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By

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APPROXIMATE WALL CORRECTIONS FOR AN OSCILLATING SWEPT WING IN A WIND TUNNEL OF CLOSED CIRCULAR SECTION

The oscillatory interference upwash for a circular tunnel is derived from the corresponding steady upwash. Corrections to measured derivatives of a slowly pitching wing are calculated by Multhopp's lifting surface theory. A satisfactory approximate method using interference parameters for a small wing is given, and an extension to rectangular tunnels is suggested.

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Approximate Wall Corrections for an Oscillating Swept Wing in a Wind Tunnel of Closed Circular section

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SUMMARY

The interference upwash due to a swept horse-shoe vortex of periodic strength in a closed circular tunnel is obtained by a streamwise integration of the corresponding upwash due to a vortex of constant strength (Ref. 1). The values so obtained are treated by Multhopp's lifting surface theory (Ref. 2) to calculate tho effect of tunnel walls on tho aerodynamic derivatives of a particular oscillating swept wing in incompressible flow (Ref. 3). The results show that frequency has a negligible effect on the corrections to tho stiffness derivatives, and that for the tests of Ref. 3 there is no need to apply interference corrections to tho damping derivatives of lift and pitching moment.

The interference upwash is also expressed by an approximate formula using four parameters δ_0, δ_1, δ_0', δ_1' relating to a wing of small span. The interference on the swept wing is calculated from this information and the result is shown to be satisfactory for practical use. In the Appendix the analogous parameters δ_0, δ_1, δ_0', δ_1' are derived for rectangular tunnels and tabulated for a range of height/breadth ratio sufficient to cover most tunnels.

Contents

Figure 1/

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Figure 1 Position of horse-shoe vortex in relation to circular tunnel

Figure 2 Plan of model used in tests at N.P.L.

§1 Notation

b/h	breadth/height ratio of a rectangular tunnel
$C_L = L/\frac{1}{2}\rho V^2 S$	Lift coefficient
$C_m = \mathcal{M}/\frac{1}{2}\rho V^2 S\bar{c}$	pitching moment coefficient
$(C_L)_1, (C_L)_2, (C_L)_3, (C_L)_4$; $(C_m)_1, (C_m)_2, (C_m)_3, (C_m)_4$	defined in equation (41)
$\bar{c}$	mean chord of wing
f	frequency of oscillation of wing
$f(x)$	a function defined in equation (A.8)
$l + i\nu l'$	tunnel-induced loading coefficient
l_1, l_2, l_1', l_3	steady loading coefficient due to W_1, W_2, W_1', W_3
M	Mach number
$m_\theta, m_{\dot\theta}$	defined by $C_m = 2\left(m_\theta \cdot \theta + \frac{\bar{c}}{V} m_{\dot\theta} \cdot \dot\theta\right)$
p	pressure
R	radius of tunnel
S	area of plan form of wing
s	semi-length of finite part of horse-shoe vortex (Fig. 1)
$s_1 = s\cos\Lambda$	semi-span of horse-shoe vortex
t	time
V	velocity far upstream in the direction of X increasing
$W = \dfrac{4\pi R^2 w}{\Gamma s_1}$	non-dimensional interference upwash due to a horse-shoe vortex
W_1, W_2	parts of W due to, and in phase with Γ_1, Γ_2, respectively
W_3	equivalent to α_3 defined in equation (64) of Ref. 2
$-i\mu W_1' = -i\mu \int_{-\infty}^{y-x_1} W_1 \, d\xi$	out-of-phase part of W due to vortex Γ_1
$w = w_r + iw_1 = \dfrac{\partial\phi'}{\partial Z}$	unsteady interference upwash

w_s/

v_s	steady interference upwash of a horse-shoe vortex
X, Y, Z	rectangular Cartesian co-ordinates, with the axis of X along the tunnel axis
$x, y, z = \left(\frac{X}{R}, \frac{Y}{R}, \frac{Z}{R}\right)$	non-dimensional rectangular Cartesian co-ordinates
$X = X_0, \ z = 0$; $= x_0, \ z = 0$	pitching axis of wing
$_1, X_1$; $_2, X_2$	co-ordinates defining positions of vortices (§5)
$_\theta, z_{\dot\theta}$	defined by $C_L = -2\left(z_\theta \cdot \theta + \frac{\bar{c} z_{\dot\theta}}{V} \dot\theta\right)$
(y)	circulation in steady flow, amplitude of circulation in unsteady flow
$_1, \Gamma_2$	strengths of horse-shoe vortices (§5)
$_0, \delta_1, \delta_0', \delta_1'$	interference parameters for a small wing
$= \theta_0 \exp(i\omega t)$	angle of incidence of wing
	angle of sweep of horse-shoe vortex (Fig. 1)
$= \omega R/V$	interference frequency parameter
$= \omega\bar{c}/V$	frequency parameter of wing
	density of fluid
(X, Y, Z)	amplitude of unsteady part of velocity potential
$'$	incremental unsteady velocity potential due to presence of tunnel walls
(X, Y, Z)	steady part of velocity potential of disturbance caused by wing
(x, y, z)	potential function defined in equation (4)
$= 2\pi f$	angular frequency
$, b$	suffixes referring the upper $(z > 0)$ and lower $(z < 0)$ sides of the wing or wake
$, t$	suffixes referring to leading and trailing edges of wing respectively
	prefix denoting increment due to tunnel interference.

§2. Introduction/

§2. Introduction

The present work has arisen on account of some tests[3] (Scruton, Woodgate and Alexander, 1953) on oscillating wings in the low-turbulence tunnel at the N.P.L. As the arrowhead wing was rather large in relation to the tunnel, it was thought desirable to have some estimate of the wall effects on the measured derivatives. For the purpose of interference correction the working section of the tunnel, a regular sixteen-sided polygon, was taken as a circle of diameter 7 ft.

Some data on wall interference for a swept wing in steady flow in a circular tunnel is available[1] (Eisenstadt, 1947). As the tabulated values in Ref. 1 are somewhat scanty, it is necessary to extrapolate extensively to obtain those needed for the configuration in Ref. 3. For this reason the interference corrections obtained here by extending the theory of Ref. 1 to unsteady flow, may be in error by about 10%. It seemed unnecessary to improve the accuracy of the calculations, since the low-turbulence tunnel is unlikely to be used for future tests of oscillating wings and the uncertainty in the tunnel wall corrections is probably within the limits of experimental error.

§3. Relation between Steady and Unsteady Flows

Consider the flow of an incompressible inviscid fluid past a wing oscillating with frequency f. Let the free stream velocity V be in the direction of increasing X. The velocity potential may be expressed as $VX + \chi(X, Y, Z) + \phi(X, Y, Z)\exp(i\omega t)$, where $\omega = 2\pi f$. From the classical theory[4] (Lomb, 1932), the local pressure p is given by

$$(p - p_\infty)/\rho = -\frac{\partial}{\partial t}\{VX + \chi + \phi\exp(i\omega t)\} - \tfrac{1}{2}(q^2 - V^2), \qquad \ldots\ldots (1)$$

where p_∞ is the pressure of the undisturbed flow and q is the magnitude of the local velocity. In the linearized theory of small perturbations equation (1) becomes

$$(p - p_\infty)/\rho = -V\frac{\partial\chi}{\partial X} + \left\{-i\omega\phi - V\frac{\partial\phi}{\partial X}\right\}\exp(i\omega t).$$

Hence

$$(p_b - p_a)/\rho = V\frac{\partial}{\partial X}(\chi_a - \chi_b) + \left\{i\omega(\phi_a - \phi_b) + V\frac{\partial}{\partial X}(\phi_a - \phi_b)\right\}\exp(i\omega t), \qquad \ldots\ldots (2)$$

where a and b denote the upper and lower surfaces of the wing and wake respectively. In the wake of the wing, where $p_a = p_b$ and $\chi_a - \chi_b$ is independent of X,

$$\frac{\partial}{\partial x}(\phi_a - \phi_b) + i\mu(\phi_a - \phi_b) = 0, \qquad \ldots\ldots\ldots (3)$$

where $\mu = \omega R/V$, $x = X/R$ and the representative length R is chosen to be the radius of the tunnel.

