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H. H. B. M. THOMAS, B.Sc. and M. LOFTS, B.A.

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1. *Summary and Introduction.*—Recent experimental investigations on small-chord controls in two-dimensional flow suggest that such controls are more efficient than wide-chord controls. The experiments also suggest that a further gain is obtained if the control or flap is broken, hinged and geared at some point along its chord.

This Note examines, on the basis of the thin aerofoil theory, the control efficiency of such double flap systems, as ailerons and as elevators. A range of values of total chord ratio is covered, and the optimum arrangement determined in each case.

The theory suffers from the limitations of the thin aerofoil theory, which fails to take account of the thickness/chord ratio and the boundary layer effects; these can be large for the thicker sections. It does, however, provide an indication of the effect of variations of the various parameters and also the ratio of the flap chords defining the optimum.

In general terms the problem considered here is to find the minimum control column force to produce a given rolling moment. Throughout the present work the lift is fixed at that produced by the 0·50 chord flap. It is shown that the smaller chord single- and double-flap systems are more efficient than a wide-chord arrangement and that a double flap is more efficient than a single flap of the same total chord.

2. *Method.*—In order to clarify the problem and avoid confusion the hinge moment is expressed in terms of a coefficient based on the moment at the control column, wing area and mean chord. This hinge moment coefficient is denoted by C_{H_s} and can be expressed in terms of the incidence and the control column deflection thus :—

$$C_{H_s} = B_1\alpha + B_2\,\delta_s$$

The expressions $-\dfrac{B_1}{A_1}$, $-\dfrac{B_2}{A_2}$, and $-(B_2 - \dfrac{1}{5}B_1)$ in the case of the ailerons, are measures of the control efficiency of a single- or double-flap system. They are evaluated for different values of the total and rear flap chords and of the gearing, using the thin aerofoil theory which is developed for a multiple-hinged flap system by Perring in R. & M. 1171[1].

*R.A.E. Techl. Note Aero. 1421, received 16th May, 1944.

Writing the coefficients in the forms given below,

$$C_{H_s} = B_1 \alpha + B_2 \delta_s ,$$
$$C_L = A_1 \alpha + A_2 \delta_s ,$$

we have

$$C_{H_s} = B_1 \alpha + B_2 \left(\frac{C_L}{A_2} - \frac{A_1}{A_2} \alpha\right)$$
$$= B_1 \left(\frac{C_L}{A_1} - \frac{A_2}{A_1} . \delta_s\right) + B_2 \delta_s ,$$

so that $$\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{\alpha = const.} = \frac{B_2}{A_2}$$

and $$\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{\delta_s = const.} = \frac{B_1}{A_1} .$$

Including response the moment which must be balanced by the control column force is equal to $\left(B_2 - \frac{1}{5} B_1\right) \delta_s$ and therefore for unit control column deflection is proportional to $\left(B_2 - \frac{1}{5} B_1\right)$. $-\frac{B_1}{A_1}$ is of importance in the design of elevators. $-\frac{B_2}{A_2}$ is a measure of the heaviness of controls, not including response, and is not of great importance. $-\left(B_2 - \frac{1}{5} B_1\right)$ is a measure of the control column force for a pair of ailerons (including response).

Consider a small displacement $d\delta_1$, $d\delta_2$, $d\delta_s$, from equilibrium. If the rear flap is geared by a link earthed to a fixed point of the aerofoil, the force in the link does no work during the displacement. The equation of virtual work is, therefore,

$$H_s d\delta_s = H_1 d\delta_1 + H_2 d\delta_2 ,$$

or in coefficient form

$$C_{H_s} = \left\{ \left(\frac{c_1}{c}\right)^2 . C_{H_1} + \left(\frac{c_2}{c}\right)^2 C_{H_2} \frac{d\delta_2}{d\delta_1} \right\} \frac{d\delta_1}{d\delta_s}$$
$$= \{E_1^2 . C_{H_1} + n E_2^2 . C_{H_2}\} . m , \qquad (1)$$

where $\frac{d\delta_2}{d\delta_1}$ is the gearing (n) of the double movement of the flap, and $\frac{d\delta_1}{d\delta_s}$ is the gearing (m) between the main flap and the control column.

