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An Investigation of High Altitude Cruising Conditions for Turbo-jet Aircraft

By

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AEROPLANE AND ARMAMENT EXPERIMENTAL ESTABLISHMENT

An investigation of high altitude cruising conditions for turbo-jet aircraft

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Summary

Specific air range has been measured on the Ashton Mk.1 at high altitude using differing techniques. A 'quasi-level' procedure consisting of runs at constant speed involving a small rate of climb or descent is recommended for performance measurements near the optimum range conditions and any speed below about m = 1.2. Optimum specific range for the Ashton is obtained at the theoretically predicted conditions for an aircraft cruising below its drag critical Mach number, i.e. maximum R.P.M. and m = 1.2; increase of air temperature reduces this specific air range by about $1\frac{3}{4}$% per 10°C. Some evidence on scale effect on specific air range during a long range flight is shown; this is likely to be more important for a long range bomber. From handling aspects flying manually in the optimum climb cruise conditions and at speeds down to m = 0.9 was quite practicable and comfortable but the static margin on this aircraft even at the aft c.g. was high; it was therefore not possible to assess a minimum acceptable limit of static stability for range flying. An Appendix derives and discusses the conditions for maximum specific air range, the argument being extended to the case with compressibility drag effects.

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/1. Introduction

1. Introduction

The cruising conditions for optimum specific air range of turbo-jet aircraft are well-known and may be shown to involve

1) constant C_L (near to the minimum drag value).

2) Constant r.p.m. at or near the maximum available

and 3) A consequent gradual increase in height as weight is reduced by consumption of fuel, the height throughout being near to the absolute ceiling of the aircraft.

This 'climb cruise' technique has been used operationally for some years, e.g. by the Comet[1], and its practicability is well-established.

The tests reported here were made on an Ashton to obtain experience of techniques of specific air range measurement at the low equivalent air speeds required for optimum range, to investigate whether any handling problems were likely to arise in the use of the 'climb cruise' technique for long cruising flights near the minimum drag speed, and to obtain evidence on any scale effects on specific air range during such a cruise.

The Ashton is a high altitude research aircraft but is non-typical in its range characteristics of current and future jet bombers in that maximum specific air range is obtained at a Mach number below the drag critical value.

In an Appendix the theoretical conditions for maximum specific air range are derived and discussed, the argument being extended to the case with compressibility drag effects.

2. Description of aircraft

The aircraft used for these tests was the first prototype Ashton Mk.1 WB.490. It is a straight wing aircraft powered by four Nene turbo-jet engines and is fully described in Ref. 2.

Details of aircraft loading, engines, test instrumentation and auto-pilot are given in Appendix 1.

3. Details of tests

3.1 Scope of tests

3.1.1 Preliminary calibrations. Prior to the main test programme tests had been made to establish the pressure error correction of the 1st pilot's pitot-static system and the speed correction factor for the balanced bridge air thermometer.

3.1.2 Stabilised level speed tests. These tests were arranged to give the high altitude specific air range up to the greatest possible height. Four nominal heights were used, 30,000, 35,000, 37,000 and 39,500 ft., the latter being the greatest height at which suitable level speed runs appeared possible. The stabilised level speed and fuel consumption were measured over the available speed range at the corresponding values of W/p, based on the aircraft weight with half fuel (62,900 lb.). At each W/p value runs were made at maximum continuous r.p.m. (11,700) and then with r.p.m. reduced in stages until a stabilised level condition could no longer be maintained.

These flights were made between 9.4.52 and 26.5.52 without any engine change or major work on the aircraft intervening.

/3.1.3

3.1.3 Climb cruise tests. In order to obtain data on fuel consumption and handling using the standard 'climb cruise' technique, full range flights were made at constant nominal m values of 1.0, 1.1, and 1.2 followed by shorter runs at m = 0.9 and 1.3. During all these runs $N/\sqrt{\theta}$ was maintained constant at the value (13,500) corresponding to maximum continuous r.p.m. (11,700) and standard air temperature (-56.5°C). Instrument readings were taken at 10-15 minute intervals and assessments made of the comfort and suitability for long range cruising in these conditions using both manual and auto-pilot control. These tests were made between 18 and 27.6.52 and from Appendix 1 it will be seen that one engine change was made between the level speed tests and these.

These tests were all made using the mid c.g. position (Appendix 1). At a later date the same conditions were repeated using the aft permissible take-off c.g. to assess the handling characteristics in this case. Brief longitudinal stability measurements were also made at the two c.g. positions to obtain the appropriate static margins.

3.1.4 Quasi-level speed tests. A further flight was made on 1.4.53 to measure the effect of change of r.p.m. on range at constant speed. Three values of r.p.m. spanning the maximum cruise value were used, at a speed corresponding approximately to m = 1.2. These runs were short 'quasi-levels' made as detailed in para.3.2.3.

There were three further engine changes between the climb cruise tests and this flight.

3.2 Test procedure

3.2.1 Stabilised level speed tests. The required W/p value was obtained on each run by selecting a test height appropriate to the estimated aircraft weight at the end of the run, based on the 'gallons gone' indicators and the rate of fuel flow. For this purpose a table of altimeter reading v gallons gone was used for each W/p value. At this height and the required r.p.m. the speed was allowed to stabilise, about 5 minutes being generally allowed to ensure this, and the conditions then maintained for about a further 4 minutes while records were taken, consisting of several auto-observer shots and readings of fuel flow and air temperature.

3.2.2 Climb cruise tests. Climbs to the initial height were made at the best climbing speed given by firm's data (210 knots A.S.I. at S.L. reducing by 2 knots per 1,000 ft.). The height to commence the cruise run was estimated taking into account the aircraft weight and the value of m used. The r.p.m. required to give the standard value of $N/\sqrt{\theta}$ was obtained from charts relating N to air thermometer reading and speed; a check was kept on the air thermometer reading throughout the flight and r.p.m. were adjusted as necessary. Constant m (i.e. constant $V_1/\sqrt{W}$) was maintained using a table of A.S.I. v gallons gone for each m, reducing A.S.I. as necessary according to the fuel consumed; it was found that an A.S.I. reduction of about 2 knots every 20 minutes was required.

Throughout each run the auto-observer was operated at intervals of 10-15 minutes, periods when conditions appeared fairly steady being chosen. Readings of the flowmeters and of the air thermometer were also taken.

The value of $\dfrac{V_{imd}}{\sqrt{W}}$ used to estimate the required A.S.I. values was derived from the drag measured in the level speed tests.

3.2.3 Quasi-level tests. These were short runs (20-30 minutes) made with r.p.m. and A.S.I. selected as above to give the desired $N/\sqrt{\theta}$ and m values. Estimates of the corresponding heights were made on the basis of brochure thrust data and the drag measured during the climb cruise tests.

/Each run..

Each run was started near this height at this A.S.I. and r.p.m. and the latter two maintained constant; the observer plotted height against time during the run to find when the aircraft had settled into a steady climb or descent and to check that this was of the required order ($\not>$ 100 ft./min.). When this had been established the auto-observer was switched on, running at 10 second intervals and observations made for 15-20 minutes, including visual readings from the flowmeters and air thermometer.

4. Results and presentation

4.1 Corrections applied to observations. The A.S.I. static pressure error correction measured at ground level was virtually constant at +1 knot through a speed range from 120 to 240 knots A.S.I. at a mean weight of 63,000 lb. It is thus unchanged when extrapolated to high altitude by the method of A.& A.E.E. Res/244 assuming the Glauert law; this was supported by some check measurements by radar tracking at 37,000 ft. This correction and the corresponding altimeter correction have been applied to all the results. Pitot pressure error was zero over the same speed range.

The speed correction coefficient for the knife edge element balanced bridge air thermometer, measured at 10,000 ft., was 0.77 and this factor was applied throughout.

The volumetric fuel flows measured by the Kent flowmeters were converted to mass flows using a fuel density deduced from the measured fuel temperatures.

Engine thrust was estimated in the usual way from the jet pipe single pitot readings assuming a final nozzle effective area derived from the maker's test bed calibrations.

4.2 Stabilised level speed tests. Table 1 gives the basic corrected observations from each run and the corresponding 'non-dimensional' parameters.

Fig.1 gives the level speed data as a carpet plot of $\frac{V}{\sqrt{\theta}}$ v $\frac{N}{\sqrt{\theta}}$ and $\frac{W}{p}$ and Fig.2 the fuel consumption as $\frac{F}{p\sqrt{\theta}}$ v $\frac{N}{\sqrt{\theta}}$. Thrust (X/p) and specific fuel consumption $\left(\frac{F}{X\sqrt{\theta}}\right)$ are also plotted against $\frac{N}{\sqrt{\theta}}$ in Figs.3 and 4 respectively. Fig.5 presents the fuel consumption data of Fig.2 in terms of engine intake conditions $\frac{F}{p_1\sqrt{\theta_1}}$ v $\frac{N}{\sqrt{\theta_1}}$ which reduces deviation between different heights due to ram effects and is a more suitable form for interpolation or limited extrapolation; for these data an intake efficiency of 80% was assumed to deduce p_1 from p and $V/\sqrt{\theta}$. Mean values from each of the climb cruise runs and the quasi-level runs are also plotted in Figs.2-5 for comparison of the engine performance in view of the engine changes which occurred between the tests.

In Fig.6 the resulting specific air range is plotted as $\frac{W}{62,900}\ \frac{V}{F}$ v $V_i\sqrt{\frac{62,900}{W}}$. In this form the curves are applicable to all weights while the scales give the actual specific air range v E.A.S. in knots at the mean weight of 62,900 lb.; a scale of m is also shown. The individual points are given and faired curves for the four nominal heights derived from the faired curves for Figs.1 and 2. Also shown for comparison is the curve at maximum cruise r.p.m. obtained from the climb cruise and quasi-level tests; the method of derivation of this curve is explained below.

/In Fig.7..

In Fig.7 the drag deduced from these results is plotted as $\frac{D}{V_i^2}$ ($\propto C_D$) v $\frac{W^2}{V_i^4}$ ($\propto C_L^2$) and the calculated best straight line through the points given. This line corresponds to

$$C_{D_Z} = 0.0234, \qquad c = 1.067$$

and $$V_{i_{md}} \sqrt{\frac{62,900}{W}} = 121 \text{ knots} \qquad (W \text{ in lb.})$$

The minimum comfortable cruising speed for level flight at the mid c.g. and mean weight was assessed during the 35,000 ft. tests at about 130 knots I.A.S. (i.e. about 1.1 x V_{md}); lower speeds were considered unacceptable for cruising because of the continual longitudinal adjustments required to maintain steady conditions. It may be noted however that in the tests at 30,000 and 39,500 ft. the lowest speeds at which stabilised runs were considered worth attempting was about 1.2 V_{md} whilst at the intermediate test heights, runs were made down to about 1.0 V_{md}.

4.3 Climb cruise and quasi-level tests

4.3.1 Climb cruise tests. Detailed results from the three full range flights at nominal m values of 1.0, 1.1, and 1.2 and from the shorter runs at m = 0.9 and 1.3 are given in Fig.8 where the variation of several parameters throughout each run is shown against weight. During the tests r.p.m. were adjusted with air temperature to keep $N/\sqrt{\theta}$ constant and A.S.I. with weight to keep $V_i/\sqrt{W}$ constant. The extent to which this was achieved is shown in Figs.8(c) and (d). The remainder of the curves show the variation with weight of several of the relevant non-dimensional parameters which would be expected, if the standard dimensional analysis method were valid, to remain sensibly constant. Departure from constancy of any of these parameters may therefore be interpreted as evidence of the effect of Reynolds number changes during the runs, which are ignored in the conventional analysis. Their significance in this respect is discussed in para.5.3. In Fig.9 values of X/V_i^2 are plotted against W^2/V_i^4 from which, with the climb correction of Appendix 3, D/V_i^2 ($\propto C_D$) was obtained for comparison with the level speed data of Fig.7.

4.3.2 Quasi-level tests. The variation of range with r.p.m. at constant speed is shown in Fig.10 where the results from the three quasi-level runs at m = 1.2 are plotted as $\frac{W}{62,900} \cdot \frac{V}{F}$ v $\frac{N}{\sqrt{\theta}}$. The range figures have been corrected to a level flight basis, i.e. zero rate of climb, by the method of Appendix 3. The measured rates of climb were of the order 60-90 ft./min.

