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SUMMARY

Fibre waviness could affect the longitudinal shear and tensile moduli of unidirectional fibre reinforced composites. This paper considers these aspects theoretically, while the degree of fibre waviness which occurs in practice is determined experimentally. Attention is concentrated on CFRP but the analysis, albeit of an approximate nature, is quite general.

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CONTENTS

1 INTRODUCTION

It is well known that certain fibres (notably those of glass, carbon or boron) have exceptional longitudinal strength and/or stiffness properties. When such fibres are embedded in a matrix to form a 'fibre reinforced composite' they can be used in a structural context. Unidirectional fibre reinforced composites are used as struts, ties or reinforcing stringers in which configurations the longitudinal properties of the fibres show up to maximum advantage. A proper understanding of the transfer of load into such members requires a knowledge of the longitudinal shear and tensile moduli of the composite.

Measured values of the longitudinal shear modulus G_l^c tend to be higher than theory predicts for a regular hexagonal array and the values also exhibit a fair degree of scatter[1]. Measured values of the longitudinal tensile modulus E_l^c show much less scatter and are in good agreement with a simple 'rule of mixtures' formula; this is to be expected as it can be shown theoretically that E_l^c is virtually independent of the actual fibre array[2].

The experimentally determined increase in G_l^c over the predictions of elementary theory could possibly be explained by taking account of the waviness of fibres which occurs in practice, although such waviness would also be expected to reduce slightly the longitudinal tensile modulus E_l^c.

The increase in G_l^c due to fibre waviness stems from the fact that, in comparison with a composite with straight parallel fibres, the matrix-filled gaps between fibres are no longer constant but varying (see Fig.1). Furthermore, the longitudinal shear modulus of a unidirectional composite is primarily dependent on the shear modulus of the matrix rather than that of the fibres. Now the varying width of the matrix between fibres results in a varying shear stress in the matrix and it is this which causes an increase in the effective shear modulus. However, the fibres themselves are no longer in a simple shear field, and the differential shears applied to them by the matrix produce varying longitudinal stresses and displacements. These have the effect of attenuating the variations in shear stress in the matrix: the longer the wavelength the greater the attenuation and the less the increase in G_l^c.

The present paper determines theoretically the influence of the amplitude and wavelength of fibre waviness on the longitudinal shear and tensile moduli, while the amplitudes and wavelengths which occur in practice are determined experimentally.

2 PRINCIPAL NOTATION

A^f section area of fibre
d diameter of fibre
E tensile (Young's) modulus
G shear modulus
I second moment of area of fibre cross-section
k foundation modulus
ℓ half wavelength
P longitudinal tensile load in fibre
t thickness of lamina containing fibre and matrix
u longitudinal displacement of fibre
U,V strain energy, nondimensional strain energy
v volume fraction ($v_m + v_f = 1$)
w_0 average gap between fibres
w_1, w_2 gaps defined by equation (1)
x longitudinal coordinate
γ shear strain
δ amplitude of sinusoidal variation, defined by equation (1)
Δ longitudinal shearing displacement of fibres
ε direct strain
ϕ fibre misalignment angle
λ nondimensional wavelength
μ δ/w_0
σ direct stress
Ω defined in equation (33)

Suffices or indices m, f, c refer to matrix, fibre and composite respectively. Suffix 1 refers to longitudinal direction, e.g. E_1^f = longitudinal tensile modulus of fibre, G_1^c = longitudinal shear modulus of composite.

3 EFFECT OF FIBRE WAVINESS ON THE LONGITUDINAL SHEAR MODULUS

We consider first the idealised two-dimensional problem shown in Fig.1a in which shear is applied to a lamina of thickness t composed of alternating layers of matrix and fibre. Alternate fibres are straight and parallel, while the others vary sinusoidally as indicated. To simplify matters further we assume that the matrix only transmits shear and the fibres are 'line elements' with finite longitudinal stiffness but zero flexural rigidity and infinite shear stiffness. Account is taken of the influence of these features in separate analyses. From symmetry the applied shear has no influence on the longitudinal strain in the straight fibres and accordingly the analysis is essentially also applicable to the arrangement shown in Fig.1b which omits these fibres.

In Fig.2 attention is focused on a typical strip bounded by adjacent (and effectively rigid) straight fibres which are sheared by amounts $\pm\Delta$. The gaps between the central fibre and the adjacent straight fibres vary sinusoidally according to the relations

$$w_1, w_2 = w_0 \pm \delta . \sin (\pi x/\ell) \quad , \qquad (1)$$

where δ is the amplitude, and ℓ is the half wavelength. The following additional notation is introduced:

A^f = section area of fibre
E_1^f = longitudinal modulus of fibre
G^m = shear modulus of matrix
u = longitudinal (x-wise) displacement of fibre
ε_f = longitudinal strain in fibre
γ_1, γ_2 = shear strains in matrix.

The matrix and fibre are assumed to behave elastically and the longitudinal variations of shear and direct stresses will be determined by the principle of minimum strain energy. The shear strains in the matrix on either side of the central fibre are given by

$$\gamma_1 = (\Delta + u)/w_1 \ , \qquad \gamma_2 = (\Delta - u)/w_2 \ , \qquad (2)$$

and the direct strain in the fibre is given by

$$\varepsilon_f = du/dx \quad . \qquad (3)$$

The strain energy U in the matrix and fibre over a length 2ℓ is given by

$$U = \frac{1}{2}\int_0^{2\ell}\left\{G^m t\left(w_1\gamma_1^2 + w_2\gamma_2^2\right) + E_1^f A^f \varepsilon_f^2\right\}dx$$

$$= \frac{1}{2}\int_0^{2\ell}\left[\frac{G^m t}{w_0}\left\{\frac{(\Delta + u)^2}{w_0 + \delta\sin(\pi x/\ell)} + \frac{(\Delta - u)^2}{w_0 - \delta\sin(\pi x/\ell)}\right\} + E_1^f A^f\left(\frac{du}{dx}\right)^2\right]dx \quad (4)$$

in virtue of equations (1), (2) and (3).

The variation of u with x is such that U is a minimum. Now the displacement u satisfies the condition of repeatability:

$$u(x) = u(x + 2\ell) ,$$

while from symmetry:

$$u(x) = u(\ell - x) , \quad (5)$$

and

$$u(x) = -u(2\ell - x) .$$

The displacement u can therefore be represented by a Fourier sine series with odd integers and in what follows we truncate such a series to two terms and write

$$u = \Delta\{\phi_1 \sin(\pi x/\ell) + \phi_2 \sin(3\pi x/\ell)\} , \quad (6)$$

where the coefficients ϕ_1, ϕ_2 are to be determined from the conditions:

$$\frac{\partial U}{\partial \phi_1} = 0 , \quad \frac{\partial U}{\partial \phi_2} = 0 . \quad (7)$$

At this stage it is convenient to introduce the following nondimensional parameters:

$$\left.\begin{aligned} V &= \left(\frac{w_0}{2G^m t\ell\Delta^2}\right)U , \\ \mu &= \delta/w_0 , \\ \lambda &= \frac{\ell}{\pi}\left(\frac{2G^m t}{E_1^f A^f w_0}\right)^{\frac{1}{2}} . \end{aligned}\right\} \quad (8)$$

V is a nondimensional measure of the strain energy and is equal to unity if δ is zero; it thus provides a direct measure of the apparent or effective increase in the shear modulus of the matrix caused by longitudinal variations in fibre spacing. The parameter μ is a nondimensional measure of the amplitude of fibre waviness, while λ is a nondimensional measure of the half wavelength ℓ .

Substituting equations (6), (8) into (4) and integrating yields

$$V = (1 - \mu^2)^{-\frac{1}{2}} + h_1\phi_1^2 + h_2\phi_2^2 - 2h_3\phi_1 - 2h_4\phi_2 + 2h_5\phi_1\phi_2 \quad , \tag{9}$$

where

$$\left.\begin{aligned} h_1 &= \frac{1}{2\lambda^2} + \frac{\alpha}{\mu^2} \quad , \\ h_2 &= \frac{9}{2\lambda^2} - \frac{(4 - 3\mu^2)\beta}{\mu^6} \quad , \\ h_3 &= \frac{\alpha}{\mu} \quad , \\ h_4 &= \frac{\beta}{\mu^3} \quad , \\ h_5 &= \frac{\beta}{\mu^4} \quad , \\ \text{and} \qquad & \\ \alpha &= \frac{1}{\sqrt{(1 - \mu^2)}} - 1 \quad , \\ \beta &= 4 - \mu^2 - \frac{(4 - 3\mu^2)}{\sqrt{(1 - \mu^2)}} \quad . \end{aligned}\right\} \tag{10}$$

Now from equation (7),

$$\left.\begin{aligned} \phi_1 &= \frac{h_2h_3 - h_4h_5}{h_1h_2 - h_5^2} \quad , \\ \phi_2 &= \frac{h_1h_4 - h_3h_5}{h_1h_2 - h_5^2} \end{aligned}\right\} \tag{11}$$

so that V is known.

