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A Method of Computing Subsonic and Transonic Plane Flows

By

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of the *Aerodynamics Division, N.P.L.*

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CORRIGENDA AND ADDENDA

Page 6. Equation (15)

$$V(\phi,\psi) = \frac{1}{m_\infty} L_i (\phi, m_\infty \psi)$$

Fig. 8. 'here is an error in the calculated velocity distribution in the region $x/c = 0.6$. The corrected curve is shown in Fig. 10.

Fig. 10. The relaxation solution is compared with results obtained by the Karman-Tsien approximation, and by experiment with transition to a turbulent boundary layer near the leading edge of the aerofoil.

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3rd September, 1953

Summary

This paper presents a relaxation treatment of a simple but exact differential equation for compressible flow. The method has advantages over other numerical treatments of the same problem and because of the simplicity of the basia differential equation should be particularly suitable for high-speed computing machines.

The flow about a 10% thick aerofoil (RAE 104 section) at zero incidence is calculated for Mach numbers of 0.70, 0.79 and 0.86. At M = 0.86 the existence, but not the position, of a trsnsonic shock wave is predicted by the relaxation technique. Satisfactory agreement with experiment is obtained.

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1./

1. Introduction

Numerous papers dealing with the numerical determination of the compressible flow about an aerofoil have been written, see for example, Refs.1-6. In most cases the differential equation solved has been quite complicated so that the difference equation, approximating to the differential equation, demands much labour in its solution. The equation, due to Woods[7], used in this paper is probably as simple in form as is possible for an exact equation for compressible flow, This simplicity is obtained at the expense of some increase in the complexity of the boundary conditions on the aerofoil surface, which are dependent on the iterations throughout the calculation. This disadvantage was not found to be important.

The (ϕ,ψ) plane is taken as the plane of the independent variables, where ϕ and ψ are the compressible potential and stream functions respectively. This choice confers the advantage, useful in the relaxation process, of having Straight boundaries in the plane of the square *mesh on* which the calculations are made. Similar transformations have been made by Thom[1] for many years.

Transonic shock waves appear in the relaxation solution of the basio differential equation in a simple manner. It is found that, for a given aerofoil and above a certain free-stream Mach number, it is no longer possible to relax all the residuals over the whole field. Individual residuals can be relaxed but, in a part of the field near the aerofoil surface, their relaxation causes the appearance of even larger residuals at neighbouring mesh points*; this is discussed more fully in §5.2. However, it *is* possible to arrange these unrelaxed residuals in pairs of opposite sign along a line starting on the aerofoil surface and running some distance into the flow. In the example calculated, for M = 0.86, the position of the shook wave was estimated from a direct shadow photograph of the flow, (Fig.9(ii)), and the lines of unrelaxed residuals arranged along equipotentials on either side of this position.

In the application of the theory it *is* found *convenient* first *to* calculate the incompressible flow past the given aerofoil. This enables a simple form of the boundary conditions on the aerofoil surface in the compressible flow plane to be used and also facilitates the treatment of the singularities at the stagnation points. There is no necessity to calculate the complete solution of the incompressible flow equation by relaxation. It is much quicker to calculate the values of L_i (cf. 82.0) on the aerofoil surface and outer boundaries by other methods (of. §4.1(a)) and then to fill in the field by relaxation.

2. List of Symbols

(x,y)	the physical plane
(ϕ,ψ)	the compressible flow plane, where ϕ = constant are the velocity equipotentials, and ψ = constant the streamlines
(q,θ)	velocity vector in polar co-ordinates
ρ,ρ_0	local and stagnation densities respectively
γ	ratio of specific heats
U	velocity at infinity
M	Mach *number*
β	$= (1 - M^2)^{\frac{1}{2}}$
s,n	distances measured along and normal to a streamline
R	$(= \cdot \partial s/\partial\theta)$ radius of curvature of the aerofoil surface

c/

*The same difficulty has been encountered by Woods[4] and Emmons[5].

c aerofoil chord

$$L = \log_e\left(\frac{U}{q}\right) \qquad \ldots(1)$$

∞ as suffix, denotes values at infinity

L,T as suffioes, denotes values at leading and trailing edges of the aerofoil respectively

i,c occasionally used as suffices to distinguish values for incompressible and compressible flow respectively

h relaxation mesh size.

3. General Mathematical Theory

3.1 Basic Differential Equation

The potential and stream functions ϕ and ψ are defined by

$$d\phi = q\,ds\,,\; d\psi = \frac{\rho}{\rho_0} q\,dn\,, \qquad \ldots(2)$$

which enables the 'intrinsic' equations, (Ref.8, p.168)

$$\frac{\partial\theta}{\partial n} + \frac{(1 - M^2)}{q}\frac{\partial q}{\partial s} = 0,$$

$$\frac{\partial\theta}{\partial s} - \frac{1}{q}\frac{\partial q}{\partial n} = 0\,,$$

to be written

$$\frac{\partial\theta}{\partial\psi} + \frac{\rho_0}{\rho} \cdot \frac{\beta^2}{q} \cdot \frac{\partial q}{\partial\phi} = 0\,,$$

$$\frac{\partial\theta}{\partial\phi} - \frac{\rho}{\rho_0} \cdot \frac{1}{q} \cdot \frac{\partial q}{\partial\psi} = 0\,.$$

By use of the substitutions

$$dV = -\frac{\rho}{\rho_0}\frac{1}{q}\,dq \qquad \ldots(3)$$

$$m = \beta\frac{\rho_0}{\rho} \qquad \ldots(4)$$

the/

the equations of flow are reduced to

$$\left.\begin{aligned} \frac{\partial\theta}{\partial\psi} - m^2\frac{\partial V}{\partial\phi} &= 0 , \\ \frac{\partial\theta}{\partial\phi} + \frac{\partial V}{\partial\psi} &= 0 , \end{aligned}\right\} \qquad \ldots(5)$$

whence

$$\frac{\partial^2 V}{\partial\psi^2} + \frac{\partial}{\partial\phi}\left(m^2 \frac{\partial V}{\partial\phi}\right) = 0 , \qquad \ldots(6)$$

which is the equation to be solved by relaxation.

Integration of equation (3) yields

$$V = -\int \frac{\rho}{\rho_0}\frac{dq}{q}$$

$$= \int \frac{\rho}{\rho_0}\, dL , \qquad \ldots(7)$$

from (I).

Now

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma-1}$$

$$= (1 + 0.2\ M^2)^{2\cdot 5}$$

with $\gamma = 1.4$. Also from the usual equations for compressible flow it can be shown that

$$dM^2 = -2M^2 (1 + 0.2M^2)\, dL ,$$

therefore equation (7) may be written

$$V = -\frac{1}{2}\int_{M=M_\infty}^{M} \frac{dM^2}{M^2(1 + 0.2M^2)^{3\cdot 5}} ,$$

$$= \left[\frac{1}{2}\log\frac{a+1}{a-1} - \frac{1}{5a^5} - \frac{1}{3a^3} - \frac{1}{a}\right]_{M_\infty}^{M} ,$$

where/

where $a = (1 + 0.2M^2)^{\frac{1}{2}}$, and use has been made of the fact that $V = 0$ when $L = 0$, i.e., when $M = M_\infty$.

Equation (4) gives

$$m = (1 + 0.2M^2)^{2.5} (1 - M^2)^{\frac{1}{2}},$$

$$= a^5 (1 - M^2)^{\frac{1}{2}}.$$

Hence V and m can be tabulated as functions of M for a given free stream Mach number; the relation between V and m will be required for the relaxation solution.

