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THE APPLICATION OF THE HILBERT TRANSFORM TO SYSTEM RESPONSE ANALYSIS

by

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SUMMARY

The application of the Hilbert transform to the analysis of the response of linear multi-degree-of-freedom systems to random forces is considered. The use of this transform in deriving modal properties of a system from the spectral density of the response of the system to a random input force which has a flat spectrum is demonstrated. It is also shown that the Fourier transform of the one-sided autocorrelogram of the response implicitly involves a Hilbert transform.

The Hilbert transform can be realised, in practice, by simple manipulations of signals in the time domain, and by Fourier transform processes available in commercial Fourier analysers.

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* Replaces RAE Technical Report 74117 - ARC 35681.

CONTENTS

1 INTRODUCTION

Several techniques are available for finding the values of natural frequency and damping ratio of the modes of a system from the response of the system to external forces[1,2,3]. If the time-histories of the forces are unknown, the techniques utilise the spectral density of the system response. In order to apply these techniques, it is generally necessary to assume that the excitation spectrum is flat in the region of each resonance frequency[2,4]. When this assumption is satisfied, the spectral density of the response is proportional to the square of the modulus of a transfer function of the system.

Analysis techniques that attempt to recover the transfer function from the spectral density all have one drawback in common, namely that many transfer functions of a system can be associated with a given spectral density, and it is not generally possible to recover the particular transfer function with which the spectral density was originally associated. This is not a significant drawback if the modal data that are sought are restricted to the values of natural frequency and damping ratio, since these can be derived from any of the transfer functions associated with a given response spectral density. However, the accuracy with which these values can be found may vary with the type of analysis which is employed, and there is, therefore, some incentive to explore new methods of analysis to assess what advantages, if any, they may offer. One such exploration that has been made is the application of the Hilbert transform to system response analysis.

Briefly, the Hilbert transform gives the imaginary part of a complex function when only the real part is known, and vice-versa. A necessary condition is that the function should be the Fourier transform of a causal function. This condition is satisfied if the complex function is a transfer function of a stable linear physically realisable system, since its inverse Fourier transform is the impulse response of the system, and is causal. If, therefore, the real part of a transfer function is known the Hilbert transform gives the imaginary part.

In applying the Hilbert transform to analyse response spectral density, a slightly devious route must be followed. This is because the response spectral density contains the modulus (and not the real part) of the transfer function and, moreover, the modulus appears in a squared form. The problem is approached, following a method due to Bode[5], by using the logarithm of the spectral density, and applying to this a Hilbert transform to derive the phase characteristic of the transfer function. This treatment leads to a 'minimum-phase-shift' transfer function, for which the inverse Fourier transforms of both the function and its logarithm are causal[6]. Thus, of the many transfer functions which can be associated with a given response spectral density, the process outlined above will give only the minimum-phase-shift function.

The above process may be compared with that of using the Fourier transform of the one-sided autocorrelogram[4] to produce a vector plot from which the modal characteristics of the system may be obtained. An example is given in which both processes are applied to determine the modal characteristics of a three degree-of-freedom system from the spectral density of the system response. The values of frequency and damping ratio obtained are equal to the known values for the system. However, the transfer functions from the two processes are dissimilar in appearance when presented in Argand diagram, or vector plot, form. In general, one process cannot be preferred to the other where the aim is solely to extract natural frequencies and damping ratios. It is also shown that the imaginary part of the Fourier transform of the one-sided autocorrelogram can be expressed as a Hilbert transform of the spectral density - a result that provides a link between the two processes, and that does not seem to have been noted in the relevant literature.

It is concluded that the application of the Hilbert transform in analysing response spectral density does not in general result in significant improvements over existing analysis techniques. There may, however, be circumstances in which it could be used to advantage, and it constitutes a useful tool in the workshop of dynamic analysis.

The application of the Hilbert transform is only of practical value if the transform can be realised without a great deal of difficulty. Accordingly, a method has been devised that enables a Hilbert transform to be carried out using standard facilities available with a Fourier analyser.

2 THEORY

2.1 Hilbert transform

Let $z(t)$ be a function of time t, and let $Z(\omega)$ be its Fourier transform:-

$$Z(\omega) = \int_{-\infty}^{\infty} z(t)e^{-i\omega t}\, dt \qquad (1)$$

$Z(\omega)$ may be expressed in its real and imaginary parts as:-

$$Z(\omega) = X(\omega) + iY(\omega) \quad . \tag{2}$$

Provided $z(t)$ is a causal function ($z(t) = 0$ for $t < 0$) and is finite at $t = 0$, the Hilbert transform relationships between $X(\omega)$ and $Y(\omega)$ are:-

$$X(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{Y(\Omega)}{(\omega - \Omega)} d\Omega \tag{3}$$

$$Y(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{X(\Omega)}{(\omega - \Omega)} d\Omega \quad . \tag{4}$$

Equations (3) and (4) show that given the real (or imaginary) part of a function $Z(\omega)$, the imaginary (or real) part can be found.