In Fig.3 the apparent or effective increase in the shear modulus of the matrix caused by longitudinal variations in fibre spacing is plotted against λ for various values of δ/w_0 using the relation

$$G^m_{eff}/G^m = V \quad . \tag{12}$$

The error involved in restricting the series for u to two terms is very small because the first term is the dominant one. Thus if we take ϕ_2 zero in equation (6) it may readily be shown that

$$\phi_1 = h_3/h_1 \quad ,$$

and hence

$$V = 1 + \frac{\mu^2}{2\lambda^2 + 1 - \mu^2 + \sqrt{(1 - \mu^2)}} \quad , \tag{13}$$

which differs from the two-term solution by less than 0.5% over the practical range of interest.

Finally we note that the present analysis is also valid for the configuration of Fig.1b if we write

$$w_0 = \tfrac{1}{2}(\text{average gap between fibres}) \quad .$$

3.1 Adaptation to the three-dimensional case

Alternative expressions for w_0 and λ are necessary if the results of this two-dimensional analysis are to be applied to the practical three-dimensional case. We make use of the relation

$$\frac{A^f}{tw_0} = \frac{v_f}{v_m} \quad , \tag{14}$$

and assume that w_0, the average gap between fibres, can be interpreted as equal to twice the width of the matrix annulus surrounding a fibre according to Hashin's model[3]. This implies that

$$\left(\frac{d}{d + w_0}\right)^2 = v_f \quad ,$$

and hence

$$w_0 = \left(\frac{1 - \sqrt{v_f}}{\sqrt{v_f}}\right) d \quad , \qquad (15)$$

and

$$\lambda = \frac{\ell}{\pi d} \left\{ \frac{2G^m}{E_1^f} \left(\frac{1 + \sqrt{v_f}}{1 - \sqrt{v_f}} \right) \right\}^{\frac{1}{2}} \quad . \qquad (16)$$

Of course, in the practical three-dimensional case the gaps between fibres vary across the section as well as longitudinally so that expression (15) above can only be regarded as the 'average of the average gaps'. By the same token the longitudinal variation is unlikely to be a simple harmonic so that there will also be some ambiguity in the interpretation of δ from actual measurements (see section 4.2). Fortunately this aspect is not serious because, as will be seen later, we are primarily interested in a narrow band of wavelengths specified, roughly speaking, by

$$1.5 \leqslant \lambda \leqslant 2.5 \quad .$$

When $\lambda < 1.5$, values of δ/w_0 sufficiently high to influence G^m_{eff} are not possible without failure of the composite, while if $\lambda > 2.5$ the increase in G^m_{eff} is negligible whatever the value of δ/w_0 .

3.1.1 The longitudinal shear modulus of the composite

If we denote by G^c_{s1} the longitudinal shear modulus of the three-dimensional composite with straight fibres - calculated, for example, for a regular hexagonal array[4] - the influence of longitudinally varying fibre spacing may now be estimated by replacing G^m by G^m_{eff} in the calculations for G^c_{s1} . If the fibres were rigid in shear this would lead to the following simple expression for the longitudinal shear modulus of the composite G^c_1 :

$$G^c_1/G^c_{s1} = G^m_{eff}/G^m \quad . \qquad (17)$$

This formula is also numerically adequate, though not strictly valid, when the finite shear rigidity of the fibres is taken into account. This is because the ratio G^c_{s1}/G^m is not sensitive to variations in the ratio G^m/G^f_1 provided that ratio is small in comparison with unity.

3.1.2 Limitation on δ/w_0 for fibres of circular cross-section

When the longitudinal fibre variations are such that adjacent fibres touch, the increase in G^m_{eff} will be somewhat less than that predicted by the two-dimensional theory because the contact zone between fibres of circular section differs markedly from that between fibres of rectangular section - as the two-dimensional model can be regarded. The effect can be accounted for by imposing an upper limit in Fig.3 for δ/w_0 which is somewhat less than unity. This upper limit, $(\delta/w_0)^*$ say, is determined here by equating G^c_1 for a composite with touching (straight) fibres of circular cross-section in a square array with G^c_1 for a two-dimensional model in which parallel fibres of width d are separated by matrix layers of width w'. The resulting value of w' represents a 'buffer zone' which is a lower limit to $(w_0 - \delta)$. The ensuing brief analysis is quite general but attention is confined numerically to the case in which $G^f_1/G^m = 20$, appropriate to CFRP.

For the square array Symm[4] and Mansfield[5] have shown that at maximum fibre packing

$$G^c_1 \sim 2.2(G^f_1 G^m)^{\frac{1}{2}} \tag{18}$$

provided $G^f_1 \gg G^m$, while for the two-dimensional model it may readily be shown that

$$G^c_1 = (d + w')\left(\frac{d}{G^f_1} + \frac{w'}{G^m}\right)^{-1} \quad . \tag{19}$$

Equating these two expressions yields, for $G^f_1/G^m = 20$,

$$w' = 0.0578\, d \quad , \tag{20}$$

which may be expressed in terms of w_0 by invoking equation (15).

Now w' is a lower limit to $(w_0 - \delta)$ so that an upper limit to δ is $(w_0 - w')$, hence

$$\begin{aligned}(\delta/w_0)^* &= 1 - w'/w_0 \\ &= 1 - \frac{0.578\sqrt{v_f}}{1 - \sqrt{v_f}} \quad . \end{aligned} \tag{21}$$

At $v_f = 0.6$, this yields

$$(\delta/w_0)^* = 0.80 \quad ,$$

which is a conveniently round number, bearing in mind the approximations involved. The corresponding curve in Fig.3 is identified by a broken line.

3.2 Limitations on δ/w_0 , λ imposed by fibre strength

If the stress in a fibre reaches a certain critical value the fibre breaks, and this imposes a restriction on the range of values of δ/w_0 , λ which are meaningful. Thus, from equation (1) the longitudinal curvature of the fibre is given by

$$\frac{d^2w}{dx^2} = \frac{\pi^2\delta}{\ell^2}\sin(\pi x/\ell) \tag{22}$$

and hence, from engineers' theory of bending - assuming the fibre to be unstressed when straight - the maximum bending strain in the fibre is given by

$$\varepsilon^f_{max} = \frac{\pi^2\delta d}{2\ell^2} \quad . \tag{23}$$

To express this in terms of λ , etc. it is necessary to invoke equations (15), (16) which yield

$$\varepsilon^f_{max} = \left(\frac{1+\sqrt{v_f}}{\sqrt{v_f}}\right)\left(\frac{G^m}{E^f_1}\right)\left(\frac{\delta}{w_0}\right)\frac{1}{\lambda^2} \quad . \tag{24}$$

This critical relationship between δ/w_0 and λ is shown in Fig.3 for various values of ε^f_{max} assuming $E^f_1/G^m = 50$, appropriate to CFRP, and $v_f = 0.6$.

3.2.1 Limitations on G_{eff}/G^m imposed by fibre strength

From Fig.3 it is seen that at $v_f = 0.6$ and $\varepsilon^f_{max} = 0.01$, say, the maximum possible value of G_{eff}/G^m is about 1.075 at $\lambda \simeq 1.9$. Similar critical values can be determined for other values of v_f by using equations (21) and (24) to define $(\delta/w_0)^*$ and λ_{crit} , which are then substituted into equations (9) or (13) to yield $(G_{eff}/G^m)_{max}$. Table 1 below shows $(\delta/w_0)^*$, λ_{crit} and $(G_{eff}/G^m)_{max}$ for a wide range of fibre volume fractions. It is seen that $(G_{eff}/G^m)_{max}$ varies with v_f in a 'flat topped' manner, reaching a maximum at about $v_f = 0.45$, and falling off markedly only at very high and very low values of v_f . In practice it is probable that very few fibres will be flexurally prestressed to such an extent and of those that are, it is unlikely that they will also exhibit the maximum amplitude of waviness $(\delta/w_0)^*$. Thus, insofar as they apply to the composite as a whole rather than localised regions between individual fibres, the values of $(G_{eff}/G^m)_{max}$ in Table 1 must be regarded as extreme upper limits which, on statistical grounds will not be approached in practice.

Table 1

Values of $(G_{eff}/G^m)_{max}$, etc. for $\varepsilon^f_{max} = 0.01$, $E^f_1/G^m = 50$

v_f	$(\delta/w_0)^*$	λ_{crit}	$(G_{eff}/G^m)_{max}$
0.10	0.973	2.85	1.057
0.15	0.963	2.63	1.067
0.20	0.953	2.49	1.071
0.25	0.942	2.38	1.076
0.30	0.930	2.29	1.079
0.35	0.916	2.22	1.081
0.40	0.901	2.16	1.082
0.45	0.882	2.10	1.082
0.50	0.860	2.04	1.082
0.55	0.834	1.98	1.080
0.60	0.801	1.92	1.077
0.65	0.760	1.85	1.073
0.70	0.704	1.76	1.067
0.75	0.626	1.64	1.058
0.80	0.510	1.47	1.044
0.85	0.317	1.15	1.022

3.3 Limitations on δ/w_0 , λ imposed by matrix stresses

In the (externally) unloaded state of CFRP the normal stresses between fibre and matrix should not exceed those necessary to break the fibre-matrix bond or the matrix itself, but the stresses are also limited by the method of manufacture. Such limitations impose further restrictions on the meaningful range of values of δ/w_0 , λ which are discussed below.