Now define Ψ by the equation

$$\Psi = \int_{-\infty}^{x}\left\{\frac{\partial\phi(\xi, y, z)}{\partial\xi} + i\mu\,\phi(\xi, y, z)\right\}d\xi\,. \qquad \ldots\ldots\ldots (4)$$

It/

It follows that

$$\frac{\partial \Psi}{\partial x} = \frac{\partial \phi}{\partial x} + i\mu\phi \quad . \qquad \ldots\ldots\ldots (5)$$

If equation (5) is integrated along any section of a thin aerofoil

$$\Psi_t - \Psi_l = \phi_t - \phi_l + i\mu \int_{x_l}^{x_t} \phi \, d\xi \ , \qquad \ldots\ldots\ldots (6)$$

where l and t denote values at the leading and trailing edges respectively. Equation (6) holds for both surfaces of the wing. Therefore in the notation of equation (2), at the trailing edge

$$\Psi_a - \Psi_b = \phi_a - \phi_b + i\mu \int_{x_l}^{x_t} (\phi_a - \phi_b) \, d\xi \ . \qquad \ldots\ldots\ldots (7)$$

From equations (3) and (5) it follows that $\partial/\partial x(\Psi_a - \Psi_b)$ vanishes in the wake, hence Ψ is the velocity potential of a steady flow.

The differential equation (5) determines

$$\phi = \int_{-\infty}^{x} \frac{\partial \Psi (\xi, y, z)}{\partial \xi} \exp\{-i\mu(x-\xi)\} \, d\xi$$

$$= \Psi (x, y, z) - i\mu \int_{-\infty}^{x} \Psi (\xi, y, z) \exp\{-i\mu(x-\xi)\} \, d\xi \ . \qquad (8)$$

Equation (8) holds for the field ϕ in the unbounded flow and for the field $\phi + \phi'$ in the presence of tunnel walls. It follows that the unsteady interference upwash is

$$w = w_r + iw_i = \partial\phi'/\partial z$$

$$= w_s - i\mu \exp(-i\mu x) \int_{-\infty}^{x} w_s \exp(i\mu\xi) \, d\xi, \quad \ldots\ldots (9)$$

where $w_s = \partial\Psi'/\partial z$ is the interference upwash from the steady velocity potential Ψ defined in equation (4).

To determine the influence of the tunnel walls it is only necessary to consider the field ϕ at a distance from the wing. For this purpose the disturbance to the uniform flow can be represented accurately enough by that due to a combination of horse-shoe vortices of periodic strength. In the limiting case of a horse-shoe vortex $x_t - x_l$ is infinitesimal and ϕ remains finite, so that the integral in equation (7) disappears and at the trailing edge

$$\Psi_a - \Psi_b = \phi_a - \phi_b = \Gamma(y). \qquad \ldots\ldots\ldots (10)$$

Thus Ψ is the potential of a steady flow with the circulation $\Gamma(y)$, since in incompressible flow ϕ, and hence Ψ, will satisfy the equation $\nabla^2\Psi = 0$.

It follows from equation (10) that the unsteady interference upwash for a horse-shoe vortex with circulation $\Gamma \exp(i\omega t)$ is related to that for a steady horse-shoe vortex with circulation Γ by equation (9). Therefore equation (9) may be used to determine the unsteady interference upwash when the unsteady wing loading is represented by a system of discrete vortices.

A/

A similar result has been obtained by Goodman[5] (1951) and has been used for small wings.

If the fluid is compressible the amplitude of the unsteady flow may be expressed in terms of a modified velocity potential

$$\phi_1 = \phi \exp\left(- \frac{i\omega}{V} \cdot \frac{M^2 x}{1 - M^2}\right),$$

where M is the Mach number. ϕ_1 satisfies the linearized equation

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2} + \mu^2 \frac{M^2}{1 - M^2} \phi_1 = 0, \qquad \ldots\ldots\ldots (11)$$

where $x = X/R \sqrt{1 - M^2}, \quad y = Y/R, \quad z = Z/R.$

If μ^2 is assumed to be negligible, ϕ_1 satisfies $\nabla^2 \phi_1 = 0$. In the wake behind a wing

$$\frac{\partial \phi_1}{\partial x} + i\mu \cdot \frac{1}{\sqrt{1 - M^2}} \phi_1 = 0,$$

so that equation (5) must be replaced by

$$\frac{\partial \psi}{\partial x} = \frac{\partial \phi_1}{\partial x} + i\mu \cdot \frac{1}{\sqrt{1 - M^2}} \phi_1,$$

and $\frac{\partial \psi}{\partial x}$ is zero in the wake.

Thus the method could be applied to compressible* flow in the case of low frequency since ψ satisfies $\nabla^2 \psi = 0$ when $\mu^2 M^2/(1 - M^2)$ is neglected in equation (11).

§4. Application to a Circular Tunnel

In order to apply equation (9) to the calculation of unsteady interference, it is necessary to know w_s at all points upstream of the wing. Such values of w_s do not occur in the calculation of steady interference and are not usually computed. However Eisenstadt[1] has considered a swept horse-shoe vortex of constant strength symmetrically situated in a circular tunnel (Fig. I) and has calculated w_s in the plane of the vortex within a distance of about R of its vertex O. These values of w_s are given for angles of sweepback, -45 deg. $\leqslant \Lambda \leqslant 45$ deg. and vortex lengths $S \leqslant 0.9$ R. Provided that the angle of sweep does not lie too for outside this range, the interpolation or extrapolation to the appropriate angle can be carried out without great difficulty.

The upwash due to a horse-shoe vortex in any closed tunnel tends to zero exponentially with distance upstream[7] (von Kármán and Burners), whereas in unlimited flow it becomes proportional to $1/X^2$. Therefore as $X \to -\infty$, the steady interference upwash $w_s = O(1/X^2)$ and is negative in the sense of Fig. 1. It is difficult to tell from the available data how large $-x$ must be before this rule can be applied, but for a circular tunnel it is certainly outside the range of values given in Ref. 1.

If/

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*It is necessary that the frequency should be small compared with the first critical frequency for resonance[6] (W. P. Jones, 1953).

If it is assumed that

$$w(x) = W(-1)/x^2 \qquad \text{..........} \quad (12)$$

for $x < -1$, it appears from the values of W for $x \geq -0.9$ that this overestimates the contribution to the infinite integral in equation (9) in the case of small frequency. The assumption that $W(x) = 0$ for $x < -1$ probably leads to a contribution that is too small. In the example considered in §8 the difference between the results based on these two assumptions was found to be small, though there is no reason to suppose that this would always be the case.

It would, of course, have been possible to extend the tables of Ref. 1 as far upstream as necessary, but the large amount of computation was not thought to be justified.

§5 Evaluation of Interference

As a first step the vorticity distribution round the wing must be expressed as the sum of a number of horse-shoe vortices of the type shown in Fig. 1. These vortices should be consistent with the quantities measured in the particular tests. For the arrowhead wing shown in Fig. 2, the theoretical (Ref. 2, Fig. 2) and experimental[3] pitching derivatives are in satisfactory agreement, so that it is sufficient to use theoretical results. In the example considered in §6, the vortices are two in number and correspond respectively to the stiffness and damping derivatives. Each vortex is specified by its length s, angle of sweep-back Λ, streamwise position X and strength Γ, independent of Y. s and Λ were chosen from the geometry of the wing so as to simplify the extrapolation from the tables of Ref. 1.

