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On the Static Thrust of Contra-Rotating Airscrews

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1. *Introduction.*—Under static conditions or at low rates of advance, the blades of a single airscrew are often stalled, except perhaps for the outer sections. At first sight, it would appear that the same conditions must apply to the blades of a contra-rotating pair of screws. However, some measurements made in America have shown that the static thrust of contra-rotating screws is considerably greater than that of a corresponding single screw; and the same conclusion has been inferred from the short take-off run of a particular aeroplane with contra-rotating screws.

The object of the present note is to suggest a possible reason for an increase of static thrust in the case of contra-rotating screws.

2. *Effects of Oscillation on Stalling Characteristics.*—Under steady conditions, the incidence at which a section of a blade of a single airscrew works is roughly constant. For low rates of advance the incidence is high, and if the rate of advance is very low, the incidence may be so high that the section stalls. In the case of a contra-rotating pair, however, the incidence at which any blade section works is never constant, even for steady rates of advance; for each blade is continually passing through the disturbed fields of flow round the blades of the other screw. The incidences at which the blades of either screw work therefore oscillate rapidly about their mean values.

Now it is known that under conditions of rapid oscillation, stalling may often be delayed to very high angles of incidence. In some experiments recorded by Denis[1] a thin aerofoil was under test in a wind tunnel and the curve of lift coefficient against incidence obtained at a speed below 27 m./sec. is given as the line A in Fig. 1; it is a curve of normal type, and shows that stalling begins at an incidence of about 10 deg. At a speed of 30 m./sec., the lift coefficient curve is practically identical up to an incidence of 17·5 deg. At this speed and incidence, however, an oscillatory instability of the wing set in, which persisted as the incidence was increased. In the range of (mean) incidence rising from 17·5 deg. to 30 deg. the lift coefficient increased rapidly, and at 30 deg. had between 2 and 3 times its static stalled value. The curve is shown as B in Fig. 1. The oscillations were accompanied by an increase in drag, which was, however, not proportionally as great as the increase in lift.

In the experiments quoted by Denis it appears that the frequency of oscillation was fairly high. From the values of chord and wind speed given, and from the estimated frequency of oscillation, it appears that the frequency parameter λ $(= 2\pi fc/V)$ was of the order of unity. The wing was oscillating in a mode having nodes at the points of suspension: two outboard at the leading edge and one centrally at the trailing edge. It is therefore difficult to estimate an amplitude of motion; however, at the central section the pitching amplitude appears to have been of the order of $\pm 3 \cdot 5$ deg.

More accurate quantitative information on this question is provided by some experiments of Bratt and Scruton[2]; their measurements, however, were of pitching moment, not lift.

A symmetrical aerofoil was subjected to forced sinusoidal oscillations of which the frequency and amplitude could be varied at will. The measurements of aerodynamical pitching moment were made by means of the magneto-striction stress indicator apparatus, which provides a complete history of the instantaneous moments.

The curve of static pitching moment is given in Fig. 2. It will be seen that the aerofoil begins to stall at about 10 deg. incidence, and beyond 13 deg. the moment falls abruptly to a low value. It might be expected that in a sinusoidal oscillation within the limits of incidence of the unstalled region, the time curve of pitching moment would also be sinusoidal, and this was found to be the case; though, as indicated by theory, there is a phase difference between displacement and moment, and the latter is a function of the frequency parameter. It would further be anticipated that the moment curve obtained in the stalled region would be an irregular periodic curve, not sinusoidal. This is true for slow oscillations, but as the frequency is increased the moment curve becomes more and more nearly sinusoidal, until at the highest values of λ it was a pure sine curve to within the limits of experimental accuracy. Moreover, as the frequency was increased, the mean value of the pitching moment increased above the static stalled value. The tendency towards a sinusoidal variation of moment with time, indicates that the moment-incidence curve is becoming linear. In this sense, the aerofoil is becoming unstalled.

