



PROCUREMENT EXECUTIVE, MINISTRY OF DEFENCE

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

A Comparison of Flight
Loads Counting Methods and their
Effects on Fatigue Life Estimates using
Data from Concorde

by

Ann G. Goodwillie

Structures Dept., R.A.E., Farnborough

LONDON: HER MAJESTY'S STATIONERY OFFICE

1974

PRICE £1.15 NET

A COMPARISON OF FLIGHT LOADS COUNTING METHODS AND THEIR EFFECTS
ON FATIGUE LIFE ESTIMATES USING DATA FROM CONCORDE

by

Ann G. Goodwillie

SUMMARY

Four load-time histories of turbulence encountered by Concorde are analysed by six different peak-counting and level-crossing counting procedures using the ICL 1904A computing facilities. The frequency distributions obtained are then used to find fatigue life ratios using the Miner-Palmgren hypothesis of cumulative damage.

The results show that those methods which consider every level crossed or every peak encountered give shorter life estimates than those which disregard all minor intermediate load fluctuations. Those which depend upon threshold conditions occupy an intermediate position. Correlation and spectral analyses are also conducted for each flight.

CONTENTS

	<u>Page</u>
1 INTRODUCTION	3
2 THE COUNTING TECHNIQUES CONSIDERED	4
2.1 Peak-count method	4
2.2 Simple peak-count method	4
2.3 Range-restricted peak-count method	5
2.4 Simple level-crossing method	5
2.5 Variable re-set method	5
2.6 Level-restricted peak-count method	6
2.7 Discussion of the methods	6
3 THE DATA	7
4 THE RESULTS	8
4.1 The numerical details	8
4.2 A comparative fatigue analysis	11
5 THE CORRELATIONS AND POWER SPECTRA	14
6 CONCLUSIONS	15
Appendix Programs for counting methods	17
Symbols	21
References	22
Illustrations	Figures 1-41
Detachable abstract cards	-

1 INTRODUCTION

In view of the forthcoming arrival of thousands of hours of mandatory recording data at the RAE it is considered useful to have at hand computer programs to analyse load data using various counting methods, so that patches of turbulence may be examined by the method which seems to describe the load sequence best for use in damage calculations. Three flights of Concorde 001 and one of 002, chosen for their turbulence encounters are used as data.

Six different counting methods called (a) the peak-count method, (b) the simple peak-count method, (c) the range-restricted peak-count method, (d) the simple level-crossing method, (e) the variable re-set method, and (f) the level-restricted peak-count method are considered here and these fall into two categories known as peak-counting and level-crossing methods. (a), (d) and (e) are examined in Sewell's report of Ref.1, where they are also used to analyse actual flight records. In Ref.2, van Dijk considers these and four other methods, categorised as range-count methods, and Schijve in Ref.3 analyses several of these methods also. However, since the techniques involved in the range-count methods are fundamentally different from those used in the six methods named above, comparison of the results would be a little meaningless so this Report restricts itself to the methods (a) to (f) only.

An important factor in designing against fatigue is to assess what loading environment is likely to be encountered by the aircraft. Ref.1 indicates that the three methods studied therein lead to substantially different fatigue life estimates, and this is because the different counting techniques do, in effect, change the environment assumed. The frequency distributions obtained by all six methods considered in this Report are used to obtain comparative estimates of fatigue life based on the Miner-Palmgren⁴ hypothesis of cumulative damage, and, for this, curves are fitted to the cumulative cycle distributions.

Using a computer program⁵ developed in the United States and modified at the RAE, correlation and spectral analyses are carried out to examine further the response of the aircraft to the turbulence encountered. The predominant frequencies are found to correspond to symmetric wing modes.

Although these counting methods are applied to centre of gravity normal accelerations in this Report, they may also be applied to measurements such as wing-root bending moment and strain values, although the interpretation and meaning of the results from the various instruments, because of their different response characteristics, even if they are positioned at the same point in the aircraft, may be quite different.

2 THE COUNTING TECHNIQUES CONSIDERED

The range of normal accelerations for the Concorde data is split up into various levels at 0.05 g intervals in both the positive and negative direction from the datum, which is considered separately as the zero mean or the actual mean. Two levels are chosen on either side of the datum and only peaks which have magnitude greater than the magnitude of these levels are considered. In this Report these are taken at ± 0.05 g, so that minor deviations from the datum are neglected.

