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by

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FORTRAN PROGRAMS FOR THE DETERMINATION OF AERODYNAMIC DERIVATIVES FROM TRANSIENT LONGITUDINAL OR LATERAL RESPONSES OF AIRCRAFT

by

A. Jean Ross
G. W. Foster

SUMMARY

Two Fortran computer programs are described, one for the analysis of longitudinal responses of aircraft, and one for the analysis of lateral responses in the presence of small longitudinal motion. The aerodynamic derivatives which affect the responses are determined by a Newton-Raphson technique to obtain iteratively the best least-squares fit to the observed data. A description is given of the numerical method, and its implementation in the programs, and then separate guides for users are provided for the two programs. An example is shown for each program.

Some of the computer output reproduced in this report may not be clearly legible in places. Readers requiring clear copies should apply direct to the authors.

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* Replaces RAE Technical Report 75090 - ARC 36303.

CONTENTS

1 INTRODUCTION

The application of digital optimisation techniques to the analysis of flight records to obtain aerodynamic derivatives has received a great deal of attention in the past few years, particularly in the United States, where a number of teams have developed computer programs based on various methods. An excellent summary of the work, together with possible extensions, is given in Ref.1, and a large number of papers is collected in Ref.2.

The possibilities of such methods were demonstrated at RAE by Waterfall[3,4], who developed programs to obtain derivatives from the coupled longitudinal and lateral responses of rocket-launched free-flight models. Two Fortran programs have been developed from the free-flight model work for application to full-scale aircraft, and are described in this Report. The program for the analysis of longitudinal response data includes all the longitudinal aerodynamic derivatives, but may be used for those pertinent to the short period oscillation for responses at constant forward speed, while the program for the analysis of lateral response data includes correction terms for the influence of measured longitudinal motion. They are easier to use than the original program in that the derivatives to be identified are chosen via input data, and modifications to the program are not needed for different sets of parameters.

The optimisation method used is that of weighted least squares, using differential correction techniques (also termed Newton-Raphson). It is hoped that the description given here is sufficiently detailed to enable the essentials of the method to be grasped and for modifications to the programs to be made to suit particular needs, but at the same time is concise enough to be a User's Guide for the programs as they stand. Section 2 summarises the basic theory, and the specific applications of the method to the longitudinal and lateral responses are described in sections 4 and 5 respectively. Section 3 will probably only be needed by actual users of the programs.

More complicated methods of parameter identification are being applied by Klein[5] at Cranfield Institute of Technology, under Ministry of Defence (PE) contract. At present, his computer programs are written in a specialised machine language developed at Cranfield, but it is hoped that Fortran versions will become available, which may supercede the RAE programs for some applications. Klein incorporates four optimisation methods (1) equation error method (2) weighted least squares, equivalent to the method described in this Report,

(3) maximum likelihood and (4) Bayesian technique. The latter two methods use more sophisticated statistical techniques, but need good first guesses for the values of the parameters to get acceptable rates of convergence. Klein has demonstrated that they can be useful for 'non-standard' responses, e.g. longitudinal short period of a slender wing aircraft with nonlinear aerodynamics, but it is felt that the RAE programs are usually sufficient for most applications, and serve as a useful introduction to the more advanced techniques.

2 THEORETICAL BACKGROUND

The aim of the program is to obtain the least-squares fit between measured response data and the calculated results from the mathematical model of the motion, by updating the values of the unknown parameters iteratively, using the method of differential corrections. The basic theory is given in Ref.3, but is summarised here for convenience, so that the steps in the computer programs may be recognised.

The response data consist of readings from n instruments (e.g. accelerometers, rate gyros, incidence probes), taken at m intervals of time. At time t_i , these instrument readings are denoted by the n-dimensional vector π_i . The equations of motion form the mathematical model, including both the force and moment equations, and the kinematic relations. These equations are written in terms of the unknown parameters, particularly the aerodynamic derivatives, and the state variables which are usually the perturbations in linear and angular velocities from a steady state. For the purpose of matching responses, the instrument readings need to be expressed in terms of the state variables, e.g. local accelerations at accelerometers away from the CG of the aircraft, and local angles of incidence at vane and probe positions. The calculation of the responses requires initial conditions of the state variables, and so these initial values must be included as unknown parameters (even if measured values are available, they cannot be assumed to be exactly correct due to instrument noise etc.). The remaining set of unknown parameters which may have to be determined are the offset errors (bias errors) of the instruments, as calibrations can vary with time or flight condition.

The p unknown parameters (aerodynamic derivatives, initial conditions and offset errors) are denoted by x_k, $k = 1 \dots p$, and the r state variables by $y_j(t)$, $j = 1 \dots r$. For a given set of values of the parameters x_k , the mathematical model is used to compute the corresponding instrument readings at time t_i ,

$$\pi_{i_c} = \Upsilon(x_1, x_2 \ldots x_p, t_i) \quad , \tag{1}$$

where the function Υ includes the equations of motion, kinematic relations and the expressions for the instrument readings.

Thus the residual errors are given by

$$R_{\pi_i} = \pi_i - \pi_{i_c} \quad . \tag{2}$$

The optimisation procedure is to determine the values of x_k which minimise the cost function

$$U = \sum_{i=1}^{m} \left(\pi_i - \pi_{i_c}\right)^* w_i \left(\pi_i - \pi_{i_c}\right) \quad , \tag{3}$$

where w_i is an $n \times n$ diagonal matrix of the weights applied to the observed response variables, and * denotes transpose. In the current programs described here, w_i are constants, independent of time, chosen according to the relative amplitudes and reliability of the response data.

The minimum of U is given by $dU = 0$, which is evaluated algebraically from first principles, by considering an updated set of parameters, x_k' , where

$$x_k' = x_k + \delta x_k \quad , \tag{4}$$

and the δx_k's are to be determined.

Then the corresponding vectors are given by

$$\pi_{i_c}' = \Upsilon\left(x_k', t_i\right) \tag{5}$$

$$R_{\pi_i}' = \pi_i - \pi_{i_c}' \tag{6}$$

$$U' = \sum_{i=1}^{m} R'^{*}_{\pi_i} w_i . R'_{\pi_i} \quad . \qquad (7)$$

In order to linearise the equations, the relation in (5) is expanded in a Taylor series,

$$\pi'_{i_c} = \pi_{i_c} + \sum_{k=1}^{p} f_{ki} . \delta x_k + 0\left((\delta x_k)^2\right) \qquad (8)$$

where $f_{ki} = \dfrac{\partial \Upsilon(x_1 \ldots x_p, t_i)}{\partial x_k}$, and are written as m-dimensional column vectors. The correction procedure assumes that $(\delta x_k)^2$ is negligible, which linearises equation (8), but necessitates an iterative solution until the assumption is valid. Then equation (6) may be written

$$R'_{\pi_i} = R_{\pi_i} - \sum_{k=1}^{p} f_{ki} . \delta x_k \quad ,$$

or in matrix form

$$D' = D - CE \qquad (9)$$

$$\text{where} \quad \left.\begin{array}{lll} D = \left[R_{\pi_i}\right] , & \text{a column vector of } mn \text{ rows} \\ C = \left[f_{ki}\right] , & \text{a } mn \times p \text{ matrix} \\ E = \left[\delta x_k\right] , & \text{a column vector of } p \text{ rows} \end{array}\right\} . \qquad (10)$$

Thus equation (7) becomes

$$\begin{aligned} U' &= D'^{*}WD' \ , \quad \text{where } W \text{ is an } nm \times nm \text{ diagonal matrix} \\ &= (D^{*} - E^{*}C^{*})W(D - CE) \quad . \qquad (11) \end{aligned}$$

Since W is a diagonal matrix, it may be shown that

$$\left[\frac{\partial U'}{\partial(\delta x_k)}\right] = -2C^*WD + 2C^*WCE$$

$$= 0 \quad , \tag{12}$$

for U' to be a minimum value of U.

Thus, with $\Psi = C^*WC$, then equation (12) gives

$$E = \Psi^{-1}C^*WD \quad , \tag{13}$$

that is, the increments δx_k required to correct the parameters x_k are known in terms of the partial derivatives f_{ki} and the residuals R_{π_i}.

The computed readings π_{i_c} are usually expressed explicitly in terms of the state variables y_j (rather than the parameters x_k), so that

$$f_{ki} = \frac{\partial \Upsilon(x_1 \ldots x_p,t_i)}{\partial x_k} = \sum_{j=1}^{r} \frac{\partial \pi_i(y_1 \ldots y_r)}{\partial y_j} \cdot \frac{\partial y_j}{\partial x_k} \quad . \tag{14}$$

The state variables satisfy the equations of motion, which may be written as

$$\dot{y}_j = g_j(y_1 \ldots y_r,x_1 \ldots x_p,t_i) \tag{15}$$

so that

$$\frac{d}{dt}\left(\frac{\partial y_j}{\partial x_k}\right) = \frac{\partial \dot{y}_j}{\partial x_k} = \frac{\partial g_j}{\partial x_k} = \sum_{j'=1}^{r} \frac{\partial g_j}{\partial y_{j'}} \cdot \frac{\partial y_{j'}}{\partial x_k} + \frac{\partial g_j}{\partial x_k} \quad , \tag{16}$$

where equations (16) are a set of pr simultaneous differential equations. These are solved, together with the equations of motion, for each t_i, to give the partial derivatives $\left(\frac{\partial y_j}{\partial x_k}\right)_{t_i}$. The remaining partial derivatives required in equation (14) are obtained algebraically.

The increments δx_k derived from equation (13) are used to give new starting values of the parameters x_k, and the iteration continues until E

is negligible and so $(\delta x_k)^2$ is negligible), and the best estimate of the parameters is found.

Two measures of accuracy are available, the best estimate of the accuracy of the observations, σ^2, and the variance S_k^2 of the parameters x_k. The former is obtained from the minimum value of U', and may be written

$$\sigma^2 = \sigma_0^2 - (E^*C^*WD)/(mn - p) \qquad (17)$$

where σ_0 is the rms of the residuals of the observations using the uncorrected parameters. If the parameters are uncorrelated, then the covariance matrix, $\mathrm{cov}(E) = \sigma^2\Psi^{-1}$, is a diagonal matrix, and so

$$S_k^2 = \sigma^2\Psi_k^{-1} , \qquad (18)$$

where Ψ_k is the kth diagonal element of Ψ^{-1}. At the 95% probability level, the probable error in x_k is then given by

$$\Delta x_k = 2S_k = 2\left(\sigma^2\Psi_k^{-1}\right)^{\frac{1}{2}} . \qquad (19)$$

If the parameters are correlated, then equations (18) and (19) are approximations, and the magnitude of the off-diagonal elements in cov(E) indicate the degree of correlation between pairs of parameters. In the RAE programs, the correlation has not as yet been used, as background experience in analysing flight records usually gives sufficient guidance in choosing the stability derivatives to be obtained. For example, it is known that q and $\dot{w}$ (pitch rate and rate of change of normal velocity respectively) are almost in phase in the longitudinal short-period oscillation, so that the aerodynamic derivatives m_q and $m_{\dot{w}}$ affect the response in the same way, i.e. the degree of correlation is near unity. In order to obtain consistent values independent of first guesses, $m_{\dot{w}}$ is set to zero, and the value obtained for m_q is taken to be equivalent to the total damping-in-pitch derivative, $m_q + m_{\dot{w}}$. In the lateral Dutch roll oscillation, the derivative ℓ_r does not usually have significant effects on the response, and so ℓ_r has to be kept constant at its estimated value, to avoid divergence of the iteration procedure. The possibility of determining n_r and/or n_p depends on the type of Dutch-roll response, whether it is predominantly yaw or roll, and so some judgement is required.

The latter situation is one in which it may be useful to have an indication of which parameters affect the computed responses most strongly. A sensitivity matrix F is therefore calculated, where the elements of F are given by

$$F_{k,\ell} = \left[\frac{1}{m}\sum_{i=1}^{m}\left(x_k \frac{\partial \Upsilon_\ell}{\partial x_k}(x_1 \ldots x_p, t_i)\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\frac{x_k^2}{m}\sum_{i=1}^{m}\left(\left\{f_{ki}\right\}_\ell\right)^2\right]^{\frac{1}{2}} , \qquad \ell = 1 \ldots n, \quad k = 1 \ldots p , \qquad (20)$$

where suffix ℓ denotes the ℓth row (instrument).

As $\pi_{i_c} \equiv \Upsilon$ is an n-dimensional vector (n being the number of instruments), F is a p × n matrix. $F_{k,\ell}$ is the rms rate of change of the computed reading of the ℓth instrument with respect to the kth parameter, over the time interval of the run, and normalised by the value of the parameter. Parameters which have small sensitivities relative to other parameters for all the instruments have a small effect on the computed response, and so cannot be identified accurately.

It has also been found[3] advantageous to analyse oscillatory responses in two steps (in the same computer run), first determining the stability derivatives which have a major influence on the frequency (m_w, or n_v and ℓ_v) and derivatives for any controls used, keeping the remaining derivatives at fixed estimated values, and then to allow the derivatives influencing the damping to vary also during later iteration cycles. This usually ensures convergence, and reduces the total number of iterations required to obtain a given accuracy. A facility for determining the first set of parameters using only the first half of the observed data is provided in the program, which then automatically includes all the observed data for the second step. The programs described here differ from those described in Refs.3 and 4, as the order in which parameters are derived is specified as input data, without changes having to be made in the program.

3 DESCRIPTION OF PROGRAMS

3.1 Types of Subroutines

The programs are written with interchangeable Subroutines, and follow the pattern of the original free-flight[3,4] programs. The flow diagram in Fig.1 shows the basic framework for the subroutines, the main decision points, and the places in the program where data may be output to the lineprinter.

The MASTER segment controls the iteration process (as discussed further in section 3.2), builds up the normal equations, and accumulates the statistical information (σ and S_k) required to give the measures of accuracy. It also controls the output of results, dependent on both requested information and on the stage of the iteration process.

The DATAREAD Subroutine is arranged to use different Label entry points, depending on the amount of data common to the flight records being analysed. The input data is described fully in section 3.2 and listed in Tables 1 and 2 for the longitudinal and lateral programs respectively, the main difference being in the observed instrument readings (Label 8), which must be made compatible with the output of the flight recorder system, and minor differences in array sizes.

The INIT Subroutine sets the initial conditions for the integration procedure, using input and computed data.

The MATHMODEL Subroutine calls the ICL Scientific Subroutine[6] F4RUNG for integrating the simultaneous differential equations of first order, which are given in the associated Subroutine F4DERY described below. MATHMODEL also contains the expressions for evaluating the computed instrument readings, π_{i_c} ($\equiv$ PI(I) in Fortran symbols), and their partial derivatives $f_{ki} \equiv$ FY(I,K), in terms of the computed state variables, $y_j \equiv$ Y(J), and the parameters, $x_k \equiv$ X(K). The forms of these expressions depend on the instrumentation used to record the flight data, typical examples being given in sections 4 and 5.

The F4DERY Subroutine which is called by F4RUNG contains two types of differential equations. The first four are the usual equations of motion, expressed as first order differential equations, DY(J) = g(Y(1), ... Y(5)), from equation (15). The remaining differential equations are those needed to evaluate the partial derivatives, $\partial y_i/\partial x_k$, from equation (16). The expressions are evaluated in terms of the parameters stored in the fixed order, and only the equations relating to the p chosen parameters are actually integrated.

The SOLNORM Subroutine solves the normal equations (13), to give incremental corrections to the parameters, and is identical to the original program[4]. The iteration process is then continued, via MASTER.

3.2 Description of data input

The programs are written so that either SI or Imperial units may be used, provided that the units of the input data are consistent.

The system of axes has origin at the centre of gravity of the aircraft, with the x-axis positive forward along the fuselage datum, and z-axis positive downward (unless the gyros and accelerometers have been aligned to a different set of geometric body axes, when this axis system is implied).

The definitions of the nondimensional aerodynamic derivatives are different for the two programs, the aeronormalised system of Ref.7 being used for the longitudinal (with $\ell_1 = \bar{c}$) , and the 'old' system of Ref.8 at present used in the lateral programs. Although this is untidy, it is recognised that several systems are in use, and so it may be helpful to see the differences affecting the programs. The main bulk of the computations use the dimensional form, and so any change of notation is readily affected by altering the appropriate multiplying factors XM(I) , relating the concise dimensional and aerodynamic nondimensional derivatives, which are evaluated in the Subroutine DATAREAD. The factors XM(I) are listed in Table 3 for longitudinal derivatives of Ref.7, and in Table 4 for lateral derivatives of Ref.8.

The order, format and notation of the input data are given in Tables 1 and 2 for ready reference, with added explanation included here.

AC(9) contain the positions of the accelerometers and probes, defined by the triads (x,y,z) for each instrument, relative to the CG along the given axes. The data is needed for calculation of instrument readings from the state variables. The rate gyros are assumed to be oriented accurately relative to the axis system, so no position information is required for them.

XE. See section 3.3.

ND is the number of instruments. The instrument readings to be matched are considered in the fixed order implied by OBS(I,J). For example, if ND = 3, the lateral program will use sideslip vane, roll and yaw rate gyros, but leave out the lateral accelerometer (see Table 4). It is possible to arrange for the case where, say, the sideslip vane is absent, but the other instruments available, by setting ND = 4 and choosing the weighting on fitting the sideslip record to some very small value, e.g. XS(1) = 0.00001 . (Choosing the weighting to be zero may produce overflow difficulties.)

YIN(5) and DTM(10) or (9) contain information defining the flight condition. The equations of motion used are essentially perturbation equations about a trimmed steady state, and so include terms dependent on that state. DTM defines steady datum values of variables associated with the instrumentation, while YIN includes the remaining steady values. The calculation of these values is explained more fully in Appendix A.

XX(22) or (20) are the first guesses for the parameter values. Guesses for the initial velocities and instrument errors have to be derived from flight data. Some suggestions are given in Appendix B for methods of obtaining first guesses for the aerodynamic derivatives, which may be useful at the beginning of a series of flight tests. Once some records have been analysed, interpolation from the results often yields better first guesses for the remaining records.

JP(20) or (22) is an integer array defining the order in which parameters are to be evaluated. The array XX(K) contains the parameters stored in the fixed order, which is related to the array X(K) with the parameters stored in a chosen order by the relation

$$X(JP(K)) = XX(K) \qquad (21)$$

i.e. JP(K) is the position in the chosen order of the parameter Kth in the fixed order. The second sections of Tables 3 and 4 give examples.

REJ and ACF are quantities related to the accuracy required, see section 3.3.

HMAX is the maximum increment of time that the numerical integration routine may cover in one step. Usually the step lengths required would simply be the time between successive observations. Two cases arise however when it may be necessary to insert intermediate steps in the integration process. The first occurs when observations at occasional time points in a sequence are missing (this is more likely to occur in telemetry data than with onboard recordings), then by setting HMAX to slightly more than the usual time interval between observations, the program uses HMAX to continue integration over the time interval where data is missing. The second case is when observations are coarsely spaced; HMAX may then be used to ensure that smaller steps (of length HMAX) are taken between observations by the numerical integration routine.

P, ... LC, see section 3.3.

XS(4) are the scaling factors to be applied to the instruments, usually chosen such that the maximum factor of 1.0 is applied to the instrument showing the smallest numerical amplitude (in actual physical units) about its steady reading. The remaining scaling factors are then obtained as the inverse ratio of the amplitudes to the smallest amplitude, all measured in physical units relative to the means. (This effectively normalises the instrument readings, and, more importantly, the errors.) Additional weighting can then be introduced to account for relative accuracies or noise levels. In relating the programs to the theory outlined in section 2, it should be noted that the diagonal elements of the weighting matrix W are the squares of XS(I).

3.3 Control of the iteration procedures

The running of the program is controlled mainly by the input data, and partly by the results obtained. Various arrangements of the input data are also possible, using the entry label facility incorporated in DATAREAD, which is best explained by examples.

For the first flight record to be analysed in a computer run, the first data card must contain the entry label 3, so that all the necessary data is read. The Data title is then followed by the data cards containing AC, SP etc. up to and including XS. The entry label 8 is then set, to read in the observed data at MD time intervals. For flight data on cards, as in the lateral program, the actual number of sets of observations included in the record may be greater than MD, as long as the first column of each card is blank, the unwanted data after MD sets have been read being skipped (for $L = 0$). With data tapes, used in the longitudinal program, the end of the time interval is specified, as described in section 4.2.

The information given at Label 6 in Tables 1 and 2 defines the user's control of the iteration procedure. For a chosen number, P , of unknown parameters the iteration continues until either

(i) a chosen number, ITM, of iterations have been completed or

(ii) each incremental change in parameter, δx_k contained in E(K), is less than a set of chosen accuracies, stored in XE(K), multiplied by a chosen accuracy factor, ACF, i.e.

$$E(K) < ACF * XE(K) \qquad \text{for all } K = 1 \text{ to } P \text{ , or}$$

(iii) divergence has occurred, so that the number of observed data points being fitted is less than the number of parameters. This is possible, since if the residual error in any instrument reading is greater than REJ, then the observations at that time are ignored in the current iteration cycle. An initial value of REJ has to be included in the data, which is then updated to the current value of $4\sigma/(\text{total number of observations})^{\frac{1}{2}}$.

For a flight record of sufficient length, the first half of the record may be analysed for the chosen number P parameters, and then the computation continues automatically after condition (i) or (ii) has been satisfied to include the complete record for a greater number, PF , of parameters. If P and PF are set equal, the complete record is analysed.

If the same flight record is to be reanalysed in the same computer run, then the next data card contains the required entry label, followed by Data title and the necessary data, then entry Label 9 to avoid repeating the observations. A likely example is that the scaling factors XS are to be changed, so that the data cards would be

```
7
Data Title
XS
9, IPLOT
```

i.e. P, PF etc. would remain unchanged from the previous computation.

To proceed to the analysis of the next flight record, the entry label is chosen so that the data preceding the label does not need to be changed. For small aircraft whose mass varies significantly with fuel state during the flight, a complete new set of data is required, so Label 3 has to be set, but for larger aircraft Labels 4 or 5 may be possible. The first card after the label contains the new Data Title, followed by the data cards.

The last case to be analysed in a computer run is identified by making LC = 2 , otherwise LC = 1 for all previous cases.

3.4 Description of output

A program heading (specified in FORMAT statement 2 in MASTER) is printed before each analysis together with the date when the program was run. The contents of the 'data title' card then define the specific data (e.g. flight number, aircraft, test number) being analysed. The next block of output reproduces input data which has changed from the previous run (this depends on the entry label, L , used), up to and including XS , but not the observations.

The first guesses of the parameters in the chosen order are then printed in the nondimensional forms as described in section 3.2. The longitudinal version also prints on the following line in brackets the values of the parameters in a dimensional concise form (see Table 3). Each parameter has an associated DELTA printed below, which at the beginning of the computation is the accuracy requirement (in nondimensional form) set as input data in the array XE(20) or (22).

If IT has been set to zero then the computed instrument readings using the first guesses for the parameters are listed, together with a count of the number of data points not rejected, and the sensitivity matrix. The results can be useful if the iterative procedure subsequently diverges, but this lengthy print out can be suppressed by setting IT = 1 . In the longitudinal version these results are also output to channel 4 which could be a paper tape punch, magnetic tape etc.; this enables the program to be used to produce 'simulated data'. If this output is not required it can be suppressed by using OUTPUT 4=/NONE in the program description statements as in the program listing of Appendix C.

For each subsequent iteration the updated parameter estimates are output, but the associated DELTA now gives the half width of the 95% confidence interval for that parameter obtained from equation (19). For parameters not being analysed DELTA appears as zero. Also after each iteration the estimated σ and the number of degrees of freedom (i.e. the difference between the total number of observations fitted and the number of parameters) are printed.

At the completion of the final iteration, the comparison of computed and actual instrument readings is output according to the value of IPLOT. For IPLOT = 0 , computed readings and their weighted errors are printed; for IPLOT = 1 , the graphs of computed and actual readings are plotted, and for IPLOT = 2 both listing and graphs are obtained. In the printed listings, if the time interval between observed readings is less than HMAX, the results are given at the observation times only, but if the time interval between two successive observations is greater than HMAX, then the computed readings are given at intervals of HMAX until the next observation time is reached. Rejection of an observation point because of a residual error greater than REJ is indicated by blank spaces in the weighted error columns and there is also a reduction in the number of degrees of freedom.

Finally the number of points not rejected is printed (this will always be at least one less than the number of observation points because the starting time point is not counted), followed by the sensitivity matrix for the instruments and parameters being considered.

Two possible messages may be output. "Not enough data left" signifies that the iterative procedure has diverged to the extent that most of the observations have been rejected. This is not an infallible trap for inhibiting the listing and plotting of large numbers for the computed instrument readings, which does occur when divergence happens on the ITMth iteration. "Changes for following parameters small" signifies that the iterative procedure has converged to within the accuracy level set before the maximum number of iterations has been completed.

3.5 Graph plotter output

IPLOT is read from column 2 of the card which starts with entry Label 8 or 9. If IPLOT = 0 no graphic output of the results of the run* will be produced; if IPLOT = 1 or 2 then the observed instrument readings and the computed instrument readings using the parameter values of the final iteration are plotted, thus showing the "goodness of fit" obtained.

The format of the graphs is as follows:- Provided the time interval being analysed is less than 16 seconds then for each instrument an A4 size box and a horizontal time axis with a scale of 2 centimetres per second are drawn. The vertical axis is 16 centimetres long and the data (observed and computed) for that instrument is suitably scaled to fit this length using HGPSCALE of Ref.9. The observations are plotted as a solid line and the computed fit as a dashed line. In both cases successive data points are joined by straight lines. The computed fit at all observation times are plotted, including those which have

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* In this section it is understood that a new 'run' starts whenever an entry label between 3 and 7 inclusive is read.

been rejected due to too poor a fit, however intermediate computed points, which appear when a time gap is greater than HMAX, are not plotted.

The first 20 characters of the 'data title' card are printed in the top right hand area of the box. These graphs are laid out so as to be suitable for inclusion in RAE Technical Reports.

Should the time interval be greater than 16 seconds then the time axis and the horizontal size of the enclosing box have to be extended beyond A4 size by the program.

The RAE computers currently use an off-line plotting system. This means that the instructions for plotting produced as the program is being executed are stored on magnetic tape. A scratch tape should be requested for this purpose; the program will open and name this FLT▽DATA▽FIT as soon as a set of data, that is a run, for which plotting is required, is encountered. The name of this tape is defined on card PL 290 in subroutine DATAREAD. For each run a set of graphs is stored as a picture on the tape. (Note that there is a dummy first picture). This magnetic tape is later used under control of #XJGA to drive the graph plotter. A single control card is also required and it is possible to specify which pictures are to be plotted on this card. Normally all the pictures are required in which case this card has the form (starting in column 1):-

```
FLT▽DATA▽FIT,0,0,6,0,1,ALL,****
```

Further details of #XJGA can be found in the 1900 Series Graph Plotter Manual[9].

4 LONGITUDINAL RESPONSE PROGRAM

4.1 Equations of motion

The linearised equations of motion for perturbations about a trim state of the aircraft used in the longitudinal version of the program are:-

$$\dot{\theta} = q \cos \Phi_e \tag{22}$$

$$\dot{w} = q_e u + u_e q + \theta g \sin \Theta_e \cos \Phi_e - \left\{ \mathring{z}_w w + \mathring{z}_u u + \mathring{z}_q q + \mathring{z}_\eta \eta \right\} \tag{23}$$

$$\dot{q} = - \left\{ \mathring{m}_u u + \mathring{m}_w w + \mathring{m}_q q + \mathring{m}_\eta \eta \right\} \tag{24}$$

$$\dot{u} = - q_e w + w_e q + \theta g \cos \Theta_e - \left\{ \mathring{x}_u u + \mathring{x}_w w + \mathring{x}_q q + \mathring{x}_\eta \eta \right\} . \tag{25}$$

In these equations θ, q, u and w are perturbations from trim values, so, for example

$$q_{total} = q_e + q \quad , \tag{26}$$

and the derivatives are in dimensional concise form[7], so, for example $\mathring{z}_w$ has the units of force/(mass × velocity).

The trim state can be steady horizontal flight, a dive, a banked turn or a steady diving banked turn (see Appendix A). As equations (22) to (25) are perturbation equations trim forces do not appear. For instance in equation (23) a_{z_e} , the normal acceleration of the aircraft in the trim state is balanced by $u_e q_e + g \cos \Theta_e \cos \Phi_e + Z_e$. At each observation point it is also necessary to calculate the partial derivatives of u, w and q with respect to each of the parameters being identified $\left(\text{to obtain the } \frac{\partial y_j}{\partial x_k} \text{ of equation (14)}\right)$. This is achieved by integrating numerically not only equations (22) to (25) but also their partial derivatives with respect to each of the parameters.

For example equation (23) differentiated with respect to q_0 , the initial value of q , is:-

$$\frac{d}{dt}\left(\frac{\partial w}{\partial q_0}\right) = q_e \frac{\partial u}{\partial q_0} + u_e \frac{\partial q}{\partial q_0} + \frac{\partial \theta}{\partial q_0} g \sin \Theta_e \cos \Phi_e - \left\{\mathring{z}_w \frac{\partial w}{\partial q_0} + \mathring{z}_u \frac{\partial u}{\partial q_0} + \mathring{z}_q \frac{\partial q}{\partial q_0}\right\}, \tag{27}$$

and as a further example, equation (24) differentiated with respect to $\breve{M}_w$ is:-

$$\frac{d}{dt}\left(\frac{\partial q}{\partial \breve{M}_w}\right) = -\left\{\mathring{m}_u \frac{\partial u}{\partial \breve{M}_w} + \left(\mathring{m}_w \frac{\partial w}{\partial \breve{M}_w} + \frac{\frac{1}{2}\rho V S \bar{c}}{m} w\right) + \mathring{m}_q \frac{\partial q}{\partial \breve{M}_w}\right\} . \tag{28}$$

It will be noted that the parameter with respect to which it is required to differentiate is in aeronormalised form, while the derivatives on the right-hand side of equations (23) to (25) are in dimensional concise form, hence

$$\begin{aligned}\frac{\partial}{\partial \breve{M}_w}\left(\mathring{m}_w w\right) &= \mathring{m}_w \frac{\partial w}{\partial \breve{M}_w} + \frac{\partial \mathring{m}_w}{\partial \breve{M}_w} w \\ &= \mathring{m}_w \frac{\partial w}{\partial \breve{M}_w} + \frac{\frac{1}{2}\rho V S \bar{c}}{m} w \quad ,\end{aligned} \tag{29}$$

where the nondimensionalising factor is stored in the array XM.

The left-hand side of equations (27) and (28), and similar equations for the other parameters (except off-set errors, see below) and equations of motion appear in the program as the ZD array of Subroutine F4DERY, while the partial derivatives which appear on the right-hand side are the Z's of F4DERY. The contents and ordering of terms in these arrays are described in Table 3.

It has been assumed in the program that terms containing partial derivatives of θ are small enough to be ignored, so in equation (27) the term $\underline{\underline{\frac{\partial \theta}{\partial q_0} g \sin \Theta_e \cos \Phi_e}}$ is taken as zero, and the partial derivatives of equation (22) are not calculated. Omitting these terms does not affect the computed instrument readings for a given set of parameters because the term containing θ in equations (23) and (25) remain and equation (22) is integrated numerically over time to give the required θ. However the omission does change the values for the updating of the parameters arrived at by the iteration procedure. If the iteration procedure converges to a set of parameters giving a good fit to the observed data the approximation is justified.

