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by

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THE CONCEPT OF LOAD DIFFUSION LENGTH IN FIBRE REINFORCED COMPOSITES

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SUMMARY

This Report considers the related problems of load diffusion and the decay of applied self-equilibrating stress systems in orthotropic sheets under conditions of plane stress. Attention is focussed on the differences between the isotropic sheet and an orthotropic sheet with the properties of a fibre reinforced composite, particularly one with unidirectional or bi-directional reinforcement. A standard is defined for a load diffusion length and it is shown how this is influenced by anisotropy.

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CONTENTS

1 INTRODUCTION

In aircraft and other structures the presence of joints, cut-outs, etc. - and even rivet holes - results in a localised redistribution of stress. A rigorous analysis of the stress distribution in actual structures is not generally possible, but over the years design experience has been built up by the accumulation of data gleaned from exact solutions of idealised structures, approximate theoretical/numerical solutions of more realistic structures, and experimental results using strain gauge or photoelastic techniques. But this experience is largely confined to materials whose elastic properties are isotropic. It has virtually no relevance in the context of unidirectional and bi-directional fibre reinforced composites which exhibit massive degrees of anisotropy. However, the theory of anisotropic bodies is well documented (e.g. Lekhnitski[1], Green and Zerna[2]) while Savin[3] has presented solutions for a wide range of stress concentration problems around holes in flat sheets. The present paper focusses attention on some load diffusion problems and the decay of self-equilibrating stress systems. For such problems the underlying intention is to relate, albeit in an approximate manner, design experience with isotropic materials to the anisotropic case. This is done by introducing the concept of a load diffusion length which typifies the lengths required for the load diffusion and stress decay processes to achieve comparable values in the isotropic and anisotropic cases.

2 NOTATION

b	semi-width of panel/strip
E_x, E_y	Young's moduli in x and y directions
G_{xy}	shear modulus
k	constant defined in equation (32)
m_1, m_2	defined by equation (10)
p	stress ratio defined in equation (1)
x, y	Cartesian coordinates, see Figs.1, 2
α_1, α_2, α_3	nondimensional stiffness parameters defined in equations (7), (28)
ε_x, ε_y, γ_{xy}	direct and shear strains
Φ	stress function
λ	nondimensional measure of load diffusion length, see equation (2)
ν_{xy}, ν_{yx}	Poisson's ratios

σ, τ magnitude of applied stresses, see equations (33), (44)

σ_x, σ_y, τ_{xy} direct and shear stresses

ξ, η nondimensional coordinates defined by equation (11).

3 ANALYSIS

The problem under consideration, which lends itself to an exact solution, is shown in Fig.1. The semi-infinite strip of width 2b has orthotropic elastic moduli specified by E_x, E_y and G_{xy} and Poisson's ratios ν_{yx}, ν_{xy}. The end at $x = 0$ is free from stress while shears are applied via tapered edge members at $y = \pm b$ so as to produce a uniform longitudinal stress $\sigma_x = \sigma$ along these edges which are otherwise unloaded. A solution to this problem was given by Mansfield[4] in the context of a sheet with 'smeared' stringers and ribs. Only simple modifications are necessary to express this solution in a form appropriate to an orthotropic sheet.

Remote from the free end at $x = 0$ the stress σ_x is constant across the strip and σ_y, τ_{xy} are zero. However, near the free end there is a 'shear lag' problem which is highlighted by the gradual build-up of the longitudinal stress along the centre line. The build-up of this stress provides a convenient measure of a 'load diffusion length'. First, we determine the value of the ratio p for an isotropic sheet, where

$$p = [\sigma_x]_{y=0,x=2b}/\sigma \quad . \tag{1}$$

For the orthotropic sheet we now determine a nondimensional measure λ of a load diffusion length such that

$$[\sigma_x]_{y=0,x=2b\lambda} = p\sigma \quad . \tag{2}$$

It will be seen that $\lambda = 1$ for the isotropic sheet. Thus, for the orthotropic sheet a value of $\lambda = 3$, for example, means that in comparison with the isotropic sheet a three-fold increase in the distance from the free end is required to achieve the same stress σ_x along the centre line.

3.1 Interpretation in the light of Saint-Venant's principle

The load diffusion problem of Fig.1 can be modified by the superposition of a uniform longitudinal compressive stress σ to yield the self-equilibrating load system shown in Fig.2(a). The boundary conditions along the edges at

$y = \pm b$ are now such that both σ_x and σ_y are zero, so that these edges must be envisaged as being supported by inextensional but transversely flexible members. Note, however, that the stresses in the panel are the same as those in the strip indicated in Fig.2(b) which forms part of a semi-infinite sheet under the self-equilibrating load system shown.

Now Saint-Venant's principle[5] states, in effect, that the stresses due to a self-equilibrating load system decrease with increasing distance from the load system. The distance necessary for a given stress reduction can, however, be markedly influenced by anisotropy and the nondimensional load diffusion distance λ is a measure of this distance expressed as a multiple of that for the isotropic sheet.

The value of λ derived from equations (1), (2) is not, however, applicable to all load diffusion problems. It applies strictly to the specific problems of Figs,1, 2 although, as will be shown later, the closely related problem of a sinusoidally varying normal stress applied to a semi-infinite sheet yields almost identical results. A greater cause for variation in the measure of a load diffusion distance lies in the choice of a reference distance. For example, it is shown later that if the reference distances of equations (1), (2) are defined by $x = b, b\lambda$ instead of $2b, 2b\lambda$ markedly different values are obtained for λ. Of course, if the stresses in the orthotropic sheet of Figs.1, 2 could be derived from those in an isotropic sheet by a simple 'stretching' in the x-direction the derived value of λ would be independent of the chosen reference distance. But such stretching is not possible because it results in stresses which violate the equations of equilibrium.

