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Some Results on the Growth of Instabilities in a Shear Layer

by

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SOME RESULTS ON THE GROWTH OF INSTABILITIES IN A SHEAR LAYER

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Ann P. Dowling**

SUMMARY

The linear spatial stability of a mixing layer between two parallel streams is considered, and a comparison is made between the stability of flows with and without a stationary splitter plate separating them upstream. The assumed velocity profile satisfies the condition of no slip on the splitter plate, but in other respects the flow is treated as inviscid.

Account is taken of the jet spreading downstream and expressions are obtained for the total amplification of an infinitesimal disturbance of a particular frequency as it travels downstream.

The stability of a hot air jet is compared to that of a cold air jet with the same velocity profile.

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CONTENTS

SYMBOLS

(x,y)	cartesian coordinates, where x is taken to be in the flow direction
$V(y)$	the shear velocity profile at a given x
ℓ	typical jet width
$r(x)$	describes the spread of the jet
U_0	typical jet velocity
ψ	the perturbation stream function
(u,v)	the perturbation velocity
p	the perturbation pressure
ϕ	the Fourier transform of ψ with respect to x and t
α	the wave number of the disturbance
ω	the frequency of disturbance
$\hat{\Phi}$	defined by $\phi = \phi_0 \exp\left[\int^y \hat{\Phi}(\xi)d\xi\right]$

$(X,Y) = (x,y)/\ell r(x)$, non-dimensional cartesian coordinates

$a \quad = \alpha\ell r(x)$, non-dimensional wave number

St Strouhal number $= \dfrac{\omega r(x)\ell}{U_0}$

$\rho(y)$ density profile

1 INTRODUCTION

The hydrodynamic stability is considered here of a mixing region between two parallel incompressible flows with different velocities. Crighton[1], amongst others, analyses spatially amplified disturbances in a jet with a simple mean velocity profile, and has obtained good agreement with the measurements made by Crow and Champagne[2]. The significance of two properties of real jets which have not been included in previous analyses are investigated here and their influence on the spatial stability of the flow is examined.

The first effect considered occurs where fluid travels out of a moving nozzle. From the frame of reference of the nozzle, we have the mixing of two parallel flows of different velocities, with a splitter plate initially separating the two. In order to investigate the effect of the wake of the splitter plate on the stability of the region two simple velocity profiles are chosen, one with and the other without a local minimum such as would represent the effect of this wake; the growth of spatial instabilities is then compared in the two cases.

The second effect we include is that of a hot jet. By considering a simple form for the density variation, it is possible to determine its effect on the stability of the shear layer.

The flow is treated as inviscid since in free boundary layers viscosity has only a damping influence (see Lin[3]). Moreover Betchov and Criminale[4] show that, for an unstable mode, the viscous theory at high Reynolds number merges with the inviscid theory.

The analysis is two-dimensional but can easily be extended to axisymmetric flows, when similar results would be expected.

2 A COMPARISON OF THE FLOW WITH AND WITHOUT A SPLITTER PLATE

2.1 The velocity profiles

(a) On leaving the splitter plate

Consider two parallel streams flowing in the x-direction, with velocities $U_0(A+1)$ and $U_0(A-1)$, separated by a shear layer. Far downstream from the splitter plate the approximate shape of the velocity profile across the mixing region may be described by

$$V = U_0\left(A - \tanh \frac{y}{2\ell}\right)$$

where ℓ defines the local shear-layer thickness.
The profile is sketched in Fig.1a.

This formula is a reasonably good approximation to a simple shear layer, with $A = 1$ corresponding to a jet flowing into still air.

In order to get a simple profile that will represent the wake of a stationary splitter plate lying along $y = \text{constant}$, $x < 0$ we choose

$$V = U_0\left(A - \tanh\frac{y}{2\ell} - \operatorname{sech}^2\frac{y}{2\ell}\right)$$

where the constant A is chosen so that the minimum value of V is zero, and occurs on the splitter plate.

Now

$$V' = -\frac{1}{2\ell}\operatorname{sech}^2\frac{y}{2\ell} + \frac{1}{\ell}\operatorname{sech}^2\frac{y}{2\ell}\tanh\frac{y}{2\ell}$$

so that $V' = 0$ corresponds to $\tanh\frac{y}{2\ell} = \frac{1}{2}$ and $V = 0$ when $V' = 0$ gives $A = 1\frac{1}{4}$. This profile is sketched in Fig.1b.

By taking the same value of $A(= 1\frac{1}{4})$ in both profiles and investigating the stability we can determine what effect the local velocity minimum induced by the stationary splitter plate has on the stability of the mixing region between parallel flows with the same overall velocity ratio of 9.

(b) <u>Variation with x</u>

The velocity profile changes downstream as the jet spreads out. In the case of the tanh-profile a suitable description of this spread is given by

$$V(x,y) = U_0\left(1\tfrac{1}{4} - \tanh\frac{y}{2r(x)\ell}\right)$$

where $r(x)$ is a slowly increasing function of x and $r(0) = 1$.

For example

$$r(x) = 1 + \varepsilon\frac{x}{\ell}, \quad x > 0$$

where ε is small, is appropriate for slow linear growth of the shear layer.

For a laminar shear layer, the thickness increases like $\sqrt{x}$ relative to some suitable origin, but since we are not concerned with instabilities in the neighbourhood of this origin it is reasonable to use the linear formula to represent the local behaviour. The linear growth is illustrated in Fig.2.

When the effect of the wake of a splitter plate is included, the earlier formula for the velocity profile becomes

$$V(x,y) = U_0\left(1\tfrac{1}{4} - \tanh \frac{y}{2r(x)\ell} - s(x)\ \mathrm{sech}^2 \frac{y}{2r(x)\ell}\right)$$

where, as before, $r(x)$ represents the spread of the jet and $s(x)$ is a slowly decreasing function of x such that $s(0) = 1$, $s(\infty) = 0$.

Here the variation in s approximately represents the filling out of the wake by viscous diffusion and will depend on the Reynolds number. The development of this profile is sketched in Fig.3.

Bouthier[5] has applied a 'multiple scales' method to show that if $V(x,y)$ is a very slowly varying function of x , such that $\partial V/\partial x = O(\varepsilon)$, where $\varepsilon \ll 1$ then to a first approximation the flow may be treated as locally parallel in the investigation of the local stability properties; we adopt this procedure in the following section.

2.2 The stability of a two-dimensional parallel shear flow

Consider a two-dimensional parallel shear flow with a velocity profile $V(y)$. We ignore viscous effects and take the flow to be incompressible.