From the slope of this line the variation of specific air range with r.p.m. or air temperature may be deduced for around the optimum conditions. An increase in air temperature of 10°C (corresponding to a reduction in r.p.m. of 310) results in a loss of $1\frac{3}{4}\%$ in specific air range at about the optimum conditions of 11,700 r.p.m. and m $\simeq$ 1.2. Decrease in air temperature produces a corresponding gain.

4.4 Comparison of all specific air range data. Comparisons of the results from the level speed tests with those from the climb cruise and quasi-level tests are complicated by changes in aircraft drag which occurred during the tests period. The evidence for these drag changes is given in Figs.7 and 9, based on thrust estimates from the jet pipe single pitot observations and is discussed in Appendix 3.

To compare the specific air range data from all the tests the results must therefore be presented corresponding to the same aircraft drag condition. The level speed test drag has been taken as this datum and corrections applied to the other test results for the measured drag difference. The derivation of the method and correction details are given in Appendix 3 from which it

/will be..

will be seen that what has in effect been done has been to use the unmodified engine performance data in the flight conditions as tested and associate them with the datum drag to find the revised aircraft weight which would be required.

The drag change between the level speed and climb cruise tests corresponds to an $8\frac{1}{2}\%$ increase of C_{D_Z} with no change in induced drag i.e. an increase in total drag of the order of 5%. The resulting correction to specific air range at the same $N/\sqrt{\theta}$ and M varies from about +3% at m = 0.9 to about +10% at m = 1.3. Corrections have also been made for the effect of the climb term to obtain results directly comparable with the level speed data; these corrections are also described in Appendix 3 and the effects of all the corrections made to the measured results are shown in Fig.11 where $\frac{WV}{F}$ v $\frac{V_i}{\sqrt{W}}$ is shown for points before and after correction.

4.5 <u>Handling during climb cruise.</u> In general from the handling aspects no serious criticisms were made of the suitability of the climb cruise technique for long range flying with this aircraft over the speed range tested (m = 0.9 to 1.3) and at both mid and aft c.g. positions.

The approximate measured static margins, both stick fixed and free at the mean weight over this speed range were:

Mid c.g. $0.13\ \bar{c}$

Aft c.g. $0.08\ \bar{c}$

It will be seen from Appendix 1 that the c.g. moves forward as fuel is consumed and no attempt was made either on the range or stability tests to keep it fixed.

The flying was shared by three pilots whose summarised comments are given below.

<u>Mid c.g.</u>

<u>Manual control.</u> The aircraft was difficult to trim at all values of m but this seemed due to the backlash present in the elevator trimmer circuit. At m values of 1.0 and 0.9 the aircraft became considerably more difficult to trim but here again, this appeared to be mainly due to the backlash. Once trimmed, however, at any value of m from 1.3 to 0.9 the aircraft handling was satisfactory and flying on the climb cruise technique easy.

<u>Auto-pilot control.</u> At m values of 1.3 and 1.2 the auto-pilot was found to control the aircraft satisfactorily and the aircraft gained height at a virtually constant A.S.I. reading. The speed was kept within ± 2 knots.

At the lower m values, with the aircraft trimmed and the auto-pilot selected, the response did not seem sufficiently quick to maintain the trimmed conditions. The divergence became a long period oscillation which however could be damped out by use of the pilot's control column. This helped to stabilise the aircraft and avoided the use of the auto-pilot controller, which usually resulted in a never-ending chase for the correct conditions.

<u>Aft c.g.</u>

<u>Manual control.</u> Before tests were commenced at the aft c.g., as much of the backlash as possible was taken out of the elevator trimmer circuit. This resulted in making it fairly easy to trim the aircraft to the desired conditions at all values of m. The aircraft could therefore be flown using the technique required for reasonably long periods without undue tiring of the pilot.

Auto-pilot control. Auto-pilot unserviceability during these tests made a satisfactory assessment of its capabilities difficult. However at the higher speeds (m = 1.2 and 1.3) the auto-pilot appeared able to control the aircraft satisfactorily and could be engaged reasonably quickly from the trimmed conditions. Least useful evidence was obtained at the lower speeds (m = 0.9 and 1.0) but its operation here appeared satisfactory once engaged correctly though this usually required several attempts at trimming; initial attempts were usually followed by fairly rapid speed increase.

5. Discussion

5.1 Specific air range data. The level speed test results plotted in Fig.6 show increasing scatter as W/p is increased and $V_i/\sqrt{W}$ reduced. At the highest altitude at which stabilised level speed runs could be made with reasonable success (W/p = 333,000) the results were very indeterminate; the lowest speed at which stabilised runs were possible at this height and the lowest height tested (W/p = 212,000) corresponded to about m = 1.15. At the two intermediate heights however, done by another pilot, surprisingly low speeds were reached, the lowest being m = 0.97 at W/p = 268,000 (Fig.6). Air conditions were apparently very calm when these tests were done and the pilot very experienced in this type of test but from previous experience it had not been thought possible to obtain genuinely stabilised results below about m = 1.1 and near this value only with considerable care under ideal conditions. It may be seen from the drag plot of Fig.11 that these low speeds yield drag results apparently very consistent with the other data; usually considerable increase of scatter is expected at such low speeds because of the difficulty of establishing what the stabilised speed is. In general however it is still considered that stabilised level speed tests at speeds below an m of about 1.2 are likely to be lengthy, requiring great care, and the results less reliable than at higher speeds so that more repeats are needed.

The data obtained from the climb cruise tests on the other hand extend down to m = 0.9 with little more difficulty at this than at higher speeds. The curve obtained at max. continuous r.p.m. is the range envelope and the absolute maximum specific air range is thus obtained at about m = 1.2 and a W/p of 344,000. This range value is some 4% higher than the best actually measured on the level speed runs although it would appear that given good air conditions, level speed runs could have been made at this condition; when the level speed tests were done however it did not appear profitable to attempt runs at a higher W/p than 333,000. It may be observed also that there are apparent discrepancies shown in Fig.6 between these highest altitude level speed runs and both the climb cruise and the remaining level speed data, e.g. the 11,700 r.p.m. lines obtained from the latter sets of data appear to be reasonably consistent with each other but not with the highest altitude level speed data.

The climb cruise results have been subjected to increases varying from 5 to 15% to obtain the comparison data for Fig.6, as shown in Fig.11. These corrections, though large are however considered quite valid given the correctness of the test measurements. It will be seen from Fig.8 that the climb cruise observations show considerable scatter in any one run with somewhat inconsistent trends in some cases. These curves are more fully discussed below but here we may note that the specific range data of Figs.6 and 11 are derived from the mean of the values on each run and some scatter is evidently to be expected. Near the optimum value however the data are very well supported by the quasi-level results.

The climb cruise runs were made primarily for the long range handling assessment and the performance measurements made, usually single observations at intervals of 10-15 mins. when conditions appeared steady, would not be expected to yield as accurate performance data as the quasi-level runs for which conditions were maintained very steady for a period of about 15 minutes whilst continuous records were taken (every 10 secs.); the quasi-level technique is recommended for performance measurements at speeds below in general about m = 1.2.

5.2 Handling on climb cruise flights. No serious difficulties attributable to the cruising technique were encountered throughout the speed range m = 0.9 to 1.3 at both mid and aft c.g. positions. Flying manually the only difficulty appeared to be associated with backlash in the elevator trimmer circuit making accurate trimming to the required speed difficult; when the backlash was reduced for the aft c.g. flights, flying was made easier despite the reduced static margin. The assessment with auto-pilot controlling was somewhat inconclusive partly from recurrent unserviceability and partly because of the low-powered servo-motors fitted (Appendix 1). For these reasons the tests gave no direct assessment of the suitability of auto-pilot control in long range cruising flights. There appears to be no reason however, given a correctly functioning and matched auto-pilot system, why any difficulty should arise.

These results were to be expected from the high stick free static margins at both c.g. positions used. For climb cruise flying where speed is required to be maintained steady and not height as in constant height cruising the relation of the speed to the minimum drag speed is irrelevant and the main requirement is that the longitudinal stability should be sufficient to enable the speed to be maintained steady without undue effort from the pilot. It is to be expected that there will be a minimum static margin below which manual control will be uncomfortably tiring but this stage is by no means reached on this aircraft even at the aft c.g. where the static margin is still about 0.08 in cruising flight conditions.

The high static margins in cruising flight for the Ashton result from the destabilising effect of engine thrust on this aircraft. Firm's stability tests at low altitude on this aircraft and on the Tudor 8 of similar configuration indicate that at the aft c.g. at similar A.S.I. values the stability is about neutral using climb r.p.m. (12,100), this being a determining factor in fixing the aft limit. With reduction of thrust however the neutral point moves aft and the cruising thrust at altitude is about a third of the sea level climb value.

5.3 Evidence on scale effect from climb cruise tests. As has been stated the most accurate performance data at low m values are to be expected from short 'quasi-level' runs at near the stabilised conditions with frequent auto-observer records to establish the rate of climb or descent accurately. The best evidence on scale effect during long range flights would thus be obtained by a series of such runs at several weights covering the available weight range, all at the same m and M. Such tests have not been made but similar evidence is provided by the series of observations taken throughout the climb cruise flights though by their nature these results contain more scatter than is desirable for the purpose. The general trends should however be shown and the inferences to be drawn from them are considered here.

In Fig.8 values of the various parameters are plotted to show their variations throughout each flight using the climb cruise technique. $N/\sqrt{\theta}$ and $V_i/\sqrt{W}$ are nominally constant through each flight, the actual variations being shown in Fig.8(c) and (d). We should therefore expect, if there are no Reynolds number effects present, that all the other non-dimensional parameters would also not show a systematic variation. The evidence provided on scale effect is shown in full in Fig.8 and the following points may be noted.

(a) $V_i/\sqrt{W}$ has been kept constant at a moderate Mach number which does not vary appreciably; D/W should therefore be constant. In the stabilised cruise conditions X = D + a small climb term; X/W will therefore not be affected by any scale effect on engine performance. Any variation in X/W not attributable to $V_i/\sqrt{W}$ changes is therefore evidence of non-stabilisation of flight conditions or of scale effect on drag. Such variations may be expected at the start of each run if the height chosen does not correspond to the selected r.p.m. and speed and may occur in the course of a run due to changing atmospheric conditions e.g. wind gradients (the distance covered in a range flight was approximately 1,000 miles). The apparent tendency of X/W to increase slightly through some runs might therefore be interpreted as scale effect on aircraft drag but may be affected by atmospheric variations.

/(b)..

(b) Scale effect on engine performance should be shown up directly by a steady variation of X/p and/or $F/p\sqrt{\theta}$ with reducing weight (i.e. increasing altitude). There appears to be a slight tendency for X/p to decrease but this is barely significant. Similarly $F/p\sqrt{\theta}$ shows a general tendency to increase but by a variable amount on each flight. The specific consumption $F/X\sqrt{\theta}$ (the ratio of the two previous parameters) reflects both these changes and shows a general tendency to increase slightly - approximately 3% over the full weight range.

(c) W/p tends to fall on each flight but by differing amounts. This parameter reflects the variations in X/p and X/W and so the variations might be considered as resulting from scale effects on thrust and drag with possible contributions to varying X/D from varying wind conditions.

(d) The nett effect of these variations on the range is shown by the parameter WV/F in Fig.8(1). This suggests a possible reduction of the range parameter by up to about 8% over the full weight range though the rate of fall-off is by no means uniform.

It is shown in Appendix 2 that at a stabilised constant m, if the drag does not change with Re, $\frac{WV}{F} \propto \frac{1}{c}\sqrt{\frac{X}{p}}$. The results indicate at the most an increase in c (i.e. in $F/X\sqrt{\theta}$) of about 4% and a reduction in X/p of about 2% which would result in a maximum reduction in range parameter throughout a flight of 5%. Where the WV/F values indicate a greater fall-off rate than 5%, it must be ascribed to scale effect on drag or to non-stabilised flight conditions due either to the initial wrong selection of height, or to relatively sudden changes of atmospheric conditions (changing of r.p.m. has accompanied any change in air temperature to maintain a constant $N/\sqrt{\theta}$).

The maximum weight ratio on these runs was about 0.79 and the greatest height change about 5,000 ft. For a long range bomber the weight and height changes in a flight might be appreciably greater than these with consequent greater effects of Reynolds number change during the run on specific air range.

6. Conclusions

1. Long range flights in the Ashton by the climb cruise technique were quite practicable using manual control apart from difficulties which arose from backlash in the elevator trimmer circuit. The static margins were high however in all cases (0.08 at the aft c.g.) and difficulties may arise in aircraft with low static longitudinal stability. Auto-pilot control should present no difficulties but the system tested was unsatisfactory for a suitable assessment to be made.

