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By

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AERONAUTICAL RESEARCH COUNCIL

Note on the Interference on a Part-Wing Mounted Symmetrically on one Wall of a Wind-Tunnel of Octagonal Section

-By-

H. C. Garner, B.A.
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6th March, 1947

Summary

(a) Reason for Inquiry.- This problem has arisen in connection with tests in the N.P.L. 9' x 7' tunnel on a part-wing with aileron, reported[1] by Halliday and the present author (1945).

(b) Range of Investigation.- This note shows quite generally the method of estimating the interference in the form of an upwash angle at any point of the plan form of a part-wing due to the system of images corresponding to any given distribution of lift. As an illustration the calculations for the 9' x 7' octagonal tunnel have been tabulated. The upwash functions are given in Tables 5 (a), 5 (b), 5 (c); and the upwash angle may be obtained by substituting these functions and parameters determined by the distribution of lift into equation (20) of §6.

(c) Conclusions.- It is discovered that an octagonal tunnel produces considerably more upwash than a rectangular tunnel near the wall where the part-wing is mounted, but that the difference is less marked in the centre of the tunnel. It is concluded that the interference of corner fillets can be ignored as far as part-wing tests on ailerons are concerned, but that it becomes appreciable if the conditions near the root section of the part-wing are under investigation.

It is found that the chordwise variation of upwash angle is practically linear, and that for all purposes of interference correction linearity may be assumed. The total upwash angle along any chord may then be represented as a local uniform incidence together with a superposed curvature of flow, which facilitate an accurate estimation of the wind-tunnel interference.

(d) Further Developments.- This note forms part of an investigation of the necessary process in order to determine tunnel interference accurately. The method of application will be published in a later report.

Summary of Contents

5./

1. Introduction

Swanson and Toll[2] (1943) have made a comprehensive investigation of methods of determining the interference on a half-wing mounted symmetrically on one wall of a rectangular tunnel. This is equivalent to the interference on a complete wing placed symmetrically in a tunnel of the same height and twice the width. The fundamental problem concerning tunnel interference is to determine the upwash velocity induced by the restricting influence of the tunnel walls. For this purpose the disturbance to a uniform free stream due to a wing supporting a given distribution of lift may be replaced to any desired degree of accuracy by a combination of horseshoe vortices; each horseshoe vortex consists of two semi-infinite trailing vortices parallel to the stream, joined by a bound vortex of equal strength perpendicular to them. At the walls of a closed rectangular tunnel the boundary condition of zero normal velocity may be satisfied identically by superposing for each horseshoe vortex a doubly infinite set of images. The upwash velocities due to each set of images may be calculated at any point of the plan form, The upwash velocity is usually considered in two parts, firstly that induced at points along the line of the bound vortex, and secondly its variation in the direction of the undisturbed stream.

An octagonal tunnel is formed by adding four isosceles triangular fillets to the corners of a rectangular tunnel (Fig. 1). At infinity in the wake the boundary conditions along the fillets are satisfied approximately by the method of vortex squares, which was first used by Batchelor[3] (1942). The complete system of images of a single horseshoe vortex in an octagonal tunnel, shown in Fig. 2, is used to calculate the induced upwash velocity along the bound vortex.

The chordwise variation in upwash velocity can only be determined exactly for rectangular tunnels. The upwash induced at the bound vortex in an octagonal tunnel is obtained by adding a small correcting term due to the vortex squares to its value in the rectangular tunnel formed by removing the fillets. It is reasonable, therefore, to treat the chordwise variation of upwash velocity by introducing a similar correction in the same ratio.

2. Notation

b	twice breadth of tunnel.
h	height of tunnel.
a	length of each isosceles fillet.
V	velocity in undisturbed stream.
c	chord of wing, measured in the direction of V.

s/

s	semi-span of wing.
x	distance along chord, measured from leading edge of centre section.
y, t	distance along wing span, measured from the centre section.
$x = R(y)$	equation of curve through mid-chord points.
r	distance along a fillet, measured from one end.
Γ	circulation round any wing section.
(K, σ)	strength of a horseshoe vortex of semi-span $t = \sigma b$.
$w(\eta, \xi)$	upwash velocity at a distance $y = \eta b$ from centre section and a distance ξb downstream of bound vortex.
$w_1(K, \sigma)$	upwash velocity at bound vortex due to horseshoe vortex (K, σ) in a rectangular tunnel.
w_2	additional upwash velocity at bound vortex due to fillets.
w_3	$w_1 + w_2$: upwash velocity at bound vortex in an octagonal tunnel.
$w_1'(\xi)$	increment to w_1 at a distance $b\xi$ downstream of bound vortex in a rectangular tunnel
$w_3'(\xi)$	increment to w_3 at a distance $b\xi$ downstream of bound vortex in an octagonal tunnel.

3. Horseshoe vortex representation of a given distribution of lift.

Consider a system of axes with origin at the leading edge of the centre section of the wing, the x - axis being in the direction of the undisturbed flow and the y-axis along the span. Let $x = R(y)$ represent a curve passing through all the midchord points, then the chord $c(y)$ is sufficient to define the plan form of the wing.

A given distribution of lift is adequately defined if the circulation $\Gamma(y)$ and the chordwise distance $l(y).c(y)$ from the leading edge to the centre of pressure are known at all sections along the wing span $-s \leq y \leq s$. At comparatively large distances from the surface of the wing, the disturbance to a uniform free stream due to the given distribution of lift is approximately equivalent to the flow arising from a distribution of horseshoe vorticity of strength $-\frac{d\Gamma(t)}{dt}$ per unit length, of semi-span t and with bound vorticity at the position

$$x = R(t) + \left\{ l(t) - \tfrac{1}{2} \right\} c(t).$$

If the same wing supports this given distribution of lift in a closed wind-tunnel, the flow arising from the horseshoe vortices would produce a flow of air across the walls of the tunnel, thus violating the boundary

conditions/

conditions; and it remains to determine the change in the flow at the surface of the wing, that is necessary to restore the boundary conditions due to horseshoe vortices of different spans.

4. Upwash velocity along the bound vortex induced by an octagonal tunnel.

In a previous report[4] (1943), the author has estimated the correction due to triangular corner fillets to the mean upwash along the span of a wing placed symmetrically in the 9' × 7' tunnel. The corresponding problem here is to determine the distribution of upwash on a wing placed symmetrically in an 18' × 7' tunnel with central fillets in addition to those at the corners (Fig. 1). The scalene fillets are again replaced by isosceles fillets of equal area.

As explained in Ref. 4 the wing and its image system for a rectangular tunnel would produce a flow of air across the fillets. For uniformly loaded wings of span not exceeding 7/8 of the breadth of the equivalent 18' × 7' tunnel the distribution of velocity V_1 into the tunnel normal to any corner fillet is almost linear with distance along the fillet. If a is the length of the fillet and r is the distance from one end of the fillet, by writing

$$V_1 = G\left(\frac{r}{a} - \frac{1}{2}\right)$$

$$\text{where}^{*}\ G = 2\left\{V_1\left(\text{at } r = \frac{3a}{4}\right) - V_1\left(\text{at } r = \frac{a}{4}\right)\right\}, \qquad \ldots\ldots\ldots(1)$$

the greatest error in the values of V_1 at the ends of the fillet is of the order 20%.

The method used by Batchelor[3] to cancel these normal components of velocity and at the same time to maintain the rectangular boundaries as streamlines is to superpose on the image system for the rectangular tunnel a distribution of vorticity k per unit length around the doubly infinite set of squares in Fig. 2. If k is given by

$$k = Ka(r - a)$$

along each side of each square, and acts in the senses indicated in Fig. 2, the vorticity produces an almost linear distribution of normal velocity V_2 across each side. To calculate V_2, it is only necessary to consider the contribution of that square, to which the particular side belongs, as the additional components amount to a negligible 1% of that. Approximately

$$V_2 = \pm \frac{0.96K}{\pi} a(r - \tfrac{1}{2}a) \qquad \ldots\ldots\ldots(2)$$

Hence by equating $(V_1 + V_2)$ to zero.

$$\pi G \pm 0.96Ka^2 = 0 \qquad \ldots\ldots\ldots(3)$$

Consider the approximation implicit in (3) in the particular case of a uniformly loaded wing with circulation Γ and of span $\frac{3b}{4}$, where b = the tunnel breadth = 18ft. In Table 1 are tabulated the calculated

values/

* This is a more convenient definition than that used in Ref. 4

values of $\frac{2bV_1\sqrt{2}}{\Gamma}$ (from (4) and (5)) along the central fillet AB and the corner fillet CD (see Fig.1) From these values G is known from (1) and K is determined for each fillet from (3). From Table 2 of Ref.3 the values of $\frac{2bV_2\sqrt{2}}{\Gamma}$ are obtained and are also given in Table 1. In Fig. 3 the curves of $-\frac{2bV_1\sqrt{2}}{\Gamma}$, $\frac{2bV_2\sqrt{2}}{\Gamma}$ and the linear substitute $-\frac{2bG\left(\frac{r}{a}-\frac{1}{2}\right)}{\Gamma}$ are compared, taking $r = 0$ at A and C. When it is taken into consideration that the additional upwashes due to the fillets are usually less than 5% of the upwashes due to the image system for the rectangular tunnel, it will be accepted that the agreement in Fig.3 justifies the use of (3).

