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Prediction of Undercarriage Reactions

By

DR. G. TEMPLE, F.R.S.

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ERRATA

Page 5
Line 9, for R_L read R_h.

Page 6
Line 27, for k_γ read K_γ.
Line 3 from bottom, for K read k.

Page 8
Line 8 from bottom, for $\ddot{x}_t$ read $\ddot{x}$.

Page 10
Lines 4 and 15, for x read z

Page 12
Line 17, for g read ng.

Page 17
Line 6 from bottom, read " a constant for a specified wheel at a prescribed landing speed on all runways."

Page 22
Line 6 from bottom, for $\dot{h}$ read h.

Page 23
Line 3, for $(1 - h)$ read $(1 - \mu h)$.

Page 24
Line 9, for $l_2\ddot{\phi}$ read $l_2\ddot{\theta}$.
Line 29, omit " in curly brackets ".

Page 25
2nd boxed equation, for $=$ read $+$.

Page 26
In box, for $_j$ read j.
Line 21, for $-$ read $=$.
Lines 3, 6 and 9 from bottom, for x_ω, read x_{w},.

Page 28
Line 26, for $\ddot{x}$ read $\ddot{x}_n$.

Page 29
Line 8, for a_n read x_n.
Line 9, for u_n read $\dot{x}_n$.

Page 31
Line 2 from bottom, for $-$ read $=$.

Page 32
Line 2, for ft./sec.^{2} read in./sec.^{2}.
Table, at $t = 0 \cdot 10$, s to read 12.675.

Aeronautical Research Council,
July, 1946.

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Prediction of Undercarriage Reactions

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DR. G. TEMPLE, F.R.S.

Communicated by the Director of Scientific Research, Ministry of Aircraft Production.

Reports and Memoranda No. 1927

September, 1944*

Summary.—The object of this report is to give a connected account of the methods which have been developed at the Royal Aircraft Establishment by Messrs. D. D. Lindsay, R. G. Thorne and S. A. Makovski (Refs. 3, 4 and 5) for the prediction of undercarriage loads under symmetric landing conditions; to extend these methods to deal with other landing manœuvres; and to formulate a simplified system of step by step computation of the loads.

CONTENTS

* R.A.E. Report. S.M.E. 3298 received 27th July, 1945.

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1. *Introduction.*—The report is divided into three parts :—

Part I, " Basic Data and Preliminary Calculations ", itemises the basic information required, either in the form of design data or of experimental results, together with the preliminary calculations which must be made before the load computation is begun.

Part II, " Equations of Motion for Standard Landing Conditions ", gives the fundamental equations for a tail wheel or nose wheel aircraft, together with the simplifications and approximations which can be made for a calculation of the landing reactions.

Part III, " Step by step Methods of Integration " describes the principle of the method of computation illustrated by reference to undercarriage prediction.

It is intended to issue a supplementary report giving some typical examples of reaction prediction worked out in detail, and extending the methods of this report for the effects of wing flexibility.

PART I

Basic Data and Preliminary Calculations

2. *General Summary.*—In the following sections, paragraphs 3–9, we itemise the basic data required for a prediction of undercarriage reactions, together with the preliminary calculations which are necessary to express the basic data in a form suitable for immediate use in the computation. It should be realised that this one set of preliminary calculations can be used for prediction in *all* the conditions of landing which are prescribed in the airworthiness requirements.

The following summary shows in tabular form the basic data required, the source from which they can be procured, and the results to be obtained by the preliminary calculations.

Basic data	Source	Results of calculations
Paragraph 3 Size and deflection curves of *pneumatic tyre.*	Data sheets provided by tyre manufacturers.	Wheel load R for tyre deflection x_t, at working inflation pressure, $R = f(x_t)$.
Paragraph 4 Dimensions and weight of *tyre-wheel assembly.*	Data sheets provided by tyre manufacturers.	Moment of inertia J of tyre-wheel assembly.
Paragraph 5 Geometry of *wheel suspension* ..	Drawings of G.A. of undercarriage.	Mechanical advantage and velocity ratio for vertical and horizontal loads at axle.
Paragraph 7 Dimensions of *shock absorber* .. or *Paragraph* 8 Acceleration curves for *axle* and *main shock absorber cylinder.*	Drawing of shock absorber unit or Drop tests	General expression for vertical axle velocity $\dot{x}$ in the form $\dot{x} = D(R-Q)^{\frac{1}{2}}$, where R is ground reaction and D, Q known functions of x.
Paragraph 9 Weight distribution in aircraft ..	Weight data sheet in type record, or designer's preliminary estimates.	Position of C.G., weight and moments of inertia of aircraft.

3. *Pneumatic Tyre.*—The "static and dynamic load deflection curves", published by the Dunlop Rubber Co., Ltd., give the tyre deflection x_t plotted against the wheel load R for a range of inflation pressures covering the recommended working conditions.

It is recommended that the "static tyre curves" should be used in all calculations of undercarriage reactions, as experience shows that they give results in better agreement with the results of drop tests.

The data sheet with the static and dynamic load deflection curves also gives in round figures the geometrical dimensions of the tyre at various inflation pressures. The only dimensions required for prediction calculations are the inflated diameter, and the inflated width.

From such a data sheet we plot, or tabulate, the wheel load R against the tyre closure x_t for the recommended inflation pressure, interpolating if necessary between the figures given on the data sheet.

As an example we give the following figures based on the data sheet for the 9·50—12 Intermediate Aero Type Tyre. (Heavy ID.11.)

Wheel load R expressed as function tyre closure x_t, $R = f(x_t)$

Tyre closure d (ins.)	Wheel load R (lbs.) (dynamic loading) at inflation pressure p (p.s.i.)		
	$p = 45$ (from data sheet)	$p = 50$ (from data sheet)	$p = 47$ (interpolated)
1	1,200	1,400	1,280
2	2,700	3,300	2,920
3	4,400	5,400	4,800
4	6,300	7,600	6,820
5	8,200	10,100	8,960
6	10,200	12,400	11,080
6·5	11,300	13,700	12,260

Inflated diameter $= 31 \cdot 25$ in., $p = 45$ p.s.i.

Inflated width $= 9 \cdot 6$ to $9 \cdot 75$ in. (the nominal inflated width is the first number in the tyre specification, e.g., a 9·50—12 tyre has a nominal width of 9·50 inches).

(To allow for growth of tyre in service add 4 per cent. of width to radius—then the tyre radius under no load $= r_0 = 15 \cdot 62 + 0 \cdot 39 = 16 \cdot 01$ in.)

4. *Tyre-wheel Assembly*.—In a data sheet issued by the Dunlop Rubber Co., Ltd. (dated 9th January, 1942) the following approximate formulae are given for the polar moments of inertia, J, of Dunlop tyre-wheel assemblies (i.e., moments of inertia about the central line of the axle.)

(i) For tyres of normal tread thickness,

$$J = 0 \cdot 60\, M_w . r_2^2 + 0 \cdot 95\, M_t . (R_1^2 + 1 \cdot 5 r_1^2)\ ;$$

(ii) For thick tread tyres,

$$J = 0 \cdot 60\, M_w . r_2^2 + 1 \cdot 01\, M_t . (R_1^2 + 1 \cdot 5 r_1^2).$$

Here

$J =$ Polar moment of inertia in lb. in.^{2}

$M_w =$ Wheel weight in lb.

$M_t =$ Tyre plus tube weight in lb.

$r_2 =$ Rim radius in ins.*

$r_0 =$ Tyre radius under no load in ins.

$R_1 = \frac{1}{2}(r_0 + r_2)$

$r_1 = \frac{1}{2}(r_0 - r_2)$

These formulae are stated to be accurate to $\pm$ 4 per cent. As the value of J is used only to estimate the time during which the wheels are skidding after touch down, this accuracy is quite sufficient.

The weights of the wheel and tyre plus tube will be supplied by the manufacturers.

* The rim diameter is given by the second number in the tyre specification, e.g., a 9·50–12 tyre has a rim diameter of 12 in., whence $r_2 = 6$ in.

5. *Geometry of Wheel Suspension.*—The geometric characteristics of the wheel suspension required for reaction prediction are its mechanical advantages and velocity ratios for vertical and horizontal loads applied at the axle when the aircraft is in some standard attitude—usually at rest with all three wheels on the ground. In a cantilever unit these mechanical advantages and velocity ratios are independent of the travel of the piston, but in an articulated unit they vary with the position of the piston. The principle of the calculation is the same in both cases, and will be clear from the sketch (Fig. 1) of a typical shock absorber linkage in an articulated unit.

Taking moments about the fork hinge, we find that a piston thrust P balances vertical and horizontal reactions at the axle, R_V and R_L, if

$$P.l = R_V l_V \text{ or } R_h l_h$$

Hence the corresponding mechanical advantages are

$$K_V = R_V/P = l/l_V \ ,$$

and

$$K_h = R_h/P = l/l_h \ .$$

The corresponding velocity ratios are

$$C_V = 1/K_V = l_V/l$$
$$C_h = 1/K_h = l_h/l$$

These velocity ratios give the ratio of axle velocity to the piston velocity, in vertical and horizontal displacements of the axle.

For displacements inclined at an angle γ to this centre line the corresponding velocity ratio is

$$C_\gamma = C_V.\cos\gamma + C_h.\sin\gamma$$

It is convenient to make the drawings and to carry out the calculations for prescribed amounts of shock absorber travel x_s. Typical results for a main undercarriage shock absorber are tabulated below :—

Attitude of aircraft :—Centre line of unit raked forward 6°, *lower end forward*

Shock absorber travel, x_s ..	0	0·4	0·8	1·2	1·6	2·0	2·4	2·61
Mechanical adv. K_V	0·308	0·297	0·288	0·200	0·274	0·270	0·265	0·263
Mechanical adv. K	0·290	0·313	0·358	0·400	0·450	0·573	0·591	0·655
Velocity ratio C_V	3·24	3·37	3·47	3·56	3·65	3·71	3·77	3·80
Velocity ratio C_h	3·45	3·10	2·79	2·50	2·22	1·95	1·69	1·55

6. *Characteristics of Shock Absorbers.*—The characteristic equation of the shock absorber unit expresses the vertical ground reaction R, in the form

$$\dot{x} = D(R-Q-F)^{1/2}.$$

where D, Q and F are functions of the vertical axle travel x only. In this section we show by a general argument that $\dot{x}$ can be expressed in this form.

In many, if not most, types of shock absorber the balance of forces between the thrust P in the shock absorber piston, the pneumatic pressure pA of the compressed air, and the hydraulic resistance $(p' - p)A$, of the damper orifices, (together with the friction f due to the offset of the ground reaction in a cantilever unit), can be expressed in the form

$$P = pA + (p' - p)A_1 + f$$

(see for example, Fig. 2), where

$$\begin{aligned} p &= \text{air pressure,} \\ A &= \text{“ air displacement area ”,} \\ p' &= \text{oil pressure,} \\ A_1 &= \text{“ oil displacement area ”.} \end{aligned}$$

Now the pressure drop, $p' - p$, through the orifices must vary as the square of the piston speed $\dot{x}_s$, so that

$$p' - p = C\dot{x}_s^2$$

where the coefficient C depends only on the piston travel x_s. Also f must be proportional to R so that

$$f = kR,$$

where the coefficient k also depends only on the piston travel x_s.

Now the total ground reaction is $K_\gamma P$ where K_γ is the mechanical advantage for a reaction inclined at an angle γ to the vertical.

Hence the vertical ground reaction is

$$R = K_\gamma P \cos \gamma.$$

where $K_\gamma = 1/C_\gamma = \{C_v \cos\gamma + C_h \sin \gamma\}^{-1}$. The ratio of the vertical axle velocity $\dot{x}$ to the piston speed $\dot{x}_s$ is the velocity ratio C_v, i.e.,

$$\dot{x} = C_v \dot{x}_s .$$

Hence, gathering together all these formulae, we find that

$$\begin{aligned} R &= K_\gamma P \cos \gamma \\ &= (k_\gamma \cos \gamma) \{pA + (p' - p)A_1 + f\} \\ &= (K_\gamma \cos \gamma) \{pA + CA_1\dot{x}_s^2 + f\} \\ &= (K_\gamma \cos \gamma) \{pA + (CA_1/C_v^2)\dot{x}^2 + f\} \end{aligned}$$

Therefore $R = (K_\gamma \cos \gamma) (pA + f) + (K_\gamma \cos \gamma . CA_1/C_v^2)\dot{x}^2$.

Hence if

$$\begin{aligned} Q &= pAK_\gamma \cos \gamma \\ F &= fK_\gamma \cos \gamma \\ D &= C_v(K_\gamma \cos \gamma . CA_1)^{-1/2} \end{aligned}$$

we can express the vertical axle velocity $\dot{x}$ in the form

$$\dot{x} = D(R-Q-F)^{1/2}.$$

Now the air pressure p, the air displacement area A, the oil displacement area A_1, the mechanical advantage K_γ, the velocity ratio C_v, the friction coefficient K and the angle γ all depend only on the instantaneous geometry of the shock absorber unit, and hence can be expressed in terms of the axle travel x. Hence D, Q and F can also be expressed in terms of x only.