3.2 Solution using von Kármán's Approximation

It can be shown that

$$m = m_\infty \left\{ 1 - \frac{\gamma + 1}{2\beta_\infty} M_\infty^4 \left(\frac{q}{U} - 1 \right) + O\left(\left[\frac{q}{U} - 1 \right]^2 \right) \right\},$$

therefore for thin aerofoils ($q/U \doteqdot 1$) at high subsonic Mach numbers or thick aerofoils at low subsonic Mach numbers, von Kármán's approximation,

$$m = m_\infty$$

is reasonable. With this approximation equation (6) becomes

$$\frac{\partial^2 V}{\partial \psi^2} + m_\infty^2 \frac{\partial^2 V}{\partial \phi^2} = 0. \qquad \ldots(8)$$

The solution of this equation may be found by a theory similar to that used in Ref.7, but using V instead of $r \left(\equiv \int \beta dL \right)$. In the case of zero circulation the solution is found to be

$$V + \frac{i\theta}{m_\infty} = \frac{1}{m_\infty \pi} \int_{-\infty}^{\infty} \frac{\theta' d\phi'}{\phi' - \phi - i m_\infty \psi}, \qquad \ldots(9)$$

where $\phi = \phi'$ and $\theta = \theta'$ are values on $\psi = 0$, the dividing streamline. Now since θ' is zero outside the range (ϕ_L, ϕ_T), i.e.,

$$\int_{\phi_{L-}}^{\phi_{T+}} d\theta' = 0, \qquad \ldots(10)$$

the real part of equation (9) can be written

$$V = \frac{1}{m_\infty \pi} \int_{\phi_L}^{\phi_T} \frac{\theta'(\phi' - \phi)\, d\phi'}{(\phi' - \phi)^2 + m_\infty^2 \psi^2}. \qquad \ldots(11)$$

One/

One further equation which will be of some value is obtained from the 'closure'[9] condition, namely

$$\int_{\phi_L-}^{\phi_T+} \theta' d\phi' = 0 , \quad \text{...(12)}$$

which can be deduced from equation (11) and the fact that there is no circulation, The numerical solution of equations of the same form as (10), (11) and (12) is described in detail in Ref.9.

3.3 Comparison with Incompressible Flow

The equation for incompressible flow can be derived from equation (6) by putting m = 1 (cf. equation (4)) and replacing V by L_i (cf. equations (1) and (3)), which give

$$\frac{\partial^2 L_i}{\partial \psi_i^2} + \frac{\partial^2 L_i}{\partial \phi_i^2} = 0 . \quad \text{...(13)}$$

The solution of this equation is (cf.(11))

$$L_i = \frac{1}{\pi} \int_{\phi_{Li}}^{\phi_{Ti}} \frac{\theta_i' (\phi_i' - \phi_i) d\phi_i'}{(\phi_i' - \phi_i)^2 + \psi_i^2} . \quad \text{...(14)}$$

If the small differences in the location of the (ϕ, ψ) mesh in the compressible and incompressible flow planes are ignored, then comparison of equations (11) and (14) yields

$$V(\phi, \psi) = \frac{1}{M_\infty} L_i (\phi, m_\infty \psi) , \quad \text{...(15)}$$

where V is a solution of the approximate flow equation (8). Equation (15) will be found useful in determining certain of the boundary conditions in the compressible flow plane.

4. Boundary Conditions

As is mentioned in the Introduction it is convenient to obtain a solution for the incompressible flow past the given aerofoil to facilitate the compressible flow calculations. This solution can be obtained quickly by making use of analytical solutions an the boundaries and completing the solution in the field by relaxation.

4.1 Incompressible Flow

(a) Aerofoil Surface

The values of L at points of the relaxation mesh (Figs.3 and 4) on the aerofoil surface were estimated by the methods of Woods[9] and Goldstein's[10] Approximation III, which agree closely with each other and with experiment. These values were kept fixed throughout the application of the relaxation process to the field.

(b)/

*This method is based on equation (14).

(b) Outer Boundary

In the relaxation process, unless inversion3 is used, it is necessary to limit the field to a finite region enclosing the aerofoil. Values of the function must then be estimated on some outer boundary. The accuracy of suchvalues can be checked by extending the boundary such that the points of the previous boundary become part of the field solved by relaxation. Then if the function is estimated on the extended outer boundary, the solution throughout the field should remain essentially unchanged. On the boundary chosen for the incompressible flow plane (Fig.3) the values of L were obtained from equation (14).

(c) Singularities at the Stagnation Points

The leading and trailing edges of the aerofoil, $(\phi_L, 0)$ and $(\phi_T, 0)$ respectively, are singular points for L, since at these points q is zero and from the definition, (11), L will have a logarithmic infinity. In the neighbourhood of the singularities an approximation to the function is (from Ref.7)

$$L = -\frac{\tau}{\pi} \log \sqrt{\phi^2 + \psi^2} \qquad \ldots(16)$$

where ϕ and ψ are measured from the singularity and τ is the discontinuity in θ at the stagnation point (e.g., at a rounded nose $\tau = \pi/2$). The singular points require special treatment in the relaxation process and this is discussed in §5.1.

4.2 Compressible Flow

(a) Aerofoil Surface

The second of equations (5) gives

$$\frac{\partial L}{\partial \psi} = \frac{\partial \theta}{\partial \phi} = -\frac{\partial \theta}{\partial s}\frac{\partial s}{\partial \phi}$$

$$= \frac{1}{Rq_c} \qquad \ldots(17)$$

For incompressible flow equation (17) reduces to

$$\frac{\partial L_i}{\partial \psi_i} = \frac{1}{Rq_i}. \qquad \ldots(18)$$

Now for convenience $\phi_T - \phi_L$ is chosen to be the same in the incompressible and compressible (ϕ, ψ) planes, in consequence the aerofoil chord with be of different lengths, c_i and c_c, in the physical, (x,y) plane. The calculation of these lengths is discussed in §5.2. Then equations (17) and (18) may be written

$$\frac{\partial L}{\partial \psi} = 0\frac{c}{R_c}\frac{1}{q_c c_c},$$

$$\frac{\partial L_i}{\partial \psi_i} = 0\frac{c}{R_i}\frac{1}{q_i c_i};$$

so/

so for a fixed position on the aercfcil surface

$$\frac{\partial V}{\partial \psi} = \frac{\partial L_i}{\partial \psi_i}\,\frac{c_i q_i}{c_c q_c} , \qquad ...(19)$$

There is a slight movement of the equipctentials from the incompressible to compressible flow planes, but the change in $c_i q_i / c_c q_c$ due to this was found to be insignificant. Thus equation (19) where $\partial V/\partial \psi$ and $\partial L_i/\partial \psi_i$ are at the same value of the potential, was used to obtain the boundary condition cm the aercfcil surface in the compressible flow plane.

(b) Cuter Boundary

The values of V on the cuter boundary of the compressible flow field ore obtained from the relation, derived in 83.3

$$V(\phi, \psi) = \frac{1}{m_\infty} L_i\,(\phi, m_\infty \psi) .$$

which although approximate is sufficiently accurate if the cuter boundaries chosen are a sufficient distance from the aerofoil. Whether this is so can be checked in the same way as that used for the incompressible flow solution (cf. §4.1(b)).

(c) Stagnation Points

The same type of singularity in V as that for the incompressible flow function, Lj, arises at the stagnation points. These were dealt with by use of the incompressible flow solution in the approximate relation, derived from equation (15),

$$V = \frac{1}{m_\infty} L_i ,$$

at points of the relaxation mesh in the Mediate neighbourhood of the stagnation points (Fig.5). This is an approximation which is thought to have little effect on the final solution.