3.2 The orthotropic sheet

The equations governing the stresses in a sheet with orthotropic properties are well known, but it is convenient to re-derive them. The stress-strain relations are given by

$$\left.\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E_x} - \nu_{yx}\frac{\sigma_y}{E_y} , \\ \varepsilon_y &= \frac{\sigma_y}{E_y} - \nu_{xy}\frac{\sigma_x}{E_x} , \\ \gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}} , \end{aligned}\right\} \qquad (3)$$

where, from the Reciprocal Theorem,

$$\nu_{yx}/E_y = \nu_{xy}/E_x \quad . \tag{4}$$

The equations of equilibrium are satisfied by the introduction of a stress function Φ such that

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2} \; , \qquad \sigma_y = \frac{\partial^2\Phi}{\partial x^2} \; , \qquad \tau_{xy} = -\frac{\partial^2\Phi}{\partial x\partial y} \quad . \tag{5}$$

The equation of compatibility in terms of strains is

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} = \frac{\partial^2\gamma_{xy}}{\partial x\partial y} \quad , \tag{6}$$

and this may be expressed in terms of Φ in virtue of equations (3) to (5):

$$\left.\begin{aligned} \alpha_1 \frac{\partial^4\Phi}{\partial x^4} + 2\alpha_2 \frac{\partial^4\Phi}{\partial x^2\partial y^2} + \frac{\partial^4\Phi}{\partial y^4} &= 0 \quad , \\ \text{where} \qquad \alpha_1 &= \frac{E_x}{E_y} \quad , \\ \alpha_2 &= \frac{E_x}{2G_{xy}} - \nu_{xy} \quad . \end{aligned}\right\} \tag{7}$$

In sheets whose boundary conditions are expressed solely in terms of stresses it follows that the material properties affect the distribution of stress only insofar as they affect the parameters α_1, α_2 . Note that for the isotropic sheet

$$\alpha_1 = \alpha_2 = 1 \quad .$$

3.3 The stress function solution

A suitable form for the stress function which satisfies the conditions along the edges at $y = \pm b$ is given by

$$\Phi = \tfrac{1}{2}\sigma y^2 + \frac{4b^2}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\left\{C_n \exp\left[-\frac{(2n+1)\pi x}{2bm_1}\right] + C_n' \exp\left[-\frac{(2n+1)\pi x}{2bm_2}\right]\right\}\cos\left\{\frac{(2n+1)\pi y}{2b}\right\} \quad (8)$$

where m_1, m_2 are the positive roots of the equation

$$m^4 - 2\alpha_2 m^2 + \alpha_1 = 0 \quad , \quad (9)$$

whence

$$\left.\begin{aligned} m_1 &= \left\{\alpha_2 + (\alpha_2^2 - \alpha_1)^{\frac{1}{2}}\right\}^{\frac{1}{2}} \quad , \\ m_2 &= \left\{\alpha_2 - (\alpha_2^2 - \alpha_1)^{\frac{1}{2}}\right\}^{\frac{1}{2}} \quad . \end{aligned}\right\} \quad (10)$$

Now for cases of orthotropic reinforcement it is sufficient to confine attention to real values of m_1, m_2, so that

$$\alpha_2^2 \geqslant \alpha_1 \quad .$$

This restriction on the possible range of values of α_1, α_2 is subsequently reflected in the discontinuous nature of certain curves in Figs.3, 4, 6.

The stresses, obtained from equation (5), are more conveniently expressed in terms of ξ and η, where

$$\left.\begin{aligned} \xi &= \pi x/2b \quad , \\ \eta &= \pi y/2b \quad . \end{aligned}\right\} \quad (11)$$

Thus,

$$\sigma_x = \sigma - \sum_n \left\{C_n \exp\left[-\frac{(2n+1)\xi}{m_1}\right] + C_n' \exp\left[-\frac{(2n+1)\xi}{m_2}\right]\right\}\cos(2n+1)\eta \quad , \quad \ldots\ldots (12)$$

$$\sigma_y = \sum_n \left[\frac{C_n}{m_1^2}\exp\left[-\frac{(2n+1)\xi}{m_1}\right] + \frac{C_n'}{m_2^2}\exp\left[-\frac{(2n+1)\xi}{m_2}\right]\right]\cos(2n+1)\eta \quad , \quad (13)$$

$$\tau_{xy} = -\sum_n \left\{ \frac{C_n}{m_1} \exp\left[-\frac{(2n+1)\xi}{m_1}\right] + \frac{C'_n}{m_2} \exp\left[-\frac{(2n+1)\xi}{m_2}\right] \right\} \sin(2n+1)\eta \quad . \qquad \ldots\ldots (14)$$

Along the end at $x = 0$, the vanishing of σ_x and τ_{xy} implies that

$$\left.\begin{aligned} \sigma - \sum_n (C_n + C'_n) \cos(2n+1)\eta &= 0 \quad , \\ \text{and} \quad \sum_n \left(\frac{C_n}{m_1} + \frac{C'_n}{m_2}\right) \sin(2n+1)\eta &= 0 \quad . \end{aligned}\right\} \qquad (15)$$

Now from Fourier analysis

$$\sigma \equiv \frac{4\sigma}{\pi} \sum_n \frac{(-1)^n \cos(2n+1)\eta}{2n+1} \quad , \qquad (16)$$

and hence C_n, C'_n are given by

$$\left.\begin{aligned} C_n &= \frac{(-1)^n 4m_1\sigma}{\pi(2n+1)(m_1 - m_2)} \quad , \\ C'_n &= \frac{-(-1)^n 4m_2\sigma}{\pi(2n+1)(m_1 - m_2)} \quad . \end{aligned}\right\} \qquad (17)$$

Substitution of equation (17) into equations (12) to (14) yields expressions for the stresses which contain series of the following type which can be expressed in closed form as follows[4]:

$$\left.\begin{aligned} \sum_n \frac{(-1)^n}{2n+1} \exp\left[-\frac{(2n+1)\xi}{m}\right] \cos(2n+1)\eta &\equiv \tfrac{1}{2} \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/m)}\right) \\ \text{and} \quad \sum_n \frac{(-1)^n}{2n+1} \exp\left[-\frac{(2n+1)\xi}{m}\right] \sin(2n+1)\eta &\equiv \tfrac{1}{4} \ln\left(\frac{\cosh(\xi/m) + \sin\eta}{\cosh(\xi/m) - \sin\eta}\right) . \end{aligned}\right\} (18)$$

Hence we obtain

$$\frac{\sigma_x}{\sigma} = 1 - \frac{2}{\pi(m_1 - m_2)}\left\{m_1 \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/m_1)}\right) - m_2 \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/m_2)}\right)\right\}, \qquad \ldots\ldots (19)$$

$$\frac{\sigma_y}{\sigma} = \frac{2}{\pi(m_1 - m_2)}\left\{\frac{1}{m_1}\tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/m_1)}\right) - \frac{1}{m_2}\tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/m_2)}\right)\right\}, \quad (20)$$

$$\frac{\tau_{xy}}{\sigma} = \frac{1}{\pi(m_1 - m_2)} \ln\left[\frac{\{\cosh(\xi/m_1) - \sin\eta\}\{\cosh(\xi/m_2) + \sin\eta\}}{\{\cosh(\xi/m_1) + \sin\eta\}\{\cosh(\xi/m_2) - \sin\eta\}}\right]. \quad (21)$$