A Runge-Kutta method is used to integrate the equations of motion and the partial derivatives with respect to the parameters numerically over time, to give computed state variables and their partial derivatives. However the fitting process (see section 2) requires the calculated instrument readings, π_{i_c}, and their partial derivatives, f_{ki}, so in MATHMODEL Subroutine the equations giving the instrument readings in terms of the state variables are used. These are:-

$$q_{calc} = q + q_e + E_q \quad , \tag{30}$$

$$a_{z_{calc}} = \frac{1}{g}\left[- \left\{\mathring{z}_w w + \mathring{z}_u u + \mathring{z}_q q + \mathring{z}_\eta \eta\right\} + x_1 \dot{q} - y_1 q r_e + 2 q_e q z_1\right] + a_{z_e} + E_{a_z} , \tag{31}$$

$$V_{calc} = \left[(w + x_2 q)(w_e - q_e x_2 + p_e y_2) + u u_e\right] \Big/ V_e + V_e + E_v \quad , \tag{32}$$

and

$$\alpha_{calc} = \frac{w}{u_e} - \frac{q x_3}{u_e} + \alpha_e + E_\alpha \quad , \tag{33}$$

where (x_1, y_1, z_1) is the position of the normal accelerometer,
(x_2, y_2, z_2) is the position of the airspeed probe,
(x_3, y_3, z_3) is the position of the angle of attack vane,

all positions being relative to the centre of gravity, q_e, a_{z_e}, V_e and α_e are the trim values, and E_q, E_{a_z}, E_V and E_α are instrument offset errors.

The partial derivatives of equations (30) to (33) are required (to obtain the f_{ki} of equation (8)), for example:-

$$\frac{\partial q_{calc}}{\partial \breve{M}_w} = \frac{\partial q}{\partial \breve{M}_w}$$

and

$$\frac{\partial \alpha_{calc}}{\partial E_\alpha} = 1 \quad .$$

These involve the use of the partial derivatives of the state variables. It will be seen that p_e and r_e appear in equations (30) to (33); these are non-zero when the trim state is a banked turn.

4.2 Data input

The data required for the running of the longitudinal program is listed in Table 1, with the units (in this case metric as an example), and the format used. Information on the sources of data, use of Labels etc. common to both programs is given in sections 3.2 and 3.3.

The longitudinal version of the program is designed to use observation data on paper tape. When an entry Label 8 is read, indicating new observations are to be read in, the next card gives the following information (see Table 1):-

NGAP Only the observations at every NGAPth point will be taken into account in the fitting process, but the control positions at every time point will be used ($\leqslant 6$).

TF The final time. Checks are made that not more than 100 points for fitting are read in, and that the end of the paper tape is not reached.

IL The paper tape loading indicator. 0 = Tape already loaded and at start. 1 = Pause, unload paper tape and load new one. 2 = Required data is later on tape already loaded and partly read during a previous run.

A print out of a typical set of data cards, and the beginning of the corresponding paper tape of observations is given in Fig.2 for a computer run which analyses for ten parameters. The elevator input is shown in Fig.3; this is taken from the flight record and is not part of the computer program's graphical output.

4.3 Output

An example of the output of the program is given in Fig.4, and the graphical output in Fig.5.

The input data is first listed; this has all been read in from cards except for the 'tape identifier' and 'original number of data points' which are read from the paper tape of observations. The first calculated results are SIGMA and DEGREES OF FREEDOM obtained using first guesses for parameters. Subsequent iterations follow, with ten parameters varying $\left(w_0, q_0, \check{M}_\eta, \check{M}_w, \check{Z}_w, \check{M}_q, E_q, E_{a_z}, \check{Z}_q \text{ and } \check{Z}_\eta\right)$. Convergence occurs after 6 iterations, so the maximum number of iterations, which was set at 8, is not now reached. The calculated instrument readings and weighted errors are listed, since IPLOT = 2. There are no gaps in the weighted errors listing, which indicates none of the data points have been rejected, confirmed by 98 out of 99 points being accepted. (The difference of 1 is due to the initial time point not being counted.)

The final line printer ouput is the sensitivity matrix (equation (20)). It should be noted that the sensitivities listed for instrument off-set errors (EQ and EAZ) are just the absolute value of the identified off-sets in the last iteration and are not comparable with the other sensitivities.

The DELTA for ZQ in the final iteration is very large indicating a large uncertainty in the value of ZQ; the DELTA for Z_η is also quite large. The sensitivities of both instruments to ZQ and Z_η are small compared to their sensitivities to other parameters.

Fig.5 shows the graphical ouput. The observed instrument readings (solid line) and the calculated fit (dashed line) are plotted.

The graphs have been reduced in size from the original CALCOMP plots.

5 LATERAL RESPONSE PROGRAM

5.1 Equations of motion

In flight tests aimed at obtaining aerodynamic derivatives from lateral responses, it is often evident that the associated longitudinal motion can be significant, and should be included in the mathematical model of the motion.

Of current interest is the determination of derivatives at high angles of attack, often achievable in flight only in steady diving turns, and so the equations of motion are expressed in terms of linearised perturbations, (v, p, r, ξ, ζ) about a steady state denoted by suffix 'e' .

$$\dot{v} = -\left\{\overset{\circ}{y}_v v + \overset{\circ}{y}_p p + \overset{\circ}{y}_r r + \overset{\circ}{y}_\xi \xi + \overset{\circ}{y}_\zeta \zeta\right\} - (r + r_e)V + (p + p_e)V \sin\alpha + g a_{ye} + g \cos\Theta_e \sin\Phi \tag{34}$$

$$\dot{p} = -\left\{\overset{\circ}{\ell}_v v + \overset{\circ}{\ell}_p p + \overset{\circ}{\ell}_r r + \overset{\circ}{\ell}_\xi \xi + \overset{\circ}{\ell}_\zeta \zeta\right\} + b_x q_e r + e_x q_e p + e_x \dot{r} \tag{35}$$

$$\dot{r} = -\left\{\overset{\circ}{n}_v v + \overset{\circ}{n}_p p + \overset{\circ}{n}_r r + \overset{\circ}{n}_\xi \xi + \overset{\circ}{n}_\zeta \zeta\right\} + b_z q_e p - e_z q_e r + e_z \dot{p} \tag{36}$$

$$\dot{\Phi} = p + r \cos\Phi_e \tan\Theta_e \quad . \tag{37}$$

The steady trim value a_{ye} is included in equation (34) to allow the total (measured) angle of attack to be used in the '$p_e V \sin\alpha$' term and the total (computed) bank angle in the gravity term. An alternative form could be used in F4DERY, (and is available in one version of the program)

$$\dot{v} = -\left\{\overset{\circ}{y}_v v + \overset{\circ}{y}_p p + \overset{\circ}{y}_r r + \overset{\circ}{y}_\xi \xi + \overset{\circ}{y}_\zeta \zeta\right\} - rV + pV \sin\alpha + g \cos\Theta_e (\sin\Phi - \sin\Phi_e)$$

It is assumed that the angle of attack is measured, but it may be set to the constant datum value, or to values estimated from the measurement of the normal acceleration, by changing the expression for the computer variable W at the beginning of the F4DERY Subroutine (Appendix D).

For level flight at one g, the steady datum values are all zero, except of course for V and α . The relationships for the steady values appropriate to a diving banked turn in terms of measured quantities are given in Appendix A, as it has been found inadvisable to use the steady instrument readings due to changes in off-set errors with g-level. It should be noted that the steady datum values and the offset errors do not have the same effect in the equations, and have to be treated separately.

Thus the state variables are v, p and r (since Φ is obtained directly from p) , and the parameters are the initial values v_0, p_0, r_0 , the aerodynamic derivatives, and the instrument errors. The aerodynamic derivatives appear in equations (34) to (36) in dimensional concise form[7], and are related to the nondimensional derivatives by the multipliers listed in Table 4, which apply to the 'old' British notation of Ref.8.

All of the first order aerodynamic derivatives are included in the equations for convenience in programming, but it is not envisaged that all will be evaluated. Derivatives having a small effect on the motion may be set to remain at zero (e.g. y_r) , or to remain at estimated values (e.g. ℓ_r) . The derivatives due to acceleration in sideslip do not usually have a distinguishable effect on lateral responses, as $\dot{\beta}$ is almost 180° out of phase with r , and so the derivatives combine, e.g. the value of n_r derived from the analysis program is then considered to be the algebraic sum of $n_r - n_{\dot{v}}$.

Examples of the differential equations for the partial derivatives are given by

$$\frac{d}{dt}\left(\frac{\partial v}{\partial v_0}\right) = \frac{\partial \dot{v}}{\partial v_0} = -\left\{\mathring{y}_v \frac{\partial v}{\partial v_0} + \mathring{y}_p \frac{\partial p}{\partial v_0} + \mathring{y}_r \frac{\partial r}{\partial v_0}\right\} - V \frac{\partial r}{\partial v_0} + \frac{\partial p}{\partial v_0} V \sin\alpha \tag{38}$$

and

$$\frac{d}{dt}\left(\frac{\partial v}{\partial y_v}\right) = \frac{\partial \dot{v}}{\partial y_v} = -\left\{\mathring{y}_v \frac{\partial v}{\partial y_v} + \mathring{y}_p \frac{\partial p}{\partial y_v} + \mathring{y}_r \frac{\partial r}{\partial y_v} - V \frac{\partial r}{\partial y_v}\right\} - V \frac{\partial r}{\partial y_v}$$

$$+ \frac{\partial p}{\partial y_v} V \sin\alpha + \rho \frac{VS}{m} v \quad . \tag{39}$$

All of these equations (34) to (39) are contained in the F4DERY Subroutine, of the lateral program, relating the arrays DY and Y .

The instrumentation is assumed to be a sideslip vane, situated at a position (x_3, y_3, z_3) relative to the CG, roll and yaw rate gyros aligned with the x and z axes respectively, and a lateral accelerometer at (x_1, y_1, z_1). Any information from the longitudinal response is used directly in equations (34) to (36), depending on the instrumentation installed. The computed instrument readings, (PI) , are given by:

$$\beta^\circ = 57.3\left\{v + v_e + x_3(r + r_e) - z_3(p + p_e)\right\}\Big/V + E_\beta \tag{40}$$

$$p^\circ/s = 57.3\left\{p + p_e\right\} + E_p \tag{41}$$

$$r^\circ/s = 57.3\left\{r + r_e\right\} + E_r \tag{42}$$

$$a_y = \Big\{\mathring{y}_v v + \mathring{y}_p p + \mathring{y}_r r + \mathring{y}_\xi \xi + \mathring{y}_\zeta \zeta + x_1[q_e(p + p_e) + \dot{r}] - y_1\left[(p + p_e)^2 + (r + r_e)^2\right] + z_1[q_e(r + r_e) - \dot{p}]\Big\}\Big/g + E_{a_y} + a_{y_e} \ . \qquad (43)$$

The partial derivatives required to calculate the necessary changes to the parameters, that is the f_{ki} defined in equations (8) and (14), are evaluated in the MATHMODEL Subroutine, together with the computed instrument readings listed above. For example, F(1,1) is given by

$$\frac{\partial\beta}{\partial v_0} = 57.3\left\{\frac{\partial v}{\partial v_0} + x_3\frac{\partial r}{\partial v_0} - z_3\frac{\partial p}{\partial v_0}\right\}\Big/V \ .$$

5.2 Data input

The data required for the running of the lateral program is listed in Table 2, with the units (in this case Imperial) and format used. Information on the sources of the data, use of labels, etc. common to both programs is given in sections 3.2 and 3.3.

The print out of a typical set of data input cards is given in Fig.6 for a computer run to analyse initially for 7 parameters, using the whole of the observed data, followed by analysis for 13 parameters. The maximum number of iterations is set at 7 in each case.

It may be of interest to note that the Dutch roll oscillation was initiated during a diving banked 3½-g turn. The rudder record was not available, and so the free oscillation is analysed, in the presence of small aileron inputs, and small changes in angle of attack. These are shown in Fig.7, as taken from flight records, and are not part of the computer program's graphical output.

5.3 Output

An example of the output is given in Fig.8 and results are shown in Figs.9 and 10 resulting from the input of Fig.7. As described in section 3.4, the first blocks of output are a repeat of the data input, the first result of the analysis being SIGMA and the DEGREES OF FREEDOM, followed by the up-dated values of the parameters at the end of each iteration cycle. At the third iteration, the solution has converged, for the first seven parameters, v_0, p_0, r_0, ℓ_v, n_v, Ep and Er , and so the observation and calculated instrument readings are plotted, as shown in Fig.9. At this stage, the frequency is determined reasonably well, but the initial guesses of the remaining derivatives do not give the correct damping. The analysis continues automatically, to include the first 13 parameters, that is, up to ℓ_ξ , using the results of the first analysis as initial values for the first seven parameters. Convergence is achieved after five iterations, and so the final matching is listed and plotted (Fig.10).

It will be noted that the number of degrees of freedom remains constant at 157 during the first analysis, decreases to 151 at the beginning of the second analysis because of the six extra unknown parameters, and then decreases to 147 during the second analysis because one residual in instrument reading is greater than $4\sigma/(XS(I)(MD*ND)^{\frac{1}{2}})$. The data at this time is rejected, and the residuals are not listed, so that inspection of the listing shows this to occur at 2.5 s. The neighbouring residuals in β and a_y are both large, but Fig.10 shows that a_y is the critical instrument. Even so, the match obtained is good, and the values of the derivatives are acceptable, with fairly small likely errors (denoted by the DELTA's). The possible exception is y_v , where DELTA is 25% of y_v , and this large uncertainty is probably related to the noise apparent on the β record (Fig.10).

6 BRIEF SUMMARY OF CURRENT APPLICATIONS OF THE PROGRAMS

The longitudinal and lateral versions are being successfully used to identify stability and control derivatives of the Hunter aircraft, which is being flown for the design and testing of manoeuvre demand systems[10]. The trim conditions are steady horizontal flight at a number of speeds and heights to cover the operational angles of attack and Mach numbers. The control inputs used to excite the longitudinal response were two-sided elevator pulses or arbitrary sequences of small elevator movements about the trim position. For the lateral response, both rudder and aileron two-sided pulses were applied, and also arbitrary sequences of both controls simultaneously. The results will be published when the work is completed.

The lateral program is also being applied in the investigation of the wing-rock phenomenon (i.e. uncommanded lateral oscillations occurring at high angle of attack) on the Gnat aircraft. Responses have been obtained of Dutch roll oscillations throughout the angle of attack range up to the onset of wing rock, by application of double-sided rudder pulses during diving banked turns, over a range of Mach numbers. It is hoped to establish the variation of the aerodynamic derivatives with angle of attack, and to see if the linear mathematical model of the aerodynamic forces and moments is valid for the wing rock oscillations. Some interim results are given in Ref.11, which also describes the parallel work on analysis of flight responses being done at Cranfield Institute of Technology and British Aircraft Corporation (Warton Division).

In the course of the work described above it has only been possible to experiment with the programs to a limited extent, so that properties such as radii of convergence have not been established. The procedure usually adopted has been to concentrate first on one or two typical responses from a series of test flights, doing a number of computer runs until a satisfactory fitted response is obtained with acceptable values of the parameters. As stated previously, it is advantageous to determine the derivatives primarily influencing the frequency first, when their first guesses can be fairly crude and the damping derivatives only need to be of the correct order of magnitude to retain convergence. The choice of control derivatives to be determined is governed by control usage. The parameter set is then extended progressively taking in derivatives which are thought to have a significant affect on the responses, to see if results are acceptable, the criteria being (i) DELTA values (likely errors) appreciably smaller than parameter values (ii) elements of sensitivity matrix not too small relatively (iii) each parameter is converging throughout successive iterations. (Two unsatisfactory characteristics may become apparent due to ill-definition, either monotonic changes in the parameter values often with increasing increments or switching between two distinct values.) Having established a set and order of parameters which gives satisfactory values, then the other responses are analysed using the same set and order, to test for consistency of results over the angle of attack and/or Mach number and/or height ranges. In such a series of computer runs the weighting matrix is usually kept unchanged, unless the amplitude ratios of the responses change markedly. A few tests have been made with different weightings, with the expected effects that the likely errors in the parameters directly affected by an increased weighting become smaller, and other likely errors increase. The effects on the values of the parameters themselves are inconclusive.

7 CONCLUSIONS AND POSSIBLE EXTENSIONS

From experience gained in using the programs, it appears that the response of 'standard' longitudinal short-period and lateral Dutch roll oscillations can be analysed readily, if the flight records are in digital form. More of the records can be used than are amenable to treatment by vector analysis methods as any small cross-coupling effects can be incorporated, and the control inputs are included in the mathematical model (provided that the control surface deflections are recorded). Much of the experience gained in analysing flight records by hand has, however, been applied in the development and use of the RAE programs, without too much emphasis being placed on the statistical information produced. It is hoped that the computer programs developed at Cranfield Institute of Technology[5] under MOD(PE) contract, which use more sophisticated techniques, will become available in FORTRAN, and so it is not envisaged that effort will be given to updating the statistical computations of the RAE programs in the near future.

Some extensions to the mathematical model of the motion are planned, one to include aerodynamic nonlinearities to use for the wing rock responses, and possibly another to include significant flexible modes.

Appendix A

DETERMINATION OF STEADY FLIGHT CONDITIONS AND INSTRUMENT OFF-SET ERRORS

Both computer programs identify the aerodynamic derivatives from a mathematical model of the motion in terms of perturbations about a trim state, which must be defined in the input data. Most flight tests are made in steady level flight, to determine the aerodynamic derivatives at one-g conditions, but it may well be that there is a need to know the derivatives at higher angles of attack. To achieve steady conditions of over one-g the flight tests have then to be made during banked turns, possibly diving turns to maintain speed. Methods of calculating the trim state are presented in this Appendix.

It should be noted that a 'pull-up' is not a steady trimmed state, as the angle of attack changes continuously to maintain constant normal acceleration.

It is usual in the flight tests to have a section of records taken whilst in the trim state just before the pilot initiates the control input to give the required transient response. These records give the steady instrument readings referred to below.

A.1 Steady, level, symmetric flight

The trim state for steady level symmetric flight is trivial as

$$\left.\begin{aligned} a_{z_e} &= -\cos\alpha_e \triangleq -1.0 \\ a_{x_e} &\triangleq \sin\alpha_e \\ v_e &= p_e = q_e = r_e = 0 \\ \Phi_e &= 0 \qquad \Theta_e = \alpha_e \text{ , by definition .} \end{aligned}\right\} \qquad \text{(A-1)}$$

If the steady instrument readings do not agree with the above, then their readings are the instrument off-set errors, e.g.

$$E_q = q \text{ (instrument reading)}$$

when an incidence vane is included in the instrumentation then it has to be assumed that it has no zero error, so that

$$w_e = V \sin \alpha_e \quad , \tag{A-2}$$

where V and α_e are the steady instrument readings. With no incidence vane, the further assumption has to be made that the lift due to elevator deflection is small, and that tunnel measurements exist for C_N , or C_L and C_D , then

$$C_N(\alpha_e) = mg/\tfrac{1}{2}\rho V^2 S \quad \text{gives the value of } \alpha_e \, , \tag{A-3}$$

where C_N is normal force coefficient. (Equation (A-3) gives more acceptable results for α_e than either of the first two equations of (A-1).)

The instrument offset errors on the deflections of control surfaces may be assumed to be zero, since the equations of motion in the analysis only use perturbations in control angles. Thus the steady instrument readings are interpreted as the trim values ξ_e, η_e, ζ_e .

A.2 Steady diving turn

At first sight, it would seem sufficient to use the steady readings of the instruments before the excitation of the transient motion as actual datum values, but these steady readings include both zero errors and the trim state, which have separate contributions in the mathematical model. Experience to date indicates that rate gyros do often have significant off-set errors, which may vary with the normal acceleration being pulled, so that p_e, q_e, r_e cannot be taken directly from the steady instrument readings. Instead, a mathematical model of the trim state has to be used, which is compatible with that for the transient motion, and which can be solved in terms of some given trim values. The equations of motion for a steady co-ordinated turn, $(Y_e = 0)$, are:

$$a_{x_e} g - g \sin \Theta_e = q_e w_e - r_e v_e \tag{A-4}$$

$$g \sin \Phi_e \cos \Theta_e = r_e u_e - p_e w_e \tag{A-5}$$

$$a_{z_e} g + g \cos \Phi_e \cos \Theta_e = p_e v_e - q_e u_e \tag{A-6}$$

$$p_e = - \Omega_e \sin \Theta_e \tag{A-7}$$

$$q_e = \Omega_e \sin \Phi_e \cos \Theta_e \tag{A-8}$$

$$r_e = \Omega_e \cos \Phi_e \cos \Theta_e \quad . \tag{A-9}$$

In the following solution it is assumed that:-

$$\left.\begin{array}{lll} \dot{V} & = 0 \,, & \text{maintained by use of thrust and dive angle} \\ v_e & = 0 \,, & \text{i.e. zero sideforce is approximated by zero sideslip} \end{array}\right\} \qquad \text{(A-10)}$$

$\frac{dh}{dt}$ is obtainable from flight records, and V_e, a_{z_e} and $w_e = V_e \sin \alpha_e$ are known from instrument readings which have been corrected for known off-set errors, neglecting the small contributions due to instrument displacements from the CG. The solution of equations (A-4) to (A-9) is obtained by introducing the attitude angles relative to flight path, suffix 'a', where for $v_e = V_e \sin \beta_e = 0$, then[7]

$$\sin \Theta_e = \sin \Theta_a \cos \alpha_e + \cos \Theta_a \cos \Phi_a \sin \alpha_e \qquad \text{(A-11)}$$

$$\cos \Theta_e \cos \Phi_e = \cos \Theta_a \cos \Phi_a \cos \alpha_e - \sin \Theta_a \sin \alpha_e \qquad \text{(A-12)}$$

$$\cos \Theta_e \sin \Phi_e = \cos \Theta_a \sin \Phi_a \quad . \qquad \text{(A-13)}$$

Then

$$\sin \Theta_a = \frac{dh}{dt} V_e \,, \quad \text{by definition,} \qquad \text{(A-14)}$$

giving Θ_a , negative for a dive.

Equations (A-5) to (A-13) are combined to give

$$\cos \Phi_a = \frac{-\cos \Theta_a \cos \alpha_e}{(a_{z_e} - \sin \alpha_e \sin \Theta_a)} \quad , \qquad \text{(A-15)}$$

where the sign of Φ_a is determined from the directions of the turn. The resultant angular velocity is obtained from equations (A-5), (A-7), (A-9)

$$\Omega_e = \frac{g}{V_e} \tan \Phi_a \quad , \qquad \text{(A-16)}$$

and the attitude angles Θ_e and Φ_e follow from equations (A-11) to (A-13) so that finally equations (A-7), (A-8), (A-9) are used to give p_e, q_e, r_e .

It is possible to obtain the trim state including the corrections to a_{z_e} and α_e for their positions relative to the CG, by an iterative process, but the corrections are usually small. The required relationships are:

$$\left.\begin{aligned} a_{z_e}\left(x_{a_z}, y_{a_z}, z_{a_z}\right) &= a_{z_e}(0,0,0) + p_e r_e x_{a_z} + q_e r_e y_{a_z} - \left(q_e^2 + r_e^2\right) z_{a_z} \\ \text{and} & \\ w_e(x_\alpha, y_\alpha, z_\alpha) &= w_e(0,0,0) - q_e x_\alpha + p_e y_\alpha \quad . \end{aligned}\right\} \quad \text{(A-17)}$$

If there is no incidence vane, then the tunnel data has to be used, usually from $C_N - v - \alpha$ plots, where

$$C_N(\alpha_e) = -\, mga_{z_e} \Big/ \tfrac{1}{2}\rho V^2 S \quad . \qquad \text{(A-18)}$$

As for the steady level flight condition, the steady values of control angles as recorded may be used to give ξ_e, η_e, ζ_e directly.

A.3 Application in computer programs

The values for the steady conditions defined in sections A.1 and A.2 are used for the data input in the arrays YIN and DTM, as listed in Tables 1 and 2 for the longitudinal and lateral programs respectively. The initial guesses for the instrument off-set errors, E_p, etc. are obtained from the actual steady instrument readings and the calculated trim values. Care must be taken to use the units specified in Tables 1 and 2, as both deg/s and rad/s, and both linear velocities and angles of incidence occur in the programs.

Appendix B

APPROXIMATIONS FOR FIRST GUESSES OF AERODYNAMIC DERIVATIVES

The methods of estimation given below are suggested for use at the beginning of a series of flight tests, when the first flight records are being analysed. Interpolation or extrapolation of previous results usually give adequate values of derivatives for the analysis of a majority of the flight tests.

B.1 Longitudinal derivatives

Theoretical estimates for most of the derivatives can be obtained from the Aerodynamics Data Sheets of the Engineering Sciences Data Unit[12] or the USAF Stability and Control Handbook (Datcom)[13]. To the degree of accuracy required, it is usually sufficient to include only wing and tailplane contributions, with downwash.

Wind-tunnel results are often available to give good first guesses for the derivatives due to angle of attack, (in notation of Ref.7)

$$\breve{Z}_w \approx -\frac{dC_L}{d\alpha} , \qquad \text{(B-1)}$$

$$\breve{M}_w \approx \frac{\bar{c}}{\ell_1}\frac{dC_{\hat{m}}}{d\alpha} . \qquad \text{(B-2)}$$

Alternatively, examination of the response records may be made, to obtain rough estimates of the frequency, ω_{SP} , amplitude ratio, $\left|\frac{a_{\hat{z}}}{\alpha}\right|$ and damping, expressed as time to half-amplitude $(T_{\frac{1}{2}})_{SP}$, of the short period oscillation. Then the following approximations give the most important derivatives:-

$$\breve{M}_w \approx -\frac{I_y}{\frac{1}{2}\rho SV^2\ell_1}\omega_{SP}^2 \qquad \text{(B-3)}$$

$$\breve{Z}_w \approx -\frac{mg}{\frac{1}{2}\rho SV^2}\left|\frac{a_z}{\alpha}\right| \qquad \text{(B-4)}$$

$$\frac{\breve{M}_q}{i_y} + \breve{Z}_w \approx -\frac{2m}{\frac{1}{2}\rho SV}\frac{\log_e 2}{(T_{\frac{1}{2}})_{SP}} . \qquad \text{(B-5)}$$

The control derivatives may be obtained from the measured initial acceleration in rate of pitch, $\dot{q}_0$, due to a near-step in elevator, $\Delta\eta$, where

$$\breve{M}_\eta \approx - I_y \dot{q}_0 \; \tfrac{1}{2}\rho S V^2 \ell_1 \; . \; \Delta\eta \qquad \text{(B-6)}$$

and

$$\check{Z}_\eta \approx \frac{\ell_1}{\ell_T} \check{M}_\eta \quad . \qquad \text{(B-7)}$$

B.2 Lateral derivatives

The theoretical estimation of lateral stability derivatives from Refs.12 and 13 should include contributions from the various surfaces, as suggested in the table below. The derivatives n_p and ℓ_r (notation of Ref.8) usually have to be the best estimate available, including all surfaces and interference effects, as they are not often obtainable from the analysis of the flight records.

Derivative/surface	Wing	Tail	Fin	Body	Other
y_v	-	-	√	√	-
ℓ_v	√	-	√	-	Wing/body
n_v	-	-	√	√	
ℓ_p	√	√	?	-	
n_r	-	-	√	√	

Wind-tunnel results are sometimes available for the sideslip derivatives. Alternatively, the measured Dutch-roll characteristics give approximate values corresponding to the relations above for the short-period oscillation:

$$n_v \approx \frac{I_z}{\rho S V^2 s} \omega_{DR}^2 \qquad \text{(B-8)}$$

$$y_v \approx - \frac{mg}{\rho S V^2} \left| \frac{a_y}{\beta} \right| \qquad \text{(B-9)}$$

$$\frac{n_r}{I_z} \cdot ms^2 + y_v \approx -\frac{2m}{\rho SV}\frac{\log_e 2}{(T_{\frac{1}{2}})_{DR}} - \frac{mg}{\rho SV^2}\left|\frac{p}{r}\right| \quad . \tag{B-10}$$

$$\frac{\ell_v}{n_v} \approx -\frac{I_x}{I_z}\left|\frac{p}{r}\right| \tag{B-11}$$

$$\ell_\xi \approx -\frac{I_x}{\rho SV^2 s}\frac{\dot{p}_0}{\Delta\xi}, \quad \text{for step } \Delta\xi \text{ in aileron} \tag{B-12}$$

$$n_\zeta \approx -\frac{I_z}{\rho SV^2 s}\frac{\dot{r}_0}{\Delta\zeta}, \quad \text{for step } \Delta\zeta \text{ in rudder.} \tag{B-13}$$

If the roll subsidence is apparent, then

$$\ell_p \approx -\frac{I_x}{\rho SVs^2}\frac{\log_e 2}{(T_{\frac{1}{2}})_{roll}} \tag{B-14}$$

These relationships assume that the cross-derivatives ℓ_r, n_p, ℓ_ζ, n_ξ are negligible and that the angle of attack is not large, but should usually give reasonable first guesses to start the iteration procedure.