For a disturbance with velocity (u,v) there exists a stream function ψ , such that

$$u = \frac{\partial\psi}{\partial y}$$

$$v = -\frac{\partial\psi}{\partial x} .$$

For an infinitesimal disturbance the linearised inviscid equations of motion give

$$\rho\left[\frac{\partial^2\psi}{\partial y\partial t} + V\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{dV}{dy}\right] = -\frac{\partial p}{\partial x}$$

$$\rho\left[-\frac{\partial^2\psi}{\partial x\partial t} - V\frac{\partial^2\psi}{\partial x^2}\right] = -\frac{\partial p}{\partial y}$$

where p is the pressure and ρ is the density, and the boundary conditions are

$$\psi \to 0 \qquad \text{as} \qquad |y| \to \infty .$$

We know that an arbitrary ψ can be expressed as an integral containing terms of the form $\phi(y)e^{i(\alpha x-\omega t)}$. As both the boundary conditions and the equations are linear, with coefficients independent of x and t, it is sufficient to consider one such Fourier component to obtain

$$\left.\begin{aligned} \rho\left[- i\omega \frac{d\phi}{dy} + V i\alpha \frac{d\phi}{dy} - i\alpha\phi \frac{dV}{dy}\right] &= - p i\alpha \\ \rho\left[- \alpha\omega\phi + \alpha^2\phi V\right] &= - \frac{\partial p}{\partial y} . \end{aligned}\right\} \quad (1)$$

Then after eliminating p we obtain Rayleigh's equation

$$\phi'' - \alpha^2\phi - \frac{V''}{V - \omega/\alpha}\phi = 0 \quad (2)$$

where a prime denotes differentiation with respect to y.

The boundary conditions are now

$$\phi \to 0 \quad \text{as} \quad |y| \to \infty .$$

We can express these in a more convenient form by noting that for large $|y|$ V is almost constant, so that ϕ satisfies

$$\phi'' - \alpha^2\phi \approx 0 .$$

This implies that as $y \to +\infty$, $\phi' + \alpha\phi \to 0$, and similarly as $y \to -\infty$, $\phi' - \alpha\phi \to 0$.

The constants α and ω are in general complex, and may be written

$$\alpha = \alpha_r + i\alpha_i$$

$$\omega = \omega_r + i\omega_i$$

where α_r is the wave-number of the disturbance, $-\alpha_i$ is the spatial growth-rate, ω_r is the frequency and ω_i the temporal growth rate.

Here we investigate the spatial growth rate for $\omega_i = 0$.

We can reduce the order of equation (2) if we introduce $\hat{\Phi}(y)$ defined by

$$\phi = \phi_0 \exp\left[\int^y \hat{\Phi}(y')dy'\right]$$

so that

$$\frac{d\phi}{dy} = \phi_0\, \hat{\Phi} \exp\left[\int^y \hat{\Phi}(y')dy'\right]$$

and

$$\frac{d^2\phi}{dy^2} = \phi_0\left[\hat{\Phi}' + \hat{\Phi}^2\right] \exp\left[\int^y \hat{\Phi}(y')dy'\right] .$$

Then equation (2) becomes

$$\frac{d\hat{\Phi}}{dy} = \alpha^2 - \hat{\Phi}^2 + \frac{V''}{V - \omega/\alpha} \tag{3}$$

which is a first-order, but non-linear, equation; see Betchov and Criminale[4] and Mickalke[6].

We can determine the boundary conditions by noting that

$$\hat{\Phi} = \frac{\phi'}{\phi}$$

as $y \to \infty$, $\phi' \to -\alpha\phi$ so that $\hat{\Phi} \to -\alpha$ and $y \to -\infty$ gives $\phi' \to \alpha\phi$ so that $\hat{\Phi} \to \alpha$.

2.3 Calculation of the eigenvalues

Following Bouthier's procedure[5] we use equation (2) to determine the stability of the velocity profile $V(x,y)$, where the slow variation of V with x is assumed not to influence the stability.

The two profiles considered are

$$V = U_0\left(1\tfrac{1}{4} - \tanh \frac{y}{2r(x)\ell}\right)$$

and

$$V = U_0\left(1\tfrac{1}{4} - \tanh \frac{y}{2r(x)\ell} - s(x)\ \mathrm{sech}^2 \frac{y}{2r(x)\ell}\right) .$$

We now non-dimensionalise lengths with respect to $r(x)\ell$ and velocities with respect to U_0 so that time is non-dimensionalised with respect to $r(x)\ell/U_0$ and define

$$X = \frac{x}{\ell r(x)}$$

$$Y = \frac{y}{\ell r(x)}$$

$$U = \frac{V}{U_0}$$

$$\Phi = \ell r(x)\hat{\Phi}$$

$$a = \alpha \ell r(x)$$

$$St = \omega \frac{r(x)\ell}{U_0}, \text{ the Strouhal number.}$$

The two non-dimensional velocity profiles are now

$$U = 1\tfrac{1}{4} - \tanh Y/2$$

and

$$U = 1\tfrac{1}{4} - \tanh Y/2 - s(X) \operatorname{sech}^2 Y/2$$

and equation (3) becomes

$$\frac{d\Phi}{dY} = a^2 - \Phi^2 + \frac{U''}{U - St/a}.$$

For the above profiles this gives

$$\frac{d\Phi}{dY} = a^2 - \Phi^2 + \frac{\tfrac{1}{2} \operatorname{sech}^2 Y/2 \tanh Y/2 + s[\tfrac{1}{2} \operatorname{sech}^4 Y/2 - \operatorname{sech}^2 Y/2 \tanh^2 Y/2]}{1\tfrac{1}{4} - \tanh Y/2 - s \operatorname{sech}^2 Y/2 - St/a} \quad (4)$$

where $s = 0$ for the tanh-profile, $s = 1$ for the profile at the splitter plate and $0 < s < 1$ for the downstream profile when the effect of the splitter plate is included.

In order to determine the eigenvalues of this equation we follow the procedure used by Michalke[6] and make the transformation

$$z = \tanh \frac{Y}{2}$$

so that equation (4) becomes

$$\frac{d\Phi}{dz} = 2\frac{(a^2 - \Phi^2)}{1 - z^2} + \frac{z + s[1 - 3z^2]}{1\tfrac{1}{4} - St/a - z - s(1 - z^2)} \quad (5)$$

with boundary conditions

$$\Phi(1) = -a \quad \text{and} \quad \Phi(-1) = a\ .$$

The problem has now been reduced to a first order boundary value problem, which can be solved numerically.