7. *Characteristics of Shock Absorbers, Deduced from Design Data.*—The characteristics of shock absorbers can be estimated with reasonable accuracy from design data. Three calculations have to be made, viz :—

(1) The pneumatic pressure p ;

(2) The coefficient C in the expression for the pressure drop through the orifices,

$$p' - p = C\dot{x}_s^{\,2} ;$$

(3) The friction coefficient k which determines the ratio of the piston friction f to the vertical ground reaction R.

(1) To calculate the pneumatic pressure p, let

p_0 = initial air pressure in shock absorber (p.s.i.)

p = air pressure when piston travel is x_s (p.s.i.)

V_0 = initial air volume (in.^{3})

V = air volume when piston travel is x_s (in.^{3})

A = air displacement area (in.^{2})

S_E = equivalent stroke = V_0/A. (in.)

Careful analysis of drop test records suggests that the pressure-volume relation follows a law of the form

$$p/p_0 = (V/V_0)^{-n}$$

where n is about 1·3, (i.e., somewhat less than the adiabatic index for air, which is 1·4). This relation is therefore used for quasi-adiabatic conditions, as in landing.

Since

$$V_0 - V = Ax_s$$

and

$$V_0 = AS_E$$

therefore

$$p/p_0 = (1 - x_s/S_E)^{-n}$$

(2) To calculate the coefficient C, let

p' = oil pressure (p.s.i.)

σ = specific weight of oil (lb./in.^{3})

g = acceleration due to gravity (386 in./sec.^{2})

v_0 = oil speed through orifice (in./sec.)

A_1 = oil displacement area (in.^{2})

a_1 = orifice area (in compression) (in.^{2})

a_2 = orifice area (in recoil) (in.^{2})

C_{D_1} = orifice coefficient in compression

C_{D_2} = orifice coefficient in recoil.

Then the condition for continuous flow is

$$\dot{x}_s A_1 = v_0 a C_D \ ,$$

where aC_D stands for $a_1 C_{D_1}$ in compression and for $a_2 C_{D_2}$ in recoil. Also by Bernoulli's equation

$$p' - p = \tfrac{1}{2}\sigma v_0^2/g$$

Hence

$$\begin{aligned} p' - p &= \tfrac{1}{2}(\sigma/g).(\dot{x}_s A_1/aC_D)^2 \\ &= C\dot{x}_s^2 , \end{aligned}$$

where

$$C = \frac{\sigma A_1^2}{2ga^2 C_D^2} .$$

The main justification for this method of calculating the coefficient C is that it is found possible to choose the orifice coefficient C_D so as to obtain reasonable agreement between calculated and measured drop test performance.

(3) It is impracticable to give any general rules for calculating f, as the method will vary so much with the design of the shock absorber.

Referring back to the results of the previous section, we can now estimate Q and D (for articulated units where $F = 0$) from the equations

$$\begin{aligned} Q &= pAK_\gamma \cos\gamma = p_0(1 - x_s/S_E)^{-n} . (AK_\gamma \cos\gamma) , \\ D &= C_v(K_\gamma \cos\gamma \, CA_1)^{-1/2} \\ &= \frac{C_v a C_D}{A_1^{3/2}} \left\{ \frac{2g}{\sigma K_\gamma \cos\gamma} \right\}^{+1/2} . \end{aligned}$$

8. *Characteristics of Shock Absorbers, Deduced from Drop Tests.*—In the calculation of the functions Q, D and F, which are characteristic of any given shock absorber, the greatest uncertainties arise in the estimation of the index " n " in the pressure-volume relation,

$$p/p_0 = (V/V_0)^{-n} ,$$

and of the orifice coefficients C_{D_1} and C_{D_2} which determine the effective area of the orifice in compression and in recoil. It would therefore be more satisfactory to determine these functions experimentally from the results of drop tests. In theory it should be sufficient to obtain accelerometer records of the axle and of the main shock absorber cylinder in two drop tests with different weights, but in practice it seems wiser to use the results of one drop test only, to assume a reasonable value for the index " n ", and to calculate the function D which depends on the orifice coefficient C_D.

In a drop test the total travel s of the main shock absorber cylinder is the sum of the axle travel x and of the tyre deflection x_t, i.e.,

$$s = x + x_t .$$

Now recording accelerometers attached to the axle and to the main shock absorber cylinder will register $\ddot{x}_t$ and $\ddot{s}$ as functions of the time t. By the methods explained in Part III, $\dot{x}$ and $\dot{s}$, together with x and s can then be calculated in terms of t. Then, by cross plotting, $\dot{x}$ and $\ddot{s}$ can be calculated in terms of x, so that we have as experimental results the equations

$$\dot{x} = V(x) \text{ and } \ddot{s} = A(x).$$

Since

$$\dot{x} = D(R - Q - F)^{1/2} \text{ and } R - W = - W\ddot{s}/g,$$

we find that

$$W - \frac{W}{g} . A(x) = \frac{V^2(x)}{D(x)} + Q(x) + F(x) \quad .$$

Since we suppose that $F(x)$ and $Q(x)$ can be calculated, this equation determines $D(x)$.

9. *Geometry of Aircraft.*—The basic geometric data required are

(1) The horizontal and vertical distances, $l_{j,0}$ and $h_{j,0}$ from the C.G. of the aircraft to the axle of the *j th* wheel in its fully extended position, relative to the agreed standard attitude of the aircraft ;

(2) The wheel track, 2b, or, in the case of multi-wheeled aircraft, the spanwise distance, b_n, of each wheel from the vertical plane of symmetry.

(3) The radii of gyration k_x, k_y, k_z of the aircraft about the forward horizontal axis, the transverse horizontal axis and the vertical axis.

From these data we can calculate the rotational factors B_{jk} and Y_{jk} appropriate to a banked and yawed landing respectively. The rotational factors for a banked landing are

$$B_{jk} = 1 + \frac{b_j b_k}{k_x^2} + \frac{l_j l_k - \mu l_j h_k}{k_y^2}$$
$$(j,k = 2, 3, \ldots)$$

where l_j and h_j are the horizontal and vertical distances from the C.G. to a wheel axle in a compressed position, in which the vertical travel is x and the horizontal travel is x_h, i.e.,

$$l_j = l_{j,0} - x_{j,h}\,, \quad h_j = h_{j,0} - x_j\,.$$

Similarly the rotational factor for a yawed landing is

$$Y = 1 + \frac{l^2 - \mu l h}{k_y^2}$$

(In this case l and h are the same for all wheels).

It will be seen later (in Part II) that the calculations are considerably simplified if the rotational factors B_{jk} and Y are sensibly constant. This question can be examined before starting the calculations by estimating the extreme values of l_j and h_j.

PART II

Equations of Motion for Standard Landing Cases

10. *General Summary.*—In the following sections we explain first the general notation which is needed to describe the undercarriage geometry, the attitude of the aircraft and the ground reactions. Next we establish the working approximations by means of which the equations of motion can be considerably simplified. We then construct the equations of motion for drop tests, and for standard landing cases, adding an account of the supplementary calculations on the spinning-up of the wheels.

11. *General Notation for all Landing Cases.*—(a) *Aircraft Axes.*—To describe the location and extension of the undercarriage we shall use a system of perpendicular axes fixed in the aircraft and described as the " aircraft axes ". The origin of this system is at the centre of gravity of the aircraft *G*. The directions of the axes are chosen so that they are " principal " axes, in the sense that the product of inertia for each pair of axes is zero. The longitudinal axis *Gx* is drawn forwards from *G* as in Fig. 3, the transverse axis *Gy* is drawn sideways to starboard, and the normal axis *Gz* is drawn downwards. The all-up weight of the aircraft will be denoted by *W* and the principal moments of inertia about *Gx*, *Gy* and *Gz* respectively by Wk_x^2, Wk_y^2 and Wk_z^2.

(b) *Tail or Nose Undercarriage.*—The axle of the wheel of the tail or nose undercarriage in its fully extended position before touch down is located with reference to the " aircraft axes " by the co-ordinates,

$$x = l_{1,0} \,, \qquad y = 0 \,, \qquad x = h_{1,0} \,,$$

$l_{1,0}$ being positive for a nose undercarriage and negative for a tail undercarriage.

After the tail or nose wheel has made contact with the ground the axle travel is $x_{h,1}$ in the negative direction of the longitudinal axis, (i.e., approximately horizontally) and x_1 in the negative direction of the normal z axis, (i.e., approximately vertically upwards). Hence the co-ordinates of the wheel axle are now

$$x = l_1 = l_{1,0} - x_{h,1} \,, \qquad y = 0 \,, \qquad z = h_1 = h_{1,0} - x_1 \,.$$

The tyre radius for the nose or tail wheel is r_1 initially. After contact has been made with the ground the tyre deflection is $x_{t,1}$ and the axle height is

$$r_1 - x_{t,1}.$$

Hence the co-ordinates of the centre of the contact region are

$$x = l_1 \,, \qquad y = 0 \,, \qquad x = h_1 + r_1 - x_{t,1}.$$

(c) *Main Undercarriage.*—We shall number the wheels from the starboard as " 2 ", " 3 ", " 4 " etc. (the number " 1 " referring to the nose wheel or tail wheel).

Considering the " k "-th wheel of the main undercarriage, the co-ordinates of the wheel axle in the fully extended equilibrium position before touch down are :—

$$x = l_{j,0} \,, \qquad y = b_j \,, \qquad z = h_{j,0} \,.$$

After touch down the axle travel is $x_{h,j}$ in the negative direction of the longitudinal x axis, (i.e. approximately horizontally), and x_j in the negative direction of the normal z axis, (i.e. approximately vertically upwards). The upward displacement of the wing at station " j ", just above the j-th leg of the main undercarriage, is $x_{w,j}$ relative to the C.G. of the aircraft. Hence the co-ordinates of the wheel axle are now—

$$x = l_j = l_{j,0} - x_{h,j} \,, \qquad y = b_j \,, \qquad z = h_j = h_{j,0} - x_j - x_{w,j}.$$

The tyre radius is r_j initially, and the subsequent tyre deflection is $x_{t,j}$ so that the axle height is

$$r_j - x_{t,j} \,,$$

and the co-ordinates of the contact region are

$$x = l_j \,, \qquad y = b_j \,, \qquad z = z_j = h_j + r_j - x_{t,j} \,.$$

(d) *Runway Axes.*—To describe the motion of the aircraft on the ground we employ a system of axes $OXYZ$, fixed relative to the runway and described as " runway axes ". The region O coincides with the $C.G.$ of the aircraft at the moment of touch down. The longitudinal axis OX is drawn forwards, horizontal and parallel to the length of the runway. The transverse axis OY is drawn horizontally sideways to starboard. The normal axis OZ is drawn vertically downwards.

(e) *Rolling, Pitching and Yawing.*—The changes in the attitude of the aircraft due to rolling, pitching and yawing are described by the relations between the aircraft axes and the runway axes. Fig. 3 shows, not the actual runway axes $OXYZ$, but a parallel system of axes $GXYZ$ drawn through the instantaneous position of the $C.G.$ of the aircraft, together with the instantaneous aircraft axes $Gxyz$.

Starting with the directions of the runway axes $GXYZ$, rotate this system about the vertical axis GZ through the angle of yaw ψ, so that GX becomes GX_1 and GY becomes GY_1. Next rotate about the horizontal transverse axis GY_1 through the angle of pitch θ; so that GX_1 becomes Gx and GZ becomes GZ_1. Finally rotate about the (nearly) horizontal longitudinal axis Gx through the angle of roll ϕ so that GY_1 becomes Gy and GZ_1 becomes Gz.

The angles ϕ, θ, ψ are all small, and their positive senses are those shown in the Fig. 4.

(f) *Ground Reactions.*—The whole purpose of the present scheme of calculation is the calculation of the reactions exerted by the ground on the undercarriage. The main reaction on the contact area of the wheel "j" is a force $-R_j$ acting in the direction Gz. In addition to this there are forces $-\mu_j R_j$ in the direction Gx and $k_j R_j$ in the direction Gy. The magnitude of these forces are conveniently described as the " vertical reaction ", the " drag force " and the " side force ", although, owing to the roll and bank of the aircraft, their lines of action are not strictly vertical and horizontal. The drag forces can be considered to act at the wheel axles, since the motion of the wheels is considered separately (see paragraph 14).

The calculation of the " vertical reaction " R_j depends upon the acceleration of the $C.G.$ in the direction Gz. This is, of course, equal and opposite to the acceleration of the point of contact (x, y, z) along Gz in the system of aircraft axes. To calculate this acceleration we proceed as follows :—

If u, v and w are the velocity components of the point (x, y, z) in the rotating system of axes, the acceleration along Gz is

$$\dot{w} + \omega_1 v - \omega_2 u \ ,$$

ω_1, ω_2 and ω_3 being the angular speeds of rotation about Gx, Gy and Gz. Now

$$\begin{aligned} u &= \dot{x} + \omega_2 z - \omega_3 y \\ v &= \dot{y} + \omega_3 x - \omega_1 z \\ w &= \dot{z} + \omega_1 y - \omega_2 x . \end{aligned}$$

Hence the acceleration along Gz is

$$\ddot{z} + 2\omega_1 \dot{y} - 2\omega_2 \dot{x} + \dot{\omega}_1 y - \dot{\omega}_2 x + \omega_1 \omega_3 x - \omega_1{}^2 z - \omega_2{}^2 z + \omega_2 \omega_3 y \ .$$

This expression for the acceleration of G along Gz is considerably simplified by the approximations introduced in paragraph 12.