5. Relaxation Procedure

5.1 Incompressible Flow

The simplest approximation to the basic differential equation of the flow, viz.,

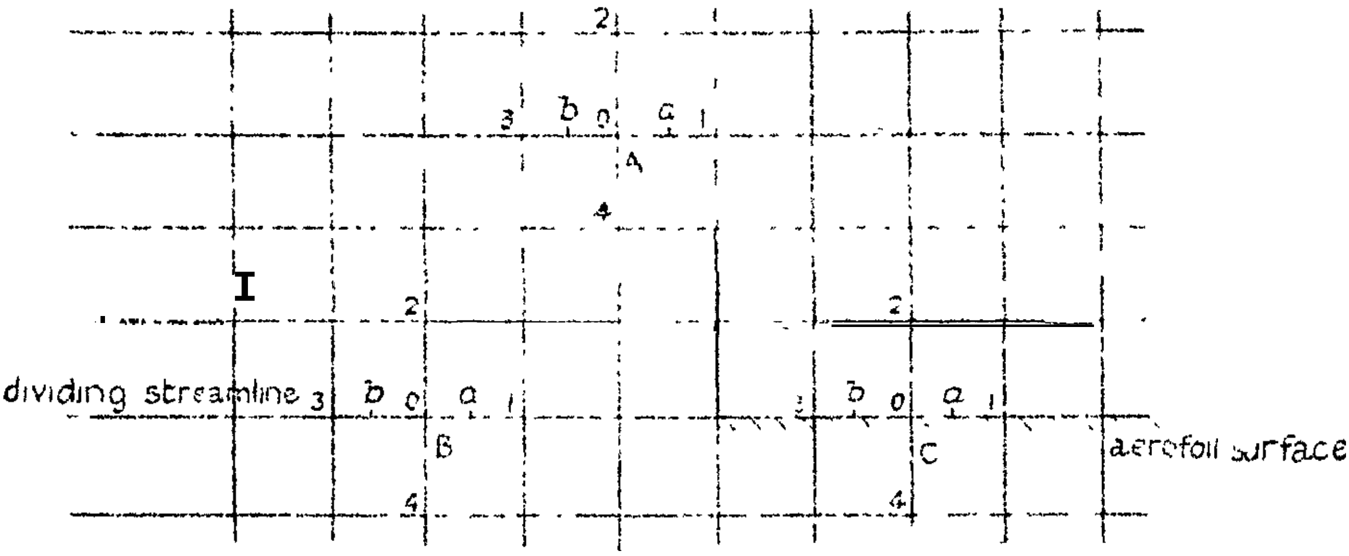
$$\frac{\partial^2 L}{\partial \psi^2} + \frac{\partial^2 L}{\partial \phi^2} = 0$$

is the difference equation

$$L_1 + L_2 + L_3 + L_4 - 4L_0 = 0 , \qquad ...(20)$$

where L_r refers to the value of L at some point r of the relaxation mesh (Fig.1). The use of equation (20) involves a possible error of $O(h^4)$.

Fig.1/



Section of relaxation mesh

a and b are the mid points of 0-1 and 0-3

FIG 1

Equation (20) must be satisfied at all points of the relaxation mesh.

As mentioned in §4.1(a) the values of L on the surface of the aerofoil and on the outer boundaries (Fig.3) were calculated and maintained during the relaxation of the interior of the field. Since the problem considered is that of a symmetrical aerofoil at zero incidence, by symmetry it is only necessary to consider the half of the field of flow bounded below by the dividing streamline. At points of the relaxation mesh on the dividing streamline but not on the aerofoil surface this symmetry gives the condition, (Fig.1, point B),

$$L_a - L_4 = 0. \qquad \ldots(21)$$

Hence for such points, from equations (20) and (21)

$$L_1 + L_3 + 2L_a - 4L_0 = 0.$$

In §4.1(c) it is noted that the stagnation points, which are singular points for L, require special treatment in the relaxation process. In the neighbourhood of the singularity L can be expressed as (cf. equation (16)),

$$L = -\frac{\tau}{\pi} \log_e \left\{ \frac{\phi^2 + \psi^2}{h^2} \right\}^{\frac{1}{2}} + \eta \qquad \ldots(22)$$

1 where ϕ and ψ are measured from the singularity and η satisfies,

$$\frac{\partial^2 \eta}{\partial \psi^2} + \frac{\partial^2 \eta}{\partial \phi^2} = 0 ,$$

and has no singularity at the stagnation points.

A relaxation treatment of such logarithmic singularities has been given in Ref.11. In this paper it is shown that 'loading factors' must be added to the residuals at points of the relaxation mesh near the singularity, (Fig.5). These factors depend on the strength of the singularity, which in this case is $-\tau/\pi$ (equation (22)). The true value of τ, obtained from the aerofoil profile, cannot be used since the conditions imposed by equations (10) and (12) of §3.2, must be satisfied. These two equations can be written

$$\left.\begin{aligned} \int_{\phi_L+}^{\phi_T-} \frac{\partial\theta}{\partial\phi}\, d\phi + \tau_L + \tau_T &= 0 , \\ \text{and} \qquad \int_{\phi_L+}^{\phi_T-} \phi \frac{\partial\theta}{\partial\phi}\, d\phi + \phi_L\tau_L + \phi_T\tau_T &= 0 . \end{aligned}\right\} \qquad \ldots(23)$$

Then, since, from the second of equations (5), for incompressible flow,

$$\frac{\partial\theta}{\partial\phi} = -\frac{\partial L}{\partial\psi} ,$$

equation (23) gives

$$\left.\begin{aligned} \tau_L + \tau_T &= \sum_j \left(\frac{\partial L}{\partial\psi}\right)_j [\phi]_j , \\ \text{and} \qquad \phi_L\tau_L + \phi_T\tau_T &= \sum_j \left(\frac{\partial L}{\partial\psi}\right)_j [\tfrac{1}{2}\phi^2]_j ; \end{aligned}\right\} \qquad \ldots(24)$$

where the summations are taken over all the mesh points on the aerofoil surface (excluding those at ϕ_L and ϕ_T). The square brackets denote a mean value of the enclosed function over the interval covered by the mesh point: e.g., for a mesh point at $\phi = 5$, with an interval $h = 1$,

$$[\tfrac{1}{2}\phi^2]_5 = \tfrac{1}{2}(5.5)^2 - \tfrac{1}{2}(4.5)^2 = 5.0 .$$

The derivative $\partial L/\partial\psi$ can be estimated at each mesh point, on the aerofoil from the equation,

$$2h \frac{\partial L}{\partial\psi} = L_2 - L_4 + O(h^2) , \qquad \ldots(25)$$

where L_4 is a fictitious value (Fig.1, point C) which can be eliminated by the use of equation (20), thus

$$2h \frac{\partial L}{\partial\psi} \doteqdot L_1 + L_3 + 2L_2 - 4L_0 .$$

Then/

Then from the simultaneous equations (24) appropriate values of τ_L and τ_T can be calculated. As the mesh interval, h, tends to zero the calculated values of τ will approach the true values.

5.2 Compressible Flow

The equation to be solved is

$$\frac{\partial^2 V}{\partial \psi^2} + \frac{\partial}{\partial \phi}\left(m^2 \frac{\partial V}{\partial \phi}\right) = 0 ,$$

which can be represented with a possible error of $O(h^4)$ by

$$V_2 + V_4 + m_a^2 V_1 + m_b^2 V_3 - V_0 (2 + m_a^2 + m_b^2) = 0 , \qquad \ldots(26)$$

where m_a^2 and m_b^2 refer to the value of m^2 corresponding to the value of V at the points a and b on Fig.1, i.e., $V_a = (V_0 + V_1)/2$.