The aim of the following analysis is to use the Hilbert transform to derive a transfer function of a system from the spectral density of the response of the system to an exciting force having a flat spectrum.

2.2 Derivation of transfer function

It has been shown[4] that the spectral density $S(\omega)$ of the response at a point in the system is proportional to the square of the modulus of the transfer function $H(\omega)$ between the points of excitation and response measurement provided the excitation has a flat spectrum. For convenience, the factor of proportionality is dropped, so that:-

$$S(\omega) = |H(\omega)|^2 \quad . \tag{5}$$

In order to apply the Hilbert transform, an equation analagous to equation (2) is required. Following Bode's[5] treatment, $H(\omega)$ may be expressed in modulus and phase form as:-

$$H(\omega) = |H(\omega)| e^{i\theta(\omega)} \tag{6}$$

where $\theta(\omega)$ is the phase function. Taking logarithms of equation (6):-

$$\log H(\omega) = \log |H(\omega)| + i\theta(\omega) \quad . \tag{7}$$

Now

$$\log S(\omega) = \log |H(\omega)|^2 = 2 \log |H(\omega)| \tag{8}$$

and therefore:-

$$\log H(\omega) = \tfrac{1}{2} \log S(\omega) + i\theta(\omega) \quad . \tag{9}$$

Equation (9) is analogous to equation (2), and given $S(\omega)$, (and hence $\frac{1}{2} \log S(\omega)$) the Hilbert transform will yield a phase function $\theta(\omega)$. However, although $\theta(\omega)$ is uniquely determined from $\frac{1}{2} \log S(\omega)$, there is not a one-to-one relationship between $H(\omega)$ and $S(\omega)$. For example, some other transfer function $H'(\omega)$ given by:-

$$H'(\omega) = H(\omega)e^{i\phi(\omega)} \tag{10}$$

will also satisfy equation (5). The transfer function $H_H(\omega)$ that is obtained from $S(\omega)$ and from $\theta(\omega)$ (which has been determined from $\frac{1}{2} \log S(\omega)$ by a Hilbert transform), i.e.

$$H_H(\omega) = S(\omega)^{\frac{1}{2}} \exp\left(-\frac{i}{\pi} \lim_{M\to\infty} \int_{-M}^{M} \frac{\frac{1}{2} \log S(\Omega)}{(\omega - \Omega)} \, d\Omega\right)$$

$$= S(\omega)^{\frac{1}{2}} \exp\left(-\frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{\omega \log S(\Omega)}{(\omega^2 - \Omega^2)} \, d\Omega\right) \tag{11}$$

will be that for which the inverse Fourier transform of $\log H_H(\omega)$ is causal:-

$$\frac{1}{2\pi} \lim_{M\to\infty} \int_{-M}^{M} \log H_H(\omega)e^{i\omega t} d\omega = 0 \quad \text{for } t < 0 \quad . \tag{12}$$

The inverse Fourier transform of a transfer function is, of course, also causal, and hence:-

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} H_H(\omega)e^{i\omega t}\, d\omega = 0 \quad \text{for } t < 0 \quad . \tag{13}$$

Functions which satisfy equations (12) and (13) are termed 'minimum-phase-shift', and their properties are dealt with in text-books, such as Ref.6. One example of a 'minimum-phase-shift' function is the transfer function between coincident excitation and response-measurement points.

2.3 Evaluation of the Hilbert transform

It will be shown in this section that the Hilbert transform can be evaluated by manipulation of functions in the time domain and by Fourier transforms.

Let

$$z(t) = z_e(t) + z_0(t) \tag{14}$$

where $z_e(t)$ is the even, and $z_0(t)$ the odd part of $z(t)$. If $z(t)$ is causal:-

$$z_e(t) = z_0(t) \text{ sgn } (t) \tag{15}$$

$$z_0(t) = z_e(t) \text{ sgn } (t) \quad . \tag{16}$$

Let the Fourier transforms of $z_e(t)$ and $z_0(t)$ be $Z_e(\omega)$ and $Z_0(\omega)$ respectively. $Z_e(\omega)$ is wholly real, and $Z_0(\omega)$ wholly imaginary. Hence, taking the Fourier transform of equation (14) and comparing it with equation (2);-

$$Z_e(\omega) = X(\omega) \tag{17}$$

$$Z_0(\omega) = iY(\omega) \quad . \tag{18}$$

It can be seen from the above that given, say, $X(\omega)$ an inverse Fourier transform yields $z_e(t)$ (from equation (17)). Multiplication of $z_e(t)$ by sgn (t) gives $z_0(t)$ (from equation (16)) and a Fourier transform of $z_0(t)$

gives $iY(\omega)$ (from equation (18)). In a similar manner, given $Y(\omega)$, $X(\omega)$ can be found.