The corresponding solution for the isotropic sheet may be obtained from this by taking the limit as $m_1 \to m_2 \to 1$, whence[4]

$$\frac{\sigma_x}{\sigma} = 1 - \frac{2}{\pi}\left\{\frac{\xi\cosh\xi\cos\eta}{\cosh^2\xi - \sin^2\eta} + \tan^{-1}\left(\frac{\cos\eta}{\sinh\xi}\right)\right\}, \quad (22)$$

$$\frac{\sigma_y}{\sigma} = \frac{2}{\pi}\left\{\frac{\xi\cosh\xi\cos\eta}{\cosh^2\xi - \sin^2\eta} - \tan^{-1}\left(\frac{\cos\eta}{\sinh\xi}\right)\right\}, \quad (23)$$

$$\frac{\tau_{xy}}{\sigma} = -\frac{2}{\pi}\left\{\frac{\xi\sinh\xi\sin\eta}{\cosh^2\xi - \sin^2\eta}\right\}. \quad (24)$$

3.4 The nondimensional measure of a load diffusion length

Along the centre line, $\eta = 0$, and hence for the isotropic sheet equations (1), (22) yield

$$p = 1 - 2\,\mathrm{sech}\,\pi - \frac{2}{\pi}\tan^{-1}(\mathrm{cosech}\,\pi)$$

$$= 0.77248\ \ldots\ . \quad (25)$$

For the orthotropic sheet the nondimensional measure λ of a load diffusion length is now given by equations (2), (19) and (25), whence

$$\frac{2}{\pi(m_1 - m_2)}\left\{m_1 \tan^{-1}\{\operatorname{cosech}(\pi\lambda/m_1)\} - m_2 \tan^{-1}\{\operatorname{cosech}(\pi\lambda/m_2)\}\right\} = 1 - p$$
$$= 0.22752 \ldots \qquad \ldots\ldots (26)$$

Before proceeding to the numerical solution of equation (26) for general combinations of the orthotropic sheet parameters it is expedient first to consider the limiting case as $E_y \to \infty$. This is because the modulus E_y has the least influence on the shear lag characteristics and the solution with infinite E_y should give a good indication of the influence of the other moduli. From equation (7) it is seen that α_1 vanishes so that, from equation (10),

$$m_1 = (2\alpha_2)^{\frac{1}{2}} ,$$
$$m_2 = 0 .$$

Substitution into equation (26) and solving for λ now yields

$$\left.\begin{aligned}\lambda &= 0.7704\alpha_2^{\frac{1}{2}} \\ &= 0.545\left(\frac{E_x}{G_{xy}}\right)^{\frac{1}{2}} ,\end{aligned}\right\} \qquad (27)$$

in virtue of equations (4), (7).

Turning now to the real life situation in which E_y is finite, it is to be expected that λ will vary approximately linearly with $\alpha_2^{\frac{1}{2}}$ but this will be modified in some way due to the presence of the parameter α_1 . For a sheet with fibre reinforcement in the x and y directions the parameters α_1, α_2 are the natural parameters to choose for specifying the sheet properties. However, for a sheet with unidirectional (x-wise) fibre reinforcement the modulus E_y is closely linked to the shear modulus G_{xy} for both are primarily dependent on the stiffness properties of the matrix. It is thus more appropriate to consider how λ varies with α_2 and α_3 , say, where

$$\left.\begin{aligned}\alpha_3 &= \alpha_2/\alpha_1 \\ &= \frac{E_y}{2G_{xy}} - \nu_{yx} .\end{aligned}\right\} \qquad (28)$$

3.5 Sheet with unidirectional fibre reinforcement

For a sheet with unidirectional fibre reinforcement the variation of λ with $\alpha_2^{\frac{1}{2}}$ for various values of α_3 is shown in Fig.3. For such sheets ν_{xy} assumes a value intermediate between the Poisson's ratios of the matrix and fibres, and typical bounds are given by

$$0.25 < \nu_{xy} < 0.35 \quad . \tag{29}$$

Furthermore, it follows from equation (4) that ν_{yx} can be neglected in the expression for α_3 in which measured values of E_y and G_{xy} for a wide range of unidirectional fibre reinforced composites yield values in the range

$$0.5 < \alpha_3 < 1.0 \quad . \tag{30}$$

These limiting values are shown in Fig.3 together with $\alpha_3 = \infty$, which corresponds to the previously considered case in which $E_y \to \infty$. The curves relating λ with $\alpha_2^{\frac{1}{2}}$ for finite values of α_3 are indistinguishable from straight lines parallel to but slightly above the line for $\alpha_3 = \infty$. Note that the curve for $\alpha_3 = 1$ originates at the point $\lambda = 1$, $\alpha_2^{\frac{1}{2}} = 1$ (the isotropic sheet) and is therefore adequately given by

$$\lambda \approx 0.23 + 0.77\alpha_2^{\frac{1}{2}} \quad . \tag{31}$$

3.6 Sheet with fibre reinforcement in x and y directions

For a sheet with orthotropic properties the variation of λ with $\alpha_2^{\frac{1}{2}}$ for various values of α_1 is shown in Fig.4. Over the practical range of values of α_2 the curves differ little from the limiting case in which $\alpha_1 = 0$, see equation (27).

3.7 Variation of λ with choice of reference distance

We have already seen that the nondimensional measure λ of a load diffusion distance is primarily dependent on the parameter α_2 and is adequately given by an equation of the form

$$\lambda \approx k\alpha_2^{\frac{1}{2}} \quad . \tag{32}$$

However, as stated in section 3.1, the choice of reference distance can have a marked influence on the values obtained for λ . Thus, if the reference distances of equations (1), (2) are defined by $x = b, b\lambda$ instead of $2b, 2b\lambda$ it can be shown that while equation (32) is still valid the constant k is now 0.5063 instead of 0.7704. This difference stems from the fact that in an orthotropic sheet the stresses σ_x can vary longitudinally in a markedly different manner from those in an isotropic sheet. To investigate this feature in greater detail it is expedient to confine attention to the simpler but closely related problem of a sinusoidally varying normal stress applied to the edge of a semi-infinite sheet.

