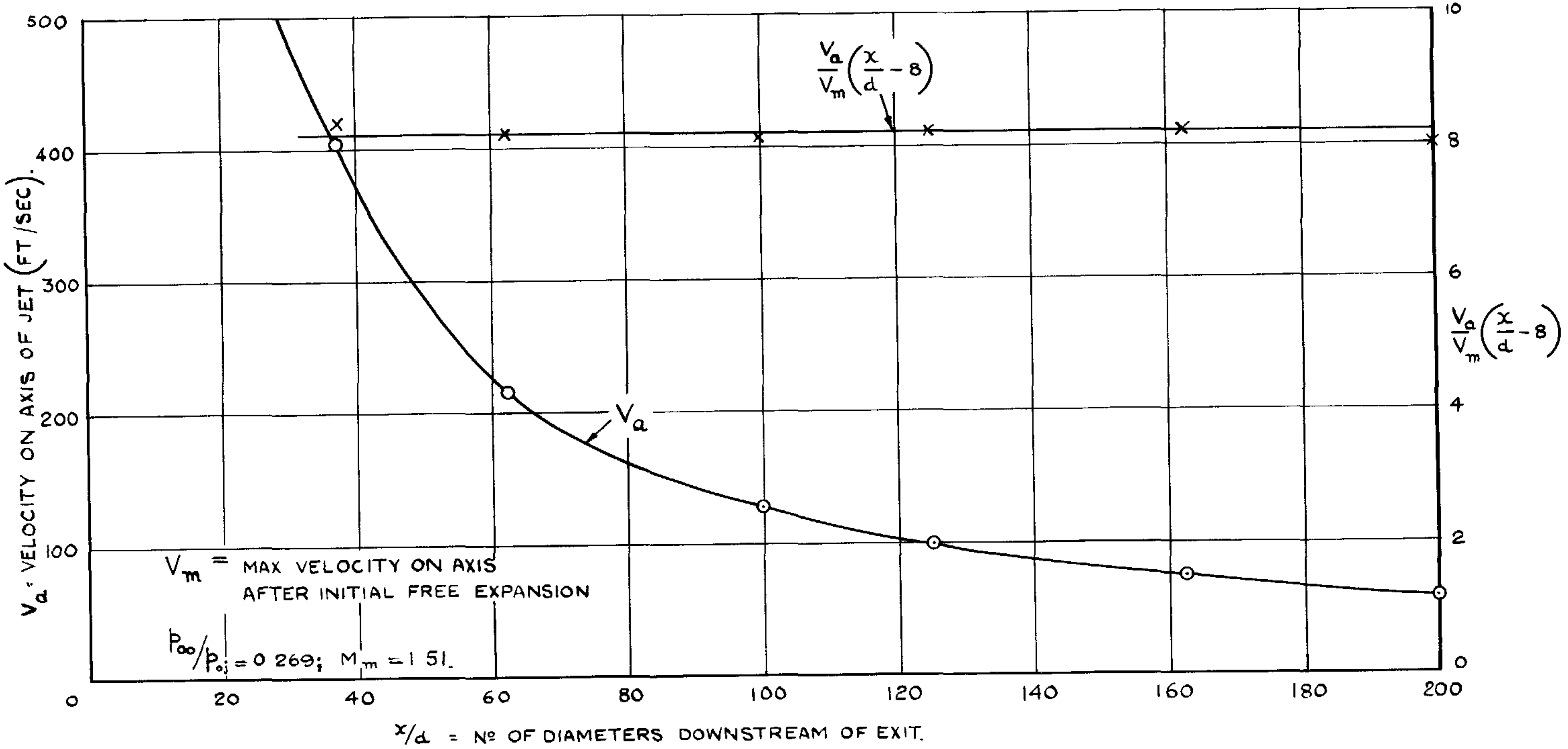


FIG. 9. VARIATION OF JET AXIAL VELOCITY.

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