These sequences of operations are effectively those of obtaining the Hilbert transforms. This will be proved by showing that the Fourier transforms of equations (15) and (16) give equations (3) and (4). Noting that the Fourier transforms of sgn (t) is $2/i\omega$, the Convolution Theorem may be used to express the Fourier transforms of equations (15) and (16) as:-

$$X(\omega) = \frac{1}{2\pi}\left(iY(\omega) \oplus \frac{2}{i\omega}\right) \tag{19}$$

$$iY(\omega) = \frac{1}{2\pi}\left(X(\omega) \oplus \frac{2}{i\omega}\right) \tag{20}$$

where $\oplus$ denotes convolution, i.e.

$$a(\omega) \oplus b(\omega) = \int_{-\infty}^{\infty} a(\Omega)b(\omega - \Omega)\,d\Omega \tag{21}$$

Hence, from equations (19), (20) and (21):-

$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} iY(\Omega)\,\frac{2}{i(\omega - \Omega)}\,d\Omega = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{Y(\Omega)}{(\omega - \Omega)}\,d\Omega \tag{22}$$

and

$$Y(\omega) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} X(\Omega)\,\frac{2}{i(\omega - \Omega)}\,d\Omega = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{X(\Omega)}{(\omega - \Omega)}\,d\Omega \tag{23}$$

which are the Hilbert transform relationships of equations (3) and (4).

2.4 Fourier transform of the one-sided autocorrelogram

The autocorrelogram $A(\tau)$ of $z(t)$ is the inverse Fourier transform of the spectral density $S(\omega)$:-

$$A(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} \, d\omega \quad . \tag{24}$$

The one-sided autocorrelogram $A'(\tau)$ (such that $A'(\tau) = 0$ for $\tau < 0$) is given by

$$A'(\tau) = \tfrac{1}{2}A(\tau) + \tfrac{1}{2}A(\tau) \operatorname{sgn} \tau \tag{25}$$

since $A(\tau)$ is an even function of τ . The Fourier transform of $A'(\tau)$, denoted by $\overline{A}'(\omega)$, is therefore given by:-

$$\overline{A}'(\omega) = \tfrac{1}{2}S(\omega) + \tfrac{1}{2}\left[\frac{1}{2\pi} S(\omega) \oplus \frac{2}{i\omega}\right] \tag{26}$$

$$= \tfrac{1}{2}S(\omega) - \frac{i}{2}\left[\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{S(\Omega)}{(\omega - \Omega)} \, d\Omega\right] \quad . \tag{27}$$

Equation (27) shows that the real part of $\overline{A}'(\omega)$ is half the spectral density (and therefore positive), whilst the imaginary part is half the Hilbert transform of the spectral density.

The one-sided autocorrelogram procedure[4] can be seen therefore to have generated no 'new' information from that contained in the original response spectral density since the real part is directly related to the spectral density by a simple factor and the imaginary part is merely a weighted average of the spectral density over the whole frequency range. It has been shown[4] however that the resulting 'plot' closely resembles the vector response plot of a transfer function and may be analysed by standard procedures.

3 EXAMPLE

In the following example, a three degree-of-freedom system has been used. The modal frequency and damping ratios were:-

Mode	Frequency (Hz)	Damping ratio
1	100	0.050
2	140	0.036
3	170	0.029

The system is excited by a time-varying force having a flat spectrum; the force is applied at one point in the system, and the response is measured at another point. The transfer function between the points is obtained by dividing the Fourier transform of the response by that of the excitation. The resulting vector plot is shown in Fig.1a, and the choice of points is such that Fig.1a is the vector plot of a non-minimum-phase-shift transfer function. The spectral density of the response can also be obtained and this is shown in Fig.2.

In the many practical situations where the time-history of the exciting force is unknown, the characteristics of the system have to be obtained from the time-history of the response alone, making the assumption that the spectrum of the exciting force is flat. Thus, in practice, the transfer function of Fig.1a is not known and the analysis is based on the spectral density of the response (Fig.2).

The method of section 2.2 has accordingly been used to find a phase function by a Hilbert transform of the logarithm of the spectral density of Fig.2. Following equation (11), a transfer function has been derived, and this is shown as a vector plot in Fig.1b. Comparison of the vector plots of Figs.1a and 1b shows that they differ markedly both in modal amplitude ratios (the ratios of the 'circle' diameters) and in pattern (the relative positions of the 'circles'). The reason for this is that the derived plot (Fig.1b) is that of a minimum-phase-shift function whereas the original plot (Fig.1a) is not. (Computer analyses[7] of both plots naturally give the same values of modal frequencies and damping ratios, which agree with the values given in the above table.) At first glance, it seems unlikely that both plots are associated with the same spectral density - particularly in view of the small modal component of mode 2 in Fig.1b - but this is indeed so.