From Ref. 1 it may be deduced that

$$C_{H_1} = - \beta_1 C_L - 2 (b_{11} + n b_{12}) \delta_1 , \qquad (2)$$
$$C_{H_2} = - \beta_2 C_L - 2 (b_{21} + n b_{22}) \delta_1 , \qquad (3)$$
$$C_L = a_1 \{\alpha + (\lambda_1 + n \lambda_2) \delta_1\} , \qquad (4)$$

where β_1, β_2, λ_1, λ_2, b_{11}, b_{12}, b_{21}, b_{22} are given in R. & M. 1171 in terms of parameters ϕ_1, ϕ_2, where

$$\cos \phi_1 = - (1 - 2E_1) \text{ and } \cos \phi_2 = - (1 - 2E_2).$$

The condition under which the control efficiencies are compared is that the lift produced per unit control column deflection at a given incidence is constant.

This gives $$(\lambda_1 + n\lambda_2) \frac{\delta_1}{\delta_s} = \text{constant},$$

i.e., $$m (\lambda_1 + n\lambda_2) = \text{constant}. \qquad (5)$$

From equations 1, 2, 3 and 4, it follows that

$$-\frac{B_1}{A_1} = -\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{\delta_s = const.} = m\,(E_1{}^2\,\beta_1 + nE_2{}^2\,\beta_2)\,,$$

$$-\frac{B_2}{A_2} = -\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{a = const.}$$

$$= m\left\{(E_1{}^2\beta_1 + nE_2{}^2\beta_2) + \frac{2}{a_1(\lambda_1+n\lambda_2)}\left[E_1{}^2(b_{11} + b_{12}n) + nE_2{}^2(b_{21} + nb_{22})\right]\right\}$$

and

$$-\left(B_2 - \frac{1}{5}B_1\right) = \Big\{m^2a_1\,(\lambda_1 + n\lambda_2)\,(E_1{}^2\beta_1 + nE_2{}^2\beta_2) + 2m^2\Big[E_1{}^2(b_{11} + nb_{12})$$

$$+ nE_2{}^2\,(b_{21} + nb_{22})\Big] - \frac{1}{5}ma_1\,(E_1{}^2\beta_1 + nE_2{}^2\beta_2)\Big\}\,.$$

Constant values of E_1 are taken and the expressions

$$-\frac{B_1}{A_1}\,,\ -\frac{B_2}{A_2}\,,\ -\left(B_2 - \frac{1}{5}\,B_1\right)$$

are evaluated for various values of E_2 and are plotted against E_2. We require the values of E_2 which give the minima of these expressions, subject to conditions given by equation 5 for a given value of E_1. In the calculations made here the constant of equation 5 is the lift produced per unit control column deflection by a single flap of 0·5 chord.

3. *Conclusions.*—From the curves obtained for $-B_1/A_1$, $-B_2/A_2$, $-(B_2 - \frac{1}{5}B_1)$ (see Figs. 1, 2, 3), it is seen that :—

(1) The smaller chord flaps working through larger angles are more efficient than the wide-chord flap.

(2) For a given total flap chord there is a gain in passing from a single- to a double-flap. This is not true for all three criteria when the rear flap is of very small chord.

(3) There is some increase of efficiency with increase of gearing of rear flap to forward flap.

(4) For any given E_1, the value of E_2 which gives a rough minimum of $-B_1/A_1$ is such that $E_2/E_1 = 0{\cdot}4$ when $n = 1$, and $E_2/E_1 = 0{\cdot}35$ when $n = 2$.

(5) For any given E_1, the values of E_2 which give a rough minimum of $-B_2/A_2$ is such that

$E_2/E_1 = 0{\cdot}7$ when $n = 1$

and $E_2/E_1 = 0{\cdot}63$ when $n = 2$.