A non-uniformly loaded wing may be represented by a combination of horseshoe vortices of different spans. In Ref.4 it was thought, that G was practically independent of the span of a horseshoe vortex; and throughout G was given its value for a small wing. However this assumption will apparently lead to serious error, whenever the section of the tunnel is much different from square. $\Lambda = \frac{G.b\sqrt{2}}{\Gamma}$ has been calculated (from (6)) for central and corner fillets of length $a = \sqrt{6}$ ft., in the 18' × 7' tunnel, and it is given in Table 2 for different spans $2\sigma b$.

4.1 General Formulae

From Ref. 3 p.8, assuming a uniform distribution of lift

$$V_1 = \frac{\Gamma}{2\,b\sqrt{2}} \sum_{l=-\infty}^{\infty} (-1)^l \left[\frac{\sin 2\pi m_1 - \sinh 2\pi n}{\cosh 2\pi n - \cos 2\pi m_1} - \frac{\sin 2\pi m_2 - \sinh 2\pi n}{\cosh 2\pi n - \cos 2\pi m_2}\right] \quad \ldots\ldots\ldots(4)$$

where along the central fillet AB (using coordinates O' X Y in Fig. 2)

$$m_1 = \frac{x - s}{b} = \frac{Y + X - s\sqrt{2}}{b\sqrt{2}} = \frac{X + \frac{a}{2} - s\sqrt{2}}{b\sqrt{2}}$$

$$m_2 = \frac{x + s}{b} = \frac{Y + X + s\sqrt{2}}{b\sqrt{2}} = \frac{X + \frac{a}{2} + s\sqrt{2}}{b\sqrt{2}}$$

$$n = \frac{y - lh}{b} = \frac{Y - X - (l + \frac{1}{2})h\sqrt{2}}{b\sqrt{2}} = \frac{\frac{a}{2} - X - (l + \frac{1}{2})h\sqrt{2}}{b\sqrt{2}}$$

where l is an integer.

It/

It is simple to show that along the corner fillet CD (using coordinates O" X Y)

$$V_1' = \frac{\Gamma}{2\ b\sqrt{2}} \sum_{l=-\infty}^{\infty} (-1)^l \left[\frac{\sin 2\pi\ m_1 + \sinh 2\pi\ n}{\cos\ 2\pi\ n + \cos 2\pi\ m_1} - \frac{\sin 2\ \pi\ m_2 + \sinh 2\pi\ n}{\cosh 2\pi\ n + \cos 2\ \pi\ m_2} \right] \qquad \ldots\ldots\ldots(5)$$

It follows that $V_1\ (s = s_1) = V_1'\ (s = \frac{1}{2}b - s_1)$

Now if we use the approximation that V_1 is a linear function of r and let

$$\Lambda = \frac{G.b\sqrt{2}}{\Gamma} = \frac{2b\sqrt{2}}{\Gamma} \left\{ V_1\left(\text{at } r = \frac{3a}{4}\right) - V_1\left(\text{at } r = \frac{a}{4}\right)\right\} \qquad \ldots\ldots\ldots(6)$$

It follows from (1) that

$$V_1 = \frac{\Gamma}{b\sqrt{2}}\Lambda\left(\frac{r}{a} - \frac{1}{2}\right)$$

Similarly for corner fillets Λ' is defined such that

$$V_1' = \frac{\Gamma}{b\sqrt{2}}\Lambda'\left(\frac{r}{a} - \frac{1}{2}\right)$$

Hence by means of (2) and (3),

$$\left.\begin{aligned} K &= \pm \frac{\pi\Gamma\Lambda}{0.96a^2b\sqrt{2}} \\ \text{and}\quad K' &= \pm \frac{\pi\Gamma\Lambda'}{0.96a^2b\sqrt{2}} \end{aligned}\right\} \qquad \ldots\ldots\ldots(7)$$

With a non-uniform distribution of lift, corresponding to a variable circulation Γ (t)

$$\left.\begin{aligned} &\text{for central fillets}\quad K = \pm \frac{\pi}{0.96a^2b\sqrt{2}} \int_0^{\frac{s}{b}} \Lambda\left(-\frac{d\Gamma}{d\sigma}\right) d\sigma \\ &\text{for corner fillets}\quad K' = \pm \frac{\pi}{0.96a^2b\sqrt{2}} \int_0^{\frac{s}{b}} \Lambda'\left(-\frac{d\Gamma}{d\sigma}\right) d\sigma \end{aligned}\right\} \qquad \ldots\ldots\ldots(7)'$$

where $t = \sigma b$ is used instead of s in the expressions for m_1 and m_2, so that Λ, Λ' and $\left(-\frac{d\Lambda}{d\sigma}\right)$ the horseshoe vortex **strength per unit** length, depend on σ.

The/

The upwash arising from the image systems for a rectangular tunnel of dimensions $b \times h$ may be calculated as in Ref. 5, p.3. At a point $y = \eta b$ along the bound vortex the upwash due to a single horseshoe vortex of strength K is

$$w_1 = -\frac{K}{4\pi b}\left(\sum_{-\infty}^{\infty}\sum\right)'\left[(-1)^n\left\{\frac{m-\eta+\sigma}{(m-\eta+\sigma)^2+\left(\frac{nh}{b}\right)^2} - \frac{m-\eta-\sigma}{(m-\eta-\sigma)^2+\left(\frac{nh}{b}\right)^2}\right\}\right] \qquad \text{.........(8)}$$

$$= -\frac{K}{4\pi b}\left[\sum_{m=-\infty}^{\infty}\frac{\pi b}{h}\left\{\operatorname{cosec} h\,\frac{\pi b}{h}(m-\eta+\sigma) - \operatorname{cosec} h\,\frac{\pi b}{h}(m-\eta-\sigma)\right\} - \left\{\frac{1}{-\eta+\sigma} - \frac{1}{-\eta-\sigma}\right\}\right]$$

Define $$\Omega(p) = \frac{1}{p} + \frac{\pi b}{h}\sum_{m=-\infty}^{\infty}\operatorname{cosec} h\,\frac{\pi b}{h}(m-p) \qquad \text{.........(9)}$$

then $$\frac{4\pi b w_1}{K} = \Omega(\sigma+\eta) + \Omega(\sigma-\eta) \qquad \text{.........(10)}$$

As in Ref. 4, p.5, for a single horseshoe vortex, the upwash due to the fillets may be calculated from the series

$$w_2 = \sum_i \frac{a^2 K_i}{8\pi}\chi(S_i, T_i)$$

where
$$\begin{aligned}\chi(S_i, T_i) = &-\frac{2}{15}\left(\frac{\pi a}{h}\right)^3\left(2S_i^3 - S_i\right)\\ &+\sqrt{\frac{2}{15}}\left(\frac{\pi a}{h}\right)^4\left(-6S_i^3 + S_i\right)T_i\\ &+\frac{1}{30}\left(\frac{\pi a}{h}\right)^5\left(24S_i^5 - 20S_i^3 + S_i\right)\\ &-\frac{\sqrt{2}}{180}\left(\frac{\pi a}{h}\right)^6\left(-120S_i^5 + 60S_i^3 - S_i\right)T_i\\ &-\frac{8}{5670}\left(\frac{\pi a}{h}\right)^7\left(720S_i^7 - 840S_i^5 + 182S_i^3 - S_i\right)\\ &+\frac{17\sqrt{2}}{113400}\left(\frac{\pi a}{h}\right)^8\left(-5040S_i^7 + 4200S_i^5 - 546S_i^3 + S_i\right)T_i\\ &+ \ldots\ldots\end{aligned} \qquad \text{.........(11)}$$

where/

where
$$\left.\begin{aligned} S_i &= \operatorname{sech} \theta_i \\ T_i &= \tanh \theta_i \end{aligned}\right\}$$

Each term in the summation includes the contribution of one column of squares.

Let $i = 0$ represent the central column of squares (containing 18, 5, 2, 11 etc in Fig. 2) and $i = \pm 1$ represent the corner columns of squares (containing 19, 6, 1, 10 etc. and 17, 4, 3, 12 etc. respectively) and so on.

$$\text{Then } \theta_i = \frac{\pi (ib + a\sqrt{2})}{2h} + \frac{\pi b \eta}{h}$$

when i is even (for central fillets) K_i is of opposite sign to K and when i is odd (for corner fillets) K_i is of the same sign as K.