In order to compare the analysis technique using the Hilbert transform with the technique using the Fourier transform of the one-sided autocorrelogram, the latter function was also computed from the response spectral density of Fig.2. The autocorrelogram was first obtained as an inverse Fourier transform of the response spectral density, and a Fourier transform of the one-sided autocorrelogram was then derived. The vector plot of this function is shown in Fig.3. Again, computer analysis of Fig.3 gave values of modal frequency and damping ratio that agreed with the above tabulated values.

Comparison of Figs.1b and 3 shows that the vector plot from the one-sided autocorrelogram (Fig.3) has an even smaller component of mode 2 than that from the Hilbert transform process (Fig.1b). The size of this component in Fig.3 reflects the relative amplitudes of the peaks in Fig.2. However, there can in general be no certainty that the Hilbert transform process will yield modal amplitudes that are more nearly equal than the one-sided autocorrelogram process. In the example given here, the minimum-phase-shift vector plot of Fig.1b is slightly preferable to Fig.3 for the extraction of modal frequencies and damping ratios, but there is not a great deal to choose between them in this respect. Certainly Fig.1a - the 'original' vector plot - is preferable to either of the others, particularly if hand-analysis of the plots has to be undertaken.

In order to demonstrate that the Hilbert transform analysis process will yield the 'original' transfer function provided the latter is a minimum-phase-shift function, the same system was again used but the minimum-phase-shift transfer function of Fig.4a was generated, by letting the excitation and response-measurement points coincide. The response spectral density corresponding to Fig.4a is shown in Fig.5, and from this starting point using the method of section 2.2, the minimum-phase-shift transfer function of Fig.4b was derived. This is obviously the same function as Fig.4a. The Fourier transform of the one-sided autocorrelogram, which was computed from the response spectral density of Fig.5, is shown in Fig.6 and may be compared with Figs.4a and 4b. Obviously, the component of the mode at 140 Hz in Fig.6 has been decreased in magnitude relative to the other two modes as a result of the squaring process inherent in computing the spectral density.

4 DISCUSSION

It has been shown, in section 2, that a minimum-phase-shift transfer function of a system can be derived from the spectral density of the response by applying a Hilbert transform. In the example of section 3 the process is applied to determine the values of modal frequency and damping ratio of a system, and is compared with what may be termed the 'one-sided autocorrelogram' process applied to the same system. It is of interest to consider in what circumstances one process would be preferable to the other.

In the 'Hilbert transform' process of section 2.2 the modulus of the derived transfer function is the square root of the spectral density. In the one-sided autocorrelogram process the modulus is approximately proportional to the spectral density. It is likely, therefore, that where the spectral density has some relatively small modal 'peaks', the transfer function derived from the

Hilbert transform process will have a better modal balance than that from the one-sided autocorrelogram. This will certainly be significant if the vector plot of the transfer function has to be analysed by hand.

Another point to be considered is that in general, the transfer functions derived from both processes may differ markedly from the actual transfer function between the excitation and response points. It is thus possible for the benefits of an apparently well-chosen response point to be lost in the processing. This is well-illustrated in the example of section 3 where the transfer function of Fig.1a is that of a well-chosen point, in that the modal 'circles' are of similar size, and the whole plot can be readily analysed to give values of natural frequency and damping ratio. In Fig.1b, however, although the vector moduli are the same as in Fig.1a, the modal circles are quite different in size, and the second mode would be difficult to analyse accurately by hand. The same criticism can be made of Fig.3, but at least the circle size can be anticipated from a study of the response spectral density. In general, whichever process is used, it is desirable that the modal peaks of the response spectral density are of broadly equal amplitude.

It may be noted that whereas the Hilbert transform process leading to equation (11) will always give a transfer function having the response spectral density from which it is derived, the one-sided autocorrelogram process of equation (27) will not do so. (In the examples of section 3 the spectral densities associated with Figs.3 and 6 are not the spectral densities of Figs.2 and 5 respectively.) It follows that the one-sided autocorrelogram process can never lead to the transfer function from which the original response spectral density was obtained. On the other hand, the Hilbert transform process can do so, provided the original response spectral density was obtained from a minimum-phase-shift transfer function as was shown in Figs.4a and 4b. Unfortunately, it is not easy to identify a minimum-phase-shift transfer function unless a mathematical test can be applied to an analytical expression of the function. As has already been stated (section 2.2) it can be shown that if the excitation and response-measurement points of a system are coincident, the transfer function between excitation and response will be a minimum-phase-shift function. If the excitation and response points are progressively separated, there will be some separation at which the transfer function will become a non-minimum-phase-shift function. Physically, the changeover will be associated with the nodal line positions of the modes of the system, but there appears to be no certain way of deciding from the vector plot of the transfer function whether the function will have minimum-phase-shift characteristics. In the restricted case, therefore, where the excitation and response points are coincident, the Hilbert transform process will derive the original transfer function associated with the spectral density. This may be advantageous in some applications.