(6) For any given E_1, the value of E_2 which gives a rough minimum of $-(B_2 - \frac{1}{5}B_1)$

is such that $E_2/E_1 = 0{\cdot}73$ when $n = 1$

and $E_2/E_1 = 0{\cdot}65$ when $n = 2$.

These conclusions may have to be modified for thick sections when the boundary layer effects on the lift and hinge moment are appreciable. They are, however, applicable to thin sections and can be taken as an indication of what gain to expect in passing over to small-chord flaps.

The experimental investigations mentioned above refer to a thin section and for this section the agreement between the theoretical and experimental results is quite good.

Assuming the results of the experimental work and of the present investigation are approximately correct for all sections it appears that the optimum arrangement is formed by a flap broken at about its quarter-chord point if the control is unbalanced. In application of the scheme it is still necessary to balance the system. It may, therefore, prove a more practical scheme to break the chord at a point further aft and to use the larger forward portion thus formed to house an internally sealed type of balance.

NOTATION

c_1, c_2, c are respectively the chords of the total flap, the rear flap and the aerofoil

$E_1 = \dfrac{c_1}{c}$ and $E_2 = \dfrac{c_2}{c}$

H_1, H_2, H_s are the hinge moments of the total flap, the rear flap, and the control column each about its own hinge

δ_1, δ_2, δ_s are the angular deflections of the total flap, the rear flap and the control column.

n = rear to forward flap gearing = δ_2/δ_1 (if linear)

m = forward flap solidus control column gearing = δ_1/δ_s (if linear)

$B_1 = \partial C_{Hs}/\partial \alpha$.

$B_2 = \partial C_{Hs}/\partial \delta_s$.

$A_1 = \partial C_L/\partial \alpha$.

$A_2 = \partial C_L/\partial \delta_s$.

β_1, β_2, λ_1, λ_2, b_{11}, b_{12}, b_{21}, b_{22} are as defined in R. & M. 1171[1].

REFERENCE

No.	*Author.*	*Title, etc.*
1	Perring	The Theoretical Relationships for an Aerofoil with a Multiply Hinged Flap System. R. & M. 1171, April, 1928.

APPENDIX

The text describes the results of the investigation using graphical methods which are made necessary by the complication of any analytical treatment.

In the case of the quantities $\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{a\,=\,const.}$ and $\left(B_2 - \frac{1}{5}B_1\right)$, to attempt an analytical solution seems too laborious. The evaluation of the conditions defining the optimum arrangement for the criterion $\left(\frac{\partial C_{H_s}}{\partial C_L}\right)_{\delta_s\,=\,const.}$ as a minimum is somewhat simpler and an approximate solution is possible.

Using the notation of the text we have to find the minimum of

$$C_{H_s} = m\,(C_{H1}E_1{}^2 + nC_{H2}E_2{}^2)$$

subject to the condition

$$ma_1\,(\lambda_1 + n\lambda_2) = \text{constant} = C,$$

where

$$C_{H1} = -\beta_1\,C_L - (2b_{11} + 2nb_{12})\delta_1\,, \qquad C_{H2} = -\beta_2 C_L - (2b_{21} + 2nb_{22})\delta_1\,;$$

$$\lambda_1 = \frac{\pi - \phi_1}{\pi} + \frac{\sin\phi_1}{\pi}, \qquad \lambda_2 = \frac{\pi - \phi_2}{\pi} + \frac{\sin\phi_2}{\pi}\,;$$

$$E_1{}^2\beta_1 = \frac{1}{2\pi}\left\{\sin\phi_1\,(1 - \tfrac{1}{2}\cos\phi_1) - (\pi - \phi_1)\,(\tfrac{1}{2} - \cos\phi_1)\right\},$$

$$E_2{}^2\beta_2 = \frac{1}{2\pi}\left\{\sin\phi_2\,(1 - \tfrac{1}{2}\cos\phi_2) - (\pi - \phi_2)\,(\tfrac{1}{2} - \cos\phi_2)\right\}.$$