4 SINUSOIDALLY VARYING NORMAL STRESS APPLIED TO SEMI-INFINITE SHEET

The problem considered here is similar to that shown in Fig.2(b) except that the applied stress varies sinusoidally:

$$[\sigma_x]_{x=0} = \sigma \cos (\pi y/2b) \quad . \tag{33}$$

The solution to this problem is readily obtained from equations (12)-(14) by confining attention to the terms containing C_0, C_0' . Thus it can be shown that

$$\frac{\sigma_x}{\sigma} = \frac{1}{m_1 - m_2}\left\{m_1 \exp\left(\frac{-\xi}{m_1}\right) - m_2 \exp\left(\frac{-\xi}{m_2}\right)\right\} \cos \eta \quad , \tag{34}$$

$$\frac{\sigma_y}{\sigma} = \frac{-1}{m_1 - m_2}\left\{\frac{1}{m_1} \exp\left(\frac{-\xi}{m_1}\right) - \frac{1}{m_2} \exp\left(\frac{-\xi}{m_2}\right)\right\} \cos \eta \quad , \tag{35}$$

$$\frac{\tau_{xy}}{\sigma} = \frac{1}{m_1 - m_2}\left\{\exp\left(\frac{-\xi}{m_1}\right) - \exp\left(\frac{-\xi}{m_2}\right)\right\} \sin \eta \quad , \tag{36}$$

where m_1, m_2 and ξ, η are given by equations (10), (11).

Similarly, by taking the limit as $m_1 \to m_2 \to 1$ we obtain the solution for the isotropic sheet:

$$\sigma_x/\sigma = (1 + \xi)e^{-\xi} \cos \eta \quad , \tag{37}$$

$$\sigma_y/\sigma = (1 - \xi)e^{-\xi} \cos \eta \quad , \tag{38}$$

$$\tau_{xy}/\sigma = \xi e^{-\xi} \sin \eta \quad . \tag{39}$$

Thus, if we adopt the same definition of λ as given by equations (1), (2) we find from equations (1), (37),

$$p = (1 + \pi)e^{-\pi} \quad , \tag{40}$$

while equations (2), (34) yield

$$p(m_1 - m_2) = m_1 \exp\left(\frac{-\pi\lambda}{m_1}\right) - m_2 \exp\left(\frac{-\pi\lambda}{m_2}\right) \quad . \tag{41}$$

In particular, the limiting case in which $E_y = \infty$, yields

$$\lambda = 0.7745\alpha_2^{\frac{1}{2}} \quad , \tag{42}$$

which is almost identical to equation (27); this is to be expected because, at the reference distance adopted, the influence of the higher harmonics implicit in the problem of Fig.2(b) should be very small.

In Fig.5 the variation of σ_x/σ with $x/2b\lambda$ is shown for the isotropic sheet, orthotropic sheets with $E_y = \infty$, and a typical unidirectionally reinforced sheet specified by $\alpha_2 = 25$, $\alpha_3 = 0.5$. The adoption of $x/2b\lambda$ as a measure of the longitudinal distance makes all the curves pass through the same (reference) point at $x/2b\lambda = 1$, and this highlights the differences in the form of the longitudinal variation. It is seen that if the reference distance adopted for the isotropic sheet exceeds $2b$ the constant k in equation (32) is increased while if the reference distance is less than $2b$ the constant is reduced. For example, if the reference distance is b, Fig.5 shows that the stress ratio σ_x/σ has dropped to 0.535. At this stress level the curve for $E_y = \infty$ passes through the point where $x/2b\lambda = 0.36$ and the constant k must be reduced by the factor 0.36/0.5. Thus, instead of equation (42) we would have

$$\lambda' \text{ , say } = 0.558\alpha_2^{\frac{1}{2}} \quad . \tag{43}$$

The fact that the revised value of k differs by some 10% from the value appropriate to the problem of Fig.2(b), see section 3.7, is due to the relatively greater influence of the higher harmonics at the shorter reference distance.

5 SINUSOIDALLY VARYING SHEAR STRESS APPLIED TO SEMI-INFINITE SHEET

Here the results of section 4 are augmented by consideration of a sinusoidally varying applied shear stress:

$$[\tau_{xy}]_{x=0} = \tau \cos(\pi y/2b) \quad . \tag{44}$$

The solution to this problem is also readily obtained from equations (12) to (14) by confining attention to the terms containing C_0, C_0' and replacing η by $(\frac{1}{2}\pi + \eta)$. Thus it can be shown that

$$\frac{\sigma_x}{\tau} = \frac{m_1 m_2}{m_1 - m_2}\left\{\exp\left(\frac{-\xi}{m_1}\right) - \exp\left(\frac{-\xi}{m_2}\right)\right\}\sin\eta \quad , \tag{45}$$

$$\frac{\sigma_y}{\tau} = \frac{-m_1 m_2}{m_1 - m_2}\left\{\frac{1}{m_1^2}\exp\left(\frac{-\xi}{m_1}\right) - \frac{1}{m_2^2}\exp\left(\frac{-\xi}{m_2}\right)\right\}\sin\eta \quad , \tag{46}$$

$$\frac{\tau_{xy}}{\tau} = \frac{-m_1 m_2}{m_1 - m_2}\left\{\frac{1}{m_1}\exp\left(\frac{-\xi}{m_1}\right) - \frac{1}{m_2}\exp\left(\frac{-\xi}{m_2}\right)\right\}\cos\eta \quad . \tag{47}$$

Similarly, by taking the limit as $m_1 \to m_2 \to 1$ we obtain the solution for the isotropic sheet:

$$\sigma_x/\tau = \xi e^{-\xi}\sin\eta \quad , \tag{48}$$

$$\sigma_y/\tau = (2 - \xi)e^{-\xi}\sin\eta \quad , \tag{49}$$

$$\tau_{xy}/\tau = (1 - \xi)e^{-\xi}\cos\eta \quad . \tag{50}$$

In defining a load diffusion length it is now natural to focus attention on the decay of the (applied) shear stress τ_{xy} . But a comparison of equations (37) and (50) shows that in the present problem the decay of τ_{xy}

with x is much more rapid than the decay of σ_x in the previous problem. Furthermore, from considerations of equilibrium the integral of the shear stresses acting on a section at constant y is necessarily zero and this is reflected in a change of sign of τ_{xy} as x increases. A natural choice for a load diffusion length is thus given by the vanishing of τ_{xy}. Hence, for the isotropic sheet

$$\xi = 1 \quad , \qquad (\text{i.e. } x = 2b/\pi) \tag{51}$$

and from equation (47) the nondimensional measure of a load diffusion length for the general orthotropic case is given by

$$\left.\begin{aligned} [\tau_{xy}]_{x=2b\lambda/\pi} &= 0 \quad , \\ \text{whence} \qquad \lambda &= \left(\frac{m_1 m_2}{m_1 - m_2}\right) \ln\left(\frac{m_1}{m_2}\right) \quad . \end{aligned}\right\} \tag{52}$$

Fig.6 shows the variation of λ with $\alpha_2^{\frac{1}{2}}$ for various values of α_3 and α_1, corresponding respectively to the cases of unidirectional and bi-directional reinforcement. The abscissa is again chosen to be $\alpha_2^{\frac{1}{2}}$ although there is no longer any linear relationship with $\alpha_2^{\frac{1}{2}}$. It is seen that as α_2 increases λ increases for the unidirectional case and decreases for the bi-directional case. The deviations from unity are, however, markedly less than for the problems considered in sections 3.4.