5 CONCLUSIONS

It has been shown how the Hilbert transform can be applied in the analysis of the response of linear multi-degree-of-freedom systems to excitation forces having flat spectra. The application enables a transfer function of the system to be obtained from the spectral density of the response. The derived transfer function will be a minimum-phase-shift function and may be used to determine the modal frequencies and damping ratios of the system by conventional methods.

A comparison of the above process with that of obtaining a transfer function from the one-sided autocorrelogram has shown that in general one process cannot be preferred to the other where the aim is solely to extract natural frequencies and damping ratios. There may be particular applications where the process employing the Hilbert transform has some comparative advantages.

The Hilbert transform can be realised in practice by using facilities available on commercial Fourier analysers.

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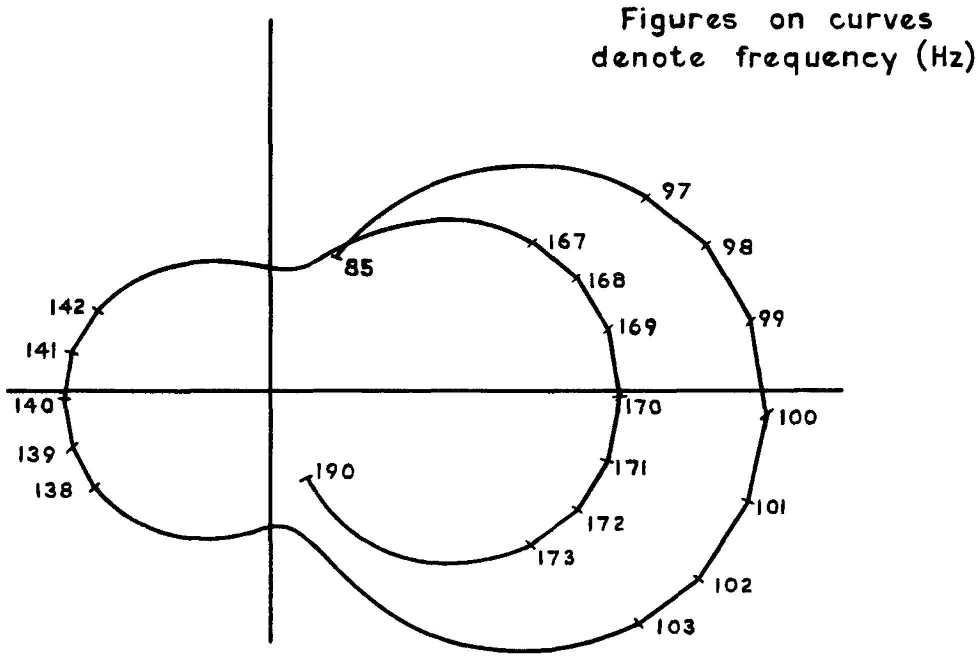
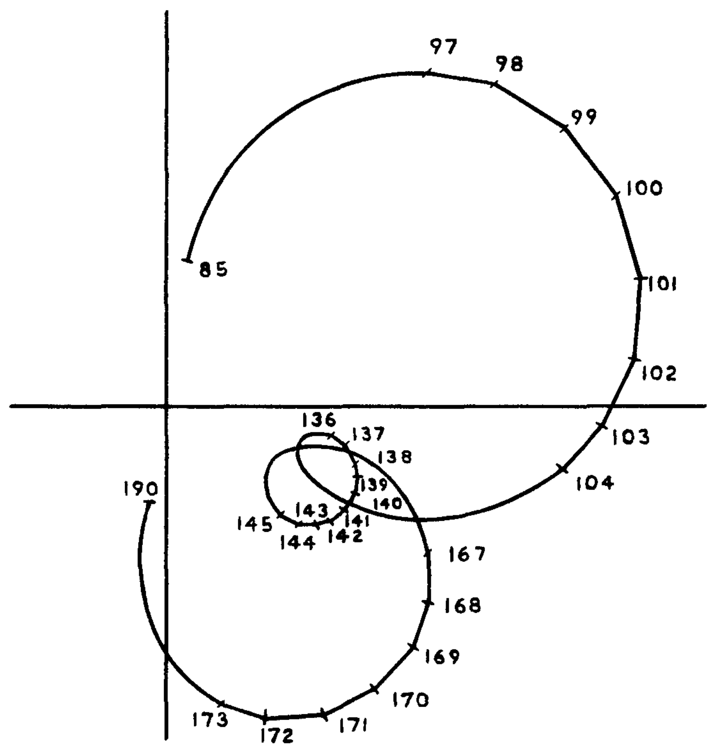


a Original transfer function (non-minimum phase shift)



b Transfer function derived by Hilbert transform of log of modulus of spectral density of Fig. 2