So from (11) and the relations

$$\left.\begin{aligned} K_i &= -\frac{\pi K \Lambda}{0.96 a^2 b \sqrt{2}} \quad \text{when } i \text{ is even,} \\ &= +\frac{\pi K \Lambda'}{0.96 a^2 b \sqrt{2}} \quad \text{when } i \text{ is odd} \end{aligned}\right\} \qquad \ldots\ldots\ldots(7)$$

$$\frac{bw_2}{0.092K} = \sum_{i=-\infty}^{\infty} (-1)^{i-1} \Lambda_i \, \chi \, (S_i, T_i) \qquad \ldots\ldots\ldots(12)$$

where
$$\begin{aligned} \Lambda_i &= \Lambda \text{ when } i \text{ is even} \\ &= \Lambda' \text{ when } i \text{ is odd} \end{aligned}$$

Then the upwash at a position $y = b\eta$ on the bound vortex induced by an octagonal tunnel due to a single horseshoe vortex of strength K and semi-span $t = b\sigma$, is $w_1 + w_2$, where w_1 and w_2 are given by (10) and (12) respectively;

$$\text{i.e. } w_3 = w_1 + w_2 = \frac{K}{b}\left[\frac{1}{4\pi}\left\{\Omega(\sigma + \eta) + \Omega(\sigma - \eta)\right\}\right.$$
$$\left. + 0.092 \sum_{i=-\infty}^{\infty} \left\{(-1)^{i-1} \Lambda_i \, \chi \, (S_i, T_i)\right\}\right] \qquad \ldots\ldots\ldots(13)$$

where Ω, Λ and χ are defined in (9), (6) and (11) respectively.

Since the series (11) does not converge rapidly, when $i = 0$ and η is small or when $i = -1$ and η is nearly $\frac{1}{2}$, it was found necessary to examine the convergence of the expression for χ (S, T) by means of the alternative series considered in the Appendix.

4.2 Results for N.P.L. 9' x 7' tunnel.

For the 9' x 7' tunnel it is necessary to substitute the values $b = 18$ft, $h = 7$ft. and $a = \sqrt{6}$ft. Values of $\frac{w_3}{K}$ (of dimensions ft.$^{-1}$) from (13) are given in Table 4(b) for different values of σ and η. These may be compared with corresponding values of $\frac{w_1}{K}$ in table 4(a) from (10), in which the effect of the fillets is not included. Although the fillets are of secondary importance, it is necessary to include their effect in order to calculate accurately the wind-tunnel interference.

For convenience of application the values of $\frac{w_3}{K}$ are also given in Table 5(a) for integral values of 36σ and 36η, at exact multiples of half a foot from the tunnel walls.

5. Chordwise variation of induced upwash.

In order to obtain a general expression for the increase in upwash velocity induced at points downstream of the bound vortex, the method and notation of Brown[5] (1939) will be used. The additional upwash velocity at (x, y, z), a distance x downstream of the bound vortex, due to a horseshoe vortex of strength K and span 2s with its centre at (0, y_1, z_1) is

$$\Delta w_1' = -\frac{Kx}{4\pi}\left[\left(\frac{1}{x^2+(z-z_1)^2} + \frac{1}{(y-y_1+s)^2+(z-z_1)^2}\right)\frac{y-y_1+s}{\left\{x^2+(y-y_1+s)^2+(z-z_1)^2\right\}^{\frac{1}{2}}}\right.$$
$$\left. -\left(\frac{1}{x^2+(z-z_1)^2} + \frac{1}{(y-y_1-s)^2+(z-z_1)^2}\right)\frac{y-y_1-s}{\left\{x^2+(y-y_1-s)^2+(z-z_1)^2\right\}^{\frac{1}{2}}}\right].$$

Imagine a wing placed symmetrically in a tunnel of breadth b and height h, and put $x = b\xi$, $y = b\eta$, $z = b\zeta$ $s = b\sigma$ and $\lambda = \frac{2h}{b}$. Then the increment to the upwash velocity due to the images of the given horseshoe vortex in a rectangular tunnel of dimensions b x h is

$$w_1' = \left(\sum_{-\infty}^{\infty}\sum\right)' (-1)^n \; \Delta w'$$

where y_1 takes the values of $y_1 + mb = b(\eta_1 + m)$

z_1 takes the values $z_1 + nh = b\left(\zeta_1 + \frac{n\lambda}{2}\right)$

m and n are positive or negative integers including

zero and $\left(\sum_{-\infty}^{\infty}\sum\right)'$ indicates that (m, n) takes all possible

pairs of values except (0,0).

Putting $\zeta = \zeta_1 = \eta_1 = 0$, it follows that in the plane of a horseshoe vortex placed symmetrically in the tunnel,

$$w_1' = -\frac{K\xi}{4\pi b}\left(\sum_{-\infty}^{\infty}\sum\right)' (-1)^n \left\{ G_n(\eta - m + \sigma) - G_n(\eta - m - \sigma) \right\}$$

where/

where $G_n(\eta) = \left(\frac{1}{\xi^2 + \left(\frac{n\lambda}{2}\right)^2} + \frac{1}{\eta^2 + \left(\frac{n\lambda}{2}\right)^2} \right) \frac{\eta}{\left\{ \xi^2 + \eta^2 + \left(\frac{n\lambda}{2}\right)^2 \right\}^{\frac{1}{2}}}$

If either $|m|$ or $|n|$ is large enough $|m| > M$ or $|n| > N$, say, it is accurate enough to put

$$G_n(\eta - m + \sigma) - G_n(\eta - m - \sigma) = 2\sigma \left\{ G_n(\eta - m + \tfrac{1}{2}) - G_n(\eta - m - \tfrac{1}{2}) \right.$$

In practice M and N may be taken as small integers (e.g. M = 1, N = 2) and w_1' may be evaluated conveniently by means of the approximation

$$w_1' = -\frac{K\xi}{4\pi b} \left(\sum_{m=-M}^{M} \sum_{n=-N}^{N} \right)' (-1)^n \left\{ G_n(\eta - m + \sigma) - G_n(\eta - m - \sigma) \right\} + R_{MN} \qquad \ldots\ldots\ldots(14)$$

where $R_{MN} = -\frac{K\xi}{4\pi b} \left(\sum_{-\infty}^{\infty} \sum \right)'' (-1)^n \, 2\sigma \left\{ G_n(\eta - m + \tfrac{1}{2}) - G_n(\eta - m - \tfrac{1}{2}) \right\}$

and $\left(\sum_{-\infty}^{\infty} \sum \right)''$ indicates that pairs (m,n) such that $|m| \leqslant M$ and $|n| \leqslant N$ are omitted.

As $\eta \to \pm \infty$, $G_n(\eta) \to \pm \frac{1}{\xi^2 + \left(\frac{n\lambda}{2}\right)^2}$

Hence it is easily shown that

$$R_{MN} = -\frac{K\sigma\xi}{\pi b} \left[\sum_{n=-\infty}^{\infty} \frac{1}{\xi^2 + \left(\frac{n\lambda}{2}\right)^2} - \sum_{n=-N}^{N} \frac{(-1)^n}{2} \left\{ G_n(\eta + M + \tfrac{1}{2}) - G_n(\eta - M - \tfrac{1}{2}) \right\} \right]$$

$$= -\frac{K\sigma\xi}{\pi b} \left[\frac{2\pi}{\lambda\xi} \operatorname{cosec} h \frac{2\pi\xi}{\lambda} - \sum_{n=-N}^{N} \frac{(-1)^n}{2} \left\{ G_n(M + \tfrac{1}{2} + \eta) + G_n(M + \tfrac{1}{2} - \eta) \right\} \right] \qquad \ldots\ldots\ldots(15)$$

Throughout the computations for the 9' × 7' tunnel, it was found that the extreme variation of $\frac{\pi b R_{MN}}{K\sigma\xi}$ with ξ and η was only 2%. So if the upwash increment is required to no greater accuracy than 0.2%, it is quite sufficient to put $\xi = \eta = 0$ inside the brackets in (15) and to take

π /

$$\frac{\pi b R_{MN}}{K\sigma\zeta} = \frac{2\pi^2}{3\lambda^2} + \frac{2}{(2M+1)^2} + 8(2M+1)\sum_{n=1}^{N}(-1)^n \frac{2n^2\lambda^2 + (2M+1)^2}{n^2\lambda^2\left\{n^2\lambda^2 + (2M+1)^2\right\}^{\frac{1}{2}}}. \quad \text{........(16)}$$

For a rectangular tunnel, therefore, the upwash velocity at any point on the plan form of the wing due to a single horseshoe vortex of strength ζ and semi-span $b\sigma$ is

$$w_1 + w_1'$$

where w_1 is defined in (10), w_1' in (14) and R_{MN} is given by either (15) or (16)

The chordwise variation in upwash velocity can only be determined exactly for rectangular tunnels. The upwash velocity w_3 induced at the bound vortex in an octagonal tunnel is obtained in (13) by adding the small quantity w_2 to the value w_1 for the rectangular tunnel formed by removing the fillets. It is reasonable to take the chordwise variation in upwash velocity for an octagonal tunnel to be

$$w_3' = w_1' + w_2',$$

where $\frac{w_2'}{w_1'} = \frac{w_2}{w_1}$

Thus the upwash velocity at a spanwise position $y = b\eta$ and at a distance $b\xi$ downstream of the bound vortex becomes

$$w = -1\left(1 + \frac{w_1'}{w_1}\right) \quad \text{........(17)}$$

where w_1, w_3 and w_1' are given in (10), (13) and (14) respectively.