The calculation of the " drag force " $\mu_j R_j$ and of the " side force " $k_j R_j$ depends upon the coefficients μ_j and k_j. These are, of course, zero when the tyre is off the ground. When it is on the ground their values depend upon whether the wheel is slipping or not, and on the angle of yaw of the wheel. Our information about the values of μ_j and k_j is very meagre. There is little doubt that μ_j approximately equals the ordinary coefficient of friction μ when the wheel is slipping and zero when it is not slipping. The value of k_j is much more uncertain. When the wheel is slipping k_j is presumably approximately equal to μa_j, where a_j is the angle between the plane of the wheel and its direction of motion. When the wheel is not slipping k_j is probably of the form $f_j a_j$, but the value of the coefficient f_j is unknown. Experiments on small tail wheel tyres suggest that the value of f_j is between 3 and 6 in lb. per lb. per radian, i.e. $\frac{1}{20}$ to $\frac{1}{10}$ in lb. per lb. per degree.

12. *Working Approximations.*—The completely general equations of motion of an aircraft landing in a banked or yawed attitude, and subject to the effects of rolling, pitching and yawing, are so complex as to be almost intractable, even if we had satisfactory information about the side forces on the wheels. It is therefore imperative to reduce these general equations to a practicable form by making every reasonable approximation.

The standard of accuracy at which we shall aim in constructing these working approximations is $\pm$ 5 per cent., a very moderate requirement from the standpoint of the computer, but one which is amply sufficient for stressing purposes.

(a) *The Vertical Motion.*—The working approximations are based on the broad general rule that in any reasonable type of landing the vertical acceleration of the *C.G.* of the aircraft rises to its peak value, of say ng, in a short interval of time T according to a roughly linear law, and then remains fairly constant at the value ng for a further interval T, during which time the vertical velocity is reduced to zero.

(A typical value of T is $0\cdot 10$ second, and typical values of n are 2 to 3).

According to this rule, when t is less than T, the acceleration is

$$ngt/T \qquad \text{upwards,}$$

the velocity is

$$V - \tfrac{1}{2}ngt^2/T \qquad \text{downwards,}$$

and the distance travelled is

$$Vt - \tfrac{1}{6}ngt^3/T \qquad \text{downwards.}$$

Hence at time $t = T$, the velocity is $V - \frac{1}{2}ngT$ and the distance travelled is $VT - \frac{1}{6}ngT^2$.

When t lies between T and $2T$, the acceleration is g upwards, the velocity is

$$(V - \tfrac{1}{2}ngT) - ng(t - T) \text{ downwards}$$

and the distance travelled is

$$(VT - \tfrac{1}{6}ngT^2) + (V - \tfrac{1}{2}ngT)\,(t - T) - \tfrac{1}{2}ng(t - T)^2.$$

Hence if the velocity is destroyed by time $t = 2T$, the initial velocity of descent is given by

$$V = \tfrac{3}{2}ngT\ ,$$

and the total distance travelled by

$$D = \tfrac{11}{6}ngT^2.$$

(With the typical values $T = 0\cdot 10$ secs. and $n = 2\cdot 5$, we find that $V = 12$ ft./sec. and $D = 17\cdot 6$ ins.)

We note that

$$D/T = \tfrac{11}{9}V.$$

Perhaps we should state explicitly that all the above formulae give merely rough approximations.

(b) *The Rotational Motion.*—If an aircraft (with no side slip) touches down simultaneously on all the wheels of the main undercarriage and the tail undercarriage (or nose undercarriage) the subsequent rotational motion is normally due to a slight rocking on the tyres. But if the aircraft touches down on the main undercarriage wheels alone, or on one only of the main undercarriage wheels, or comes in with side slip, the subsequent rotational motion will be much more violent. In these latter circumstances it seems reasonable to take the angular accelerations to be of the same order of magnitude as the vertical acceleration of the *C.G.* divided by some representative length, such as the semi-wheel track, b.

According to this rule

$$b\ddot{\theta} = ngt/T, \quad \text{if } t \text{ is less than } T,$$
$$\text{or } ng, \quad \text{if } t \text{ lies between } T \text{ and } 2T.$$

Now, initially the angular velocities are zero. Hence

$$b\dot{\theta} = \tfrac{1}{2}ngt^2/T, \qquad \text{if } t \text{ is less than } T,$$
$$\text{or } \tfrac{1}{2}ngT + ng(t - T), \quad \text{if } t \text{ lies between } T \text{ and } 2T.$$

Also, if the initial value of θ is θ_0, its value at any subsequent time is given by

$$b(\theta - \theta_0) = \tfrac{1}{6}ngt^3/T \text{ if } t \text{ is less than } T$$

or

$$\tfrac{1}{6}ngT^2 + \tfrac{1}{2}ngT(t - T) + \tfrac{1}{2}ng(t - T)^2 \text{ if } t \text{ lies between } T \text{ and } 2T.$$

Thus the angular velocity is

$$\tfrac{1}{2}ngT/b = \tfrac{1}{3}V/b \text{ at } t = T,$$

and

$$\dot{\theta} = \tfrac{3}{2}ngT/b = V/b \text{ at t} = 2T.$$

The change in the angular displacement is

$$\theta - \theta_0 = \frac{1}{11}\frac{D}{b} \text{ at } t = T$$

and

$$\theta - \theta_0 = \frac{7}{11}\frac{D}{b} \text{ at } t = 2T \ .$$

All these values are, of course, merely rough approximations to the absolute values of $\ddot{\theta}$, $\dot{\theta}$ or $\theta - \theta_0$.

(c) *The Axle Motion.*—In order to estimate the importance of the various terms in the full expression for the acceleration of G along zG we still need some rough idea of the magnitude of the axle velocity, $\dot{x}$.

Initially $\dot{x}$ is zero, but as explained later in Part III, it varies as $t^{\frac{1}{2}}$ for small values of t. A representative state of affairs is obtained by taking $\dot{x}$ to vanish when $t = 2T$ and we are thus lead to write as a simple approximation to the axle velocity,

$$\dot{x} = C\left\{\frac{t}{2T}\right\}^{1/2}\cdot\left\{1 - \frac{t}{2T}\right\}$$

whence

$$\ddot{x} = \frac{C}{2T}\left\{\frac{1}{2}\left(\frac{t}{2T}\right)^{-1/2} - \frac{3}{2}\left(\frac{t}{2T}\right)^{1/2}\right\} .$$

Thus $\dot{x}$ attains its maximum value when $t = \dfrac{2}{3}T$, and its maximum value is

$$\dot{x}_{max.} = 2C/3\sqrt{3} \ .$$

The axle travel is given by

$$\dot{x} = 2TC\left\{\frac{2}{3}\left(\frac{t}{2T}\right)^{3/2} - \frac{2}{5}\left(\frac{t}{2T}\right)^{5/2}\right\} ,$$

and the total travel is

$$\dot{x}_{max.} = 8TC/15.$$

Hence

$$\dot{x}_{max.} = 4\frac{5}{4\sqrt{3}}\frac{x_{max.}}{T} = 0\cdot 7\,\frac{x_{max.}}{T} .$$

Once again we must emphasize that these formulae are the roughest approximations and give only a general idea of the size of the quantities involved.

(d) *Effect of Rotation on Vertical Acceleration.*—We can now return to the exact expression for the acceleration of G along zG obtained in paragraph 11 (f) and estimate the order of magnitude of the various terms which it explains. We rewrite the full expression in the form

$$\begin{gathered}\ddot{z} + \dot{\omega}_1 y - \dot{\omega}_2 x \\ + 2\omega_1 \dot{y} \\ + \omega_1\omega_3 x - (\omega_1^2 + \omega_2^2)\, z + \omega_2\omega_3 y \\ - 2\omega_2 \dot{x}\end{gathered}$$

and we note from paragraph 11 (c) that at the point of contact of the j-th wheel,

$$\begin{aligned} x &= l_j, \\ y &= b_j, \\ z &= h_j + r_j - x_{t,j} \; . \end{aligned}$$

We expect $\ddot{z}$ to have the order of magnitude given by

$$\begin{aligned} \ddot{z} &= ngt/T \quad \text{if } t \text{ is less than } T \\ &\text{or } ng \quad \text{if } t \text{ lies between } T \text{ and } 2T. \end{aligned}$$

The angular accelerations $\dot{\omega}_1$ and $\dot{\omega}_2$ are expected (by paragraph 12(b)) to have the order of magnitude

$$\dot{\omega}_1\,,\; \dot{\omega}_2 = \ddot{z}/b\,.$$

Now x and y are comparable with b. Hence the term

$$\dot{\omega}_1 y - \dot{\omega}_2 x \text{ must be retained.}$$

y has the constant value b, so that the term $2\omega_1\dot{y}$ is always zero.

From the results of (b) the angular velocities, ω_1, ω_2, ω_3, are of order $\frac{1}{2}\ddot{z}\; t/b$ if t is less than T, or $\ddot{z}\;(t - \frac{1}{2}T)/b$ if t lies between T and $2T$, i.e., the angular velocities do not exceed $\frac{3}{2}\;\ddot{z}\;T/b$.

Also, x, y and z are of order b. Hence any one of the terms

$$\omega_1\omega_3 x - (\omega_1^2 + \omega_2^2)z + \omega_2\omega_3 y$$

is of order $\frac{9}{4}\;\ddot{z}\;.\;ng.\;T^2/b = \dfrac{27}{22}\cdot\dfrac{D}{b}$.

Now the ratio of D, the total vertical travel of the $C.G.$, to b, the semi-wheel track, is clearly a " small quantity " so that all these four terms can be neglected.

Finally we come to the term $\omega_2\dot{x}$. We have shown that, as regards orders of magnitude,

$$\begin{aligned} b\,\omega_2 &= \tfrac{1}{2}\, ngt^2/T \quad \text{if } t \text{ is less than } T. \\ &\text{or } ng(t - \tfrac{1}{2}T) \quad \text{if } t \text{ lies between } T \text{ and } 2T. \end{aligned}$$

Also

$$\dot{x} = \frac{15\; x_{max.}}{8T}\left\{\frac{t}{2T}\right\}^{1/2} . \left\{1 - \frac{t}{2T}\right\} .$$

After some calculation we now find that

$$\omega_2\dot{x}/\ddot{z} \text{ never exceeds } \tfrac{1}{2}\; x_{max.}/b \;,$$

an undoubtedly small quantity.

Hence our final conclusion is that a sufficient approximation to the acceleration of G along zG is

$$\ddot{z} + \dot{\omega}_1 y - \dot{\omega}_2 x$$
$$= - \ddot{x}_j - \ddot{x}_{w,j} - \ddot{x}_{t,j} + b_j\ddot{\phi} - l_j\ddot{\theta} .$$

l_j is not a constant. Its value is $l_{j,0} - x_{h,j}$ but it is usually sufficient to write

$$l_j \doteqdot \text{mean value of } (l_{j,0} - x_{h,j})$$

unless the horizontal travel $x_{h,j}$ of the axle is a substantial fraction of $l_{j,0}$, the horizontal distance of the axle forward of the $C.G.$

(e) *Rolling Moments.*—The rolling moment due to the vertical ground reaction on the j-th wheel is $- R_j b_j$, and the rolling moment due to the side force is

$$\mu\alpha_j R_j z_j \text{ if the wheel is slipping, or}$$
$$f_j\alpha_j R_j z_j \text{ if the wheel has ceased to slip.}$$

In the first case the ratio of the moment due to the side force to the moment due to the vertical reaction is

$$\mu\alpha_j(z_j/b_j) .$$

(With the typical values $\mu = 0{\cdot}8$, $\alpha_j = 6$ degrees, $z_j = \frac{2}{3}\, b_j$, this ratio is about 5 per cent.)

In the second case the angle α_j, which is approximately the angle of yaw ψ, will have diminished a certain amount. The ratio will now be $f_j\psi\ (z_j/b_j)$, and, in our present state of ignorance as to the value of f_j we can only assume that as before this ratio is small.

We shall therefore always take for the rolling moment due to ground reaction the approximate expression $-R_j b_j$.

(f) *Yawing Forces and Moments.*—The calculation of the yawing forces and moments acting on the aircraft is difficult as it involves a reasonably accurate knowledge of the magnitude of the side forces on the tyres. But fortunately the yawing forces and moments either do not enter or can be neglected in the equations of vertical motion, or of rolling or of pitching, and are therefore not required for the main part of the calculations. This is perhaps the most fortunate circumstance in the whole scheme of prediction.

(g) *Changes in Lift.*—The rapid destruction of the vertical velocity of descent after touch down produces a rapid change in the effective angle of incidence of the wings and tailplane. (The effect of the pitching motion is negligible in comparison with the effect of the vertical acceleration.) The corresponding change in the lift is approximately

$$\Delta L = - L_0\,(V/U\alpha)$$

where
V = vertical speed of descent just before touch down,
U = forward speed of aircraft,
L_0 = lift just before touch down = W = all up weight,
and
α = effective angle of incidence just before touch down.