The methods are described for positive excursions from the datum only, but the negative excursions are handled in an analogous way. Several methods require a peak and trough detection process and this is described in the Appendix, section A.2.

Most electro-mechanical counting accelerometers give cumulative counts and the peak-counting methods described below are of this type.

2.1 Peak-count method

This method is the most selective one considered as only the maximum excursion from the datum is counted between successive datum crossings. Then a count is made of each level up to and including the maximum level crossed by the excursion considered. Thus it can be seen that secondary load variations are disregarded completely.

The name adopted here is the same as in Ref.1, but in Refs.2 and 3 the method is called the 'peak-between-mean crossings' count method and the 'mean-crossing-peak-count method' respectively. The programming details are discussed in the Appendix, section A.2. The method is illustrated in Fig.1a.

2.2 Simple peak-count method

This method is the least restrictive of all those considered here. Every peak value above, or every peak value below the datum is noted and a cumulative count made, i.e. a count is made of every level up to and including the maximum level encountered by each peak. The results are, therefore, seriously influenced by small load variations which are counted as if they were significant. These small variations may be disregarded by imposing certain restrictions such as those described in sections 2.3 and 2.6 below. The simple peak-count method is illustrated in Fig.1b and the program is discussed in the Appendix, section A.2.

This method is one of those considered by van Dijk², who names it 'the peak-count method', as does Schijve³.

2.3 Range-restricted peak-count method

Only those peaks which are both preceded and followed by drops of at least a certain magnitude (0.05 g) or a fixed percentage (50%) of the incremental peak value, whichever is the greater, are counted. It can be seen that intermediate load variations are rigorously disregarded but that in so doing some significant deviations may be neglected. Adaptations of both this method and the one described in section 2.6 are used in VGH records⁸; however the percentage drop requirement is usually greater than 50%. It is illustrated in Fig.2a, and the program is described in the Appendix, section A.3.

2.4 Simple level-crossing method

As the name suggests the method counts every crossing of the predetermined acceleration levels, and, in this Report, when on the positive side of the datum only the positive slope level crossings are counted, and on the negative side those crossings with negative slope are the ones considered. When the record starts and ends at the datum there is no difference to the resulting frequency distributions if the opposite slopes are used. Widely different load patterns can yield the same number of counts at each level with this method of counting. Every intermediate load variation is considered, however irrelevant it might be in the actual effect it has on fatigue life. To compensate to some extent for this those level crossings which are associated with load variations smaller than a certain range value may be disregarded and this is described in section 2.5 below. The technique is illustrated in Fig.2b, and the program is described in the Appendix, section A.4.

2.5 Variable re-set method

This method is also a level-crossing method, but here thresholds are introduced. The crossing of a certain acceleration level (primary level) is counted only if a lower level (secondary level) is subsequently crossed in the opposite direction. The threshold is defined as the distance between the primary and secondary levels. The primary and secondary counting levels applied in this Report are tabulated below. It can be seen that the threshold is related to the amplitude of the primary counting level, i.e. the greater the size of the primary counting level being considered the greater the drop must be

to the secondary counting level before a count is made. The numbers given below are relative to the datum.

	Acceleration levels (+ve) in 'g'										
Primary	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
Secondary	0.0	0.05	0.05	0.10	0.10	0.15	0.15	0.20	0.20	0.25	0.25

The negative primary levels have secondary levels in analogous positions to the positive ones.

The method excludes counts from small amplitude load variations about some mean level, but it is still influenced to some extent by intermediate fluctuations and also the same counts may result from very different load patterns. This method is that named as the restricted level-crossing method and the fatigometer count method respectively in Refs.2 and 3. These methods are associated with the fatigometer developed at RAE^{6,7}.

The method is illustrated in Fig.3a, and the program described in the Appendix, section A.5.

2.6 Level-restricted peak-count method

This method is a peak-counting type method which has the same type of threshold requirement as the previous level-crossing type method. As before only peaks which are preceded and followed by the required threshold are considered, but now a count is made of each level from the datum up to the maximum level encountered by the peak. The method is illustrated in Fig.3b and the program is described in the Appendix, section A.6.