5.1 Results for N.P.L. 9' × 7' tunnel.

For the 9' × 7' tunnel it is necessary to substitute $b = 18$ft. and $\lambda = \frac{7}{9}$ in (14) in order to determine w_1'. Values of $\frac{w_1'}{K}$ (of dimensions (ft.)$^{-1}$) are given in Table 5(b) for different values of σ and η at points 1ft. and 2ft. downstream of the bound vortex. It will be noted that $\frac{w_1'}{K}$ is practically linear with ξ, and that it is accurate enough to assume a linear chordwise variation of upwash. By estimating $\frac{w_3}{w_1}$ for the appropriate values of σ and η, approximate values of

$$\frac{w_3'}{K} \cdot \frac{1}{b\xi} \text{ (of dimensions (ft.)}^{-2}\text{)} = \frac{w_1'}{K} \cdot \frac{w_3}{w_1} \cdot \frac{1}{b\xi}$$

have been tabulated in Table 5(c) and are convenient for use in (17).

6. Method of calculation of total induced upwash angle

From § 3, a given distribution of lift is approximately equivalent to a distribution of horseshoe vorticity of strength $-\frac{d\Gamma(t)}{dt}$ per unit length, of semi-span t and with bound vorticity at the position

$$x = R(t) + \left\{ l(t) - \tfrac{1}{2} \right\} c(t)$$

From § 4, the upwash velocity at the position

$$(x, y) = \left\{ R + (l - \tfrac{1}{2})c \, , \, y \right\}$$

due to one element of horseshoe vorticity $-\frac{d\Gamma}{dt}\delta t$ is

$$\delta w_3 = \frac{K}{b}\left[\frac{1}{4\pi}\left\{\Omega(\sigma+\eta) + \Omega(\sigma-\eta)\right\} + 0.092\sum_{i=-\infty}^{\infty}\left\{(-1)^{i-1}\Lambda_i\,\chi(S_i, T_i)\right\}\right] \quad \text{.........(13)}$$

where $K = -\frac{d\Gamma}{dt}\delta t$,

$$\sigma = \frac{t}{b}, \quad \eta = \frac{y}{b}$$

and Ω, Λ and χ are defined in (9), (6) and (11) respectively.

From § 5, equation (17), the upwash velocity at any position (x,y) due to the element of horseshoe vorticity is

$$\delta w = \delta w_3\left(1 + \frac{w_1'}{w_1}\right) = \delta w_3 \left[1 + \frac{-\xi\left(\sum_{m=-M}^{M}\sum_{n=-N}^{N}\right)'(-1)^n\left\{G_n(\eta - m + \sigma) - G_n(\eta - m - \sigma)\right\} + \frac{4\pi b R_{MN}}{K}}{\Omega(\sigma+\eta) + \Omega(\sigma-\eta)}\right] \quad \text{........(18)}$$

where δw_3 is given in (13) above,

$$\xi = \frac{x - R - (l - \tfrac{1}{2})\,c}{b},$$

G_n/

$$G_n(\eta) = \left(\frac{1}{\xi^2 + \left(\frac{n\lambda}{2}\right)^2} + \frac{1}{\eta^2 + \left(\frac{n\lambda}{2}\right)^2}\right) \frac{\eta}{\left\{\xi^2 + \eta^2 + \left(\frac{n\lambda}{2}\right)^2\right\}^{\frac{1}{2}}}$$

R_{MN} is given by either (15) or (16) and $\left(\sum_{m=-M}^{M} \sum_{n=-N}^{N}\right)$ indicates that (m, n) takes all possible pairs of integral values such that $|m| \leqslant M$, $|n| \leqslant N$ except (0,0).

In practice M and N may be taken as small integers (e.g. $M = 1$, $N = 2$).

$$w = \int_0^s \left(\frac{\delta w}{\delta t}\right) dt$$

$$\therefore \frac{w}{V} = \int_0^s \frac{W}{V}\left(-\frac{d\Gamma}{dt}\right) dt \qquad \text{........(19)}$$

where $W = \frac{w_3}{K}\left(1 + \frac{w_1'}{w_1}\right)$ from (18).

The total induced upwash angle is easily evaluated by dividing the range of integration into about 10 arbitrary intervals $0 = t_0 \leqslant t \leqslant t_1,\ t_1 \leqslant t \leqslant t_2,\ \ldots\ldots,\ t_9 \leqslant t \leqslant t_{10} = s$, and by summing

$$\frac{w}{V} = \sum_{r=1}^{10} \left[\frac{\Gamma(t)}{V}\right]_r W_r \qquad \text{........(20)}$$

where $\left[\frac{\Gamma(t)}{V}\right]_r$ denotes $\frac{\Gamma(t_{r-1}) - \Gamma(t_r)}{V}$

and W_r denotes a mean value of W in the integral $t_{r-1} \leqslant t \leqslant t_r$, for which it is usually good enough to substitute $t = \frac{1}{2}(t_{r-1} + t_r)$. However for the last interval $t_9 \leqslant t \leqslant t_{10} = s$, $\Gamma(t)$ is of the form $k\sqrt{1 - \frac{t^2}{s^2}}$

and it is easily shown that it is preferable to substitute

$$t = \frac{t_9 + 2s}{3}$$

in order to obtain the best mean value.

The/

The process of evaluation of the integral (19) for $\frac{w}{V}$ is

(a) to tabulate the functions $\frac{w_1}{K}$, $\frac{w_3}{K}$, $\frac{w_1'}{K}$ for suitable values of $t = b\sigma$, $y = b\eta$. These quantities (of dimensions $(ft)^{-1}$) for the N.P.L. 9' x 7' tunnel are given in Tables 4 (a), (b) and 5 (a), (b);

(a) to choose $t_1, t_2, \ldots t_9$ so that the mean values $\frac{1}{2}(t_{r-1} + t_r)$ for $1 \leqslant r \leqslant 9$, $\frac{1}{3}(t_9 + 2s)$ correspond to the values of $b\sigma$ for which the functions have been tabulated,

(c) to evaluate (20) for suitable values of $y = b\eta$, remembering that $\frac{w_1'}{K}$ depends on $b\xi = x - R(t) - \{l(t) - \frac{1}{2}\} c(t)$ as in (18). For the N.P.L. 9' x 7' tunnel, W may be determined directly from Tables 5(a), 5(c).

7. Conclusions

It is discovered that an octagonal tunnel produces considerably more upwash than a rectangular tunnel near the wall, where the part-wing is mounted, but that the difference is less marked in the centre of the tunnel. Although the influence of the triangular fillets on the distribution of lift near the root of the part-wing is appreciable, the overall effect on the total rolling moment due to an aileron deflection is trivial. This has already been demonstrated in Ref. 1, Fig. 8. It is concluded that the interference of corner fillets can be ignored as far as part-wing tests on ailerons are concerned, but that it becomes appreciable if the conditions near the root section are under investigation.

It is found that the chordwise variation of upwash angle is practically linear, and that for all purposes of interference correction linearity may be assumed. The total upwash angle along any chord may then be represented as a local uniform incidence together with a superposed curvature of flow, which facilitate an accurate estimation of the wind tunnel interference.

References

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2.	R. S. Swanson and T. A. Toll.	"Jet-Boundary Corrections for Reflection-Plane Models in Rectangular Wind-Tunnels." - A.R.C. 7136.

No.	Author	Title
3.	G. K. Batchelor.	"Interference in a Wind Tunnel of Octagonal Section." - Australian Council for Aeronautics. Report ACA - 1.
4.	H. C. Garner	"Note on Interference in a Wind-Tunnel of Octagonal Section." - A.R.C. 6659.
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APPENDIX

Convergence of $\chi(S, T)$

Since the series (11) does not converge rapidly when $i = 0$ and η is small or when $i = -1$ and η is nearly $\frac{1}{2}$, it was considered necessary to examine the convergence of the expression for $\chi(S, T)$ by means of an alternative calculation.