Let Γ_1 and Γ_2 be the constant strengths of the vortices corresponding to the stiffness and the damping respectively. Then the amplitude of the lift in phase with the pitching motion is identified with

$$\rho V \Gamma_1 \,.\, 2s \cos\Lambda = -\rho V^2 S\, z_\theta\, \theta_0 , \qquad \text{..........} \quad (13)$$

where θ_0 is the amplitude of the pitching motion. Hence

$$\Gamma_1 \,.\, s\cos\Lambda = -\tfrac{1}{2} VS z_\theta \theta_0 \,. \qquad \text{..........} \quad (14)$$

Since this lift is assumed to act at the midpoint of the vortex, the pitching moment about an axis $X = Xc$ is identified with

$$\rho V \Gamma_1 \,.\, 2s\cos\Lambda\,(X_0 - X_1 - \tfrac{1}{2} s \sin\Lambda) = \rho V^2 S \bar{c}\, m_\theta\, \theta_0 , \quad \text{......} \quad (15)$$

where $X = X_1$ at the apex of the vortex. Then from equations (13) and (15)

$$X_1 = X_0 - \tfrac{1}{2} s \sin\Lambda + \bar{c} m_\theta / z_\theta ,$$

so that in non-dimensional co-ordinates

$$x_1 = x_0 - \frac{s}{2R}\sin\Lambda + \frac{\bar{c} m_\theta}{R z_\theta} \,. \qquad \text{..........} \quad (16)$$

The amplitude of the lift out of phase with the pitching motion is identified with

$$\rho V \Gamma_2 \,.\, 2s\cos\Lambda = -\rho VS\,\bar{c}\, z_{\dot\theta} \,.\, \omega\, \theta_0 \qquad \text{..........} \quad (17)$$

where $\omega\theta_0$ is the amplitude of the pitching velocity. Hence

$$\Gamma_2\, Rs \cos\Lambda = -\tfrac{1}{2}\mu VS\,\bar{c}\, z_{\dot\theta}\, \theta_0 \,. \qquad \text{..........} \quad (18)$$

The position $X = X_2 = Rx$, at the apex of the vortex of strength Γ_2 is given similarly to equation (16) by

x_2/

$$x_2 = x_0 - \frac{s}{2R} \sin \Lambda + \frac{\bar{c} m_{\dot\theta}}{R z_{\dot\theta}} . \qquad (19)$$

Thus, if s and Λ are given, equations (14), (16), (18) and (19) determine the strengths and positions (Γ_1 , x_1); (Γ_2 , x_2) of the two vortices.

Eisenstadt[1] gives tabulated values of the tunnel-induced velocity parameter

$$W = \frac{4\pi R^2 N_s}{\Gamma s \cos \Lambda} \qquad (20)$$

for a range of values of s/R, Λ, X/R. (Fig. 1). The unsteady tunnel-induced upwash is related to w_s by equation (9). Since the present example is associated with Multhopp's[2] theory, which is restricted to cases of low frequency, terms of order μ^2 in equation (9) are ignored.* Thus for each vortex

$$\left.\begin{aligned} w_r &= w_s \\ w_i &= -\mu \int_{-\infty}^{x} w_s \, d\xi \end{aligned}\right\} \qquad (21)$$

where x is measured from the apex of the vortex. From equation (18), Γ_2 is of the first order in frequency, so that the corresponding $(w_i)_2$ may be ignored. It follows that the two vortices combine to give

$$\left.\begin{aligned} w_r &= (w_s)_1 \\ w_i &= (w_s)_2 - \mu \int_{-\infty}^{x-x_1} (w_s)_1 \, d\xi \end{aligned}\right\} \qquad (22)$$

where w_r and w_i are the resultant interference upwashes respectively in phase and out of phase with the pitching motion. From equations (14), (20) and (22),

$$w_r = - \frac{VS z_\theta \theta_0}{8\pi R^2} W_1 , \qquad (23)$$

where the value of W_1 is obtained from the tables of Ref. 1 for $\sigma = s/R$ and angle of sweepback Λ as a function of y and $\xi = x - x_1$, where x_1 is given in equation (16). Similarly from equations (18), (20), (22) and (23),

$$w_i = - \frac{\mu VS\theta_0}{8\pi R^2} \left[\bar{c} z_{\dot\theta} W_2 - R z_\theta \int_{-\infty}^{x-x_1} W_1 \, d\xi \right], \qquad (24)$$

where W_2 corresponds to $\sigma = s/R$, Λ and $\xi = x - x_2$ with x_2 from equation (17), and W_1 as a function of ξ is obtained as in equation (23) with the use of equation (12), when $\xi < -0.9$.

It is now necessary to compute the effect of the interference upwash $w = (w_r + i w_i) e^{i\omega t}$ on the quantities measured in the tests. A convenient method of calculation is that of Ref. 2. The plan form and the choice of the odd

number/

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*Equation (9) has been evaluated numerically for the present example with $\mu = 0.5(\omega\bar{c}/V = 0.347)$. The various contributions to the interference upwash were altered appreciably, but the resulting changes in the final interference corrections to C_L and C_m were unimportant.

number m determine the (m + 1) points at which w is required. sets of (m + 1) linear simultaneous equations then determine values of local lift and local centre of pressure at $\frac{1}{2}$(m + 1) sections of the wing and hence the effect of tunnel walls on the pitching derivatives. These calculated effects are then subtracted from the measured derivatives.

§6 Example

Fig. 2 shows the shape and dimensions of a model which has been the subject of experimental work in the low-turbulence tunnel at the N.P.L. In these tests the model was made to pitch about the axes shown, which were at 0.613 and 0.738 of the root-chord from the apex. Since the root-chord happens to equal the tunnel radius R, the axes correspond to x_0 = 0.613 and 0.738.

As calculations have been performed on this plan form using Multhopp's unsteady lifting surface theory', it was decided to use these to express the vorticity distribution round the wing in terms of horse-shoe vortices of the type considered above. These were taken to have sweepback Λ = 60° and length S = 0.70 R. Their positions were obtained from equations (16) and (19) where $\bar{c}$ = 0.694 R, and z_θ, $z_{\dot\theta}$, m_θ and $m_{\dot\theta}$ are determined theoretically as in Ref. 2, Table III (m = 5).

Table I

x_0	$-z_\theta$	$-m_\theta$	x_1	$-z_{\dot\theta}$	$-m_{\dot\theta}$	x_2
0.613	0.852	0.055	0.355	0.892	0.301	0.545
0.738	0.852	- 0.099	0.355	0.738	0.159	0.584

The interference upwashes in phase and out of phase with the pitching motion are then given by equations (23) and (24) respectively. From Table I it is clear that w_r in equation (23) and the integral in equation (24) are independent of x_0. Only W_2 needs separate computation for each pitching axis. In the method of Ref. 2 with m = 5, W_1 and W_2 are required at two chordwise positions on each of three streamwise lines. These six points (x'_ν, y_ν), (x''_ν, y_ν) (ν = 0, 1, 2) are indicated in Fig. 2. The interference upwashes were calculated along y = 0 and y = 0.5 by using the method described above, and the values at the Multhopp positions were found by interpolation as given in Table II.