From equation (1), again assuming that the fibres are free from stress when straight, the longitudinal distribution of transverse load/unit length $p(x)$ acting on a single fibre is given by

$$p(x) = E_1^f I \frac{d^4 w}{dx^4}$$

$$= \frac{\pi^5 E_1^f \delta d^4}{64\ell^4} \sin(\pi x/\ell) \qquad (25)$$

for a fibre of circular cross-section.

These loads/unit length result from 'built-in' or manufacturing stresses and they are therefore very dependent on the viscous and visco-elastic properties of the matrix during the curing process. If the matrix were inviscid prior to curing, individual fibres would be free to straighten themselves unless they were misaligned and tangling with other fibres. However, although such tangling may well occur, the liquid matrix is highly viscous so that for values of $v_f \geqslant 0.6$, say, the inter-fibre gaps are sufficiently small to retard the stress relieving flow. The resultant pattern of internal stress in the hardened matrix will be complex; in particular, the stresses acting at the fibre-matrix interface, which maintain the fibre in its curved state, will have normal and shearing components which vary circumferentially and longitudinally. In what follows we arbitrarily ignore the shearing component and assume that the circumferential distribution of normal stress acting at any fibre-matrix interface is constant over half the circumference and zero elsewhere; the magnitude varies longitudinally as $\sin(\pi x/\ell)$. On this basis the maximum normal stress is given by

$$\sigma^m_{max} = \frac{\pi^5 E_1^f \delta d^3}{64\ell^4} , \qquad (26)$$

from equation (25).

The corresponding value of the strain ε^m_{max} can be expressed in terms of δ/w_0, λ as

$$\varepsilon^m_{max} = \frac{\pi(1 + \surd v_f)^2}{16(1 - \surd v_f)\surd v_f}\left(\frac{G^m}{E_1^f}\right)\left(\frac{G^m}{E^m}\right)\left(\frac{\delta}{w_0}\right)\frac{1}{\lambda^4} , \qquad (27)$$

which is shown in Fig.3 for various values of ε^m_{max} assuming $v_f = 0.6$, $E_1^f/G^m = 50$, $G^m/E^m = 0.4$.

By comparing these curves with those for given values of ε^f_{max}, it can be concluded that the limitations imposed on δ/w_0, λ by matrix stresses are less severe than those imposed by fibre stresses.

4 EXPERIMENTAL DETERMINATION OF FIBRE WAVINESS

4.1 Preparation of the specimen

A typical 2mm thick laminate of carbon fibre reinforced plastic (high strength, type II carbon fibre/ERCA 4617 resin) was selected for examination. The composite was fabricated from ten resin preimpregnated carbon fibre sheets by an autoclave process[6] which produces a composite of high quality.

A random sample 10 × 10 × 2 mm was cut from the laminate, mounted in a resin block and polished in a plane perpendicular to the fibre axes. Successive operations of abrading and polishing the surface led to a series of photographs at sections separated by small intervals along the fibre axes. Because of the difference in hardness between the fibres and the matrix, a small amount of differential polishing was inevitable, leaving the fibre ends slightly rounded and proud of the matrix. In optical photomicrography this leads to some ambiguity in determining the exact boundary of a fibre and for this reason a scanning electron microscope was employed. The large depth of field possible with this instrument allowed adequate measurement of the separation of the fibres. Fig.4 shows part of the section at ×100 magnification. Lines of resin-rich composite between the original individual laminae are clearly visible. The light circle at C is a fine wire mounted in the resin block as an identification mark. Area A, which has a local fibre volume fraction of 0.6, was chosen to be representative of the more uniform fibre distribution between the laminating lines.

The section was then abraded and re-polished as previously described to obtain six sectional photographs of area A at ×1000 magnification, shown in Fig.5a-f. It was difficult to obtain equal gaps between sections and the distances of successive sections from the first are 0.025, 0.06, 0.10, 0.15 and 0.18 mm with an accuracy of ±0.005 mm. The average of the fibre diameters is approximately 8 μm so that, with interest centred on values of λ of about 2, the corresponding half wavelength ℓ is approximately 0.09 mm.

4.2 Measurement of fibre waviness

Fig.5a-f are typical of CFRP sections at a fibre volume fraction of 0.6. At this value of v_f the theoretical value of the average gap between fibres, as defined by equation (15), is 0.29 d whereas the minimum gap, w_{min} say, between fibres in a regular hexagonal array is 0.23 d. This follows from the general relation:

$$w_{min}/d = \left(\frac{\pi}{2v_f\sqrt{3}}\right)^{\frac{1}{2}} - 1 \quad . \tag{28}$$

The fact that w_0 and w_{min} differ is a consequence of the application of two-dimensional analysis to a three-dimensional problem. However, the fact that $(w_0 - w_{min})$ is almost identically equal to w', introduced in section 3.1.2, is a fortuitous but reassuring check on the arguments employed there.

From a practical standpoint the actual minimum gap, w^*_{min} say, between adjacent fibres is the simplest to measure and, insofar as this differs from the theoretical value of w_{min}, the difference $|w_{min} - w^*_{min}|$ could be interpreted as a deviation from the average gap. However, measurements of w^*_{min} show that approximately 60% of 'adjacent' fibres are actually in contact (i.e. $w^*_{min} = 0$), while for the remainder w^*_{min} varies up to about 2d. It is thus more realistic to recognise the fact that the *local* fibre volume fraction varies considerably across a section. The influence of fibre waviness on the longitudinal shear modulus is governed largely by the variation in the matrix gap between fibres and is thus essentially dependent on localised phenomena. Thus a plot of the longitudinal variation of w^*_{min} is more meaningfully interpreted as a variation - assuming there is one - about a local mean.

For those fibres in Fig.5d which are in contact, or nearly so, measurements at surrounding sections indicate only small or negligible changes in w^*_{min}. This is to be expected because within local regions of high fibre volume fraction, fibre waving is necessarily more restricted. Indeed, when

$$w^*_{min}/d \leqslant 0.05 \quad ,$$

which corresponds to a range of local fibre volume fractions given by

$$v_f \geqslant 0.82 \quad ,$$

it follows from Table 1 that the influence on G^m_{eff} is much reduced, even if the fibre waving is at its most extreme.

In what follows we have confined attention to those pairs of adjacent fibres in Fig.5d for which

$$w^*_{min} \approx w_{min} \quad ,$$

so that Fig.3 is directly applicable.

There are 17 such pairs identified by the adjacent fibres ab, bc, de, fg, ... etc. and Fig.6 shows for each pair the variation of $(w_{min} - w^*_{min})/w_0$ with x, the distance along the fibres from the first section (Fig.5a). It is seen that x varies from 0 to 0.18 mm, a distance of approximately $2\ell_{crit}$; thus it should be possible to identify half sine wave components of length ℓ_{crit}, or thereabouts, if they occur within the measured region. Furthermore, if the gaps between fibres varied in the simple manner of equation (1), it would be possible to identify the term $(w_{min} - w^*_{min})/w_0$ with $(\delta/w_0) \sin (\pi x/\ell)$.

Now the measured gap between fibres w^*_{min} is necessarily positive or zero, so that $(w_{min} - w^*_{min})/w_0$ cannot exceed w_{min}/w_0, i.e. 0.8. However, w^*_{min} is not restricted to the upper limit imposed by the assumed theoretical variation of the gap between fibres. Thus values of $(w_{min} - w^*_{min})/w_0$ more negative than -0.8 are possible and, indeed, frequently occur. In practice this means that initially adjacent fibres have wandered sufficiently far apart for them to be either no longer adjacent (e.g. fibres fg, ℓm) or still adjacent but in a region of very low local fibre volume fraction (e.g. fibres qr). For these reasons we confine attention in Fig.6 to positive values of $(w_{min} - w^*_{min})/w_0$. In this region it is seen that the gap between fibres varies approximately sinusoidally but that the half wavelength ℓ is markedly more than ℓ_{crit}. Indeed, the shortest half wavelength, which occurs for fibres de, is approximately $2\ell_{crit}$.

If this sample is typical of CFRP it can be concluded that the effect of fibre waviness on the longitudinal shear modulus is negligible.