In the notation of Fig. 6 of Ref. 3, the velocity due to the vorticity round square 4 is given by

$$\frac{4\pi}{K}(v + i\,u) = -4az_0 + 2a^2(1+i) + z_0(i-1)(z_0 - a - ia)\log_e\left\{1 - \frac{a(1+i)}{z_0}\right\}$$

$$+(1+i)(z_0 - a)(z_0 - ia)\log_e\frac{z_0 - ia}{z_0 - a} \qquad \ldots\ldots\ldots(21)$$

Combine this with the velocity due to square 1, which is of opposite sign and is given by a similar equation replacing

z_0 by $z_1 = z_0 + \frac{h}{\sqrt{2}}(i - 1)$

At a point along the bound vortex

$$z_0 = p(1 + i) + q(1 - i)$$

$$z_1 = p(1 + i) - q(1 - i)$$

where
$$p = \frac{1}{2\sqrt{2}}(a\sqrt{2} + b - 2b\eta)$$

$$q = \frac{1}{2\sqrt{2}}(h)$$

It follows from (21) and a similar equation that due to squares 4 and 1,

$$\frac{4\pi}{K}(v + iu) = (1 - i)\Big[-8aq - 4\{p(p - a) - q^2\}\phi$$

$$-2(2pq - aq)\log_e R + 4(2pq - aq)\psi$$

$$-\{2p(p - a) - 2q^2 + a^2\}\log_e S\Big] \qquad \ldots\ldots\ldots(22)$$

where/

$$\left.\begin{aligned}
\text{where } \tan\phi &= \frac{aq}{p(p-a)+q^2} \\
R &= \frac{\{p(p-a)+q^2\}^2 + (aq)^2}{(p^2+q^2)^2} \\
\tan\psi &= \frac{2ap - a^2}{2\{p(p-a)+q^2\}} \\
S &= \frac{2p^2 - 2ap + 2q^2 + a^2 + 2aq}{2p^2 - 2ap + 2q^2 + a^2 - 2aq}
\end{aligned}\right\}$$

$$\text{Then } \frac{4\pi \quad w\sqrt{2}}{K} = \frac{4\pi}{K}(v-u) = (1+i)\frac{4\pi}{K}(v+iu)$$

$$= 2\left[-8aq - 4\{p(p-a) - q^2\}\phi - 2(2pq - aq)\log_e R\right.$$

$$\left. + 4(2pq - aq)\psi - \{2p(p-a) - 2q^2 + a^2\}\log_e S\right] = 2B, \text{ say.} \qquad \text{........(23)}$$

Hence the upwash arising from the double infinity of squares is given by

$$\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty} \frac{4\pi \quad w\sqrt{2}}{K}$$

$$\text{where } p = \frac{1}{2\sqrt{2}}\{a\sqrt{2} - 2b\eta + (2m-1)b\}$$

$$q = \frac{1}{2\sqrt{2}}(2n-1)h$$

$\therefore$ An alternative form for $\chi(S, T)$, when $\theta = \dfrac{\pi(kb + a\sqrt{2})}{2h} \pm \dfrac{\pi b\eta}{h}$

$$\text{is } \frac{2\sqrt{2}}{a^2}\sum_{n=1}^{\infty}(-1)^{n-1}B \quad (\text{see } (11)) \qquad \text{........(24)}$$

where/

where $p = \frac{}{2\sqrt{2}}(a\sqrt{2} \pm 2 b \eta + kb)$

$$q = \frac{1}{2\sqrt{2}}(2n - 1) h.$$

For the central fillets k is even and for the corner ones k is odd.

Expanding (23) in powers of $\frac{a}{q}$,

$$B = - \frac{a^2}{15}\left(\frac{a}{q}\right)^3 + 0\left(\frac{a}{q}\right)^5 \approx - \frac{a^2}{15}\left(\frac{a}{h}\right)^3 \frac{(2\sqrt{2})^3}{(2n - 1)^3}$$

if n is large enough,

For the values of η for which the series (11) converges too slowly, it is accurate enough to take

$$\chi(S, T) = \frac{2\sqrt{2}}{a^2} \sum_{n=1}^{4} (-1)^{n-1} B - \frac{64}{15}\left(\frac{a}{h}\right)^3 \sum_{n=5}^{\infty} \frac{(-1)^{n-1}}{(2n - 1)^3}. \qquad \ldots\ldots\ldots(25)$$

Values of $\chi(S, T)$ when $b = 18$, $h = 7$, $a = \sqrt{6}$ calculated from (25) are compared in Table 3(a) with the values obtained from (11). When $i = 0$, appreciable errors will arise due to the slow convergence of (11), when

$\eta \leqslant \frac{1}{8}$, and it is necessary to use the alternative formula (25). The final values of $\chi(S_i\ T_i)$ are tabulated completely in Table 3(b).

Table 1.

(a) Central fillet AB

	$\frac{r}{a}$	$\frac{2bV_1\sqrt{2}}{\Gamma}$	$\frac{2bV_2\sqrt{2}}{\Gamma}$
A	0	-0.302	+0.149
	1/8		+0.226
	1/4	-0.170	+0.182
	3/8		+0.098
	1/2	-0.016	0
	5/8		-0.098
	3/4	0.162	-0.182
	7/8		-0.226
B	1	0.364	-0.149

(b) Corner fillet CD

	$\frac{r}{a}$	$\frac{2bV_1\sqrt{2}}{\Gamma}$	$\frac{2bV_2\sqrt{2}}{\Gamma}$
C	0	-2.225	0.883
	1/8		1.344
	1/4	-0.954	1.079
	3/8		0.583
	1/2	+0.149	0
	5/8		-0.583
	3/4	1.022	-1.079
	7/8		-1.344
D	1	1.659	-0.883

Table 2/

Table 2.

Values of $\Lambda'_i = \Lambda$, when i is even
$= \Lambda'$, when i is odd

32σ	Central fillets Λ	Corner fillets Λ'
1	0.85922	0.07151
2	1.55687	0.14756
3	1.93392	0.23279
4	1.97585	0.33236
5	1.79843	0.45206
6	1.52868	0.59801
7	1.24774	0.77670
8	0.99320	0.99320
9	0.77670	1.24774
10	0.59801	1.52868
11	0.45206	1.79843
12	0.33236	1.97585
13	0.23279	1.93392
14	0.14756	1.55687
15	0.07151	0.85922

Table 3(a)

32η	χ from equation (11)	χ from equation (25)
0		-0.17983
1	-0.14717	-0.15171
2	-0.09573	-0.08758
3	-0.02968	-0.02353
4	+0.01801	+0.02018
5	+0.04167	+0.04181
6	+0.04836	+0.04799
7	+0.04603	+0.04570
8	+0.04005	+0.03978
9	+0.03321	+0.03306
10	+0.02681	+0.02672
11	+0.02131	+0.02125
12	+0.01678	+0.01672

Table 3(b)/

Table 3(b)

Values of $\chi(S_i, T_i)$

32η	i = - 2	i = - 1	i = 0	i = + 1	i = + 2
0	0.00011	0.00622	-0.17983	0.00622	0.00011
1	0.00014	0.00799	-0.15171	0.00484	0.00009
2	0.00018	0.01025	-0.08758	0.00376	0.00007
3	0.00023	0.01314	-0.02353	0.00292	0.00005
4	0.00030	0.01675	+0.02018	0.00227	0.00004
5	0.00039	0.02125	0.04181	0.00176	0.00003
6	0.00050	0.02672	0.04799	0.00137	0.00002
7	0.00064	0.03306	0.04570	0.00106	0.00002
8	0.00083	0.03978	0.03978	0.00083	0.00001
9	0.00106	0.04570	0.03306	0.00064	0.00001
10	0.00137	0.04799	0.02672	0.00050	0.00001
11	0.00176	0.04181	0.02125	0.00039	0.00001
12	0.00227	+0.02018	0.01675	0.00030	0.00001
13	0.00292	-0.02353	0.01314	0.00023	0.00000
14	0.00376	-0.08758	0.01025	0.00018	0.00000
15	0.00484	-0.15171	0.00799	0.00014	0.00000

Table 4(a)/

Table 4(a)

Interference on a half-wing in a rectangular 9' × 7' tunnel (without fillets)

$$\frac{w_1}{K} = 0.0044210 \left(\frac{4\pi.bw_1}{X}\right) \text{ (of dimensions ft.}^{-1}\text{)}$$