Table II

ν	(x_ν, y_ν)	W_1 x_1 = 0.355	W_2 x_0 = 0.613, x_2 = 0.545	W_2 x_0 = 0.738, x_2 = 0.584	$\int_{-\infty}^{x_\nu - x_1} W_1\, d\xi$ using eqn (12)	$\int_{-\infty}^{x_\nu - x_1} W_1\, d\xi$ replacing $-\infty$ by -1	W_3
0'	(0.948,0)	2.61	2.23	2.14	2.07	1.86	1.88
0''	(0.417,0)	1.53	1.20	1.13	0.98	0.77	0.30
1'	(1.163,0.229)	3.00	2.66	2.58	2.66	2.45	1.15
1''	(0.775,0.229)	2.26	1.06	1.78	1.64	1.43	- 0.25
2'	(1.352,0.397)	3.32	3.03	2.96	3.27	3.06	- 0.25
2''	(1.089,0.397)	2.90	2.52	2.44	2.44	2.23	- 1.42

Thus/

Thus, from (equations (23) and (24) the tunnel-induced angle of upwash is

$$\frac{w_r + iw_i}{V} = \frac{S}{8\pi R^2}\left\{(-z_\theta\ \theta_o)W_1 + \frac{i\mu\bar{c}}{R}(-z_{\dot\theta}\ \theta_o)W_2 - i\mu(-z_\theta\ \theta_o)W_1'\right\}, \qquad (25)$$

where $-z_\theta$ and $-z_{\dot\theta}$ are evaluated in Table I,

$$W_1,\ W_2 \text{ and } W_1' = \int_{-\infty}^{x-x_1} W_1\, d\xi$$

are evaluated in Table II.

When $(w_r + iw_i)/V$ is substituted for the right hand side of equation (55) of Ref. 2, the real part of the tunnel-induced loading is determined as in steady flow by putting $\omega = \mu V/R = 0$. This loading is in phase with the pitching motion and will be denoted by

$$l = \frac{S}{8\pi R^2}(-z_\theta\ \theta_o)l_1, \qquad \ldots\ldots\ldots (26)$$

so that from equations (25) and (26), $l = l_1$ corresponds to a distribution of incidence $a = W_1$ in steady flow. By following the treatment of equation (55) in Ref. 2, it is seen that the loading $l = l_1$ produces an additional term

$$i\nu W_3 = \frac{i\omega\bar{c}}{V}W_3 = \frac{i\omega c}{V}a_3$$

in equation (64) of Ref. 2. This effectively contributes to the imaginary terms in equation (25), so that the tunnel-induced loading out of phase with the pitching motion may be mitten as

$$\frac{i\omega\bar{c}}{V}l' = \frac{S}{8\pi R^2}\left[i\nu(-z_{\dot\theta}\ \theta_o)l_2 - i\mu(-z_\theta\ \theta_o)l_1'\right]$$

$$+\ i\nu\frac{S}{8\pi R^2}(-z_\theta\ \theta_o)l_3, \qquad \ldots\ldots\ldots (27)$$

where $l = l_2,\ l_1',\ l_3$ correspond to $a = W_2,\ W_1',\ W_3$ respectively in steady flow, and the values of W_3 are included in Table II. The tunnel-induced lift and pitching about each pitching axis, calculated for $a = W_1,\ W_2,\ W_1',\ W_3$, arc given in Table III. In the case of W_1' the average of two columns in Table II has been taken.

Table III

a	W_1	W_2 $x_o = 0.613$	W_2 $x_o = 0.738$	W_1' average	W_3
δC_L	4.75	4.14	4.00	3.86	1.64
δC_m $x_o = 0.613$	- 0.78	- 0.76	-	- 0.85	- 0.53
δC_m $x_o = 0.738$	+ 0.07	-	- 0.03	- 0.16	- 0.23

From/

From the dimensions of the model and the tunnel in Fig. 2, $S/8\pi R^2 = 0.0253$. Then from equations (13), (26) and Table III,

$$\delta z_\theta = -\frac{\delta C_L}{2\theta_o} = \frac{0.0253 \times 4.75}{2} z_\theta = 0.0601\ z_\theta = -\ 0.0512. \qquad \ldots (28)$$

From the definition of m_θ in equation (15),

$$\left.\begin{aligned} \delta m_\theta = \frac{\delta C_m}{2\theta_o} &= 0.0099\ z_\theta = -\ 0.0084, \text{ when } x_o = 0.613, \\ &= -\ 0.0009\ z_\theta = +\ 0.0008, \text{ when } x_o = 0.738 \end{aligned}\right\} \qquad (29)$$

From equations (17), (27) and Table III,

$$\delta z_{\dot\theta} = -\frac{\delta(\text{lift})}{\rho v s c\ \theta_o} = -\frac{(\delta C_L)'}{2\theta_o}$$

$$= -\frac{S}{16\pi R^2}\left[-z_{\dot\theta}(\delta C_L)_2 + \frac{R}{c} z_\theta (\delta C_L)_1' - z_\theta (\delta C_L)_3\right] \qquad \ldots\ldots (30)$$

$$= -\ 0.0127\ [(3.69 \text{ or } 2.95) - 4.74 + 1.401$$

$$\left.\begin{aligned} &= -\ 0.004 \text{ when } x = 0.613 \\ &= +\ 0.005 \text{ when } x = 0.738 \end{aligned}\right\}.$$

Thus $\delta z_{\dot\theta}$, the tunnel-induced contribution to the measured $z_{\dot\theta}$, consists of three terms, which individually give approximate percentages + 5, - 7 and + 2. These cancel each other to the extent that the total $\delta z_{\dot\theta}$ is negligible. Similarly to equation (30)

$$\delta m_{\dot\theta} = \frac{(\delta C_m)'}{2\theta_o} = \frac{S}{16\pi R^2}\left[-z_{\dot\theta}(\delta C_m)_2 + \frac{R}{c} z_\theta (\delta C_m)_1' - z_\theta (\delta C_m)_3\right].$$

Hence for the pitching axis $x_o = 0.613$,

$$\delta m_{\dot\theta} = 0.0127[-\ 0.68 + 1.04 - 0.45] = -\ 0.001. \qquad \ldots\ldots (31)$$

For the pitching axis $x_o = 0.738$,

$$\delta m_{\dot\theta} = 0.0127[-\ 0.02 + 0.19 - 0.20] = -\ 0.000. \qquad \ldots\ldots (32)$$

In both cases $\delta m_{\dot\theta}$ is negligible, but for the forward axis the individual terms contribute about 3 per cent to the measured $m_{\dot\theta}$. The calculated quantities δz_θ, δm_θ, $\delta z_{\dot\theta}$, $\delta m_{\dot\theta}$ are subtracted from the so-called measured values in Table I to obtain the corrected values.

§7 Use of Small Horse-shoe Vortices

A useful approximation is obtained by considering small horse-shoe vortices for the purpose of estimating the order of magnitude of the interference upwash. Moreover it will be shown that a good approximation to the results of the calculated example of §6 can be obtained in terms of four interference parameters associated with a small wing. The values of these parameters for a circular tunnel are given in this Section, and corresponding values for rectangular tunnels are given in Table AII in the Appendix, so that tunnel wall interference may be estimated by the method given below for wings in rectangular tunnels.

When/

When the wing is small, the tunnel interference will depend only on the total lift. The interference upwash due to a small wing is therefore the limiting value of that of a horse-shoe vortice of semi-span s_1 and circulation $\Gamma \exp(i\omega t)$ when s_1 tends to zero and Γs_1 remains finite. The values of the upwash in the steady case are known for a circular tunnel[1], so that the oscillating interference upwash may be obtained by equation (9). The numerical value of the upwash at the wing is given, by

$$\frac{4\pi R^2}{\Gamma s_1}(w_r + iw_i) = 2 - 1.15\, i\mu + O(\mu^2), \qquad \text{..........(33)}$$

where the interference upwash is $(w_r + iw_i)\exp(i\omega t)$. Owing to the difficulty of estimating upstream values of W mentioned in §4 the value 1.15* in equation (33) may be in error by an amount probably not more than 10%.