2. Maximum specific air range for the Ashton is obtained using maximum available r.p.m. and a speed corresponding to m = 1.2. This is the theoretically expected condition for an aircraft for which the corresponding Mach number is below the drag critical value.

3. A change of 10°C in air temperature resulted in a change of 1½% in max. specific air range, increase of temperature reducing range.

4. Although apparently satisfactory stabilised level speed measurements have been made at speeds near m = 1 it is recommended in general that performance measurements at speeds less than about m = 1.2 should be obtained by 'quasi-level' tests.

5. Owing to scale effects on engine performance and aircraft drag the 'non-dimensional' specific air range fell by the order of 5-8% during a range flight. The corresponding weight ratio was about 0.8 and the effect is likely to be greater for a long range bomber with a greater proportional disposable load and consequent greater height increase during a sortie.

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/Notation

Notation

W – Aircraft weight, lb.

N – Engine speed, r.p.m.

V – True airspeed, knots

V_i – Equivalent airspeed $= V\sqrt{\sigma}$, knots

$V_{i_{md}}$ – Theoretical value of equivalent airspeed for minimum drag below drag critical Mach number

m – Ratio of V_i to $V_{i_{md}}$

M – Mach number

p – Ambient air pressure, lb./sq.in. $\div$ 14.7

θ – Ambient air temperature, °K $\div$ 288

σ – Ambient air density - slugs/cu.ft. $\div$ 0.002378 $= p/\theta$

p_1 – Intake air pressure, lb./sq.in. $\div$ 14.7

θ_1 – Intake air temperature, °K $\div$ 288

F – Total fuel consumption, lb./hr.

X – Total nett thrust, lb.

D – Aircraft drag, lb.

c – Engine specific fuel consumption $= F/X$, lb. fuel/hr./lb. thrust

C_{D_Z} – Value of drag coefficient obtained by linear extrapolation to $C_L = 0$ of curve of C_D against C_L^2

e – N.A.C.A. 'efficiency factor' $= 1/\pi A \dfrac{dC_D}{dC_L^2}$, where

A is aspect ratio

R – WV/F, non-dimensional specific air range parameter.

Table 1

Level Speed Test Data

Run No.	h_p Height ft.	W Weight lb.	N R.P.M.	VR ASI. kts.	T_a Air temp. °K	F Fuel flow lb/hr	X Thrust lb.	$\frac{W}{p}$	$\frac{N}{\sqrt{\theta}}$	$\frac{V}{\sqrt{\theta}}$	$\frac{F}{p\sqrt{\theta}}$	$\frac{X}{p}$	$\frac{V}{W}\frac{p}{62900}$	$V\sqrt{\frac{62900}{W}}$
					Nominal W/p = 212,000									
1	27855	69600	11680	229	228½	8420	6112	213100	13130	391	29000	18710	.0456	212
2	28360	68500	11645	225	226½	8200	5952	214700	13130	389	29000	18660	.0458	211
3	28605	67450	11420	217	225½	7450	5532	213800	12920	378	26680	17530	.0481	205
4	28710	66500	11215	205	225½	6930	5018	211800	12670	358	24950	15970	.0483	195
5	29140	65600	11010	190	225½	6180	4534	213000	12440	336	22690	14710	.0498	182½
6	29630	64750	10800	172	225½	5600	4108	214700	12210	309	21000	13600	.0501	167
7	29990	63750	10575	150½	225½	5080	3676	215000	11960	274	19380	12380	.0484	148
8	27930	70000	11715	233	225	8720	6336	215600	13250	396	30330	19490	.0447	214½
9	28210	68200	11295	214	223½	7320	5396	212600	12820	369	25880	16800	.0481	201
10	28680	67900	11040	196	223	6510	4822	211500	12550	344	23520	15330	.0492	185½
11	29120	66500	10800	182	222	5860	4318	215500	12300	323	21640	14000	.0510	174½
12	29330	64900	10605	168½	222	5410	3982	216000	12080	301	20150	13020	.0514	164
13	29715	62000	10420	151½	221	4990	3650	206500	11900	274	18980	12150	.0475	151
					Nominal W/p = 268,000									
14	32665	68650	11705	192½	225½	6870	4975	262000	13240	368	29610	18980	.0516	180
15	32820	67450	11625	183½	226½	6540	4797	259000	13100	352	28350	18410	.0512	173½
16	32995	66600	11400	175	224½	6120	4452	258000	12900	338	26820	17220	.0518	167
17	33155	68700	11690	201½	217	7190	5222	268400	13490	386	32400	20400	.0508	188
18	33685	67500	11620	195½	215½	6870	5005	270600	13430	382	31860	20050	.0516	184
19	34090	66650	11415	182	215½	6280	4615	272000	13200	360	29650	18840	.0525	173
20	34420	65650	11230	174	215½	5800	4230	272000	12980	347	27800	17500	.054	167
21	34860	64950	11050	160	214½	5280	3838	275200	12820	324	25920	16250	.0546	155
22	34945	64000	10820	134½	214½	4860	3476	272400	12550	275	23980	14770	.0510	132
23	33430	69300	11675	202	216½	7160	5170	275000	13470	392	32720	20480	.0523	188
24	33730	67800	11330	186	216	6150	4550	272000	13085	364	28540	18260	.0551	174½
25	33620	66900	11085	171	215½	5640	4118	267000	12820	336	26020	16430	.0547	163
26	33970	65900	10940	154	215	5180	3848	267000	12670	306	24370	15600	.0534	148½
27	34310	64600	10755	120½	215	4665	3410	266000	12440	243	22270	14050	.0463	118½
28	35280	61700	11320	173	218½	5760	4212	267000	13000	353	28690	18240	.0524	171
					Nominal W/p = 294,000									
29	35350	68400	11724	169	228	5960	4174	296700	13140	345	29000	18110	.056	159
30	35750	67250	12026	178½	227½	6480	4532	297600	13500	366	32200	20050	.0539	168
31	36165	66400	11795	160½	231	5830	4020	299000	13170	340	29350	18100	.0552	156
32	36515	65600	11612	147	231	5340	3694	300200	12970	315	27350	16900	.0551	144
33	36640	64950	11432	127½	231	4925	3404	298000	12760	274	25280	15650	.0513	125
34	37190	62700	11442	120½	231	4730	3304	296400	12790	262	25000	15620	.0495	120½
35	34980	69250	11720	193	211	6840	4922	295300	13680	390	34100	21000	.0536	179½
36	35440	68000	11275	173	213½	5700	4168	296300	13095	354	28800	18150	.0576	162½

/contd

Table 1 contd.

Run No.	h_p Height ft.	W Weight lb.	N R.P.M.	V_R ASI. kts.	T_a Air temp. °K	F Fuel flow lb/hr	X Thrust	$\frac{W}{p}$	$\frac{N}{\sqrt{\theta}}$	$\frac{V}{\sqrt{\theta}}$	$\frac{F}{p\sqrt{\theta}}$	$\frac{X}{p}$	$\frac{F}{V}\cdot\frac{W}{62900}$	$V_i\sqrt{\frac{62900}{W}}$
37	35585	67100	11105	158½	211	5310	3394	294300	12975	326	27220	17090	.056	150
38	35810	66000	10975	146	210	5020	3644	293000	12860	302	26100	16160	.054	140
39	36010	64300	10910	130½	210½	4730	3456	287700	12760	274	24750	15450	.0508	127½
40	36860	62750	11310	160	215	5215	3848	292000	13100	339	28100	17900	.056	157
41	37160	61850	11150	146	214½	4925	3486	292100	12930	312	26950	16460	.0539	145
					Nominal W/p = 333,000									
42	37980	67700	11700	168	211	5860	4188	333000	13670	365	33640	20540	.0576	158
43	38470	66300	11605	157½	210½	5540	3943	334000	13525	346	32500	19840	.0566	150
44	38600	65600	11505	154	211	5340	3796	332000	13440	339	31670	19220	.0566	148

NOTE: Subsequent tests on other aircraft have indicated a slight fall in the recovery factor of the air thermometer type used here with increased altitude which if present on these tests would result in these air temperatures being slightly under estimated. This would not affect any conclusions drawn here but would require small adjustments to some of the parameters as tabled and plotted which should be borne in mind if these figures are used for other purposes.

Appendix 1

Details of aircraft relevant to tests

1. Loading

The majority of the tests were made using a mid c.g. position, some repeats being made with take-off at the aft permissible c.g. limit. With consumption of fuel the c.g. moved forward. Total fuel capacity is 3250 gallons. The loading details are summarised in the following table.

Loading condition		Weight lb.	C.G. position - u/c down	
			Ins. aft of datum	S.M.C.
Aft limit		-	95.0	28.20
Forward limit		-	79.0	16.95
Mid c.g. of tests	Take-off	76,220	88.2	23.45
	All fuel gone	50,125	80.0	17.60
Aft c.g. of tests	Take-off	78,220	95.0	28.2
	All fuel gone	52,125	90.5	25.0

Raising the undercarriage moves the c.g. forward 1.6" (1.1% S.M.C.)

2. Engine details

The engines fitted were Nenes Mks.5 and 6, the difference between the marks being only in the positioning of the gearbox to enable them to be mounted in pairs.

The appropriate limitations were as follows:-

Rating	R.P.M.	Time limit (mins)	Jet pipe temperature limitation
Take-off	12,400	15	720
Max. Intermediate	12,100	30	680
Max. Cruise	11,700	Unlimited	620
Idling	2500 ± 100	10	550

Several engines were change during the trials and the engine numbers with dates fitted are given below:-

	Start of tests - 30.5.52	30.5.52 -22.9.52	22.9.52 -30.1.53	30.1.53 -23.4.53	23.4.53 -5.6.53
P.O.	32	32	13	13	468
P.I.	471	471	471	9	9
S.I.	461	461	150	150	150
S.O.	10	477	477	477	477

The fuel used throughout was AVTUR.

3. Auto-pilot

The aircraft was fitted with a Smith's Mk.9 auto-pilot. This is an all electric system providing 3-axis control. It differs fundamentally from previous Marks in that it functions on a 'rate/rate' principle, as opposed to the more usual "displacement" principle.

A full description of the Mk.9 auto-pilot is given in ref.3. The fact that it will control the aircraft at constant incidence makes it theoretically suitable for use in the 'climb cruise' range flying technique.

The installation was given normal servicing with repeated checks and adjustments if necessary before flight use. It was unrepresentative of a service installation in that the servomotors were limited to an output

/torque..

torque of 30 lb. ft. to meet aircraft stressing requirements in the event of a 'runaway', whilst it was understood from I.A.P., R.A.E., that motors of 66 lb. ft. torque would be required to make the control representative of that for a modern bomber.

4. Test instrumentation

The majority of the instruments used in these tests were contained in an automatic observer using an F.24 camera. The relevant instruments were:

Clock
Frame counter
A.S.I.
Altimeters - 2 off
Engine speed indicators - 4 off
Jet pipe temperature gauges - 4 off
Jet pipe pitot differential pressure gauges - 4 off
Gallons gone veeder counters - 4 off
Fuel temp. gauges - 2 off

The A.S.I. and altimeter were connected to the first pilot's pitot static system together with the static side of the jet pipe pitot differential pressure gauges.

The first pilot's system consisted of a pitot head on the port side of the aircraft just forward of the cockpit with a static vent on the port side of the fuselage.

The fuel temperature elements were located in the fuel lines forward of the engine main bulkhead and within 4-5 ins. of a combustion chamber, and doubts arose on their reliability but on repositioning the elements in the fuel line in the undercarriage bay, the readings appeared to be unaffected.

Further instruments were on the flight observer's panel and read visually. These instruments were:-

(1) Balanced bridge thermometer - the M.O.4 knife-edge element was placed under the port wing about 10 ft. outboard of the engine nacelle.

(2) Fuel flowmeters. 4 off. Of the Kent type with a range of 120-1300 g.p.h.

All the instruments were calibrated at about monthly intervals during the test period.