Suppose E_1 is fixed, then $\lambda_1 = \text{constant} = A$

and $E_1{}^2\beta_1 = \text{constant} = B$ (say)

Thus
$$\left(\frac{-\,\partial C_{H_s}}{\partial C_L}\right)_{\delta_s} = m\,(E_1{}^2\beta_1 + nE_2{}^2\beta_2) = m\,(B + nE_2{}^2\beta_2)$$

and
$$a_1 m(A + n\lambda_2) = C,$$

or

$$\left(-\,\frac{\partial C_{H_s}}{\partial C_L}\right)_{\delta_s} = \frac{m}{2\pi}\left[2\pi B + n\left\{\sin\phi_2\,(1 - \tfrac{1}{2}\cos\phi_2) - (\pi - \phi_2)\,(\tfrac{1}{2} - \cos\phi_2)\right\}\right]$$
$$= f\,(\phi_2, m)$$

and
$$\frac{ma_1}{\pi}\left[\pi A + n\{(\pi - \phi_2) + \sin\phi_2\}\right] = C.$$

Further suppose n is fixed. For a minimum of $f\,(\phi_2, m)$

$$\left.\begin{aligned} \frac{\partial f}{\partial m} - \mu\,.\,\frac{a_1}{\pi}\left[\pi A + n\{(\pi - \phi_2) + \sin\phi_2\}\right] &= 0 \\ \frac{\partial f}{\partial \phi_2} - \mu\,.\,\frac{a_1}{\pi}\,n\,(\cos\phi_2 - 1) &= 0 \end{aligned}\right\}$$

or

$$\left\{\begin{aligned} &2\pi B + n\,\{\sin\phi_2\,(1 - \tfrac{1}{2}\cos\phi_2) - (\pi - \phi_2)\,(\tfrac{1}{2} - \cos\phi_2)\} \\ &\quad - 2\mu a_1\,[\pi A + n\,\{(\pi - \phi_2) + \sin\phi_2\}] = 0 \\ &\sin\phi_2\{\sin\phi_2 - (\pi - \phi_2)\} - 2\mu a_1\,(\cos\phi_2 - 1) = 0. \end{aligned}\right.$$

Eliminating μ gives,

$$2\,\pi B\,(\cos\phi_2 - 1) + n\,(\cos\phi_2 - 1)\,\{\sin\phi_2\,(1 - \tfrac{1}{2}\cos\phi_2) - (\pi - \phi_2)\,(\tfrac{1}{2} - \cos\phi_2)\}$$
$$- \sin\phi_2\,\{\sin\phi_2 - (\pi - \phi_2)\}\{\pi A - n\,(\pi - \phi_2 + \sin\phi_2)\} = 0$$

Put

$$\pi - \phi_2 = \xi\,;\ \sin\phi_2 = \xi - \frac{\xi^3}{6},$$
$$\cos\phi_2 = -1 + \frac{\xi^2}{2} - \frac{\xi^4}{24}.$$

Then

$$2\pi B\left(\frac{\xi^2}{2} - \frac{\xi^4}{24} - 2\right) + n\left(\frac{\xi^2}{2} - \frac{\xi^4}{24} - 2\right)\left\{\left(\xi - \frac{\xi^3}{6}\right)\left(\frac{3}{2} - \frac{\xi^2}{4} + \frac{\xi^4}{48}\right)\right.$$
$$\left. - \xi\left(\frac{3}{2} - \frac{\xi^2}{2} + \frac{\xi^4}{24}\right)\right\} + \left(\xi - \frac{\xi^3}{6}\right)\frac{\xi^3}{6}\left\{\pi A - n\left(2\xi - \frac{\xi^3}{6}\right)\right\} = 0$$

Retaining powers $<$ 5th

$$2\pi B\left(\frac{\xi^2}{2} - \frac{\xi^4}{24} - 2\right) + \frac{\pi A}{6}\,\xi^4 = 0$$

which gives

$$\left(\frac{2A}{B} - 1\right)\xi^4 + 12\xi^2 - 48 = 0$$

Solving,

$$\xi^2 = \frac{-12 \pm \sqrt{144 + 192\left(2\,\dfrac{A}{B} - 1\right)}}{2\left(2\,\dfrac{A}{B} - 1\right)}$$
$$= \frac{6B}{B - 2A} \pm \frac{2B^{1/2}}{2A - B}\sqrt{9B + 12\,(2A - B)}$$

This shows that the value of E_2 for the minimum is practically independent of the gearing n, and gives results in good agreement with those attained graphically.