On the aerofoil surface the form of equation (25) for V can be used with equation (26) to give,

$$2V_2 + V_1 m_a^2 + V_3 m_b^2 - V_0 (2 + m_a^2 + m_b^2) \doteq 2h \frac{\partial V}{\partial \psi} .$$

Now in §4.2(a) it is shown that the derivatives satisfy the relation

$$\frac{\partial V}{\partial \psi} = \frac{c_i}{c_c} \cdot \frac{q_i}{q_c} \frac{\partial L_i}{\partial \psi_i} ,$$

and it seems reasonable to suppose that the difference approximations to the derivatives will be related quite accurately in a similar manner. Therefore to obtain the boundary condition on the aerofoil in the compressible flow plane the following relation, for the same mesh size, was used,

$$2V_2 + m_a^2 V_1 + m_b^2 V_3 - V_0 (2 + m_a^2 + m_b^2) = \frac{c_i \; q_i}{c_c \; q_c} (L_1 + L_3 + 2L_2 - 4L_0) .$$

This form of the boundary condition reduces any errors introduced in the representation of derivatives by differences.

The outer boundary and stagnation points were treated as described in §4.2(b) and §4.2(c), the incompressible flow solution being used to obtain L_i, and hence V, at mesh points near the singularities.

One further point which remains to be discussed is the calculation of the chord, c. From the aerofoil co-ordinates it is possible to estimate the ratio s/c, where s is the perimeter distance along the aerofoil surface. Now

s/

$$s = \int_{\phi_L}^{\phi_T} ds \, ,$$

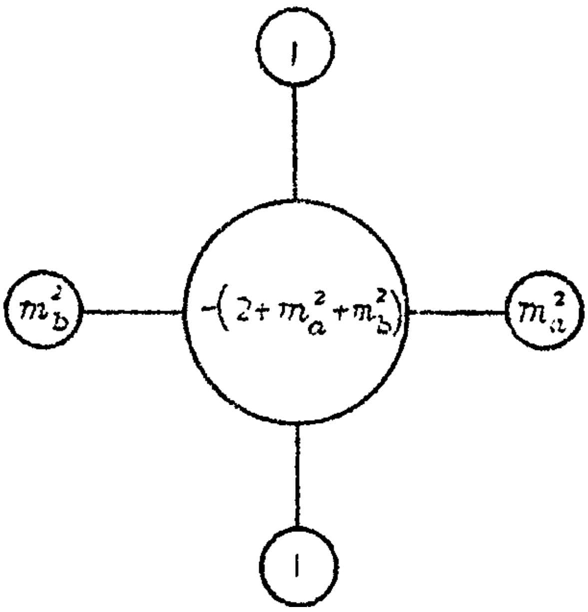
$$= \int_{\phi_L}^{\phi_T} \frac{d\phi}{q} \, , \qquad \ldots(27)$$

(from equation (2)), which can be integrated numerically by use of the values of q obtained from the partial solution at ony stage in the calculation. At the stagnation points q is zero, and, as in Ref.7, the equation

$$s = \frac{\bar{\phi}}{\bar{q}\left(1 - \frac{\tau}{\pi m_{\infty}}\right)} \, ,$$

where $\bar{\phi}$ is measured from the singularity, and $\bar{q}$ is the value of q at $\phi = \bar{\phi}$, was used for the mesh intervals adjacent to the stagnation points.

As is mentioned in the Introduction the existence of a shock-wave is revealed by the relaxation technique. It is found to be impossible to relax over the whole field because, for a sufficiently high free-stream Mach number and with q/U greater than unity, m^2 assumes large negative values. In this instance it is easily seen from the relaxation pattern (Fig.2), derived from equation (26), that the essential feature of relaxation, the fact that the elimination of a residual at one point shall not produce larger residuals at neighbouring points, is lost.



Relaxation pattern for equation (26)

FIG 2.

The/

The large unrelaxed residuals can be moved about the domain by making alterations to V at mesh points on the same streamline. In this my they can be collected in pairs of apposite sign along two neighbouring equipotentials of the mesh as required.

6. Application of Method

The flow about a 10% thick symmetrical aerofoil of RAE 104 section*, at zero incidence and for three subsonic Mach numbers, was calculated by the method discussed. above. The solution on the aerofoil surface in the incompressible flow plane was calculated by the method of Ref.9. The corresponding velocity distribution is shown in Mg.6, together with that obtained by the method of Ref.10, and by experiment (measured at M = 0.40 and reduced to M = 0 by the Glauert Law).

The velocity distribution8 obtained from the relaxation solutions for Mach numbers of 0.70, 0.79, and 0.86 are shown in Figs.7, 8 and 9, and compared with corresponding experimental values. The experimental values of q/U were measured in the NPL 20" x 8" High Speed Tunnel using streamline walls, i.e., walls shaped to the streamlines of the free-air flow about the model. In Fig.7 the velocity distribution calculated by the method of Ref.9 is also shown; this method is not applicable to the higher Mach numbers, where there are local regions of supersonic flow.

At a Mach number of 0.70 the flow is entirely subsonic, at M = 0.79 (just above the pressure critical Mach number) there is a small region of supersonic flow, extending from about 0.30c to 0.55c (Fig.8(ii)), which presented no difficulties in the relaxation process. The shock wave present at M = 0.86 was dealt with as discussed above, the flow photograph, (Fig.9(ii)), being used to determine the position of the foot of the shock.

The experimental results for M = 0.79, between 0.50c and 0.700, indicate a local separation of the boundary layer, transition taking place at about 0.70c, with turbulent reattachment. At M = 0.86 the experimental results in that region indicate a laminar boundary layer upstream of the shook wave, with separation at about 0.50c, transition under the shock, and possibly turbulent reattachment. Since the theoretical problem considered is that of the flow of an inviscid fluid it is thought that the discrepancies between the relaxation solution and the experimental results are due in part to boundary-layer effects.

7. Acknowledgement

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*Co-ordinates given in Ref.13.