Appendix 2

Notes on conditions for optimum specific air range for turbo-jet aircraft

1. General

From the usual 'non-dimensional' relations for turbo-jet aircraft performance we have, assuming no scale effects,

$$a_o M = \frac{V}{\sqrt{\theta}} = f_1\left(\frac{N}{\sqrt{\theta}}, \frac{W}{p}\right) \text{ and } \frac{F}{p\sqrt{\theta}} = f_2\left(\frac{N}{\sqrt{\theta}}, \frac{V}{\sqrt{\theta}}\right)$$

whence the non-dimensional form of specific air range may be written as

$$\frac{WV}{F} = \frac{W}{p} \times \frac{V}{\sqrt{\theta}} \div \frac{F}{p\sqrt{\theta}} = f_3\left(\frac{N}{\sqrt{\theta}}, \frac{V}{\sqrt{\theta}}\right)$$

Any two of the interdependent variables $\frac{N}{\sqrt{\theta}}$, $\frac{V}{\sqrt{\theta}}$ (or M), $\frac{W}{p}$, and $\frac{V_i}{\sqrt{W}}$ (or m) will therefore define the flight conditions including the range parameter, $R = \frac{WV}{F}$

$$\text{Now } R = \frac{WV}{F} = \frac{WV}{cD} = \frac{V}{c} \cdot \frac{C_L}{C_D} \qquad \text{..........(1)}$$

Note that this assumes X = D. For the climb cruise at constant W/p, X exceeds D by a small climb term of the order of 1-2%; the effects of this may be ignored in considerations of range technique.

2. Without compressibility drag effects

2.1 In this case we assume $C_D = C_{D_Z} + k\,C_L^2$ where C_{D_Z} and k are constants and thus find

$$\frac{X}{W} \doteq \frac{C_D}{C_L} = \sqrt{k\,C_{D_Z}}\left(m^2 + \frac{1}{m^2}\right) \text{ where } m = \sqrt[4]{\frac{C_{D_Z}}{kC_L^2}} \propto \frac{V_i}{\sqrt{W}}$$

$$\text{also } V = \frac{V_i}{\sqrt{W}}\sqrt{\frac{W}{p}}\sqrt{\theta} \propto m\sqrt{\frac{W}{p}}\sqrt{\theta}$$

Hence from (1)

$$R \propto \frac{\sqrt{\theta}}{c} \cdot \sqrt{\frac{W}{p}} \frac{m}{m^2 + 1/m^2} \qquad \text{..........(2a)}$$

$$\text{or} \propto \frac{\sqrt{\theta}}{c} \cdot \sqrt{\frac{X}{p}} \frac{m}{(m^2 + 1/m^2)^{3/2}} \qquad \text{..........(2b)}$$

Note that $c/\sqrt{\theta}$ is the 'non-dimensional' form of specific fuel consumption and is independent of air temperature for a given $N/\sqrt{\theta}$ and $V/\sqrt{\theta}$ (or m and W/p or X/p).

2.2 If we now assume as a simplifying approximation that specific consumption is independent of thrust and speed the usual values for optimum m may be deduced.

/Thus..

Thus from 2(a) we have at constant W/p, i.e. constant height for a given weight

$$R \propto \frac{m^3}{1 + m^4} \quad \therefore \quad \frac{m}{R} \cdot \frac{\partial R}{\partial m} = \frac{3 - m^4}{1 + m^4}$$

Thus optimum specific range at a given height and weight is obtained when $m = \sqrt[4]{3} = 1.32$ or at the highest possible speed if the thrust for this is not available.

Note that

(i) a 10% change of speed from the optimum reduces specific range by only about 2% for a less and 1½% for a greater speed.

(ii) Cruising at constant height at the optimum speed (or any other constant m) results in increasing specific range as fuel is consumed, since $\frac{V}{F} \propto W^{-\frac{1}{2}}$, but also requires decreasing speed, since $V \propto W^{\frac{1}{2}}$, and decreasing r.p.m. ($X \propto W$). If speed is maintained constant specific range still increases with weight reduction but at a decreasing rate as m increases above the optimum.

(iii) Since, at constant m, $R \propto \sqrt{W/p}$ greatest specific range at any constant m is obtained at the greatest possible height i.e. using maximum permissible r.p.m.; equation 2(a) does not however tell us directly which combination of height and speed gives the absolute maximum specific air range.

From 2(b) we have at constant X/p, i.e. constant r.p.m. if air temperature is constant and thrust is independent of speed,

$$R \propto \frac{m}{(m^2 + 1/m^2)^{3/2}} \quad \therefore \quad \frac{m}{R} \frac{\partial R}{\partial m} = \frac{2\,(2 - m^4)}{1 + m^4}$$

Thus optimum specific range at a given r.p.m. is obtained when $m = \sqrt[4]{2} = 1.19$.

Note that

(i) a 10% change in m (or E.A.S.) from the optimum reduces range by about 2½%.

(ii) Height will be reduced as speed increases (for $m > 1$).

(iii) Cruising at constant r.p.m. and air temperature at the optimum speed (or any constant m) WV/F is constant so that specific range, $V/F \propto W^{-1}$ and increases with weight reduction at twice the rate for the constant height case; in this case W/p is constant so that height increases continuously but the true airspeed remains constant in the standard stratosphere since $V \propto m \cdot \sqrt{\frac{W}{p}} \cdot \sqrt{\theta}$

(iv) Since X/p increases with r.p.m. at a given speed and air temperature the absolute maximum specific air range is obtained at the maximum permissible r.p.m. and $m = 1.19$.

(v) Since any 'non-dimensional' performance parameter may be expressed as $f\,(N/\sqrt{\theta}\,,\ m)$ the effect of a change in air temperature, ΔT_a, in the specific range at constant r.p.m. and m (e.g. on the absolute maximum value) is exactly equivalent to a change in r.p.m., ΔN, such that $\Delta N/N = -\frac{1}{2}\,\Delta T_a/T_a$; increase of air temperature thus reduces the maximum possible specific range.

/Thus..

Thus summarising the conditions for maximum range in cruising flight when the assumptions made above are relevant we have

(a) WV/F must be maintained at its maximum value as weight falls; this involves keeping W/p or X/p, and m at their optimum values.

(b) These optimum values are obtained with X/p a maximum i.e. maximum possible r.p.m. and $m = 1.19$.

(c) Height therefore increases as weight falls.

(d) I.A.S. is reduced as weight falls to keep $V_i/\sqrt{W}$ constant.

(e) Mach number and hence true airspeed remain constant in the standard stratosphere.

(f) C_L and thus incidence remain constant as the weight falls so that the required cruising conditions can be suitably maintained by an auto-pilot controlling at constant attitude.

(g) The actual values of specific air range and Mach number so obtained will depend on air temperature, both being reduced by an increase of temperature.

2.3 In a practical case where specific fuel consumption varies with r.p.m. and speed, and thrust at a given r.p.m. also varies with speed, the above deductions on the optimum condition may be slightly modified.

Thus from 2(b) we have

$$\frac{\delta R}{R} = -\frac{\delta c}{c} + \frac{1}{2}\,\frac{\delta (X/p)}{X/p} + 2\,\frac{(2-m^4)}{1+m^4}\cdot\frac{\delta m}{m}$$

$\therefore$ at constant m, range increases with increased X/p provided

$$\left[\frac{X/p}{c}\cdot\frac{\partial c}{\partial (X/p)}\right]_V < 0.5$$

The value of this derivative for a particular engine may be derived from the firm's performance data; it is invariably positive and usually of the order 0.2 to 0.3 at max. cruise r.p.m. at typical cruising speeds so that maximum range is in practice usually obtained at the maximum available r.p.m.

The best speed for range (at constant r.p.m.) is then given approximately by

$$\frac{2(2-m^4)}{1+m^4} = \left[\frac{V}{c}\cdot\frac{\partial c}{\partial V}\right]_N - \frac{1}{2}\left[\frac{V}{X/p}\,\frac{\partial (X/p)}{\partial V}\right]_N$$

Estimates made of these derivatives for several current engines at max. cruise r.p.m. gave values of the optimum m varying between 1.1 and 1.2.

3. With compressibility drag effects

A detailed general investigation of the effects of compressibility would be unprofitable without generalised data on the variation of drag with Mach number. However making some simplifying assumptions some indication of the probable effects can be shown.

From (1) $\quad R = \dfrac{V}{c}\cdot\dfrac{C_L}{C_D} = a_o\, M\cdot\dfrac{\sqrt{\theta}}{c}\cdot\dfrac{C_L}{C_D}$.

/If we..

If we represent drag with compressibility by $C_D = E.\ C_{DS}$ where C_{DS} is the subcritical drag coefficient at the same C_L and E is a function of Mach number only, equal to unity below the drag critical Mach number, M_c, then, again assuming constant specific consumption

$$R \propto \frac{M}{E} \cdot \frac{C_L}{C_{DS}}$$

Thus for any constant Mach number optimum specific range is obtained at the maximum value of $\frac{C_L}{C_{DS}}$ i.e. when $m = 1$ (this result is applicable of course below and above M_c).

Note that with this type of assumption on the effect of compressibility on drag, where the proportional increases of profile and induced drag are the same for a given M, this optimum C_L (or $V_i/\sqrt{W}$) at constant M is independent of M. With other (and more realistic) assumptions this would not be so and the concept of 'minimum drag speed' used here and the significance of m would become ambiguous for $M > M_c$.

The optimum Mach number is then obtained when M/E is a maximum or

$$\frac{M}{E} \cdot \frac{dE}{dM} = 1.$$

If we write $E = 1 + K(M - M_c)^2$ for $M > M_c$

$$\text{Then } \frac{M}{E} \cdot \frac{dE}{dM} = \frac{2KM(M - M_c)}{1 + K(M - M_c)^2} = 1$$

$$\therefore\ M_{opt}^2 = M_c^2 + 1/K$$

$$\text{and } E_{opt} = 1 + \left(\sqrt{KM_c^2 + 1} - \sqrt{KM_c^2}\right)^2$$

Thus as $K \longrightarrow \infty$, $M_{opt} \longrightarrow M_c$ and $E_{opt} \longrightarrow 1$.

A typical value of K to represent the start of the drag rise for a current swept-wing aircraft would be about 20.

Taking $M_c = 0.8$ we get $M_{opt} = 0.83$ and $E_{opt} = 1.019$, i.e. an optimum Mach number of 0.03 above the critical and a compressibility drag increase of about 2% giving an increase of range of about 2% over that obtained by cruising at the critical Mach number.

We should therefore expect that unless the drag increase with Mach number is very gradual the optimum range Mach number would be only slightly above the drag critical Mach number.

In practice variations of engine specific consumption with thrust and speed will again slightly modify these results (tending generally to reduce slightly the optimum Mach number).

It may be observed that the optimum specific range value thus obtained is effectively an absolute optimum for the airframe and is obtained at a particular height and thrust (for a given weight). Specific range is not in this case increased by providing greater thrust but only if the particular thrust required can be provided with a reduced specific consumption.

Thus the value of the product $M_c(L/D)_{max}$ (where $(L/D)_{max}$ is the sub critical value) for an airframe may be regarded as a figure of merit for its range capabilities since it is closely proportional to the maximum possible specific air range obtainable at a given weight with engines of a given specific consumption.

/For a..

For a given airframe-engine combination the optimum range condition requires a particular $N/\sqrt{\theta}$ value and if this can be maintained if the air temperature rises, without exceeding the engine limitations, then the actual specific air range will be independent of temperature.

4. Scale effects on specific air range

It has so far been assumed that the usual non-dimensional relations are applicable over the full weight range of the aircraft i.e. that WV/F is a function only of any two of m. M, $N/\sqrt{\theta}$, W/p. This results from the engine performance relations X/p and $F/p\sqrt{\theta}$ = fns $(N/\sqrt{\theta}, M)$ and the assumption of an aircraft drag of the form $C_D = fn(C_L, M)$ giving $D/p = fn(m, M)$.

In practice Reynolds number (or height) is a further variable in each of these relations. At high altitudes the performance of turbo-jet engines falls off with increasing altitude below that which would be predicted from the usual non-dimensional relations at lower altitudes so that the thrust is lower and the specific fuel consumption is higher; the reducing Reynolds number as height is increased at constant speed tends also to result in an increase in profile drag. The nett effect during a standard climb cruise will be a reduction in the range parameter WV/F with weight reduction which may be quite marked for a long range aircraft with a large disposable load.

This effect must be measured in flight by tests at the optimum conditions at at least the highest and lowest possible weights.

Appendix 3

Corrections applied to results for drag and rate of climb differences

1. Reasons for corrections

Corrections have been required to the test results in presenting the specific air range data on a comparable basis for the following reasons.

(a) Changes of drag occurred during the period of the trials, which spanned a minor inspection and several engine changes; where comparison is required between range data therefore the results have all been presented as corresponding to the drag measured in the first series of tests, the level speed tests.

(b) Part of the programme consisted of stabilised level speed runs and part of climb cruise and quasi-level tests involving a small rate of climb. The measured specific air range will evidently be lower in the latter cases for the same airspeed and engine speed; allowances have therefore been made to these results to 'correct' them to a level speed basis for direct comparison with the level speed tests.