5 EFFECT OF MISALIGNMENT OF STRAIGHT FIBRES ON THE LONGITUDINAL SHEAR MODULUS

The measurements of fibre waviness discussed in section 4.2 indicate a predominance of wavelengths yielding values of λ considerably in excess of λ_{crit}. At the same time fibre misalignment angles dw/dx of about 0.05 (see Fig.6) are not uncommon, and it is appropriate to augment the previous theoretical analysis of fibre waviness by an analysis of the effect of misalignment of straight fibres on the longitudinal shear modulus. We consider a basically unidirectional composite built up from a number of laminates in each of which the fibres are straight and parallel. Alternate laminates, however, have small misalignments specified by

$$\frac{dw}{dx} = \pm\tfrac{1}{2}\phi \quad , \tag{29}$$

where ϕ is the total misalignment angle between adjacent laminates.

Under an applied longitudinal shear it may be shown from symmetry that the deformation of the composite is one of pure shear strain γ , say. In terms of this shear strain the extensional strain in the fibres is therefore given by

$$\begin{aligned} \varepsilon_f &= \pm\gamma \sin \tfrac{1}{2}\phi \cos \tfrac{1}{2}\phi \\ &\approx \pm\tfrac{1}{2}\phi\gamma \quad . \end{aligned} \tag{30}$$

The extensional energy in the fibres/unit volume of composite is thus $\frac{1}{8}v_f E_1^f \phi^2 \gamma^2$ and the shear energy in the composite/unit volume is $\frac{1}{2}G_1^c\gamma^2$. The total strain energy/unit volume is therefore

$$\tfrac{1}{2}\gamma^2\left\{G_1^c + \tfrac{1}{4}v_f E_1^f \phi^2\right\} \quad ,$$

and it is seen that the influence of such fibre misalignment is to increase the longitudinal shear modulus of the composite by a factor K , say, where

$$K = 1 + \tfrac{1}{4}v_f \phi^2 \left(\frac{E_1^f}{G_1^c}\right) \quad . \tag{31}$$

For CFRP at $v_f = 0.6$, we have

$$E_1^f/G^m = 50 \quad , \quad \text{say}$$

$$G_1^c/G^m \approx 3.5 \quad ,$$

so that

$$K \approx 1 + 2.1\phi^2 \quad .$$

Thus, for example, if

$$\phi = 0.05 \quad ,$$

this yields

$$K = 1.005 \quad ,$$

so that the increase is negligible; but if

$$\phi = 0.1 \quad , \quad \text{say,}$$

$$K = 1.02$$

and the effect is perhaps just noticeable.

6 EFFECT OF FIBRE WAVINESS ON LONGITUDINAL TENSILE MODULUS

For the fibre matrix arrays shown in Fig.1 the longitudinal direct stiffness of the curved fibres is necessarily less than for the straight fibres. However, for the range of values of δ/w_0, λ that we are considering, it is shown below that the reduction in stiffness is negligible. Thus, confining attention to a single curved fibre and regarding it as a curved beam on an elastic foundation we can write

$$E_1^f I \frac{d^4}{dx^4}\left\{w - \delta \sin(\pi x/\ell)\right\} + k\left\{w - \delta \sin(\pi x/\ell)\right\} = P\frac{d^2 w}{dx^2}, \quad (32)$$

where w = transverse deflexion of 'beam' measured from a straight centre-line,

P = longitudinal tensile load,

k = foundation modulus (defined later).

Equation (32) may be integrated to yield:

$$\left.\begin{aligned} w &= \frac{\delta \sin(\pi x/\ell)}{1 + \Omega} \\ \text{where} \quad \Omega &= \frac{P\pi^2/\ell^2}{k + \pi^4 E_1^f I/\ell^4} \,. \end{aligned}\right\} \quad (33)$$

Now the average longitudinal strain due to fibre bending, ε^b, say, is given by

$$\begin{aligned} \varepsilon^b &= \frac{1}{2\ell}\int_0^\ell \left[\frac{\pi^2\delta^2}{\ell^2}\cos^2(\pi x/\ell) - \left(\frac{dw}{dx}\right)^2\right]dx \\ &= \frac{\pi^2\delta^2}{4\ell^2}\left(1 - \frac{1}{(1+\Omega)^2}\right) \,. \end{aligned} \quad (34)$$

The longitudinal strain due to fibre extension is given by

$$\varepsilon^t = P/(E_1^f A^f) \,, \quad (35)$$

so that the total longitudinal strain is given by

$$\varepsilon = \varepsilon^t + \varepsilon^b \,. \quad (36)$$

The longitudinal stiffness is accordingly given by

$$\frac{dP}{d\varepsilon} = \left(\frac{d\varepsilon^t}{dP} + \frac{d\varepsilon^b}{dP}\right)^{-1}$$

$$= \left[\frac{1}{E_1^f A^f} + \frac{\pi^4 \delta^2}{2\ell^4 (k + \pi^4 E_1^f I/\ell^4)(1 + \Omega)^3}\right]^{-1} \quad . \qquad (37)$$

In order to determine the relative importance of terms in the above expression for $dP/d\varepsilon$ it is convenient first to confine attention to k and $\pi^4 E_1^f I/\ell^4$. The foundation modulus k is such that

$$kW = p(x)$$

where $W(= w - \delta \sin (\pi x/\ell))$ is the transverse displacement of the fibre under end load and $p(x)$ is the resulting transverse load/unit length. Now we can write

$$\frac{W^{\bullet}}{w_0} \approx \frac{\sigma_2^m}{E^m}$$

and

$$p(x) \approx 2d\sigma_2^m$$

where σ_2^m is the transverse stress in the matrix averaged across a diameter and the factor 2 is introduced because of the presence of restraining matrix on either side of the fibre. Accordingly,

$$k \approx \frac{8}{\pi}\left(\frac{v_f}{v_m}\right)E^m \qquad (38)$$

and the ratio

$$(\pi^4 E_1^f I/\ell^4)/k \approx \frac{\pi^2 v_m}{128 v_f}\left(\frac{1 + \sqrt{v_f}}{1 - \sqrt{v_f}}\right)^2\left(\frac{G^m}{E_1^f}\right)\left(\frac{G^m}{E^m}\right)\frac{1}{\lambda^4} \qquad (39)$$

which is negligible in comparison with unity for the range of values of λ under consideration. This means that the matrix is much more effective than the fibre flexural rigidity in maintaining the curved shape of a fibre under end load. The expression for Ω is thus adequately given by

$$\Omega \approx \frac{P\pi^2}{k\ell^2}$$

$$\approx \frac{\pi^2(1 + \sqrt{v_f})^2}{16v_f}\left(\frac{G^m}{E^m}\right)\left(\frac{\sigma_1^f}{E_1^f}\right)\frac{1}{\lambda^2} \tag{40}$$

which is also negligible in comparison with unity. This means that the curved fibre embedded in matrix does not exhibit any noticeable nonlinear behaviour under end load. The expression for $dP/d\varepsilon$ can therefore be further simplified to give

$$\frac{dP}{d\varepsilon} \approx \frac{E_1^f A^f}{1 + \dfrac{\pi^4\delta^2 E_1^f A^f}{2k\ell^4}} \tag{41}$$

and finally we note that

$$\frac{\pi^4\delta^2 E_1^f A^f}{2k\ell^4} \approx \frac{\pi^2 v_m(1 + \sqrt{v_f})^2}{16v_f^2}\left(\frac{G^m}{E_1^f}\right)\left(\frac{G^m}{E^m}\right)\left(\frac{\delta}{w_0}\right)^2\frac{1}{\lambda^4} \quad , \tag{42}$$

which is again small in comparison with unity, so that

$$\frac{dP}{d\varepsilon} \approx E_1^f A^f \quad . \tag{43}$$

The fibres thus behave as if they were straight.

7 CONCLUSIONS

Simplified analyses have been made of a two-dimensional model of a unidirectional fibre reinforced composite under longitudinal shear and tension. Account is taken of fibre waviness by assuming that the fibres are displaced laterally by an amount that varies sinusoidally. The solutions have been adapted to describe the practical three-dimensional composite in which fibre waviness is introduced, albeit unintentionally, in the process of manufacture. It is shown that an increase in the amplitude of the fibre waviness causes an increase in the longitudinal shear modulus G_1^c, but the increase is smaller the longer the wavelength. For very short wavelengths increases in G_1^c in excess of 30% are theoretically possible. However, for fibres which were straight in the stress-free state prior to manufacture of the composite there are restrictions on the degree of waviness which the fibre or matrix can tolerate without breaking. Because of this, interest is restricted to a narrow band of wavelengths. In unidirectional carbon fibre reinforced plastic (CFRP), for example, increases in G_1^c of up to 8% are theoretically possible at half wavelengths ℓ_{crit} of about ten times the fibre diameter. The degree of waviness which occurs in practice in CFRP has been estimated from measurements of fibre positions at successive sections. These show marked variations in the local density of fibres with about half the fibres in contact with others, thus effectively prohibiting any waviness. For the remaining fibres the amplitude of waving can be severe but the half wavelengths exceed ℓ_{crit} by factors of 2 or more. It follows that fibre waviness in CFRP has a negligible influence on the longitudinal shear modulus. A separate analysis on the influence of misalignment of straight fibres also predicts a negligible increase in the longitudinal shear modulus for misalignment angles comparable to those which occur in practice.