However, the boundary conditions are not in a suitable form for computation, so we introduce z_1 and z_2 where

$$z_1 = -1 + h$$

and

$$z_2 = 1 - h$$

so that

$$\Phi(z_1) = \Phi(-1) + h\Phi'(-1) + 0(h^2)$$

and

$$\Phi(z_2) = \Phi(1) - h\Phi'(1) + 0(h^2)\ .$$

The advantage of transforming the equation to one of first order is that it simplifies the numerical determination of the derivatives on the boundary. These can easily be obtained from equation (5) by using de l'Hopital's rule, giving

$$\Phi'(1) = -2a\Phi'(1) + \frac{1 - 2s}{\frac{1}{4} - St/a}$$

or

$$\Phi'(1) = \frac{1 - 2s}{(\frac{1}{4} - St/a)(1 + 2a)}$$

and similarly

$$\Phi'(-1) = \frac{-1 - 2s}{(2\frac{1}{4} - St/a)(1 + 2a)}\ .$$

Now for a fixed value of the Strouhal number St and a guessed value of a, equation (5) can be integrated numerically from z_1 to 0 giving a value of Φ at the origin, $\Phi_1(0)$ say, and also from z_2 to 0 giving a second value $\Phi_2(0)$ at the origin. The eigenvalue a_0 for this particular Strouhal number has

$$\Phi_1(0) = \Phi_2(0) ,$$

and this condition is achieved by iteration.

We introduce $F(a)$, where F is defined by $F(a) = \Phi_1(0) - \Phi_2(0)$, and evaluate it for three different values of a. Then three improved values of a are calculated from the approximated zeros of F by linear interpolation on each pair of the previous values of a. This procedure is repeated until $|F|$ is sufficiently small. The convergence is rapid and in general only three or four iterations have been required.

It is useful to know the eigenvalue a for one particular Strouhal number, so giving a point from which to start the investigation, instead of searching for eigenvalues throughout the complex plane. Fortunately we can identify one eigenvalue immediately.

Since for

$$s = 0 \quad \text{and} \quad \frac{St}{a} = 1\tfrac{1}{4} , \quad a = \tfrac{1}{2}$$

we see that $\Phi = -z/2$ is an eigenvector of equation (5) and it follows that $St = 0.625$ has an eigenvalue $a = 0.5$, which is, in fact, the well-known neutral solution. We now wish to determine a set of complex eigenvalues a in this neighbourhood, and those with a negative imaginary part will correspond with a disturbance that grows in the downstream direction. This is found to occur for $St < 0.625$ and so we restrict our attention to this range of Strouhal number. The procedure is to reduce St in small steps from 0.625 and at each stage a new complex eigenvalue is found by use of the preceding solution. The process is continued down to $St = 0$.

For $s \neq 0$, we do not have the convenience of being able to identify a starting solution. However for disturbances of long wavelength we should not expect the small scale detail to make a significant difference. For low Strouhal number therefore, we search for eigenvalues near those for $s = 0$. Once one eigenvalue for $s \neq 0$ has been obtained in this way we can find the complete range of unstable wavelengths and their corresponding growth rates by the procedure described above.

This determines one eigenvalue for each Strouhal number. It does not exclude the possible existence of other eigenvalues lying elsewhere in the complex plane. For example, Crighton[1] was able to find another branch in the same quadrant of the complex plane (branch 4 in his Fig.10.4), but in the present calculations trial solutions that were started near this branch were found to converge on solutions in another quadrant. There does not seem to be any general method of determining whether all required branches have been found, nor of indicating where to look for new branches.

2.4 Results

The eigenvalues obtained for the two velocity profiles corresponding to $s = 0$ and $s = 1$ are listed in Table 1 in the order in which they were found. The curve of growth-rate $-a_i$ against Strouhal number for these two profiles is plotted in Fig.4a, and that of wave-number a_r against Strouhal number is shown in Fig.5. It seems reasonable to expect that the growth-rates and the wave-numbers for values of s between 0 and 1 will lie between the two curves.

The effect of the wake of the splitter plate on the stability can now be assessed. Fig.4 shows that the peak growth-rates are more than doubled, and although non-linear effects will alter the total growth, it may be hoped that the linear theory will predict the most amplified disturbances without much error. On this basis the most unstable Strouhal numbers have changed from about 0.275 to about 0.34, and the most rapidly growing non-dimensional wave-numbers from 0.2 to 0.3. Also the flow becomes stable at a slightly lower Strouhal number.

In particular, we want to investigate the amplification of a disturbance at a fixed frequency, i.e. a particular timewise Fourier component, as it travels down the mixing region.

Following Crighton[1] we can write

$$\psi(x) = f(x)\,\phi(x,y)\,\exp\left\{i\int^{x}\alpha(x')dx' - i\omega t\right\}$$

where f is an amplitude function, $\alpha(x)$ is the eigenvalue for that ω and is obtained by considering the flow as locally parallel, and $\phi(x,y)$ is the corresponding eigenvector.

Crighton argues that, if f and ϕ are slowly varying functions of x, their variation will have little effect on the growth-rate, which is dominated by the exponential term.

It was noted from Fig.4 that there is no growth for

$$St > 0.625 \ .$$

With

$$r(x) = 1 + \varepsilon \frac{x}{\ell} \ ,$$

then

$$St = \frac{\omega\ell}{U_0}\left(1 + \varepsilon \frac{x}{\ell}\right) \ ,$$

so that for $\omega\ell/U_0 > 0.625$ there is no growth at any x and the corresponding waves are everywhere damped. Moreover a disturbance whose frequency ω satisfies $\omega < 0.625\ U_0/\ell$, so that it is initially amplified, will become damped after travelling a distance d_c downstream. This distance will depend on how the shear layer thickens with x and provided this is known, the growth rate can be determined as a function of x up to the point $x = d_c$ where it vanishes. We illustrate the procedure for the simplest case where the shear layer thickness linearly with x, although for a laminar shear layer it would strictly be more logical to allow the thickness to increase like $\sqrt{x - x_o}$, where x_o is some constant. With the linear assumption it follows that

$$d_c = \frac{\ell}{\varepsilon}\left[0.625 \frac{U_0}{\omega\ell} - 1\right] \ .$$

We can estimate the growth of disturbances by

$$\psi(x) \approx \psi(0) \exp\left[-\int_0^x \alpha_i(x')dx'\right]$$

so that at a distance d downstream, the fractional increase in amplitude is given by

$$\text{Fractional increase} = \frac{\psi(d)}{\psi(0)}$$

$$= \exp\left[-\int_0^d \alpha_i(x')dx'\right] .$$

Set

$$W = \frac{\omega\ell}{U_0}\left(1 + \varepsilon\,\frac{x'}{\ell}\right) ;$$

then, in terms of the non-dimensional growth rate $-a_i$ the fractional increase becomes,

$$\frac{\psi(d)}{\psi(0)} = \exp\left[-\frac{1}{\varepsilon}\int_{\frac{\omega\ell}{U_0}}^{\frac{\omega\ell}{U_0}\left(1+\varepsilon\frac{d}{\ell}\right)} a_i(W)\,\frac{dW}{W}\right]$$

In the case of the tanh-profile

$$-\int_{\frac{\omega\ell}{U_0}}^{\frac{\omega\ell}{U_0}\left(1+\varepsilon\frac{d}{\ell}\right)} a_i(W)\,d(\ln W)$$

is the area under the full-line curve C_1 in Fig.4b between the limits

$$W = \frac{\omega\ell}{U_0} \quad \text{and} \quad \frac{\omega\ell}{U_0}\left(1 + \varepsilon\,\frac{d}{\ell}\right) ;$$

Fig.4b is, in fact, Fig.4a redrawn to a logarithmic base.