2. Drag correction

2.1 Method of correction. In adjusting the data to correspond to a different drag to that of the test it is desirable to make the adjustment at the same engine conditions so that the measured engine data are used unchanged.

This involves keeping $N/\sqrt{\theta}$ and $V/\sqrt{\theta}$ constant as the drag is changed; thus X/p and $F/p\sqrt{\theta}$ will also be unchanged and hence pV/F. The correction required is in effect therefore to aircraft weight so that in the new drag configuration the same speed is obtained for the same thrust i.e. corrections are required to W/p and hence to $V_i/\sqrt{W}$ and WV/F, i.e. to the specific air range at the same weight.

A small incidence change may thus be involved and the only assumption made is that this and the differences responsible for the drag change do not affect the engine intake efficiency and hence the engine performance.

Thus X/p and $V_i/\sqrt{p}$ $(= V/\sqrt{\theta})$ are unchanged hence also X/V_i^2 and D/V_i^2 $(\propto C_D)$.

The measured engine data can thus be easily associated with any given aircraft drag when the $C_D - C_L$ relation is given (or if compressibility effects are present if the datum drag is given in the form $C_D = f\ (C_L, M)$ since the observed engine data give C_D and M). This method has been applied to the present results to obtain correction factors to $V_i/\sqrt{W}$, W/p and WV/F.

Thus from Fig.7 giving $\frac{D}{V_i^2}$ $(\propto C_D)$ v $\frac{W^2}{V_i^4}$ $(\propto C_L^2)$ for all the test results we read off $\left[\frac{W^2}{V_i^4}\right]_a$, the actual test value, and $\left[\frac{W^2}{V_i^4}\right]_c$, the datum (level speed test) value, for each test value of $\frac{D}{V_i^2}$ and obtain the ratio of these, r

/contd.

i.e. $$\left[\frac{W^2}{V_i^4}\right]_a = r \left[\frac{W^2}{V_i^4}\right]_c$$

$$\therefore \quad \left[\frac{V_i}{\sqrt{W}}\right]_c = r^{\frac{1}{4}} \left[\frac{V_i}{\sqrt{W}}\right]_a$$

$$\left[\frac{W}{p}\right]_c = r^{-\frac{1}{2}} \left[\frac{W}{p}\right]_a \quad \text{since} \quad \frac{V_i}{\sqrt{p}} \quad \text{is unchanged}$$

$$\left[\frac{WV}{F}\right]_c = r^{-\frac{1}{2}} \left[\frac{WV}{F}\right]_a \quad \text{since} \quad \frac{pV}{F} \quad \text{is unchanged.}$$

2.2 Application to results. In Fig.9 values of X/V_i^2 corresponding to the readings taken throughout each climb cruise when conditions were apparently steady (i.e. generally omitting the first few readings of each run before the rate of climb settled to a fairly steady value) are plotted against W^2/V_i^4 (nominally constant for each run). The calculated best straight line through these points is given and an allowance applied corresponding to subtraction of the calculated extra thrust required for the small rate of climb (about 2%, see para. 3.2 below) to obtain the corresponding line for D/V_i^2 v W^2/V_i^4. This line has been replotted in Fig.7 for comparison with the level speed test drag; a drag increase corresponding to an increase in C_{D_Z} of $8\frac{1}{2}$% with no change of induced drag is seen to have occurred between the two sets of tests. Also plotted in Fig.9 are the values of X/V_i^2 obtained on the quasi-level tests but corrected to the rate of climb corresponding to the climb cruise conditions (see para.3.2) to show directly the drag change between these two sets of tests.

That these changes are genuine drag changes and not due to a change in the single pitot v thrust relation is supported by the thrust v engine speed plot of Fig.2 where there are no signs of such systematic discrepancies in the thrust values as are shown by the drag data.

The correction factor r has been obtained for each set of climb cruise points by taking the mean values of X/V_i^2 and W^2/V_i^4 for each run and correcting the former for the mean rate of climb to give D/V_i^2. These are thus the values of D/V_i^2 and $(W^2/V_i^4)_a$; $(W^2/V_i^4)_c$ is obtained from the best linear relation of D/V_i^2 v W^2/V_i^4 for the level speed tests given in Fig.7; similar corrections were obtained for the three quasi-level points.

The effect of these corrections on the range picture is shown in Fig.11 where WV/F v $V_i/\sqrt{W}$ is shown for the points before and after correction. The correction factor r varied from about 0.8 at m = 1.3 to 0.95 at m = 0.9 so that the corresponding corrections made to specific air range varied from about 10% to 3% increase.

3. Climb correction

3.1 Correction to level flight conditions. For any run where rate of climb, v, and thrust are measured the drag may be estimated from

$$D = X - W\frac{v}{V}$$

To correct the results to level flight at the same speed and height we thus have

$$\text{Thrust correction} \quad \triangle X = - W\frac{v}{V}$$

/Fuel flow..

Fuel flow correction $\Delta F = c \, . \, \Delta X$ where for c, the specific fuel consumption, we can use the measured value, F/X or an estimate from the firm's performance data.

R.P.M. correction $\Delta N = - \Delta X \Big/ \left[\frac{\partial X}{\partial N}\right]_V$ where $\left[\frac{\partial X}{\partial N}\right]_V$ may be estimated from firm's data

$$\therefore \quad \frac{\Delta R}{R} = - \frac{\Delta F}{F} = - c \frac{\Delta X}{F} \text{ or } - \frac{\Delta X}{X}$$

Correction from a slow climb to level flight conditions at the same speed, weight and height thus involves an increase in the specific air range and a decrease in r.p.m.

A further correction to bring the r.p.m. back to the test value or to the nominal test value may be made if $\left[\frac{N}{R} \, . \, \frac{\partial R}{\partial N}\right]_m$ is known experimentally; the quasi-level test results as plotted in Fig.10 give this derivative for the present case and have been used for this purpose.

3.2 Correction to climb cruise conditions. To correct level speed or quasi-level run results to the climb cruise condition (C_L, $N/\sqrt{\theta}$ constant) we must first derive the corresponding rate of climb, v_o, and use the above process to correct to $v = v_o$ instead of $v = 0$.

We have for the climb cruise

$\frac{W}{p}$ = constant; $\frac{dW}{dt} = - \frac{F}{3600}$ lb./sec. where F may be the measured fuel flow (in lb./hr.)

$$\therefore \quad \frac{dp}{dt} = \frac{dW}{dt} \div \frac{W}{p} = - \frac{F}{3600} \div \frac{W}{p}$$

$$v_o = \frac{dh}{dt} = - F \frac{dh}{dp} \div 3600 \frac{W}{p}$$

Also $dP = - \rho \, g \, dh$ where P, ρ, g, h are in consistent units

$$\therefore \quad \frac{dh}{dp} = - \frac{2.76 \times 10^4}{\sigma} = - 2.76 \times 10^4 \, . \, \frac{\theta}{p}$$

$$\therefore \quad v_o = 7.66 \, \theta \, . \, \frac{F}{W} \text{ ft./sec.}$$

$$\simeq 7.66 \, \theta \, . \, c \, . \, \frac{D}{W}$$

$$= 5.76 \, c \, \frac{D}{W} \quad \text{in the standard stratosphere}$$

For the standard climb cruise at constant C_L and N, D/W and c are constant so that v_o is constant throughout a cruising flight in the standard stratosphere.

Hence also the proportion of power expended on climbing remains constant throughout the cruise.

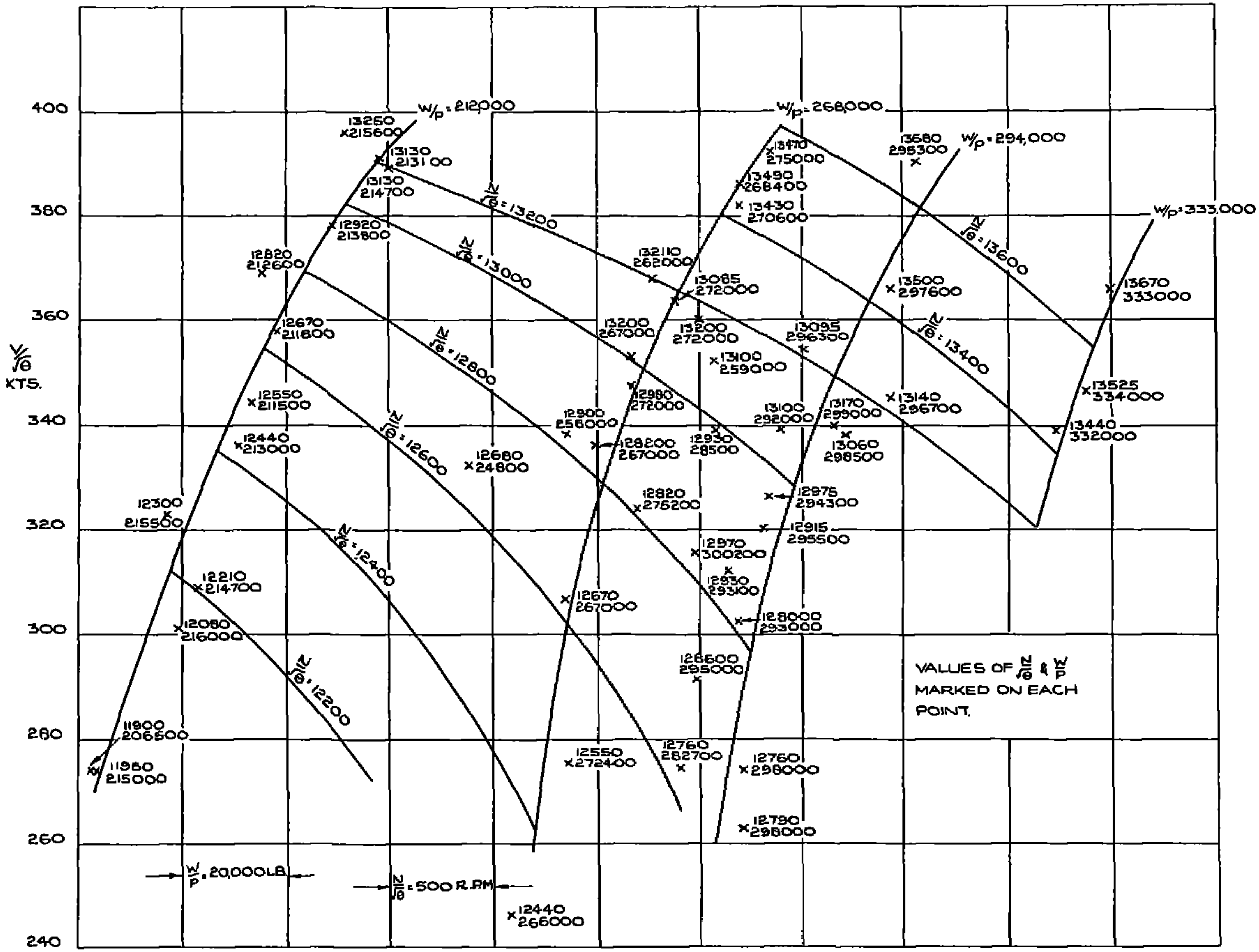
Thus power to climb $= Wv_o = 5.76 \, c \, X$ ft. lb./sec.