2.7 Discussion of the methods

Distinct relationships exist between the six methods described above. Let us define a general peak-counting type method in which only those peaks which are preceded and followed by rises of at least a certain magnitude Q or a fixed percentage R of the peak value, whichever is the greater, are counted (*viz.* section 2.3). Then:

- (a) the peak-count method has $Q = 0$ and $R = 100\%$ of the maximum peak value encountered between datum crossings,
- (b) the simple peak-count method has $Q = R = 0$,

(c) the range-restricted peak-count method has $Q = 0.05 g$ and $R = 50\%$ of the incremental peak value,

(d) the level-restricted peak-count method has Q defined in terms of number of levels between a primary and secondary counting level which depends on the value of the primary level; $R = 0$.

The remaining two methods, the simple level-crossing method and the variable re-set method, are level-crossing type methods. It can be seen from the descriptions that the simple level-crossing method has the same counting conditions as the simple peak-count method. They differ only in that the former counts levels crossed and the latter makes a cumulative count of peaks. The same analogy applies to the variable re-set method and the level-restricted peak-count method respectively.

3 THE DATA

Before the data could be used in a Fortran program it was necessary to tabulate them in strict format on punched cards. The Concorde data were recorded at the rate of 25 readings per second. The actual time scale is ignored and it is assumed that for fatigue evaluation the load-time histories are completely characterized by all peak values in their actual sequence no matter what time has elapsed between successive peaks.

The term 'zero mean' is taken to refer to the 1.0g reference datum corresponding to level unaccelerated flight and normal acceleration values are given in 'g' units from the zero mean. However, especially in the patch of turbulence encountered by Concorde 002, the acceleration readings are seen to have a definite bias, so the mean acceleration levels for each set of data are found and the counting analyses are performed both from the zero mean and from the actual mean level for the patch. The rms acceleration for each set of data is found and converted to a rms gust velocity by the Kaynes⁹ spectral method for continuous turbulence. This method assumes that the atmospheric turbulence may be regarded as continuous and homogeneous in the horizontal plane. The energy of the vertical component is assumed to be distributed according to the Von Karman two-dimensional spectrum, and the scale length used is given by the inverse density ratio times 1000 ft.

A synopsis of the conditions as far as they are known is given below, and the time-histories are plotted in Figs.4-7.

Flight No.	Duration of turbulence in seconds	Height in ft	Mean level in g units	Eas in knots	Rms acceleration in g units	Rms gust velocity in m/s
10071	14.92	33000	-0.0224	400	0.1212	1.556
10113	32.64	40000	-0.0154	432	0.0855	1.060
10121	27.6	51200	0.0890	512	0.1534	1.394
20047	70.6	26900	0.1653	380	0.0990	1.251

The plots were made using the program of Ref.10 and the order of the flights as given in the table is maintained throughout this Report.

4 THE RESULTS

4.1 The numerical details

The following counts at each acceleration level are found for the datum of 1.0 g. The table uses the following abbreviations:-

- PCM - peak-count method
- SPCM - simple peak-count method
- RRPCM - range-restricted peak-count method
- SLCM - simple level-crossing method
- VRSM - variable re-set method
- LRPCM - level-restricted peak-count method

Flight 10071	Acceleration levels in g																	
	-0.35	-0.30	-0.25	-0.20	-0.15	-0.10	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
PCM			2	2	4	5	9		13	8	8	3	2	1	1	1		
SPCM			2	10	16	21	25		26	18	9	3	2	1	1	1		
RRPCM			2	2	5	7	11		17	12	9	3	2	1	1	1		
SLCM			2	9	8	7	10		16	16	9	3	2	1	1	1		
VRSM			2	2	4	6	9		13	9	8	3	2	1	1	1		
LRPCM			2	2	4	6	10		15	10	8	3	2	1	1	1		
Flight 10113																		
PCM		1	2	3	11	20	34		26	11	6	1	1	1	1			
SPCM		1	3	5	19	40	68		45	23	10	1	1	1	1			
RRPCM		1	3	5	17	31	49		31	14	7	1	1	1	1			
SLCM		1	3	5	19	37	53		35	17	8	1	1	1	1			
VRSM		1	3	5	16	28	34		26	14	7	1	1	1	1			
LRPCM		1	3	5	17	30	44		29	14	7	1	1	1	1			
Flight 10121																		
PCM	2	2	2	4	8	16	28		32	21	13	3	2	1	1	1	1	
SPCM	2	3	4	6	12	21	44		91	72	57	33	25	17	12	7	1	
RRPCM	2	2	3	5	9	18	34		42	30	20	7	5	2	2	2	1	
SLCM	2	3	3	5	10	18	37		41	30	29	11	10	5	8	7	1	
VRSM	2	2	3	5	9	18	28		32	25	15	5	3	2	2	2	1	
LRPCM	2	2	3	5	9	18	30		39	28	18	6	4	2	2	2	1	
Flight 20047																		
PCM	1	1	2	2	2	3	8		22	20	16	13	11	10	9	6	3	1
SPCM	1	2	3	3	3	5	10		222	216	201	141	78	40	37	17	6	1
RRPCM	1	1	2	2	2	4	9		86	83	70	55	33	18	16	6	3	1
SLCM	1	2	2	2	2	4	9		42	57	116	106	61	30	31	15	6	1
VRSM	1	1	2	2	2	4	8		22	38	31	25	22	16	14	6	3	1
LRPCM	1	1	2	2	2	4	9		51	49	42	31	26	17	15	6	3	1