Fig. 1a&b Transfer functions for 3 degree of freedom system

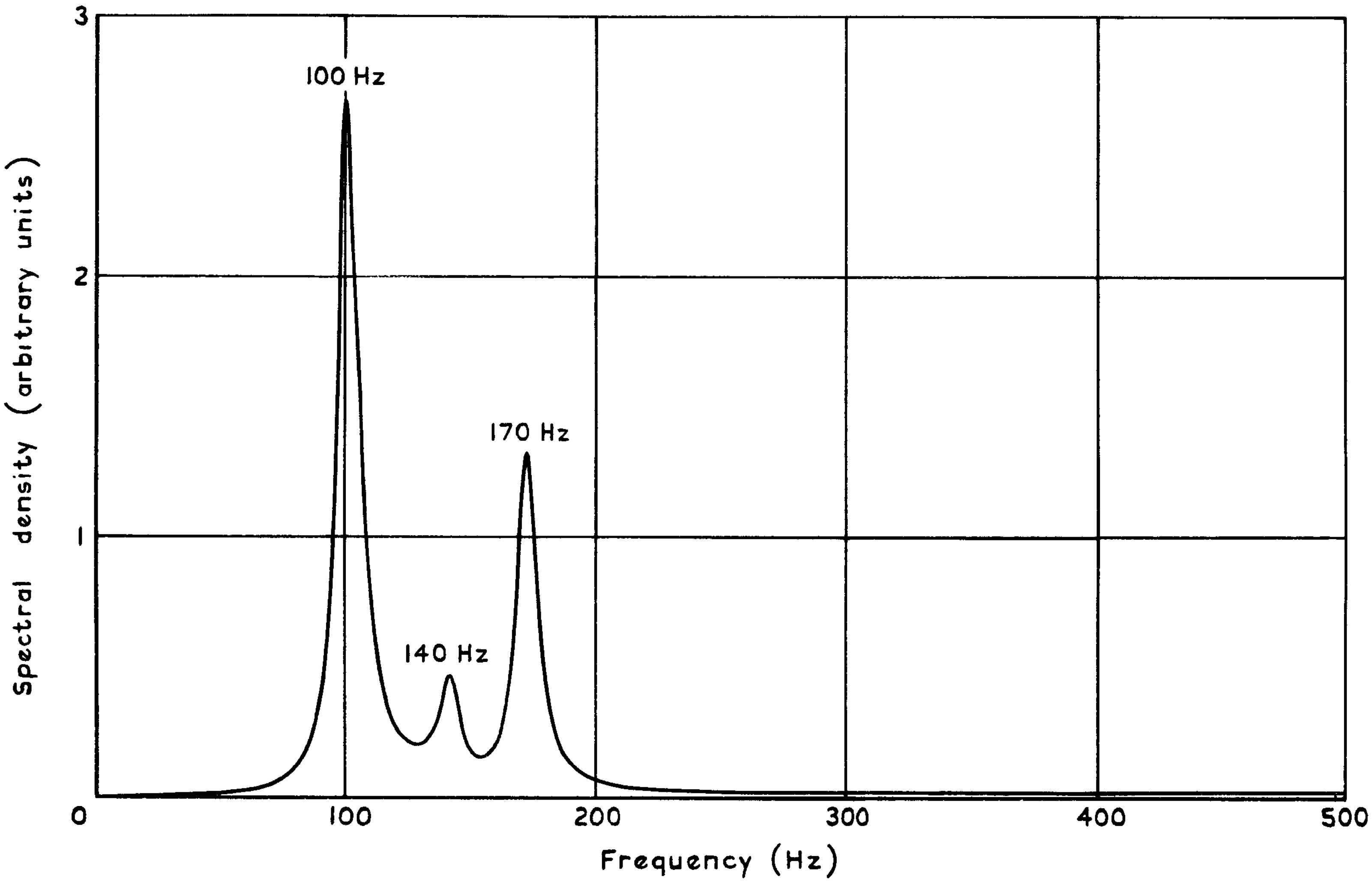


Fig. 2 Spectral density derived from Figs. 1a & 1b

Figures on curves
denote frequency (Hz)