Appendix C

```
FORTRAN COMPILATION BY #XFAT MK 4C    DATE  20/09/74  TIME  06/18/20


 0001                  LIST(LP)
 0002                  LIBRARY(SUBGROUPSRF7)                                         PL     A
 0003                  LIBRARY(SUBGROUPSRGP)                                         PL     B
 0004                  LIBRARY(SUBGROUPS-RS)                                         PL     C
 0005                  LIBRARY (ED,SUBGROUPFSCE.SUBROUTINES)
 0006                  PROGRAM(A53E)
 0007                  COMPACT DATA
 0008                  OUTPUT 2,102=LPO
 0009                  INPUT 103=CR0
 0010                  USE 1,101=ED1/FORMATTED/128
 0011                    INPUT 3=TR0
 0012                  OUTPUT 4=/NONE
 0013                  OUTPUT 5=TY0
 0014                  TRACE 0
 0015                  END
```

```
      MASTER LONGITUDINAL  PLOT
      INTEGER P,PF,PM,PC,SD,Q                                                101
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),       2
     1PSI(625),E(25),TIME(100),OBS(5 ,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(10),
     3 YIN(5),ITD(100),PARAM(20), XC(20),COBS(100,6,2)
      DIMENSION FAF(200) ,DPI(4,25)                                     PL    10
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY       5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                  6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,        7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,COBS,OBS,XS,NGAP/
     4 MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP+ES(22)                9
      COMMON/AVIONIC/XMAV(20),RUNAME,MDO
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL     1
     1 TAXIS,TMOVE                                                      PL     2
      EQUIVALENCE(COBS(1,1,1),TIME(1))
      DATA PARAM/8HTHETAO  ,8HWO      ,8HQO      ,8HUO      ,8HXW
     1,8HXQ      ,8HXU      ,8HX ETA   ,8HZW      ,8HZQ      ,8HZU
     2 ,8HZ ETA   ,8HMW      ,8HMQ      ,8HMU      ,8HM ETA   ,8HEQ
     3   ,8HEAZ     ,8HEV      ,8HEALPHA  /
      DATA PTI/5HSIM 3/
      JPLOT=0                                                           PL    20
      CALL GFIN
C  PRINT PROGRAM NAME AND HEADINGS                                           160
      CALL DATE(TODAY)
    1 WRITE(2,2)  TODAY                                                      170
    2 FORMAT('1 LONGITUDINAL ANALYSIS FOR FULL-SCALE A/C'/
     1' G.W.FOSTER,AERO.DEPT. R141C.',60X,'RUN ON..',A8/ )
C
C  READ DATA AND PROGRAMME CONTROL PARAMETERS                                200
    3 CALL DATAREAD                                                          210
C  SET PRINT CONTROL AND TEST FOR SIMULATION MODE                            220
    4 PC=0                                                                   230
    5 DO 50 I=1,20
  200 XC(JP(I))=XMAV(I)*X(JP(I))
   50 CALL COPY8(ORDER(JP(I)),PARAM(I))
      WRITE(2,90)((ORDER(J+J1),J1=1,5),(X(J+J1),J1=1,5),
     1            (XC(J+J1),J1=1,5),(CWD(J+J1),J1=1,5),J=0,15,5)
C  TEST FOR INITIAL PRINTOUT                                                 280
      IF(IT.EQ.0)GO TO 6                                                     290
C  TEST FOR FINAL PRINTOUT                                                   300
      IF(PC.NE.1)GO TO 7                                                     310
C  TEST FOR FINAL NUMBER OF PARAMETERS                                       320
      IF(P.EQ.PF)GO TO 6                                                     330
C  CHECK VALUE OF ITERATION COUNTER                                          340
      IF(IT.GT.ITM)GO TO 6                                                   350
      P=PF                                                                   360
      N=NF                                                                   365
      MD=2*MD                                                                370
      PC=0                                                                   380
      IT=1                                                                   390
      GO TO 7                                                                400
C  WRITE MAIN HEADINGS                                                       410
    6 IF(IPLOT.NE.1)WRITE(2,92)
      IF(IT.NE.0)GO TO 7
      CALL RUNOUT(4)
      WRITE(4,97)PTI,MD
   97 FORMAT(1X,A5//1X,I4)
C  SET UP INITIAL CONDITIONS                                                 430
    7 CALL INIT                                                              440
C  CLEAR MATRIX STORES AND SET COUNTERS                                      450
      SD=0                                                                   460
      S=1
      J=P*P                                                                  480
```

```
      DO 8 I=1,J                                                       500
    8 PSI(I)=0.0                                                       510
      DO 9 I=1,K                                                       520
    9 CWD(I)=0.0                                                       530
      SIGMA=0.0                                                        535
      NAP=0
      DO 830 I=1,4
      DO 830 J=1,25
  830 DPI(I,J)=0.0
C  MAIN COMPUTING CYCLE SETTING UP NORMAL EQUATIONS                    540
   10 DO 22 Q=2,MD                                                     550
      IF((MOD(Q,10).EQ.1).AND.(IT.EQ.0.OR.(PC.EQ.1.AND.IPLOT.NE.1)))
     1               WRITE(2,501)
  501 FORMAT(1H )
C  CALCULATE MODEL PREDICTIONS AND PARTIAL DERIVATIVES                 560
      CALL MATH MODEL                                                  570
C  TEST ERRORS AND CALCULATE RMS                                       580
   11 J=Q                                                              590
      ER=0.0                                                           600
      DO 12 I=1,ND                                                     610
      E(I)=(OBS(I,J)-PI(I))*XS(I)                                      620
      ZED=ABS(E(I))                                                    630
      IF(ZED.GT.REJ)GO TC 22                                           64
   12 ER=ER+E(I)*E(I)                                                  650
      SIGMA=SIGMA+ER                                                   660
      SD=SD+ND                                                         670
      NAP=NAP+1
C TEST IF PRINTING OF RESIDUALS IS REQUIRED
      IF(IT.EQ.0.CR.(PC.EQ.1.AND.IPLOT.NE.1))WRITE(2,93)(E(I),I=1,ND)
C  ADD TO EACH ELEMENT OF NORMAL EQUATIONS                             750
C SUM OVER INSTRUMENTS ('DO 22' LOOP SUMS OVER TIMES)
      DO 21 L=1,ND
C FOR EACH ROW....
      DO  20 I=1,P
C  ANDFOR EACH COLUMN TO LEFT OF DIAGONAL......
      DO 19 J=1,I
      KD=(I-1)*P+J
      KDT=(J-1)*P+I
      PSI(KD),PSI(KDT)=PSI(KD)+FY(L,I)*FY(L,J)*XS(L)**2
   19 CONTINUE
      CWD(I)=FY(L,I)*XS(L)+E(L) + CWD(I)
      DPI(L,I)=DPI(L,I)+FY(L,I)**2                                RMSSENSE
   20 CONTINUE
   21 CONTINUE
   22 CONTINUE
      IF((PC.NE.1.AND.IT.NE.0).OR.NAP.EQ.0)GOTO 16
      DO 40 L=1,ND
      DO 40 I=1,P
   40 DPI(L,I)=ABS(X(I)*SQRT(DPI(L,I)/NAP))                        RMSSENSE
      WRITE(2,1012) NAP,MD,SD
 1012 FORMAT(1H0/1H0,I3,' OUT OF ',I3,' POINTS ACCEPTED',10X,'DEGREES OF
     1 FREEDOM ',I3/'0 RMS SENSITIVITY MATRIX'/1H0,13X,1HQ,9X,2HAZ,8X,
     11H0,      7X,5HALPHA /)
      DO 41 I=1,P
   41 WRITE(2,1013) ORDER(I),(DPI(L,I),L=1,ND)
 1013 FORMAT(1H ,A8,4(2X,F8.5))
C  FINISH IF FINAL PRINT OUT                                           950
      IF(PC.EQ.1)GO TO 28                                              960
C  CHECK FOR SUFFICIENT DATA ACCEPTED                                  970
   16 IF(P.LT.SD)GO TO 80
      WRITE(2,1011)SD
 1011 FORMAT('0 NOT ENOUGH DATA LEFT    SD=',I5)
      IF(IPLOT.EQ.0)GO TC 28                                      PL    23
      IPLOT=0                                                     PL    24
      CALL HGPSYMBL(0.0,0.0,1.0,TITLE,270.0,20)                   PL    25
```

```
      CALL HGPSYMBL(0.0,-10.5,1.0,20HNOT ENOUGH DATA LEFT,2,0,0,20)        PL      26
      CALL HGPLOT(0.0,0.0,3,0)                                             PL      27
      CALL HGPLOT(-1.0,0.0,0,4)                                            PL      28
      CALL HGPLOT(0.0,0.0,3,0)                                             PL      29
      GO TO 28
   80 CONTINUE
C  RESET DATA REJECTION LEVEL                                                     990
      REJ=SQRT(16*SIGMA/SD)                                                      1000
C  SOLVE NORMAL EQUATIONS                                                        1010
      CALL SOLNORM(P,SIGMA,SD)                                                   1020
C  INCREASE ITERATION COUNTER                                                    1030
      IT=IT+1                                                                    1040
C  IMPROVE PARAMETER VALUES                                                      1050
      DO 25 I=1,P                                                                1060
   25 X(I)=X(I)+E(I)                                                             1070
      DO 30 I=1,20
      K=JP(I)
   30 XX(I)=X(K)
C  PRINT SIGMA AND NO OF DEG. FREEDOM                                            1080
      WRITE(2,94)SIGMA,SD                                                        1100
C  CHECK FOR MAX NO OF ITERATIONS                                                1110
      IF(ITM.GT.IT)GO TO 26                                                      1120
      PC=1                                                                       1130
      GO TO 5                                                                    1140
C  CHECK PARAMETER CHANGES                                                       1150
   26 DO 27 I=1,20
      IF(JP(I).GT.P)GO TO 27
      ER=ACF*XE(I)
      IF(ABS(E(JP(I))).GT.ER) GO TO 5
   27 CONTINUE
      WRITE(2,1010)
 1010 FORMAT('0 CHANGES FOR FOLLOWING PARAMETERS ALL SMALL')
      PC=1                                                                       1190
      GO TO 5                                                                    1200
C IS THERE ANY PLOTTING WANTED....                                     PL      30
   28 IF(IPLOT.EQ.0)GO TO 98                                           PL      40
      DO 1201 INS=1,ND                                                 PL      50
      DO 1200 II=1,MD                                                  PL      60
      FAF(II)=OBS(INS,II)                                              PL      70
 1200 FAF(II+MD)=FIT(INS,II)                                           PL      80
      CALL HGPLOT(0.0,22.0,0,4)                                        PL      90
      CALL AXISCALE(FAF,2*MD-1,XMIN,DX,INS)                            PL     100
      CALL PLOT(TD(1),FAF(1),TD(2)     ,FAF(MD+1),MD,0.25,0.25)        PL     110
      IF(MOD(INS,2).EQ.0)CALL HGPLOT(-TMOVE,-44.0,0,4)                 PL     120
 1201 CALL HGPLOT(0.0,0.0,3,0)                                         PL     130
      IF(MOD(ND,2).EQ.1)CALL HGPLOT(-TMOVE,-22.0,0,4)                  PL     140
      CALL HGPLOT(0.0,0.0,3,0)                                         PL     150
   98 IF(LC.LT.2)GOTO1                                                 PL     160
   99 IF(JPLOT.EQ.0)STOP                                               PL     170
      CALL HGPLOT(0.0,0.0,0,2)                                         PL     180
      CALL HGPTAPE(2,12H            ,0,0,0)                            PL     190
      STOP
   90 FORMAT(1H0,4(/1H ,10X,5(A8,3X)/7H VARI. ,5(F8.3,3X),16H;NON-DIMENS
     1IONAL/1H ,5X,5(1H(,F8.3,1H),1X),9H :AVIONIC/7H DELTA ,5(F8.3,3X)/)
     2)
   92 FORMAT(9H0 T(SECS) ,5X,3HETA,10X,1HQ,11X,2HAZ, 7X,9HAIR SPEED,4X,
     1 5HALPHA,5X,15(1H-),18HERRORS IN MATCHING ,15(1H-)/
     2 1H ,76X,1HQ,11X,2HAZ, 7X,9HAIR SPEED,4X,5HALPHA)
   93 FORMAT(1H+,70X,4(3X,F9.5))
   94 FORMAT(9H0SIGMA = ,F6.4,22H DEGREES OF FREEDOM = ,I3//)                   1280
      END                                                                        1490

END OF SEGMENT, LENGTH  1054, NAME  LONGITUDINALPLOT
```

```
      SUBROUTINE DATAREAD                                                   1500
      INTEGER P,PF,PM,PC,SD,Q                                               1501
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),       2
     1PSI(625),E(25),TIME(100),OBS(5 ,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(10),
     3 YIN(5),ITD(100)      ,COBS(100,6,2)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY       5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                  6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,        7
     3N1,PC,AC,PB,DTM,YIN,T0,XA,COBS,OBS,XS,NGAP/
     4 MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP,JES(22)               9
      COMMON/AVIONIC/XMAV(20),RUNAME,MDO
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL     1
     1 TAXIS,TMOVE                                                      PL     2
      EQUIVALENCE(COBS(1,1,1),TIME(1))
C  READ ENTRY LABEL   (FROM COLUMN 1)                                   1560
    1 READ(1,80)L                                                           1570
   80 FORMAT(I1)                                                            1580
      IF(L.EQ.0)GO TO 1                                                     1590
C  READ DATA TITLE    (FROM   COL.2 OF NEXT CARD)                       1600
      READ(1,81)TITLE                                                   PL   200
   81 FORMAT(10A8)                                                      PL   210
      WRITE(2,81)TITLE                                                  PL   220
    2 GO TO (1,2,3,4,5,6,7,8,9),L                                           1640
C READ INSTRUMENT POSITIONS....
    3 READ(1,84)(AC(I),I=1,9)                                               1661
   84 FORMAT(3F10.3)                                                        1662
      WRITE(2,85)(AC(I),I=1,9)                                              1663
   85 FORMAT(21H0INSTRUMENT POSITIONS  /  5H   X1,8X,2HY1,8X,2HZ1,8X,
     1 2HX2,8X,2HY2,8X,2HZ2,8X,2HX3,8X,2HY3,8X,2HZ3/1H ,9(F7.3,3X))
C  READ MODEL CONSTANTS                                                 1670
      READ(1,86)(SP(I),I=1,4)
   86 FORMAT(4F10.3)
      WRITE(2,87)(SP(I),I=1,4)
   87 FORMAT(5H0  IY,11X,1HM,9X,1HS,5X,4HCBAR/ F12.1,2F10.1,F7.2)
C  READ PARAMETER ACCURACIES                                            1710
      READ(1,88)(XE(I),I=1,20)
   88 FORMAT(8F10.3)                                                        1712
C  READ AND PRINT NUMBER OF INSTRUMENTS                                 1720
    4 READ(1,89)ND                                                          1721
   89 FORMAT(I2)                                                            1722
      WRITE(2,90)ND                                                         1723
   90 FORMAT(25H0NUMBER OF INSTRUMENTS = ,I1)                               1724
C  READ INITIAL CONDITIONS ETC.                                         1740
    5 READ(1,93)T0,YIN,DTM                                                  1741
      FIRSTIME=IFIX(T0)                                                 PL   230
   93 FORMAT(6F10.4/8F10.4/2F10.4)
      WRITE(2,94)T0,YIN,DTM
   94 FORMAT(1H0,4X,2HT0,18X,2HVT,17X,3HRHO,9X,1HG/1H , 3F10.3, F10.1,
     1 F10.7,F10.2//101H THETA.TRIM    W.TRIM    Q.TRIM    U.TRIM  PHI.TRIM
     2RIM  ETA.TRIM    P.TRIM    R.TRIM   AZ.TRIM ALFA.TRIM /1H ,10F10.4
     3)
C  READ MODEL PARAMETERS                                                1760
      READ(1,96)(XX(I),I=1,20)
   96 FORMAT(8F10.4)                                                        1752
C READ ORDER ARRAY.....
      READ(1,105)(JP(I),I=1,20)
  105 FORMAT(20I0)
      DO 21 I=1,16
      JES(I)=JP(I)
      DO 22 IE=17,20
   22 IF(JP(IE).LT.JP(I))JES(I)=JES(I)-1
   21 CONTINUE
C  EVALUATE MULTIPLIERS XM                                              1760
```

```
      HRVS=YIN(4)*YIN(2)*SP(3)/2.0
      XM(5),XM(7),XM(9),XM(11)=HRVS/SP(2)
      XM(6),XM(10)=HRVS*SP(4)/SP(2)
      XM(8),XM(12)=HRVS*YIN(2)/SP(2)
      XM(13),XM(15)=HRVS*SP(4)/SP(1)
      XM(14)=HRVS*SP(4)**2/SP(1)
      XM(16)=HRVS*SP(4)*YIN(2)/SP(1)
      DO 41 I=1,4
      XM(16+I)=1.0
   41 XM(I)=1.0
      DO 42 I=1,20
   42 XMAV(I)=XM(I)
      XMAV(13)=XM(13)*YIN(2)
      XMAV(15)=XM(15)*YIN(2)
C  READ PROGRAM CONTROL PARAMETERS                                      1770
    6 READ(1,97)REJ,ACF,HMAX,P,PF,IT,ITM,LC
   97 FORMAT(3F10.4/5I0)
      WRITE(2,98)REJ,ACF,HMAX,P,PF,IT,ITM                               1773
   98 FORMAT(8H0  E MAX,6X,8HACCURACY,3X,7HDELTA T/3F10.4//28H INITIAL N   1774
     1O OF PARAMETERS = ,I2,2X,11HFINAL NO = ,I2//17H INITIAL ITRN. = ,   1775
     2I2,9X,16HITRN. MAXIMUM = ,I2)                                     1776
C CALCULATE N AND NF....
      N=3*P+6
      NF=3*PF+6
      DO 52 IE=17,20
      IF(JP(IE).LE.P)N=N-3
   52 IF(JP(IE).LE.PF)NF=NF-3
C  READ SCALING FACTORS                                                 1780
    7 READ(1,99)(XS(I),I=1,ND)                                          1781
   99 FORMAT(7F10.2)                                                    1782
      WRITE(2,100)(XS(I),I=1,ND)                                      1830
  100 FORMAT(11H0SCALING OF/5X,1HQ,8X,2HAZ,9X,1HV,7X,5HALPHA/4F10.2//)
C IS THERE ANY PLOTTING NEEDED....                                    PL   240
      READ(1,401)L,IPLOT                                              PL   250
  401 FORMAT(I1,I1)                                                   PL   255
      IF(IPLOT.EQ.0)GO TO 2                                           PL   260
      IF(JPLOT.NE.0)GOTO 400                                          PL   270
C BEFORE FIRST PLOT OPEN TAPE...                                      PL   280
      CALL HGPTAPE(0,12HFLT DATA FIT,0,0,7)                           PL   290
      CALL HGPTAPE(1,12H            ,0,0,0)                           PL   300
      CALL HGPLOT(0.0,0.0,16,1)                                       PL   310
      CALL HGPLOT( 0.0,2.0,0,4)                                       PL   320
      CALL HGPLOT(0.0,0.0,3,0)                                        PL   330
      JPLOT=1                                                         PL   340
C BEFORE EACH SET OF DATA PLOTTED START NEW PICTURE...,               PL   350
  400 CALL HGPTAPE(1,12H            ,0,0,0)                           PL   360
      CALL HGPLOT(-4.0,0.0,0,4)                                       PL   370
      CALL HGPLOT(0.0,0.0,3,0)                                        PL   380
      GO TO 2                                                           1792
C
    8 READ(1,200)NGAP,TF   ,IL
      NGAP=MINO(NGAP,6)
  200 FORMAT(I2,F8.4,I1)
      IL=IL+1
      GO  TO (300,301,302),IL
C  NEW DATA LATER ON PRESENT TAPE,,,,,
  302 MDO=MDO-MD
      GO TO 8889
C NEW PAPER TAPE TO  BE LOADED
  301 CALL  RLEASE(3)
      WRITE(5,310)
  310 FORMAT(' PLEASE UNLOAD PAPER TAPE,LOAD NEXT ONE' )
      PAUSE
  300 READ(3,202)RUNAME,MDO
  202 FORMAT(1X,A5//1X,I4)
 8889 WRITE(2,207)RUNAME,MDO
```

```
      207 FORMAT(19H0 TAPE IDENTIFIER    ,A3,5X,32H0RIGINAL NUMBER OF DATA PO
         2INTS   ,I4)
          WRITE(2,208)NGAP
      208 FORMAT(12H0ONLY EVERY ,I4,11H POINT USED)
    C READ DATA                                                               1800
     8888 READ(3,203)TIME(1),OBS(1,1),OBS(2,1), COBS(1,1,2)
          IF(TIME(1).GE.T0)GOTO 210
          MDO=MDO-1
          GO TO 8888
      210 MD=1
      212 IF((MD*NGAP+1).GT.MDO.OR.TIME(MD).GE.TF.OR.MD.EQ.100) GOTO 215
          IF(NGAP.EQ.1)GO TO 213
          DO 214 NN=2,NGAP
      214 READ(3,206) COBS(MD,NN,1),COBS(MD,NN,2)
      206 FORMAT(/F7.5,18X,F9.5)
      213 MD=MD+1
          READ(3,203) TIME(MD),OBS( 1,MD),OBS(2,MD),COBS(MD,1,2)
      203 FORMAT(/F7.3,3F9.5)
          GO TO 212
      215 CONTINUE
          DO 402 I=1,MD                                                    PL   400
      402 TD(I)=(TIME(I)-FIRSTIME)*2.0                                     PL   410
          TAXIS=2.0*IFIX(TIME(MD)-FIRSTIME+0.99)                           PL   420
          TMOVE=AMAX1(32.0,TAXIS+5.0)                                      PL   430
    C  ENTRY POINT FOR REPEAT WITH SAME DATA                                  1820
        9 DO 10 I=1,20
          K=JP(I)                                                             1833
          X(K)=XX(I)                                                          1836
       10 CWD(K)=XE(I)                                                        1840
          DO 11 I=20,25
       11 CWD(I)=0.0
          IF(P.NE.PF)MD=MD/2                                                  1850
          RETURN                                                              1860
          END                                                                 1870

END OF SEGMENT, LENGTH   894, NAME  DATAREAD
```

```
      SUBROUTINE F4DERY
      INTEGER P,PF,PM,PC,SD,Q                                           1501
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),   2
     1PSI(625),E(25),TIME(100),OBS(5 ,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(10),
     3 YIN(5),ITD(100)     ,COBS(100,6,2)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY   5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                              6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,    7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,COBS,OBS,XS,NGAP/
     4 MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP,JES(22)           9
      EQUIVALENCE(COBS(1,1,1),TIME(1))
      CPHI=COS(DTM(5))
      STHE=SIN(DTM(1))
      CTHE=COS(DTM(1))
      IF(NGAP.EQ.1)GO TO 602
      DO 600 NN=2,NGAP
      IF(Y(1).GT.COBS(Q-1,NN,1)) GO TO 600
      GO TO 601
  600 CONTINUE
C WHEN IN FINAL REGION OF TIME INTERVAL OR IF NGAP=1.....
  602 D=(Y(1)-COBS(Q-1,NGAP,1))/(COBS(Q,1,1)-COBS(Q-1,NGAP,1))
      Y(6)=(COBS(Q,1,2)-COBS(Q-1,NGAP,2))*D +COBS(Q-1,NGAP,2)-DTM( 6)
      GO TO 603
C WHEN NOT IN FINAL REGION.......
  601 D=(Y(1)-COBS(Q-1,NN-1,1))/(COBS(Q-1,NN,1)-COBS(Q-1,NN-1,1))
      Y(6)=(COBS(Q-1,NN,2)-COBS(Q-1,NN-1,2))*D +COBS(Q-1,NN-1,2)-DTM( 6)
  603 CONTINUE
      DY(2)= Y(4)*CPHI
      DY(3)= DTM(3)*Y(5) +DTM(4)*Y(4) +Y(2)*STHE*CPHI*G +XP(9)*Y(3)
     1       +XP(10)*Y(4) +XP(11)*Y(5) +XP(12)*Y(6)
      DY(4)= XP(13)*Y(3) +XP(14)*Y(4) +XP(15)*Y(5) +XP(16)*Y(6)
      DY(5)= -DTM(3)*Y(3) -DTM(2)*Y(4) +Y(2)*CTHE*G +XP(5)*Y(3)
     1        +XP(6)*Y(4) +XP(7)*Y(5) +XP(8)*Y(6)
      DY(6)=0.0
C GET Z'S FROM Y'S.......
      DO 91 I=1,16
      IF(JP(I).GT.P)GO TO 94
      DO 93 IV=1,3
   93 Z(IV,I)=Y(3+IV+3*JES(I))
      GOTO 91
   94 Z(1,I),Z(2,I),Z(3,I)=0.0
   91 CONTINUE
      DO 1 I=1,16
      IF(JP(I).GT.P)GOTO 2
      ZD(1,I)= DTM(3)*Z(3,I) +DTM(4)*Z(2,I) +XP(9)*Z(1,I) +XP(11)*Z(3,I)
     1         +XP(10)*Z(2,I)
      IF(I.GE.9.AND.I.LE.12)ZD(1,I)=ZD(1,I)+Y(I-6)*XM(I)
      ZD(2,I)= XP(13)*Z(1,I) +XP(14)*Z(2,I) +XP(15)*Z(3,I)
      IF(I.GE.13.AND.I.LE.16)ZD(2,I)=ZD(2,I)+Y(I-10)*XM(I)
      ZD(3,I)= -DTM(3)*Z(1,I) -DTM(2)*Z(2,I) +XP(5)*Z(1,I) +XP(6)*Z(2,I)
     1          +XP(7)*Z(3,I)
      IF(I.GE.5.AND.I.LE.8)ZD(3,I)=ZD(3,I)+Y(I-2)*XM(I)
      GOTO 1
    2 ZD(1,I),ZD(2,I),ZD(3,I)=0.0
    1 CONTINUE
C
      DO 81 I=1,16
      DO 83 IV=1,3
   83 DY(3+IV+3*JES(I))=ZD(IV,I)
   81 CONTINUE
      RETURN
      END

END OF SEGMENT, LENGTH   717, NAME   F4DERY
```

```
      SUBROUTINE MATHMODEL                                                  2500
      INTEGER P,PF,PM,PC,SD,Q                                               2501
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),       2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(10),
     3 YIN(5),ITD(100)    ,COBS(100,6,2)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY       5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                  6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,        7
     3N1,PC,AC,PB,DTM,YIN,T0,XA,COBS,OBS,XS,NGAP/
     4 MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP,JES(22)               9
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)   ,FIRSTIME,JPLOT,IPLOT, PL    1
     1 TAXIS,TMOVE                                                     PL    2
      EQUIVALENCE(COBS(1,1,1),TIME(1))
      IF(Q.EQ.2)H=TIME(MD)-Y(1)                                             2540
    1 ING=0                                                                 2550
C  CALCULATE STEP LENGTH                                                    2560
      A=TIME(Q)-Y(1)                                                        2570
      IF(ABS(A-H).LT.0.0001)ING=1                                           2580
      B1=A                                                                 2590
      IF(HMAX.GT.A)GO TO 2                                                  2600
      B1=HMAX                                                               2610
      IF(ABS(H-HMAX).LT.0.0001)ING=1                                        2612
    2 H=B1                                                                  2614
C  INTEGRATE 1 STEP                                                         2620
    3 CALL F4RUNG(N1,ING,H,Y,DY,HQ)                                         2630
      IF(ING.EQ.1)GO TO 4                                                   2632
      ING=1                                                                 2634
      GO TO 3                                                               2636
    4 GO TO(21,22,23,24),ND
   24 PI(4)=DTM(10)+ (Y(3)-Y(4)*AC(7))/DTM(4) +XX(20)
   23 PI(3)=YIN(2) +((Y(3)+AC(4)*Y(4))*(DTM(2)-DTM(3)*AC(4)+DTM(7)*AC(5)
     1          )+DTM(4)*Y(5))/YIN(2) +XX(19)
   22 PI(2)=DTM(9)+(XP(9)*Y(3)+XP(10)*Y(4) +XP(11)*Y(5) +XP(12)*Y(6)
     1          +AC(1)*DY(4) -AC(2)*Y(4)*DTM(8) +2.0*AC(3)*DTM(3)*Y(4))
     2         /YIN(5)   +XX(18)
   21 PI(1)=Y(4)+DTM(3)+XX(17)
C
      IF(A.GT.HMAX) GO TO 20
      DO 71 I=1,16
      JE=JP(I)
      DO 76 ND1=1,4
  76  FY(ND1,JE)=0.0
      IF(JF.GT.P) GO TO 71
      JE=3*JES(I)
      GOTO(11,12,13,14),ND
   14 FY(4,JP(I))= (Y(4+JE)-Y(5+JE)*AC(7))/DTM(4)
   13 FY(3,JP(I))=((Y(4+JE)+AC(4)*Y(5+JE))*(DTM(2)-DTM(3)*AC(4)+DTM(7)*
     1                  AC(5))+ DTM(4)*Y(6+JE))/YIN(2)
   12 FY(2,JP(I))=(XP(9)*Y(4+JE) +XP(10)*Y(5+JE) +XP(11)*Y(6+JE)
     1                +AC(1)*DY(5+JE)-AC(2)*DTM(8)*Y(5+JE)+2.0*AC(3)*DTM(3)
     2                *Y(5+JE)  )/G
      IF(I.GE.9.AND.I.LE.12)FY(2,JP(I))=FY(2,JP(I))+Y(I-6)*XM(I)/G
   11 FY(1,JP(I))= Y(5+JE)
   71 CONTINUE
   77 DO 172 I=17,(16+ND)
      DO 172 INS=1,ND
      FY(INS,JP(I))=0.0
  172 IF(INS.EQ.(I-16))FY(INS,JP(I))=1.0
C  IF REQUIRED PRINT RESULT                                                 2800
   20 IF(IT.EQ.0)GO TO 121
      IF(PC.NE.1)GO TO 30                                                   2820
      IF(IPLOT.EQ.1)GO TO 30
  121 WRITE(2,50) Y(1),Y(6),( PI(I) ,I=1,ND)
```

```
   50 FORMAT(1H ,F8.3,5(5X,F9.5))
      IF(IT.NE.0)GO TO  30
      WRITE(4,451) Y(1),PI(1),PI(2),Y(6)
  451 FORMAT(1X/F7.3,3F9.5)
   30 IF(A.GE.HMAX)GO TO 1                                                2940
      IF(IT.EQ.0.OR.IPLOT.EQ.0)GO TO 31
      DO 51 INS=1,ND                                                 PL    450
   51 FIT(INS,Q-1)=PI(INS)                                           PL    460
   31 RETURN                                                              2950
      END                                                                 2960

END OF SEGMENT, LENGTH   708, NAME  MATHMODEL
```

```
      SUBROUTINE INIT                                                     6000
      INTEGER P,PF,PM,PC,SD,Q                                             6001
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),     2
     1PSI(625),E(25),TIME(100),OBS(5 ,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(10),
     3 YIN(5),ITD(100)    ,COBS(100,6,2)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY     5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,      7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,COBS,OBS,XS,NGAP/
     4 MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP,JES(22)             9
      EQUIVALENCE(COBS(1,1,1),TIME(1))
    1 IF(P.NE.PF)N1=N                                                     6060
      IF(P.EQ.PF)N1=NF                                                    6070
    2 DO 5 I=1,20
    5 XP(I)=XX(I)*XM(I)                                                   6096
      Y(1)=TO                                                             6110
      DO 10I=1,4
   10 Y(I+1)=XX(I)
      DO 4 I=6,70                                                         6170
    4 Y(I)=0.0
      DO 11 I=2,4
      IF(JP(I).GT,P)GO TO 11
      Y(3*JES(I)+I+2)=1.0
   11 CONTINUE
      U=YIN(2)
      G=YIN(5)
      RETURN                                                              6300
      END                                                                 6310

END OF SEGMENT, LENGTH   128, NAME  INIT
```

```
                 SUBROUTINE AXISCALE(S,N,XMIN,DX,LL)
                 COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL      1
                1 TAXIS,TMOVE                                                         PL      2
                 DIMENSION S(200),Z(4)
                 DATA Z/4H  Q ,4H AZ ,4H SPD,4HALFA/
                 CALL HGPAXISV(0.0,0.0,4HTIME,-4,TAXIS,0.0,FIRSTIME,1.0,2.0,-4)
                 CALL HGPSCALE(S,N,15.0,XMIN,DX,1)
                 CALL HGPAXISV(0.0,0.0,Z(LL),4,15.0,0.0,XMIN,2*DX,2.0,3)
                 CALL HGPSYMBL(16.0,13.0,0.35,TITLE,0.0,20)
                 CALL HGPRECT(-1.3,-4.0,21.0, TMOVE-2.4,0.0,3)
                 CALL HGPLOT(0.0,0,0,3,0)
                 RETURN
                 END

END OF SEGMENT, LENGTH   115, NAME  AXISCALE

                 SUBROUTINE PLOT(X,Y,XPNTS,YPNTS,N,UD,UG)
                 DIMENSION X(100),Y(100),XPNTS(100),YPNTS(100)
                 CALL HGPLINE(X,Y,N,1)
                 CALL HGPLOT(0.0,0.0,3,0)
                 CALL DASHED(XPNTS,YPNTS,N-1,UD,UG)
                 CALL HGPLOT(0.0,0.0,3,0)
                 RETURN
                 END

END OF SEGMENT, LENGTH    75. NAME  PLOT
```

```
      SUBROUTINE DASHED(X,Y,N,DL2,DL3)
      DIMENSION X(N),Y(N),DL(3)
      DL(2)=DL2
      DL(3)=DL3
      CALL HGPLOT(X(1),Y(1),3,0)
      REM=DL(2)
      IC=2
      XP=X(1)
      YP=Y(1)
      DO 1 I=2,N
    2 PL=SQRT((X(I)-XP)**2+(Y(I)-YP)**2)
      IF(REM-PL)4,4,3
C          IF NOT ENOUGH DASH OR SPACE LEFT ......
    4 XP=XP+(X(I)-XP)*REM/PL
      YP=YP+(Y(I)-YP)*REM/PL
      CALL HGPLOT(XP,YP,IC,0)
      IC=5-IC
      REM=DL(IC)
      GOTO 2
C          IF ENOUGH DASH OR SPACE LEFT ......
    3 IF(IC.EQ.2.AND.PL.NE.0.0)CALL HGPLOT(X(I),Y(I),2,0)
      REM=REM-PL
      XP=X(I)
    1 YP=Y(I)
C
      CALL HGPLOT(0.0,0.0,3,0)
      RETURN
      END
```

```
END OF SEGMENT, LENGTH   178, NAME  DASHED
```

```
            SUBROUTINE  SOLNORM(N,SIGMA,ND)                                         000 4010
            INTEGER P,Q,R,S,T,RD,SD                                                 000 4020
            COMMON/COMP/COV(650),A(25),B(26),C(25),D(25),X(25),Y(26),Z(26)/             4040
           1NORME/PSI(625),CWD(25),E(25)                                                4045
C            SUBROUTINE  SOLNORM                                                    000 4000
C
          1 AD=0                                                                    000 4050
            T=N-1                                                                   000 4060
            L=N+1                                                                   000 4070
            DO  2  I=1,N                                                            000 4080
          2 Z(I)=CWD(I)                                                             000 4090
            KD=1                                                                    000 4100
            DO  7  I=1,N                                                            000 4110
            X(I)=I                                                                  000 4120
            JD=N*(I-1)+1                                                            000 4130
            DO  3  ID=1,N                                                           000 4140
            A(ID)=PSI(JD)                                                           000 4150
          3 JD=JD+1                                                                 000 4160
            DO  4  J=1,N                                                            000 4170
            YD=ABS(A(J))                                                            000 4180
            IF(AD.GE.YD)GOTO 4                                                      000 4190
            AD=YD                                                                   000 4200
            R=I                                                                     000 4210
            S=J                                                                     000 4220
          4 CONTINUE                                                                000 4230
C           PUT UNIT MATRIX AND CWD INTO COV                                        000 4235
C
            DO  6  J=1,N                                                            000 4240
            JD=KD+J-1                                                               000 4250
            IF(I.NE.J)GO TO 5                                                       000 4260
            COV(JD)=1                                                               000 4270
            GOTO 6                                                                  000 4280
          5 COV(JD)=0                                                               000 4290
          6 CONTINUE                                                                000 4300
            KD=KD+N                                                                 000 4310
            COV(KD)=CWD(I)                                                          000 4320
          7 KD=KD+1                                                                 000 4330
C
C           CONSTRUCT TRIANGULAR MATRIX USING ROW & COLUMN INTERCHANGE,             000 4335
C           TO LABEL 29.                                                            000 4336
C
            P=N-1                                                                   000 4340
            IF(P.LT.1)GOTO 30                                                       000 4350
            M=N                                                                     000 4360
            DO 29 Q=1,P                                                             000 4370
            AD=0                                                                    000 4380
            JD=M*(R-1)+Q                                                            000 4390
            DO 8 ID=1,M                                                             000 4400
            KD=Q+ID-1                                                               000 4410
            A(KD)=PSI(JD)                                                           000 4420
          8 JD=JD+1                                                                 000 4430
            JD=L*(R-1)+1                                                            000 4440
            DO  9  ID=1,L                                                           000 4450
            Y(ID)=COV(JD)                                                           000 4460
          9 JD=JD+1                                                                 000 4470
            CD=A(S)                                                                 000 4480
            A(S)=A(Q)                                                               000 4490
            A(Q)=CD                                                                 000 4500
            D(Q)=1.0/CD                                                             000 4510
            CD=X(S)                                                                 000 4520
            X(S)=X(Q)                                                               000 4530
```

```
      IF(R.EQ.Q)GOTO 16
      JD=L*(Q-1)+1
      DO  10  ID=1,L
      B(ID)=COV(JD)
   10 JD=JD+1
      JD=N*(Q-1)+G
      DO  11  ID=1,M
      KD=Q+ID-1
      C(KD)=PSI(JD)
   11 JD=JD+1
      CD=C(S)
      C(S)=C(Q)
      C(Q)=CD
C
      DO  12  J=Q,N
      C(J)=C(J)-A(J)*CD*D(Q)
      YD=ABS(C(J))
      IF(AD.GE.YD)GOTO  12
      AD=YD
      RD=K
      SD=J
   12 CONTINUE
      DO  13  J=1,L
   13 B(J)=B(J)-Y(J)*CD*D(Q)
      JD=N*(R-1)+G
      DO  14  ID=1,M
      KD=Q+ID-1
      PSI(JD)=C(KD)
   14 JD=JD+1
      JD=L*(R-1)+1
      DO  15  ID=1,L
      COV(JD)=B(ID)
   15 JD=JD+1
   16 DO  19  J=1,Q
      IF(J.EQ.Q)GOTO 19
      JD=N*(J-1)+G
      DO  17  ID=1,M
      KD=Q+ID-1
      C(KD)=PSI(JD)
   17 JD=JD+1
      CD=C(S)
      C(S)=C(Q)
      C(Q)=CD


      JD=N*(J-1)+G
      DO  18  ID=1,M
      KD=Q+ID-1
      PSI(JD)=C(KD)
   18 JD=JD+1
   19 CONTINUE
      K=Q+1
      DO  26  I=K,N
      IF(I.EQ.R)GOTO 26
      JD=N*(I-1)+G
      DO  20  ID=1,M
      KD=Q+ID-1
      C(KD)=PSI(JD)
   20 JD=JD+1
      JD=L*(I-1)+1
      DO  21  ID=1,L
      B(ID)=COV(JD)
   21 JD=JD+1
      CD=C(S)
      C(S)=C(Q)
```

```
      DO  22  J=Q,N                                                    000 5180
      C(J)=C(J)-CD*A(J)*D(Q)                                           000 5190
      YD=ABS(C(J))                                                     000 5200
      IF(AD.GE.YD)GOTO 22                                              000 5210
      AD=YD                                                            000 5220
      RD=I                                                             000 5230
      SD=J                                                             000 5240
   22 CONTINUE                                                         000 5250
      DO 23 J=1,L                                                      000 5260
   23 B(J)=B(J)-CD*Y(J)*D(Q)                                           000 5270
      JD=N*(I-1)+Q                                                     000 5280
      DO  24  ID=1,M                                                   000 5290
      KD=Q+ID-1                                                        000 5300
      PSI(JD)=C(KD)                                                    000 5310
   24 JD=JD+1                                                          000 5320
      JD=L*(I-1)+1                                                     000 5330
      DO  25  ID=1,L                                                   000 5340
      COV(JD)=B(ID)                                                    000 5350
   25 JD=JD+1                                                          000 5360
   26 CONTINUE                                                         000 5370
      JD=N*(Q-1)+Q                                                     000 5380
      DO  27  ID=1,M                                                   000 5390
      KD=Q+ID-1                                                        000 5400
      PSI(JD)=A(KD)                                                    000 5410
   27 JD=JD+1                                                          000 5420
      JD=L*(Q-1)+1                                                     000 5430
      DO  28  ID=1,L                                                   000 5440
      COV(JD)=Y(ID)                                                    000 5450
   28 JD=JD+1                                                          000 5460
      M=M-1                                                            000 5470
      R=RD                                                             000 5480
   29 S=SD                                                             000 5490
C
C     SUBSTITUTE, PUTTING SOLUTION INTO B(I) & PROBABLE ERRORS         000 5495
C     FROM DIAGONAL OF COV INTO CWD(I).                                000 5496
C
   30 ID=N*N                                                           000 5500
      C(N)=PSI(ID)                                                     000 5510
      D(N)=1.0/C(N)                                                    000 5520
      DO  42  Q=1,L                                                    000 5530
      DO  31  I=1,N                                                    000 5540
   31 C(I)=0.0                                                         000 5550
      I=N                                                              000 5560
   32 K=N-I+1                                                          000 5570
      JD=L*(I-1)+Q                                                     000 5580
      B(Q)=COV(JD)                                                     000 5590
      JD=N*(I-1)+I                                                     000 5600
      DO  33  ID=1,K                                                   000 5610
      KD=ID+I-1                                                        000 5620
      A(KD)=PSI(JD)                                                    000 5630
   33 JD=JD+1                                                          000 5640
      AD=0                                                             000 5650
      DO  34  J=1,N                                                    000 5660
   34 AD=AD-A(J)*C(J)*D(I)                                             000 5670
      C(I)=B(Q)*D(I)+AD                                                000 5690
      I=I-1                                                            000 5690
      IF(I.GE.1)GOTO 32                                                000 5700
      DO  36  I=1,N                                                    000 5710
   36 Y(I)=X(I)                                                        000 5720
      AD=1.3
      DO  41  I=1,N                                                    000 5740
      J=I                                                              000 5750
   37 IF(0.5.GT.ABS(AD-Y(J)))GO TO 38                                  000 5760
      J=J+1                                                            000 5770
      GOTO 37                                                          000 5780
```

```
      C(J)=C(I)
      C(I)=A(I)
      AD=AD+1
      IF(Q.EQ.L)GOTO 39
      IF(Q.NE.I)GOTO 40
      CWD(I)=C(I)
      GOTO 40
   39 E(I)=C(I)
   40 A(I)=Y(J)
      Y(J)=Y(I)
   41 Y(I)=A(I)
   42 CONTINUE
      DO  43 I=1,N
   43 SIGMA=SIGMA+E(I)*Z(I)
      ND=ND-N
      DO 44  I=1,N
      AD=ABS(SIGMA*CWD(I)/ND)
   44 CWD(I)=SQRT(AD)
      SIGMA=SQRT(ABS(SIGMA/ND))
      RETURN
      END

END OF SEGMENT, LENGTH  1778, NAME  SOLNORM
```

```
          SUBROUTINE GFIN
          DIMENSION ST(10)
          EXTERNAL GFERR
          CALL FTRAP(GFERR)
        1 FORMAT(10A8)
        2 READ(103,1,END=3)ST
          WRITE(101,1)ST
          GO TO 2
        3 CALL RLEASE(103)
          WRITE(101,4)
        4 FORMAT(4H****)
          CALL RUNOUT (101)
          ENDFILE 101
          REWIND 101
          RETURN
          END

END OF SEGMENT, LENGTH    41, NAME  GFIN

          ERROR TRAP
          SUBROUTINE GFERR(IER)
          DIMENSION ST(10)
          COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL     1
         1 TAXIS,TMOVE                                                      PL     2
          IF(JPLOT.EQ.0)GO TO 8
          CALL HGPLOT(0.0,0.0,0,2)
          CALL HGPTAPE(2,12H            ,0,0,0)
        8 READ(101,1,END=2)ST
        1 FORMAT(10A8)
          WRITE(102,3)ST
        3 FORMAT(1H0///'0FIRST CARD NOT READ....'//1H ,10A8////)
          GO TO 5
        2 WRITE(102,4)
        4 FORMAT(1H0///'0ALL CARDS,INCLUDING ****,HAVE BEEN READ')
        5 RETURN
          END

END OF SEGMENT, LENGTH    56, NAME  GFERR
```

```
 0835
 0836                   FINISH

END OF COMPILATION -   NO ERRORS


S/C SUBFILE;             125 BUCKETS USED



CONSOLIDATED BY XPCK 12C   DATE     20/09/74   TIME 06/20/12

ICLFORTRAN  (2)    RENAMED  ICLA-DEFAULT(2)

PROGRAM A53E
COMPACT DATA (15AM)
COMPACT PROGRAM (DBM)
CORE              25472
```