Similarly, for the data taken from the various mean levels as datum the following counts are obtained at each level:-

Flight 10071	Acceleration levels in g relative to various means																			
	-0.50	-0.45	-0.40	-0.35	-0.30	-0.25	-0.20	-0.15	-0.10	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
PCM							2	3	4	9		10	7	6	4	3	1	1	1	1
SPCM							5	12	18	24		30	22	13	5	3	1	1	1	1
RRPCM							3	4	6	11		18	13	10	5	3	1	1	1	1
SLCM							5	8	7	9		18	16	13	5	3	1	1	1	1
VRSM							2	3	4	9		10	11	8	5	3	1	1	1	1
LRPCM							2	3	4	9		14	11	8	5	3	1	1	1	1
Flight 10113																				
PCM							3	3	10	21	32		29	15	8	1	1	1	1	
SPCM							3	3	13	33	55		52	28	13	2	1	1	1	
RRPCM							3	3	12	30	42		37	19	9	2	1	1	1	
SLCM							3	3	13	32	43		38	21	10	2	1	1	1	
VRSM							3	3	12	28	32		29	19	9	2	1	1	1	
LRPCM							3	3	12	28	39		33	19	9	2	1	1	1	
Flight 10121																				
PCM			2	2	3	4	10	19	21	25		23	9	5	2	2	2	1		
SPCM			3	3	5	8	16	37	59	73		62	37	27	19	15	9	2		
RRPCM			2	2	4	6	13	23	30	37		27	11	6	2	2	2	1		
SLCM			3	2	4	7	13	34	44	40		31	15	9	5	10	8	2		
VRSM			2	2	4	5	12	21	27	25		23	11	6	2	2	2	1		
LRPCM			2	2	4	6	13	23	31	35		25	11	6	2	2	2	1		
Flight 20047																				
PCM	1	1	2	2	3	4	6	15	37	74		56	31	17	6	5	3	1		
SPCM	2	2	3	3	5	7	10	30	70	114		140	77	40	17	15	6	1		
RRPCM	1	1	2	2	3	5	8	21	50	93		102	47	25	10	8	4	1		
SLCM	2	1	2	2	4	6	8	28	60	88		106	62	33	14	13	6	1		
VRSM	1	1	2	2	3	5	7	18	44	74		56	42	20	9	7	3	1		
LRPCM	1	1	2	2	3	5	7	18	44	81		71	46	24	10	8	4	1		

As stated previously, the methods fall into two groups which can be directly compared. The SLCM and VRSM are level-crossing methods, remembering that another name for VRSM is 'restricted level-crossing method'. The names suggest and the results endorse the idea that at each level the VRSM results

should be less than or equal to the SLCM in every case. The other methods are 'peak-counting' methods. It can be seen that the counts given by the SPCM are, at every level and in every case greater than or equal to those given by every other method and this is what might have been expected from the descriptions. Similarly the PCM results form the lower bound for those given by every other method at every level.

It should be noted that all the methods which require secondary counting conditions of any kind are dependent upon the restrictions chosen, and a change in the restriction might radically alter the results. The restrictions applied in the various methods, however, have been kept to roughly the same order of magnitude so that direct comparisons could be meaningful.