For the profile with the stationary plate present the situation is more complicated. For now α depends on x not only through the scaling factor $r(x)$ but also through the function $s(x)$. This function s represents the effect of viscosity in smoothing out the velocity profile and hence depends on Reynolds number. However without making any assumptions about $s(x)$, we expect that the curves of growth-rate against frequency for any values of s , $(0 < s < 1)$, will lie between those of $s = 0$ and $s = 1$. We therefore use these two curves to obtain bounds on the growth of disturbances of a particular frequency. At a distance d downstream, the bounds of the fractional increase in amplitude are

$$\exp\left[-\frac{1}{\varepsilon}\int_{C_1\ \frac{\omega\ell}{U_0}}^{\frac{\omega\ell}{U_0}\left(1+\varepsilon\frac{d}{\ell}\right)} a_i(W)d\ \ln W\right] < \frac{\psi(d)}{\psi(0)} < \exp\left[-\frac{1}{\varepsilon}\int_{C_2\ \frac{\omega\ell}{U_0}}^{\frac{\omega\ell}{U_0}\left(1+\varepsilon\frac{d}{\ell}\right)} a_i(W)d\ \ln W\right]$$

where C_1 and C_2 are the bounding curves illustrated in Fig.4b.

3 THE EFFECT OF DENSITY VARIATIONS

As a further example, the jet fluid is allowed to have a higher temperature and therefore a lower density than the external fluid; the density ratio assumed here is 4. For simplicity, the analysis is confined to the (tanh) shear layer.

Assuming that the density and velocity in the mixing region can both be described by a tanh-profile with the same length scale, we consider a profile with

$$\rho = \rho_1 \frac{1}{2}\left\{5 + 3\tanh\left(\frac{y}{2\ell r(x)}\right)\right\}$$

$$V = U_0\left\{\frac{5}{4} - \tanh\left(\frac{y}{2\ell r(x)}\right)\right\}$$

where ρ_1 is the density of the inner stream.

Now elimination of p from the linearised equations of motion (1) gives

$$\phi'' - \alpha^2\phi - \frac{V''\phi}{V - \omega/\alpha} + \frac{\rho'}{\rho}\left[\phi' - \phi\frac{V'}{V - \omega/\alpha}\right] = 0$$

with boundary conditions $\phi \to 0$ as $|y| \to \infty$. After making the same substitutions as before, and using non-dimensional co-ordinates, we have

$$\frac{d\Phi}{dz} = 2\frac{(a^2 - \Phi^2)}{1 - z^2} + \frac{z}{1\frac{1}{4} - z - St/a} - \frac{3}{5 + 3z}\left[\Phi + \frac{1 - z^2}{2(1\frac{1}{4} - z - St/a)}\right]$$

where the last term on the right-hand side represents the effect of the density variation.

The boundary conditions are

$$\Phi(-1) = a \quad \text{and} \quad \Phi(1) = -a .$$

The eigenvalues for this equation were found for different values of the Strouhal number in exactly the same way as those for equation (5). The results are listed in Table 2, and sketched in Figs.6 and 7 where they are compared with those for the same velocity profile but with no density variation.

We see that the most unstable Strouhal number has been decreased from 0.275 to 0.2 by the effect of the density variation, but there is very little change in the wave-number corresponding to this most amplified disturbance. Although the maximum growth-rate has been increased by the density stratification, the density change has a stabilizing effect at the higher Strouhal numbers by reducing the critical value at which the flow first becomes stable.

Again assuming a linear spread of the shear layer, an initially growing disturbance becomes damped after travelling a distance d_c downstream where

$$d_c = \frac{\ell}{\varepsilon}\left[0.55\,\frac{U_0}{\omega\ell} - 1\right]$$

for a density ratio of 4 compared with

$$d_c = \frac{\ell}{\varepsilon}\left[0.625\,\frac{U_0}{\omega\ell} - 1\right]$$

for a density ratio of 1, and with the same velocity ratio of 9 in each case.

4 CONCLUSIONS

The spatial growth-rates and associated properties of unstable disturbances to a shear flow have been compared for two velocity profiles by a method based on the work of Michalke[6] and Crighton[1]. The profiles chosen were the familiar tanh-profile and another incorporating a local velocity minimum typical of a profile just downstream of a splitter plate. The maximum growth rates for the latter were found to be more than twice those of the tanh-profile for the same conditions at infinity and the same width of shear flow. The effect of a hot jet was also considered and this was found to give a slight increase in the growth rate.

There are several limitations in applying this method. One is that when Rayleigh's equation is solved numerically one cannot be sure of finding all the eigenvalues. Another point is that regions of exponential growth arise so that in practice the non-linear effects soon become important. Previous studies[7] have suggested that vortices develop downstream of the splitter plate. However if the effect of the non-linear terms is not strongly frequency-dependent this analysis should give an estimate of the most amplified frequency. The initial rate of production, and hence the spacing, of the vortices in the situations studied, can then be inferred.

Acknowledgments

The author would like to thank Dr. E.G. Broadbent for his help and advice.