Appendix D

```
FORTRAN COMPILATION BY #XFAT MK 4C    DATE  20/09/74  TIME  07/20/34


 0001                   LIST(LP)
 0002                   LIBRARY(SUBGROUPSRF7)
 0003                   LIBRARY(SUBGROUPSRGP)
 0004                   LIBRARY(SUBGROUPS-RS)
 0005                   LIBRARY (ED,SUBGROUPFSCE.SUBROUTINES)                           4
 0006                   PROGRAM(A53D)
 0007                   COMPACT DATA                                                    6
 0008                   OUTPUT 2,102=LP0
 0009                   INPUT 103=CR0
 0010                   USE 1,101=ED1/FORMATTED/128
 0011                   TRACE 0
 0012                   END
```

```
      MASTER A25D PLOT 2
      INTEGER P,PF,PM,PC,SD,Q
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),       2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(9),YIN(5)
     3,PARAM(22)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY       5
     1 RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                  6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,        7
     3N1,PC,AC,PB,DTM,YIN,T0,XA,TIME,OBS,XS/                                   8
     4MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL     1
     1 TAXIS,TMOVE                                                      PL     2
      DIMENSION FAF(200),DPI(4,25)                                      PL    10
C  HEADINGS FOR PRINT OUT OF PARAMETERS,E.G. V0    ,LP   ,ETC.             1772
      DATA PARAM(1)/176H  V0       P0       R0       YV       YP       YR
     1  YXI      YZT      LV       LP       LR       LXI      LZT      NV
     2NP       NR       NXI      NZT     EBETA     EP       ER       EAY   /
      CALL GFIN
      JPLOT=0                                                           PL    20
C  PRINT PROGRAM NAME AND HEADINGS                                           160
      CALL DATE (TODAY)
    1 WRITE(2,2)TODAY                                                        170
    2 FORMAT(75H1 PROGRAM  D1   JULY 1973      DUTCH ROLL ANALYSIS FOR
     1FULL SCALE AIRCRAFT//35H A.J.ROSS AERO. R141BLDG. PROG A25D,60X,
     2'RUN ON  ',A8//)

C  READ DATA AND PROGRAMME CONTROL PARAMETERS                                200
    3 CALL DATAREAD                                                          210
C  SET PRINT CONTROL AND TEST FOR SIMULATION MODE                            220
    4 PC=0                                                                   230
      IF(MD.EQ.1)PC=1                                                        240
C  PRINT PARAMETER AND ACCURACY VALUES IN TWO ROWS                           250
      DO 50 I=1,22
      K=JP(I)
      CALL COPY(8,ORDER(K),1,PARAM(I),1)
   50 CONTINUE
    5 WRITE(2,90)(ORDER(I),I=1,8),(X(I),I=1,8),(CWD(I),I=1,8),               260
     1(ORDER(I),I=9,15),(X(I),I=9,15),(CWD(I),I=9,15),(ORDER(I),I=16,22)     261
     2,(X(I),I=16,22),(CWD(I),I=16,22)                                       262
C  TEST FOR INITIAL PRINTOUT                                                 280
      IF(IT.EQ.0)GO TO 6                                                     290
C  TEST FOR FINAL PRINTOUT                                                   300
      IF(PC.NE.1)GO TO 7                                                     310
C  TEST FOR FINAL NUMBER OF PARAMETERS                                       320
      IF(P.EQ.PF)GO TO 6                                                     330
C  CHECK VALUE OF ITERATION COUNTER                                          340
      IF(IT.GT.ITM)GO TO 6                                                   350
      P=PF                                                                   360
      N=NF                                                                   365
      MD=2*MD                                                                370
      PC=0                                                                   380
      IT=1                                                                   390
      GO TO 7                                                                400
C  WRITE MAIN HEADINGS                                                       410
    6 IF(IPLOT.NE.1) WRITE(2,92)
C  SET UP INITIAL CONDITIONS                                                 430
    7 CALL INIT                                                              440
C  CLEAR MATRIX STORES AND SET COUNTERS                                      450
      SD=0                                                                   460
      S=1
      J=P*P                                                                  480
```

```
          DO 8 I=1,J                                                          500
        8 PSI(I)=0.0                                                          510
          DO 9 I=1,MINO(K,25)                                            NB MODIF
        9 CWD(I)=0.0                                                          530
          SIGMA=0.0                                                           535
          NAP=0
          DO 830 I=1,4
          DO 830 J=1,25
      830 DPI(I,J)=0.0
C  MAIN COMPUTING CYCLE SETTING UP NORMAL EQUATIONS                           540
   10 DO 22 Q=2,MD                                                            550
C  CALCULATE MODEL PREDICTIONS AND PARTIAL DERIVATIVES                        560
          CALL MATH MODEL                                                     570
C  TEST ERRORS AND CALCULATE RMS                                              580
   11 J=Q                                                                     590
          ER=0.0                                                              600
          DO 12 I=1,ND                                                        610
          E(I)=(OBS(I,J)-PI(I))*XS(I)                                         620
          ZED=ABS(E(I))                                                       630
          IF(ZED.GT.REJ)GO TO 22                                              64
   12 ER=ER+E(I)*E(I)                                                         650
          SIGMA=SIGMA+ER                                                      660
          SD=SD+ND                                                            670
          NAP=NAP+1
C TEST IF PRINTING OF RESIDUALS IS REQUIRED
          IF((PC.EQ.1.AND.IPLOT.NE.1).OR.IT.EQ.0) WRITE( 2,93)(E(I),I=1,ND)
C SUM OVER INSTRUMENTS ('DO 22' LOOP SUMS OVER TIMES)
          DO 21 L=1,ND
C FOR EACH ROW....
          DO  20 I=1,P
C  ANDFOR EACH COLUMN TO LEFT OF DIAGONAL......
          DO 19 J=1,I
          KD=(I-1)*P+J
          KDT=(J-1)*P+I
          PSI(KD),PSI(KDT)=PSI(KD)+FY(L,I)*FY(L,J)*XS(L)**2
   19 CONTINUE
          CWD(I)=FY(L,I)*XS(L)*E(L) + CWD(I)
          DPI(L,I)=DPI(L,I)+FY(L,I)**2
   20 CONTINUE
   21 CONTINUE                                                                930
   22 CONTINUE                                                                940
          IF((PC.NE.1.AND.IT.NE.0).OR.NAP.EQ.0)GOTO 16
          DO 40 L=1,ND
          DO 40 I=1,P
   40 DPI(L,I)=ABS(X(I) *SQRT(DPI(L,I)/NAP))
          WRITE(2,1012) NAP,MD,SD
 1012 FORMAT(1H0/1H ,I3,' OUT OF ',I3, ' POINTS ACCEPTED',5X,'DEGREES OF
     1 FREEDOM ',I3  /
     2        'ORMS  SENSITIVITY MATRIX'//1H ,12X,4HBETA,7X,1HP,9X,1HR,
     1         9X,2HAY/)
          DO 41 I=1,P
   41 WRITE(2,1013) ORDER(I),(DPI(L,I),L=1,ND)
 1013 FORMAT(1H ,A8,4(2X,F8.5))
C  FINISH IF FINAL PRINT OUT                                                  950
          IF(PC.EQ.1)GO TO 28                                                 960
C  CHECK FOR SUFFICIENT DATA ACCEPTED                                         970
   16 IF(P.LT.SD)GO TO 80
          WRITE(2,1011)SD
 1011 FORMAT('0 NOT ENOUGH DATA LEFT     SD=',I5)
          IF(IPLOT.EQ.0)GO TO 28                                        PL    23
          IPLOT=0                                                       PL    24
          CALL HGPSYMBL(0.0,0.0,1.0,TITLE,270.0,20)                     PL    25
          CALL HGPSYMBL(0.0,-18.5,1.0,20HNOT ENOUGH DATA LEFT,270.0,20) PL    26
          CALL HGPLOT(0.0,0.0,3,0)                                      PL    27
          CALL HGPLOT(-1.0,0.0,0,4)                                     PL    28
```

```
      GO TO 28
   80 CONTINUE
C  RESET DATA REJECTION LEVEL                                            990
      REJ=SQRT(16*SIGMA/SD)                                             1000
C  SOLVE NORMAL EQUATIONS                                               1010
      CALL SOLNORM(P,SIGMA,SD)                                          1020
C  INCREASE ITERATION COUNTER                                           1030
      IT=IT+1                                                           1040
C  IMPROVE PARAMETER VALUES                                             1050
      DO 25 I=1,P                                                       1060
   25 X(I)=X(I)+E(I)                                                    1070
      DO 30 I=1,22
      K=JP(I)
   30 XX(I)=X(K)
C  PRINT SIGMA AND NO OF DEG. FREEDOM                                   1080
      WRITE(2,94)SIGMA,SD                                               1100
C  CHECK FOR MAX NO OF ITERATIONS                                       1110
      IF(ITM.GT.IT)GO TO 26                                             1120
      PC=1                                                              1130
      GO TO 5                                                           1140
C  CHECK PARAMETER CHANGES                                              1150
   26 DO 27 I=1,22                                                      1160
      IF(JP(I).GT.P)GO TO 27
      ER=ACF*XE(I)
      IF(ABS(E(JP(I))).GT.ER) GO TO 5
   27 CONTINUE
      WRITE(2,1010)
 1010 FORMAT('0 CHANGES FOR FOLLOWING PARAMETERS ALL SMALL')
      PC=1                                                              1190
      GO TO 5                                                           1200
C IS THERE ANY PLOTTING WANTED....                                PL      30
   28 IF(IPLOT.EQ.0)GO TO 98                                      PL      40
      DO 1201 INS=1,ND                                            PL      50
      DO 1200 II=1,MD                                             PL      60
      FAF(II)=OBS(INS,II)                                         PL      70
 1200 FAF(II+MD)=FIT(INS,II)                                      PL      80
      CALL HGPLOT(0.0,22.0,0,4)                                   PL      90
      CALL AXISCALE(FAF,2*MD-1,XMIN,DX,INS)                       PL     100
      CALL PLOT(TD(1),FAF(1),TD(2)     ,FAF(MD+1),MD,0.25,0.25)   PL     110
      IF(MOD(INS,2).EQ.0)CALL HGPLOT(-TMOVE,-44.0,0,4)            PL     120
 1201 CALL HGPLOT(0.0,0.0,3,0)                                    PL     130
      IF(MOD(ND,2).EQ.1)CALL HGPLOT(-TMOVE,-22.0,0,4)             PL     140
      CALL HGPLOT(0.0,0.0,3,0)                                    PL     150
   98 IF(LC.LT.2)GOTO1                                            PL     160
   99 IF(JPLOT.EQ.0)STOP                                          PL     170
      CALL HGPLOT(0.0,0.0,0,2)                                    PL     180
      CALL HGPTAPE(2,12H            ,0,0,0)                       PL     190
      STOP
   90 FORMAT(1H ,8X,8A8/7H VARI. ,8F8.3/7H DELTA ,8F8.3//1H ,8X,7A8/
     17H VARI. ,7F8.3/7H DELTA ,7F8.3//1H ,8X,7A8/7H VARI. ,7F8.3/
     27H DELTA ,7F8.3//)
   92 FORMAT(1H0,1X,7HT(SECS),6X,1HW,6X,1HQ,4X,2HXI,4X,4HZETA//         1260
     11H ,12X,3HPHI,3X,4HBETA,6X,1HP,6X,1HR,5X,2HJY,5X,5HRBETA          1261
     2,5X,2HRP,5X,2HRR,4X,3HRJY)                                        1262
   93 FORMAT(1H+,45X,5F7.3)                                             1270
   94 FORMAT(9H0SIGMA = ,F6.4,22H DEGREES OF FREEDOM = ,I3//)           1280
      END                                                               1490

END OF SEGMENT, LENGTH  1047, NAME  A25DPLOT2
```

```
      SUBROUTINE DATAREAD                                                 1500
      INTEGER P,PF,PM,PC,SD,Q                                             1501
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),     2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(9),YIN(5)
     3,ITD(100)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY     5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                                6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,      7
     3N1,PC,AC,PB,DTM,YIN,T0,XA,TIME,OBS,XS/                                 8
     4MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL   1
     1 TAXIS,TMOVE                                                     PL    2
C     READ ENTRY LABEL   (FROM COLUMN 1)                                  1560
    1 READ(1,80)L                                                         1570
   80 FORMAT(I1)                                                          1580
      IF(L.EQ.0)GO TO 1                                                   1590
C     READ DATA TITLE    (FROM   COL.2 OF NEXT CARD)                      1600
      READ(1,81)TITLE                                                  PL  200
   81 FORMAT(10A8)                                                     PL  210
      WRITE(2,81)TITLE                                                 PL  220
    2 GO TO (1,2,3,4,5,6,7,8,9),L                                         1640
C     READ ACCELEROMETER POSITIONS                                        1660
    3 READ(1,84)(AC(I),I=1,9)                                             1661
   84 FORMAT(3F10.3)                                                      1662
      WRITE(2,85)(AC(I),I=1,9)                                            1663
   85 FORMAT(55H0 X.AY     Y.AY      Z.AY      X.AZ     Y.AZ     Z.AZ     ,  1664
     123H X.PB    Y.PB     Z.PB/9(F7.3,2X))                               1665
C     READ MODEL CONSTANTS                                                1670
      READ(1,86)(SP(I),I=1,8)                                             1671
   86 FORMAT(8F10.3)                                                      1672
      WRITE(2,87)(SP(I),I=1,8)                                            1673
   87 FORMAT(5H0  IX,11X,2HIY,10X,2HIZ,9X,3HIXZ,9X,1HM,9X,1HS,5X,2HL1,    1674
     15X,2HL2/3F12.1,3F10.1,2F7,2)                                        1675
      BX=(SP(2)-SP(3))/SP(1)                                              1680
      BY=(SP(3)-SP(1))/SP(2)                                              1681
      BZ=(SP(1)-SP(2))/SP(3)                                              1682
      EX=SP(4)/SP(1)                                                      1683
      EY=SP(4)/SP(2)                                                      1684
      EZ=SP(4)/SP(3)                                                      1685
C     READ PARAMETER ACCURACIES                                           1710
      READ(1,88)(XE(I),I=1,22)                                            1711
   88 FORMAT(8F10.3)                                                      1712
C     READ AND PRINT NUMBER OF INSTRUMENTS                                1720
    4 READ(1,89)ND                                                        1721
   89 FORMAT(I2)                                                          1722
      WRITE(2,90)ND                                                       1723
   90 FORMAT(25H0NUMBER OF INSTRUMENTS = ,I1)                             1724
    5 READ(1,93)T0,YIN,DTM
      FIRSTIME=IFIX(T0)                                                PL  230
C     READ INITIAL CONDITIONS ETC.                                        1740
   93 FORMAT(8F10.4/7F10.4/2F10.4)
      WRITE(2,94)T0,YIN,DTM                                               1743
   94 FORMAT(55H0       T0        VT       THETA       RHO        G      ,  1744
     14H PHI/F10.3,F10.1,F10.3,F10.7,F10.2,F10.3//14H0   JZ.TRIM    ,     1745
     2 'JY.TRIM    U.TRIM    Q.TRIM    V.TRIM    P.TRIM    R.TRIM  XSI.
     3TRIM ZETA.TRIM' /9F10.3)
C     READ MODEL PARAMETERS                                               1760
      READ(1,96)(XX(I),I=1,22)                                            1751
   96 FORMAT(8F10.4)                                                      1752
 5555 READ(1,105)(JP(I),I=1,22)                                           1753
  105 FORMAT(22I0)
C     [illegible]
```

```
C   EVALUATE MULTIPLIERS AF                                               1760
      DO 40 I=1,3                                                         1761
   40 XM(I)=1.0                                                           1762
      RHOV=YIN(3)*YIN(1)                                                  1763
      XM(4)=RHOV*SP(6)/SP(5)                                              1764
      XM(9)=RHOV*SP(6)*SP(8)/SP(1)                                        1765
      XM(14)=RHOV*SP(6)*SP(8)/SP(3)                                       1766
      DO 41 I=4,14,5                                                      1767
      XM(I+1)=XM(I)*SP(8)                                                 1768
      XM(I+2)=XM(I+1)                                                     1769
      XM(I+3)=XM(I)*YIN(1)                                                1770
   41 XM(I+4)=XM(I+3)                                                     1771
C   READ PROGRAM CONTROL PARAMETERS                                       1770
    6 READ(1,97)REJ,ACF,HMAX,P,PF,IT,ITM,LC
   97 FORMAT(3F10.4/5I0)
      WRITE(2,98)REJ,ACF,HMAX,P,PF,IT,ITM                                 1773
   98 FORMAT(8HO  E MAX,6X,8HACCURACY,3X,7HDELTA T/3F10.4//28H INITIAL N  1774
     1O OF PARAMETERS = ,I2,2X,11HFINAL NO = ,I2//17H INITIAL ITRN. = ,   1775
     2I2,9X,16HITRN. MAXIMUM = ,I2)                                       1776
C CALCULATE N AND NF....
      N=3*P+5
      NF=PF*3+5
      DO 52 IE=19,22
      IF(JP(IE).LE.P)N=N-3
   52 IF(JP(IE).LE.PF)NF=NF-3
C   READ SCALING FACTORS                                                  1780
    7 READ(1,99)(XS(I),I=1,ND)                                            1781
   99 FORMAT(7F10.2)                                                      1782
      WRITE(2,100)(XS(I),I=1,ND)                                         1830
  100 FORMAT(11HOSCALING OF/7X,4HBETA,7X,1HP,9X,1HR,9X,2HJY/              1784
     14F10.2//)
C IS THERE ANY PLOTTING NEEDED....                                     PL  240
      READ(1,401)L,IPLOT                                               PL  250
  401 FORMAT(2I1)                                                      PL  255
      IF(IPLOT.EQ.0)GOTO 2                                             PL  260
      IF(JPLOT.NE.0)GOTO 400                                           PL  270
C BEFORE FIRST PLOT OPEN TAPE...                                       PL  280
      CALL HGPTAPE(0,12HFLT DATA FIT,0,0,7)                            PL  290
      CALL HGPTAPE(1,12H            ,0,0,0)                            PL  300
      CALL HGPLOT(0.0,0.0,16,1)                                        PL  310
      CALL HGPLOT( 0.0,2.0,0,4)                                        PL  320
      CALL HGPLOT(0.0,0.0,3,0)                                         PL  330
      JPLOT=1                                                          PL  340
C BEFORE EACH SET OF DATA PLOTTED START NEW PICTURE...                 PL  350
  400 CALL HGPTAPE(1,12H            ,0,0,0)                            PL  360
      CALL HGPLOT(-4.0,0.0,0,4)                                        PL  370
      CALL HGPLOT(0.0,0.0,3,0)                                         PL  380
      GO TO 2                                                             1792
C
C READ DATA                                                               1800
    8 READ(1,102)MD,(ITD(I),(OBS(K,I),K=1,9),I=1,MD)
  102 FORMAT(I3/(I3,9F8.3))
      DO 402 I=1,MD                                                    PL  400
   45 TIME(I)=0.1*ITD(I)
  402 TD(I)=(TIME(I)-FIRSTIME)*2.0                                     PL  410
      TAXIS=2.0*IFIX(TIME(MD)-FIRSTIME+0.99)                           PL  420
      TMOVE=AMAX1(32.0,TAXIS+5.0)                                      PL  430
C
      PM=12                                                               1810
C
C   ENTRY POINT FOR REPEAT WITH SAME DATA                                 1820
    9 DO 10 I=1,22                                                        1830
      K=JP(I)                                                             1833
      X(K)=XX(I)                                                          1836
   10 CWD(K)=XE(I)                                                        1840
      DO 11 I=23,25
   11 CWD(I)=0.0
      IF(P.NE.PF)MD=MD/2                                                  1850
      RETURN                                                              1860
      END                                                                 1870

END OF SEGMENT, LENGTH   735, NAME  DATAREAD
```

```
      SUBROUTINE F4DERV
      INTEGER P,PF,PM,PC,SD,Q                                           3001
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),   2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(9),YIN(5)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY   5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                              6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,    7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,TIME,OBS,XS/                               8
     4MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP
C  RESPONSE EQNS.                                                       3100
      D=(Y(1)-TIME(Q-1))/(TIME(Q)-TIME(Q-1))                            3110
      DY(2)=Y(4)+DTM(6)                                                 3120
      SPHI=SIN(Y(2))                                                    3122
C FOR VARYING ANGLE OF ATTACK.....
      W=YIN(1)*SIN(((OBS(9,Q)-OBS(9,Q-1))*D+OBS(9,Q-1))/57.3)
      TQ=DTM(4)                                                         3126
      B= OBS(8,Q)/57.3 -DTM(8)
      C=0.0                                                             3132
      DO 20 I=3,5                                                       3134
      J=5*I                                                             3136
   20 DY(I)=XP(J-11)*Y(3)+XP(J-10)*Y(4)+XP(J-9)*Y(5)+XP(J-8)*B+XP(J-7)  3138
     1*C                                                                3139
      DY(3)=DY(3)-U*(Y(5)+DTM(7))+W*(Y(4)+DTM(6))+32.2*DTM(2)+G*SPHI    3140
      ROL=DY(4)+BX*TQ*Y(5)+EX*TQ*Y(4)                                   3142
      YAW=DY(5)+BZ*TQ*Y(4)-EZ*TQ*Y(5)                                   3145
      EXZ=1.0-EX*EZ                                                     3146
      DY(4)=(ROL+EX*YAW)/EXZ                                            3147
      DY(5)=(YAW+EZ*ROL)/EXZ                                            3148
C  DIFF.EQNS. OF PARTIAL DERIVATIVES OF V,P,AND R.                      3160
      DO 70 I=1,3
      DO 71 J=1,22
   71 Z(I,J)=0.0
   70 CONTINUE
      JT=0
      N2=N1-2
      DO 90 K=6,N2,3
      J=(K-3)/3+JT
   83 JJ=1
   81 IF(JP(JJ).EQ.J)GO TO 80
      JJ=JJ+1
      GO TO 81
   80 IF(JJ.GT.18)GO TO 82
      DO 84 I=1,3
   84 Z(I,JJ)=Y(K+I-1)
      GO TO 90
   82 JT=JT+1
      J=J+1
      GO TO 83
   90 CONTINUE
C   BASIC PART. DIFF.                                                   3180
      DO 10 J=1,18                                                      3190
      ZD(1,J)=XP(4)*Z(1,J)+XP(5)*Z(2,J)+XP(6)*Z(3,J)-U*Z(3,J)+W*Z(2,J)
      ZD(2,J)=XP(9)*Z(1,J)+XP(10)*Z(2,J)+XP(11)*Z(3,J)+BX*TQ*Z(3,J)     3210
     1+EX*TQ*Z(2,J)                                                     3220
   10 ZD(3,J)=XP(14)*Z(1,J)+XP(15)*Z(2,J)+XP(16)*Z(3,J)+BZ*TQ*Z(2,J)    3230
     1-EZ*TQ*Z(3,J)
C   ADD CONTRIBUTIONS FROM DERIV.                                       3250
      DO 30 I=1,3                                                       3260
      L=5*I                                                             3270
      ZD(I,L-1)=ZD(I,L-1)+XM(L-1)*Y(3)
      ZD(I,L)=ZD(I,L)+XM(L)*Y(4)                                        3290
```

```
      ZD(I,L+1)=ZD(I,L+1)+XM(L+1)*Y(5)
      ZD(I,L+2)=ZD(I,L+2)+XM(L+2)*B
   30 ZD(I,L+3)=ZD(I,L+3)+XM(L+3)*C
C   CORRECT FOR IXZ
      DO 31 J=1,18
      Z2=ZD(2,J)
      Z3=ZD(3,J)
      ZD(2,J)=(Z2+EX*Z3)/EXZ
   31 ZD(3,J)=(Z3+EZ*Z2)/EXZ
C   SPURIOUS VALUES FOR D/D(EB) ETC
      DO 32 I=1,3
      DO 33 J=19,22
      Z(I,J)=0.0
   33 ZD(I,J)=0.0
   32 CONTINUE
C   REARRANGE FOR DY AND Y
      JT=0
      N2=N1-2
      DO 60 K=6,N2,3
      J=(K-3)/3+JT
   53 JJ=1
   51 IF(JP(JJ).EQ.J)GO TO 50
      JJ=JJ+1
      GO TO 51
   50 IF(JJ.GT.18)GO TO 52
      DO 61 I=1,3
      Y(K+I-1)=Z(I,JJ)
   61 DY(K+I-1)=ZD(I,JJ)
      GO TO 60
   52 JT=JT+1
      J=J+1
      GO TO 53
   60 CONTINUE
      RETURN
      END

END OF SEGMENT, LENGTH   808, NAME  F4DERY
```

```
      SUBROUTINE PATHMODEL                                               2500
      INTEGER P,PF,PM,PC,SD,Q                                            2501
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),    2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(9),YIN(5)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY    5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                               6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,     7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,TIME,OBS,XS/                                8
     4MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP
      COMMON/GRAPH/FIT(4,100),TD(100),TITLE(10)  ,FIRSTIME,JPLOT,IPLOT, PL   1
     1 TAXIS,TMOVE                                                     PL   2
CC
      IF(Q.EQ.2)H=TIME(MD)-Y(1)                                          2540
    1 ING=0                                                              2550
C  CALCULATE STEP LENGTH                                                 2560
      A=TIME(Q)-Y(1)                                                     2570
      IF(ABS(A-H).LT.0.0001)ING=1                                        2580
      B1=A                                                              2590
      IF(HMAX.GT.A)GO TO 2                                               2600
      B1=HMAX                                                            2610
      IF(ABS(H-HMAX).LT.0.0001)ING=1                                     2612
    2 H=B1                                                               2614
C  INTEGRATE 1 STEP                                                      2620
    3 CALL F4RUNG(N1,ING,H,Y,DY,HQ)                                      2630
      IF(ING.EQ.1)GO TO 4                                                2632
      ING=1                                                              2634
      GO TO 3                                                            2636
C  CALCULATE MODEL VECTOR  (NB. TQ = Q + Q(TRIM), ETC.)                  2640
    4 TP=Y(4)+DTM(6)                                                     2643
      TR=Y(5)+DTM(7)                                                     2644
      C4=TP*TQ+DY(5)                                                     2649
      C5=TP*TP+TR*TR                                                     2650
      C6=TQ*TR-DY(4)                                                     2651
      FI=57.3*Y(2)                                                       2671
      PI(1)=57.3*(Y(3)+DTM(5)+AC(7)*Y(5)-AC(9)*Y(4))/U+XX(19)
      PI(2)=57.3*TP+XX(20)                                               2675
      PI(3)=57.3*TR+XX(21)                                               2677
      PI(4)=      (XP(4)*Y(3)+XP(5)*Y(4)+XP(6)*Y(5)+XP(7)*B+XP(8)*C+     2679
     1C4*AC(1)-C5*AC(2)+C6*AC(3))/YIN(4)+ XX(22)+DTM(2)
    7 IF(A.GT.HMAX)GO TO 20                                              2690
C  CALCULATE PARTIAL DERIVATIVES OF MODEL VECTOR                         2710
    9 DO 11 I=1,ND                                                       2712
      DO 10 J=1,P                                                        2714
   10 FY(I,J)=0.0                                                        2716
   11 CONTINUE                                                           2718
      DO 12 I=1,4
      J=18+I
   12 FY(I,JP(J))=1.0
      K=P                                                                2730
      J=3                                                                2732
      I=1                                                                2734
   13 J=J+3                                                              2736
      C4=TQ*Y(J+1)+DY(J+2)                                               2744
      C5=2*TP*Y(J+1)+2*TR*Y(J+2)                                         2745
      C6=TQ*Y(J+2)-DY(J+1)                                               2746
      GINV=1.0/YIN(4)
      FY(1,I)=57.3*(Y(J)+AC(7)*Y(J+2)-AC(9)*Y(J+1))/U                    2751
      FY(2,I)=57.3*Y(J+1)                                                2753
      FY(3,I)=57.3*Y(J+2)                                                2755
      FY(4,I)=GINV *(XP(4)*Y(J)+XP(5)*Y(J+1)+XP(6)*Y(J+2)+C4*AC(1)-      2757
     1C5*AC(2)+C6*AC(3))                                                 2758
```

```
      101 I=I+1                                                         [illegible]
          IF(I.EQ.JP(19).OR.I.EQ.JP(20).OR.I.EQ.JP(21).OR.I.EQ.JP(22))GO        2748
         1TO 101                                                                2749
          IF(I.LE.K)GO TO 13                                                    2762
          FY(4,JP(4))=FY(4,JP(4))+GINV *XM(4)*Y(3)                              2770
          FY(4,JP(5))=FY(4,JP(5))+GINV *XM(5)*Y(4)                              2772
          FY(4,JP(6))=FY(4,JP(6))+GINV *XM(6)*Y(5)                              2774
          FY(4,JP(7))=FY(4,JP(7))+GINV *XM(7)*B                                 2776
          FY(4,JP(8))=FY(4,JP(8))+GINV *XM(8)*C                                 2778
C    IF REQUIRED PRINT RESULT                                                   2800
       20 IF(IT.EQ.0)GO TO 21                                                   2810
          IF(PC.NE.1)GO TO 30                                                   2820
          IF(IPLOT.EQ.1) GO TO 30
       21 WRITE(2,50)Y(1),W,TQ,B,C,FI,(PI(I),I=1,4)                             2830
       50 FORMAT(1H0,F8.3,4F7.2/1H ,8X,4F7.2,F7.3)                             2840
       30 IF(A.GE.HMAX)GO TO 1                                                  2940
          IF(IT.EQ.0.CR.IPLOT.EQ.0)GO TO 31                               PL     440
          DO 51 INS=1,ND                                                  PL     450
       51 FIT(INS,Q-1)=PI(INS)                                            PL     460
       31 RETURN                                                                2950
          END                                                                   2960

END OF SEGMENT, LENGTH   759, NAME  MATHMODEL
```

```
      SUBROUTINE INIT                                                   6000
      INTEGER P,PF,PM,PC,SD,Q                                           6001
      DIMENSION PI(10),Y(100),FY(10,25),DY(100),HQ(100),SP(242),CWD(25),   2
     1PSI(625),E(25),TIME(100),OBS(10,100),XS(10),XM(22),XX(22),XE(22),X
     2(22),JP(22),Z(3,22),ZD(3,22),ORDER(22),XP(22),AC(12),DTM(9),YIN(5)
      COMMON/COMP/PI,Y,DY,HQ,SP,A,B,C,D,F,G,H,U,V,W,C1,C2,C3,C4,C5,C6,FY   5
     1,RHO,RHOV,PHI,SG,CG,SPHI,CPHI,TP,TQ,TR/                              6
     2NORME/PSI,CWD,E/DATA/ITM,IT,MD,LC,SD,Q,HMAX,ACF,REJ,P,PF,PM,N,NF,    7
     3N1,PC,AC,PB,DTM,YIN,TO,XA,TIME,OBS,XS/                               8
     4MODEL/XE,X,XX,XM,JP,ORDER,BX,BY,BZ,EX,EY,EZ,ND,XP
    1 IF(P.NE.PF)N1=N                                                   6060
      IF(P.EQ.PF)N1=NF                                                  6070
    2 DO 3 I=1,22                                                       6090
    3 XP(I)=XX(I)                                                       6092
      DO 5 I=4,18                                                       6094
    5 XP(I)=XX(I)*XM(I)                                                 6096
      Y(1)=TO                                                           6110
      Y(2)=YIN(5)                                                       6130
      Y(3)=XX(1)                                                        6132
      Y(4)=XX(2)                                                      6134
      Y(5)=XX(3)                                                        6136
      DO 4 I=6,70                                                       6170
    4 Y(I)=0.0                                                          6180
      Y(6)=1.0                                                          6190
      Y(10)=1.0                                                         6192
      Y(14)=1.0                                                         6194
      U=YIN(1)                                                          6220
      G=YIN(4)*COS(YIN(2))                                              6230
      RETURN                                                            6300
      END                                                               6310

END OF SEGMENT, LENGTH   124, NAME  INIT
```