The simplest method of computing the interference is to replace the finite wing by two small horse-shoe vortices, whose strengths are determined by z_θ and $z_{\dot\theta}$ and to assume that the interference upwash is constant over the wing as given by equation (33). This approximation is much too crude for the example considered in §6, and in fact Table II shows that the interference upwash is far from constant over the wing.

A better approximation is obtained by assuming that the interference upwash varies linearly in the chordwise direction and is constant in the spanwise direction. To this approximation

$$\frac{4\pi R^2}{\Gamma s_1}(w_r + iw_i) = 16\left\{\delta_0 + \frac{X}{2R}\delta_1 + i\mu\left(\delta_0' + \frac{X}{2R}\delta_1'\right)\right\}$$

$$= 16\left\{\delta_0 + \frac{x}{2}\delta_1 + i\mu\left(\delta_0' + \frac{x}{2}\delta_1'\right)\right\}, \qquad \text{.......... (34)}$$

Here δ_1 may be found from Ref. 1: δ_1' is proportional to the gradient of the out-of-phase interference upwash and so by equation (21) $\delta_1'/2 = -\delta_0$. The factor 16 in equation (34) is introduced to make δ_0, δ_1, δ_0' and δ_1' analogous to the quantities used in steady interference theory (see for example Ref. 8). The numerical values for a circular tunnel are:-

$$\delta_0 = 0.125, \quad \delta_1 = 0.250, \quad \delta_0' = -0.07 \text{ and } \delta_1' = -0.250. \qquad \text{. . . . . (35)}$$

The positions of the two small horse-shoe vortices used to represent the model are obtained by putting s = 0 in equations (16) and (19), so that the one representing the in-phase part of the lift distribution is at

$$x_1 = x_0 + \frac{\bar c}{R}\cdot\frac{m_\theta}{z_\theta} \qquad \text{.......... (36)}$$

and the one representing the out-of-phase part is at

$$x_2 = x_0 + \frac{\bar c}{R}\frac{m_{\dot\theta}}{z_{\dot\theta}}, \qquad \text{.......... (37)}$$

Then, ignoring terms in μ^2, the interference upwash $(w_r + iw_i)$ is given by equations (23) and (24.) with

$$W_1 = 16\left\{\delta_0 + \delta_1 \,.\, \tfrac{1}{2}(x - x_1)\right\}, \qquad \text{.......... (38)}$$

W_2/

---

*The corresponding value deduced from Ref. 5 is about - 10 x 1.15. This discrepancy is believed to be due to incorrect labelling in Goodman's diagram.

$$W_2 = 16\{\delta_0 + \delta_1 \cdot \tfrac{1}{2}(x - x_2)\}\ , \quad \ldots\ldots \quad (39)$$

and

$$\int_{-\infty}^{x - x_1} W_1 d\xi = W_1' = - 16\{\delta_0' + \delta_1' \cdot \tfrac{1}{2}(x - x_1)\}\ . \quad \ldots\ldots \quad (40)$$

In each case the upwash consists of two parts the first being constant over the wing and the second being proportional to x, where x is conveniently measured from the apex of the wing. The corresponding aerodynamic loading on the wing can therefore be expressed in terms of the coefficients of four basic lift distributions, i.e.,

$$\left.\begin{array}{llll} (C_L)_1, & (C_m)_1, & \text{the in-phase coefficients corresponding to} & w = Ve^{i\omega t} \\ (C_L)_2, & (C_m)_2, & \text{" in-phase " " "} & w = \dfrac{xRV}{\bar{c}} e^{i\omega t} \\ i\nu(C_L)_3, & i\nu(C_m)_3, & \text{the out-of-phase " " "} & w = Ve^{i\omega t} \\ i\nu(C_L)_4, & i\nu(C_m)_4, & \text{" out-of-phase " " "} & w = \dfrac{xRV}{\bar{c}} e^{i\omega t} \end{array}\right\} \quad \ldots.(41)$$

These quantities are obtainable by Multhopp's lifting surface theory[2]; the first three, being required in the calculation of pitching derivatives, are tabulated in Ref. 2 for several plan forms.

The upwash distribution W_1 in equation (38) gives rise to an in-phase contribution

$$(\delta C_L)_1 = 16(\delta_0 - \tfrac{1}{2} x_1 \delta_1)(C_L)_1 + 16 \cdot \tfrac{1}{2} \delta_1 \cdot \frac{\bar{c}}{R} (C_L)_2 \quad \ldots. \quad (42)$$

and an out-of-phase contribution

$$(\delta C_L)_3 = \nu \left\{16(\delta_0 - \tfrac{1}{2} x_1 \delta_1)(C_L)_3 + 16 \cdot \tfrac{1}{2} \delta_1 \cdot \frac{\bar{c}}{R} (C_L)_4\right\}. \quad \ldots \quad (43)$$

The out-of-phase upwash distributions W_2 and W_1' in equations (39) and (40) give rise to respective out-of-phase contributions

$$(\delta C_L)_2 = 16(\delta_0 - \tfrac{1}{2} x_1 \delta_1)(C_L)_1 + 16 \cdot \tfrac{1}{2} \delta_1 \cdot \frac{\bar{c}}{R} (C_L)_2 \quad \ldots \quad (44)$$

and

$$(\delta C_L)_1' = - 16(\delta_0 - \tfrac{1}{2} x_1 \delta_1')(C_L)_1 - 16 \cdot \tfrac{1}{2} \delta_1' \cdot \frac{\bar{c}}{R} (C_L)_2 . \quad \ldots \quad (45)$$

The in-phase loadings due to W_2 and W_1' are of order μ^2 and are therefore neglected. Similar expressions for the moment derivatives may be obtained by changing suffix L to suffix m throughout the last four equations. The interference effects on the derivatives may now be obtained by using equations (23) and (24) and equations (42) to (45).

For the arrowhead wing of §6 the values of $(C_L)_1$, $(C_L)_2$, $(C_L)_3$ and $(C_L)_4$ are

$$(C_L)_1 = 1.704, \quad (C_L)_2 = 2.571, \quad (C_L)_3 = 0.717, \quad (C_L)_4 = 0.818 \ \ldots \ (46)$$

Consider/

Consider the case when the pitching axis is at $x_0 = 0.613$. By using the values in Table I, and equations (36) and (37), it follows that

$$\left.\begin{aligned} x_1 &= 0.658 \\ \text{and} \quad x_2 &= 0.847 \end{aligned}\right\} . \qquad \ldots\ldots\ldots\ldots (47)$$

With the aid of equations (47) and (35), equations (42) to (45) give

$$\begin{aligned} (\delta C_L)_1 &= 4.74, \quad (\delta C_L)_2 = 4.09 \\ -(\delta C_L)_1' &= -3.24, \quad (\delta C_L)_3 = 1.63 . \end{aligned} \qquad \ldots\ldots\ldots\ldots (48)$$

Then from equation (26) the in-phase increment to C_L is

$$-2\theta_0 \, \delta z_\theta = -\frac{S z_\theta \theta_0}{8\pi R^2} (\delta C_L)_1 ,$$

whence $\delta z_\theta = 0.0600 \, z_0 = 0.0511.$ .... (49)

From equations (27) and (30) the out-of-phase increment to C_L determines

$$\begin{aligned} \delta z_{\dot\theta} &= \frac{S}{16\pi R^2} \left\{ z_{\dot\theta} (\delta C_L)_2 - \frac{R}{c} z_\theta (\delta C_L)_1' + z_\theta \cdot (\delta C_L)_3 \right\} \\ &= -0.0135. \end{aligned} \qquad \ldots\ldots\ldots\ldots (50)$$

The equations for pitching moments analogous to (46) and (48), for the pitching axis at $x_0 = 0.613$, are

$(C_m)_1 = -0.110, \quad (C_m)_2 = -0.521, \quad (C_m)_3 = -0.178, \quad (C_m)_4 = -0.292;$

and $(\delta C_m)_1 = 0.799, \quad (\delta C_m)_2 = -0.757, \quad -(\delta C_m)_1' = +0.702, \quad (\delta C_m)_3 = -0.527.$

Calculations similar to equations (49) and (50) yield the values

$$\delta m_\theta = -0.0086, \quad \delta m_{\dot\theta} = -0.0033.$$

The same method holds for the other axis position; the results are presented in Table IV together with the values obtained by the method of §6 for comparison.