Table 1

TABLE OF RESULTS

For the velocity $U = 1\frac{1}{4} - \tanh Y/2$, corresponding to $s = 0$

Strouhal number	Non-dimensional wave-number	Non-dimensional growth rate
St	a_r	$-a_i$
0.625	0.5	0
0.6	0.484	0.008
0.55	0.443	0.021
0.5	0.415	0.036
0.45	0.376	0.051
0.4	0.335	0.067
0.35	0.299	0.076
0.3	0.244	0.082
0.25	0.194	0.083
0.2	0.145	0.077
0.15	0.098	0.062
0.1	0.059	0.042
0.05	0.027	0.022

For a profile $U = 1\frac{1}{4} - \tanh Y/2 - \operatorname{sech}^2 Y/2$, corresponding to $s = 1$

St	a_r	$-a_i$
0.0725	0.017	0.04
0.1	0.036	0.062
0.15	0.072	0.098
0.2	0.119	0.135
0.25	0.173	0.171
0.3	0.247	0.210
0.35	0.344	0.218
0.4	0.443	0.181
0.45	0.515	0.147
0.5	0.594	0.087
0.55	0.646	0.029

Table 2

EIGENVALUES FOR THE tanh-VELOCITY PROFILE
WITH A DENSITY VARIATION OF 4

Strouhal number	Non-dimensional wave-number	Non-dimensional growth-rate
0.5	0.516	0.028
0.45	0.476	0.046
0.4	0.431	0.063
0.35	0.384	0.079
0.3	0.332	0.092
0.25	0.277	0.101
0.2	0.215	0.103
0.15	0.152	0.094
0.1	0.073	0.088

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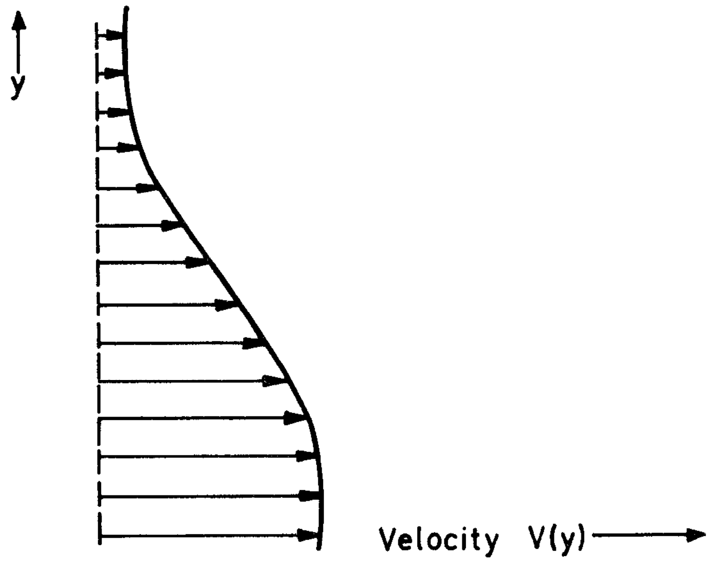
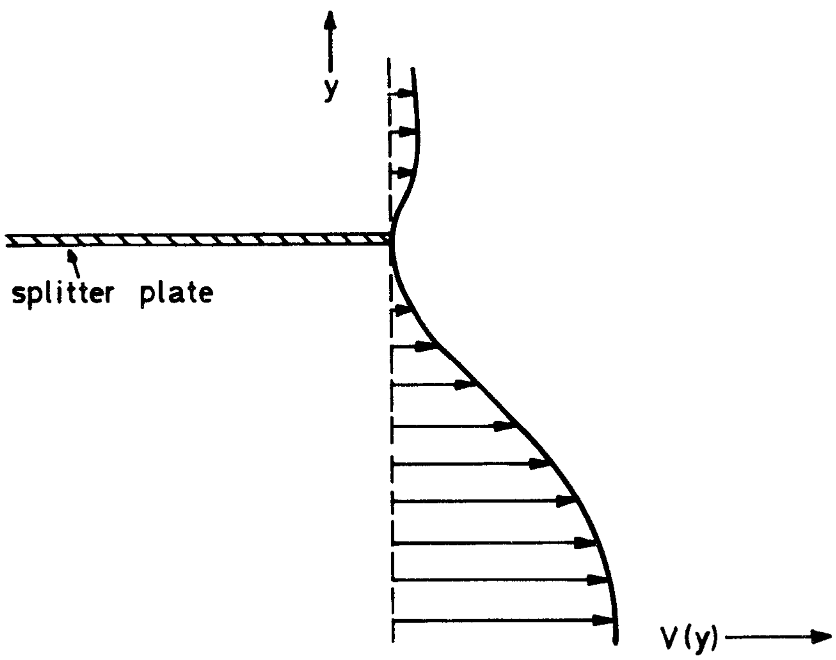
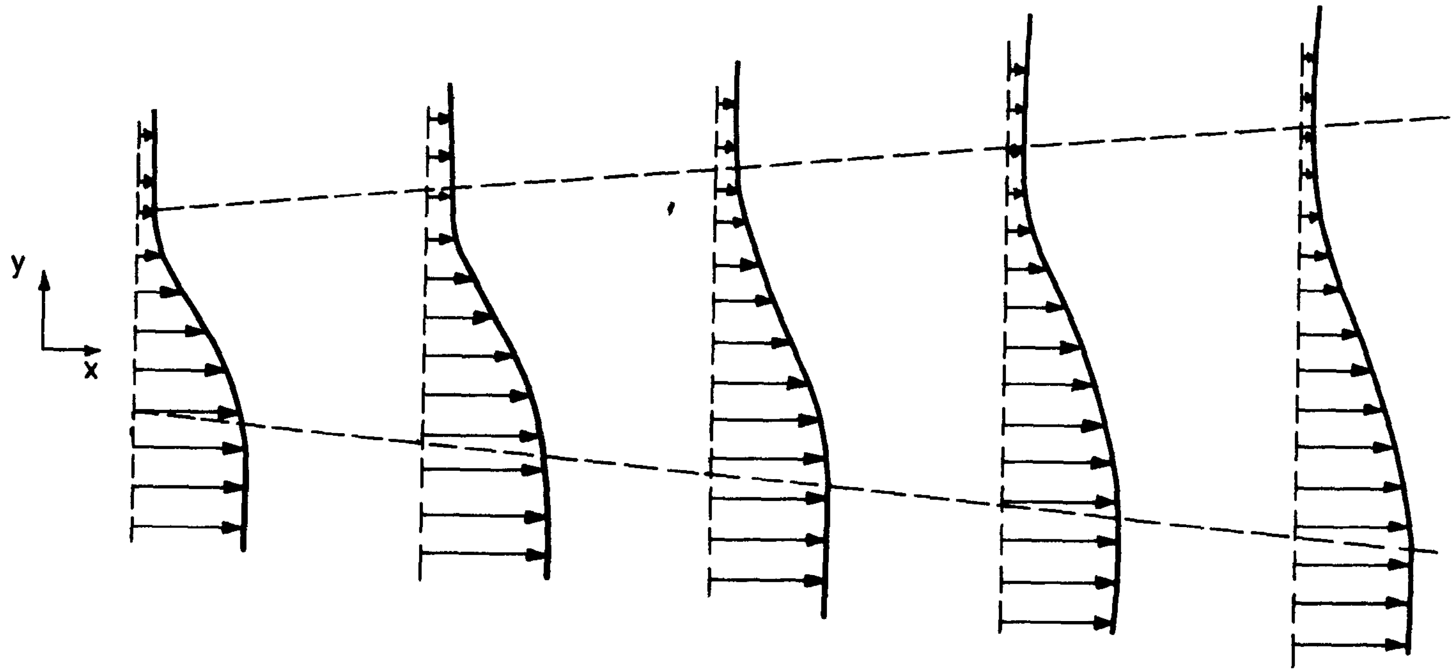