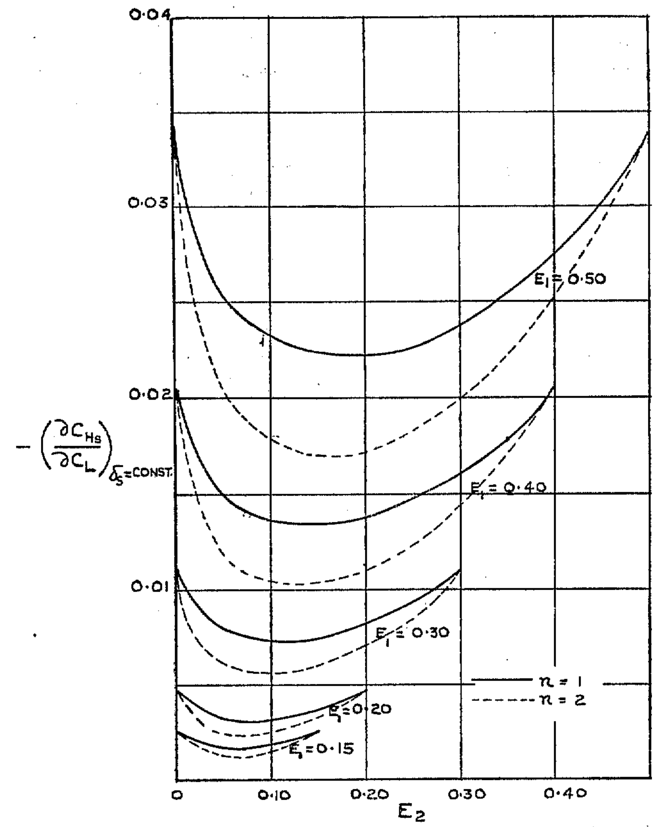

FIG. 1.

VARIATION OF $\left(\frac{\delta C_{H_s}}{\delta C_L}\right)_{\delta_s = CONST.}$ WITH E_1, E_2 AND GEARING OF DOUBLE FLAP.

Note: — Control Column Gearing to Flap, etc.