6 DISCUSSION AND CONCLUSIONS

The concept of a load diffusion length is a convenient design tool but the relative magnitude of such a length depends markedly on the standard adopted. This is because the stress distribution in an anisotropic sheet or strip cannot be derived from the corresponding isotropic case by a simple 'stretching' process. Differences in the *form* of the stress decay, as opposed to the overall *rate* of decay, are typified by the differences in the curves shown in Fig.5. For this reason there is no need for much accuracy in the determination of a load diffusion length because the concept can only be used as a rough guide. By the same token, advantage can be taken of simplifications arising from the neglect of small Poisson's ratio terms. The adoption of equations (1), (2) for defining a nondimensional measure λ of a load diffusion length in problems where unidirectional (x-wise) loads are applied to fibre reinforced composites with either unidirectional or bi-directional reinforcement yields the following simple approximate expression:

$$\lambda \approx 0.55 \left(\frac{E_x}{G_{xy}}\right)^{\frac{1}{2}} .$$

This simple expression is based on some exact solutions to load diffusion problems in orthotropic sheets. Typical values of the moduli for CFRP yield values of λ in the range $3 < \lambda < 4$.

Finally it is to be noted that in any design procedure the use of the concept of a load diffusion length must by augmented by considerations of *strength*, paying particular attention to the relatively low shear strength of unidirectional or bi-directional composites.

REFERENCES

No.	Author	Title, etc.
1	S.G. Lekhnitski	*Anisotropic Plates.* Translated from the second Russian edition by S.W. Tsai and T. Cheron. Gordon and Breach (1968)
2	A.E. Green W. Zerna	*Theoretical elasticity.* Second edition, Clarendon Press, Oxford (1968)
3	G.N. Savin	*Stress concentration around holes.* Pergamon Press (1961)
4	E.H. Mansfield	The stress distribution in panels bounded by constant-stress edge members. ARC Rep. and Memo. No.2965 (1954)
5	Saint-Venant	*Mém. savants étrangers.* Vol.14 (1855)