Some points showing the increase of mean moment with λ are given in Fig. 2; they relate to an amplitude of $\pm 5{\cdot}0$ deg. about a mean incidence of $17{\cdot}9$ deg., and an amplitude of $\pm 2{\cdot}9$ deg. about a mean incidence of $20{\cdot}0$ deg. The increase in moment is not quite as regular as the points suggest, since there are sometimes two flow regimes at one value of λ; however, they serve to indicate the general trend. As might be expected, when the sinusoidal moment curve is reached, further increase in λ has little effect; again, it is to be expected that a large amplitude produces a greater effect than a small amplitude. It may be assumed that, since the effect of oscillation appears to be to unstall the flow, the lift coefficient will behave similarly to the moment coefficient. As suggested by the results of Denis, a value of λ of the order of unity and an amplitude of the order of ± 5 deg. are sufficient to produce a considerable unstalling effect.

3. *Application to the Contra-Rotating Airscrew.*—It appears to be of interest to see whether the oscillatory changes of incidence on the blades of a contra-rotating pair of screws, referred to at the beginning of §2, are of sufficient magnitude and rapidity to give rise to an appreciable unstalling effect.

4. *Development into an Infinite Cascade in the z-Plane.*—Consider a single screw having N blades; let its angular velocity be Ω. Attention will be restricted to some typical radius r. At this radius, let the axial velocity of inflow be U; then, if the rotational inflow factor is neglected, the components of wind velocity relative to a blade section are U axially and Ωr circumferentially, so that if the resultant is W,

$$W^2 = U^2 + \Omega^2 r^2. \qquad \text{(1)}$$

Suppose the local lift coefficient to be C_L, the chord c and the circulation K. Then

$$\rho W K = \tfrac{1}{2}\rho W^2 c C_L,$$

or

$$K = \tfrac{1}{2} W c C_L. \qquad \text{(2)}$$

Imagine these blade sections to be developed into an infinite two-dimensional cascade; its spacing will be $2\pi r/N$. Let each section be replaced by a single vortex of strength K, disposed in the z-plane along the imaginary axis at the points $z = 0$, $z = \pm 2\pi i r/N$, $z = \pm 4\pi i r/N$, . . . and so on. At a general point z the velocity components due to the vortices will be given by

$$\frac{dw}{dz} = \frac{iK}{2\pi}\left\{\frac{1}{z} + \frac{1}{z - 2\pi i r/N} + \frac{1}{z + 2\pi i r/N} + \dots\right\}$$

$$= \frac{iKN}{4\pi r}\coth\left(\frac{Nz}{2r}\right),$$

which on substitution from (2) for K becomes

$$\frac{dw}{dz} = \frac{iWNcC_L}{8\pi r}\coth\left(\frac{Nz}{2r}\right). \qquad (3)$$

If velocities corresponding to the axial and circumferential components of the incident stream are superimposed on the vortex system, the result is

$$\frac{dw}{dz} = U - i\Omega r + \frac{iWNcC_L}{8\pi r}\coth\left(\frac{Nz}{2r}\right). \qquad (4)$$

Now suppose there is a second screw, coaxial with and similar to the first but rotating in the opposite sense with angular velocity Ω; let the distance between the screw discs be d. The angular velocity of either screw relative to the other is 2Ω; hence in the cascade analogy, the blade sections of the second screw will be passing along the line $x = d$ with speed $2\Omega r$. Accordingly, the rate at which the sections of the second screw pass the vortices corresponding to the first screw is

$$f = \frac{2\Omega r}{2\pi r/N} = \frac{\Omega N}{\pi}. \qquad (5)$$

This is the frequency at which disturbances of the fluid flow repeat with respect to the second screw. For the purpose of estimating the frequency parameter λ it is sufficient to assume that the wind speed relative to the blade is Ωr; accordingly, on use of (5) we have approximately

$$\lambda = \frac{2\pi fc}{\Omega r} = \frac{2Nc}{r}. \qquad (6)$$

The components of velocity relative to the blades of the second screw due to the flow pattern of (4) are found by adding $2i\Omega r$ to the expression in (4) and putting $z = d + iy$. Here y is a parameter which varies from 0 to $2\pi r/N$ as one blade section moves into the position vacated by its neighbour. The velocity components become

$$u = U - \frac{WNcC_L}{8\pi r}\cdot\frac{\sin(Ny/r)}{\cosh(Nd/r) - \cos(Ny/r)},$$

$$v = \Omega r + \frac{WNcC_L}{8\pi r}\cdot\frac{\sinh(Nd/r)}{\cosh(Nd/r) - \cos(Ny/r)}.$$