Table 1

INPUT DATA FOR LONGITUDINAL VERSION

Entry label	Fortran symbol	Format	Item	Units	Comments
1	L	I1			Entry label of next READ instruction
	Title	1X,A79	Data Title		
3	AC(9)	3 cards in 3F10.3	Instrument positions	m	Positions (x,y,z) of normal accelerometer, airspeed indicator and angle of attack vane
	SP(4)	4F10.3	I_y m S ℓ	kgm^2 kg m^2 m	Moment of inertia Mass Reference area Reference length (usually $\bar{c}$) Aircraft constants
	XE(22)	3 cards in 8F10.3			Accuracy level to define convergence for each parameter, listed in fixed order of parameters
4	ND				Number of instruments
5	TO YIN(2) YIN(4) YIN(5)	1 card in (F10.3,10X, F10.3,10X, 2F10.3)	t_0 V ρ g	s m/s kg/m^3 m/s^2	Initial time Trim speed Air density Acceleration due to gravity
	DTM(10)	2 cards 1 in 8F10.3	θ_e w_e q_e u_e Φ_e η_e p_e r_e	rad m/s rad/s m/s rad rad rad/s rad/s	The trim values of:- pitch angle normal velocity (along z axis) pitch rate forward velocity (along x axis) bank angle elevator deflection roll rate yaw rate
		1 in 2F10.3	a_{z_e} α_e	g rad	normal acceleration (-1.0 in steady horizontal, zero α flight) angle of attack
	XX(20)	3 cards in 8F10.3	First guesses		Parameters listed in fixed order
	JP(20)	20I0	Chosen order		
6	REJ ACF HMAX	1 card in 3F10.3			Initial data rejection level Factor on parameter accuracies Maximum integration interval
	P PF IT ITM LC	1 card in 5I0			Initial number of parameters to be identified using half of observations Final number of parameters to be identified using all of observations Initial value of iteration counter Maximum number of iterations Last case indicator
7	XS(4)	4F10.3			Weightings on instrument readings
	L IPLOT	2I1			Next entry label (8 or 9) Indicator for plotting requirements
8	NGAP TF IL	(I2, F8.4, I1)			Spacing of observations points to be fitted ($\leqslant 6$) Final time Paper tape loading indicator
9					Entry label for continuing with same observations
		****	Terminator		Must be final card in deck

Paper tape input

Fortran symbol	Format	Item	Units	Comments
RUNAME MDO	1 line in A5,I4			5 alphanumeric character tape name Number of observed data points on tape
TIME(I) OBS(1,I) OBS(2,I) COBS(I,1,2)	(F6.3, 3F9.5)	t_i q_i a_{z_i} η_i	s rad/s g rad	For each time point:- time of observation pitch rate normal acceleration elevator position NOTE: For this particular aircraft instrumentation only q and a_z observed

Table 2

INPUT DATA FOR LATERAL VERSION

Entry label	Fortran symbol	Format	Item	Units	Comments
1	L	I1			Label number of next READ instruction
	TITLE	1X,A79	Data Title		Title starts in column 2, ending at column 80 Columns 2-20 are title put on graphs
3	AC(9)	3 cards 3F10.3	Instrument positions	ft	Positions (x,y,z) of lateral accelerometer, normal accelerometer and sideslip vane
	SP(8)	8F10.3	I_x, I_y		Aircraft constants:-
			I_z, I_{xz}	slug ft^2	Moments and products of inertia
			m	slug	Mass
			S	ft^2	Reference area
			ℓ_1, ℓ_2	ft	Reference lengths
	XE(22)	3 cards 8F10.3			Accuracy level to define convergence for each parameter, listed in fixed order of parameters
4	ND	I2			Number of instruments
5	TO	6F10.4	t_0	s	Initial time
	YIN(5)		V	ft/s	Trim speed
			Θ_e	rad	Trim pitch angle, relative to earth axes
			ρ	slug/ft^3	Air density
			g	ft/s^2	Acceleration due to gravity
			Φ_e	rad	Trim bank angle, relative to earth axes
	DTM(9)	2 cards	a_{z_e}		Normal acceleration Trim values, all relative to geometric body axes
		7F10.4	a_{y_e}		Lateral acceleration
		2F10.4	w_e	ft/s	Normal velocity
			q_e	rad/s	Pitch rate
			v_e	ft/s	Sideslip velocity
			p_e	rad/s	Roll rate
			r_e	rad/s	Yaw rate
			ξ_e	rad	Aileron deflection
			ζ_e	rad	Rudder deflection
	XX(22)	3 cards 8F10.4	First guesses		Parameters listed in fixed order
	JP(22)	22I0	Chosen order		
6	REJ	3F10.3			Initial data rejection level
	ACF				Factor on parameter accuracies
	HMAX				Maximum integration interval
	P	5I0			Initial number of parameters to be identified
	PF				Final number of parameters to be identified
	IT				Initial value of iteration counter
	ITM				Maximum value of iterations
	LC				Last case indicator
7	XS(4)	4F10.2			Weighting factors on sideslip vane, roll rate, and yaw rate gyros and lateral accelerometer
	L	2I1			Next entry label, 8 or 9
	IPLOT				Indicator for plotting requirements
8	MD	I3			Number of data cards to be read
	ITD(I)	MD cards in 9F8.3	Time		Number of time point
	OBS(1,I)		β	deg	Sideslip
	OBS(2,I)		p	deg/s	Roll rate
	OBS(3,I)		r	deg/s	Yaw rate
	OBS(4,I)		a_y	g	Lateral acceleration factor
	OBS(5,I)		$-a_z$	g	Normal acceleration factor
	OBS(6,I)		V	ft/s	Forward speed
	OBS(7,I)		h	ft	Height
	OBS(8,I)		ξ	deg	Aileron angle
	OBS(9,I)		α	deg	Angle of attack

Table 3

ARRAYS USED IN LONGITUDINAL VERSION

I	AC(I)	SP(I)	YIN(I)	DTM(I)	XS(I)	ΦBS(I,J)	CΦBS(J,K,I)
1	x_{a_z}	I_y	-	Θ_e	$W^{\frac{1}{2}}_q$	q	t
2	y_{a_z}	m	V	ω_e	$W^{\frac{1}{2}}_{a_z}$	a_z	η
3	z_{a_z}	S	-	q_e	$W^{\frac{1}{2}}_v$	V_{AS}	
4	x_v	$\bar{c}$	ρ	u_e	$W^{\frac{1}{2}}_\alpha$	α	
5	y_v		g	Φ_e			
6	z_v			η_e			
7	x_α			p_e			
8	y_α			r_e			
9	z_α			a_{z_e}			
10				α_e			

J is count over times at which fitting is required (1 to MD)
K is count over intermediate time points (1 to NGAP)

Parameters in fixed order						
I	XX(I)	XP(I)	XM(I)	Z(1,I)	Z(2,I)	Z(3,I)
1	θ_0	θ_0	1	$\partial w/\partial\theta_0$	$\partial q/\partial\theta_0$	$\partial u/\partial\theta_0$
2	w_0	w_0	1	$\partial w/\partial w_0$	$\partial q/\partial w_0$	$\partial u/\partial w_0$
3	q_0	q_0	1	$\partial w/\partial q_0$	$\partial q/\partial q_0$	$\partial u/\partial q_0$
4	u_0	u_0	1	$\partial w/\partial u_0$	.	.
5	$\check{X}_w$	$-\overset{\circ}{x}_w$	$\frac{1}{2}\rho VS/m$	$\partial w/\partial\check{X}$	.	.
6	$\check{X}_q$	$-\overset{\circ}{x}_q$	$\frac{1}{2}\rho VS\bar{c}/m$	$\partial w/\partial\check{X}_q$	.	.
7	$\check{X}_u$	$-\overset{\circ}{x}_u$	$\frac{1}{2}\rho VS/m$	$\partial w/\partial\check{X}_u$	etc.	
8	$\check{X}_\eta$	$-\overset{\circ}{x}_\eta$	$\frac{1}{2}\rho V^2S/m$	$\partial w/\partial\check{X}_\eta$		
9	$\check{Z}_w$	$-\overset{\circ}{z}_w$	$\frac{1}{2}\rho VS/m$	$\partial w/\partial\check{Z}_w$		
10	$\check{Z}_q$	$-\overset{\circ}{z}_q$	$\frac{1}{2}\rho VS\bar{c}/m$	$\partial w/\partial\check{Z}_q$		
11	$\check{Z}_u$	$-\overset{\circ}{z}_u$	$\frac{1}{2}\rho VS/m$	$\partial w/\partial\check{Z}_u$		
12	$\check{Z}_\eta$	$-\overset{\circ}{z}_\eta$	$\frac{1}{2}\rho V^2S/m$	$\partial w/\partial\check{Z}_\eta$		
13	$\check{M}_w$	$-\overset{\circ}{m}_w$	$\frac{1}{2}\rho VS\bar{c}/I_y$	$\partial w/\partial\check{M}_w$		
14	$\check{M}_q$	$-\overset{\circ}{m}_q$	$\frac{1}{2}\rho VS\bar{c}^2/I_y$	$\partial w/\partial\check{M}_q$		
15	$\check{M}_u$	$-\overset{\circ}{m}_u$	$\frac{1}{2}\rho VS\bar{c}/I_y$	$\partial w/\partial\check{M}_u$		
16	$\check{M}_\eta$	$-\overset{\circ}{m}_\eta$	$\frac{1}{2}\rho V^2S\bar{c}/I_y$	$\partial w/\partial\check{M}_\eta$		
17	E_q	E_q	1	0	0	0
18	E_{a_z}	E_{a_z}	1	0	0	0
19	E_v	E_v	1	0	0	0
20	E_α	E_α	1	0	0	0
	*	*				

Also ZD(J,I) = $\frac{d}{dt}$ Z(J,I)

* The notation is that of Ref.5. The marking ˇ denotes a nondimensional (aeronormalised) quantity, and ° for a dimensional quantity. Lower case letters denote concise derivatives, and upper case for non-concise.

Table 3 (concluded)

An example of array arrangements for a particular chosen order in longitudinal program, as used for results in Fig.4:-

JP(I)	XX(I)	I	X(I)	XC(I)
13	θ_0	1	w_0	w_0
1	w_0	2	q_0	q_0
2	q_0	3	$\check{M}_\eta$	$-\mathring{m}_\eta$
12	u_0	4	$\check{M}_w$	$-V\mathring{m}_w$
16	$\check{X}_w$	5	$\check{Z}_w$	$-\mathring{z}_w$
17	$\check{X}_q$	6	$\check{M}_q$	$-\mathring{m}_q$
11	$\check{X}_u$	7	E_q	E_q
18	$\check{X}_\eta$	8	E_{a_z}	E_{a_z}
5	$\check{Z}_w$	9	$\check{Z}_q$	$-\mathring{z}_q$
9	$\check{Z}_q$	10	$\check{Z}_\eta$	$-\mathring{z}_\eta$
15	$\check{Z}_u$	11	$\check{X}_u$	$-\mathring{x}_u$
10	$\check{Z}_\eta$	12	u_0	u_0
4	$\check{M}_w$	13	θ_0	θ_0
6	$\check{M}_q$	14	$\check{M}_u$	$-V\mathring{m}_u$
14	$\check{M}_u$	15	$\check{Z}_u$	$-\mathring{z}_u$
3	$\check{M}_\eta$	16	$\check{X}_w$	$-\mathring{x}_w$
7	E_q	17	$\check{X}_q$	$-\mathring{x}_q$
8	E_{a_z}	18	$\check{X}_\eta$	$-\mathring{x}_\eta$
19	E_v	19	E_v	E_v
20	E_α	20	E_α	E_α
			*	*
	↑ Fixed order		↑ Chosen order	

* X(I) and XC(I) are labelled 'non-dimensional' and 'avionic' respectively in output.

** See Fig.4.

	Y(I)
1	t
2	$\theta(t)$
3	$w(t)$
4	$q(t)$
5	$u(t)$
6	$\eta(t)$
7	$\partial w/\partial w_0$
8	$\partial q/\partial w_0$
9	$\partial u/\partial w_0$
10	$\partial w/\partial q_0$
11	$\partial q/\partial q_0$
12	$\partial u/\partial q_0$
13	$\partial w/\partial \check{M}_\eta$
14	$\partial q/\partial \check{M}_\eta$
15	$\partial u/\partial \check{M}_\eta$
16	$\partial w/\partial \check{M}_w$
17	$\partial q/\partial \check{M}_w$
18	$\partial u/\partial \check{M}_w$
19	$\partial w/\partial \check{Z}_w$
20	$\partial q/\partial \check{Z}_w$
21	$\partial u/\partial \check{Z}_w$
22	$\partial w/\partial \check{M}_q$
23	$\partial q/\partial \check{M}_q$
24	$\partial u/\partial \check{M}_q$
25	$\partial w/\partial \check{Z}_q$
26	$\partial q/\partial \check{Z}_q$
27	$\partial u/\partial \check{Z}_q$
28	$\partial w/\partial \check{Z}_\eta$
29	$\partial q/\partial \check{Z}_\eta$
30	$\partial u/\partial \check{Z}_\eta$
10 parameters varying**	

also DY(I) $= \frac{d}{dt}$ Y(I)

Table 4

ARRAYS USED IN LATERAL VERSION

I	AC(I)	SP(I)	YIN(I)	DTM(I)	XS(I)	OBS(I,J)
1	x_{ay}	I_x	V	a_{z_e}	$\sqrt{W}_\beta$	β
2	y_{ay}	I_y	Θ_e	a_{y_e}	$\sqrt{W}_p$	p
3	z_{ay}	I_z	ρ	w_e	$\sqrt{W}_r$	r
4	x_{az}	I_{xz}	g	q_e	$\sqrt{W}_{a_y}$	a_y
5	y_{az}	m	Φ_e	v_e		$-a_z$
6	z_{az}	S		p_e		V
7	x_{probe}	$\ell_1 = \ell_T$		r_e		h
8	y_{probe}	$\ell_2 = S$		ξ_e		ξ
9	z_{probe}			ζ_e		α

Parameters in fixed order						
I	XX(I)	XP(I)	XM(I)	Z(1,I)	Z(2,I)	Z(3,I)
1	v_0	v_0	1	$\partial v/\partial v_0$	$\partial p/\partial v_0$	$\partial r/\partial v_0$
2	p_0	p_0	1	$\partial v/\partial p_0$	$\partial p/\partial p_0$	$\partial r/\partial p_0$
3	r_0	r_0	1	$\partial v/\partial r_0$	$\partial p/\partial r_0$	$\partial r/\partial r_0$
4	y_v	$-\mathring{y}_v$	VS/m	$\partial v/\partial y_v$	.	.
5	y_p	$-\mathring{y}_p$	VSs/m	$\partial v/\partial y_p$	.	.
6	y_r	$-\mathring{y}_r$	VSs/m	$\partial v/\partial y_r$	.	.
7	y_ξ	$-\mathring{y}_\xi$	V^2S/m	$\partial v/\partial y_\xi$	.	.
8	y_η	$-\mathring{y}_\zeta$	V^2S/m	$\partial v/\partial y_\zeta$	.	.
9	ℓ_v	$-\mathring{\ell}_v$	VSs/I_x	$\partial v/\partial \ell_v$	.	.
10	ℓ_p	$-\mathring{\ell}_p$	VSs^2/I_x	$\partial v/\partial \ell_p$	.	.
11	ℓ_r	$-\mathring{\ell}_r$	VSs^2/I_x	$\partial v/\partial \ell_r$	.	.
12	ℓ_ξ	$-\mathring{\ell}_\xi$	V^2Ss/I_x	$\partial v/\partial \ell_\xi$	.	.
13	ℓ_η	$-\mathring{\ell}_\zeta$	V^2Ss/I_x	$\partial v/\partial \ell_\zeta$	.	.
14	n_v	$-\mathring{n}_v$	VSs/I_z	$\partial v/\partial n_v$	.	.
15	n_p	$-\mathring{n}_p$	VSs^2/I_z	$\partial v/\partial h_p$	.	.
16	n_r	$-\mathring{n}_r$	VSs^2/I_z	$\partial v/\partial n_r$	.	.
17	n_ξ	$-\mathring{n}_\xi$	V^2Ss/I_z	$\partial v/\partial n_\xi$	.	.
18	n_ζ	$-\mathring{n}_\zeta$	V^2Ss/I_a	$\partial v/\partial n_\zeta$	$\partial p/\partial n_\zeta$	$\partial r/\partial n_\zeta$
19	E_β	E_β	V^2Ss/I_z	0	0	0
20	E_p	E_p	V^2Ss/I_z	0	0	0
21	E_r	E_r	V^2Ss/I_z	0	0	0
22	E_{a_y}	E_{a_y}	V^2Ss/I_z	0	0	0
	*	*				

Also $ZD(J,I) = \frac{d}{dt} Z(J,I)$

* Note on notation, XX(I) and X(I) (see Table 4 (cont.)) store aerodynamic derivatives non-dimensionalised as in Ref.8, and so that notation is used. XP(I) stores concise dimensional derivatives, and so the notation of Ref.7 is convenient.

Table 4 (concluded)

An example of array arrangement for a particular chosen order in lateral program, as used for results of Fig.8:-

JP(I)	XX(I)	I	X(I)
1	v_0	1	v_0
2	p_0	2	p_0
3	r_0	3	r_0
12	y_v	4	ℓ_v
17	y_p	5	n_v
18	y_r	6	E_p
19	y_ξ	7	E_r
20	y_ζ	8	n_r
4	ℓ_v	9	ℓ_p
9	ℓ_p	10	E_β
15	ℓ_r	11	E_{a_y}
13	ℓ_ξ	12	y_v
21	ℓ_ζ	13	ℓ_ξ
5	n_v	14	n_ξ
16	n_p	15	ℓ_r
8	n_r	16	n_p
14	n_ξ	17	y_p
22	n_ζ	18	y_r
10	E_β	19	y_ξ
6	E_p	20	y_ζ
7	E_r	21	ℓ_ζ
11	E_{a_y}	22	n_ζ
	↑ Fixed order		↑ Chosen order

Y(I)			
1	t	21	$\partial v/\partial n_r$
2	$\phi(t)$	22	$\partial p/\partial n_r$
3	$v(t)$	23	$\partial r/\partial n_r$
4	$p(t)$	24	$\partial v/\partial \ell_p$
5	$r(t)$	25	$\partial p/\partial \ell_p$
6	$\partial v/\partial v_0$	26	$\partial r/\partial \ell_p$
7	$\partial p/\partial v_0$	27	$\partial v/\partial y_v$
8	$\partial r/\partial v_0$	28	$\partial p/\partial y_r$
9	$\partial v/\partial p_0$	29	$\partial r/\partial y_v$
10	$\partial p/\partial p_0$	30	$\partial v/\partial \ell_\xi$
11	$\partial r/\partial p_0$	31	$\partial p/\partial \ell_\xi$
12	$\partial v/\partial r_0$	32	$\partial r/\partial \ell_\xi$
13	$\partial p/\partial r_0$	Added for continuation with 13 parameters varying*	
14	$\partial r/\partial r_0$		
15	$\partial v/\partial \ell_v$		
16	$\partial p/\partial \ell_v$		
17	$\partial r/\partial \ell_v$		
18	$\partial v/\partial n_v$		
19	$\partial p/\partial n_v$		
20	$\partial r/\partial n_v$		
7 parameters varying*			

also $DY(I) = \frac{d}{dt} Y(I)$.

* See Fig.8.

Table 5

FORTRAN SYMBOL USED IN COMPUTATIONS

COMMON BLOCK COMP (must contain 828 variables, as it is overwritten by SOLNORM)

Y(100)	variables in F4RUNG Subroutine	
DY(100)	derivatives of Y(I) with respect to time	
HQ(100)	computing space in F4RUNG	
PI(10)	model vector of calculated instrument readings	
FY(10,25)	partial derivatives matrix	
SP(242)	spare computing space, 242 balances space in SOLNORM	
H	step length for integration	
U	mean trim speed	
C1,C2,C3,C4,C5,C6	computing space	lateral program only
A	check on step length	
B	perturbation of aileron angle, ξ	
C	perturbation of rudder angle, ζ	
G	component of acceleration due to gravity, $g \cos \Theta_e$	
W	observed normal speed	
RHO	air density, ρ	
RHOV	ρU	
PHI	bank angle, Φ	
SPHI	$\sin \Phi$	
TP	total roll rate	
TQ	total pitch rate	
TR	total yaw rate	
CPHI	$\cos \Phi$	Longitudinal program only
D,F,V,SG,CG	computing space not used in current programs	

COMMON BLOCK GRAPH

FIT(4,100)	calculated instrument readings
TD(100)	times, scaled for plotting (2cm to 1s)
TITLE(10)	contents of DATA TITLE in alphanumeric code
FIRSTIME	time at which axis is to start
TAXIS	length of time axis (cm)
TMOVE	determines length of box enclosing graphs

The symbols introduced in the other COMMON BLOCKS are either defined in Tables 1 to 4, or are self-explanatory.