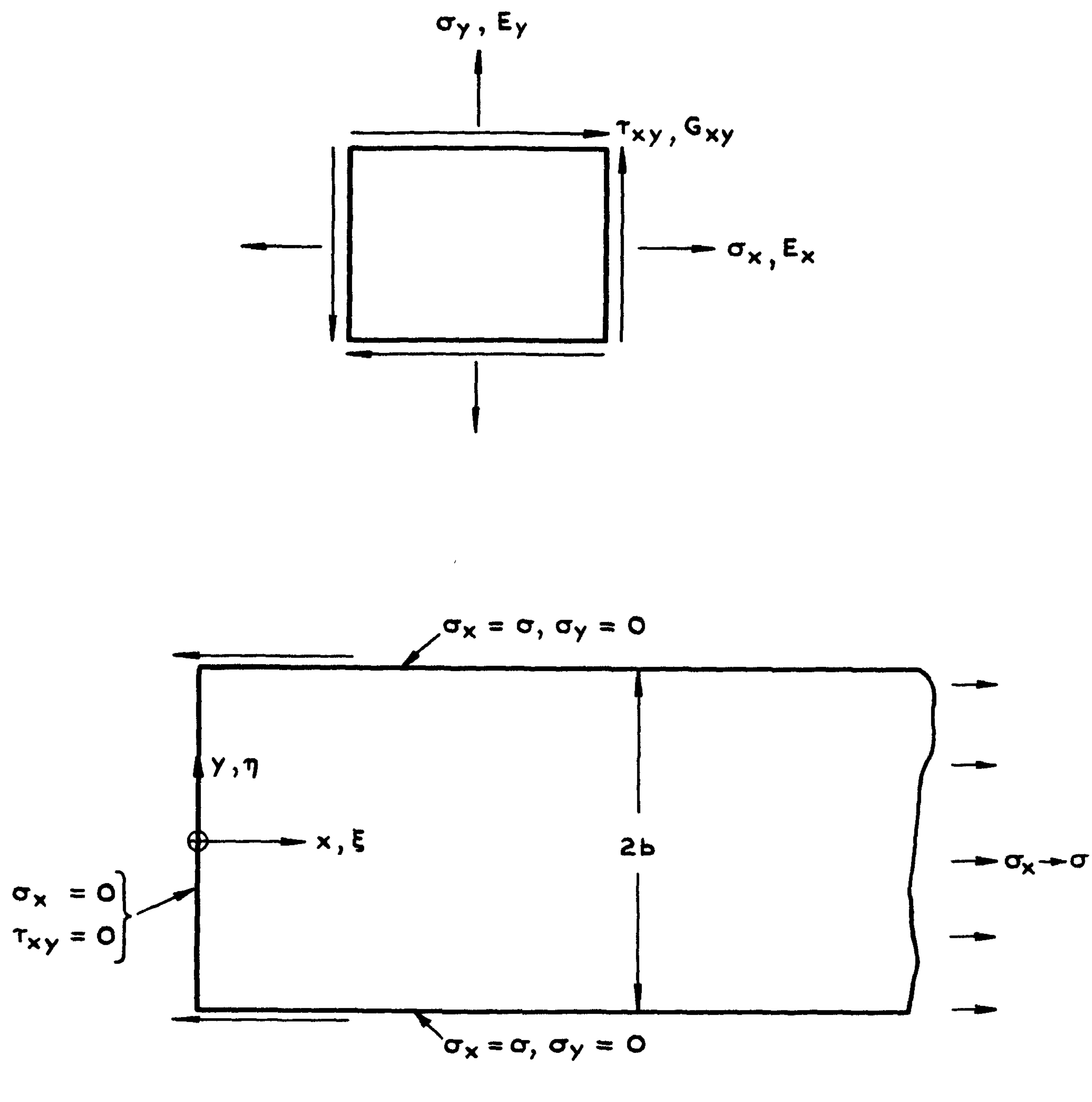

Fig. I Figure showing notation

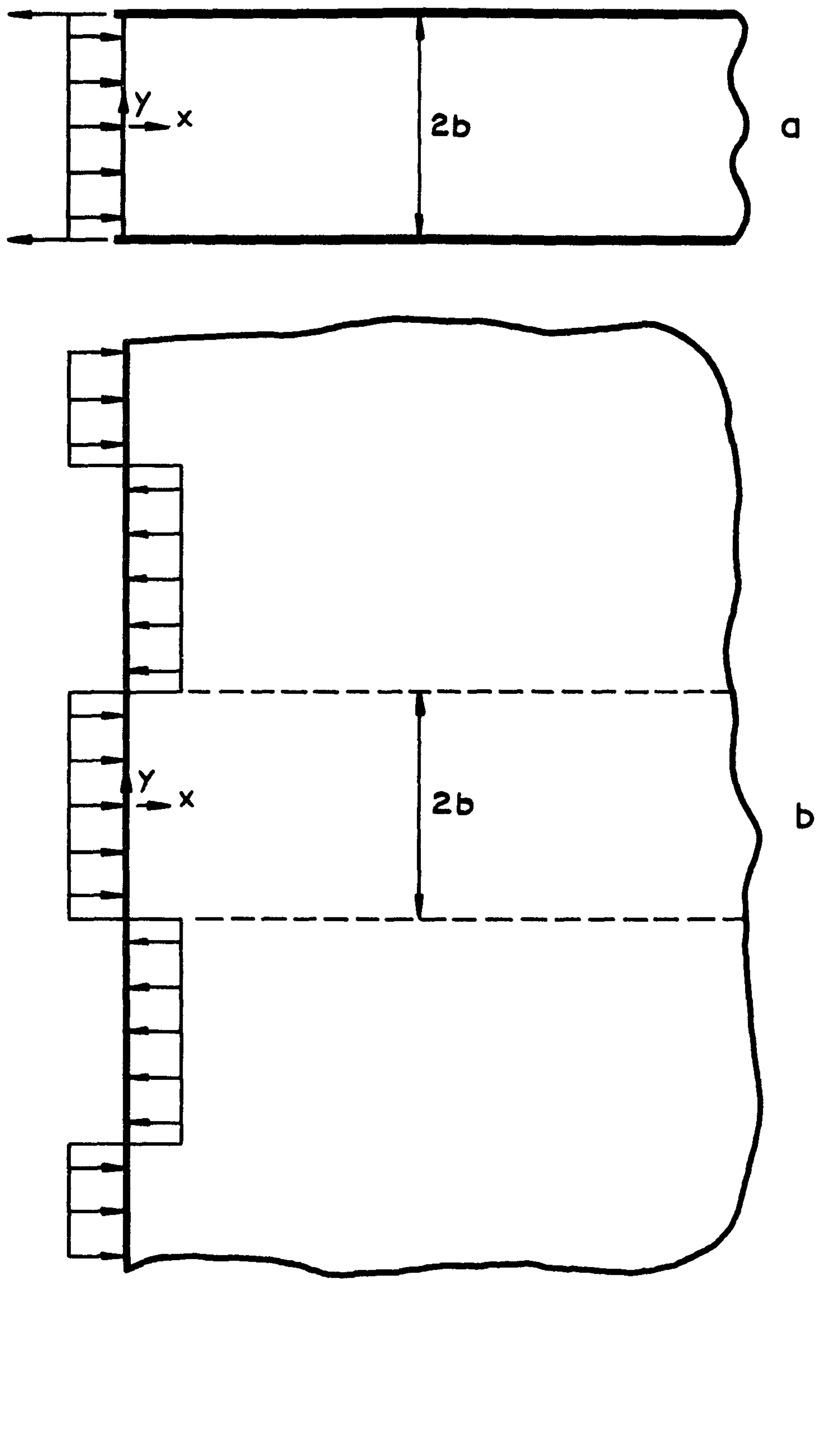

Fig.2 a&b Self-equilibrating load systems applied to panel and semi-infinite sheet