Hence if $\tan\phi = u/v$,

$$\phi = \tan^{-1}\left[\frac{U\left\{\cosh(Nd/r) - \cos(Ny/r)\right\} - kW\sin(Ny/r)}{\Omega r\left\{\cosh(Nd/r) - \cos(Ny/r)\right\} + kW\sinh(Nd/r)}\right], \qquad (7)$$

where

$$k = NcC_L/8\pi r. \qquad (8)$$

The variation in ϕ during a complete cycle is the variation in incidence on the blade section. For static conditions and at low rates of advance, U is small compared with Ωr, so that it is sufficiently accurate to write

$$U = \varepsilon W,$$

$$\Omega r = W,$$

and then (7) reduces to

$$\phi = \tan^{-1}\left[\frac{\varepsilon\left\{\cosh(Nd/r) - \cos(Ny/r)\right\} - k\sin(Ny/r)}{\cosh(Nd/r) - \cos(Ny/r) + k\sinh(Nd/r)}\right]. \qquad (9)$$

This equation provides a simple way of estimating roughly the variations in blade incidence throughout a cycle if it is assumed that k, and therefore C_L, is constant ; i.e. if the changes in circulation on each blade due to the passage through the disturbed fields of flow are assumed to be negligible. For the purpose of the present investigation such an assumption can be made without serious error.

5. *A Numerical Example.*—As a numerical case, let

$$N = 3$$
$$c = 0 \cdot 5 \text{ ft.}$$
$$r = 4 \text{ ft.}$$
$$C_L = 1 \cdot 4.$$

The values of c and r are roughly appropriate to $0 \cdot 7$ of the tip radius of a normal 3-blader screw of 11 ft. 6 in. diameter. The value of C_L is the maximum usually attainable for a blade section at a steady incidence.

Equation (6) shows the frequency parameter for a contra-rotating pair with the above dimensions to be

$$\lambda = 0 \cdot 75 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (10)$$

For smaller radii, the parameter would be rather higher, since the ratio c/r usually increases as r is reduced.

Now suppose the distance between the screw discs* to be

$$d = 0 \cdot 5 \text{ ft.}$$

Equation (9) then becomes

$$\phi = \tan^{-1} \left[\frac{\varepsilon \left\{ 1 \cdot 0711 - \cos 0 \cdot 75 y \right\} - 0 \cdot 02089 \sin 0 \cdot 75 \, y}{1 \cdot 0791 - \cos 0 \cdot 75 \, y} \right].$$

Graphs of ϕ for a complete cycle are given in Fig. 3, for values of $0 \cdot 1$ and $0 \cdot 2$ of ε. The latter quantity is not zero even at static, since there is always a slight inflow. It will be seen, however, that the variation in ϕ is not very dependent on ε ; and the maximum variation in ϕ is 3 deg. on each side of the mean in both cases.

6. *Conclusion.*—It is concluded from §5 that the magnitude of the variation in incidence and the frequency parameter are quite sufficient to produce a noticeable unstalling effect, judged by the results of Denis and of Bratt and Scruton. On the other hand, the curve of variation of incidence with time, as shown in Fig. 3, is far from sinusoidal : the effect of this fact cannot be assessed.

It is probable that the disturbances of incidence experienced by the blades are severer than the approximate theory set out above would indicate, since in the theory the blades are replaced by points, whereas in practice they have finite magnitude and the clearance between the trailing edges of the front blades and the leading edges of the rear blades is usually quite small.

REFERENCES

Ref.	*Author*	*Title, etc.*
1	Denis	The Aerodynamics of a Vibrating Wing. *Comptes Rendus*, February, 1938.
2	Bratt & Scruton	Measurements of Pitching Moment Derivatives for an Aerofoil Oscillating about the Half-Chord Axis. R. & M. 1921. 1938.