Total power $= 1.69 \, VX$ ft. lb./sec. (V in knots)

$$\therefore \quad \frac{\text{Climb power}}{\text{Total power}} = \frac{3.4c}{V}$$

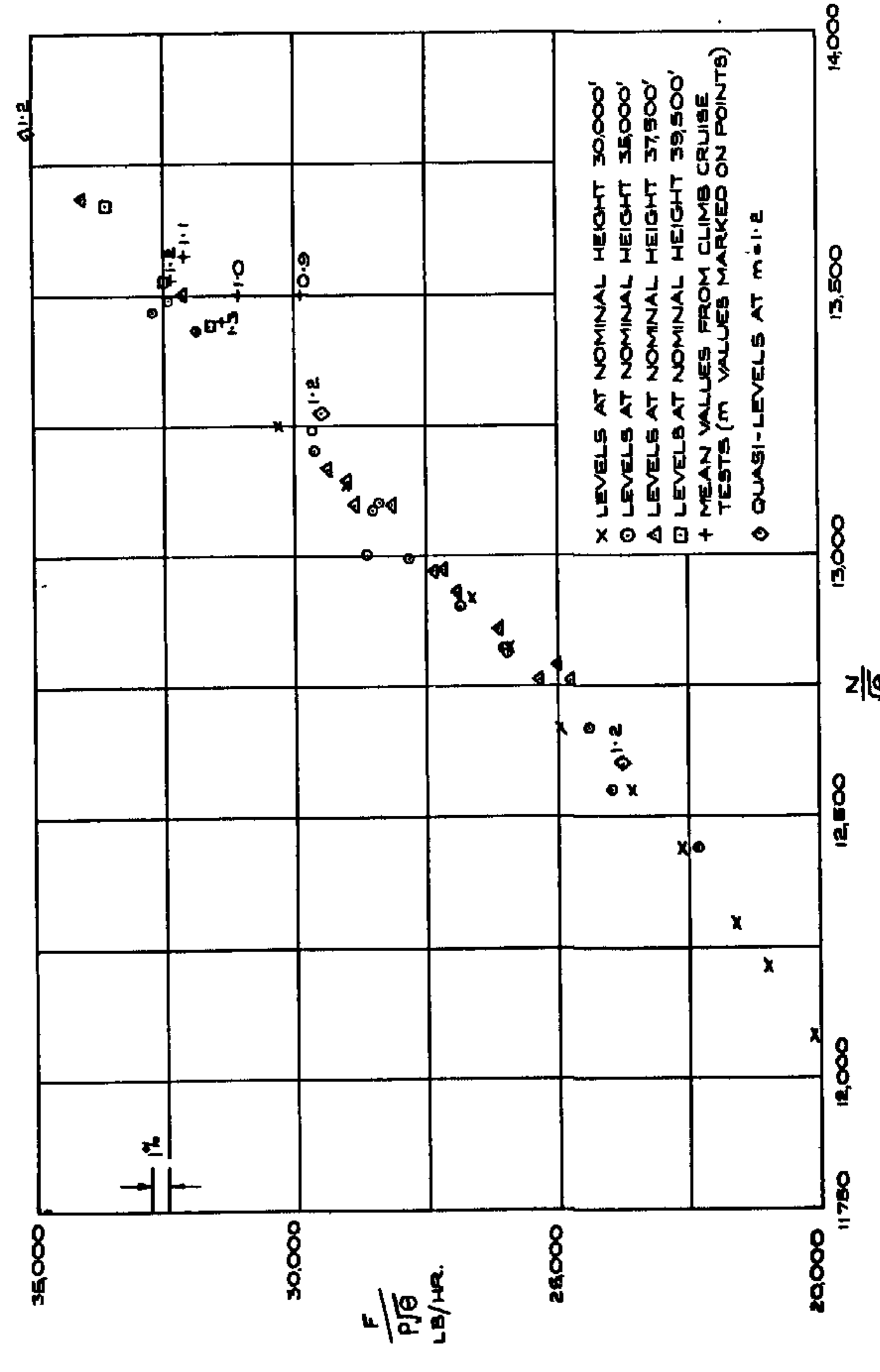
where c is in lb./hr./lb. thrust
V is in knots

FIG. 1



LEVEL SPEED GRID $V/\sqrt{\theta}$ v $\frac{N}{\sqrt{\theta}}$ & $\frac{W}{P}$ FROM STABILISED SPEED TESTS.

FIG. 2



NON-DIMENSIONAL FUEL CONSUMPTION v R.P.M.
$\frac{F}{P\sqrt{\theta}}$ v $\frac{N}{\sqrt{\theta}}$ FOR ALL TESTS.

NON-DIMENSIONAL NETT THRUST V R.P.M. FOR ALL TESTS.

$\frac{X}{P}$ v $\frac{N}{\sqrt{\theta}}$

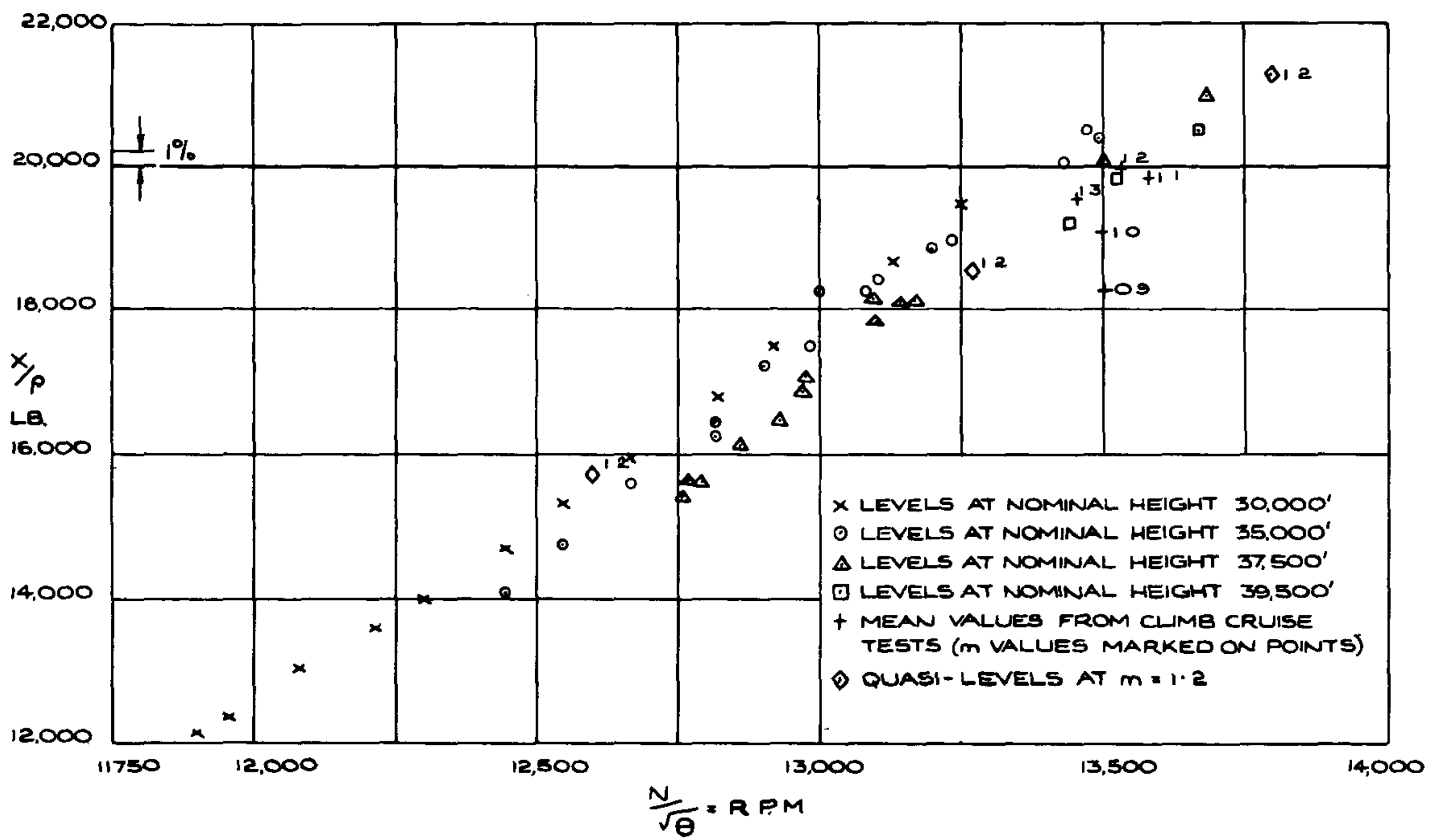
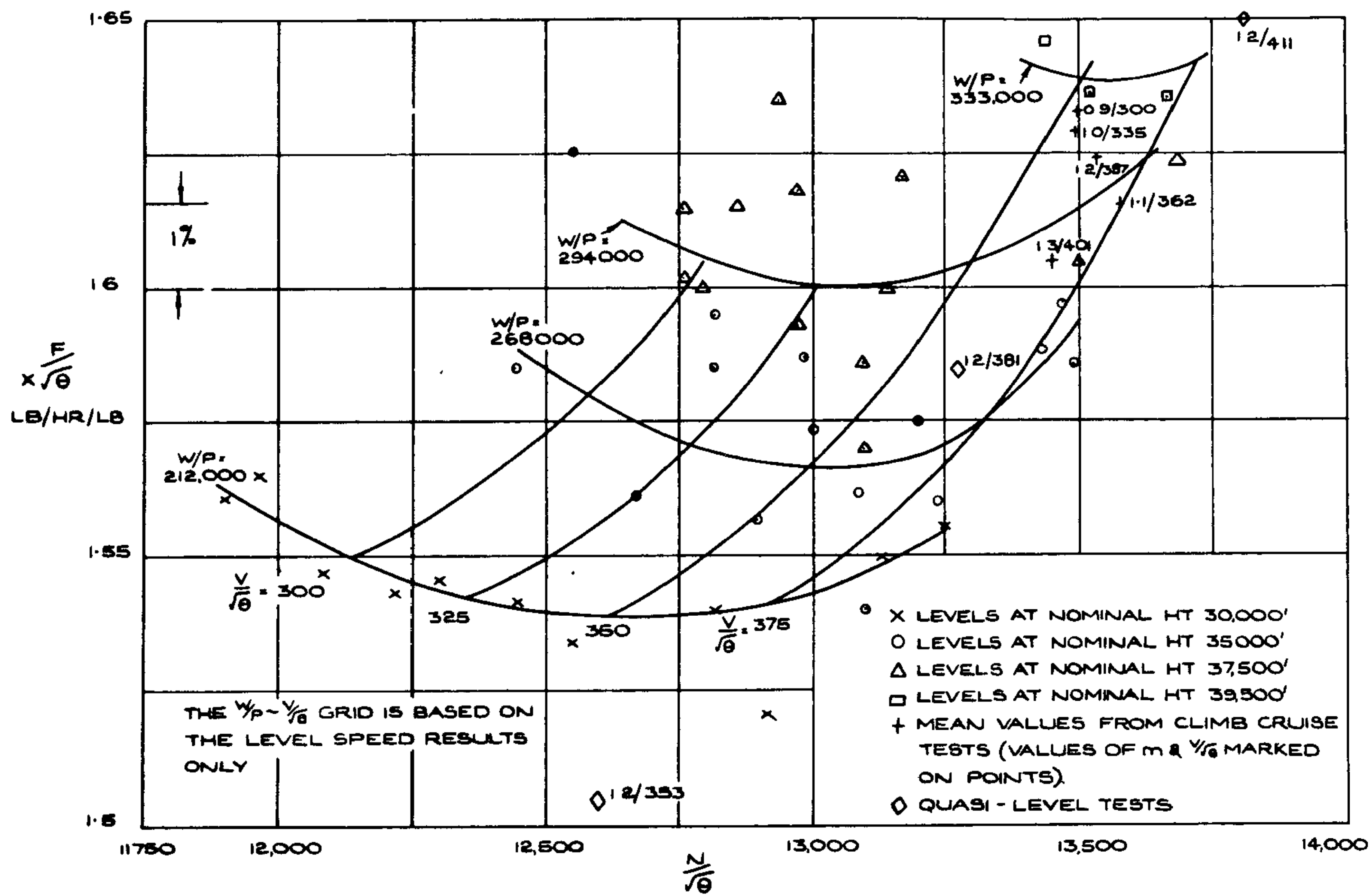