SYMBOLS

AXES: A geometric-body system of axes is used (with x-axis forward and z-axis down), corresponding to the alignments of the accelerometers and rate gyros, with origin at the cg of the aircraft.

Units: Dimensional quantities may be expressed in SI or Imperial units, provided that the data is consistent. Angles are expressed in radians, unless otherwise stated.

a_x, a_y, a_z	forward, lateral and normal accelerations
b	wing span
b_x, b_y, b_z	$(I_y - I_z)/I_x$, $(I_z - I_x)/I_y$, $(I_x - I_y)/I_z$
$\bar{c}$	mean chord
C	$[f_{ki}]$, an $mn \times p$ matrix
C_L,C_D,C_N	coefficients of lift, drag, normal force
D	$[R_{\pi i}]$, a column vector of mn rows
E	$[\delta x_k]$, a column vector of p rows
E_p, E_r etc.	off-set errors in instruments
e_x, e_y, e_z	I_{xz}/I_x, I_{xz}/I_y, I_{xz}/I_z
f_{ki}	$\partial\Upsilon(\underline{x},t_i)/\partial x_k$, the partial derivative of the calculated instrument readings vector at time t_i , with respect to the parameter x_k
g_j	time rate of change of state variable, y_j , as a function of parameters, state and time
g	acceleration due to gravity
I_x, I_y, I_z	moments of inertia
I_{xz}	product of inertia
L_v, L_p etc	rolling moment derivative due to sideslip, roll rate etc.
ℓ_1, ℓ_2	characteristic lengths of aircraft
M_w, M_q etc.	pitching moment derivative due to rate of heave, pitch rate etc.
m	aircraft mass
m	number of data points (section 2)
N_v, N_p etc.	yawing moment derivative due to rate of sideslip, roll rate, etc.
n	number of instruments (section 2)
p	roll rate
p	number of parameters (section 2)
q	pitch rate
r	yaw rate
r	number of state variables (section 2)

SYMBOLS (continued)

R_π	residual errors
s	semi-span
S	reference area (wing area)
S_k	standard deviation of k'th parameter
t_i	i'th time point in data
t_0	time at which analysis starts
u	forward speed (speed along x-axis)
v	sideslip speed (speed along y-axis)
V_e	aircraft speed in trim condition
w	heaving speed (speed along z-axis)
W	instrument weighting matrix
x_1,y_1,z_1 etc.	instrument positions
$x_1,\ldots,x_p$	parameters
X_u, X_w etc.	longitudinal force derivative due to forward velocity, heaving etc.
$y_1,\ldots,y_r$	state variables
Y_v,Y_p etc.	side force derivative due to sideslip, roll rate etc.
Z_w,Z_q etc.	normal force derivative due to heaving, pitching etc.
α	angle of attack
β	angle of sideslip
δx_k	correction to k'th parameter
ζ	rudder angle
η	elevator angle
θ	pitch angle
ξ	aileron angle
π_i	vector of instrument readings at time t_i
π_{ic}	vector of calculated instrument readings at time t_i
ρ	air density
σ	rms of the residuals of the observations
Υ	complete set of relations required to calculate π_{ic}
ϕ	bank angle
Φ_e	trim bank angle
Ψ	C*WC (section 2)
Ω_e	trim rate of rotation about vertical axis

SYMBOLS (concluded)

Suffices etc.	these have been applied to the arbitrary letter X so as to indicate their positions
X_e	trim values
$\overset{\circ}{X}$	dimensional quantity
X_0	value at initial time
X_a	relative to flight path axes
$\check{X}$	aeronormalised quantity
$\dot{X}$	derivative with respect to time
X^*	transpose of matrix

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4	A.P. Waterfall	Fortran programs for the automatic digital analysis of free-flight dynamic response data. Unpublished MOD(PE) material (1971)
5	V. Klein	Parameter identification applied to aircraft. Cranfield Institute of Technology, CIT-FI-73-018 (1973)
6		Scientific Subroutines Manual. ICL 1900 Series 4096
7	H.R. Hopkin	A scheme of notation and nomenclature for aircraft dynamics and associated aerodynamics. ARC R & M 3562 (1966)
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10	D. Kimberley P.W.J. Fulham	Flight control system development in the United Kingdom. Unpublished MOD(PE) material (1973)
11	A. Jean Ross	Determination of aerodynamic derivatives from transient responses in manoeuvring flight. Unpublished MOD(PE) material (1974)

REFERENCES (concluded)

No.	Author	Title, etc.
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13		USAF Stability and Control Datcom. McDonnell Douglas Corporation (1970)

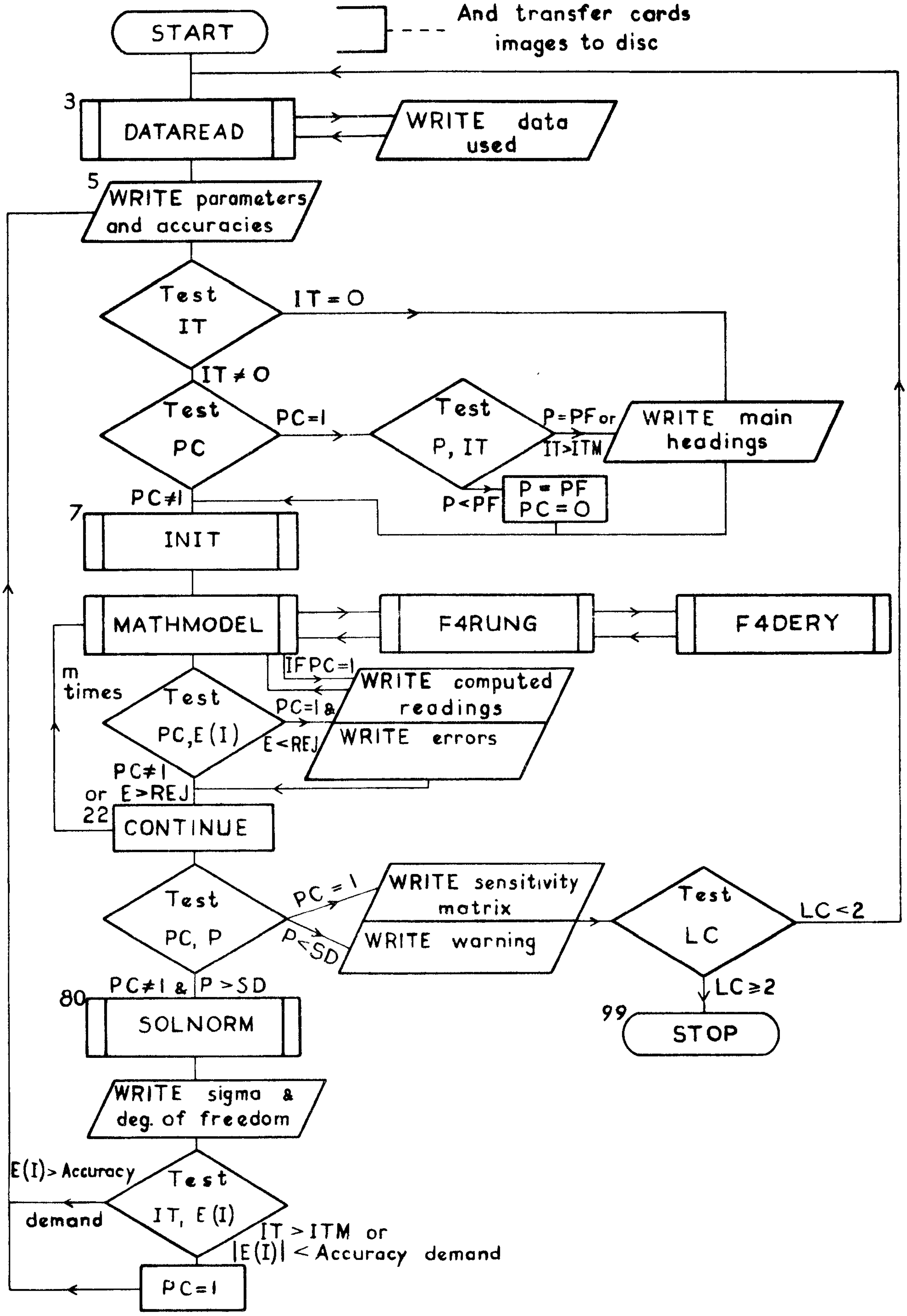

Fig. 1 Flow chart of MASTER segment (omitting graph plotter subroutines)

```
3
 HUNTER AG227                  DEMONSTRATION RUN


51680.0   7937.8    32.423    3.17
0.02      1.0       0.005     1.0       0.001     1.0       0.0       1.0
0.05      0.5       0.1       0.02      0.5       0.1        1.0      0.005
0.005     0.005     1.0       1.0
 2
5.0                 190.0               0.654     9.81
0.0       0.0                 190.0               -0.1185
-1.0

    -5.893  6.527             -0.237    -0.365    -2.834              -0.493
-0.105    -0.493
13 1 2 12 16 17 11 18 5 9 15 10 4 6 14 3 7 8 19 20
10.0      1.0       0.25
10 10 1 8 2
1.0       0.1       0.0       0.0
82
 4 24.6   0
****
```

Start of paper tape

```
AG227
 600
 0.000 -0.15020 -1.23610 -0.10442 -0.62187
 0.050 -0.14715 -1.22032 -0.10442 -0.62296
 0.100 -0.13996 -1.21506 -0.10355 -0.61969
 0.150 -0.13625 -1.21769 -0.10246 -0.62187
 0.200 -0.13233 -1.22558 -0.10224 -0.61969
 0.250 -0.12797 -1.16772 -0.10181 -0.61641
 0.300 -0.12557 -1.17824 -0.10115 -0.61314
 0.350 -0.12121 -1.17035 -0.10115 -0.60987
 0.400 -0.11837 -1.14668 -0.10050 -0.60660
 0.450 -0.11510 -1.16772 -0.10093 -0.60223
 0.500 -0.11227 -1.18350 -0.09984 -0.60223
 0.550 -0.10900 -1.17824 -0.09941 -0.60550
 0.600 -0.10617 -1.18876 -0.09854 -0.60550
 0.650 -0.10529 -1.18350 -0.09788 -0.60223
 0.700 -0.10355 -1.21769 -0.09745 -0.60441
 0.750 -0.10268 -1.23873 -0.09679.-0.60660
 0.800 -0.10290 -1.24662 -0.09483 -0.60878
 0.850 -0.10050 -1.26766 -0.09243 -0.61314
 0.900 -0.10333 -1.27292 -0.09025 -0.61096
 0.950 -0.10464 -1.28344 -0.08916 -0.60769
 1.000 -0.10420 -1.28607 -0.08873 -0.60223
 1.050 -0.10813 -1.29396 -0.08894 -0.60441
```

Fig.2 Data input for longitudinal program

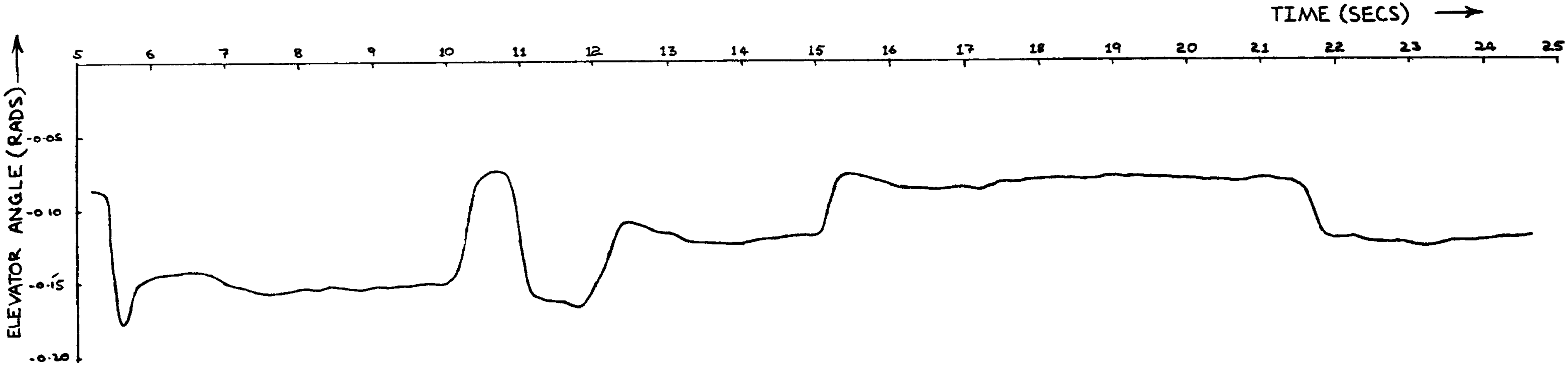

Fig.3 Elevator input for longitudinal example

```
 LONGITUDINAL ANALYSIS FOR FULL-SCALE A/C
G.W.FOSTER,AERO.DEPT. R141C.                                                                                 RUN ON..20/09/74

HUNTER AG227                DEMONSTRATION RUN

INSTRUMENT POSITIONS
  X1         Y1         Z1         X2         Y2         Z2         X3         Y3         Z3
 0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000

  IY          M          S      CBAR
   51680.0    7937.8      32.4   3.17

NUMBER OF INSTRUMENTS = 2

    TO                  VT                    RHO         G
     5.000     0.000   190.000         0.0 0.6540000      9.81

THETA.TRIM    W.TRIM    Q.TRIM    U.TRIM  PHI.TRIM  ETA.TRIM    P.TRIM    R.TRIM   AZ.TRIM ALFA.TRIM
    0.0000    0.0000    0.0000  190.0000    0.0000   -0.1185    0.0000    0.0000   -1.0000    0.0000

  E MAX      ACCURACY   DELTA T
 10.0000     1.0000     0.2500

INITIAL NO OF PARAMETERS = 10  FINAL NO = 10

INITIAL ITRN. =  1          ITRN. MAXIMUM =  8

SCALING OF
    Q         AZ         V        ALPHA
    1.00      0.10

 TAPE IDENTIFIER  AG227     ORIGUNAL NUMBER OF DATA POINTS   600

ONLY EVERY    4 POINT USED


          WO          QO          M ETA        MW          ZW
VARI.    0.000       0.000       -0.493      -0.365      -5.893    :NON-DIMENSIONAL
     (   0.000) (    0.000) ( -11.574) (   -8.569) (   -1.496)    :AVIONIC
DELTA    1.000       0.005        0.005       0.500       0.050

          MQ          EQ          EAZ         ZQ          Z ETA
VARI.   -2.834      -0.105       -0.493       6.527      -0.237    :NON-DIMENSIONAL
     (  -1.110) (   -0.105) (   -0.493) (    5.251) ( -11.428)    :AVIONIC
DELTA    0.100       0.005        0.005       0.500       0.020

          XU          UO          THETAO      MU          ZU
VARI.    0.000       0.000        0.000       0.000       0.000    :NON-DIMENSIONAL
     (   0.000) (    0.000) (    0.000) (    0.000) (    0.000)    :AVIONIC
DELTA    0.000       1.000        0.020       1.000       0.100

          XW          XQ          X ETA       EV          EALPHA
VARI.    0.000       0.000        0.000       0.000       0.000    :NON-DIMENSIONAL
     (   0.000) (    0.000) (    0.000) (    0.000) (    0.000)    :AVIONIC
DELTA    0.001       1.000        1.000       1.000       0.000

SIGMA = 0.0084 DEGREES OF FREEDOM = 186



          WO          QO          M ETA        MW          ZW
VARI.   -2.094      -0.033       -0.246      -0.249      -4.263    :NON-DIMENSIONAL
     (  -2.094) (   -0.033) (   -5.778) (   -5.851) (   -1.082)    :AVIONIC
```

Fig.4a Results for longitudinal example

```
           MQ          EQ          EAZ         ZG          Z ETA
VARI.    -4.501      -0.102      -0.548     -12.655      -0.565    :NON-DIMENSIONAL
       ( -1.763) (   -0.102) (   -0.548) ( -10.181) ( -27.250)   :AVIONIC
DELTA     0.176       0.001       0.009       3.178       0.159

           XU          UO          THETAO      MU          ZU
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
       (  0.000) (    0.000) (    0.000) (    0.000) (    0.000)   :AVIONIC
DELTA     0.000       0.000       0.000       0.000       0.000

           XW          XQ          X ETA       EV          EALPHA
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
       (  0.000) (    0.000) (    0.000) (    0.000) (    0.000)   :AVIONIC
DELTA     0.000       0.000       0.000       0.000       0.000

SIGMA = 0.0056 DEGREES OF FREEDOM = 186



           WO          QO          M ETA       MW          ZW
VARI.    -5.392      -0.015      -0.202      -0.140      -2.393    :NON-DIMENSIONAL
       ( -5.392) (   -0.015) (   -4.750) (   -3.295) (   -0.607)   :AVIONIC
DELTA     0.423       0.006       0.008       0.006       0.143

           MQ          EQ          EAZ         ZG          Z ETA
VARI.    -3.269      -0.103      -0.546     -19.019      -0.286    :NON-DIMENSIONAL
       ( -1.281) (   -0.103) (   -0.546) ( -15.301) ( -13.806)   :AVIONIC
DELTA     0.251       0.001       0.006       4.504       0.111

           XU          UO          THETAO      MU          ZU
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
       (  0.000) (    0.000) (    0.000) (    0.000) (    0.000)   :AVIONIC
DELTA     0.000       0.000       0.000       0.000       0.000

           XW          XQ          X ETA       EV          EALPHA
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
       (  0.000) (    0.000) (    0.000) (    0.000) (    0.000)   :AVIONIC
DELTA     0.000       0.000       0.000       0.000       0.000

SIGMA = 0.0053 DEGREES OF FREEDOM = 186



           WO          QO          M ETA       MW          ZW
VARI.    -4.666      -0.009      -0.214      -0.185      -3.092    :NON-DIMENSIONAL
       ( -4.666) (   -0.009) (   -5.017) (   -4.349) (   -0.785)   :AVIONIC
DELTA     0.474       0.005       0.005       0.003       0.073

           MQ          EQ          EAZ         ZG          Z ETA
VARI.    -3.134      -0.103      -0.544      -5.617      -0.224    :NON-DIMENSIONAL
       ( -1.228) (   -0.103) (   -0.544) (   -4.518) ( -10.824)   :AVIONIC
DELTA     0.138       0.001       0.005       2.853       0.079

           XU          UO          THETAO      MU          ZU
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
       (  0.000) (    0.000) (    0.000) (    0.000) (    0.000)   :AVIONIC
DELTA     0.000       0.000       0.000       0.000       0.000

           XW          XQ          X ETA       EV          EALPHA
VARI.     0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
```

Fig.4 b

```
    (   0.000) (   0.000) (   0.000) (   0.000) (   0.000)  :AVIONIC
DELTA   0.000      0.000      0.000      0.000      0.000

SIGMA = 0,0052 DEGREES OF FREEDOM = 186



          WO          QO          M ETA       MW          ZW
VARI.   -5.019      -0.013      -0.202      -0.187      -3.087    :NON-DIMENSIONAL
    (   -5.019) (   -0.013) (   -4.746) (   -4.381) (   -0.783)  :AVIONIC
DELTA    0.400       0.005       0.005       0.003       0.088

          MQ          EQ          EAZ         ZQ          Z ETA
VARI.   -2.625      -0.103      -0.544      -0.740      -0.094    :NON-DIMENSIONAL
    (   -1.028) (   -0.103) (   -0.544) (   -0.595) (   -4.533)  :AVIONIC
DELTA    0.149       0.001       0.005       2.955       0.082

          XU          UO          THETAO      MU          ZU
VARI.    0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
    (    0.000) (    0.000) (    0.000) (    0.000) (    0.000)  :AVIONIC
DELTA    0.000       0.000       0.000       0.000       0.000

          XW          XQ          X ETA       EV          EALPHA
VARI.    0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
    (    0.000) (    0.000) (    0.000) (    0.000) (    0.000)  :AVIONIC
DELTA    0.000       0.000       0.000       0.000       0.000

SIGMA = 0,0052 DEGREES OF FREEDOM = 186


 CHANGES FOR FOLLOWING PARAMETERS ALL SMALL

          WO          QO          M ETA       MW          ZW
VARI.   -5.076      -0.013      -0.203      -0.187      -3.086    :NON-DIMENSIONAL
    (   -5.076) (   -0.013) (   -4.756) (   -4.382) (   -0.783)  :AVIONIC
DELTA    0.380       0.004       0.005       0.003       0.084

          MQ          EQ          EAZ         ZQ          Z ETA
VARI.   -2.651      -0.103      -0.544      -0.976      -0.098    :NON-DIMENSIONAL
    (   -1.038) (   -0.103) (   -0.544) (   -0.785) (   -4.735)  :AVIONIC
DELTA    0.133       0.001       0.005       2.755       0.078

          XU          UO          THETAO      MU          ZU
VARI.    0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
    (    0.000) (    0.000) (    0.000) (    0.000) (    0.000)  :AVIONIC
DELTA    0.000       0.000       0.000       0.000       0.000

          XW          XQ          X ETA       EV          EALPHA
VARI.    0.000       0.000       0.000       0.000       0.000    :NON-DIMENSIONAL
    (    0.000) (    0.000) (    0.000) (    0.000) (    0.000)  :AVIONIC
DELTA    0.000       0.000       0.000       0.000       0.000

 T(SECS)     ETA          Q            AZ          AIR SPEED    ALPHA      ---------------ERRORS IN MATCHING---------------
                                                                                 Q            AZ         AIR SPEED     ALPHA
   5.200     0.03261     -0.12145     -1.16493                                -0.00455     -0.00817
   5.400     0.02781     -0.12458     -1.16002                                -0.00012     -0.00261
   5.600    -0.05895     -0.08773     -1.15533                                -0.00230     -0.00150
   5.800    -0.03279     -0.03527     -1.35683                                 0.00410     -0.00081
   6.000    -0.02800     -0.01978     -1.60499                                 0.00234      0.00191
   6.200    -0.02625     -0.02228     -1.83032                                 0.00135     -0.00028
```

Fig.4c

6.400	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6.600	-0.02407	-0.05398	-2.09575	-0.00052	0.00023
6.800	-0.02582	-0.07263	-2.12369	-0.00498	-0.00014
7.000	-0.03127	-0.08377	-2.10400	-0.00888	-0.00105
7.200	-0.03323	-0.08861	-2.06773	-0.00775	0.00111
7.400	-0.03715	-0.08850	-2.02946	-0.00916	0.01201
7.600	-0.03868	-0.08383	-2.00603	-0.00577	0.00493
7.800	-0.03781	-0.07937	-2.00056	-0.00434	0.01149
8.000	-0.03563	-0.07719	-2.00586	-0.00129	0.00491
8.200	-0.03584	-0.07659	-2.01248	-0.00189	0.01373
8.400	-0.03497	-0.07657	-2.01966	0.00180	0.00366
8.600	-0.03497	-0.07697	-2.02462	0.00176	0.00574
8.800	-0.03650	-0.07703	-2.02721	-0.00298	0.00442
9.000	-0.03584	-0.07666	-2.03144	-0.00095	0.00300
9.200	-0.03475	-0.07742	-2.03458	-0.00215	0.00042
9.400	-0.03588	-0.07935	-2.03302	-0.00414	0.00027
9.600	-0.03545	-0.08075	-2.02682	-0.00449	0.00570
9.800	-0.03301	-0.08234	-2.01719	-0.00595	0.00184
10.000	-0.03192	-0.08377	-2.00512	-0.00517	0.00300
10.200	-0.01404	-0.08806	-1.99807	-0.00328	0.00229
10.400	0.03501	-0.12662	-1.94537	0.00367	0.00018
10.600	0.04594	-0.17344	-1.75108	0.00558	-0.00373
10.800	0.04307	-0.20080	-1.47650	0.00809	-0.00305
11.000	-0.00380	-0.19271	-1.18899	0.00458	-0.00208
11.200	-0.04129	-0.12834	-1.06360	0.00255	-0.00200
11.400	-0.04435	-0.06377	-1.16248	0.00382	-0.00552
11.600	-0.04544	-0.01944	-1.39983	0.00069	-0.00519
11.800	-0.04783	0.00431	-1.69511	-0.00017	-0.00354
12.000	-0.03584	0.00567	-1.98908	-0.00109	-0.00150
12.200	-0.01186	-0.02438	-2.20170	-0.00352	0.00030
12.400	0.00841	-0.07595	-2.26084	-0.00231	-0.00115
12.600	0.00754	-0.12331	-2.15719	0.00036	-0.00152
12.800	0.00274	-0.15049	-1.96338	-0.00037	-0.00223
13.000	0.00122	-0.15988	-1.75090	0.00096	0.00151
13.200	-0.00162	-0.15489	-1.56480	0.00098	-0.00211
13.400	-0.00532	-0.13967	-1.43588	-0.00050	-0.00159
13.600	-0.00511	-0.12112	-1.37709	0.00166	-0.00221
13.800	-0.00532	-0.10442	-1.37655	0.00327	-0.00173
14.000	-0.00532	-0.09192	-1.41712	0.00603	0.00206
14.200	-0.00314	-0.08534	-1.47941	0.00642	-0.00434
14.400	-0.00249	-0.08463	-1.54096	0.00702	-0.00370
14.600	-0.00096	-0.08801	-1.58959	0.00648	-0.00726
14.800	-0.00009	-0.09333	-1.61799	0.00460	-0.00521
15.000	0.00056	-0.09930	-1.62589	0.00294	-0.00415
15.200	0.02520	-0.11476	-1.61671	0.00336	-0.00586
15.400	0.04133	-0.14435	-1.53654	0.00200	-0.00494
15.600	0.04046	-0.16708	-1.38359	-0.00034	-0.00682
15.800	0.03740	-0.17448	-1.21128	0.00030	-0.00301
16.000	0.03435	-0.16959	-1.06267	-0.00110	-0.00498
16.200	0.03239	-0.15693	-0.96284	-0.00395	-0.00287
16.400	0.03261	-0.14231	-0.91826	-0.00680	-0.00075
16.600	0.03217	-0.12928	-0.91888	-0.01089	0.00115
16.800	0.03261	-0.11950	-0.95196	-0.01304	-0.00054
17.000	0.03261	-0.11405	-1.00086	-0.01326	0.00751
17.200	0.03195	-0.11217	-1.05197	-0.01122	0.01078
17.400	0.03435	-0.11372	-1.09758	-0.00640	0.00771
17.600	0.03610	-0.11861	-1.12638	-0.00238	0.01480
17.800	0.03675	-0.12447	-1.13475	0.00239	0.01038
18.000	0.03697	-0.12558	-1.12612	0.00532	0.00873
18.200	0.03740	-0.13331	-1.10653	0.00752	0.00519
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Fig.4 d

```
  18.400      0.03704     -0.13565     -1.08139                                                0.00659     0.00215
  18.600      0.03762     -0.13634     -1.05544                                                0.00619     0.00140
  18.800      0.03849     -0.13601     -1.03366                                                0.00347     0.00027

  19.000      0.04002     -0.13555     -1.01671                                               -0.00092    -0.00379
  19.200      0.03958     -0.13498     -1.00238                                               -0.00149     0.00240
  19.400      0.03915     -0.13341     -0.99340                                               -0.00284     0.00413
  19.600      0.03849     -0.13130     -0.99090                                               -0.00473     0.00178
  19.800      0.03762     -0.12918     -0.99462                                               -0.00336     0.00636
  20.000      0.03806     -0.12751     -1.00401                                               -0.00220    -0.00638
  20.200      0.03740     -0.12677     -1.01455                                               -0.00381     0.00046
  20.400      0.03740     -0.12628     -1.02572                                               -0.00321     0.00368
  20.600      0.03653     -0.12618     -1.03543                                               -0.00375    -0.00139
  20.800      0.03740     -0.12661     -1.04431                                               -0.00157     0.00291

  21.000      0.03849     -0.12809     -1.04915                                                0.00100     0.00261
  21.200      0.03806     -0.12958     -1.04805                                                0.00249     0.00697
  21.400      0.03719     -0.13034     -1.04370                                                0.00325     0.00311
  21.600      0.02694     -0.12765     -1.03676                                                0.00492     0.00768
  21.800      0.00165     -0.10675     -1.05778                                                0.00407     0.00216
  22.000     -0.00162     -0.08291     -1.15432                                                0.00290    -0.00108
  22.200     -0.00249     -0.06824     -1.29160                                                0.00982    -0.00129
  22.400     -0.00489     -0.06205     -1.43608                                                0.00820    -0.01209
  22.600     -0.00532     -0.06293     -1.56711                                                0.00407    -0.00819
  22.800     -0.00532     -0.07004     -1.66675                                                0.00551    -0.00690

  23.000     -0.00707     -0.07960     -1.72642                                                0.00461    -0.00699
  23.200     -0.00903     -0.08673     -1.75399                                                0.00302    -0.01186
  23.400     -0.00685     -0.09434     -1.75909                                               -0.00049    -0.00924
  23.600     -0.00576     -0.10161     -1.74159                                               -0.00194    -0.00021
  23.800     -0.00554     -0.10683     -1.70862                                               -0.00130    -0.00167
  24.000     -0.00489     -0.10966     -1.66968                                                0.00306    -0.00293
  24.200     -0.00358     -0.11035     -1.63229                                                0.00375    -0.00746
  24.400     -0.00358     -0.10984     -1.59942                                                0.00433    -0.00259
  24.600     -0.00336     -0.10784     -1.57537                                                0.00582    -0.00947


 98 OUT OF  99 POINTS ACCEPTED          DEGREES OF FREEDOM 196

 RMS SENSITIVITY MATRIX

              Q          AZ          U          ALPHA

WO          0.00609    0.04436
QO          0.00139    0.01050
M ETA       0.03902    0.40960
MW          0.03654    0.37650
ZW          0.01944    0.33408
MQ          0.01786    0.12622
EQ          0.10308    0.00000
FAZ         0.00000    0.54413
ZQ          0.00015    0.00241
Z ETA       0.00062    0.01205
```