Table IV

Pitching axis	Increment	Values from §7	Values from §6
Both Axes	δz_θ	- 0.0512	- 0.0511
$x_0 = 0.613$	$\delta z_{\dot\theta}$	- 0.0135	- 0.0044
	δm_θ	- 0.0086	- 0.0084
	$\delta m_{\dot\theta}$	- 0.0033	- 0.0011
$x_0 = 0.738$	$\delta z_{\dot\theta}$	- 0.0042	+ 0.0050
	δm_θ	+ 0.0006	+ 0.0008
	$\delta m_{\dot\theta}$	- 0.0010	- 0.0001

The/

The largest difference between the two methods is in z'_θ and even this, being of order 0.01 is negligible for practical purposes. Equation (50) gives numerically for x_o = 0.613

$$\delta z'_\theta = -0.01266 \{3.65 - 3.97 + 1.39\}.$$

The numerical form of equation (30) for x_o = 0.613 is

$$\delta z'_\theta = -0.01266 \{3.69 - 4.74 + 1.40\} \text{ for the 0.613 axis.}$$

The major portion of the difference is due to the central term in the bracket, i.e., to the term containing $(\delta C_L)'_1$. While part of the difference may be due to inaccuracy in W'_1, it is unlikely that this accounts for the whole. However even with this difference the values obtained are probably good enough for most tunnel correction requirements. In the Appendix corresponding values of δ_0, δ_1, δ'_0, δ'_1 are given for rectangular tunnels of various shapes so that the process may also be applied for rectangular tunnels.

§8 Concluding Remarks

(1) One of the basic steps is the evaluation of the interference upwash due to any oscillatory horse-shoe vortex from the corresponding upwash due to a steady horse-shoe vortex. Tables of the steady upwash for a circular tunnel are incomplete so that the numerical results are approximate only.

(2) Multhopp's lifting surface theory has been used to compute the corrections to the derivatives and this implies restriction to small values of the frequency parameter. However this restriction is not fundamental, since any theory of oscillating wings capable of dealing with higher frequencies could be used.

(3) For the particular wing taken as an example the wall interference on the damping derivatives is zero to the order of accuracy of the calculations (§6). It should not however be assumed that these corrections are negligible for all plan forms.

(4) The use of small horse-shoe vortices is adequate for the example considered, in which the plan form was large relative to the tunnel. It is thought that the data for small wings in rectangular tunnels given in the Appendix can be used to estimate the interference in most low-speed tests of oscillating wings.

(5) As mentioned in the footnote to 65 the effect of frequency is probably small provided that the interference frequency parameter, μ, does not exceed 0.5.

(6) The effect of compressibility could be taken into account by the method indicated at the end of §3 provided that the frequency is small.

§9 Acknowledgements

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APPENDIX/

APPENDIX

Small Wings in Closed Rectangular Tunnels

Consider the case of an unswept steady horse-shoe vortex (A = 0 in Fig. 1). The upwash due to such a vortex is (Ref. 9, § 12.2)

$$w = \frac{\Gamma}{4\pi}\left\{\frac{(Y-s)}{(Y-s)^2+Z^2}\left[\frac{X}{\sqrt{X^2+(Y-s)^2+Z^2}}+1\right]\right.$$

$$-\frac{(Y+s)}{(Y+s)^2+Z^2}\left[\frac{X}{\sqrt{X^2+(Y+s)^2+Z^2}}+1\right]$$

$$\left.-\frac{X}{X^2+Z^2}\left[\frac{(Y+s)}{\sqrt{X^2+(Y+s)^2+Z^2}}-\frac{(Y-s)}{\sqrt{X^2+(Y-s)^2+Z^2}}\right]\right\} \qquad \ldots\ldots \text{(A1)}$$

If $\Gamma \to \infty$ and $s \to 0$ so that Γs remains constant,

$$w = -\frac{\Gamma s}{2\pi}\left\{\frac{Z^2-Y^2}{(Y^2+Z^2)^2}\left[1+\frac{X}{\sqrt{X^2+Y^2+Z^2}}\right]+\frac{Z^2}{(Y^2+Z^2)}\cdot\frac{X}{(X^2+Y^2+Z^2)^{3/2}}\right\} \qquad \ldots\ldots \text{(A2)}$$

Now consider the case of a rectangular tunnel of breadth b and height h and take h as the representative length so that X = xh, Y = yh, Z = zh. Then

$$w = -\frac{\Gamma s}{2\pi h^2}\left\{\frac{z^2-y^2}{(y^2+z^2)^2}\left[1+\frac{x}{\sqrt{x^2+y^2+z^2}}\right]+\frac{z^2}{(y^2+z^2)}\cdot\frac{x}{(x^2+y^2+z^2)^{3/2}}\right\}$$

$$= -\frac{\Gamma s}{2\pi h^2}\cdot\frac{z^2-y^2}{(y^2+z^2)^2} \quad \text{on } x = 0, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(A3)}$$

and

$$-\mu\int_{-\infty}^{0} w(\xi, y, z)\,d\xi = -\frac{\mu\Gamma s}{2\pi h^2}\,\frac{y^2}{(y^2+z^2)^{3/2}}\,. \qquad \ldots\ldots\ldots\ldots \text{(A4)}$$

In order to obtain the unsteady interference upwash the quantities have to be summed over the image system. Thus

$$w_r = \frac{-\Gamma s}{2\pi h^2}\sum_{m=-\infty}^{m=+\infty}\ \sum_{n=-\infty}^{n=+\infty}{}_{(m,n)\neq(0,0)}(-1)^n\frac{(n^2-m^2b^2/h^2)}{(n^2+m^2b^2/h^2)^2}, \qquad \ldots\ldots\ldots\ldots \text{(A5)}$$

which is the usual steady interference, and

w_i/

$$w_i = -\frac{\mu\Gamma s}{2\pi h^2} \sum_{m=-\infty}^{m=+\infty} \sum_{n=-\infty}^{n=+\infty} (-1)^n \frac{m^2 \cdot b^2/h^2}{\left(\frac{m^2b^2}{h^2} + n^2\right)^{3/2}} \qquad (m,n) \neq (0,0) \quad \ldots\ldots\ (A6)$$

$$= -\frac{\mu\Gamma s}{2\pi h^2} \sum_{m=-\infty}^{+\infty} m^2 \frac{b^2}{h^2} \sum_{n=-\infty}^{+\infty} \frac{(-1)^n}{\left(\frac{m^2b^2}{h^2} + n^2\right)^{3/2}}$$

$$= -\frac{\mu\Gamma s}{2\pi h^2} \sum_{m=-\infty}^{+\infty} m \frac{b}{h} f\left(m \frac{b}{h}\right), \quad \ldots\ldots\ (A7)$$

$$\text{where } f(x) = \frac{x}{|x|^3} + 2 \sum_{n=1}^{\infty} (-1)^n \frac{x}{(x^2 + n^2)^{3/2}} \quad \ldots\ldots\ (AG)$$

f(x) and its differential coefficient f'(x) are tabulated in Table AI; the method of obtaining these values is described in Ref. 8.