In terms of the maximum ground reaction R, the change of lift is

$$\Delta L/R = - \frac{V}{U\alpha} \cdot \frac{W}{R}$$
$$= - V/(n\alpha U)$$
$$= - \tfrac{3}{2} gT/(U\alpha) \text{ by § } 12(a).$$

Taking the typical values $T = 0{\cdot}1$ sec., $U = 100$ m.p.h., and $\alpha = 17$ degs. or $0{\cdot}3$ radians, we find that

$$\Delta L/R = - 10\%.$$

If we also consider the "ground effect" we find that the lift increases somewhat as the aeroplane glides down to earth. The combined result of the "ground effect" and of the loss of lift due to destruction of vertical velocity is that the lift may be taken as constant and equal to W to the order of approximation required in these calculations.

(h) *Wing Flexibility*.—The effect of wing flexibility is usually negligible in a level landing in which the reactions are the same on the port and starboard undercarriages, but it may be substantial in a banked landing in which all the reaction is initially on one of the main undercarriage legs.

The reason for the small effect of wing flexibility in a level landing is that the main undercarriage legs are usually attached to the wings near the nodes of the fundamental symmetric mode of wing vibration, so that this mode cannot be elicited by the ground reactions transmitted up the legs. The symmetric flexural overtones are usually ineffective because of their relatively high frequency and the antisymmetric modes are naturally not elicited by symmetrically distributed reactions. The effective modes are usually the torsional modes in which engine noding is the dominant feature. However in the present types of aircraft with weights up to 60,000 lb., it is usually sufficient to treat the wings as rigid.

This conclusion may be no longer true in the case of much larger aircraft. The importance of wing flexibility in those conditions is still unknown, but calculations are in hand to assess its importance.

13. *Equations of Motion for an Idealised Landing*.—In constructing the equations of motion, we begin with the simplest possible case, viz. an idealised landing in which the aircraft touches down without bank or yaw and in which no subsequent pitching motion occurs. We shall find that many of the complex landing cases can be resolved by reference to this idealised case, and that the variation in the vertical reaction which occurs in the actual landing cases can be obtained by applying a suitable scale factor to the results of the idealised case. Moreover it will be shown that the idealised landing case closely simulates a drop test "under airborne conditions", so that it is possible to use the experimental results of such a drop test instead of the theoretical calculations for the idealised landing case.

As we have shown in paragraph 11(c) the height of the $C.G.$ of the aircraft above the ground at any instant is

$$\begin{aligned} z &= h + r - x_t \\ &= h_0 + r - x - x_w - x_t \end{aligned}$$

where we have suppressed the suffix "j", since in an idealised landing the aircraft lands symmetrically, and all the main undercarriages experience equivalent reactions and displacements. The nose (or tail) undercarriage is supposed not to make contact with the ground. The actual vertical travel of the $C.G.$ since the moment of touch down is

$$\begin{aligned} s &= h_0 + r - z \text{ (downwards)} \\ &= x + x_w + x_t \\ &= \text{axle travel} + \text{wing travel} + \text{tyre deflection.} \end{aligned}$$

The equation of vertical motion of the aircraft is

$$(W/g)\,\ddot{s} = -R\ ,$$

R being the ground reaction vertically upwards. The initial conditions are

$$\begin{aligned} x &= 0, \quad & x_w &= 0, \quad & x_t &= 0, \\ \dot{x} &= 0, \quad & \dot{x}_w &= 0, \quad & \dot{x}_t &= V, \end{aligned}$$

where V = vertical rate of descent of aircraft.

Furthermore, we have the " tyre equation "

$$R = f(x_t) ;$$

the " shock absorber equation ",

$$\dot{x} = D(R - Q - F)^{1/2}.$$

where D, Q, F are tabulated functions of x; and finally the " wing equation ", which will reduce to the simple form

$$x_w = 0 ,$$

if the positions of attachment of the main undercarriage legs were chosen so that the lowest symmetric modes of wing vibration are not excited. We shall suppose this condition to be fulfilled as otherwise the values of x, x_w and x_t would vary from leg to leg by reason of the variation of wing deflection along the span.

In carrying out the calculations it is necessary to allow for the effect of ground friction as shown in the following section.

14. *Spinning-up of Wheels and Braking.*—From the first instant that it touches the ground each wheel skids longitudinally until the drag force exerted by the ground has spun it up to a speed of rotation at which it can roll without skidding. It will be assumed that the drag force is equal to μR where μ is the coefficient of friction between the tyre and the runway, while R is the vertical ground reaction. Typical values of μ are

Dry, ribbed concrete	$\mu = 0 \cdot 8$ to $1 \cdot 0$
Hard dry grass	$\mu = 0 \cdot 6$ to $0 \cdot 8$
Wet concrete	$\mu = 0 \cdot 4$ to $0 \cdot 5$
Wet grass	$\mu = 0 \cdot 2$ to $0 \cdot 4$

It has been shown by Mr. Strang of the Bristol Aeroplane Company that the total torque on the wheel, allowing for the backward displacement of the line of action of the vertical reaction is $\mu R_j \varrho_j$ where ϱ_j is the effective rolling radius under load R_j (ref. 9). Hence the angular momentum of the wheel after time t is $\int_0^t \mu R_j \, . \, \varrho_j dt$. When the wheel is fully spun up its angular speed is V/ϱ_j, where V is the forward speed of the aircraft. Experiments (ref. 9) at low speeds and light loads suggest that approximately

$$\varrho_j = r_j - \tfrac{1}{3} x_{t,j}.$$

Hence, if J is the polar moment of inertia of the wheel and tyre, the angular momentum when it is fully spun is

$$(J/g) \, . \, (V/\varrho_j).$$

Therefore the time t taken to spin up the wheel is given by the equation

$$\varrho_j \int_0^t \mu R_j \, . \, \varrho_j \, . \, dt = \frac{JV}{g}$$

= a constant for all wheels at a prescribed landing speed on a specified runway.

ϱ_j and R_j all vary with the time, and the integral is easily evaluated numerically at each 1/100th of a second during the landing run, the calculation being continued until the expression on the left-hand side of this equation attains the value of the right-hand side. The wheel is then fully spun up and the frictional drag force sinks to zero.

This spinning up calculation is mainly of importance because it determines the drag forces acting on the undercarriage. It is also of subsidiary importance in determining the variation of the angle γ which the total reaction R_j makes with the vertical. During spinning up, $\tan \gamma = \mu$; afterwards during rolling $\gamma = 0$. Reference to paragraph 6 will show that the coefficients Q and D depend on the angle γ, and will therefore experience abrupt changes in value at the instant when the wheel becomes fully spun up.

There are two special cases in which this calculation is not required.

(1) In a landing with " brakes on "—on emergency precaution occasionally required to prevent over-shooting the runway—the drag force remains equal to μR_j throughout the landing run, and the angle γ is constantly equal to $tan^{-1}\mu$.

(2) In a landing after a bounce the wheels may be presumed to be already spun up during the previous touch down. Hence the drag force is now zero and the angle γ is also zero throughout the landing run.

It is convenient to note here the effect of the brakes on the reaction transmitted through the undercarriage legs. When the brakes are not applied the torque due to any drag force at the contact area of the tyre is almost completely balanced by the rotary inertia of the wheel so that almost no torque is transmitted to the undercarriage leg. But when the brakes are applied so as to produce a braking torque C, there will be a drag force D acting at the contact area and a force D_1 acting forwards on the wheel at the axle. The reaction on the undercarriage leg will therefore consist of a torque C plus a drag force D_1 acting at the axle in addition to the vertical reaction R.

The equations of motion of the wheel are

$$(J/g)\ddot{\theta} = D\varrho - C$$

and

$$D - D_1 = (w/g)\,\dot{u} \;,$$

where w is the weight of wheel and tyre and u is the forward velocity of the axle. Now, considering the whole aircraft

$$(W/g)\,\dot{u} = -D - D_A \;,$$

where D_A is the net aerodynamic drag. Hence

$$\begin{aligned} D_1 &= D - (w/g)\dot{u} \\ &= D + \frac{w}{W}\left\{D + D_A\right\} \;, \end{aligned}$$

so that D_1 is approximately equal to D. (These equations apply to each wheel considered separately).

When the wheel is slipping D is equal to μR, but, when the wheel is fully spun up, its rotary inertia is negligibly small* and

$$D\varrho = C$$

The time taken to spin up the wheel will of course be lengthened since the spinning couple is reduced from $\mu R\varrho$ to $\mu R\varrho - C$.

---

* For $(J/g)\ddot{\theta}$ is approximately $J\dot{u}/(rg)$ or say nJ/r, while C is of the order of $\frac{1}{4}W/r$, and the ratio $(nJ/r) \div (\frac{1}{4}W/r)$ or $4nJ/Wr^2$ is negligibly small.

15. *Equations of Motion for a Drop Test.*—The present standard form of drop test differs in two important respects from an idealised landing :—

(1) In an idealised landing the weight W is neutralised by the lift L, but in the drop test there are no forces corresponding to the lift ;

(2) In an idealised landing on horizontal ground a drag force μR acts until the wheels have been spun up, but in the drop test the unit falls on to an inclined plane of angle $\gamma = \tan^{-1}\mu$ and a drag force approximately equal to μR acts throughout the whole of the test.

In forming the equations of motion for a drop test we must therefore allow for the vertical distance x_g through which the contact region of the tyre descends as it rolls down the inclined plane. The height of the mass dropped above the initial point of impact is therefore

$$\begin{aligned} z &= h + (r - x_t) \cos \gamma - x_g \\ &= h_0 - x + (r - x_t) \cos \gamma - x_g . \end{aligned}$$

The actual vertical travel of the mass is

$$\begin{aligned} s &= x + x_t \cos \gamma + x_g \\ &= \text{axle travel} + \text{vertical component of tyre deflection} + \text{ground sinkage.} \end{aligned}$$

The equation of vertical motion is

$$(W/g)\, \ddot{s} = - R + W \ ,$$

and the initial conditions are

$$\begin{aligned} x &= 0, \quad & x_t &= 0, \quad & x_g &= 0 \ , \\ \dot{x} &= 0, \quad & \dot{x}_t &= V \ . \end{aligned}$$

We also have the geometrical condition :—

horizontal axle velocity $\dot{x}_h = \dot{x}_g \cos \gamma - \dot{x}_t \sin \gamma$ (see Fig. 4).

Hence

$$\begin{aligned} \dot{s} &= \dot{x} + \dot{x}_t \cos \gamma + (\dot{x}_h \tan \gamma + \dot{x}_t \sin \gamma \tan \gamma) \\ &= \dot{x} + \dot{x}_t \sec \gamma + \mu \dot{x}_h = \dot{y} + \mu \dot{x}_h \ , \end{aligned}$$

say where y = vertical mass travel relative to point of impact.

16. *Comparison of Idealised Landings and Drop Tests.*—In spite of the differences between an idealised landing and a drop test it is possible to choose the conditions under which a drop test is carried out so that the behaviour of the shock absorber closely simulates its behaviour in an idealised landing. These conditions are :—

(1) that the same amount of energy is absorbed in each case by the tyre and shock absorber in bringing the mass dropped to rest, and

(2) that the time of operation is the same in each case.

When these conditions are satisfied we say that the drop test is carried out " under airborne conditions ".

The fact that a drop test carried out under airborne conditions does exhibit shock absorber characteristics very similar to those shown in an idealised landing can be generally substantiated by carrying out step by step calculations and comparing the variation of the vertical ground reaction R with the vertical axle travel x in the two cases. The agreement between the $R, x \sim$ curves is so satisfactory that the present standard form of drop test can be accepted as an experimental representation of an idealised landing.

The detailed form of the " airborne conditions" can be obtained as follows, extending the analysis given by Makovski (Ref. 8).

In an idealised landing the energy absorbed by the tyre and shock absorber is

$$E_1 = \int R dx_t + \int P dx_s \text{ (in the notation of para. 5)}$$

$$= \int R dx_t + \int (R dx + F dx_h) \ ,$$

where F is the horizontal force on the wheel axle.

Now $x_t + x = s$, and $R = -(W/g)\ddot{s}$
so that

$$\int R dx_t + \int R dx = -(W/g)\int \ddot{s} \,.\, \dot{s}\,, dt = \tfrac{1}{2} W_1 V_1^2/g$$

where W_1 is the weight and V_1 the vertical rate of descent.

Also

$$F - \mu R = (w/g)(\dot{u} - \ddot{x}_h) \ ,$$

where w = weight of wheel and tyre, and u = forward velocity of aircraft. Hence

$$\int F dx_h - \mu \int R dx_h = (w/g)\int \dot{u}\, dx_h - (w/g)\left[\tfrac{1}{2}\dot{x}_h^2\right] .$$

Now the horizontal axle velocity $\dot{x}_h$ is initially zero and is usually small by the time that $\dot{s} = 0$. The retardation of the aircraft is approximately $\mu g R/W$, so that

$$(w/g)\int \dot{u}\, dx_h \doteqdot -\mu \frac{w}{W}\int R\, dx_h \text{ , and}$$

$$\int F\, dx_h = \left(1 - \frac{w}{W}\right)\mu \int R\, dx_h$$

$$\doteqdot \mu \int R\, dx_h \ .$$

Therefore in an idealised landing the energy absorption is

$$E_1 = \tfrac{1}{2} W_1 V_1^2/g + \mu \int R\, dx_h \ ,$$

V_1 being the impact velocity.