**of the New Zealand Scientific Defence Corps, at present seconded to the Aerodynamics Division, N.P.L.

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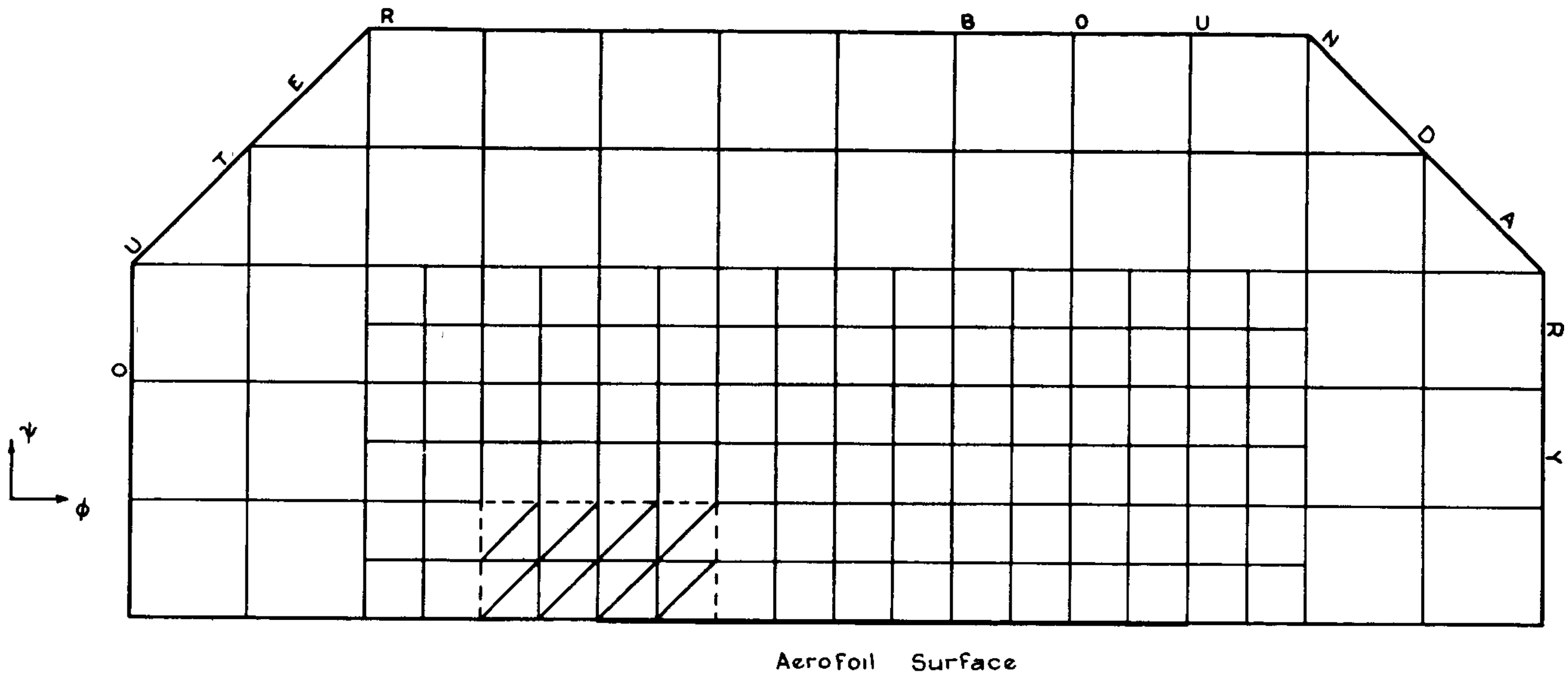
JDS.

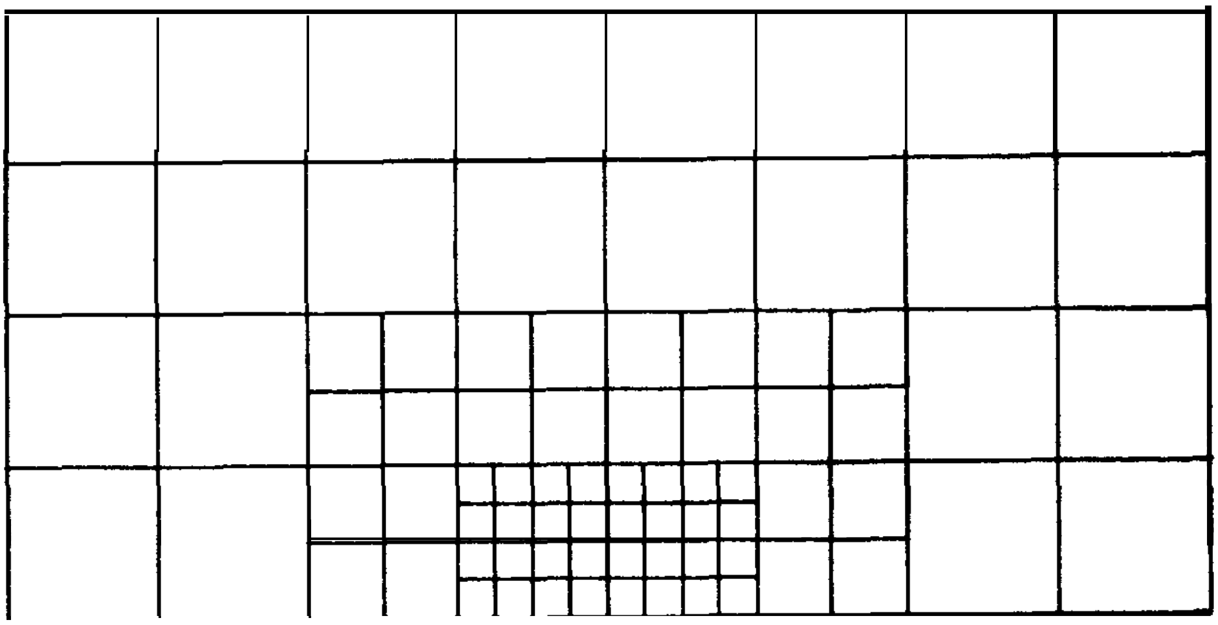
FIG. 3.



<u>Relaxation Mesh in (φ, ψ) plane</u>

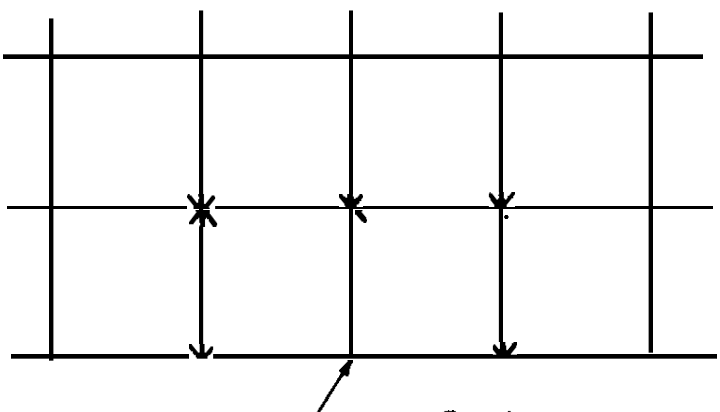
The shaded area was covered by a finer mesh which is shown in Fig 4.

Fig. 4.



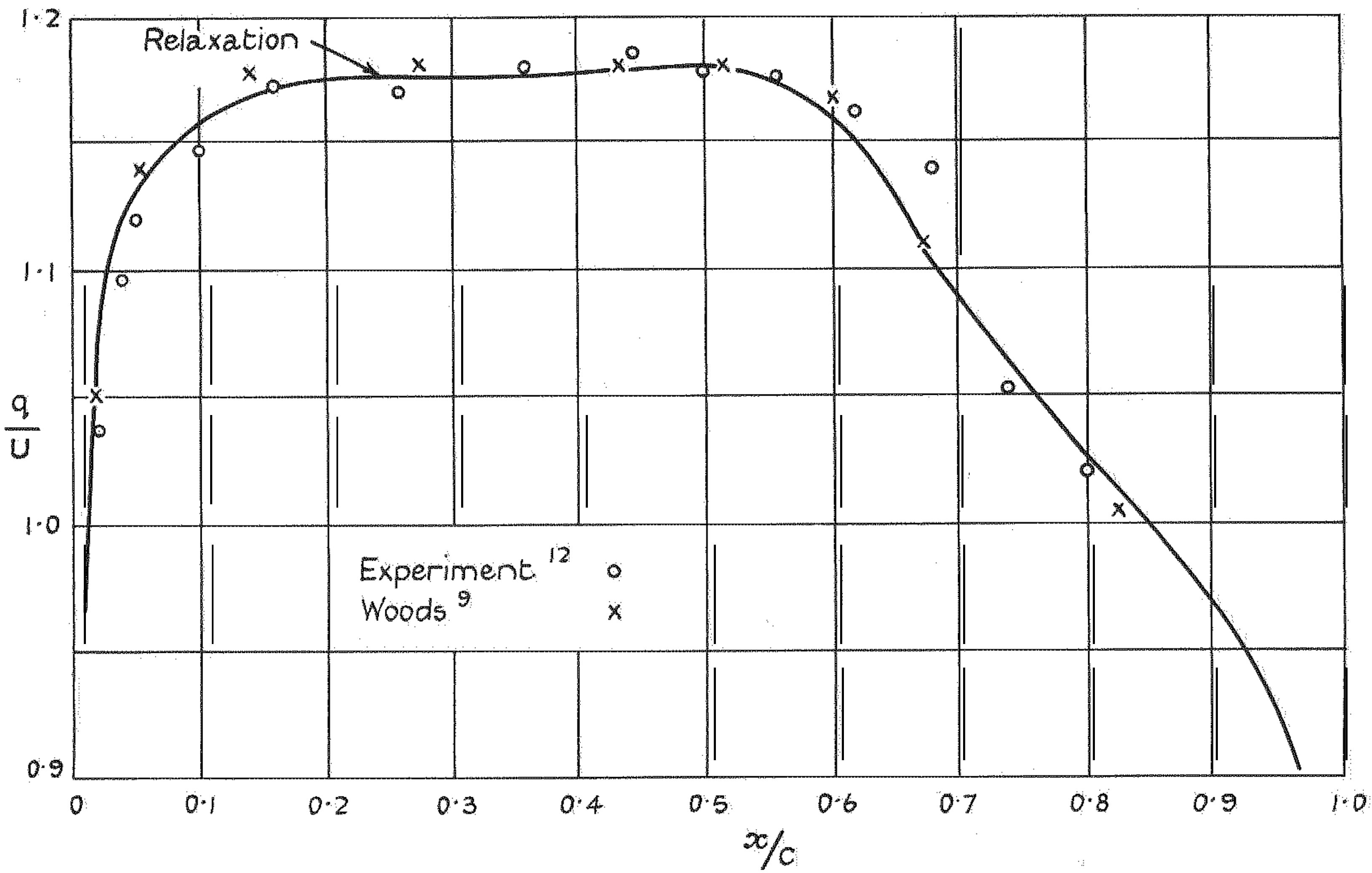
Part of the relaxation mesh covering the leading edge of the aerofoil