$$\text{Hence } w_1 = -\frac{\mu\Gamma s}{\pi h b} \cdot \frac{b^2}{h^2} \sum_{m=1}^{\infty} m\, f(m\,.\,b/h). \quad \ldots\ldots\ (A9)$$

The above argument applies to horse-shoe vortices of zero span but a moderately small wing can, for the purpose of calculating tunnel interference, be regarded as equivalent to such a vortex. If we write

$$\frac{w}{v} = \frac{SC_L}{bh} (\delta_0 + x\delta_1 + i\mu[\delta_0' + x\delta_1']), \quad \ldots\ldots\ (A.10)$$

then by comparison with equation (A.9)

$$\delta_0' = -\frac{1}{4X} \cdot \frac{b^2}{h^2} \sum_{m=1}^{\infty} m\, f\,(m\,.\,b/h). \quad \ldots\ldots\ (11.11)$$

δ_0 and δ_1 can easily be evaluated from functions tabulated in Ref. 8. Their values are

$$\delta_0 = -\frac{1}{8} \cdot \frac{b}{h} \left[f_2(0) \ t\ 2\sum_{m=1}^{\infty} f_1\,(m\,.\,b/h)\right] \quad \ldots\ldots\ (A.12)$$

and

$$\delta_1 = -\frac{1}{8\pi} \cdot \frac{b}{h} \left[f_4(0) + 2\sum_{m=1}^{\infty} f_3\,(m\,.\,b/h)\right], \quad \ldots\ldots\ (A.13)$$

where f_1, f_2, f_3, f_4 are discussed and tabulated in Ref. 8, and equations (A.12) and (A.13) are limiting forms of the expressions given there for the interference upwash for a horse-shoe vortex of finite span as the span tends to zero. The quantities δ_0 and δ_1 are, of course, already known for several shapes of tunnel.

By/

By an argument similar to that used for the circular tunnel in §7 it follows that $\delta_1' = -\delta_0$ in equation (A.10).

The four quantities δ_0, δ_1, δ_0' and δ_1' are tabulated in Table A.II so that calculations similar to those of §7 can be carried out for a rectangular tunnel. Interpolation in the range $b/h \geqslant 1$ using Bessel's interpolation formula with second differences should give values correct to about four decimals, which is ample for calculations of tunnel interference.

TABLE A.I/

TABLE A.I

f(x), and f'(x)

x	f(x)	- f'(x)	x	f(x)	- f'(x)
0	+ ∞	+ ∞	2.00	+ 0.012 403	+ 0.042 388
0.05	399.910 209	16001.781 328	2.05	0.010 457	0.035 660
0.10	99.822 571	2001.717 426	2.10	0.008 819	0.030 014
0.15	44.183 549	594.207 908	2.15	0.007 440	0.025 273
0.20	24.661 576	251.481 051	2.20	0.006 279	0.021 289
0.25	15.591 411	129.322 128	2.25	0,005 301	0,017 941
0.30	10.640 749	75.220 799	2.30	0.004 476	0.015 124
0.35	7.640 142	47.610 199	2.35	0.003 781	0.012 755
0.40	5.683 356	32.028 317	2.40	0.00j 194	0.010 760
0.45	4.337 225	22.547 000	2.45	0.002 700	0.009 080
0.50	3.373 268	16.430 405	2.50	0.002 282	0.007 665
0.55	2.661 463	12.296 792	2.60	0.001 632	0.005 466
0.60	2.123 197	9.396 704	2.70	0.001 168	0.003 902
0.65	1.708 506	7.299 224	2.80	0.000 836	0.002 769
0.70	1.384 285	5.744 169	2.90	0.000 600	0.001 994
0.75	1.127 810	4.567 702	3.00	0.000 430	0.001 428
3.80	0.923 008	3.662 816	3.10	0,000 309	0.001 023
0.85	0.758 217	2.957 335	3.20	0.000 222	0.000 733
0.90	0.624 793	2.401 178	3.30	0.000 159	0.000 526
0.95	0.516 210	1.958 712	3.40	0.000 115	0.000 378
1.00	0.427 463	1.604 024	3.50	0.000 082	0.000 271
1.05	0.354 667	1.317 908	3.60	0.000 059	0.000 195
1.10	0.294 772	1.085 886	3.70	0.000 043	o.coo 140
1.15	0.245 362	0.896 894	3.80	0.000 031	0.000 101
1.20	0.204 509	0.742 366	3.90	0.000 022	0.000 073
1.25	0.170 664	0.615 608	4.00	0.000 016	0.000 052
1.30	0.142 574	0.511 337	4.10	0.000 012	0.000 038
1.35	0.119 226	0.425 352	4.20	0.000 008	0.000 027
1.40	0.099 791	0.354 295	4.30	0.000 006	0.000 020
1.45	0.083 593	0.295 461	4.40	0.000 004	0.000 014
1.50	0.070 078	0.246 665	4.50	0.000 003	0.000 010
1.55	0.058 789	0.206 133	4.60	0.000 002	0.000 co7
1.60	0.049 351	0.172 418	4.70	0.000 002	0.000 005
1.65	0.041 453	0.144 339	4.80	0.000 001	0.000 004
1.70	0.034 839	0.120 926	4.90	0.000 001	0.000 003
1.75	0.029 296	0.101 384	5.00	0.000 001	0.000 002
1.80	0.024 647	0.085 058	5.10	0.000 000	0,000 002
1.85	0.020 746	0.071 405	5.20	0.000 000	0.000 001
1.90	0.017 470	0.059 979	5.30	0.000 000	0.000 001
1.95	0.014 717	0.050 409	5.40	0.000 oco	0.000 001
2.00	+ 0.012 403	+ 0.042 388	5.50	0.000 000	0.000 000