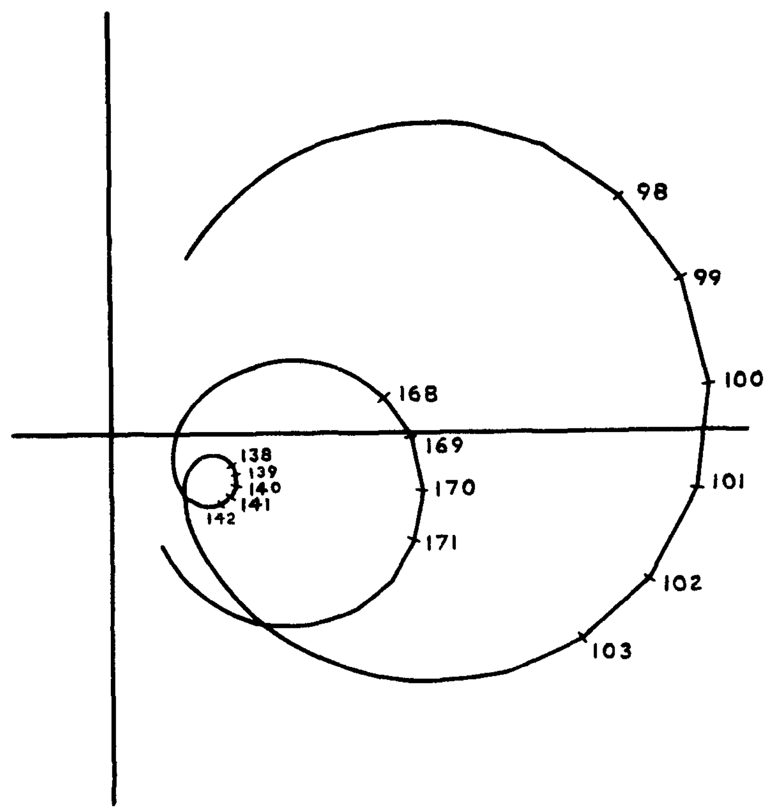
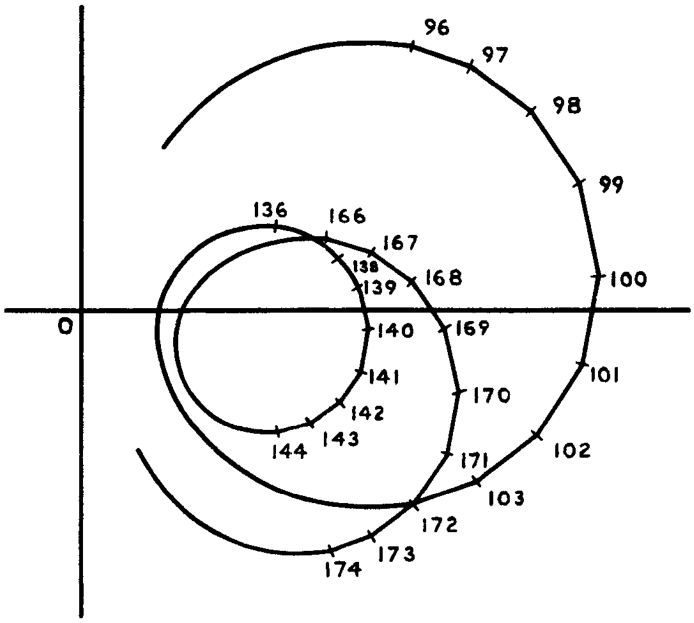
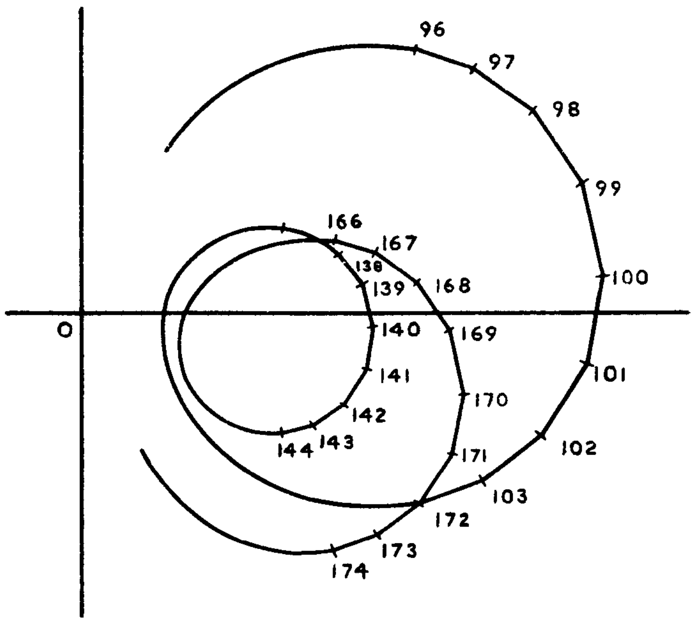


Fig.3 Transfer function from one-sided autocorrelogram derived from spectral density of Fig. 2

Figures on curves
denote frequency (Hz)



a Original transfer function. (minimum phase shift)



b Transfer function derived by Hilbert transform of log of modulus of spectral density of Fig. 5