Fig. 5.

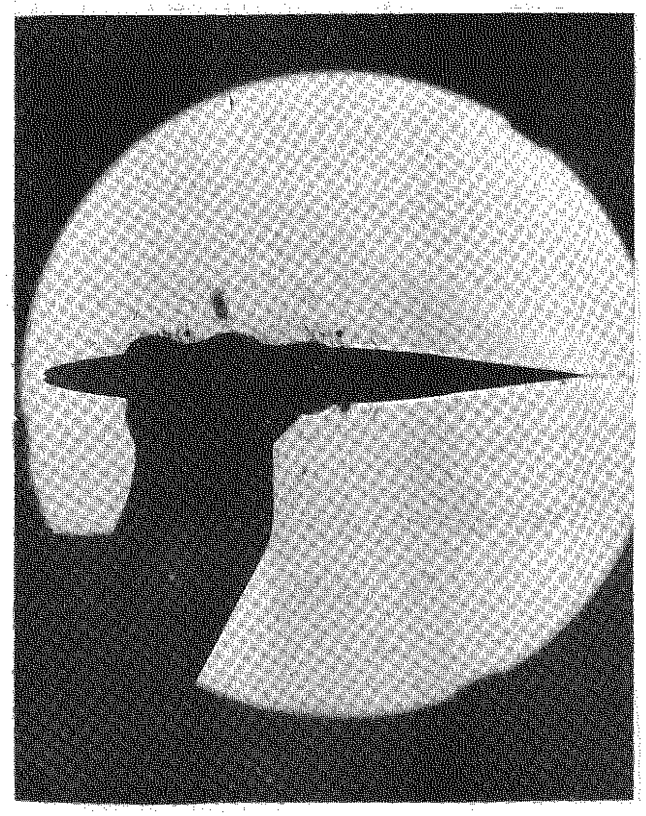


Showing the points of the relaxation mash at which 'loading factors' were applied (cf section 5·1)

FIG. 7.

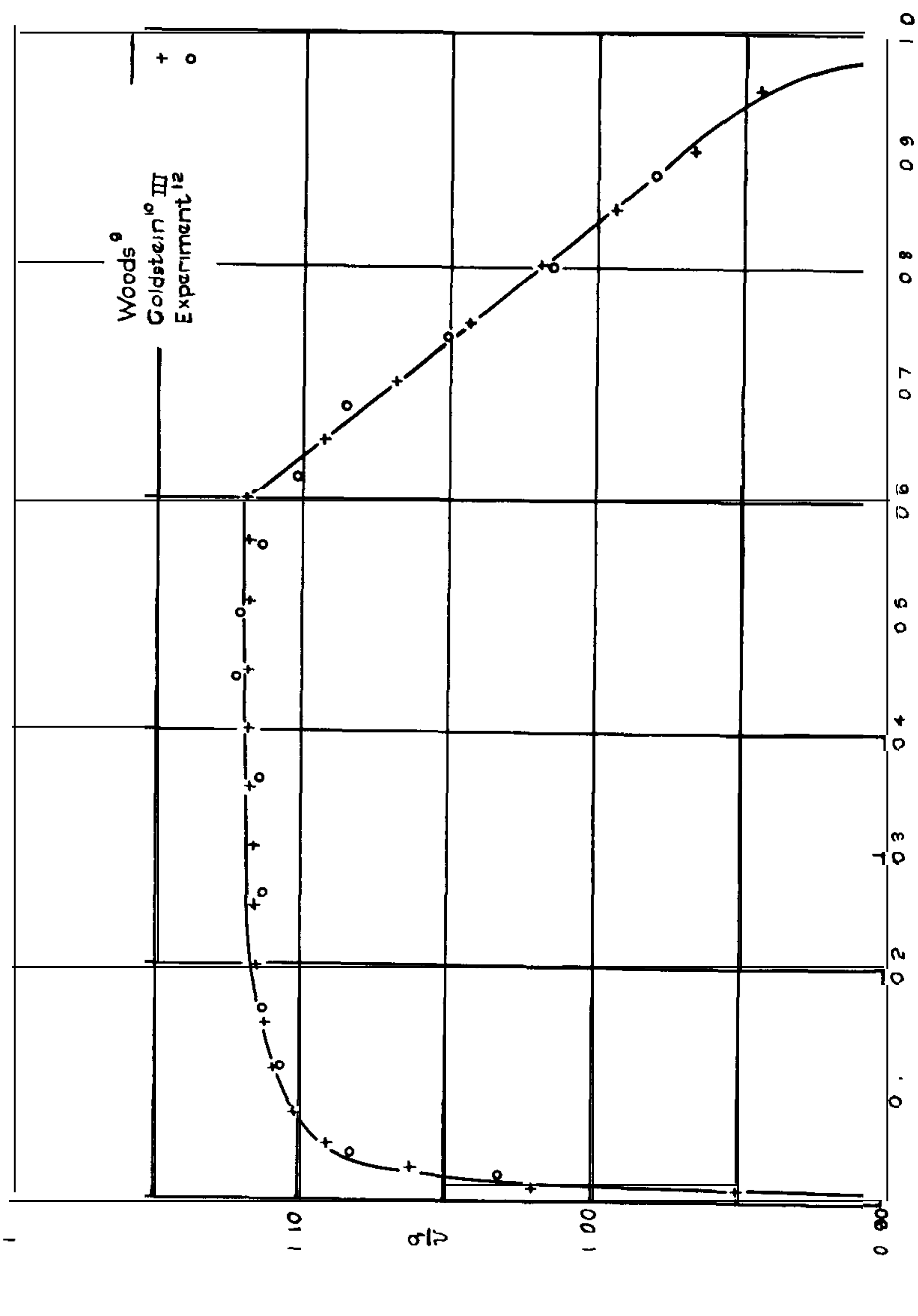


(i) Velocity distribution, 10% thick RAE 104 section. M=0·70.