To estimate the small correction $\mu \int R\, dx_h$ to the main term $\frac{1}{2} W_1 V_1^2/g$, we may assume that

$$\int R\, dx_h \div \int R ds \doteqdot x_{hm}/s_m$$

where x_{hm}, s_{1m} are the final values of x_h and s_m, i.e. the total horizontal axle travel and the total vertical travel of the $C.G.$ respectively. Then

$$\int R dx_h \doteqdot (x_{hm}/s_{1m}) \,.\, \tfrac{1}{2} W_1 V_1^2/g$$

and

$$E_1 \doteqdot (1 + \mu x_{hm}/s_{1m}) \,.\, \tfrac{1}{2} W_1 V_1^2/g \ .$$

To calculate the energy absorption in a drop test we first consider the forces acting on the wheel as it rolls down the inclined plane (see Fig. 4). Let T be the reaction of the plane perpendicular to its surface, S_1 the reaction parallel to the plane and S_2 the force exerted in the same direction by the wheel fork. Then, ignoring the difference between rolling radius and axle height,

$$\varrho S_1 = J\ddot{\theta},$$

where θ is the angular rotation of the wheel. Also

$$S_1 - S_2 = (w/g)\,\ddot{x}_h = (w/g)\,\varrho\ddot{\theta}$$

so that

$$S_2 = (J/\varrho - w\varrho/g)\,\ddot{\theta}\ .$$

Hence

$$\int S_2 dx_h = \int (J - w\varrho^2/g)\,\ddot{\theta}\dot{\theta}\,dt \doteqdot (J - w\varrho^2/g)\,(\tfrac{1}{2}\dot{\theta}^2) = 0\,,$$

i.e., the force S_2 does no appreciable work.

The energy absorption in a drop test is

$$\begin{aligned} E_2 &= \int T.dx_t + \int P dx_s \\ &= \int R \sec\gamma\,.\,dx_t + \int (R\,dx + \mu R\,dx_h) \end{aligned}$$

But

$$\mu dx_h = dx_g - \mu \sin\gamma\,.\,dx_t$$

and

$$\sec\gamma\,dx_t + \mu dx_h = dx_g + \cos\gamma\,.\,dx_t\ ,$$

so that

$$\begin{aligned} E_2 &= \int R(dx + dx_g + \cos\gamma\,dx_t) \\ &= \int R ds \\ &= -\,(W_2/g)\int \ddot{s}\,.\,\dot{s}\,dt + W_2\int ds \\ &= \tfrac{1}{2}\,W_2 V_2^2/g + W_2 s_{2m} \end{aligned}$$

s_{2m} being the total travel of the mass from the moment of impact, V_2 the impact velocity, and W_2 the weight of the mass dropped.

Now in a drop test

$$s_{2m} = y_m + \mu x_{hm}\ ,$$

y_m being the maximum vertical mass travel relative to the point of impact. y_m will be approximately the same as s_{1m} in an idealised landing. Hence the first "airborne condition", $E_1 = E_2$, becomes

$$\left(1 + \frac{\mu x_{hm}}{y_m}\right)\frac{1}{2}\,\frac{W_1 V_1^2}{g} = \frac{1}{2}\,\frac{W_2 V_2^2}{g} + W_2(y_m + \mu x_{hm})\,.$$

We also note that

$$\begin{aligned} 1 + \frac{\mu x_{hm}}{y_m} &= 1 + \frac{\mu x_{hm}}{s_{2m} - \mu x_{hm}} \\ &= \frac{1}{1 - \mu x_{hm}/s_{2m}} \end{aligned}$$

To deal with the second condition, viz. that the times of operation should be the same, we assume that these times are proportional to the total travel of the mass dropped divided by the impact velocity. This implies that

$$\frac{y_m}{V_1} = \frac{y_m + \mu x_h}{V_2} ,$$

i.e., that

$$V_2 = \frac{V_1}{1 - \mu x_{hm}/s_{2m}}$$

From the preceding equations we can determine the values of W_2 and V_2 in a drop test which correspond to an idealised landing with prescribed values of W_1 and V_1.

Laboratory drop tests on to an inclined plane are usually carried out for angles γ which correspond to values of μ of about $0 \cdot 4$. In the corresponding idealised landing the time taken to spin up the wheels would exceed the time for a complete stroke of the shock absorber, so that comparable drag forces μR act in both cases for a complete stroke.

17. *Classification of Actual Landing Cases.*—We now pass from the idealised landing case, in which there is no rotational motion, to the actual landing cases, and we consider these in order of increasing complexity. The factor which determines the mathematical complexity of a landing case is the number of degrees of freedom which are involved. We shall therefore discuss the actual landing cases in the following order, remembering that although a nosewheel aircraft may pitch forward on to its nose wheel, a tail wheel aircraft hardly ever pitches backwards on to its tail wheel during the initial stages of the landing run, with which we are alone concerned.

A. *Aircraft with Rigid Wings, or Wings in which no Effective Modes of Vibration are excited by Landing Shock.*—(1) A symmetric landing, free from yaw and bank, in which a tail-wheel aircraft pitches forward on all the main wheels.

(2) A level yawed landing, in which a tail-wheel aircraft pitches forward on all the main wheels.

(3) A banked landing, free from yaw, in which a tail-wheel aircraft rolls and pitches forward on one main wheel.

(4) The next stage in a symmetric, yawed or banked landing of a nosewheel aircraft, after it has pitched forward on to the nose wheel, and is pitching and rolling on all its wheels.

B. *Aircraft with Flexible Wings.*—Investigations are now proceeding with a view to simplifying the theory and standardising the calculations, and a further report will be issued later.

18. *Symmetric Landing of a Tail-Wheel Aircraft with Rigid Wings.*—If all the undercarriage units are similar to one another and are similarly mounted, they will all behave in exactly the same way and need not be distinguished by special suffices.

In a symmetric landing there is no tendency to roll or yaw but there will be a tendency to pitch. Reference to Fig. 3 and paragraph 12(d) and (e) will show that the upward vertical acceleration of the $C.G.$ of the aircraft is

$$- \ddot{x} - \ddot{x}_t - l\ddot{\theta} ,$$

while the pitching moments are

$$Rl \text{ and } - \mu R\dot{h}$$

using R for the vertical reaction on any one undercarriage unit.

Hence the equations of vertical motion and of pitching motion are

$$NR = -(W/g)(\ddot{x} + \ddot{x}_t + l\ddot{\theta}),$$

(N being the number of main undercarriage legs) and

$$NR(l - \mu h) = (W/g)k_y^2\ddot{\theta}.$$

Therefore, eliminating the pitching acceleration $\ddot{\theta}$, we find the working equation

$$\boxed{NYR = -(W/g)\,(\ddot{x} + \ddot{x}_t)}$$

where $Y = 1 + l\,(l - h)/k_y^2$, is the rotational factor, as in a yawed landing, introduced in paragraph 9.

The forces acting on each undercarriage unit are the vertical reaction R, and the drag force μR.

It is clear that the vertical reaction R can be deduced immediately from idealised landing calculations or from drop test observations, if the rotational factor Y is sensibly constant, by dividing the calculated or observed values of R by Y.

19. *Level Yawed Landing of a Tail-Wheel Aircraft with Rigid Wings.*—It has been shown in paragraph 12(*e*) that we can neglect the effect of the side forces on the rolling moments. The total rolling moment is therefore

$$-M = -\,(b_2R_2 + b_3R_3 + b_4R_4 + \ldots) \; .$$

In the case of a level yawed landing the reactions R_j will be distributed symmetrically over the port and starboard undercarriages. Hence the moment, M, of these reactions about the longitudinal axis of the aircraft must always vanish. It therefore follows that there is *no rolling motion in a level yawed landing.*

We shall assume that all the undercarriage units are similar to one another and similarly mounted, so that they will all behave in exactly the same way and need not be distinguished by suffices.

Reference to Fig. 3 and paragraph 12(d) and (e) will show that the upward vertical acceleration of the $C.G.$ of the aircraft is

$$-\ddot{x} - \ddot{x}_t - l\ddot{\theta} \;,$$

while the pitching moments are

$$Rl \text{ and } -\mu Rh \;,$$

using R for the vertical reaction on any one undercarriage unit.

Hence the equations of vertical motion and of pitching motion are (exactly as in the previous case of a symmetric landing)

$$NR = -\,(W/g)\,(\ddot{x} + \ddot{x}_t + l\ddot{\theta}) \;,$$

(N being the number of main undercarriage legs) and

$$NR(l - \mu h) = (W/g)k_y^2\ddot{\theta} \; .$$

Therefore, eliminating the pitching acceleration $\ddot{\theta}$, we find the working equation

$$\boxed{NYR = -(W/g)\,(\ddot{x} + \ddot{x}_t)}$$

where $Y = 1 + l(l - \mu h)/k_y^2$ is the rotational factor for a level yawed landing.

The vertical ground reaction in a yawed landing can therefore be deduced immediately from idealised landing calculations or drop test observations, by dividing the ground reactions in the drop test by the factor Y, which is approximately constant.

The other forces on the wheels are the drag force μ_jR_j and the side force k_jR_j, the values of the coefficients μ_j and k_j are briefly discussed in paragraph 11(f).

20. *Banked Landing of Tail-Wheel Aircraft with Rigid Wings.*—In a banked landing the aircraft touches down on one main wheel, here taken to be the outer starboard wheel, and then rolls over until the other main wheel touches the ground. Thereafter both wheels usually remain on the ground, although there is a possibility of rocking from one main wheel to the other.

In the first stage of the landing, while only one main wheel is in contact with the ground, the equations of motion can be formed as follows :—

Reference to Fig. 3 and paragraph 12(d) and (e) will show that the upward vertical acceleration of the *C.G.* of the aircraft is

$$-\ddot{x}_2 - \ddot{x}_{2,t} + b_2\ddot{\phi} - l_2\ddot{\phi} \ ,$$

while the only rolling moment is $-R_2b_2$. The pitching moments are

$$R_2l_2 \text{ and } -\mu_2R_2h_2$$

where

$$l_2 = l_{2,0} - x_{2,h}$$

and

$$h_2 = h_{2,0} - x_2 \ ,$$

(the drag force being considered to act at the wheel axle).

Hence the equations of vertical motion, rolling motion and pitching motion are

$$R_2 = (W/g)\left\{ -\ddot{x}_2 - \ddot{x}_{2,t} + b_2\ddot{\phi} - l_2\ddot{\theta} \right\} ,$$

$$-R_2b_2 = (W/g)k_x{}^2\ddot{\phi} \ ,$$

and

$$R_2l_2 - \mu_2R_2h_2 = (W/g)k_y{}^2\ddot{\phi} \ .$$

If we substitute the values of $\ddot{\phi}$ and $\ddot{\theta}$ in the equation of vertical motion we obtain the working equation

$$\boxed{B_{22}R_2 = -(W/g)\ (\ddot{x}_2 + \ddot{x}_{2,t})}$$

where B_{22} is the first rotational factor for a banked landing,

$$B_{22} = 1 + \frac{b_2{}^2}{k_x{}^2} + \frac{l_2{}^2 - \mu l_2h_2}{k_y{}^2}$$

introduced in paragraph 9.

This equation is of exactly the same *form* as the equation of vertical motion for an idealised landing, the only difference being that R_2 is now multiplied, not by unity, but by the "rotational" factor in curly brackets which allows for the effects of rolling and pitching. This factor is not constant, since l_2 and h_2 both vary with the time, but it may often be treated as constant, if $x_{2,h}/l_{2,0}$ and $x_2/h_{2,0}$ are small, or if the uncertainty in the value of k_y makes it superfluous to consider variations in l_2 and h_2.

If the " rotational factor " B_{22} is treated as constant we can immediately use either.

(a) the experimental results of a drop test (with appropriate load and initial vertical speed), or

(b) the numerical calculations for an idealised landing, and obtain at once the variation in R_2 with time by dividing the drop test results by the value of the rotational factor B_1.

In the second stage of the landing, when two or more main wheels are on the ground, the formation of the equations of motion is a little more complex.

If we consider the case of an aircraft with only two main undercarriage legs, we shall have two expressions for the upward vertical acceleration $\ddot{H}$ of the *C.G.*, viz.,

$$\ddot{H} = -\ddot{x}_2 - \ddot{x}_{2,t} + b_2\ddot{\phi} - l_2\ddot{\theta}$$

and

$$\ddot{H} = -\ddot{x}_3 - \ddot{x}_{3,t} + b_3\ddot{\phi} - l_3\ddot{\theta} \quad ,$$

(b_3 being negative and equal to $-b_2$).