Attention is also given to the influence of fibre waviness on the longitudinal tensile modulus. It is shown that for all practical composites the matrix is sufficiently rigid to prevent any effective change in the amplitude of fibre waviness under end load. Thus, and in contrast to the behaviour of an unsupported wavy fibre, the matrix-embedded wavy fibre does not exhibit any noticeable nonlinear behaviour under end load and the influence of fibre waviness on the longitudinal tensile modulus is negligible.

With fibre waviness eliminated as a possible cause of the discrepancy between theory and experiment in predicting the longitudinal shear modulus of unidirectional fibre reinforced composites, a further paper will consider the influence of fibre distribution on the composite moduli, focusing attention on the ways in which the distributions observed in practice differ from the idealised hexagonal array.

Finally, we note that the techniques developed for adapting simplified two-dimensional models to describe the practical three-dimensional composite will also be relevant in simplified load diffusion studies of composites with broken fibres. In addition, the simplified analysis and experimental observation of fibre waviness, with its attendant built-in stresses, will be relevant in the fields of fracture and fatigue.

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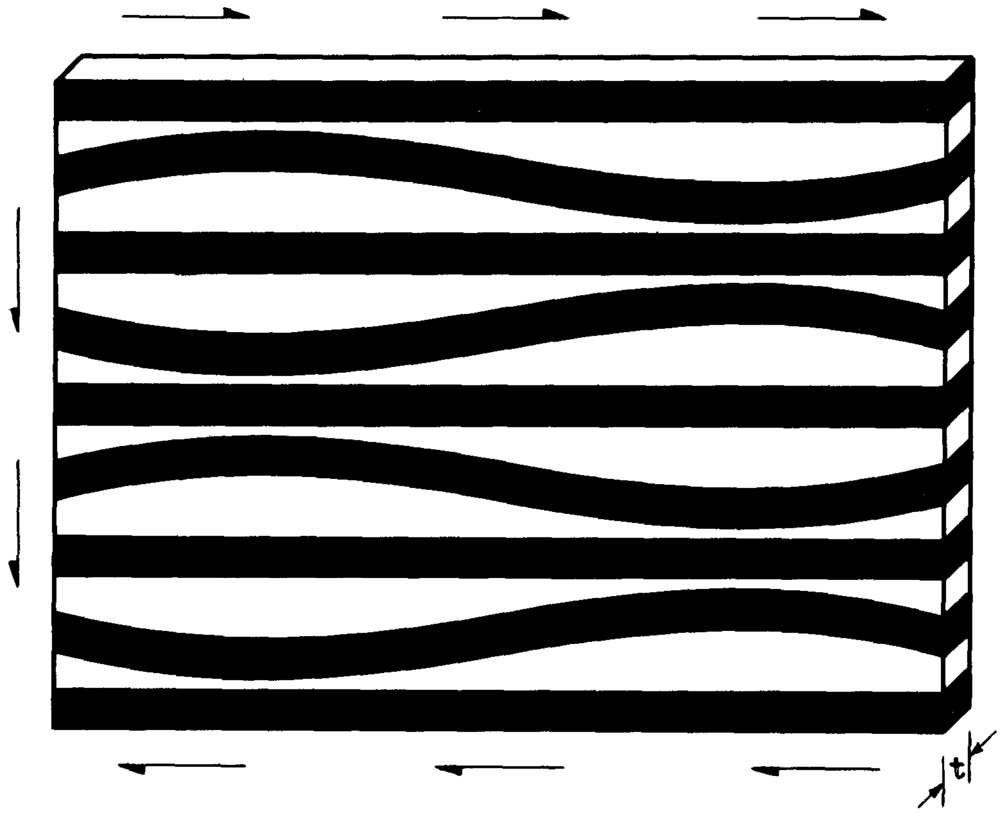
REFERENCES

No.	Author	Title, etc.
1	W. Paton A.H. Lockhart	Longitudinal shear characteristics of unidirectional carbon fibre composites. NEL Report 547 (1973)
2	Z. Hashin B.W. Rosen	The elastic moduli of fibre-reinforced materials. J. Appl. Mech., 31, 223 (1964)
3	Z. Hashin	Theory of fibre reinforced materials. NASA CR-1974 (March 1972)
4	G.T. Symm	The longitudinal shear modulus of a unidirectional fibrous composite. J. Composite Materials, 4, 426 (1970)
5	E.H. Mansfield	On the elastic moduli of unidirectional fibre reinforced composites. ARC R & M 3782 (1974)
6	P.D. Ewins R. Childs B. Clarke	RAE Report (to be published)



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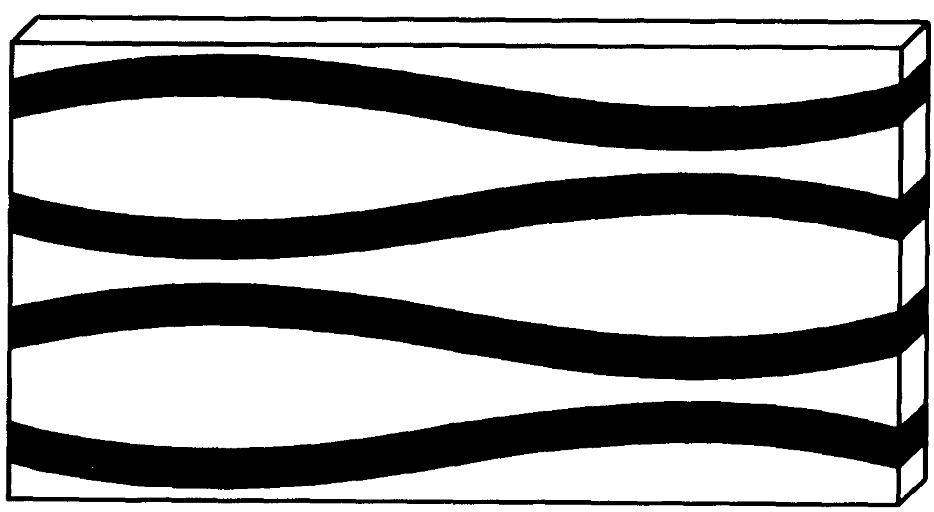

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Fig. 1a&b Sinusoidally varying fibres in lamina under shear