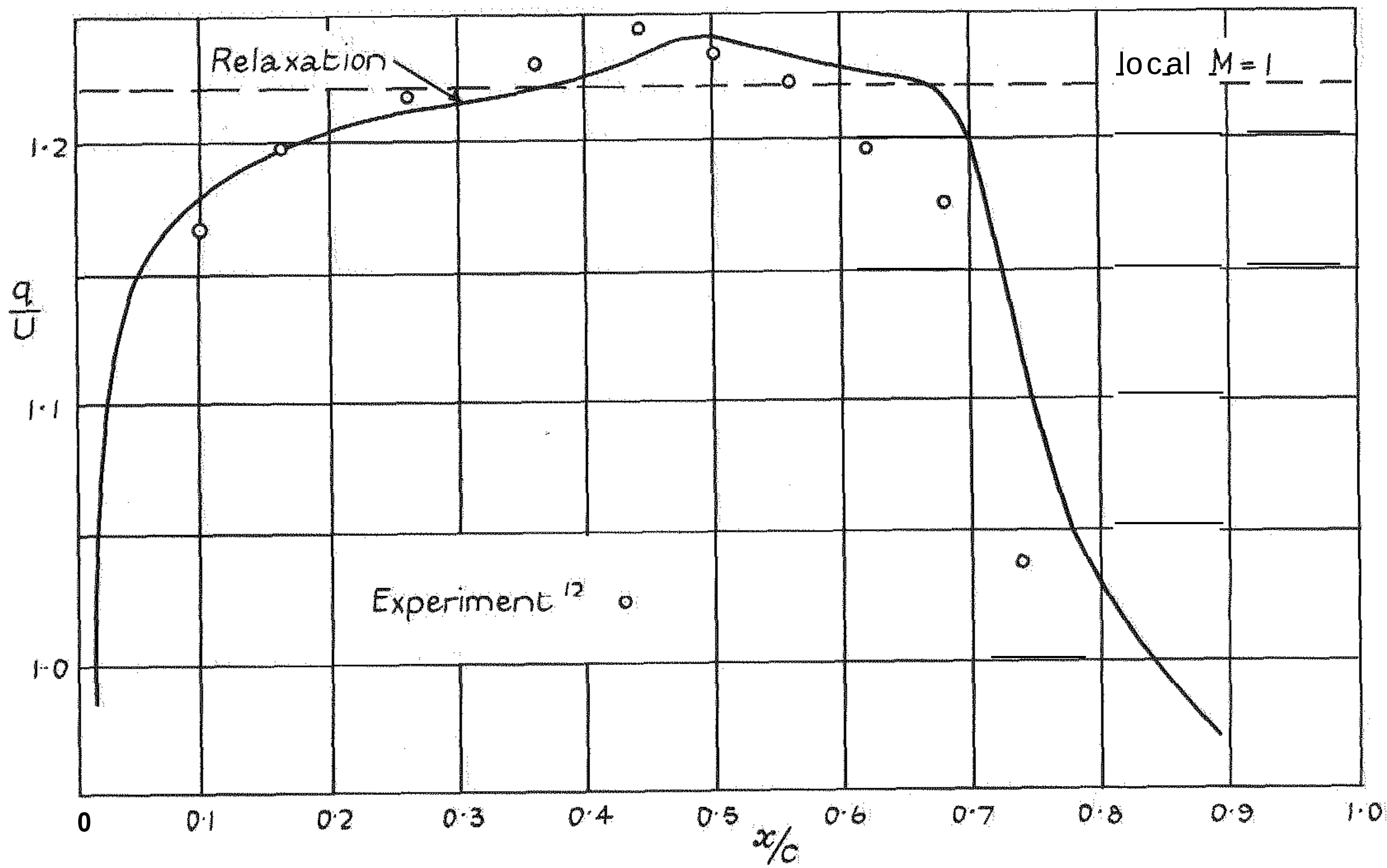


(ii) Flow photograph.

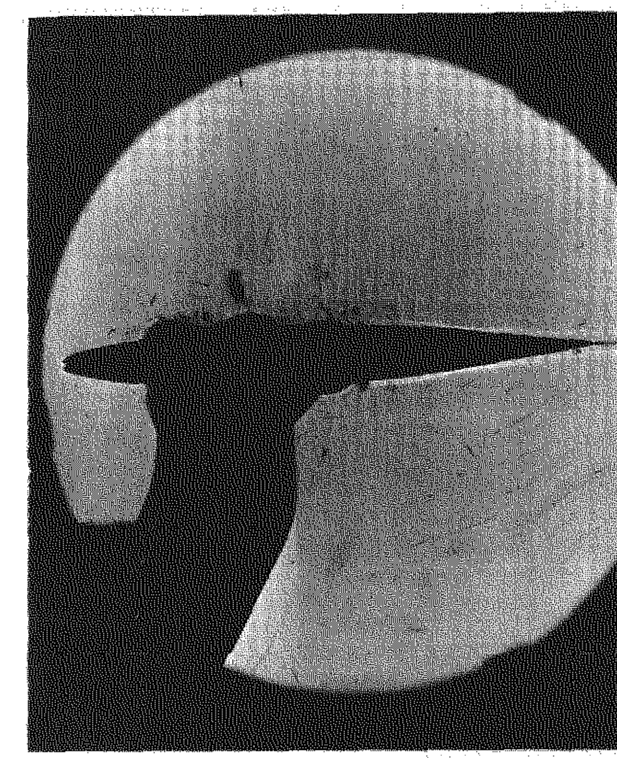
FIG 6



Velocity distribution, 10% thick RAE 104 section, M = 0



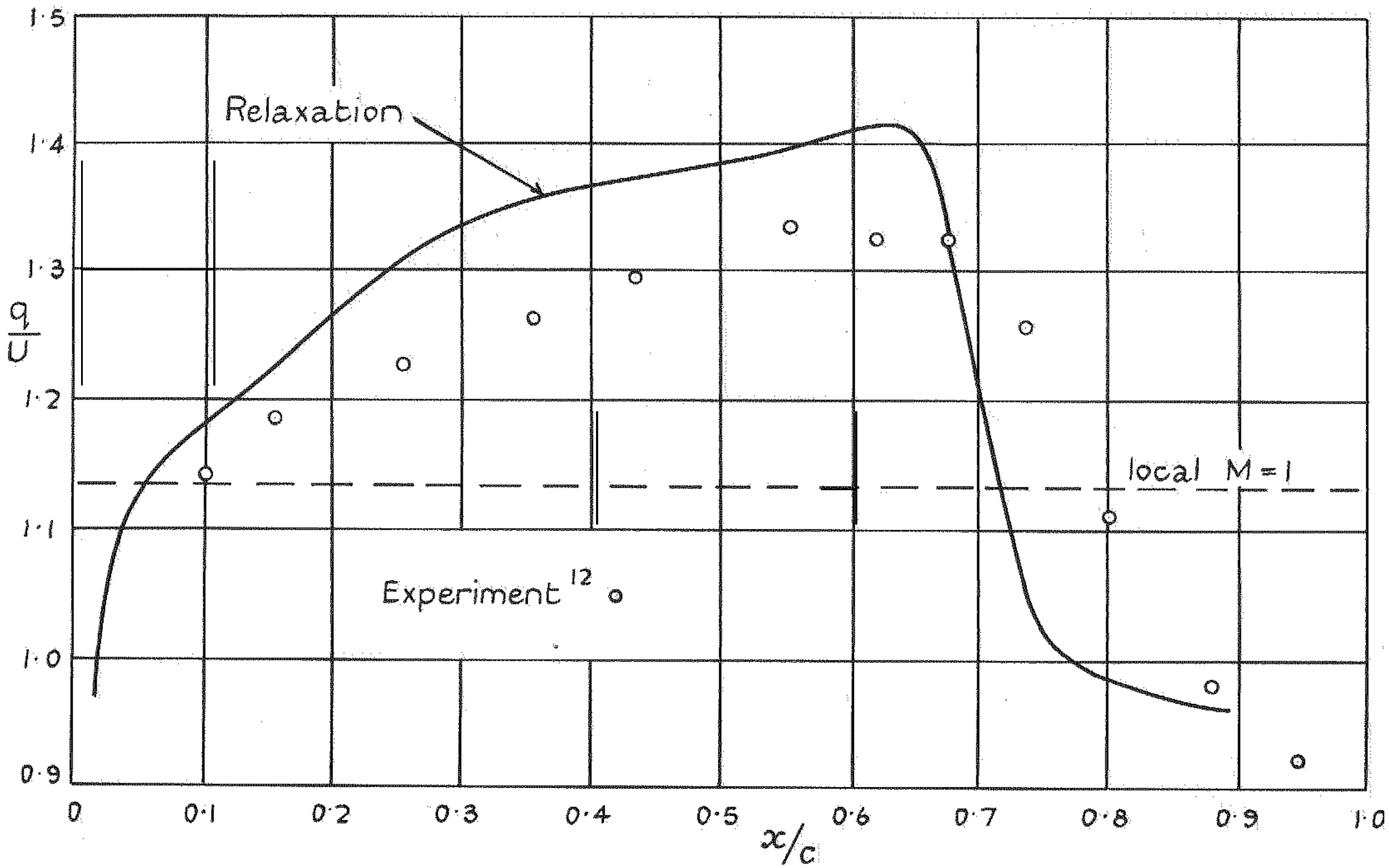
(i) Velocity distribution, 10% thick RAE 104 section. M = 0·79