The equations of vertical motion, of rolling motion and of pitching motion are now

$$R_2 + R_3 = (W/g) \, . \, \ddot{H} \; ,$$

$$-R_2b_2 - R_3b_3 = (W/g)k_x{}^2\ddot{\phi} \; ,$$

$$R_2(l_2 - \mu_2h_2) + R_3(l_3 - \mu_3h_3) = (W/g)k_y{}^2\ddot{\theta} \; .$$

We now substitute the values of $\ddot{\phi}$ and $\ddot{\theta}$ first in one of the expressions for $\ddot{H}$, and then in the other. We thus obtain the working equations

$$\boxed{\begin{aligned} B_{22}R_2 + B_{23}R_3 &= -(W/g) \, . \, (\ddot{x}_2 + \ddot{x}_{2,t}) \\ B_{32}R_2 + B_{33}R_3 &= -(W/g) \, . \, (\ddot{x}_3 = \ddot{x}_{3,t}) \end{aligned}}$$

where B_{22}, B_{23}, B_{32}, B_{33} are the " rotational factors " of paragraph 9.

Finally we consider the case of an aircraft with three main undercarriages. After touching down on the starboard main wheel the aircraft will roll over on to the central wheel (in accordance with the first working equation for R_2), then roll further with both the starboard wheel and the central wheel on the ground (in accordance with the second pair of working equations for R_2 and R_3), and finally rock on all three of the main wheels.

In the last stage of the motion there will be three expressions for the upward vertical acceleration of the *C.G.* viz.

$$\ddot{H} = -\ddot{x}_k - \ddot{x}_{k,t} + b_k\ddot{\phi} - l_k\ddot{\theta} \; , \; (k = 2, 3, 4)$$

where

$$b_4 = -b_2 \text{ and } b_3 = 0.$$

The equations of vertical motion, of rolling motion and of pitching motion are

$$R_2 + R_3 + R_4 = (W/g) \, \ddot{H} \; ,$$

$$-R_2b_2 - R_3b_3 - R_4b_4 = (W/g) \, k_x{}^2\ddot{\phi} \; ,$$

$$R_2(l_2 - \mu_2h_2) + R_3(l_3 - \mu_3h_3) + R_4(l_4 - \mu_4h_4) = (W/g)k_y{}^2\ddot{\theta} \; .$$

Following the same procedure as before we find the working equations

$$B_{k2}R_2 + B_{k3}R_3 + B_{k4}R_4 \quad = -(W/g) \, (\ddot{x}_k + \ddot{x}_{k,t})$$

$$(\text{for } k = 2,3,4) \; .$$

Similar methods can obviously be applied to the banked landing of a tail-wheel aircraft with rigid wings with any number of main undercarriage legs.

Perhaps we should add one word of caution viz.

B_{jk} and B_{kj} are, in general, different in value (unless of course $k = j$).

21. *Rocking of a Nose-Wheel Aircraft* (*with rigid wings*) *on its Main Wheels and Nose Wheel.*— In the initial stages of a symmetrical, yawed or banked landing of a nose-wheel aircraft the same equations of motion are valid as developed in paragraphs 18, 19, 20 for a tail-wheel aircraft.

After the lapse of a certain time however the nose wheel will make contact with the ground and thereafter the aircraft will rock on all its tyres. In this section we shall construct the equations of motion allowing for both pitching and rolling.

Each of the undercarriage legs, including the nose wheel, will furnish a different expression for the upward vertical acceleration $\ddot{H}$ of the *C.G.* of the aircraft. Reference to Fig. 3 and paragraph 12(d) shows that these expressions are

$$\ddot{H} = -\ddot{x}_j - \ddot{x}_{t,j} + b_j\ddot{\phi} - l_j\ddot{\theta} \ ,$$
$$\text{for } j = 1,2,3 \ \ldots\ldots$$

Hence the equations of vertical motion are

$$R_1 + R_2 + \ldots = -(W/g)\left\{\ddot{x}_j + \ddot{x}_{t,j} - b_j\ddot{\phi} + l_j\ddot{\theta}\right\}.$$

As before (see paragraph 12(e)) the rolling moments are $-R_jb_j$, while the pitching moments are R_jl_j and $-\mu R_jh_j$. Hence the equations of rolling and pitching motion are

$$-R_1b_1 - R_2b_2 - \ldots = (W/g)k_x{}^2\ddot{\phi},$$

and

$$R_1(l_1 - \mu h_1) + R_2(l_2 - \mu h_2) + \ldots = (W/g)k_y{}^2\ddot{\theta}.$$

We substitute the values of $\ddot{\phi}$ and $\ddot{\theta}$ in the equation of vertical motion and thus obtain (by the same process as in paragraph 20) the working equations

$$\boxed{\begin{array}{c} B_{j1}.R_1 + B_{j2}R_2 + \ \ldots = -(W/g)\,(\ddot{x}_j + \ddot{x}_{t,j}) \ , \\ \text{for } _j = 1,2, \ \ldots \ , \end{array}}$$

the symbol B_{jk} denoting the rotational banking factor

$$B_{jk} - 1 + \frac{b_jb_k}{k_x{}^2} + \frac{l_j(l_k - \mu h_k)}{k_y{}^2} \ .$$

22. *Determination of the Instant at which a Wheel makes Contact with the Ground.*—In carrying out the calculations for a landing in which the aircraft rolls over on one wheel or pitches forward on the main wheels until another wheel or wheels come into contact with the ground, it is necessary to determine the instant at which this contact occurs, as a new reaction comes into play at this instant and a new set of equations of motion must be used.

The co-ordinates of the lowest point of the tyre on the j-th. undercarriage unit (while it is still in the air) are

$$x = l_j \ , \qquad y = b_j \ , \qquad z = h_j + r_j$$

where

$$h_j = h_{j,0} - x_{\omega,j} \, .$$

Hence the vertical distance of this point below the *C.G.* is

$$D_j = z + y\phi - x\theta$$
$$= h_{j,0} - x_{\omega,j} + r_j + b_j\phi - l_j\theta.$$

If the k-th. wheel is in contact with the ground, a similar calculation shows that the height of the *C.G.* above the ground is

$$H = h_{k,0} - x_k - x_{\omega,k} + r_k - x_{t,k} + b_k\phi - l_k\theta \ .$$

Hence to determine the instant at which the j-th wheel makes contact with the ground, it is sufficient to keep a tally of the values of H and of D_j and to note when D_j becomes equal to H.

PART III

Step-by-Step Methods of Integration

23. *Advantages and Disadvantages of Step-by-Step Methods.*—In the following sections, paragraphs 23–26, we give an account of a step-by-step method of calculating the undercarriage loads which are developed during various types of landing. The method explained here is a development of the original method given by E. Jones and F. C. R. Cook, (Ref. 1), as extended by D. D. Lindsay and R. G. Thorne (Refs. 3 and 4), and as simplified by S. A. Makovski (Ref. 5). The present development of these methods has been the outcome of a number of experiments with various step-by-step methods in which special attention has been paid to ease of operation by unskilled computers and to the determination of the accuracy of the results.

Before speaking of the advantages of step-by-step methods in general it may be as well to meet one obvious criticism. This criticism is that our knowledge of the basic data is far too uncertain to justify prolonged and laborious numerical calculations, and that it would be preferable to employ some rapid and approximate method based on considerations of energy balance.

In order to appreciate the force of this criticism we have made a number of attempts to use such rapid and approximate energy-methods, working on lines similar to those sketched by R. Hadekel (Ref. 2); but our experience has been that such methods, when applied, over a wide range of conditions, to shock absorber units, which are not necessarily of the optimum design and characteristics, are more laborious than step-by-step methods, and, are not only less accurate, but do not provide any means of estimating the degree of accuracy. Moreover, we have found in practice step-by-step methods are by no means prolonged or laborious, having due regard to the fact that they provide a complete picture of the time variation of all the geometrical and dynamical characteristics of the problem.

Turning now to the advantages and disadvantages of step-by-step methods we may tabulate these as follows :—

(1) Step-by-step methods are simple in principle, since they are constructed from the standard equations of motion by replacing ordinary instantaneous time derivatives, such as $dx(t)/dt$, by average rates of increase, such as

$$\frac{x(t+\tau)-x(t-\tau)}{2\tau}$$

where τ is the fundamental time interval adopted in the investigation.

(2) Step-by-step methods are also simple in practice, provided they are expressed as a straightforward routine of operations. The method explained below has been tested by a number of computers, none of whom had any specialised knowledge of undercarriage or shock-absorber problems, and it has been approved by them as simple and direct.

(3) These methods are self checking, in the sense that an arithmetical slip is noticed almost at once, if the results are kept plotted as a graph while the calculation proceeds.

(4) The errors introduced by substituting average rates of increase for instantaneous derivatives are easily estimated, and they can be reduced to any initially prescribed value by choosing the fundamental time interval τ to be sufficiently small.

(5) The inevitable boredom induced by any computation occupying a whole day is considerably alleviated by the interest aroused when a running record of the results is kept as a graph or series of graphs. This has the effect of providing a very slow motion picture of the aircraft manoeuvre which is being studied.

Against these advantages we must offset what appears to be the one and only disadvantage of step-by-step methods. To determine the effect of varying any parameter, such as the vertical rate of descent of the aircraft, it is necessary to make a separate and complete computation for each value of the parameter which is to be considered. Fortunately the number of cases which need be considered by the designer is small so that this disadvantage does not seriously affect step-by-step methods applied to undercarriage problems. It is however a serious limitation of these methods when they are applied to general theoretical problems.

24. *Principle of the Method and Accuracy of its Results.*—This section describes an extremely simplified method of step-by-step calculation—so simple, indeed, that it may excite the scorn of the professional computer accustomed to work with elaborate difference tables and complex interpolation formulae. Nevertheless it is entirely adequate for our present purposes in which 5 per cent. accuracy is all that is required.

Let us suppose that the velocity dx/dt of some moving part of the mechanism under examination can be calculated for any position x and at any time t. Let the initial value of x at time $t = 0$ be a_0. It is proposed to calculate x at intervals of time τ, i.e. at the instants

$$t = \tau,\ 2\tau,\ 3\tau,\ 4\tau,\ \ldots.,\ n\tau,\ (n + 1)\tau,\ \ldots.$$

up to the final instant at the end of the period in which we are interested. (Usually $\tau = 0{\cdot}01$ or $0{\cdot}005$ second and the period covers about $0{\cdot}15$ seconds).

The calculation proceeds in three stages—the start, the run and the finish.

The Start of the Calculation.—To start the calculation we must calculate the value of x, say x_1, when $t = \tau$. This is easily achieved if we can determine the value of the velocity dx/dt, say $\dot{x}_0$, and of the acceleration d^2x/dt^2, say $\ddot{x}_0$, at $t = 0$. For, approximately, $x_1 = x_0 + \tau\dot{x}_0 + \frac{1}{2}\tau^2\ddot{x}_0$, the error being about $\frac{1}{6}\tau^3{}_0\dddot{x}$ where $\dddot{x}_0$ is the value of d^3x/dt^3 at $t = 0$, or say, "of order τ^3".

If, however, the initial velocity or acceleration is infinite or zero special ad hoc methods must be employed. An example is given in paragraph 25 below.

The Run of the Calculation.—When $t{=}n\tau$, let $x = x_n$, $dx/dt = \dot{x}_n$, $d^2x/dt^2 = \ddot{x}_n$. and $d^3x/dt^3 = \dddot{x}_n$. Then we start with approximate equations of the form

$$x_{n+1} = x_n + \tau\dot{x}_n + \tfrac{1}{2}\tau^2\ddot{x}_n + \tfrac{1}{6}\tau^3\dddot{x}_n,$$

and

$$\tau\dot{x}_{n+1} = \tau(\dot{x}_n + \tau\ddot{x} + \tfrac{1}{2}\tau^2\dddot{x}_n),\ \text{and}\ \tau\dot{x}_{n-1} = \tau(\dot{x}_n - \tau\ddot{x}_n + \tfrac{1}{2}\tau^2\dddot{x}_n)\,.$$

From this equation we can form a number of other relations which involve only the positions and velocities, such as

$$x_{n+1} = x_{n-1} + 2\tau\dot{x}_n + (\tfrac{1}{3}\tau^3\dddot{x}_n), \qquad \text{(T)}$$

and

$$x_{n+1} = x_n + \tau\dot{x}_n + \tfrac{1}{2}\tau(\dot{x}_n - \dot{x}_{n-1}) + (\tfrac{5}{12}\tau^3\dddot{x}_n), \qquad \text{(M)}$$

neglecting in each case the terms written in brackets at the end.

Each of these equations seems at first sight to be quite satisfactory, but practical experience shows that equation (T), originally used by Temple, possesses a kind of intrinsic instability which leads to a divergent oscillation in the resulting calculations, while equation (M), devised by Makovski, is essentially stable and gives satisfactory smooth curves, and is therefore recommended for the calculation of the axle displacement x. For the calculation of the total travel s we recommend the equations,

$$\dot{s}_{n+1} = \dot{s}_n + \tfrac{1}{2}\tau(\ddot{s}_n + \ddot{s}_{n+1}) \quad \text{and} \quad s_{n+1} - s_n = s_n - s_{n-1} + \tau^2\ddot{s}_n$$

as satisfactory and convenient.