Fig. 4 a & b Transfer functions for 3 degree of freedom system

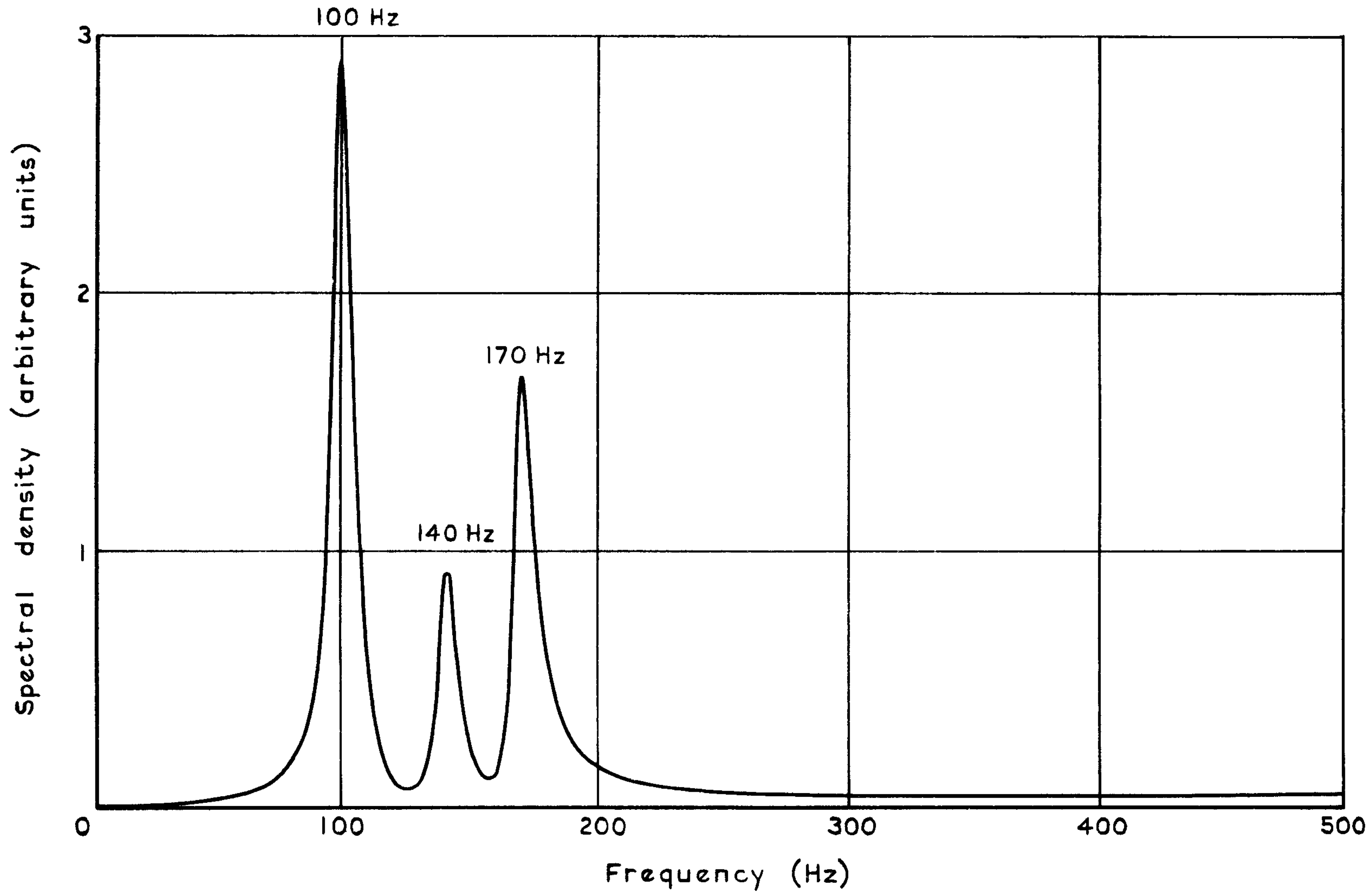


Fig.5 Spectral density derived from Fig. 4 a

Figures on curves
denote frequency (Hz)

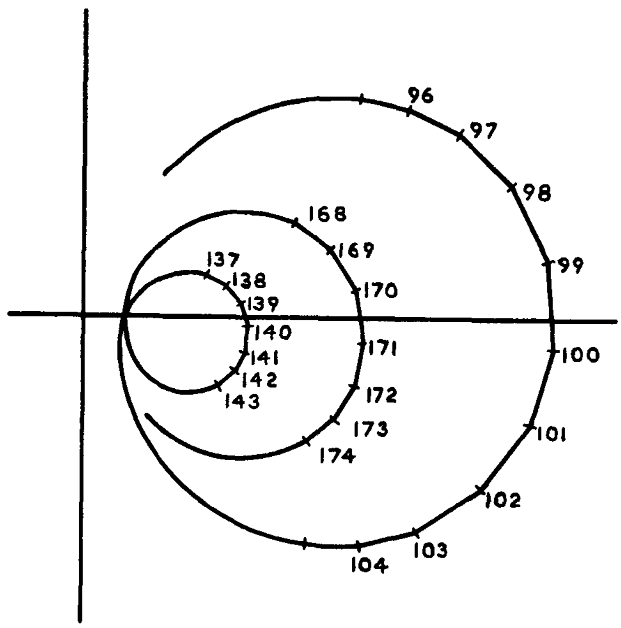


Fig. 6 Transfer function from one-sided autocorrelogram derived from spectral density of Fig. 5

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