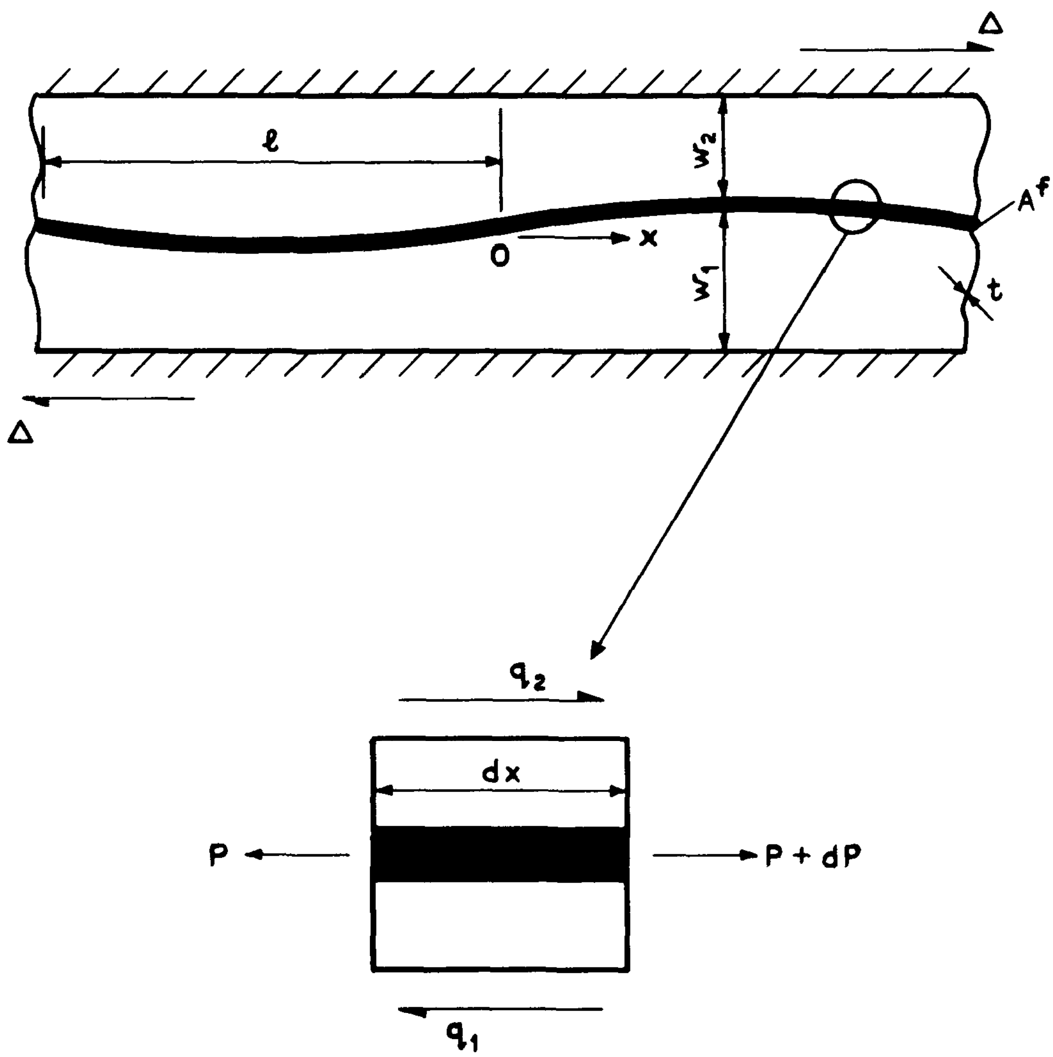

Fig.2 Notation for sinusoidally varying fibre

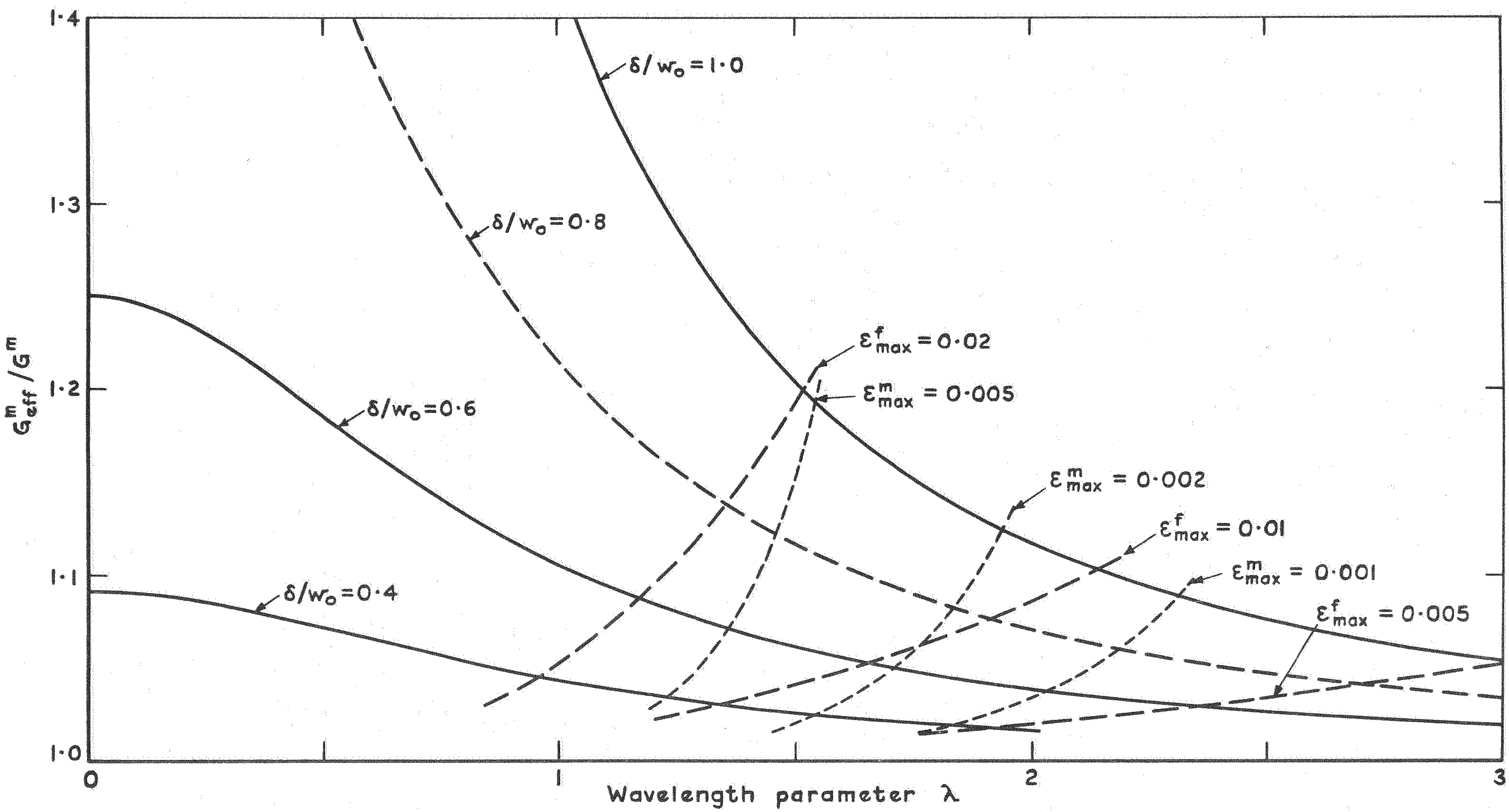

Fig.3 Variation of G^m_{eff}/G^m with δ/w_o and λ

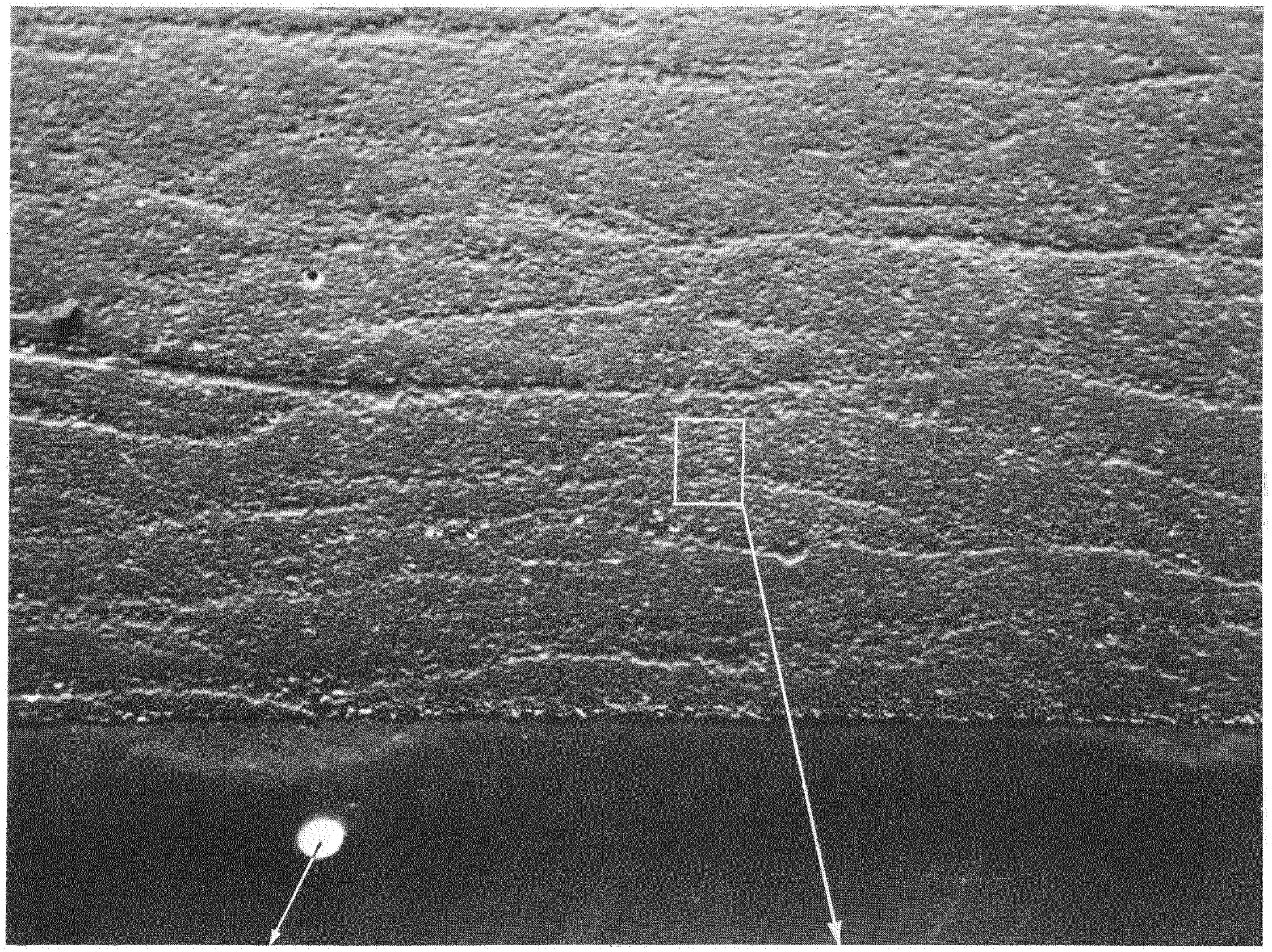

Fig.4 Section of CFRP specimen (x100)