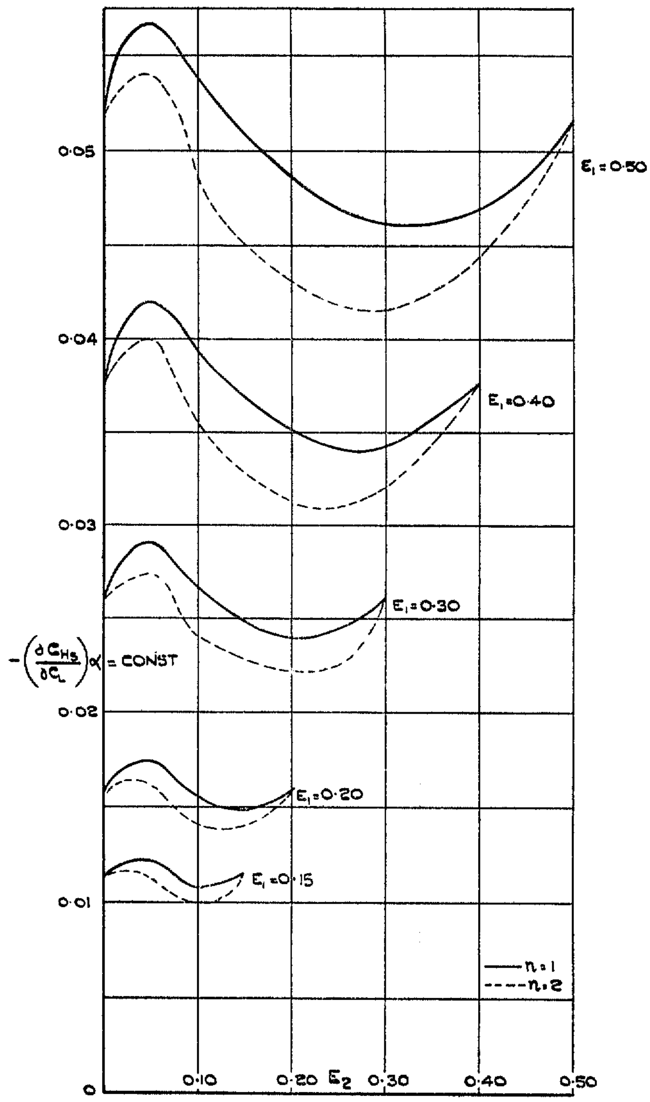

FIG. 2.

VARIATION OF $\left(\frac{\delta C_{H_s}}{\delta C_L}\right)_{\alpha = const.}$ WITH E_1, E_2 AND GEARING OF DOUBLE FLAP.

Note:—Control Column Gearing to Flap adjusted to give the same rolling power as 0·5c Single Flap.

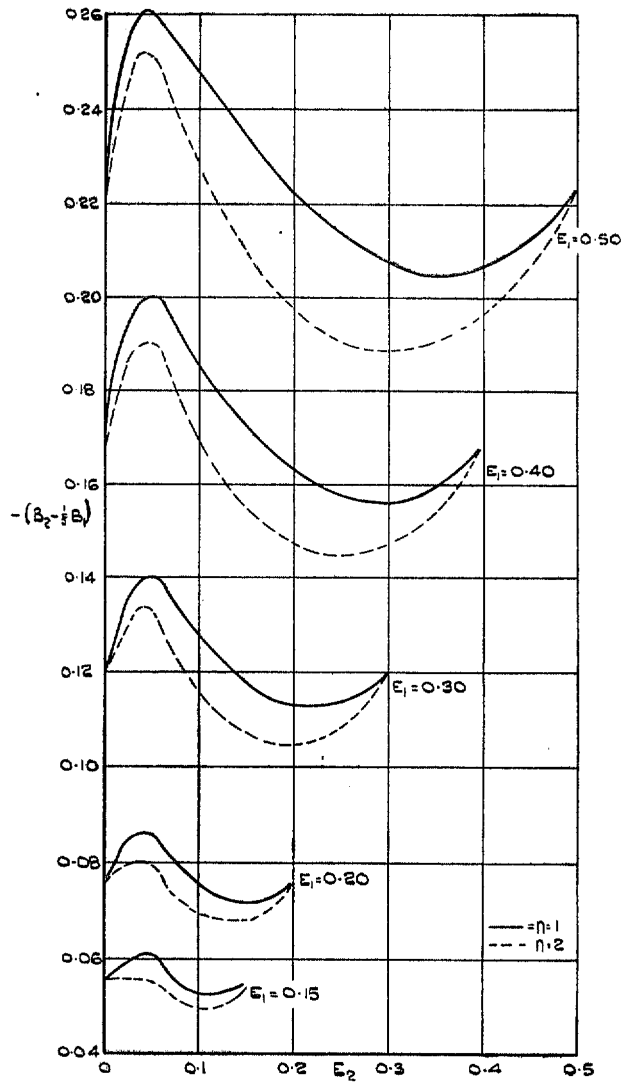

FIG. 3.

VARIATION OF $\left(B_2 - \frac{1}{5} B_1\right)$ WITH E_1, E_2 AND GEARING OF DOUBLE FLAP.

Note:—Control Column Gearing to Flap adjusted to give the same rolling power as the 0·50c Single Flap.

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