The effect of the non-cumulative level-crossing methods is that in some cases the higher magnitude acceleration levels have more counts than the lower ones, and therefore the resulting distribution has a peak. Indeed, in the case where the mean is assumed to be 1.0 g in flight 20047 the high frequency of level-crossings in the SLCM at 0.15 g and 0.20 g suggests that the mean lies around these values. In fact the mean value for the patch is found to be 0.1653 g.

When we consider how to interpret this effect when the distribution is used to estimate fatigue life, difficulties arise, since these level-crossing methods, when used in fatigue life estimates are usually considered to be peak-count type methods, i.e. if there are N_n crossings of level x_n and N_{n+1} crossings of level x_{n+1} then there are considered to be $(N_n - N_{n+1})$ peaks in the class between levels x_n and x_{n+1} . When $N_{n+1} > N_n$ this is meaningless. However, using the arithmetic mean of the positive and negative count distributions in the comparison of fatigue life estimates alleviates this effect to some extent. Also, examination of the results shows that the effect of these cases where $N_{n+1} > N_n$ is most significant where the datum is taken as 1.0 g (see Figs.8 and 9, which are illustrated on 3 cycle log/linear graph paper) and therefore for fatigue analysis the distributions used are those taken from data relative to the true mean for the patch. Even then cases do occur where $N_{n+1} > N_n$ but in each case $(N_{n+1} - N_n) < 1$ so the problem is surmounted by putting $N_{n+1} = N_n$, which infers that, instead of assuming we have a negative number of peaks between levels, we assume we have none.

For large scale analysis on many hours of data these non-cumulative methods are often used in fatigue analysis as, in the limit, it usually happens that, for N_n and N_{n+1} defined above, $N_n \geq N_{n+1}$ at all levels.

4.2 A comparative fatigue analysis

As has been stated previously, the arithmetic means of the positive and negative distributions, obtained using the various means as datum, are used in the ensuing analysis. This involves the assumption that the loads measured are turbulence loads only and that there are no manoeuvre loads present; if this were actually the case it would be expected that the positive and negative distributions would be approximately symmetric about the datum. This is not obtained in general, but the ratio of the number of times the counts at the negative levels are greater than those at the corresponding positive levels is 63/52. It is therefore felt that the assumption stated is sufficiently small to give reasonable comparative fatigue estimates derived from the distributions using the various counting techniques. The arithmetic mean distributions are superimposed on Figs.10-33.

The method of performing the damage calculations is that derived by Chilver¹¹, and the formula used is given as

$$D = \int_{x_1}^{x_{m+1}} \frac{1}{N(x)} \left| \frac{dF(x)}{dx} \right| dx, \quad (1)$$

where D is the total damage done
 $[x_{m+1} - x_1]$ is the interval of normal acceleration over which the total damage is to be calculated
 m is the number of levels at which counts are made
 $N(x)$ is the S-N curve for the aircraft
 $F(x)$ is the frequency distribution of cumulative cycles per hour.

The method of fitting a curve $F(x)$ to the arithmetic mean of the distributions is that described by Keynes¹², and is piecewise continuous; let the number of counts at a given load level x_1 be N_1 , x_2 be N_2 , ..., x_n be N_n . Then exponential functions of the form $A_n \exp(a_n x)$ are fitted between the consecutive points with coordinates (x_n, N_n) and (x_{n+1}, N_{n+1}) . The interval between the last load level at which a positive count is made and the first one at which no count is made is fitted by a parabola with its vertex at the first load level at which no count is made. Therefore the total damage D is the sum of the damage done over the various intervals in the set $([x_1, x_2], [x_2, x_3], \dots, [x_m, x_{m+1}])$.

The piecewise continuous distribution of counts thus obtained is then converted to a frequency distribution per hour by multiplying by a constant particular to each flight. The method of calculating the damage assumes the distribution F to be cumulative.

$N(x)$, the S-N curve for Concorde is given in Ref.13 as:

$$N(y) = C' / \left(\frac{1}{2} (y + \sqrt{y^2 + 3m''y}) \right)^5, \quad (2)$$

where C' - constant, having dimensions (stress)⁵

N - number of cycles to failure

m'' - mean stress

y - alternating stress.

The curve is derived in terms of stress amplitudes. However, since the stress (y) is linearly related to acceleration (x) then $N(y)$ is linearly related to $N(x)$ viz.:-

Let $y = \gamma x$

y - unit of stress

x - unit of acceleration

Let m' = mean acceleration corresponding to m'' , the mean stress, i.e.