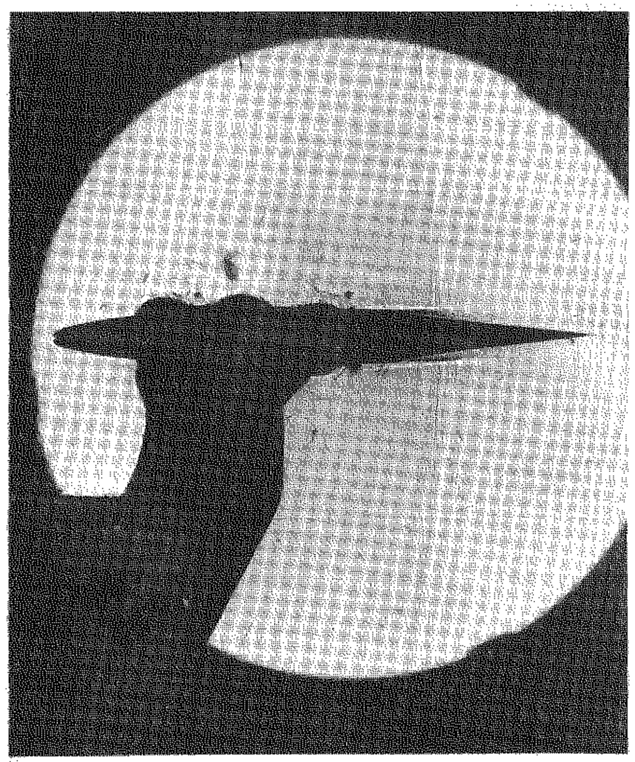
(ii) Flow photograph.

FIG. 3.

FIG. 9.

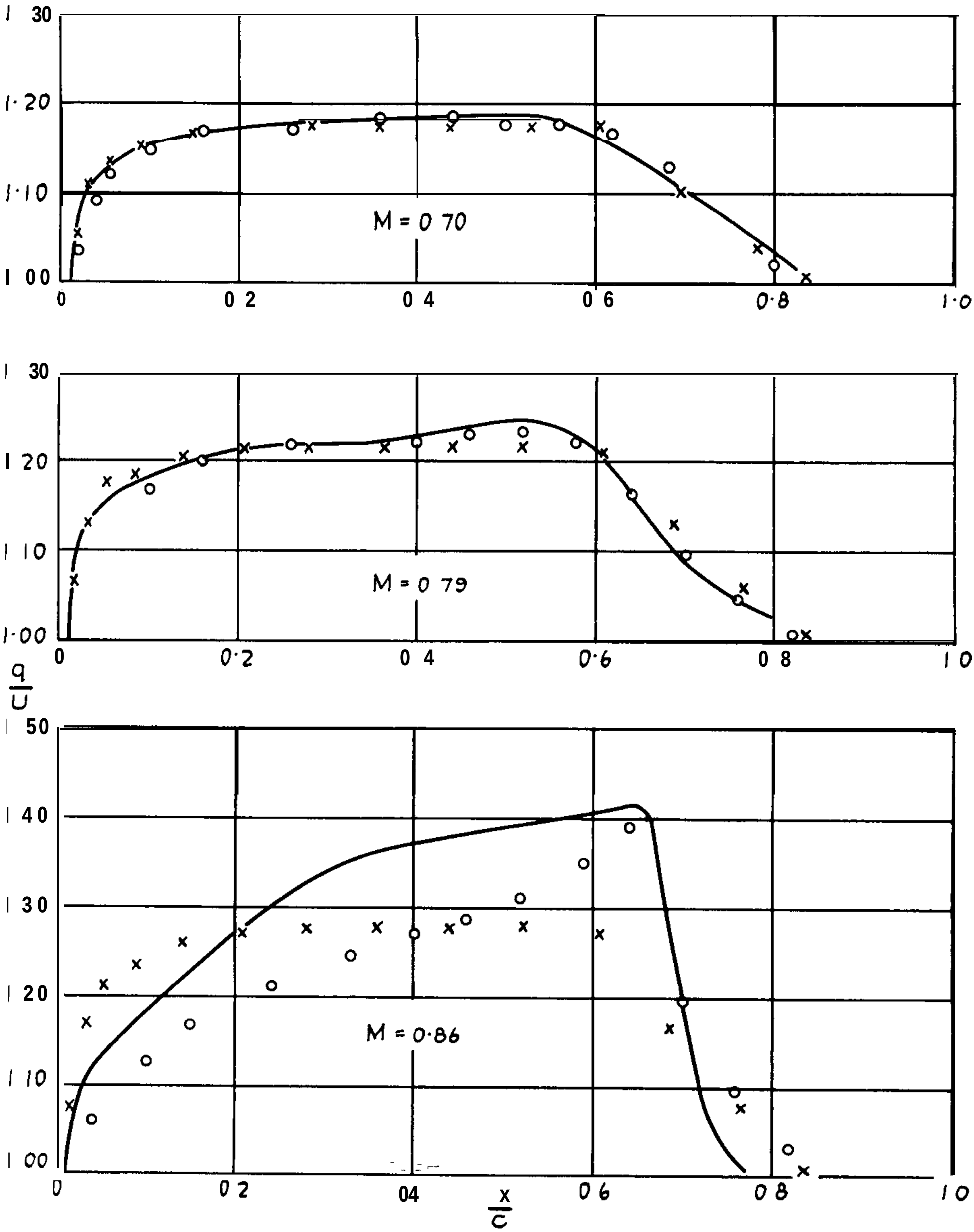


(i) Velocity distribution, 10% thick RAE 104 section. M = 0·86.



(ii) Flow photograph.

FIG 10.



Comparison of velocity distributions, 10% RAE 104.

Relaxation ——— Kármán - Tsien x

Experiment with forward transition o

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