To use equation (M) we start with the values of x_0, x_1, $\dot{x}_0$, $\dot{x}_1$ and then use (M) to calculate x_2. From this $\dot{x}_2$ is calculated, and then x_3 is obtained by the use of (M). Hence step by step we can determine x_4, x_5, ..., x_n, x_{n+1}, for as far as we please.

The Finish of the Calculation.—In many cases we have to cover only the period during which x is rising to a maximum, and we want to determine the maximum value of x, say A, and the instant $t = T$ at which it is attained. At $t = T$, $dx/dt = 0$. Let $d^2x/dt^2 = -F$. Then, near the instant $t = T$,

$$x = A - \tfrac{1}{2}F(T - t)^2,$$

and $dx/dt = F(T - t)$.

Now let us suppose that u_n is small and that the maximum value of x is near $t = n\tau$. Then

$$A - a_n = \tfrac{1}{2}F(T - n\tau)^2 \ ,$$
$$u_n = F(T - n\tau) \ ,$$

and, approximately

$$F = f_n = (u_n - u_{n-1})/\tau \ .$$

From these equations we can calculate F, T and A.

The Accuracy of the Calculation.—The error in the working equation recommended for x is about $\frac{5}{12}\dddot{x}_n\tau^3$. Now $\dddot{x}_n$ is roughly equal to $(\ddot{x}_{n+1} - \ddot{x}_n)/\tau$, so that the error in $\overline{x_{n+1} - x_n}$ is approximately $\frac{5}{12}\tau^2(\ddot{x}_{n+1} - \ddot{x}_n)$.

This error is cumulative and the resultant error in $x_n - x_0$ is therefore approximately

$$\tfrac{5}{12}\tau^2(\ddot{x}_{n+1} - \ddot{x}_n) + \tfrac{5}{12}\tau^2(\ddot{x}_n - \ddot{x}_{n-1}) + \dots ,$$

i.e., approximately $\frac{5}{12}\tau^2(\ddot{x}_{n+1} - \ddot{x}_0)$.

We usually have a rough idea of the values of $\ddot{x}_n$ which are likely to be found and hence we can begin by choosing the fundamental interval τ to make the resultant error sufficiently small. Suppose for example that we know that $\ddot{x}_n - \ddot{x}_0$ never exceeds $3g$. Then, with a fundamental interval of $0 \cdot 01$ second, the error in the calculated values of x will never exceed $\frac{5}{12} \times 0 \cdot 0001 \times 3 \times 32 \times 12$ in. $= 0 \cdot 05$ in. Similarly, with a fundamental interval of $0 \cdot 005$ second, the error in x is less than $0 \cdot 0125$ in.

25. *Undercarriage Calculations.*—In order to make the method of step-by-step calculation perfectly clear we shall now give a detailed example of an undercarriage calculation. The conditions are that the load is fully airborne throughout, that the unit is free from friction, and that due allowance is made for spinning-up the wheel. Under these conditions the calculations are directly applicable to the initial stages of yawed and banked landings. The numerical values adopted are

Load $W = 5{,}500$ lb.

Initial velocity downwards $= 144$ in./sec.

Coefficient of friction for spinning-up calculations $= \mu = 0 \cdot 4$.

As a result of preliminary calculations, made as described in Part I, the coefficients Q and D in the formula

$$\dot{x} = D(R - Q)^{1/2}$$

are known as functions of the vertical axle displacement x, so that the vertical ground reaction R can be calculated in terms of x and $\dot{x}$. It is convenient to plot Q and D against x as shown in Fig. 5. Furthermore we have a curve giving R plotted against the tyre closure x_t .

The fundamental equation of motion is

$$R = -(W/g)\ \ddot{s}$$

where

$$s = x + x_t \ .$$

We now proceed to explain the start, run and finish of the undercarriage calculation.

The Start of the Calculation.—The shock absorber does not start to move until R rises to the value Q. During this initial period of time, say from $t = -t_0$ to $t = 0$, the tyre only is acting, and the fundamental equations are

$$x = 0, \quad R = -(W/g)\ddot{x}_t\,, \quad R = f(x_t)\ ,$$

while at $t = -t_0$, $x_t = 0$ and $\dot{x}_t = 144$ ft./sec. (It is found to be very convenient to take the zero hour from which time is reckoned as the instant when the shock absorber begins to function).

Let the initial slope of the tyre deflection curve, $R = f(x_t)$, be r_0, so that approximately $R = r_0 x_t$, r_0 being 1,640 lb./in. Then

$$\ddot{x}_t = -\,(r_0 g/W)x = -\,115{\cdot}2x_t\,.$$

The initial motion is therefore adequately represented by the formulae

$$x_t = 144\,(t + t_0)\left\{1 - \frac{1}{6} \times 115{\cdot}2 \times (t + t_0)^2\right\}$$

and

$$\dot{x}_t = 144\left\{1 - \tfrac{1}{2} \times 115{\cdot}2 \times (t{+}t_0)^2\right\}$$

The initial value of Q when $x = 0$ is 2,100 lb., and R reaches this value when $x_t = 1{\cdot}28$ in. Hence, approximately

$$t_0 = 0{\cdot}0089 \text{ sec.}$$

Therefore, when $t = 0$,

$$\dot{x}_t = 143{\cdot}34 \text{ in./sec.}$$

This is of course the value of $\dot{s}$ when $t = 0$. The higher powers of $(t + t_0)$ omitted from the formulae for x_t and $\dot{x}_t$ are negligibly small.

We can now commence the prediction of the shock absorber loads, but first we must dispose of an initial difficulty, viz., that the initial value of $\ddot{x}$ is infinite, as appears at once from the equation

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d}{dt}\ D(R - Q)^{1/2}$$

$$= \dot{D}(R - Q)^{1/2} + \tfrac{1}{2}D\ \frac{\dot{R} - \dot{D}}{(R - Q)^{1/2}}$$

when we remember that $R = Q$ at $t = 0$.

To circumvent this difficulty, which was alluded to above, we obtain an approximate expression for $\dot{x}^2$ by writing

$$\dot{x}^2 = D^2(R - Q) = D^2\left(rx_t - x\frac{dQ}{dx}\right)$$

$$= D^2 rs - D^2\left(r + \frac{dQ}{dx}\right)x = \lambda t - \mu x,$$

where $\lambda = D^2 r\dot{s}$ and $\mu = D^2\left(r + \dfrac{dQ}{dx}\right)$, both calculated at $t = 0$. Then, where t is small, an approximate expression for x is

$$x = \frac{2}{3}\,\lambda^{1/2}\,.\,t^{3/2} - \frac{1}{6}\,\mu\,.\,t^2\,,$$

as may be verified by substitution, neglecting higher powers of t.

In the present case

$$\lambda = (0{\cdot}592)^2 \times 2{,}100 \times 144 = (325)^2 \text{ in.}^2\text{/sec.}^3,$$

and

$$\mu = 0{\cdot}35 \times (2{,}100 + 300) \quad = 840 \text{ in./sec.}^2.$$

Hence the initial values of x are

$$x_1 = 0\cdot216 - 0\cdot014 = 0\cdot202 \text{ in. at } t = \tau,$$

and
$$x_2 = 0\cdot611 - 0\cdot056 = 0\cdot555 \text{ in. at } t = 2\tau,$$

the fundamental interval τ being 0.01 sec.

The calculation of s_1 is much simpler, since at $t = 0$,

$$s = 1\cdot28, \quad \dot{s} = 144 \text{ in./sec.,}$$

$$\text{and} \quad \ddot{s} = -gR/W = -386\cdot4 \times 2{,}100/5{,}500 = -147\cdot5 \text{ in./sec.}^2$$

Therefore $s_1 = 1\cdot28 + 143\cdot3\times 0\cdot01 - \frac{1}{2} \times 147\cdot5\times 0\cdot0001 = 2\cdot706$ in. We can now calculate in succession

$$x_{t,1} = s_1 - x_1 = 2\cdot504 \text{ in.}$$

$$R_1 = 4{,}550 \text{ lb. (from tyre deflection curve),}$$

$$\left.\begin{aligned} Q_1 &= 2{,}150 \text{ lb.} \\ D_1 &= 0\cdot592 \end{aligned}\right\} \text{(from data curves),}$$

$$\dot{x}_1 = 29\cdot2 \text{ in./sec. (from } \dot{x} = D(R - Q)^{1/2}) ,$$

$$\ddot{s}_1 = -320 \text{ in./sec.}^2 \text{ (from } -gR/W).$$

The Run of the Calculation.—We can now continue with the run of the calculation using the following scheme of operation :—

Data, s_n, x_n, s_{n-1}, x_{n-1}, $\dot{x}_n$, $\dot{x}_{n-1}$.

Calculate

$$x_{t,n} = s_n - x_n,$$

$$R_n = f(x_{t,n}),$$

$$\dot{x}_n = D(R_n - Q_n)^{1/2},$$

and then x_{n+1}.

Also, from $\ddot{s}_n = -gR_n/W$,

we get $\dot{s}_n$,

and s_{n+1}.

This gives all the directions for one complete step from $n\tau$ to $(n + 1)\tau$. The numerical integrations are all to be carried out by the formulae

$$x_{n+1} = x_n + \tau\dot{x}_n + \tfrac{1}{2}\tau(\dot{x}_n - \dot{x}_{n-1}),$$

$$\dot{s}_n = \dot{s}_{n-1} + \tfrac{1}{2}\tau(\ddot{s}_n + \ddot{s}_{n-1}),$$

$$\text{and} \quad s_{n+1} - s_n = s_n - s_{n-1} + \tau^2\ddot{s}_n .$$

The results of these calculations are all shown in Table I.

The Finish of the Calculation.—Towards the end of the calculation, as $\dot{x}$ approaches zero, the method used for the run of the calculation is liable to break down due to the small difference between R and Q. When this occurs, the damping effect of the oil is small and may be neglected, and R may be taken to be equal to Q. Since R and Q are functions of x_t and x respectively, we can plot R against $s(-x+x_t)$, and then we determine R_{n+1} directly from the value of s_{n+1} obtained by the normal method.

To obtain the final column for $\dot{s} = 0$, we have to destroy the vertical velocity of 3·20 in./sec. With the acceleration —987 ft./sec.^{2}, the time taken is $3\cdot20/987 \simeq 0\cdot003$ sec., and the distance travelled is the mean velocity 1·60 in./sec. times 0·003 sec. or about 0·005 in., which is very small, and the calculation is rounded off as shown.

The Accuracy of the Calculation.—The acceleration $\ddot{s}$ rises from zero to about 1,000 in./sec.^{2}.

Hence the formula of paragraph 24 shows that the error in s will not exceed

$\frac{1}{3} \times (0\cdot01)^2 \times 1{,}000 = \frac{1}{30}$ in.

Similarly the values of $\ddot{x}$ listed in the table indicate a maximum acceleration $\ddot{x}$ of 1,200 in./sec.^{2} after the initial period of 0·01 sec. to which we gave especial treatment. Hence the error in x will not exceed $\frac{1}{3} \times (0\cdot01)^2 \times 1{,}200 = 0\cdot04$ in.

These results are extremely satisfactory and show that a time interval of 0·01 sec. is quite small enough for our purposes.

26. *The Choice of a Fundamental Interval.*—The fundamental interval τ must be chosen before commencing the calculations, so as to be small enough to ensure sufficient accuracy in the results and so as to be large enough to make the number of steps in the calculation not too numerous. The two criteria for the choice of τ are

(1) that τ must be small enough to ensure that the functions Q and D are reasonably constant in each interval, and

(2) that τ is small enough to yield a reasonably small value for the estimated error

$$\tfrac{1}{3}\tau^2 [\ddot{x}_n - \ddot{x}_0]$$

in each variable x.