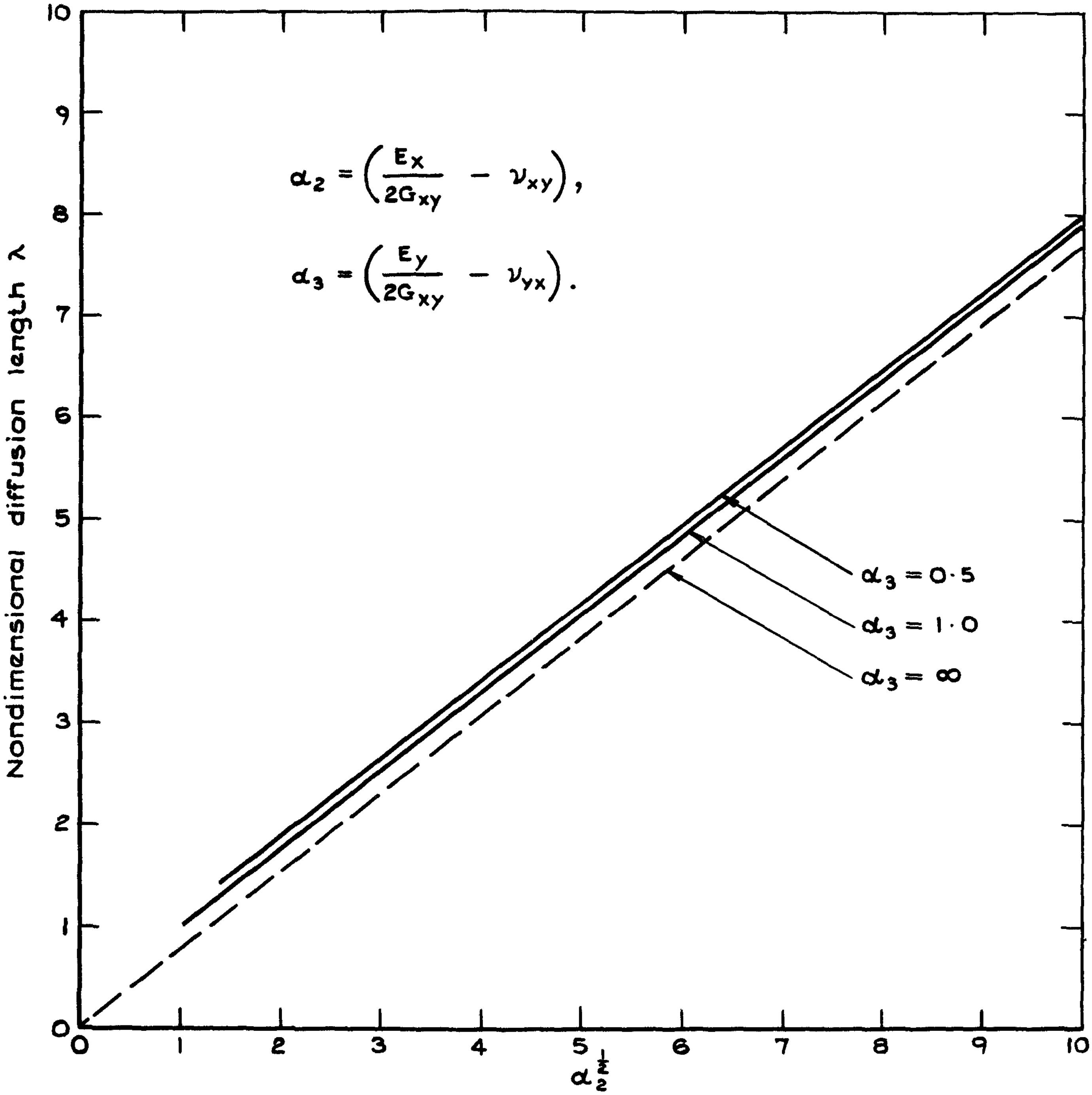

Fig.3 Variation of diffusion length with material properties: unidirectional fibre reinforcement

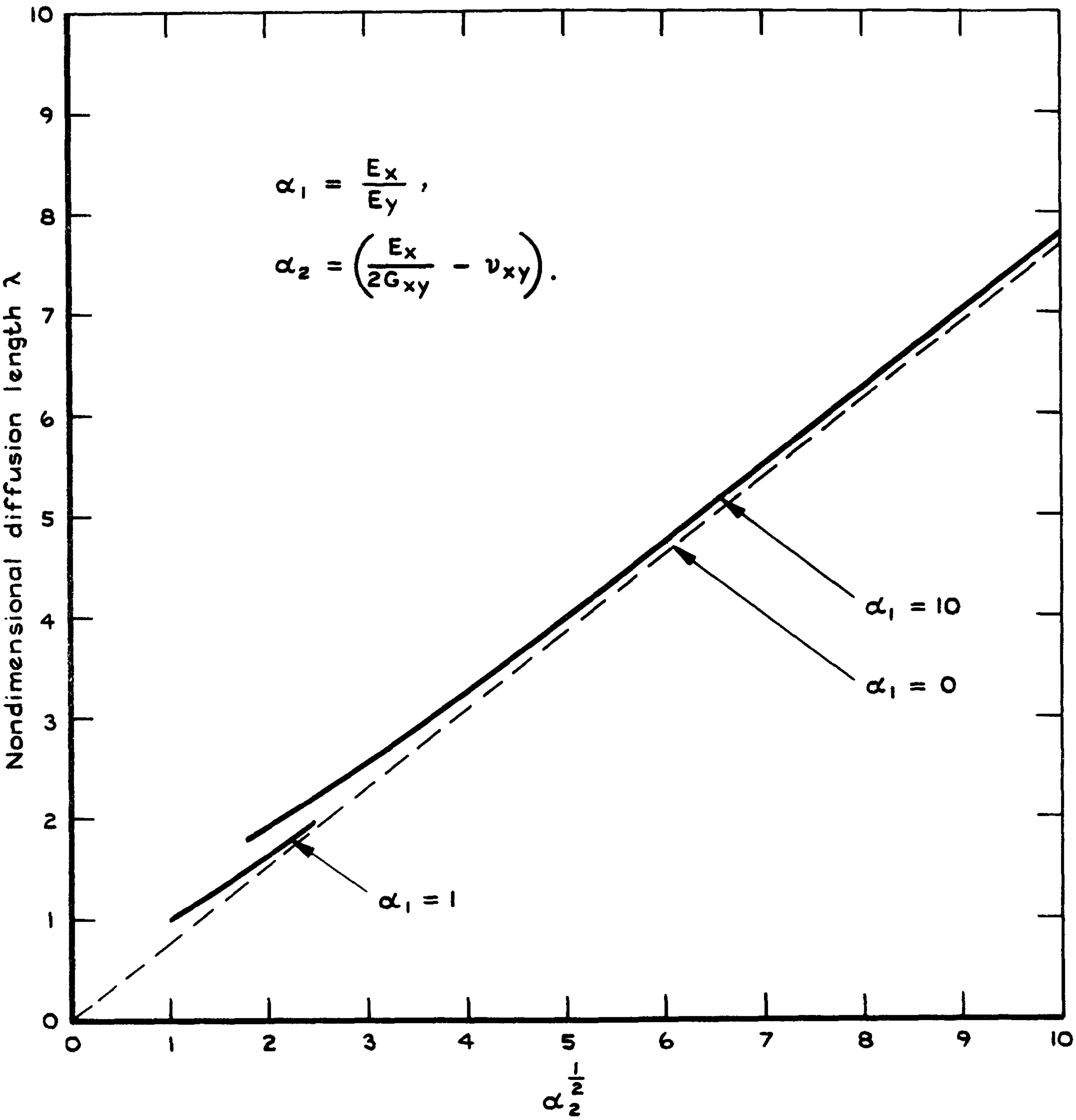

Fig.4 Variation of diffusion length with material properties: orthogonal fibre reinforcement