Fig.4 e

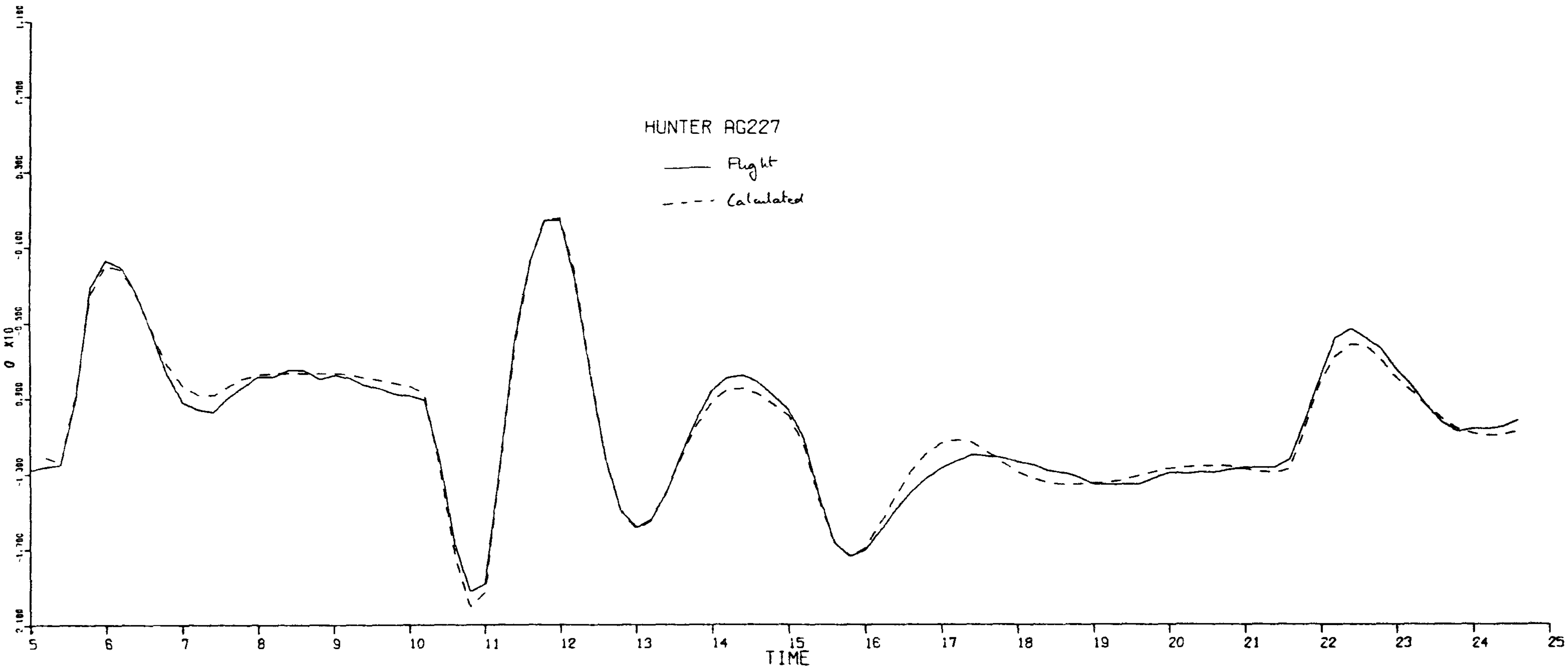

Fig.5a Computed and recorded rate of pitch

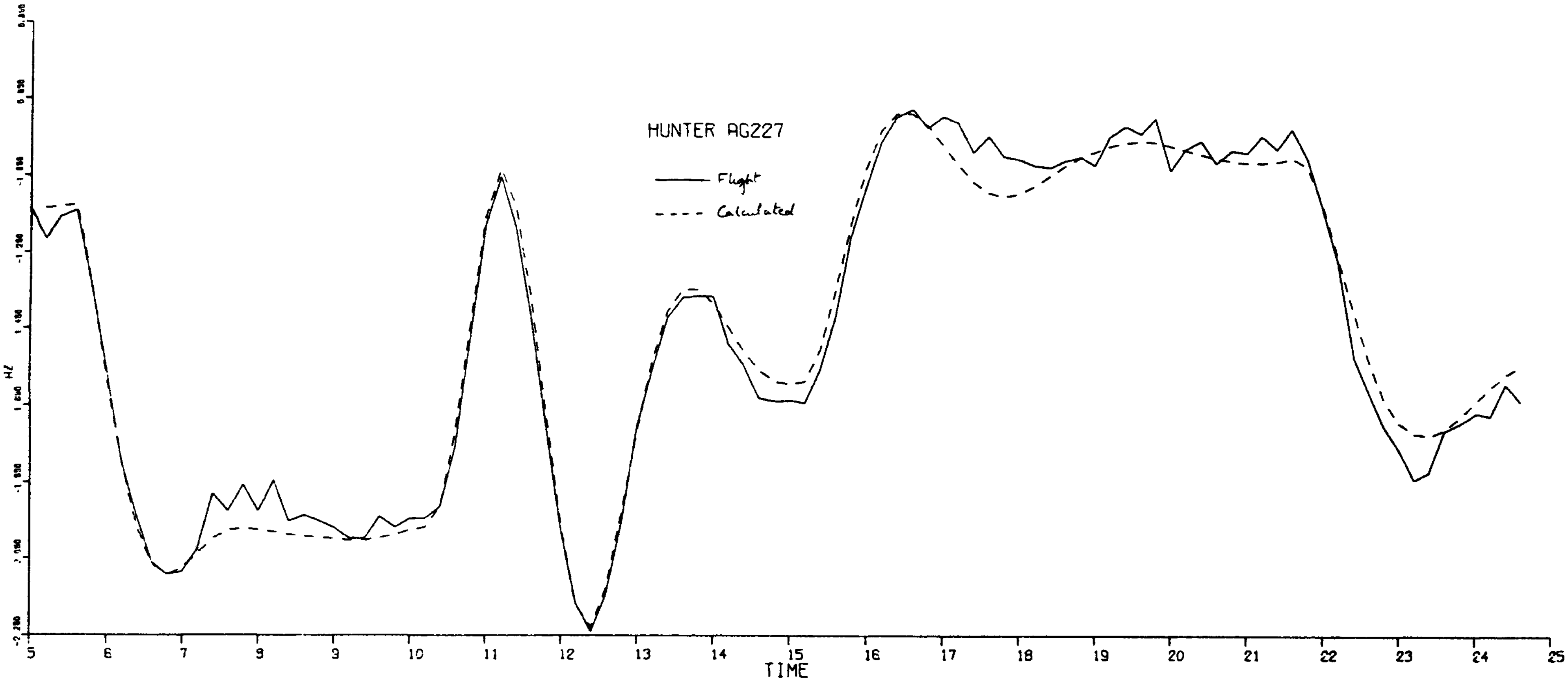

Fig.5b Computed and recorded normal acceleration

```
3
 GNAT 11407
-1.750    0.580     -1.000
2.250     -1.00     0.00
15.670    0.0       0.0
1403.1    8012.8    9180.7    -115.8    205.13    175.0      11.5     12.0
1.0       0.1       0.02      0.05      0.1       0.1       0.1       0.1
0.005     0.015     0.05      0.1       0.1       0.005     0.05      0.03
0.1       0.1       0.005     0.1       0.05      0.005
 4
1.6       751.0     0.111     0.00114   32.2      -1.192
3.47      0.031     53.6      0.099     -1.44     0.119      -0.039
0.0049
-3.4      0.527     -0.183    -0.223
-0.105    -0.331    0.033     -0.066              0.091               -0.442
                              -1.762    0.137     0.031
1 2 3 12 17 18 19 20 4 9 15 13 21 5 16 8 14 22 10 6 7 11
10.0      0.2       0.15
7 7 1 7 1
0.20      0.10      0.20      1.00
81
 42
 16   -0.37   29.08  -12.62  -0.017   2.093   749.6  23003.    0.49    1.97
 17    0.46    7.70  -10.69  -0.114   2.060   749.8  23020.    0.49    1.62
 18    1.14   -0.31   -7.84  -0.172   2.060   750.2  23003.    0.49    1.67
 19    1.97  -15.57   -5.25  -0.195   2.180   750.8  23003.    0.49    1.72
 20    2.04  -24.34   -3.62  -0.186   2.311   751.9  23003.    0.40    2.07
 21    1.97  -34.64    0.85  -0.158   2.409   751.9  23003.    0.40    2.22
 22    1.29  -37.31    2.71  -0.089   2.572   749.6  23003.    0.40    2.41
 23    0.76  -31.97    4.27  -0.012   2.724   751.9  23003.    0.40    2.41
 24    0.46  -25.10    4.27   0.094   2.713   751.3  23003.    0.40    2.41
 25   -0.59   -7.56    2.93   0.163   2.681   749.8  23020.    0.31    2.31
 26   -0.52   -2.98    1.89   0.176   2.626   750.2  23003.    0.13    2.36
 27   -1.19    4.26   -0.34   0.165   2.539   749.6  23003.    0.04    2.22
 28   -1.27   14.46   -1.83   0.153   2.528   750.8  23053.    0.04    2.22
 29   -0.97   21.77   -3.91   0.129   2.485   751.6  23070.    0.04    2.22
 30   -0.97   20.55   -5.40   0.078   2.452   753.0  23087.    0.04    2.12
 31   -0.44   16.49   -6.33   0.025   2.419   750.7  23087.    0.04    2.07
 32    0.01   12.83   -6.59  -0.012   2.332   752.4  23087.   -0.05    1.87
 33    0.39    5.79   -5.48  -0.049   2.234   750.5  23070.    0.04    1.87
 34    0.69   -2.22   -4.06  -0.075   2.223   751.6  23070.    0.04    1.92
 35    0.99   -8.70   -2.87  -0.075   2.321   751.3  23087.   -0.05    1.97
 36    0.99  -11.37   -1.46  -0.063   2.387   752.4  23087.   -0.05    2.02
 37    0.61  -11.76    0.33  -0.040   2.409   750.7  23087.   -0.05    2.02
 38    0.61   -9.85    1.67  -0.008   2.485   751.6  23070.   -0.05    1.92
 39    0.31   -7.56    2.04   0.018   2.485   753.0  23087.    0.04    2.02
 40    0.01   -6.42    1.59   0.055   2.496   752.5  23137.    0.04    1.87
 41   -0.37   -1.08    1.22   0.073   2.485   750.8  23137.    0.04    1.87
 42   -0.74    3.50    0.48   0.080   2.463   751.9  23137.    0.04    1.77
 43   -0.52    7.70   -0.71   0.080   2.452   753.4  23120.    0.04    1.67
 44   -0.44    9.22   -2.05   0.069   2.398   752.4  23170.    0.04    1.58
 45   -0.14    9.22   -3.24   0.057   2.376   751.1  23204.    0.04    1.67
 46    0.09    8.08   -3.69   0.032   2.321   751.1  23204.    0.04    1.67
 47    0.09    4.26   -3.62   0.011   2.300   751.1  23204.    0.04    1.82
 48    0.31    2.36   -3.54  -0.010   2.267   751.3  23221.    0.13    1.92
 49    0.46    0.83   -3.09  -0.022   2.267   750.5  23204.    0.04    1.97
 50    0.46   -1.46   -2.50  -0.026   2.267   749.9  23204.    0.04    1.97
 51    0.39   -2.22   -1.61  -0.022   2.289   749.9  23204.    0.04    2.12
 52    0.61   -3.75   -0.94  -0.008   2.354   750.5  23204.   -0.05    1.97
 53    0.61   -3.75   -0.56   0.015   2.398   750.5  23204.   -0.23    1.97
 54    0.46   -3.37   -0.27   0.018   2.398   752.0  23271.   -0.14    1.92
 55    0.39   -2.98   -0.19   0.036   2.365   750.9  23322.   -0.14    1.82
```

Fig.6a Data input for lateral example

```
 56    0.09     0.07    -0.42    0.039    2.321    750.9  23322.    -0.32     1.72
 57   -0.14     3,12    -0.94    0.050    2.311    750.1  23305.    -0.05     1.58
6
 GNAT 11407 CONT.
10.0       0.2        0.15
 13 13 1 7 2
0.20      0.10        0.20       1.00
92
```

Fig.6b

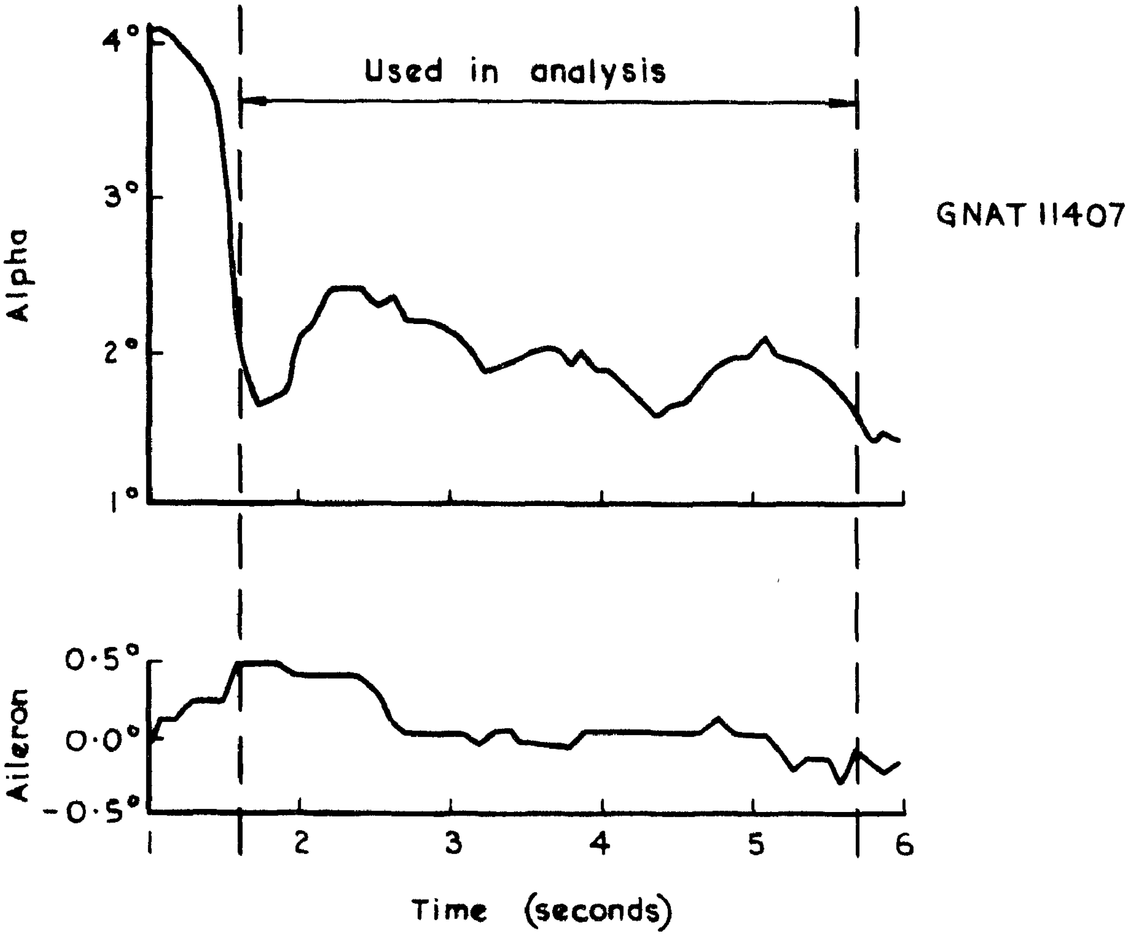

Fig.7 Aileron and angle of attack inputs for lateral example

```
 PROGRAM  D1   JULY 1973      DUTCH ROLL ANALYSIS FOR  FULL SCALE AIRCRAFT
A.J.ROSS AERO. R141BLDG. PROG A25D                                                                  RUN ON  24/09/74

GNAT 11407                          DEMONSTRATION RUN

 X.AY       Y.AY      Z.AY      X.AZ      Y.AZ      Z.AZ      X.PB      Y.PB      Z.PB
-1.750     0.580    -1.000     2.250    -1.000     0.000    15.670     0.000     0.000

  IX          IY          IZ          IXZ          M          S      L1     L2
     1403.1      8012.8      9180.7      -115.8      205.1      175.0  11.50  14.00

NUMBER OF INSTRUMENTS = 4

      TO         VT        THETA      RHO          G        PHI
   1.600      751.0      0.111 0.0011400      32.20     -1.192


   JZ.TRIM    JY.TRIM    W.TRIM     Q.TRIM     V.TRIM     P.TRIM     R.TRIM  XSI.TRIM ZETA.TRIM
   5.470      0.031     53.600      0.099     -1.440      0.119     -0.039     0.005     0.000

  E MAX       ACCURACY    DELTA T
  10.0000     0.2000     0.1500

INITIAL NO OF PARAMETERS =  7  FINAL NO =  7

INITIAL ITRN. =  1          ITRN. MAXIMUM =  7

SCALING OF
      BETA        P         R         JY
     0.20       0.10      0.20       1.00


           VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -3.400    0.527    -0.183    -0.105     0.091    -1.762     0.137    -0.442
DELTA     1.000    0.100     0.020     0.005     0.005     0.100     0.050     0.030

           LP       EBETA      EAY       YV        LXI       NXI       LR
VARI.    -0.331    0.000     0.031     0.223    -0.066     0.000     0.033
DELTA     0.015    0.005     0.005     0.050     0.100     0.100     0.050

           NP        YP        YR        YXI       YZT       LZT       NZT
VARI.     0.000    0.000     0.000     0.000     0.000     0.000     0.000
DELTA     0.050    0.100     0.100     0.100     0.100     0.100     0.100


 SIGMA = 0.1303 DEGREES OF FREEDOM = 157


           VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -8.169    0.377    -0.186    -0.098     0.094    -8.172     0.127    -0.442
DELTA     0.868    0.039     0.005     0.003     0.001     0.229     0.103     0.000

           LP       EBETA      EAY       YV        LXI       NXI       LR
VARI.    -0.331    0.000     0.031     0.225    -0.066     0.000     0.033
DELTA     0.000    0.000     0.000     0.000     0.000     0.000     0.050

           NP        YP        YR        YXI       YZT       LZT       NZT
VARI.     0.000    0.000     0.000     0.000     0.000     0.000     0.000
DELTA     0.050    0.100     0.100     0.100     0.100     0.100     0.100


 SIGMA = 0.1301 DEGREES OF FREEDOM = 157
```

Fig.8 a   Results for lateral example

```
             VO        PO        RO        LV        NV        EP        ER        NR
VARI.     -8.667     0.372    -0.184    -0.099     0.094    -8.158     0.139    -0.442
DELTA      0.876     0.039     0.005     0.003     0.001     0.228     0.103     0.000

             LP      EBETA      EAY        YV       LXI       NXI        LR
VARI.     -0.331     0.000     0.031     0.223    -0.006     0.000     0.033
DELTA      0.000     0.000     0.000     0.000     0.000     0.000     0.050

             NP        YP        YR       YXI       YZT       LZT       NZT
VARI.      0.000     0.000     0.000     0.000     0.000     0.000     0.000
DELTA      0.050     0.100     0.100     0.100     0.100     0.100     0.100


SIGMA = 0.1196 DEGREES OF FREEDOM = 153


             VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -10.051     0.344    -0.185    -0.100     0.095    -8.209     0.187    -0.442
DELTA      0.832     0.036     0.005     0.003     0.001     0.211     0.096     0.000

             LP      EBETA      EAY        YV       LXI       NXI        LR
VARI.     -0.331     0.000     0.031     0.223    -0.006     0.000     0.033
DELTA      0.000     0.000     0.000     0.000     0.000     0.000     0.050

             NP        YP        YR       YXI       YZT       LZT       NZT
VARI.      0.000     0.000     0.000     0.000     0.000     0.000     0.000
DELTA      0.050     0.100     0.100     0.100     0.100     0.100     0.100


SIGMA = 0.1194 DEGREES OF FREEDOM = 153


 CHANGES FOR FOLLOWING PARAMETERS ALL SMALL
             VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -10.154     0.340    -0.185    -0.100     0.095    -8.272     0.195    -0.442
DELTA      0.819     0.037     0.005     0.003     0.001     0.211     0.096     0.000

             LP      EBETA      EAY        YV       LXI       NXI        LR
VARI.     -0.331     0.000     0.031     0.225    -0.006     0.000     0.033
DELTA      0.000     0.000     0.000     0.000     0.000     0.000     0.050

             NP        YP        YR       YXI       YZT       LZT       NZT
VARI.      0.000     0.000     0.000     0.000     0.000     0.000     0.000
DELTA      0.050     0.100     0.100     0.100     0.100     0.100     0.100


 40 OUT OF  42 POINTS ACCEPTED      DEGREES OF FREEDOM 160

RMS  SENSITIVITY MATRIX

               BETA         P            R           AY

   VO        0.24337     3.49560     0.94247     0.01080
   PO        0.02886     2.19133     0.11522     0.00650
   RO        0.83717    13.16474     2.91251     0.03720
   LV        0.41347    11.67623     1.39537     0.01951
   NV        1.66022    25.07389     6.12565     0.07547
   EP        0.00000     8.27224     0.00000     0.00000
   ER        0.00000     0.00000     0.19468     0.00000
```

Fig.8 b

```
 PROGRAM  D1   JULY 1973      DUTCH ROLL ANALYSIS FOR  FULL SCALE AIRCRAFT
A.J.ROSS AERO, R141BLDG, PROG A25D                                                                RUN ON  24/09/76

GNAT 11407 CONT.              DEMONSTRATION RUN

  E MAX      ACCURACY   DELTA T
  10.0000     0,2000    0,1500

INITIAL NO OF PARAMETERS = 13  FINAL NO = 13

INITIAL ITRN. =  1          ITRN. MAXIMUM =  7

SCALING OF
     BETA        P          R          JY
    0,20       0,10       0,20       1,00


            VO      PU      RO      LV      NV      EP      ER      NR
VARI.   -10,154   0,340  -0,185  -0,100   0,095  -8,272   0,195  -0,442
DELTA     1,000   0,100   0,070   0,005   0,005   0,100   0,050   0,030

            LP    EBETA    EAY      YV     LAI     NXI      LR
VARI.    -0,331   0,000   0,031   0,223  -0,066   0,000   0,033
DELTA     0,015   0,005   0,005   0,050   0,100   0,100   0,050

            NP      YP      YR     YXI     YZT     LZT     NZT
VARI.     0,000   0,000   0,000   0,000   0,000   0,000   0,000
DELTA     0,050   0,100   0,100   0,100   0,100   0,100   0,100


SIGMA = 0,1054 DEGREES OF FREEDOM = 151


            VO      PU      RO      LV      NV      EP      ER      NR
VARI.    -5,049   0,339  -0,170  -0,088   0,092  -7,707   0,153  -0,264
DELTA     0,967   0,044   0,006   0,005   0,001   0,490   0,086   0,028

            LP    EBETA    EAY      YV     LAI     NXI      LR
VARI.    -0,263   0,147  -0,002  -0,223  -0,038   0,000   0,033
DELTA     0,018   0,083   0,017   0,068   0,012   0,000   0,000

            NP      YP      YR     YXI     YZT     LZT     NZT
VARI.     0.000   0,000   0,000   0,000   0,000   0,000   0.000
DELTA     0,000   0.000   0,000   0,000   0,000   0,000   0.000


SIGMA = 0,0905 DEGREES OF FREEDOM = 147


            VO      PU      RO      LV      NV      EP      ER      NR
VARI.    -5.686   0,354  -0,172  -0,087   0,093  -8,459   0,165  -0,272
DELTA     0,826   0,034   0,005   0,003   0,001   0,462   0,075   0,023

            LP    EBETA    EAY      YV     LAI     NXI      LR
VARI.    -0,261   0,148  -0,006  -0,204  -0,034   0,000   0,033
DELTA     0,012   0,072   0,015   0,056   0,004   0,000   0,000

            NP      YP      YR     YXI     YZT     LZT     NZT
VARI.     0,000   0,000   0,000   0,000   0,000   0,000   0.000
DELTA     0,000   0,000   0,000   0,000   0,000   0,000   0,000
```

Fig.8 c

```
SIGMA = 0.0909 DEGREES OF FREEDOM = 147

            VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -5.647     0.355    -0.171    -0.087     0.093    -8.457     0.143    -0.271
DELTA     0.828     0.034     0.005     0.003     0.001     0.460     0.075     0.023

            LP      EBETA      EAY        YV       LXI       NXI        LR
VARI.    -0.262     0.146    -0.006    -0.206    -0.055     0.000     0.033
DELTA     0.012     0.072     0.015     0.056     0.009     0.000     0.000

            NP        YP        YR       YXI       YZT       LZT       NZT
VARI.     0.000     0.000     0.000     0.000     0.000     0.000     0.000
DELTA     0.000     0.000     0.000     0.000     0.000     0.000     0.000


SIGMA = 0.0909 DEGREES OF FREEDOM = 147


 CHANGES FOR FOLLOWING PARAMETERS ALL SMALL
            VO        PO        RO        LV        NV        EP        ER        NR
VARI.    -5.642     0.355    -0.171    -0.087     0.093    -8.456     0.142    -0.272
DELTA     0.828     0.034     0.005     0.003     0.001     0.460     0.075     0.023

            LP      EBETA      EAY        YV       LXI       NXI        LR
VARI.    -0.261     0.146    -0.006    -0.206    -0.055     0.000     0.033
DELTA     0.012     0.072     0.015     0.056     0.009     0.000     0.000

            NP        YP        YR       YXI       YZT       LZT       NZT
VARI.     0.000     0.000     0.000     0.000     0.000     0.000     0.000
DELTA     0.000     0.000     0.000     0.000     0.000     0.000     0.000


 T(SECS)        W         Q       XI      ZETA

              PHI      BETA         P         R        JY      RBETA        RP        RR       RJY

   1.700    21.23      0.10      0.00      0.00
           -65.99      0.45      9.75    -11.18    -0.091      0.002    -0.205     0.097    -0.023

   1.800    21.88      0.10      0.00      0.00
           -64.78      1.34     -2.75     -9.11    -0.161     -0.039     0.244     0.254    -0.011

   1.900    22.54      0.10      0.00      0.00
           -64.87      1.94    -15.81     -6.15    -0.194      0.007     0.024     0.180    -0.001

   2.000    27.12      0.10      0.00      0.00
           -66.17      2.18    -26.50     -2.85    -0.187     -0.028     0.216    -0.155     0.001

   2.100    29.09      0.10      0.00      0.00
           -68.35      2.05    -33.30      0.25    -0.148     -0.017    -0.134     0.120    -0.010

   2.200    31.58      0.10      0.00      0.00
           -70.97      1.61    -35.24      2.66    -0.082     -0.064    -0.207     0.010    -0.007

   2.300    31.58      0.10      0.00      0.00
           -73.54      0.96    -32.24      4.07    -0.004     -0.040     0.027     0.041    -0.008

   2.400    31.58      0.10      0.00      0.00
           -75.59      0.23    -25.16      4.35     0.071      0.046     0.006    -0.015     0.023
```

Fig.8 d

```
2,500  30.27   0,10    0.00    0,00
      -76.77  -0,45  -15.16    3.57   0,132

2,600  30,92   0,10   -0.00    0,00
      -76,88  -0,98   -3,75    1,98   0,170      0,091   0,077  -0,019   0,006

2.700  29.09   0,10   -0,00    0,00
      -75.86  -1,27    6.82   -0,07   0,177      0,017  -0,256  -0,054  -0,012

2.800  29.09   0,10   -0,00    0,00
      -73.92  -1,32   14,69   -2,20   0,156      0,011  -0,023   0,074  -0,003

2.900  29.09   0,10   -0,00    0,00
      -71,35  -1,14   19.04   -4,04   0,117      0,034   0,273   0,027   0,012

3,000  27.78   0,10   -0,00    0,00
      -68.54  -0,78   19,68   -5,33   0,067     -0,038   0,087  -0,013   0,011

3,100  27.12   0,10   -0,00    0,00
      -65,84  -0,32   17,01   -5,92   0,016     -0,024  -0,052  -0,082   0,009

3,200  24.50   0,10   -0,01    0,00
      -63.52   0,15   12,22   -5,78  -0,026     -0,029   0,061  -0,162   0,014

3.300  24.50   0,10   -0,00    0,00
      -61.77   0,57    5,69   -5,01  -0,061     -0,036   0,010  -0,094   0,012

3.400  25.16   0,10   -0,00    0,00
      -60,69   0,87   -1,04   -3,79  -0,077     -0,036  -0,118  -0,055   0,002

3.500  25.81   0,10   -0,01    0,00
      -60.24   1,02   -6,49   -2,35  -0,075     -0,006  -0,221  -0,104  -0,000

3,600  26,47   0,10   -0,01    0,00
      -60.24   1,01  -10,24   -0,95  -0,060     -0,003  -0,113  -0,103  -0,003

3,700  26.47   0,10   -0,01    0,00
      -60.52   0,85  -11,87    0,21  -0,034     -0,048   0,011   0,023  -0,006

3,800  25,16   0,10   -0,01    0,00
      -60.85   0,59  -11,29    0,97  -0,001      0,004   0,144   0,140  -0,007

3,900  26.47   0,10   -0,00    0,00
      -61,04   0,28   -9,10    1,25   0,030      0,007   0,154   0,157  -0,012

4.000  24.50   0,10   -0,00    0,00
      -60,93  -0,03   -5,48    1,06   0,059      0,009  -0,094   0,106  -0,004

4,100  24.50   0,10   -0,00    0,00
      -60.42  -0,30   -1,11    0,47   0,079     -0,015   0,003   0,151  -0,006

4,200  23.19   0,10   -0,00    0,00
      -59.46  -0,47    3,10   -0,39   0,087     -0,053   0,031   0,174  -0,007

4,300  21,88   0,10   -0,00    0,00
      -58,11  -0,54    6,74   -1,35   0,084      0,005   0,096   0,128  -0,004

4.400  20.71   0,10   -0,00    0,00
      -56.47  -0,51    9,05   -2,20   0,071      0,014   0,017   0,041  -0,002

4.500  21.88   0,10   -0,00    0,00
      -54,66  -0,38    9,84   -2,97   0,052      0,049  -0,062  -0,055   0,005

4.600  21.88   0,10   -0,00    0,00
      -52.86  -0.19    9.12   -3.38   0.029      0.056  -0.104  -0.062   0.003
```

Fig.8 e

```
4.700  23.85    0,10   -0.00    0.00
      -51.19    0,03    7.11   -3.46   0,006     0,011 -0,285 -0,032  0.005

4.800  25.16    0,10   -0.00    0.00
      -49.79    0,25    3.88   -3.21  -0,014     0,012 -0,152 -0,067  0.004

4.900  25.81    0,10   -0.00    0.00
      -48.72    0,43    0.71   -2.69  -0,024     0,007  0,012 -0,080  0.002

5.000  25.81    0,10   -0.00    0.00
      -47.95    0,54   -2.24   -2.02  -0,028    -0,015  0,078 -0,096  0.002

5.100  27.78    0,10   -0.00    0.00
      -47.45    0,57   -4.55   -1.30  -0,024    -0,035  0,233 -0,061  0.002

5.200  25.81    0,10   -0.01    0.00
      -47.12    0,52   -5.57   -0.60  -0,011     0,017  0,182 -0,055  0.003

5.300  25.81    0,10   -0.01    0.00
      -46.81    0,42   -4.89   -0.20   0,006     0,037  0,114 -0,072  0.009

5.400  25.16    0,10   -0.01    0.00
      -46.40    0,29   -3.75    0.06   0,018     0,035  0,038 -0,066 -0.000

5.500  23.85    0,10   -0.01    0.00
      -45.85    0,14   -2.01    0.09   0,032     0,050 -0,097 -0,055  0.004

5.600  22.54    0,10   -0.01    0.00
      -45.07    0,01    0.80   -0.10   0,046     0,017 -0,073 -0,064 -0,007

5.700  20.71    0,10   -0.01    0.00
      -44.05   -0,09    2.53   -0.43   0,045    -0,009  0,059 -0,102  0.005

40 OUT OF  42 POINTS ACCEPTED     DEGREES OF FREEDOM 160

RMS  SENSITIVITY MATRIX

           BETA        P          R          AY

 V0       0.14019   1,99366   0,54500   0,01413
 P0       0.03689   2,72696   0,14217   0,00917
 R0       0.81872  12,97960   2,87014   0,08051
 LV       0.44306  12,17920   1,49483   0,05339
 NV       1.83092  27,48627   6,74980   0,18058
 EP       0.00000   8,45648   0,00000   0,00000
 ER       0.00000   0,00000   0,14220   0,00000
 NR       0.31919   4,75694   1,22721   0,03216
 LP       0.32021   8,64769   1,10744   0,04001
EBETA     0.14570   0,00000   0,00000   0,00000
 EAY      0.00000   0,00000   0,00000   0,00601
 YV       0.08312   1,20519   0,30030   0,04808
 LXI      0.03022   2,81573   0,10361   0,00279
```