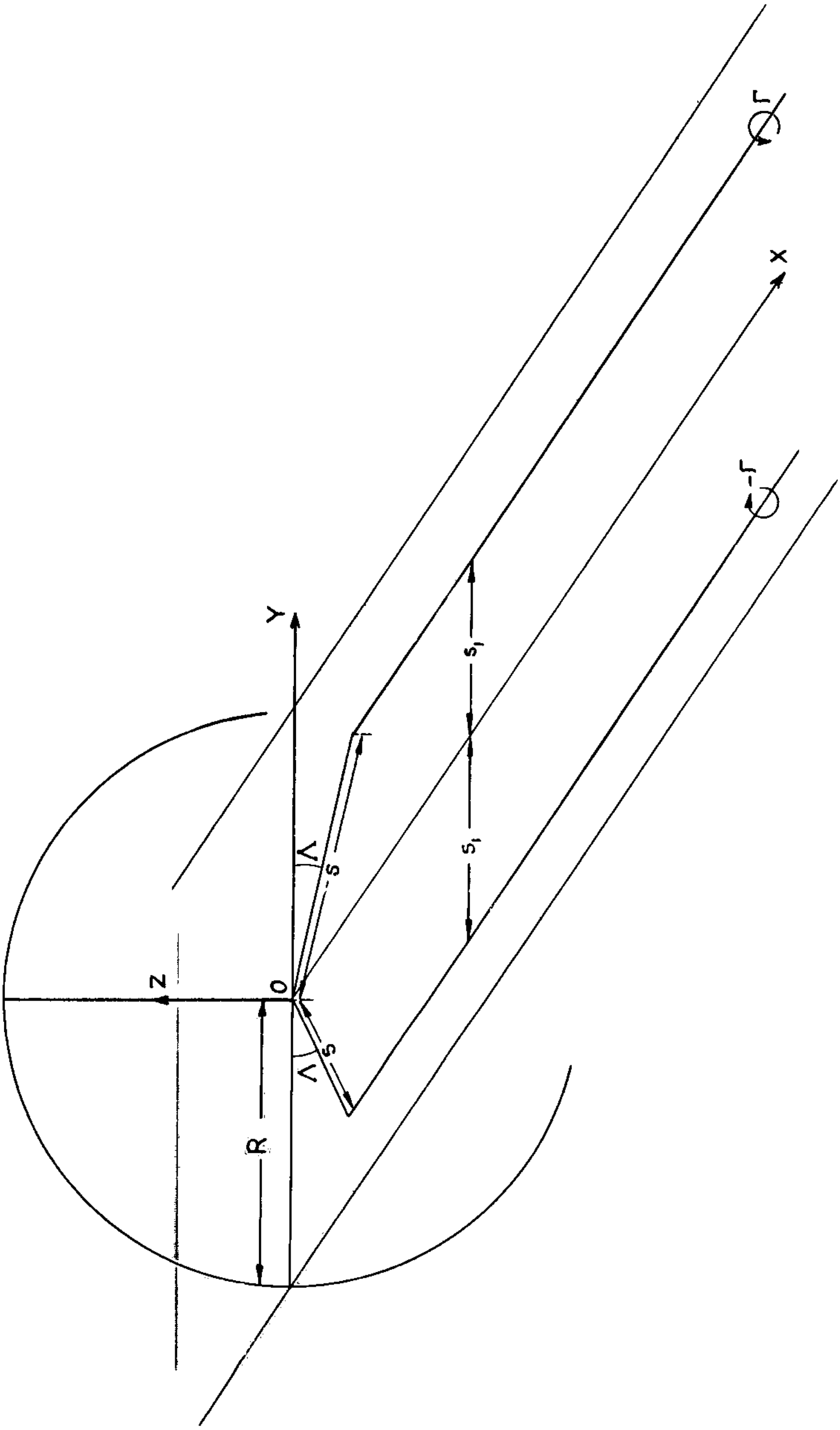
TABLE A.II

Values of δ_0, δ_1, δ_0', δ_1' for Rectangular Tunnel as Functions of Breadth/height Ratio, b/h.

b/h	δ_0	δ_1	-	δ_0'	-	δ_1'
0.5	0.261 821	0.513 593		0.069 580		0.261 821
0.6	0.218 314	0.397 029		0.075 100		0.218 314
0.7	0.187 567	0.327 613		0.062 940		0.187 567
0.8	0.165 150	0.284 178		0.052 573		0.165 150
0.9	0.148 690	0.256 762		0.043 692		0.148 690
1.0	0.136 778	0.240 099		0.036 099		0.136 778
1.1	0. 128 474	0.231 086		0.029 640		0.128 474
1.2	0.123 090	0.227 716		0.021, 183		0.123 090
1.3	0.120 089	0.228 570		0.019 622		0.120 089
1.4	0.119 026	0.232 690		0.015 829		0.119 026
1.5	0.119 538	0.239 249		0.012 703		0.119 538
1.6	0.121 322	0.247 637		0.010 145		0.121 322
1.7	0.124 125	0.257 558		0.008 065		0.124 125
1.8	0.127 742	0.268 521		0.006 385		0.127 742
1.9	0.132 005	0.280 314		0.005 036		0,132 005
2.0	0.136 778	0.292 737		0.003 958		0.136 778
2.1	0.141 950	0.305 636		0.003 101		0.141 950
2.2	0.147 436	0.318 897		0.002 421		0.147 436
9/7	0.120 390	0.228 247		0.020 224		0.120 390
13/9	0.119 078	0.235 340		0.014 363		0.119 078

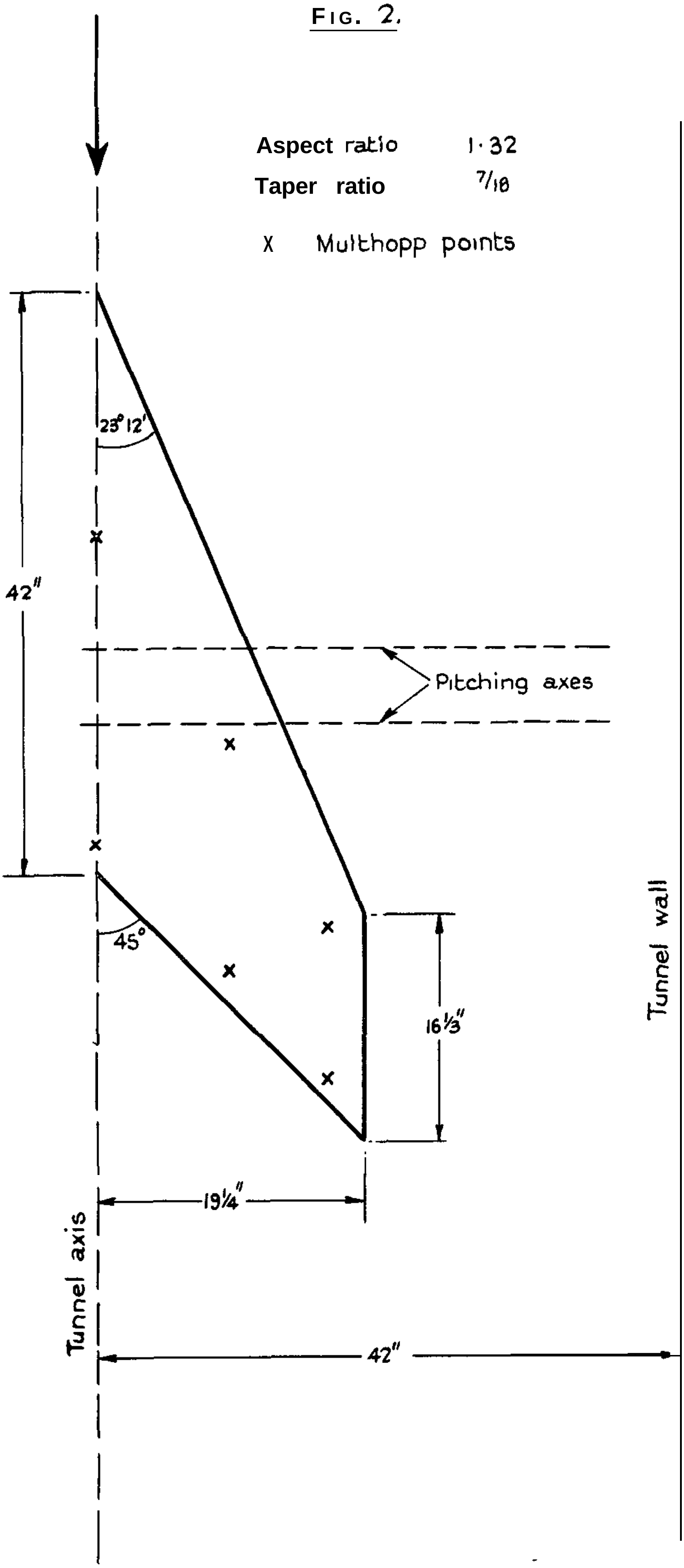
GMW.

FIG. 1.



Position of horse-shoe vortex in relation to circular tunnel

FIG. 2.



Plan of model used in tests at N.P.L. (1/10 scale)

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