	23 = 0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8
2σ = 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1/16	0.00301	0.00294	0.00276	0.00248	0.00213	0.00175	0.00138	0.00103	0.00072	0.00046	0.00027	0.00012	0.00003	-0.00001	+0.00000
1/8	0.00588	0.00576	0.00542	0.00489	0.00423	0.00351	0.00278	0.00210	0.00149	0.00099	0.00059	0.00030	0.00011	+0.00003	0.00006
3/16	0.00852	0.00836	0.00789	0.00717	0.00627	0.00526	0.00423	0.00325	0.00236	0.00161	0.00102	0.00058	0.00030	0.00018	0.00021
1/4	0.01084	0.01065	0.01011	0.00927	0.00820	0.00699	0.00572	0.00449	0.00337	0.00239	0.00161	0.00102	0.00064	0.00048	0.00052
5/16	0.01278	0.01259	0.01203	0.01114	0.00999	0.00866	0.00725	0.00584	0.00453	0.00336	0.00240	0.00167	0.00120	0.00099	0.00106
3/8	0.01435	0.01416	0.01362	0.01275	0.01161	0.01026	0.00879	0.00728	0.00584	0.00453	0.00342	0.00257	0.00201	0.00178	0.00188
7/16	0.01554	0.01537	0.01488	0.01408	0.01301	0.01173	0.01029	0.00878	0.00729	0.00590	0.00471	0.00377	0.00315	0.00290	0.00308
1/2	0.01640	0.01626	0.01584	0.01514	0.01420	0.01304	0.01172	0.01029	0.00884	0.00746	0.00625	0.00528	0.00466	0.00445	0.00474
9/16	0.01698	0.01686	0.01652	0.01596	0.01518	0.01420	0.01305	0.01178	0.01047	0.00919	0.00804	0.00714	0.00658	0.00650	0.00699
5/8	0.01733	0.01724	0.01698	0.01655	0.01595	0.01518	0.01426	0.01322	0.01213	0.01105	0.01008	0.00934	0.00897	0.00912	0.00996
11/16	0.01751	0.01745	0.01727	0.01698	0.01656	0.01601	0.01536	0.01460	0.01380	0.01302	0.01235	0.01191	0.01188	0.01244	0.01384
3/4	0.01757	0.01754	0.01744	0.01728	0.01704	0.01673	0.01636	0.01593	0.01550	0.01510	0.01486	0.01488	0.01538	0.01660	0.01891
13/16	0.01757	0.01756	0.01754	0.01751	0.01745	0.01739	0.01731	0.01725	0.01723	0.01733	0.01764	0.01832	0.01960	0.02185	0.02564
7/8	0.01756	0.01757	0.01763	0.01772	0.01785	0.01803	0.01828	0.01861	0.01909	0.01977	0.02080	0.02236	0.02479	0.02865	0.03500

Table 4(b)/

Table 4(b)

Interference on a Half Wing in the N.P.L. 9' × 7' Tunnel.

$$\frac{w_3}{K} = \frac{w_1}{K} + \frac{w_2}{K} = 0.0044210\left(\frac{4\pi b w_1}{K}\right) + 0.005111\left(\frac{b w_2}{0.092K}\right) \text{(of dimensions ft.}^{-1}\text{)} \quad \text{Values of } \frac{w_3}{K}$$

2σ	$2\eta = 0$	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8
1/16	0.00380	0.00361	0.00315	0.00259	0.00205	0.00158	0.00118	0.00084	0.00056	0.00033	0.00016	0.00004	-0.00004	-0.00009	-0.00009
1/8	0.00732	0.00698	0.00612	0.00509	0.00408	0.00319	0.00242	0.00176	0.00120	0.00075	0.00040	0.00015	-0.00002	-0.00011	-0.00012
3/16	0.01031	0.00987	0.00877	0.00742	0.00609	0.00487	0.00378	0.00283	0.00201	0.00133	0.00080	0.00040	+0.00014	-0.00001	-0.00003
1/4	0.01267	0.01220	0.01102	0.00953	0.00802	0.00660	0.00528	0.00408	0.00303	0.00213	0.00140	0.00086	0.0004	+0.00028	+0.00023
5/16	0.01446	0.01401	0.01286	0.01139	0.00985	0.00833	0.00687	0.00550	0.00425	0.00315	0.00225	0.00156	0.00107	0.00079	0.00072
3/8	0.01579	0.01538	0.01434	0.01298	0.01150	0.01000	0.00849	0.00702	0.00564	0.00440	0.00335	0.00252	0.00193	0.00158	0.00150
7/16	0.01673	0.01639	0.01549	0.01429	0.01296	0.01155	0.01009	0.00862	0.00719	0.00587	0.00472	0.00379	0.00311	0.00271	0.00264
1/2	0.01737	0.01709	0.01635	0.01534	0.01420	0.01295	0.01162	0.01023	0.00884	0.00753	0.00635	0.00538	0.00467	0.00425	0.00423
9/16	0.01777	0.01755	0.01696	0.01615	0.01522	0.01418	0.01304	0.01182	0.01057	0.00935	0.00824	0.00731	0.00664	0.00628	0.00638
5/8	0.01797	0.01781	0.01736	0.01675	0.01604	0.01523	0.01433	0.01335	0.01232	0.01130	0.01037	0.00960	0.00908	0.00889	0.00924
11/16	0.01804	0.01792	0.01760	0.01718	0.01668	0.01613	0.01550	0.01481	0.01408	0.01337	0.01273	0.01225	0.01202	0.01219	0.01301
3/4	0.01800	0.01793	0.01773	0.01748	0.01720	0.01689	0.01656	0.01621	0.01584	0.01552	0.01530	0.01527	0.01556	0.01634	0.01800
13/16	0.01791	0.01787	0.01778	0.01769	0.01762	0.01756	0.01753	0.01753	0.01759	0.01775	0.01809	0.01871	0.01978	0.02160	0.02476
7/8	0.01779	0.01779	0.01781	0.01786	0.01799	0.01818	0.01846	0.01885	0.01938	0.02012	0.02116	0.02268	0.02494	0.02845	0.03430

Table 5(a)/

Table 5(a)

Interference at the Bound Vortex on a Half-wing in the N.P.L. 9' x 7' Tunnel.

Values of $\frac{w_3}{K}$ at exact multiples of half a foot from the tunnel walls (Distance from the wall is 18σ or 18η ft.)

18σ	18η = 0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
0.5	0.00339	0.00325	0.00291	0.00247	0.00203	0.00163	0.00127	0.00097	0.00071	0.00049	0.00031	0.00017	0.00006	-0.00001	-0.00006	- 0.00008
1.0	0.00658	0.00633	0.00568	0.00486	0.00403	0.00326	0.00258	0.00199	0.00148	0.00104	0.00068	0.00039	0.00018	+0.00002	-0.00008	- 0.00012
1.5	0.00938	0.00905	0.00819	0.00709	0.00596	0.00491	0.00396	0.00311	0.00236	0.00172	0.00117	0.00074	0.00046	0.00015	-0.00000	- 0.00008
2.0	0.01170	0.01133	0.01037	0.00912	0.00781	0.00656	0.00540	0.00434	0.00339	0.00255	0.00183	0.00123	0.00077	0.00043	+0.00021	+ 0.00010
2.5	0.01353	0.01316	0.01219	0.01090	0.00953	0.00818	0.00690	0.00568	0.00456	0.00354	0.00266	0.00191	0.00132	0.00088	0.00058	0.00043
3.0	0.01495	0.01460	0.01368	0.01245	0.01110	0.00974	0.00839	0.00709	0.00584	0.00469	0.00366	0.00278	0.00207	0.00153	0.00116	0.00097
3.5	0.01603	0.01571	0.01488	0.01376	0.01250	0.01118	0.00985	0.00852	0.00721	0.00597	0.00484	0.00385	0.00304	0.00240	0.00197	0.00174
4.0	0.01682	0.01655	0.01583	0.01484	0.01370	0.01249	0.01123	0.00994	0.00863	0.00737	0.00618	0.00513	0.00424	0.00354	0.00305	0.00280
4.5	0.01737	0.01715	0.01654	0.01570	0.01472	0.01365	0.01251	0.01131	0.01007	0.00884	0.00767	0.00659	0.00569	0.00495	0.00444	0.00420
5.0	0.01773	0.01755	0.01706	0.01637	0.01555	0.01465	0.01367	0.01261	0.01150	0.01037	0.00927	0.00825	0.00735	0.00667	0.00619	0.00600
5.5	0.01794	0.01780	0.01741	0.01686	0.01624	0.01550	0.01470	0.01383	0.01289	0.01193	0.01097	0.01008	0.00931	0.00871	0.00834	0.00828
6.0	0.01803	0.01792	0.01763	0.01722	0.01675	0.01621	0.01561	0.01495	0.01424	0.01350	0.01276	0.01207	0.01150	0.01110	0.01093	0.01113
6.5	0.01803	0.01795	0.01775	0.01747	0.01715	0.01680	0.01642	0.01599	0.01553	0.01506	0.01460	0.01421	0.01394	0.01385	0.01405	0.01468
7.0	0.01796	0.01792	0.01780	0.01764	0.01748	0.01731	0.01713	0.01696	0.01677	0.01661	0.01650	0.01649	0.01663	0.01702	0.01777	0.01910
7.5	0.01787	0.01785	0.01780	0.01776	0.01774	0.01775	0.01779	0.01786	0.01799	0.01818	0.01847	0.01893	0.01962	0.02068	0.02226	0.02475

Table 5(b)/

Table 5(b)

Interference on a half wing in a rectangular 9' x 7' Tunnel.