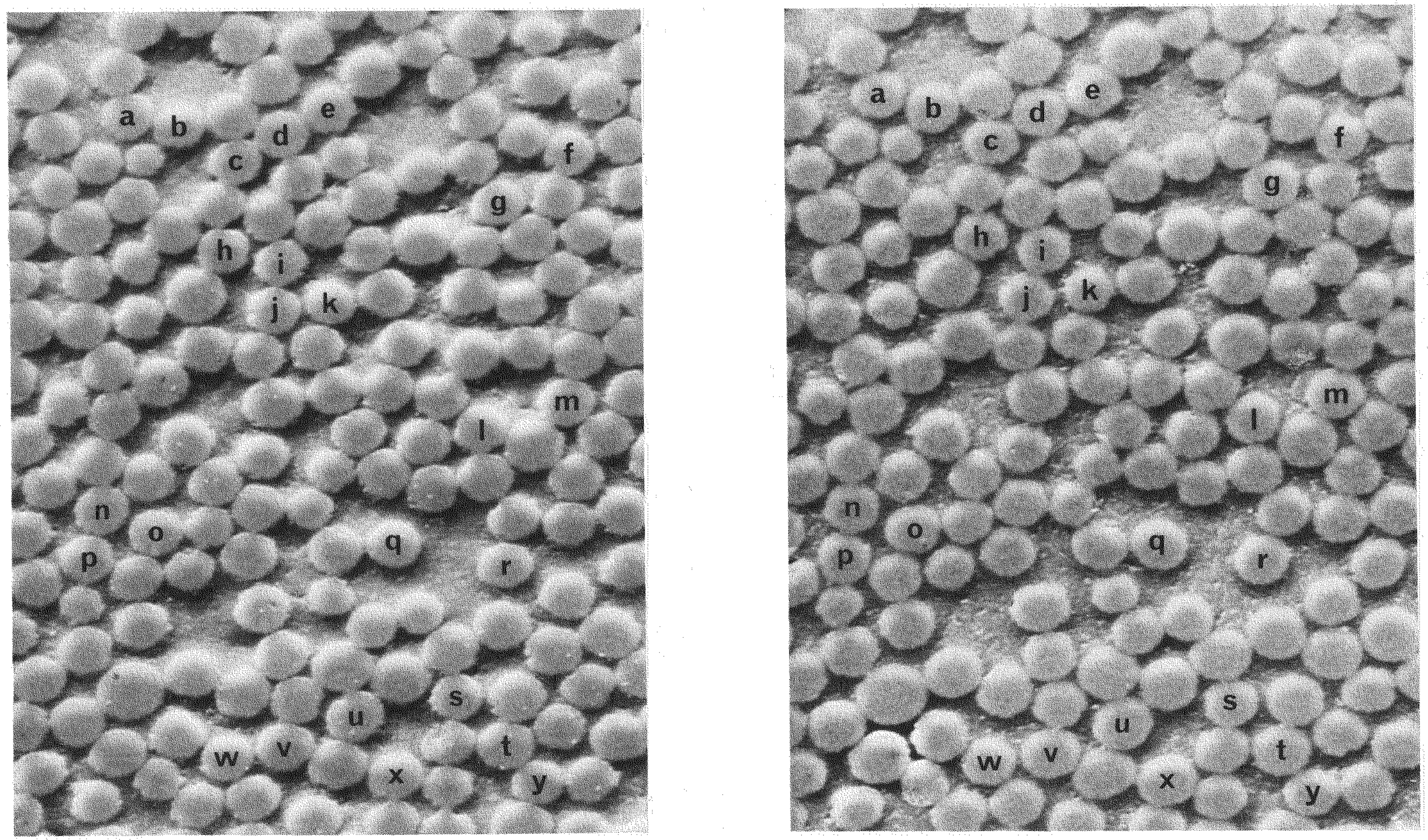

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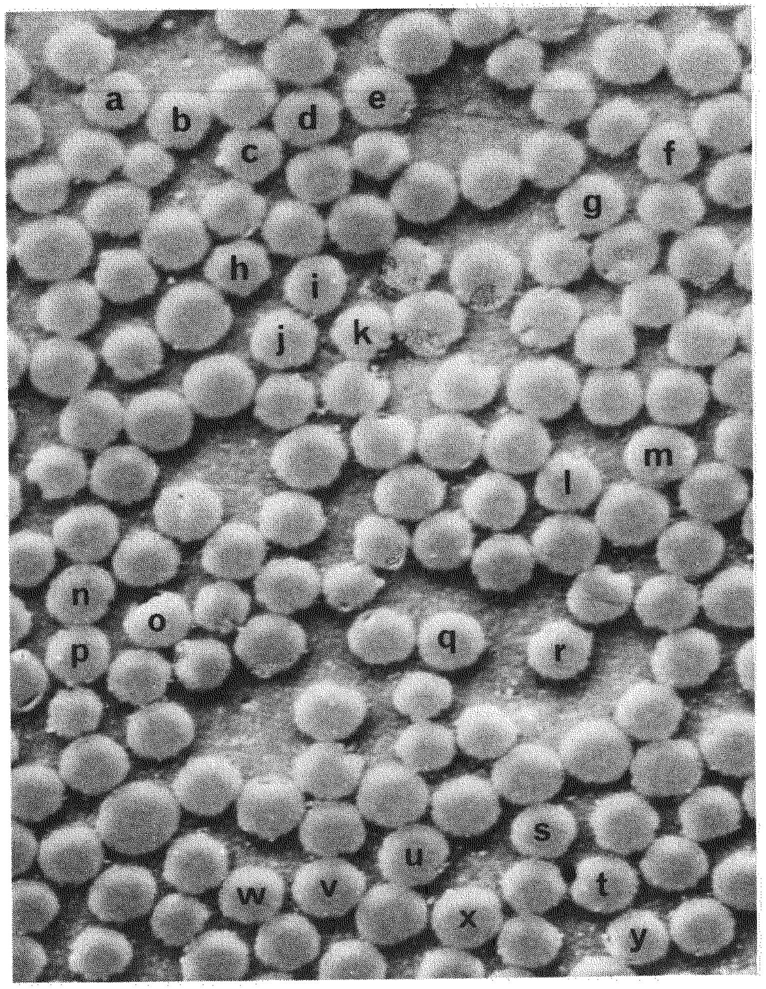
Fig.5a-f Successive CFRP sections at $v_f = 0.6$, x1000



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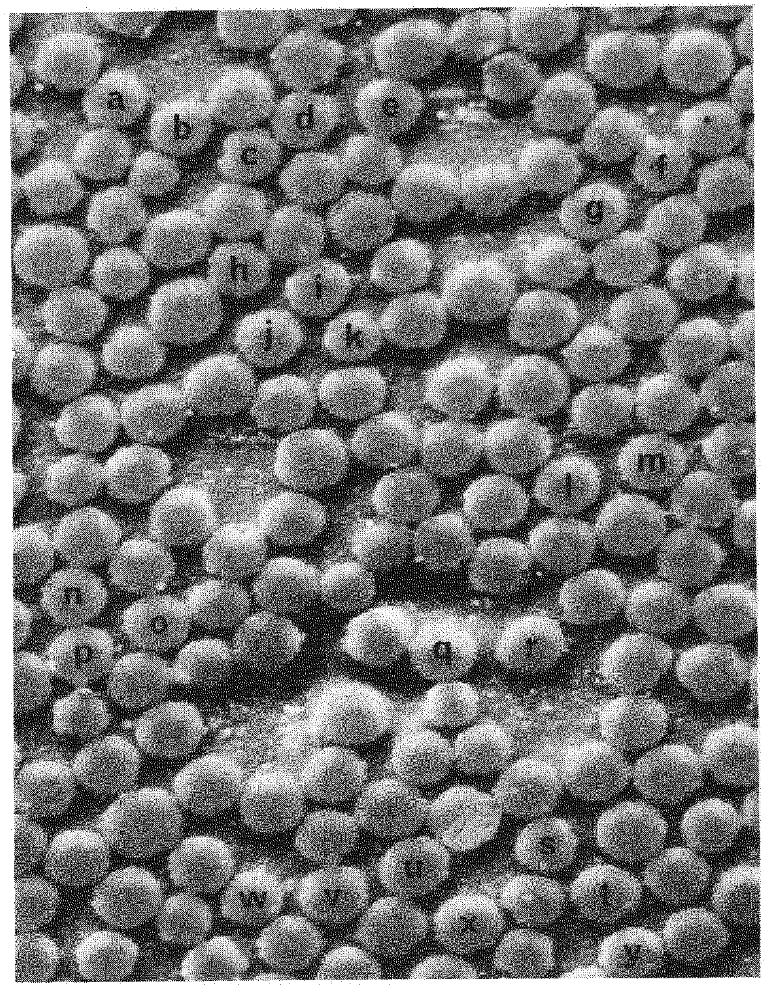