a The velocity profile neglecting the effect of the splitter plate



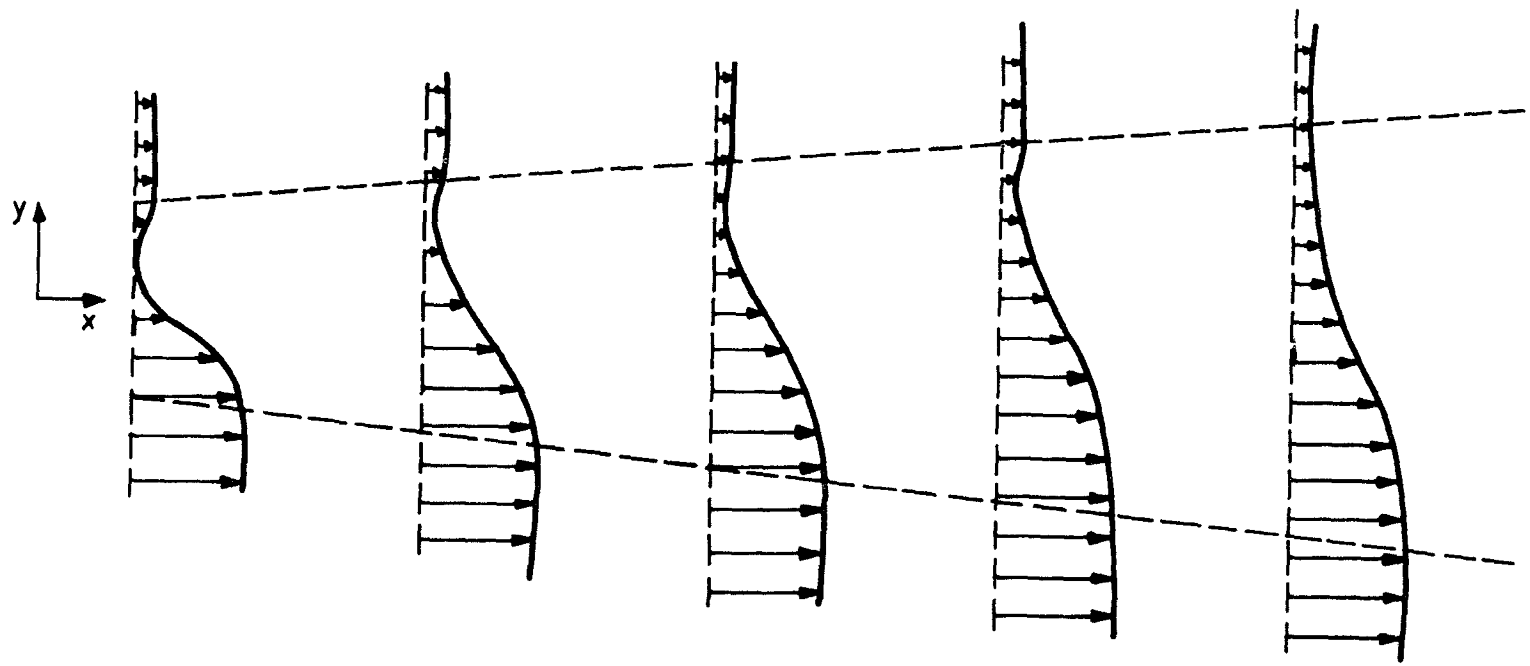
b The velocity profile including the effect of the splitter plate

Fig. 1a&b The velocity profiles at the trailing edge of the plate



$$V(x,y) = Uo\left(1\tfrac{1}{4} - \tanh \frac{y}{2lr(x)}\right)$$

Fig. 2 The development of the velocity profile downstream neglecting the effect of the splitter plate



$$V(x,y) = 1\tfrac{1}{4} - \tanh \frac{y}{2r(x)L} - S(x)\,\mathrm{sech}^2 \frac{y}{2r(x)L}$$

Fig. 3 The development of the velocity profile downstream including the effect of the splitter plate