Values of w_1'/K (of dimensions ft.$^{-1}$) at points 1' and 2' downstream of bound vortex.

18σ	18η = 0	18η = 2.5	18η = 4.5	18η = 6.0	18η = 7.0
			1 ft. downstream		
1.5	0.002345	0.001650	0.000797	0.000397	0.000266
2.5	0.003613	0.002705	0.001446	0.000782	0.000553
3.0	0.004124	0.003200	0.001818	0.001036	0.000756
3.5	0.004554	0.003662	0.002221	0.001340	0.001014
4.0	0.004905	0.004085	0.002651	0.001700	0.001337
5.0	0.005412	0.004797	0.003555	0.002596	0.002222
6.0	0.005721	0.005336	0.004464	0.003731	0.003521
6.5	0.005826	0.005552	0.004908	0.004392	0.004376
7.0	0.005911	0.005742	0.005349	0.005126	0.005417
7.5	0.005983	0.005915	0.005794	0.005956	0.006723
			2 ft. downstream		
1.5	0.004409	0.003112	0.001510	0.000753	0.000507
2.5	0.006799	0.005099	0.002734	0.001483	0.001053
3.0	0.007765	0.006031	0.003436	0.001966	0.001441
3.5	0.008576	0.006900	0.004197	0.002542	0.001932
4.0	0.009242	0.007697	0.005005	0.003223	0.002545
5.0	0.010201	0.009041	0.006708	0.004919	0.004225
6.0	0.010787	0.010060	0.008428	0.007064	0.006681
6.5	0.010987	0.010470	0.009265	0.008318	0.008289
7.0	0.011148	0.010831	0.010101	0.009698	0.010237
7.5	0.011286	0.011164	0.010947	0.011261	0.012651

Table 5(c)/

Table 5(c)

Estimated chordwise variation of upwash velocity due to interference on a half-wing in the N.P.L. 9' x 7' Tunnel.

Values of $\frac{w_3'}{K} \cdot \frac{1}{b\xi}$ (of dimensions (ft.)$^{-2}$)

$$= \frac{w_3}{w_1} \times \frac{w_1'}{K} \cdot \frac{1}{b\xi}$$

where $b\xi$ is the distance downstream of the bound vortex and $b = 18$ ft.

18σ	$18\eta = 0$	$18\eta = 2.5$	$18\eta = 4.5$	$18\eta = 6.0$	$18\eta = 7.0$
0.5	0.001068	0.000514	0.000181	(0.00012)*	(-0.00008)*
1.0	0.002056	0.001048	0.000390	0.000171	(-0.00002)*
1.5	0.002930	0.001590	0.000643	0.000294	(+0.00008)*
2.0	0.003658	0.002132	0.000962	0.000474	(0.00023)*
2.5	0.004238	0.002668	0.001345	0.000711	0.000435
3.0	0.004709	0.003188	0.001762	0.001002	0.000681
3.5	0.005088	0.003677	0.002212	0.001350	0.000972
4.0	0.005380	0.004130	0.002688	0.001750	0.001319
4.5	0.005605	0.004512	0.003175	0.002200	0.001736
5.0	0.005780	0.004891	0.003676	0.002712	0.002237
5.5	0.005915	0.005193	0.004172	0.003283	0.002844
6.0	0.006028	0.005465	0.004659	0.003910	0.003581
6.5	0.006103	0.005697	0.005135	0.004595	0.004461
7.0	0.006158	0.005900	0.005586	0.005348	0.005532
7.5	0.006205	0.006074	0.006024	0.006185	0.006869

*The values in brackets were obtained by extrapolation, since the values of $\frac{w_3}{w_1}$ could not be determined easily.

PI.

ADDENDUM

In presenting the following note as an A.R.C. Current Paper it is necessary to explain that the basic method of treating the boundary conditions at the wall of an octagonal wind tunnel is due to Batchelor (Ref.3), and that Ref.4 is a generalization of Ref.3 used to compute the interference on complete wings placed symmetrically in the N.P.L. 9 ft. × 7 ft. and 13 ft. × 9 ft. wind tunnels.

The chordwise variation of upwash is not included in Ref.4 and the mean corrections to incidence and drag based on an elliptic distribution of lift are given in the form

$$(\Delta\alpha) = \frac{\delta}{2}\frac{S}{C} C_L$$

$$(\Delta C_D) = \frac{\delta}{2}\frac{S}{C} C_L^2 ,$$

where C is the sectional area of the tunnel and S is the area of the plan form of the wing.

To give an idea of the numerical significance of corner fillets, Figs. 6 and 7 of Ref.4 are reproduced here as Fig.4. For both tunnels δ is compared with δ_R for the corresponding rectangular tunnel of area hb and with

$$\Delta_R \quad \delta_R \cdot \frac{C}{hb} ,$$

where $C = hb - a^2$.

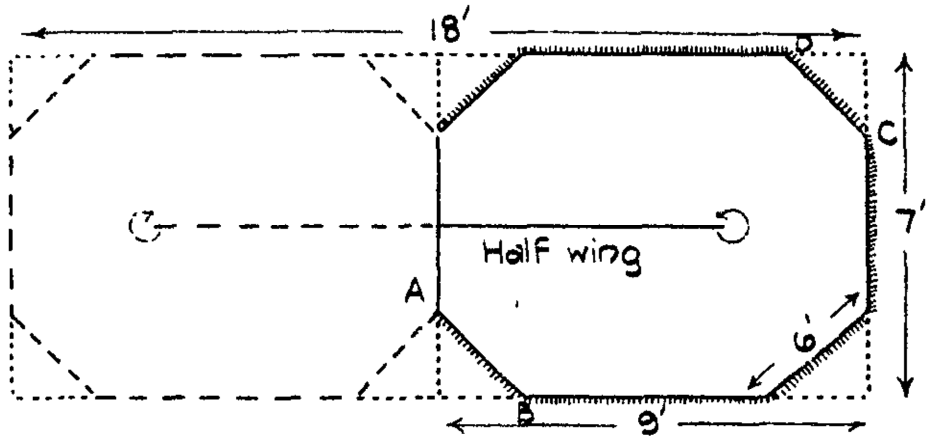
It is recommended that, for wings of moderate span $2s < 0.6b$, it is accurate enough to take

$$\frac{\delta}{\delta_R} = \frac{hb + C}{2hb}$$

and that the same ratio should be used for the chordwise variation of upwash.

The following note is an extension of the same basic method to the problem of a part-wing mounted symmetrically on one wall of an octagonal tunnel.