$m'' = \gamma m'$.

Then

$$\begin{aligned} N(y) &= 2^5 C' \left(y + \sqrt{y^2 + 3m''y} \right)^{-5} \\ &= 2^5 \gamma^{-5} C' \left(x + \sqrt{x^2 + 3m'x} \right)^{-5} \end{aligned}$$

i.e.

$$N(y) = \gamma^{-5} N(x) \quad (3)$$

As the particular value of γ is unknown for these flights, we can only find the ratios of the damage assumed by the distribution of counts obtained from the various counting methods considered.

In all of these damage calculations it is assumed that all excursions from the datum to any given positive maximum is matched by a similar negative excursion to the equivalent negative amplitude, i.e. the loads occur in complete cycles. Since the acceleration trace is typified in the damage calculations by the results of the counting techniques it is also assumed that the counting procedures are reasonably consistent with this assumption.

The integral given in equation (1) can easily be evaluated on the ICL 1904A computer, and programming in Fortran enables us to use the standard subroutine called F4INTGSS. This subroutine uses Gauss's four-point and six-point integration formulae, varying the steplength until the two formulae give results which agree within a limit specified by the user.

In each case, the mean m' is taken as the true mean for the patch. x_1 is 0.05 g in each case, i.e. the minimum level considered and x_{m+1} is the first level at which the arithmetic mean is zero, which may be different for different flights.

The results of the damage calculations described above are shown in the table below. It must be remembered that we can only have a comparison between the various results for the different counting methods, since each number in the table must be multiplied by an unknown constant, which may vary from patch to patch, and also divided by C' , the constant from the S-N curve equation used. The result of the integral (1) is in terms of the proportion of the life of the aircraft exhausted by the turbulence encounter, and for each flight these proportions have been normalised.

The normalised proportions of the life of the aircraft exhausted by the turbulence encountered in each flight.						
Flight No.	PCM	SPCM	RRPCM	SLCM	VRSM	LRPCM
10071	0.636	1	0.758	0.865	0.687	0.692
10113	0.734	1	0.866	0.905	0.834	0.849
10121	0.377	1	0.451	0.704	0.423	0.451
20047	0.500	1	0.643	0.847	0.570	0.610

In each case it can be seen that the simple peak-count method results and the peak-count method results form the upper and lower bounds respectively for the results using the other methods. For these methods with threshold conditions it turns out that, with the restriction values stated in section 2 the range-restricted results are greater than the level-restricted peak-count method results, which are, in turn, greater than those given by the variable re-set method. The non-restrictive simple level-crossing method gives estimates which are less than those of the simple peak-count method but greater than the range-restricted peak-count method.

For each flight the range-restricted peak-count method gives the result which is nearest the 'average' and an adaptation of it is, of course, one of

those widely used in VGH recording programs *viz.* section 2.3. If we now consider the results relative to this method we find that, on average:-

SPCM over-estimates by 47%
 SLCM over-estimates by 22%
 LRPCM under-estimates by 4%
 VRSM under-estimates by 7%
 PCM under-estimates by 18%

It can be seen that those methods which involve threshold conditions, whether they are of peak-counting or level-crossing type, give estimates of damage which are within 7%. As has already been noted, methods involving threshold conditions of this type have been incorporated for many years in VCH records, fatiguemeters etc. It is hoped that, by prudent choice of the threshold conditions, these methods, when applied to measurements such as wing root bending moment and strain values, will give equally close agreement in similar calculations.

It must be borne in mind that the results given above are derived from only four small sets of data, the largest illustrating only 70 seconds of turbulence. Also errors are inherent in these calculations, e.g.

- (1) those perpetrated by forcing the non-cumulative methods into cumulative form,
- (2) those resulting from using the arithmetic mean of the positive and negative distributions in the damage calculations.

Thus error in fatigue evaluation may be minimised by choosing a method which results in the same number of counts being made at corresponding positive and negative load levels and by dealing with cumulative counting techniques only.

Another consideration in the search to find a counting method consistent with the 'cycles' assumption may be manoeuvre loading. Perhaps on certain sets of data the gusts superimposed on manoeuvre loads are cyclic, so that it would be helpful if some method of removing trends and using an adjusted mean could be found before the counting analysis is started.