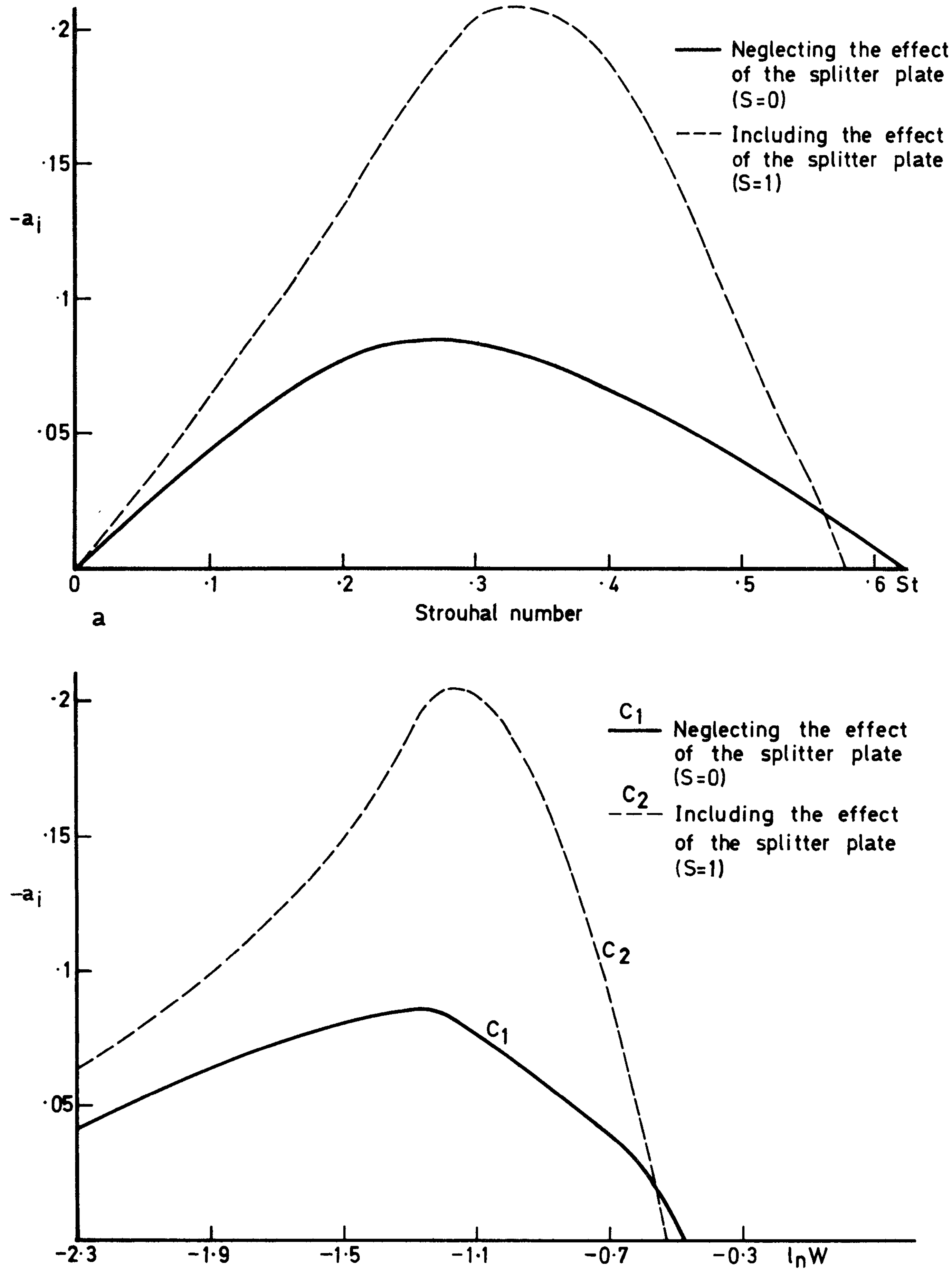


Fig.4a&b The effect of the splitter plate on the growth rate

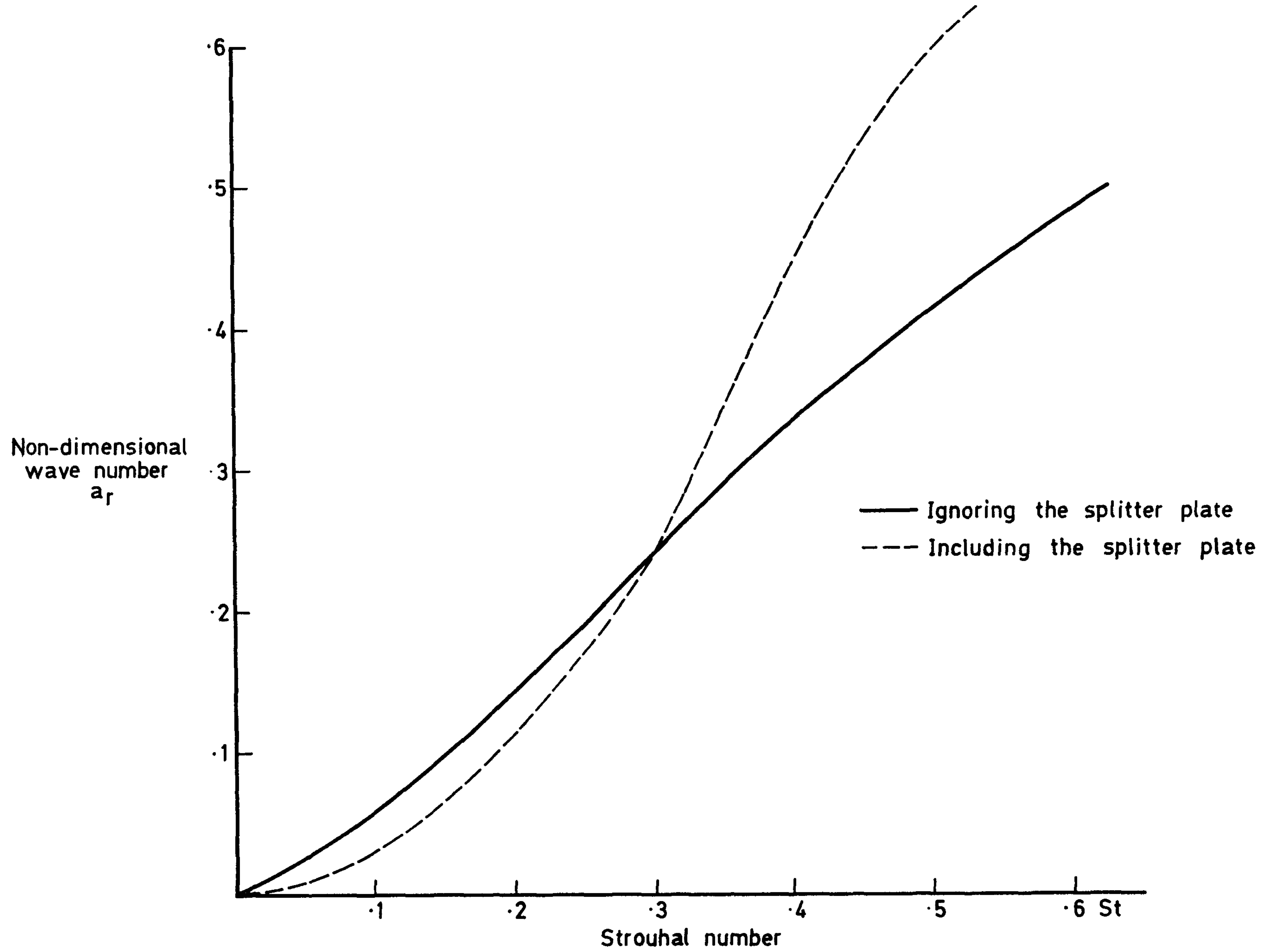


Fig. 5 The effect of the splitter plate on the wave number

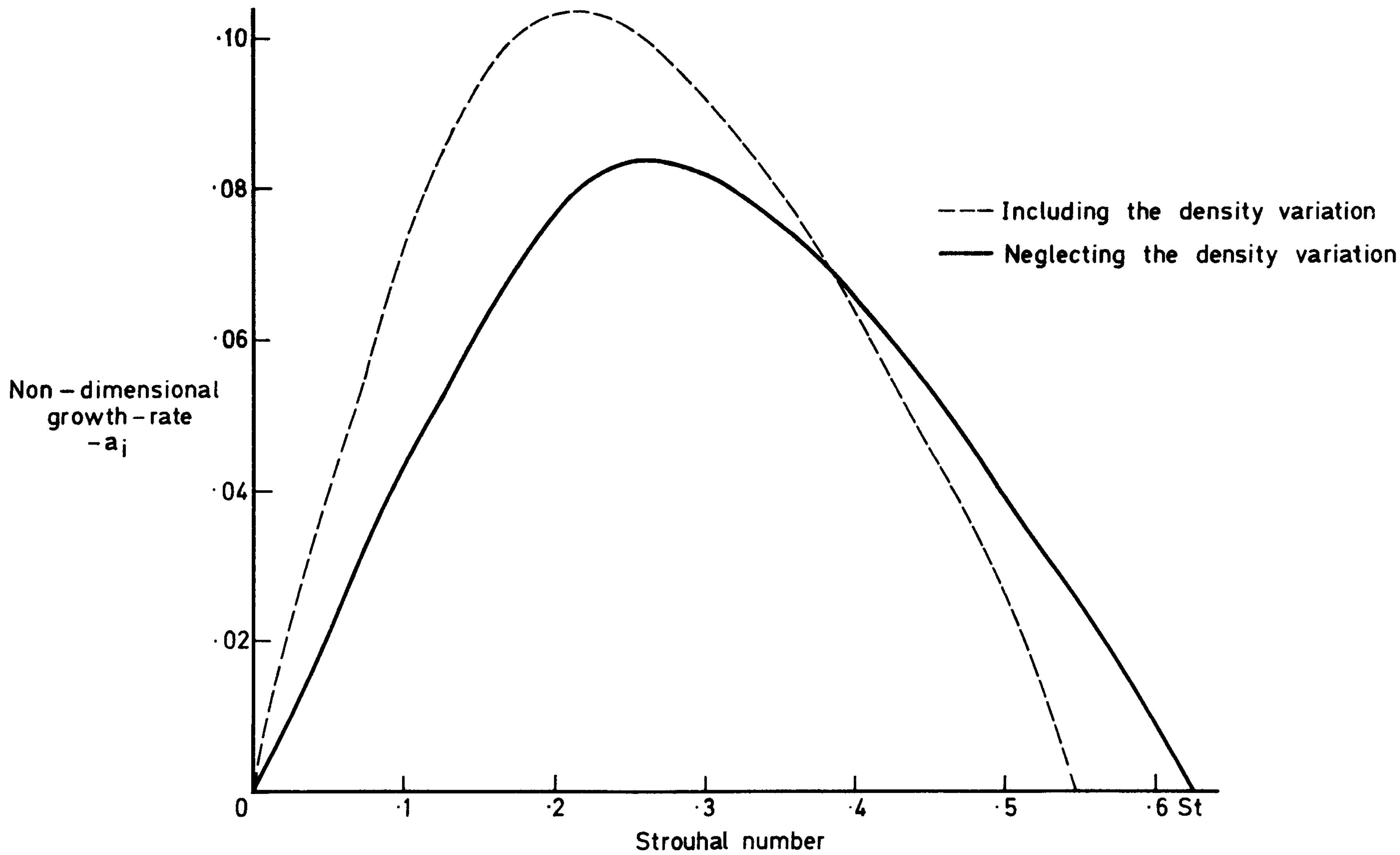