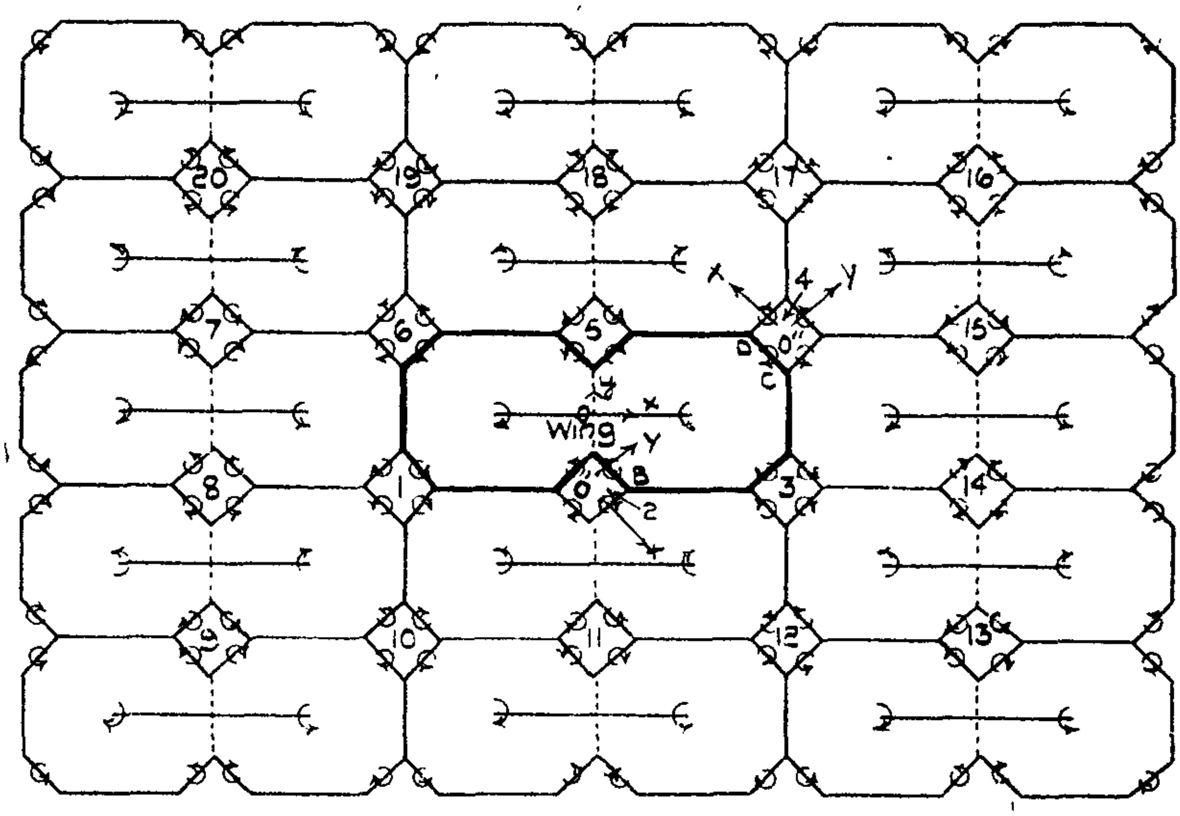
Fig 1



A B = central fillet
C D = corner fillet

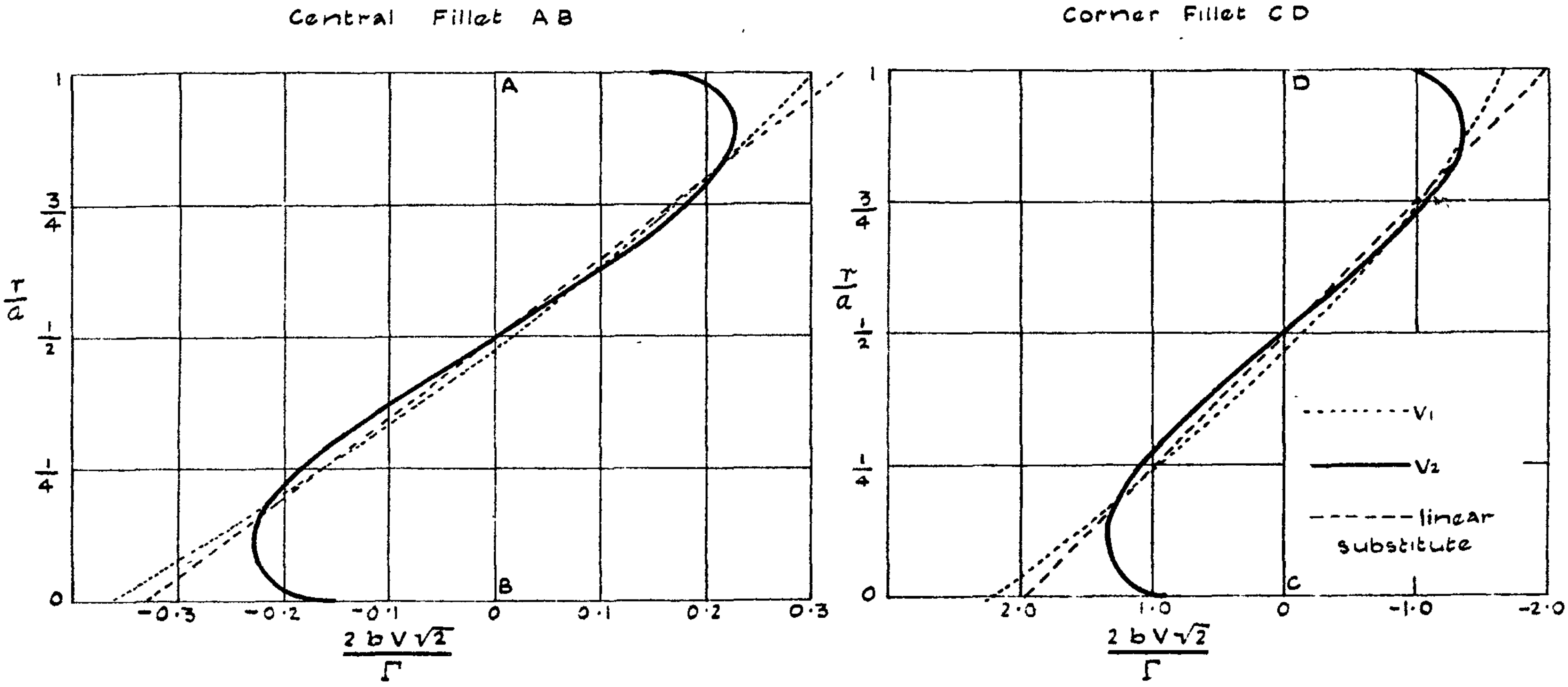
Equivalent 18' x 7' tunnel.

Fig 2



Complete system of images

"Interference on a Part-Wing with Aileron in a Wind Tunnel of Octagonal Section."

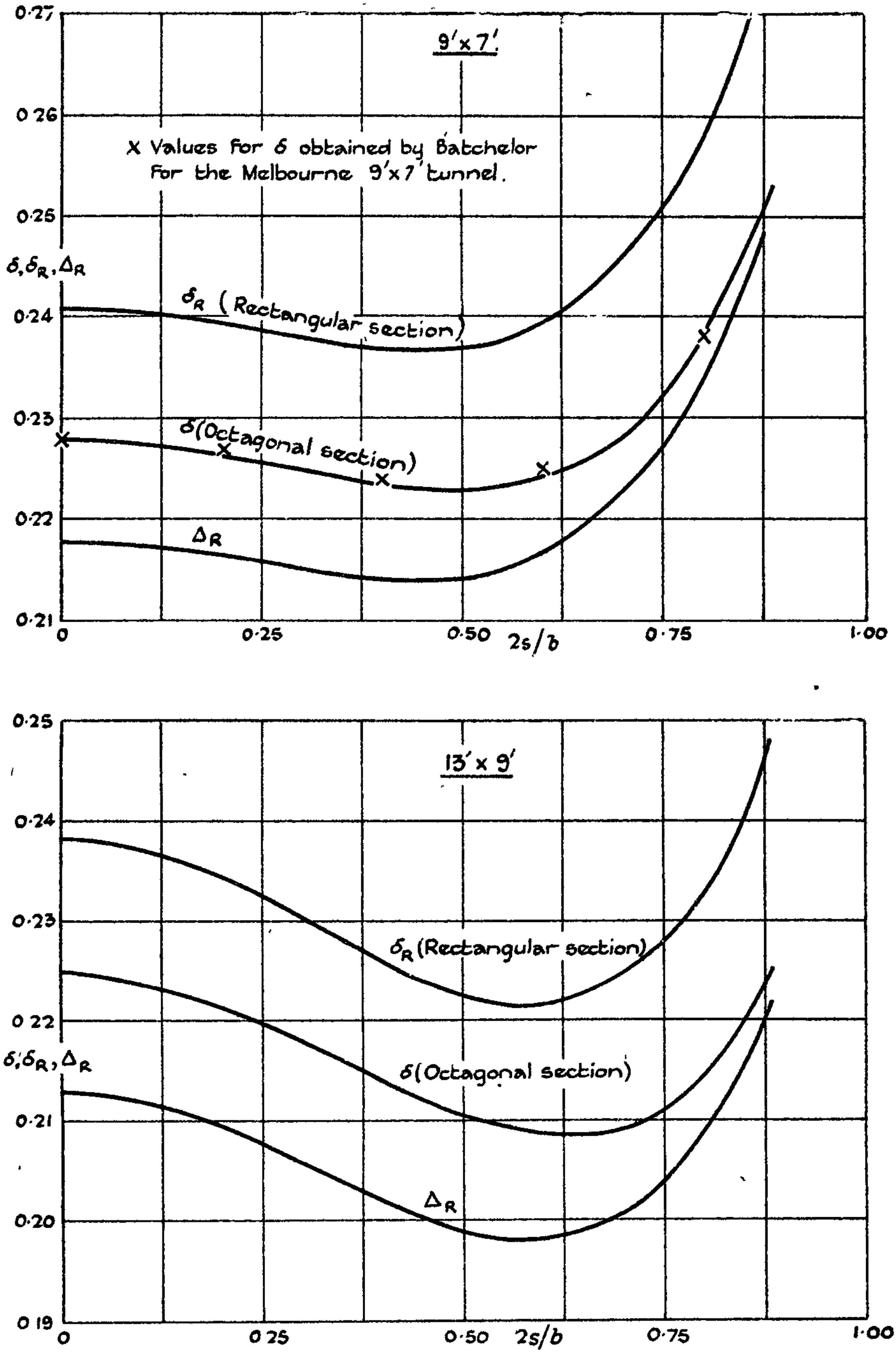


18 ft. x 7 ft. Octagonal tunnel with isosceles fillets of length √6 ft.
Distribution of normal velocity for uniformly loaded wing of span 13.5 ft.

10413. Fig 3.

M.C.

Fig. 4.



Interference factors for N.P.L. 9' x 7' tunnels and 13' x 9' tunnel against wing span.

E.E.M

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(10413)
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