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* The actual distance between the screw discs would be greater than the figure chosen. However, in order to make some allowance for the finite size of the blades it has been assumed that the equivalent vortices would be somewhere near the trailing edges of the front screw and somewhere near the leading edges of the rear screw ; see remarks in §6.

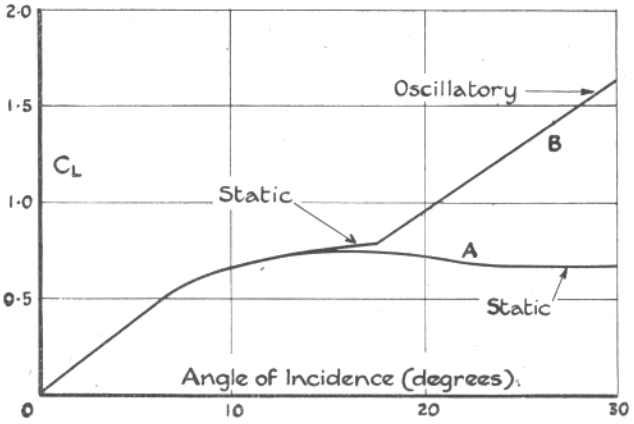

FIG. 1

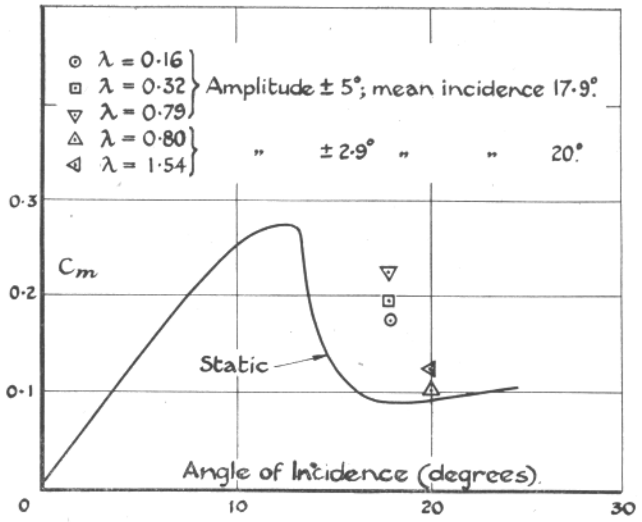

FIG. 2

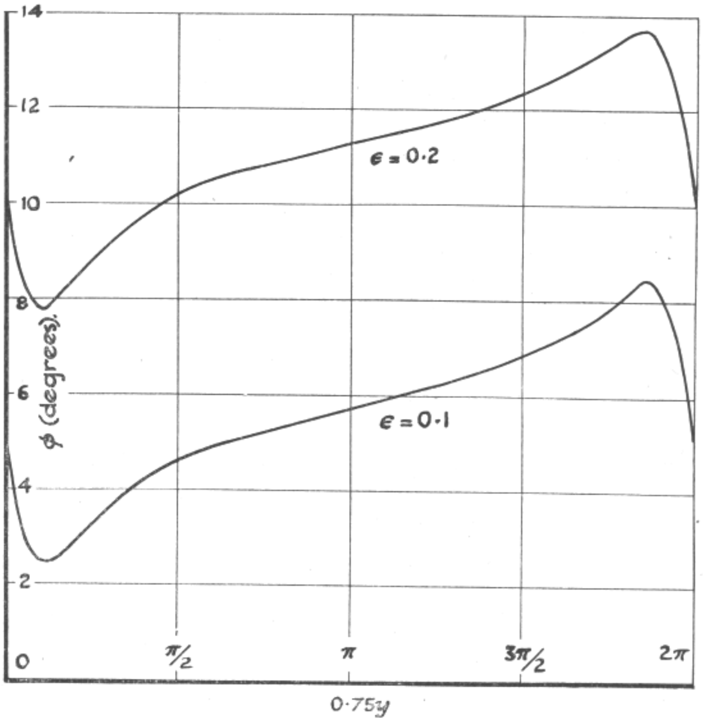

FIG. 3

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