Fig.8 f

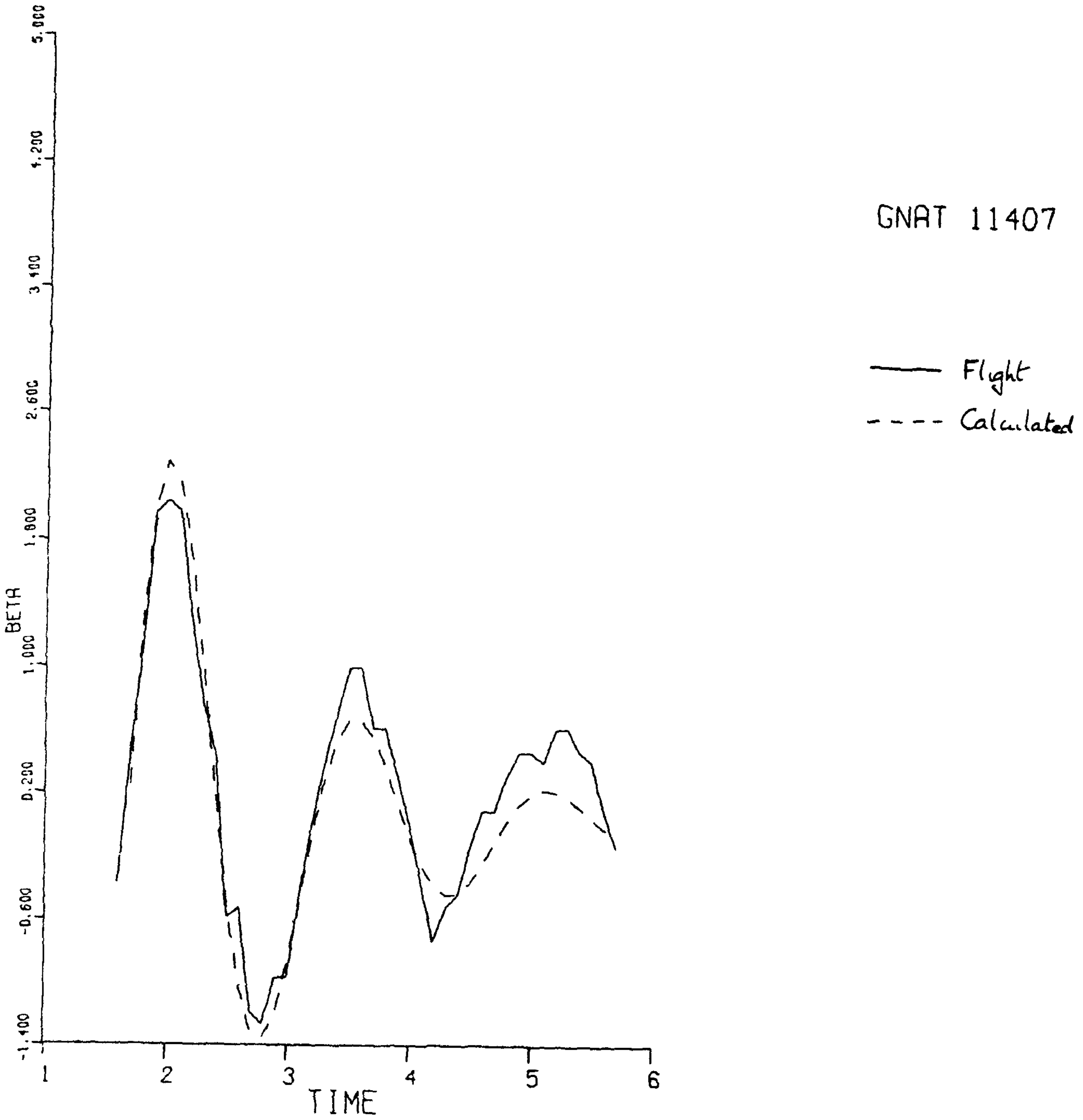

Fig.9a Computed and recorded sideslip angle (interim result)

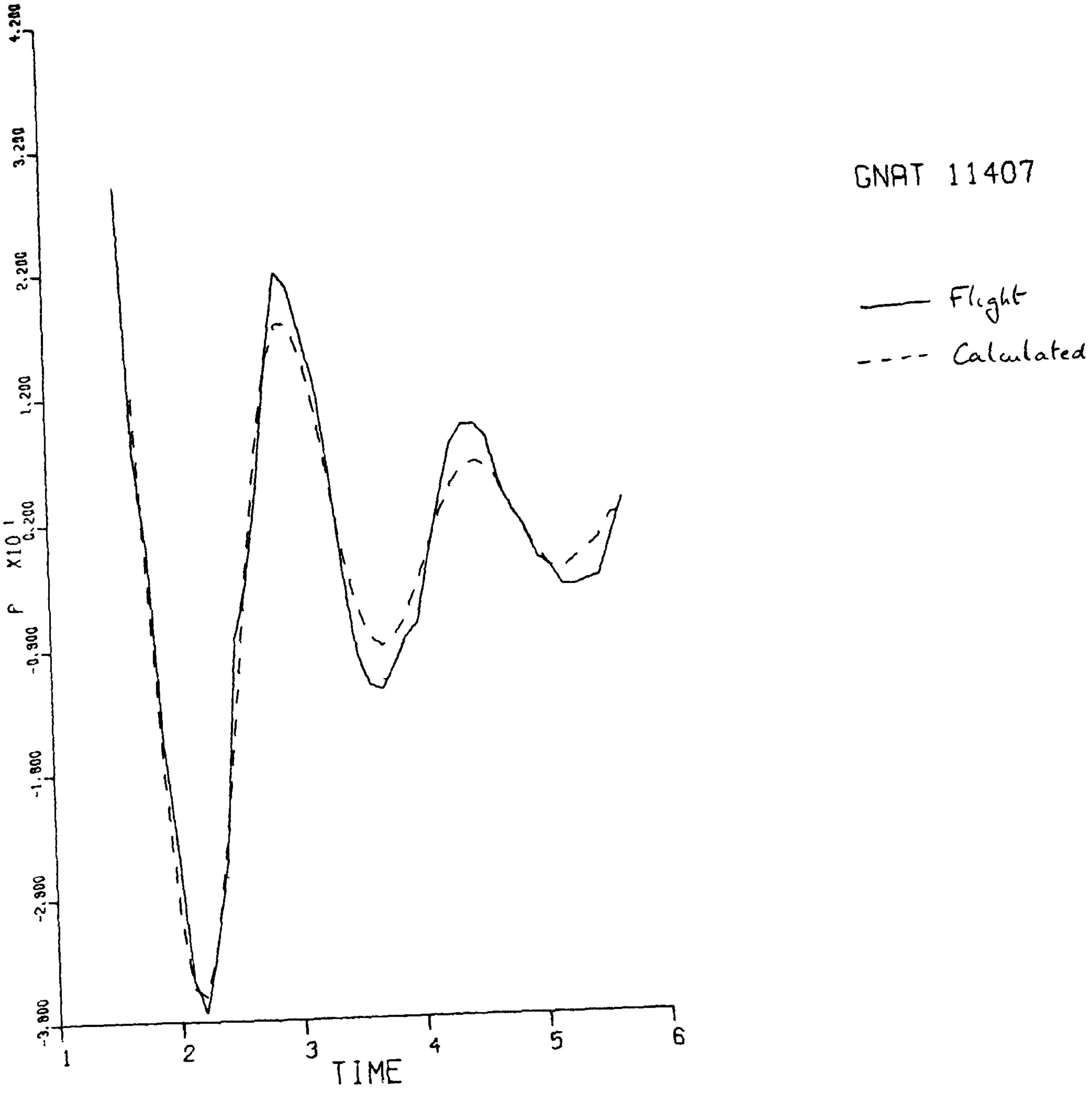

Fig.9b Computed and recorded roll rate (interim result)

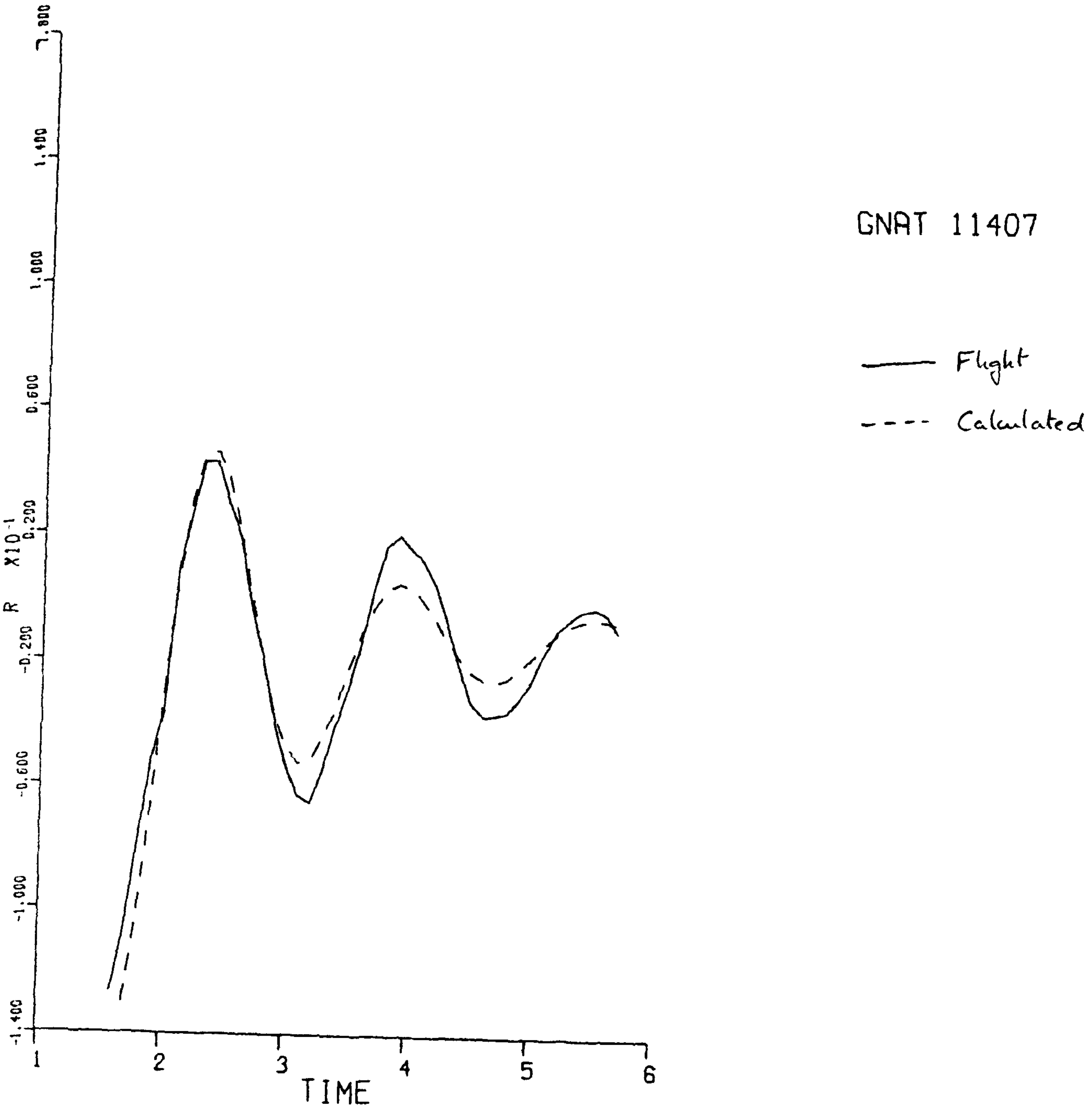

Fig.9c Computed and recorded yaw rate (interim result)

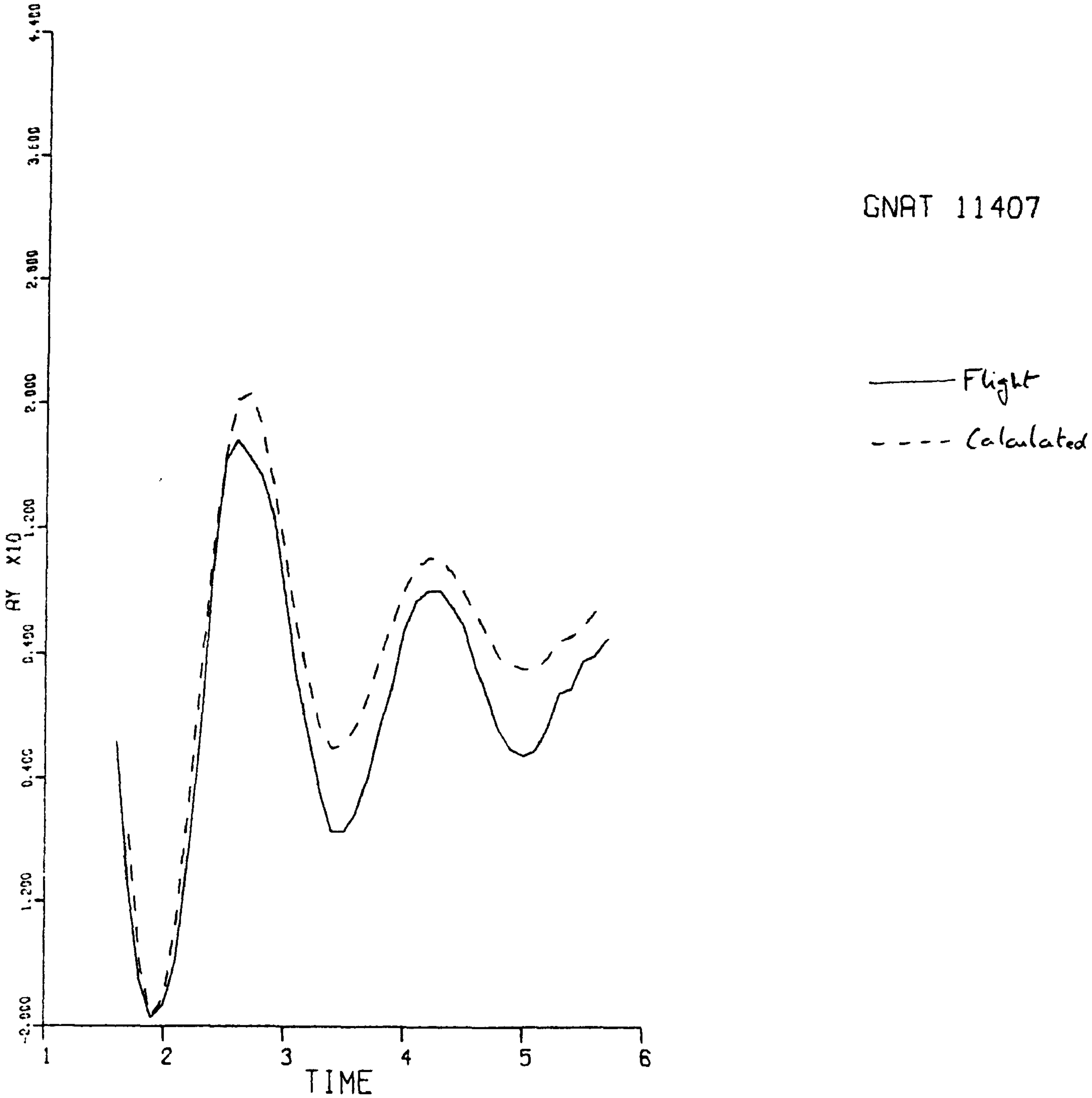

Fig.9d Computed and recorded lateral acceleration (interim result)

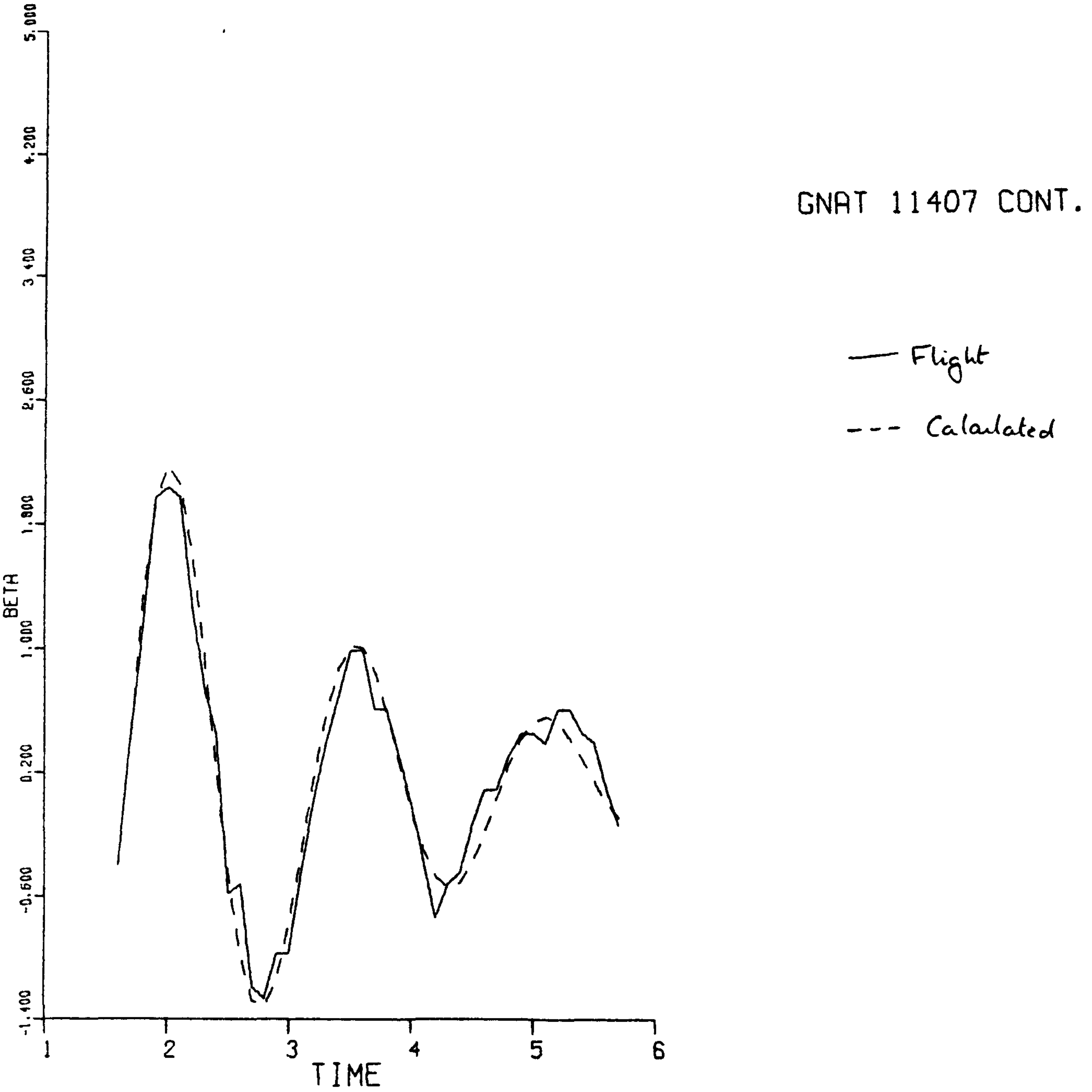

Fig.10a Computed and recorded sideslip angle (final result)

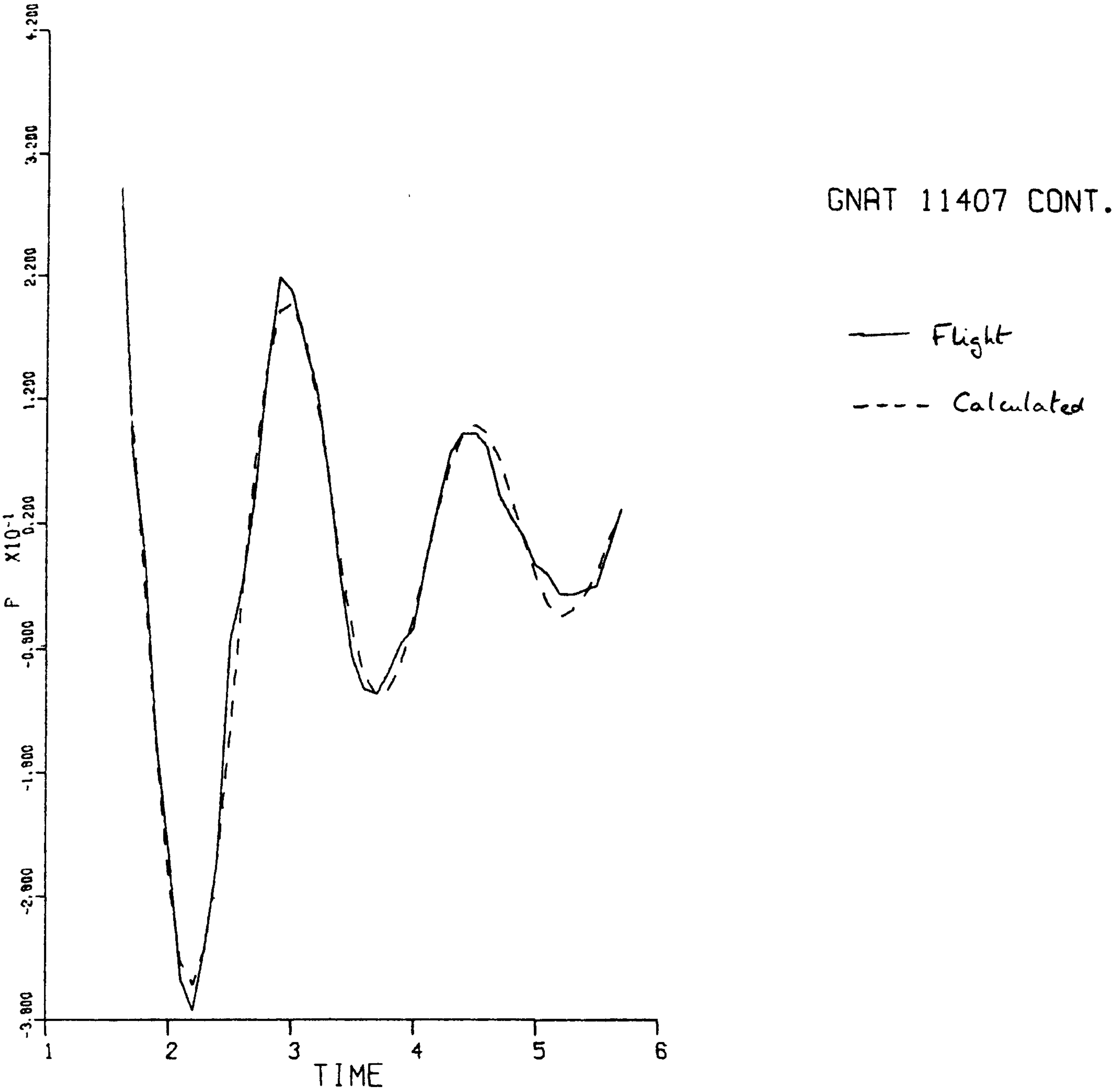

Fig.10b Computed and recorded roll rate (final result)

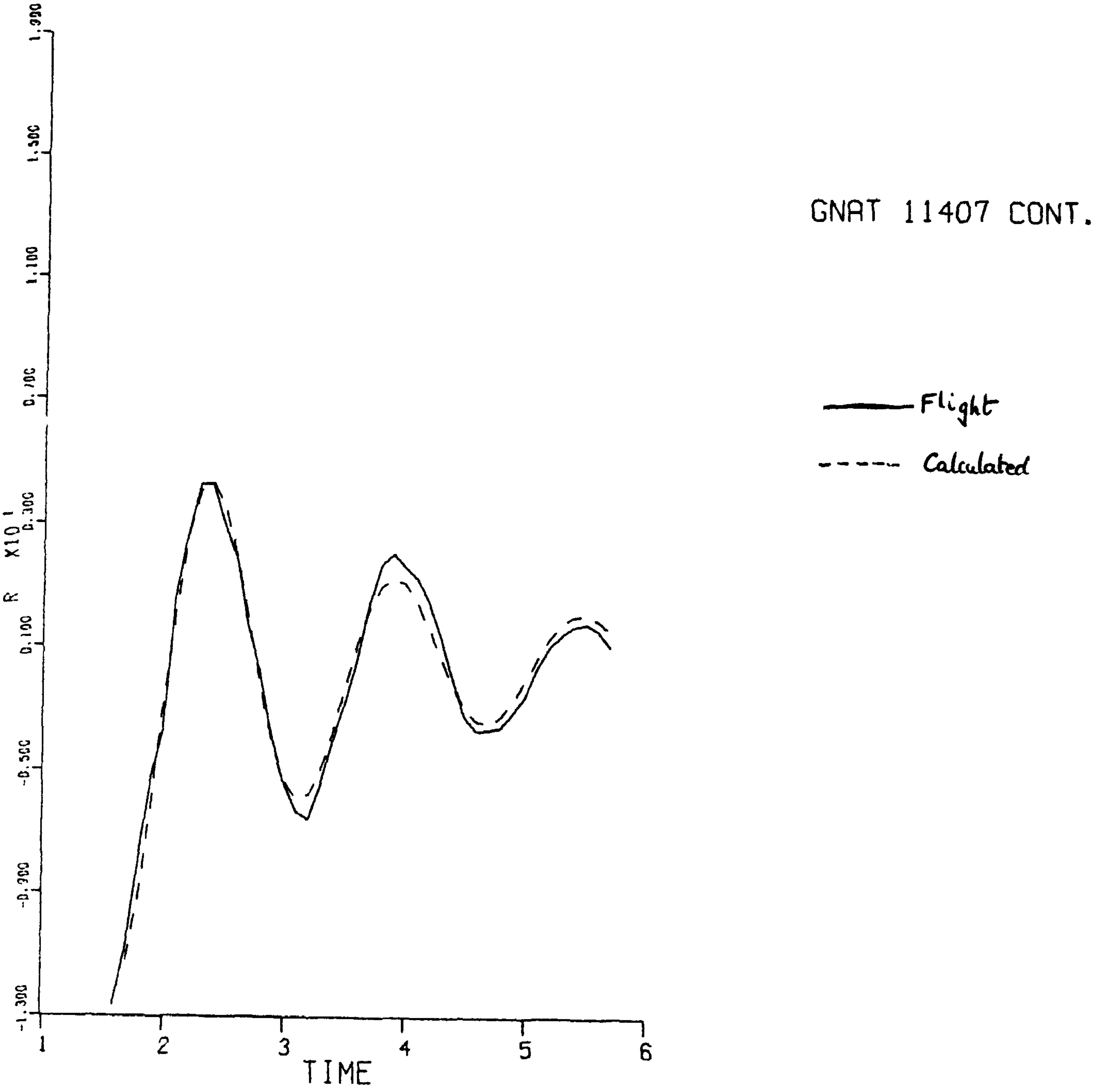

Fig.10c Computed and recorded yaw rate (final result)

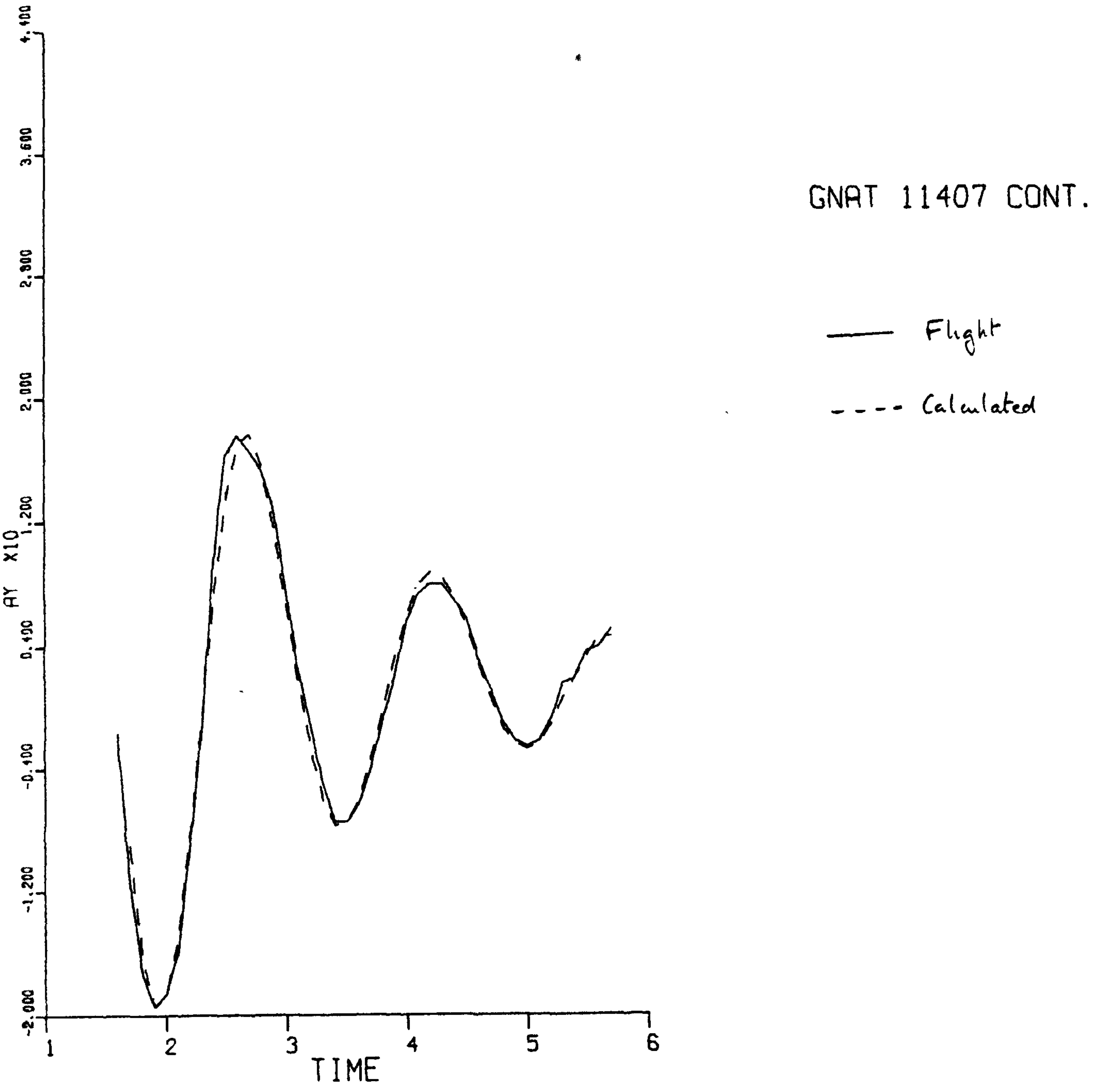

Fig.10d Computed and recorded lateral acceleration (final result)

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ARC CP No.1344
July 1975

A. Jean Ross
G. W. Foster

533.6.013.417 .
533.6.013.412 :
533.6.013.413 :
533.6.013.47 :
518.5

FORTRAN PROGRAMS FOR THE DETERMINATION OF AERODYNAMIC DERIVATIVES FROM TRANSIENT LONGITUDINAL OR LATERAL RESPONSES OF AIRCRAFT

Two Fortran computer programs are described, one for the analysis of longitudinal responses of aircraft, and one for the analysis of lateral responses in the presence of small longitudinal motion. The aerodynamic derivatives which affect the responses are determined by a Newton-Raphson technique to obtain iteratively the best least-squares fit to the observed data. A description is given of the numerical method, and its implementation in the programs, and then separate guides for users are provided for the two programs. An example is shown for each program.

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DETACHABLE ABSTRACT CARDS

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