Fig. 6 The effect of a density variation on the growth-rate for the tanh – velocity profile

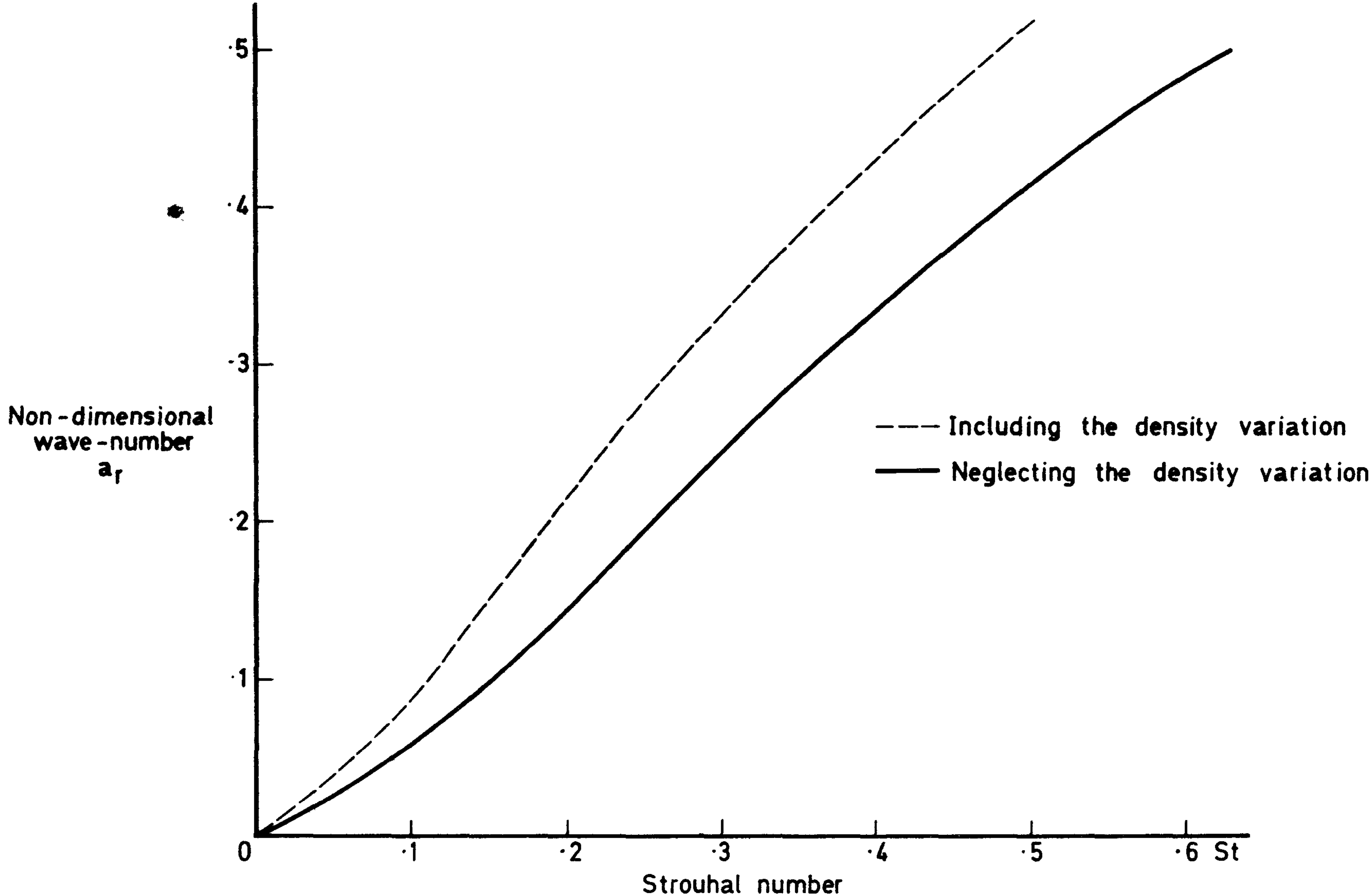


Fig.7 The effect of the density variation on the wave-number for the tanh – velocity profile

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