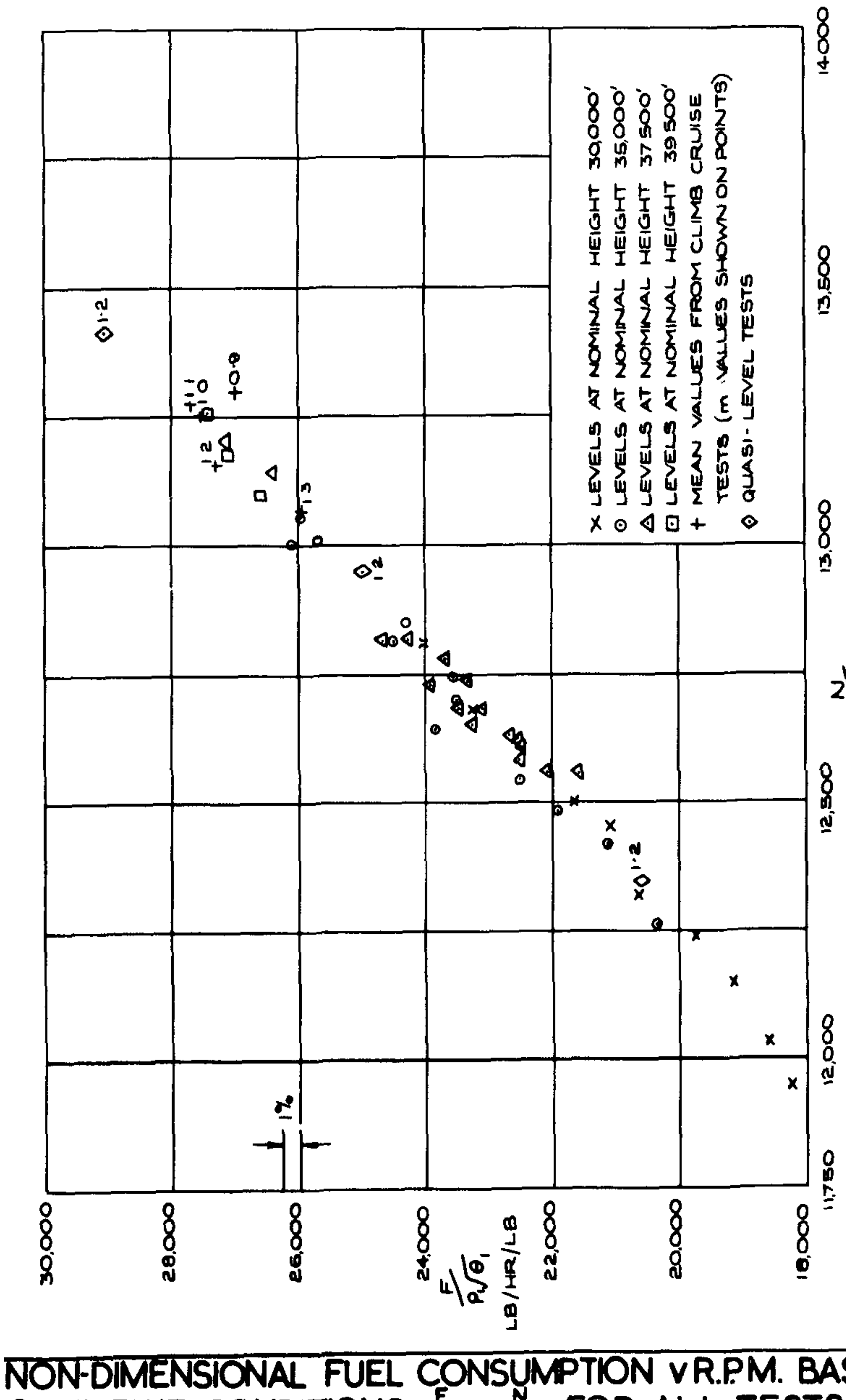
FIG. 3



NON-DIMENSIONAL SPECIFIC FUEL CONSUMPTION v R.P.M. $\times \frac{F}{\sqrt{\theta}}$ v $\frac{N}{\sqrt{\theta}}$ FOR ALL TESTS.

FIG. 4

FIG. 5



NON-DIMENSIONAL FUEL CONSUMPTION v R.P.M. BASED ON INTAKE CONDITIONS $\frac{F}{P_1\sqrt{\theta_1}}$ v $\frac{N}{\sqrt{\theta_1}}$ FOR ALL TESTS.

SPECIFIC AIR RANGE IN LEVEL FLIGHT.

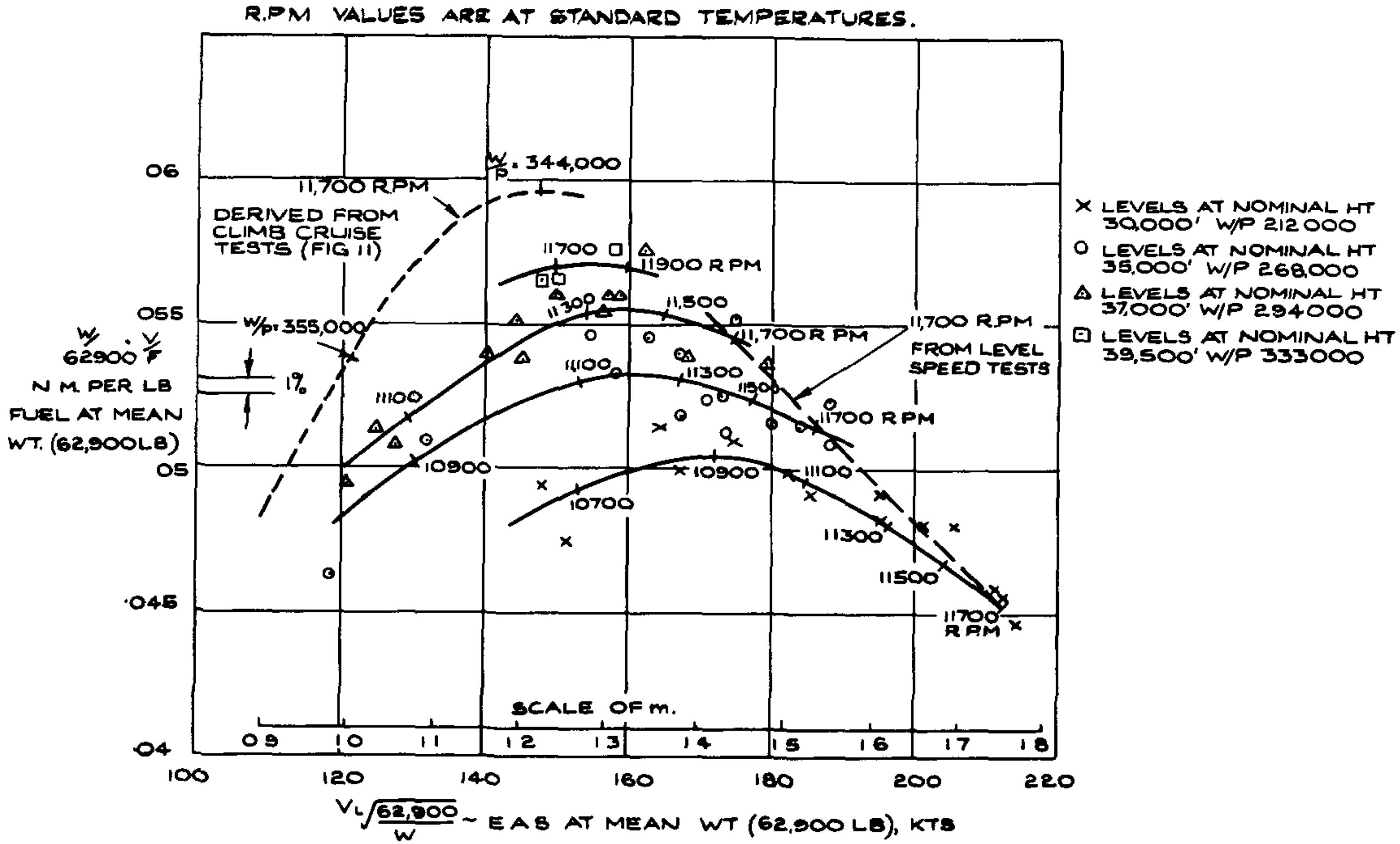
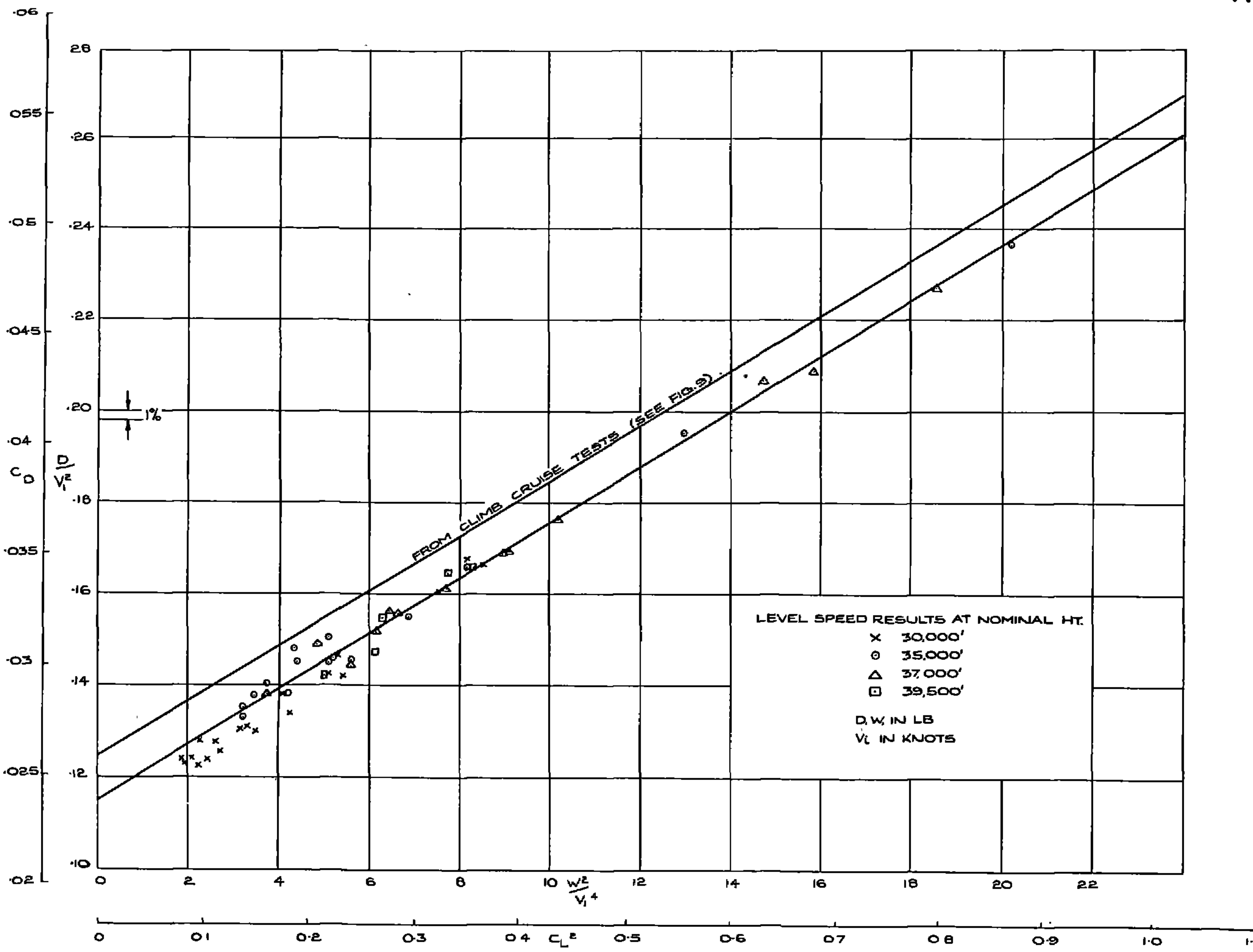
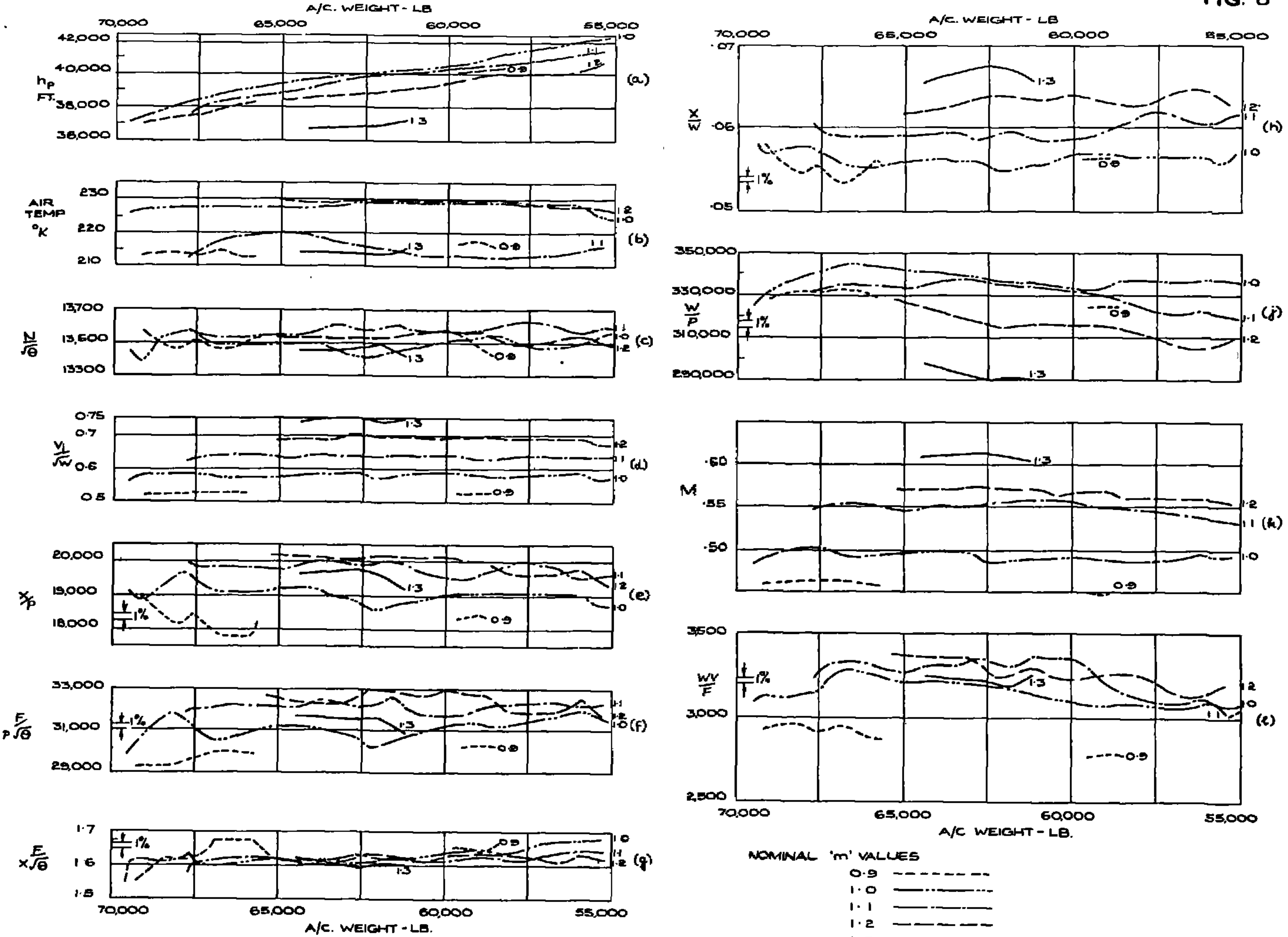