TABLE 1

Specimen Undercarriage Calculation

(IN.—LB.-SEC. UNITS)

SLIPPING CEASES (between the two ·05 columns)

t	-·0089	0	·01	·02	·03	·04	·05	·05	·06	·07	·08	·09	·10	·11	·12	·13	·14	·15	·16	·17	·173
s	0	1·28	2·706	4·100	5·445	6·727	7·935	7·935	9·063	10·105	11·056	11·913	11 675	13·341	13·910	14·382	14·757	15·035	15·215	15·297	15·30
x		0	·202	0·655	1·039	1·704	2·545	2·545	3·337	4·121	4·921	5·714	6 419	7·034	7·578	8·045	8·419	8·667	8·814	8·89	8·89
x_t	0	1·28	2·504	3·545	4·406	5·023	5·390	5·390	5·726	5·984	6·135	6·199	6 256	6·307	6·332	6·337	6·338	6·368	6·401	6·41	6·41
R	0	2100	4550	6900	9000	10500	11400	11400	12250	12900	13350	13500	13650	13800	13850	13850	13850	13900	14000	14050	14050
Q		2100	2150	2250	2450	3600 / 2700	3000	3900	4300	4800	5500	6400	7350	8300	9500	10900	12300	13400	14000	14050	14050
$R-Q$		0	2400	4650	6550	6900 / 7800	8400	7500	7950	8100	7850	7100	6300	5500	4350	2950	1950	500	—	—	—
D		·592	·595	·615	·72	·72 / ·885	·968	·840	·857	·875	·892	·87	825	·784	·765	·77	·775	·775	—	—	—
$\ddot{x}$		0	29·2	42·0	58·3	59·8 / 75·5	88·7	72·7	76·5	78·8	79·1	73·4	65 5	58·1	50·5	41·8	30·5	17 3	—	—	—
$\ddot{s}$		-147·5	-320	-485	-632	-738	-800	-800	-861	-906	-938	-948	-960	-970	-974	-974	-974	-976	-984	-987	-987
$\dot{s}$	144	143·34	141·00	136·97	131·38	124·53	116·84	116·84	108·54	99·70	90·48	81·05	71 51	61·86	52·14	42·40	32·66	22·91	13·08	3·20	0
$s_n - s_{n-1}$			1·426	1·394	1·345	1·282		1·208	1·128	1·042	0·951	·857	762	·666	·569	·472	·375	·278	·180	·082	
$\tau^2 \ddot{s}_n$			-·032	-·049	-·063	-·074		-·08	-·086	-·091	-·094	-·095	-096	-·097	-·097	-·097	-·097	-·098	-·098	-·099	
$\tau \dot{x}_n$				0·420	0·583	0·755		·727	·765	·788	·791	·734	·655	·581	·505	·418	·305				
$\frac{1}{2}\tau(\ddot{x}_n - \ddot{x}_{n-1})$				·064	·022	·086		·065	·019	·012	·002	-·029	-040	-·037	-·038	-·044	-·057				
+				0·484	·665	0·841		·792	·784	·800	·793	·705	615	·544	·467	·374	·248				

N.B.—In column for $t =$ ·04, the superscribed figures represent values obtained using Q and D curves which apply after slipping ceases. The value of $\dot{x}$ so obtained is used in determining x at $t =$ ·06 and the effect of the discontinuity at $t =$ ·05 is thus avoided.

NOTATION

A = " air " displacement area
$A = \alpha + W/U$ = effective angle of incidence (Part II paragraph 12)
A_1 = " oil " displacement area
A_{jk} = stiffness coefficient
a_1 = orifice area in compression
a_2 = orifice area in recoil
α = effective angle of incidence just before touch down
α_j = angle between plane of wheel and direction of motion
B_{jk} = rotational factor for banked landing
b_j = spanwise distance of j-th. wheel from $C.G.$
C = pressure drop through orifice for unit $\dot{x}_s$
C_{D_1}, C_{D_2} = orifice coefficients in compression and recoil
C_v, C_h, C_γ = velocity ratios of shock absorber for vertical, horizontal and inclined reactions at axle
c = chord length
γ = angle of inclination of resultant reaction R to vertical
D = orifice function for shock absorber
D = drag force at contact area during braking (Part II)
D_A = net aerodynamic drag
d = tyre closure
E_1 = energy absorption in idealised landing
E_2 = energy absorption in drop test
F = horizontal force on wheel axle
f = friction force in shock absorber unit
ϕ = angle of roll
g = acceleration due to gravity (= 386 in./sec.^{2})
H = height of $C.G.$ above ground
$h_{j,0}$ = vertical distance from $C.G.$ of aircraft to axle of j-th. wheel in its fully extended position
J = moment of inertia of tyre wheel assembly about central line of axle
k_v, k_h, k_γ = mechanical advantages of shock absorber unit for vertical horizontal and inclined reactions at axle
k_x, k_y, k_z = radii of gyration of aircraft about forward, transverse and vertical axes
L = lift
$l_{j,0}$ = horizontal distance from $C.G.$ of aircraft to axle of j-th. wheel in its fully extended position
l_v, h_v = orifice coefficients in compressed position
M = total rolling moment
μ = coefficient of friction
N = number of main undercarriage legs
$n = 1 \cdot 3$ = " quasi-adiabatic index "
P = piston thrust
p = air pressure
p' = oil pressure
p_0 = initial air pressure in shock absorber

ψ = angle of yaw
Q = air pressure function for shock absorber
R = vertical ground reaction on wheel
r = tyre radius
R_v, R_h = vertical and horizontal reactions on axle
ϱ = air density
s = total travel of shock absorber unit = $x + x$
σ = specific weight of oil
T = time for vertical acceleration of $C.G.$ to reach maximum
t = time interval
τ = fundamental time interval
θ = angle of pitch
U = forward speed of aircraft
u, v, w = velocity components of point in rotating system of axes
V = air volume in shock absorber (Part I)
V = vertical rate of descent of aircraft (Part II)
V_0 = initial air volume in shock absorber
v_0 = oil speed through orifice
W = all up weight
w = weight of wheel and tyre
w_1, w_2, w_3 = angular speeds of rotation about forward, transverse and vertical axes
Wk_x^2, Wk_y^2, Wk_z^2 = principal moments of inertia about forward, transverse and vertical axes
x = vertical axle displacement
x_s = shock absorber travel
x_t = tyre deflection
Y = rotational factor for yawed landing

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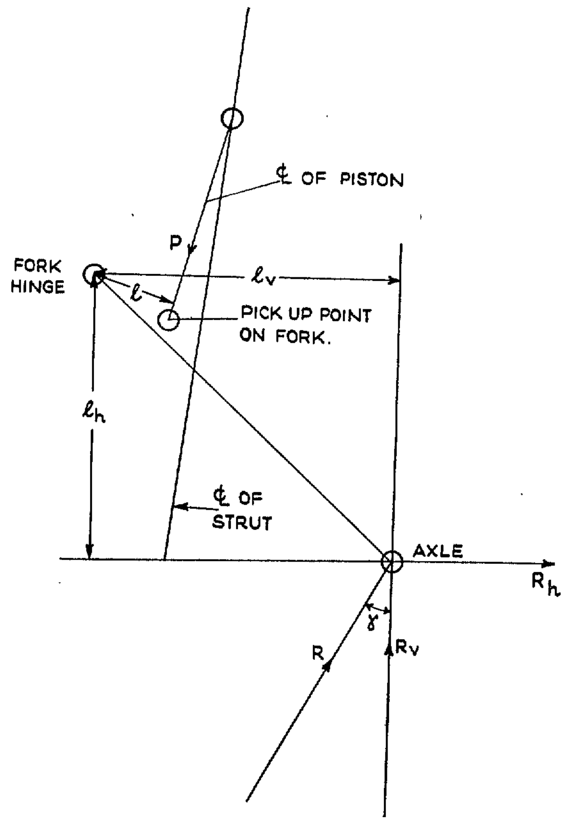

Fig. 1. Typical Shock Absorber Linkage in an Articulated Unit.

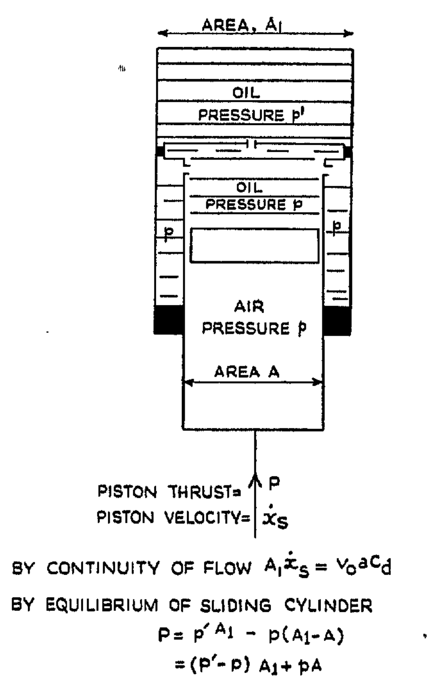

Fig. 2. Typical Shock Absorber Design.

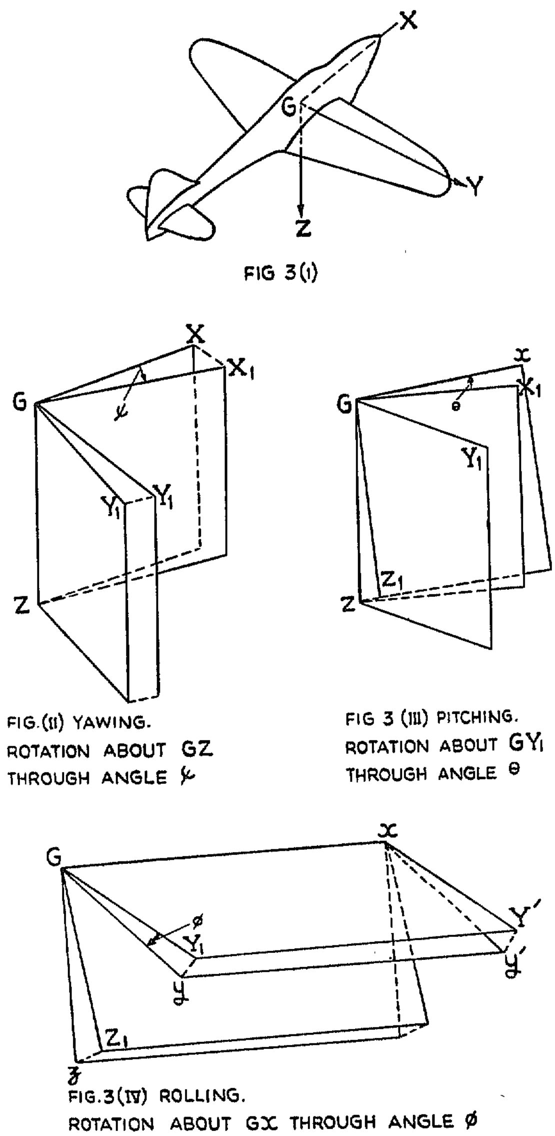

Fig. 3.

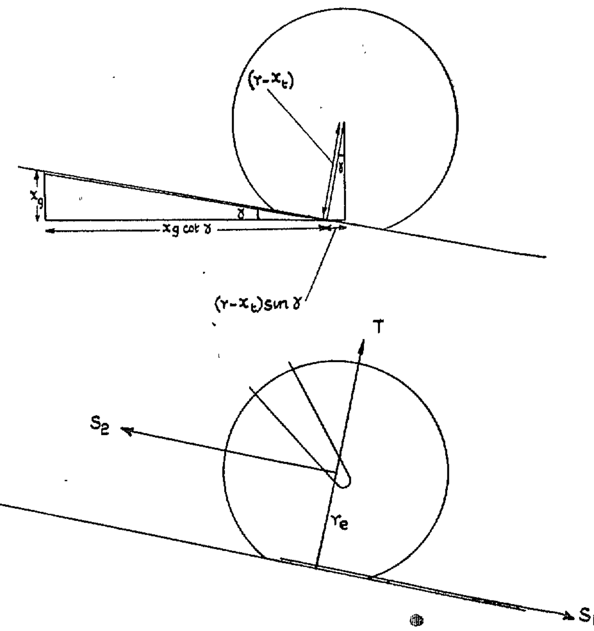

Fig. 4. Geometry of Drop Test Using Wedge.

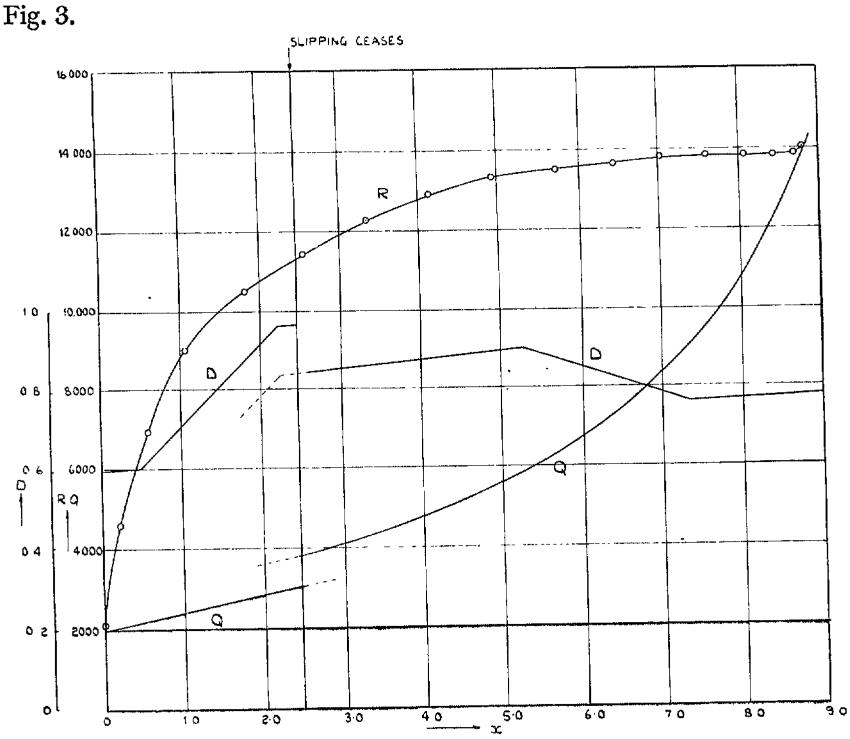

Fig. 5. Curves of R and Functions Q and D.

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