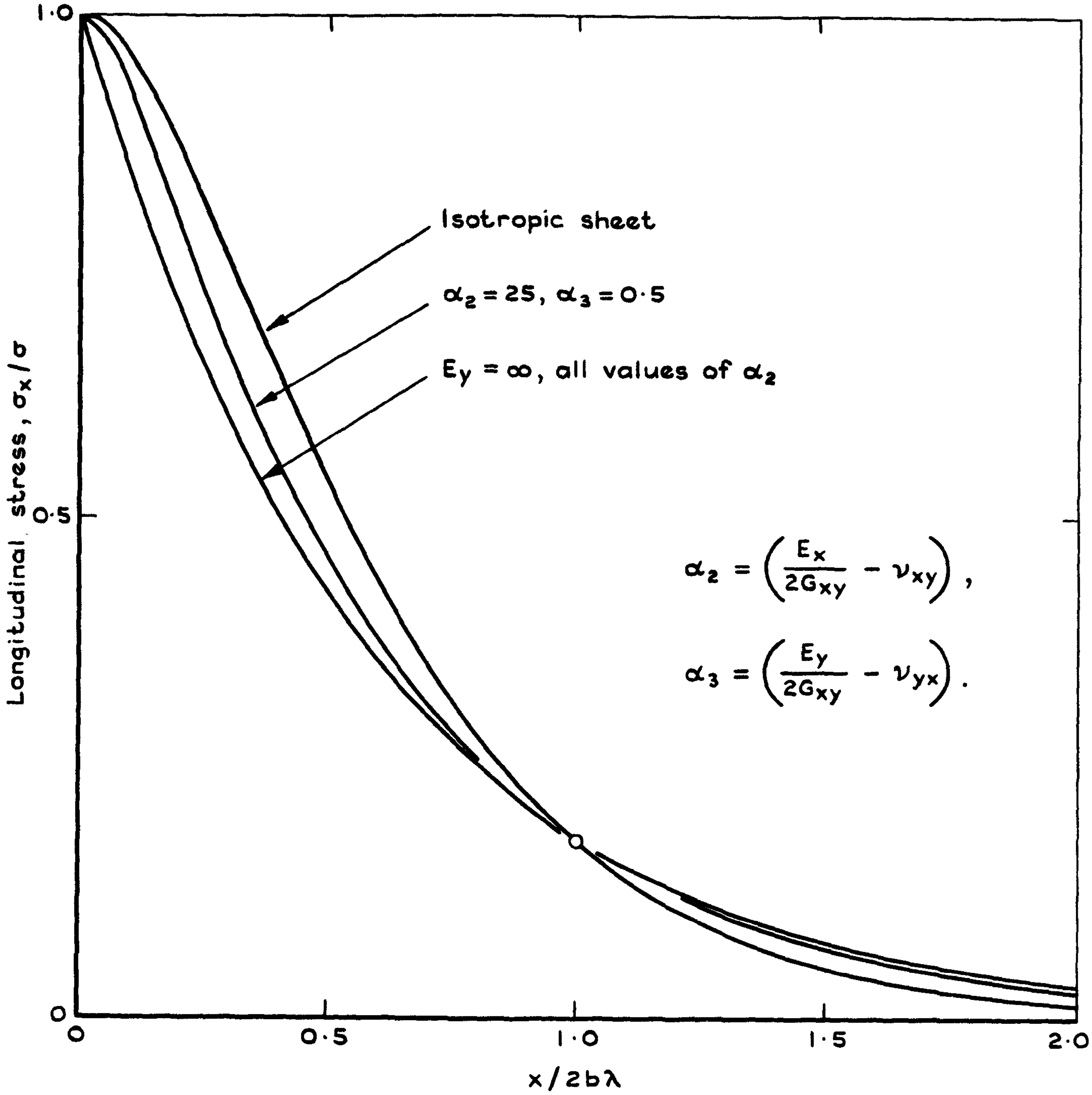

Fig. 5 Variation of longitudinal stress with distance x

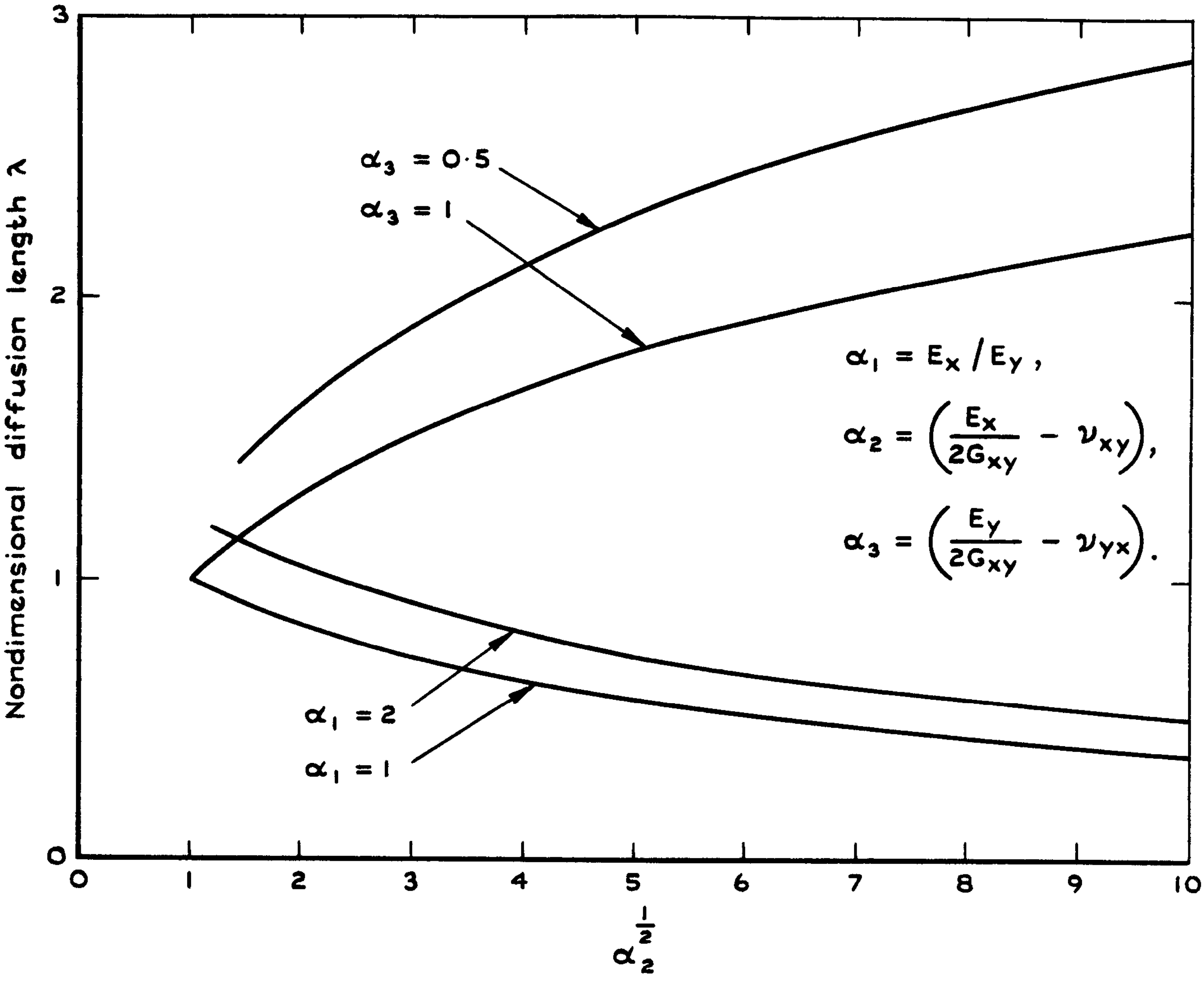

Fig.6 Variation of diffusion length with material properties : applied self-equilibrating shear

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