FIG. 6

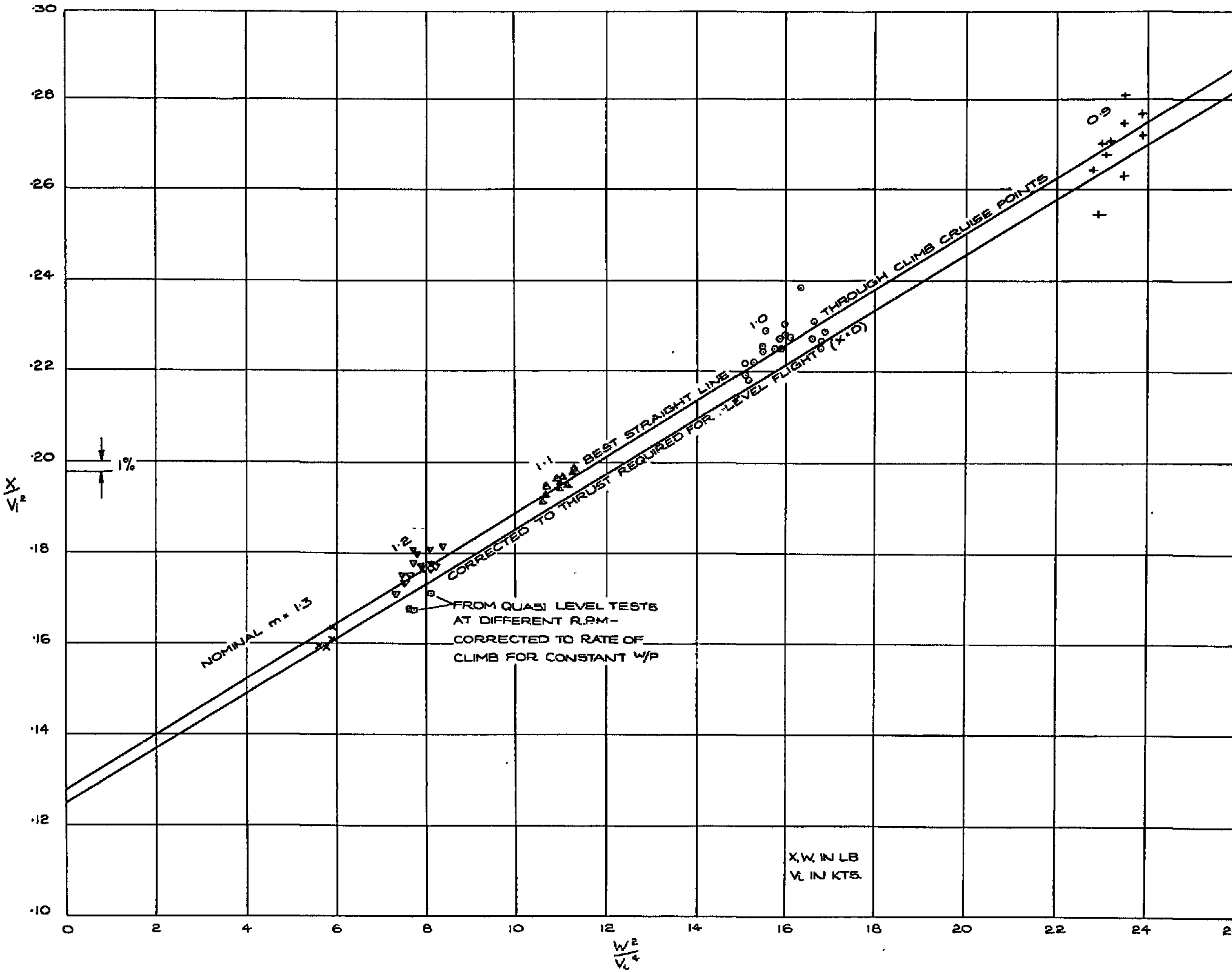


DRAG MEASURED IN LEVEL SPEED TESTS.

FIG. 8



VARIATION OF VARIOUS PARAMETERS DURING CLIMB CRUISE FLIGHTS.



$\frac{X}{V_i^2}$ v $\frac{W}{V_i^4}$ FOR CLIMB CRUISE & QUASI-LEVEL TESTS

FIG. 10 & 11

FROM QUASI-LEVEL TESTS AT m ≈ 1·2
CORRECTED TO LEVEL FLIGHT AT SAME DRAG

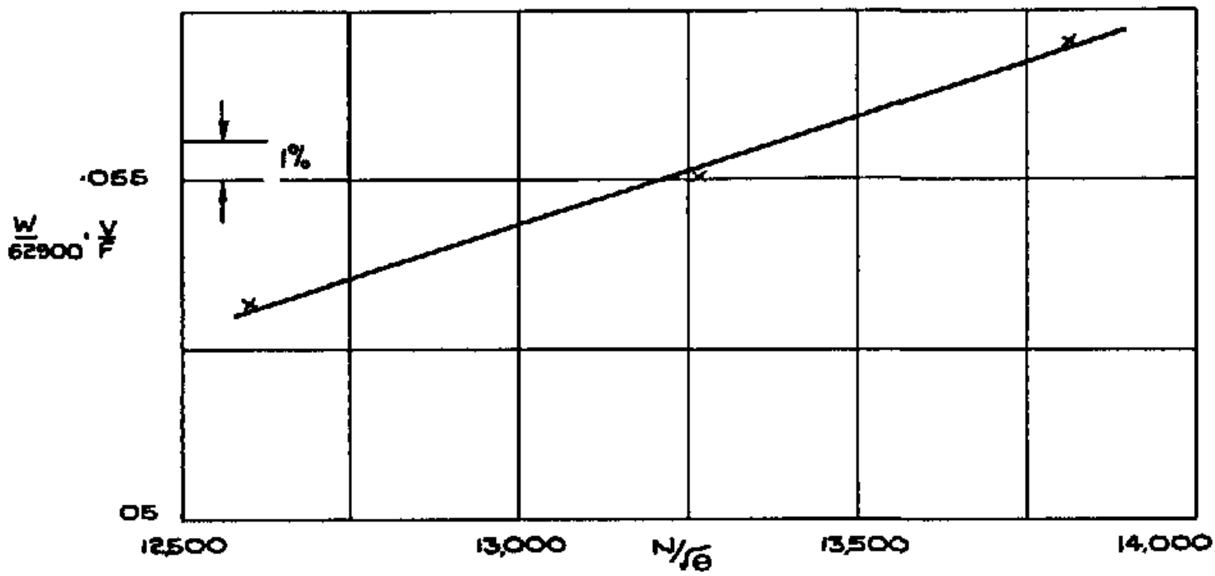


FIG 10 VARIATION OF SPECIFIC AIR RANGE WITH R P M OR AIR TEMPERATURE AT CONSTANT m

CURVES REPRESENT RANGE ENVELOPES AT N/√θ = 13500 (N = 11700 @ -56.5 °C)

(a) FOR CLIMB CRUISE IN DRAG CONDITION TESTED
(b) FOR CLIMB CRUISE CORRECTED TO DRAG OF LEVEL SPEED TESTS
(c) AS WOULD BE MEASURED IN LEVEL FLIGHT AT DRAG OF LEVEL SPEED TESTS

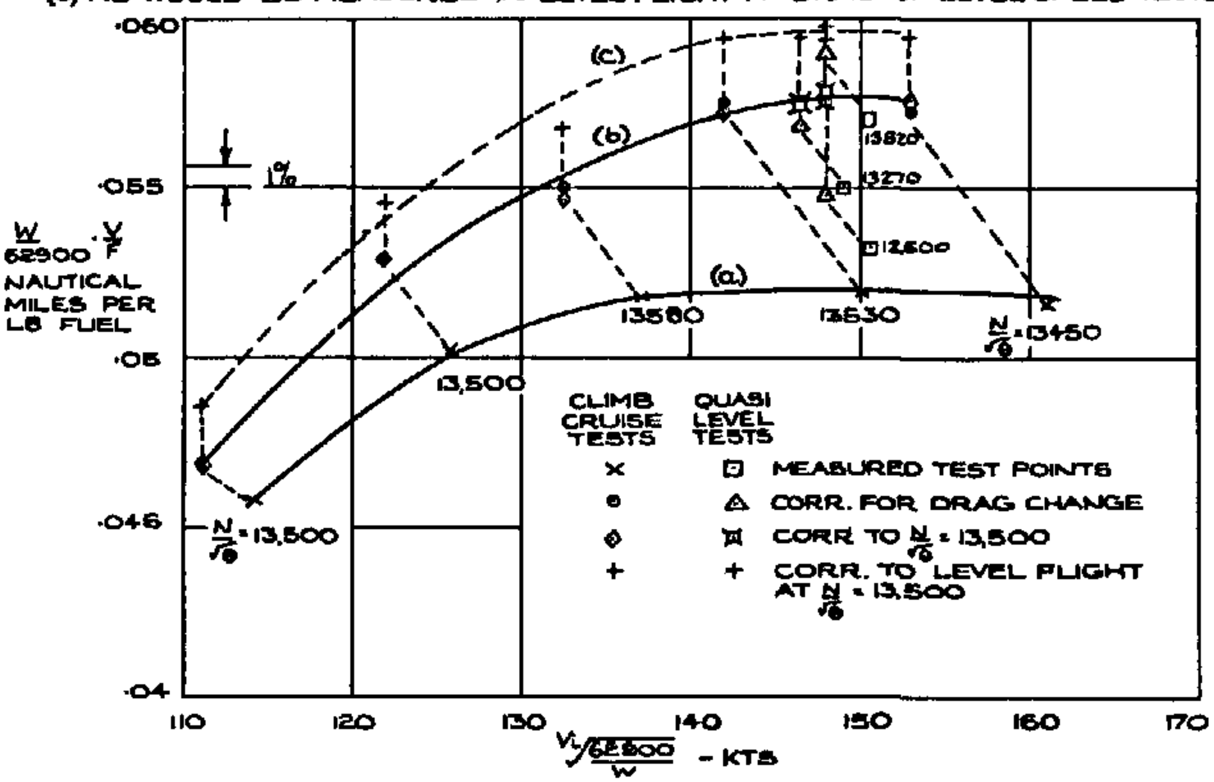


FIG 11 SPECIFIC AIR RANGE RESULTS FROM CLIMB CRUISE & QUASI-LEVEL TESTS SHOWING CORRECTIONS MADE FOR DRAG CHANGE AND CLIMB

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