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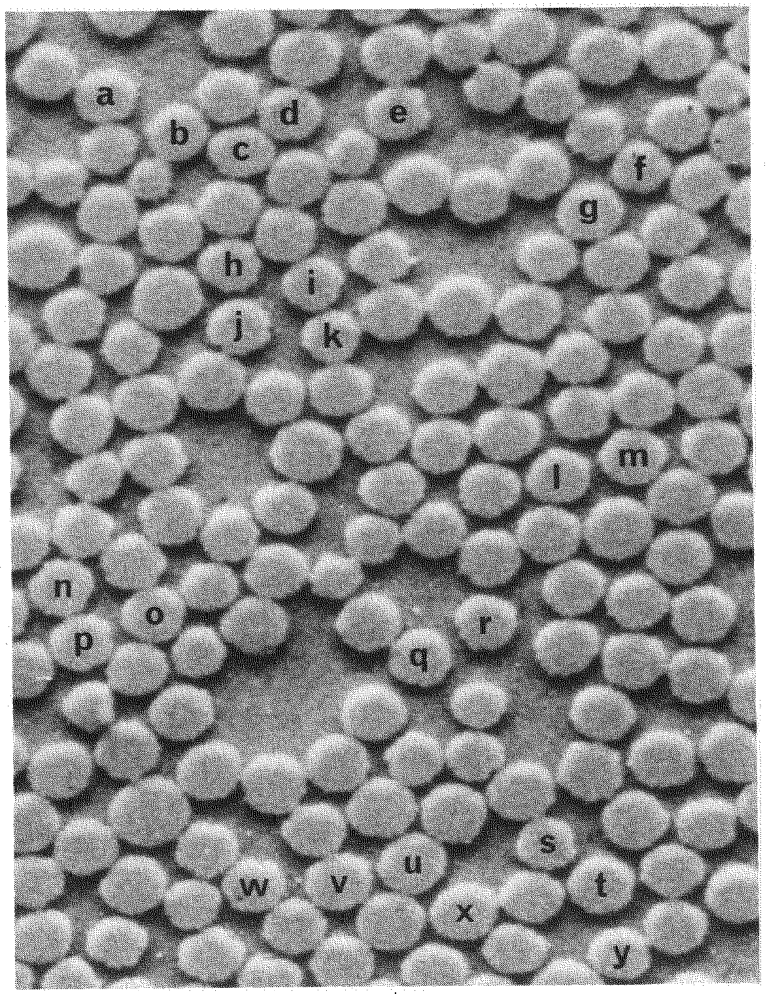
Fig.5 continued



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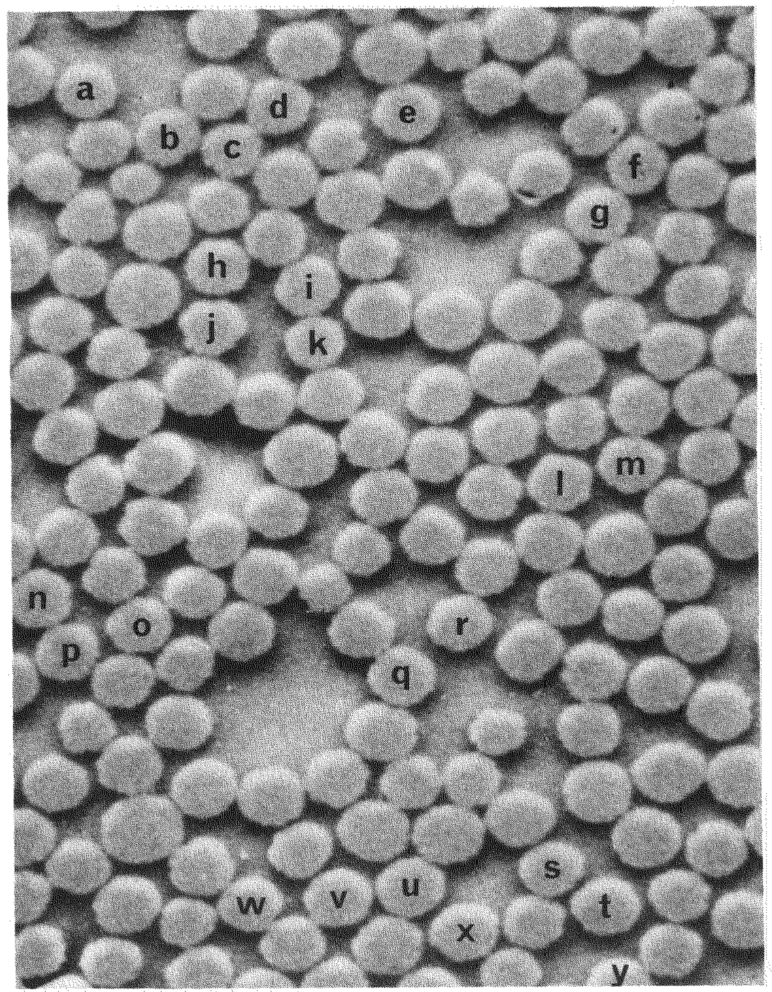

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Fig.5 concluded

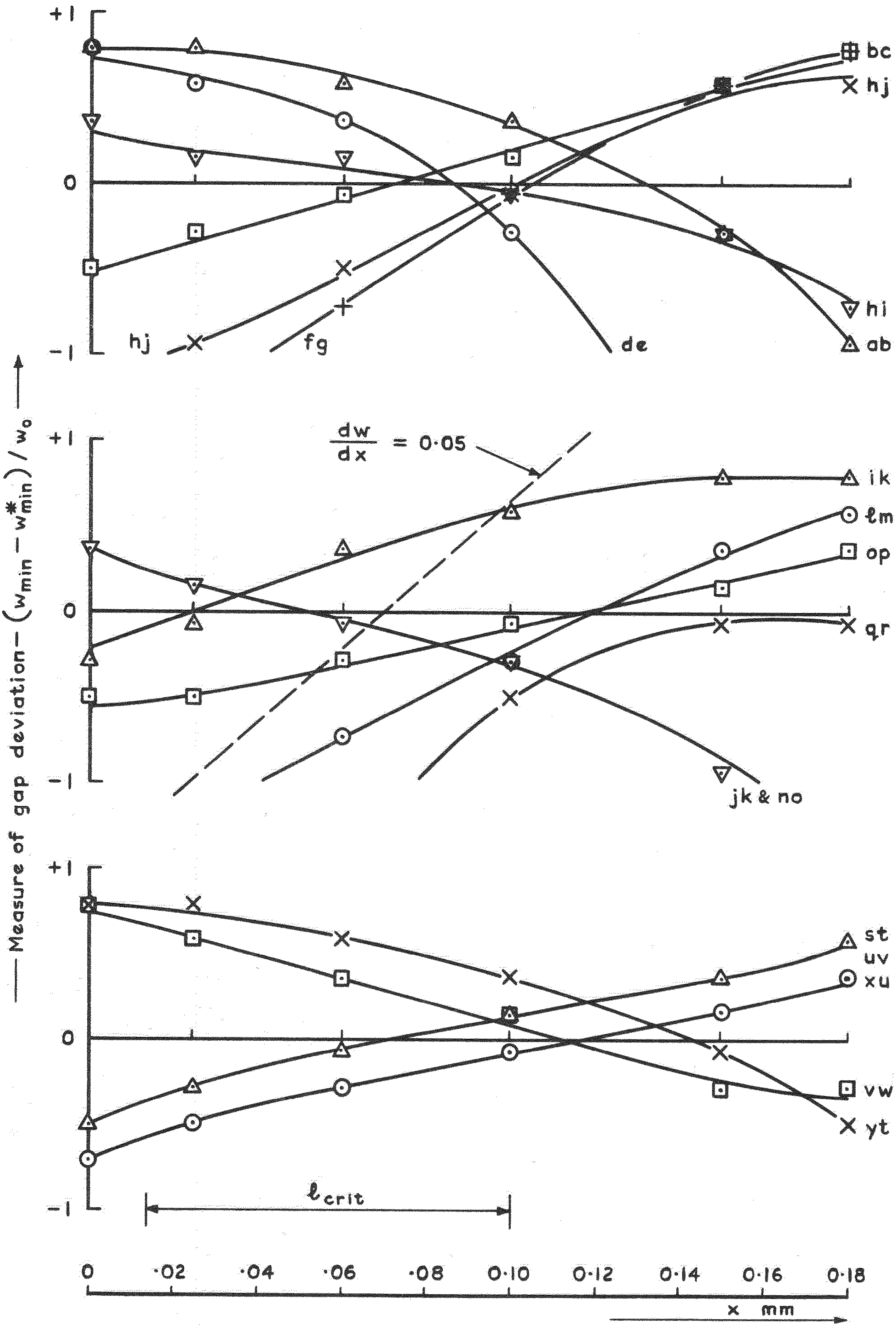

Fig. 6 Variation of $(w_{min} - w^*_{min}) / w_o$ with x

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