5 THE CORRELATIONS AND POWER SPECTRA

Correlogram and power spectral analyses are performed on each flight and the resulting graphs are shown in Figs.34-41. The analyses are performed as stated previously using a program developed in the USA. Additional data

required are the calibration constant which is unity for these sets of data and the number of lags required, which is taken to be 50. Also the format, in which the data to be analysed is read in, is set as data. The number of readings and the time between successive readings are also given explicitly.

It can be seen that the four correlograms and spectra obtained are remarkably similar. The conditions of flight 10071 were, in addition to those given in section 3, $M = 1.2$, the centre of gravity was 57%, tank 7 was full and tank 10 had 6000 kg. These conditions correspond to those of certain flight flutter tests performed on 001 and the most outstanding spectral densities found at frequencies of about 2.75 Hz and 6.5 Hz corresponded, within experimental error, to the second and sixth symmetric wing modes found in the flutter tests. The frequency of 11.5 Hz was also found in the tests, but all that can be said is that it lies between the sixth and ninth symmetric modes.

The weight distribution of the fuel was unknown for the other three flights considered, and as different fuel distributions make a significant difference to the frequencies of the wing modes no conclusions could be reached. However, as all the spectra are similar it may be inferred that the large peaks in the spectra for the remaining 3 flights also correspond to symmetric modes.

6 CONCLUSIONS

Most of the counting methods considered require threshold conditions which can be chosen by the individual in a way which he considers best suits the data being analysed. The choice may radically alter the counting results obtained. The methods which have threshold conditions, whether they be of the level-crossing or peak-counting type, tend to give the same number of counts at the higher amplitude load levels and the difference in the damage calculations resulting from use of the different methods will vary only within $\pm 3\frac{1}{2}\%$.

The simple peak-count and simple level-crossing methods tend to give rather high counts especially at the lower levels, and this leads to the resulting pessimistic life estimates obtained from them, and so for most useful analyses on load data these methods may be ignored.

For estimating spectra for future aircraft designs or selecting loads for fatigue testing the peak-count method serves a useful purpose (Ref.2) but it tends to give optimistic fatigue estimates due to its exclusion of all but the maximum peak between zero-crossings. However the method is simple to use and the difference in the fatigue life estimates calculated using the distribution of counts obtained from it and from these methods with threshold conditions may be at most 18%.

However, for giving valid estimates of fatigue life, especially in comparative work where most of the errors inherent in the assumptions cancel out, these methods with threshold conditions may be used to best advantage as it may be possible to adapt them to describe the load distribution in a more 'cyclic' way. Care must be taken if any attempt is made to find a true estimate of life, in finding threshold conditions which are meaningful for the load system being examined, however, and, for use in damage calculations, the method which results in the positive count distribution and negative count distribution being most alike should be chosen.

Appendix

PROGRAMS FOR COUNTING METHODS

A.1 Peak-count method

This program is used to obtain the distributions of counts using the method described in section 2.1 on the sets of data being considered in this Report, and it forms the foundation for all five others which are described below.

All the data, K in number, are read in as real numbers given to 4 decimal places and the maximum value, the minimum value and the mean are found. The data are scanned and if two or more successive readings are equal, all but one of the equal set are discarded, and the data are stored in a new five-figure integer array called C. It is found to be less troublesome if integer manipulations are adhered to throughout. The data are further manipulated by subtracting 1.0000 from the original numbers (i.e. 10000 from each of the new integers - considering true normal accelerations) and the results put in C. The levels considered are now at '500' intervals (i.e. 0.05 g), level 1 is attributed to the minimum negative level encountered, and the number of the mean level (MB) is found from:-

$$MB = \left(- \frac{\text{MIN } C(I)}{I=1,K} \right) / 500 + 1 ,$$

remembering that the result of integer division in Fortran is integer.

The positions of the zero crossings in the new array, C, are then found and noted. Successive sets of data lying between zero crossings are then examined. *If the numbers are positive, the maximum is found and converted to the number of the largest level crossed. 1 is then added to all levels from the datum up to this level. Similarly, if the numbers are negative, the minimum is found and converted to the number of the lowest level number crossed. 1 is added in this case to all levels from this one up to the datum.** The operation is complete when the last datum crossing is reached. The intervals before the first and after the last datum crossing are analysed as if there had been datum crossings before the first and after the last data points.

The data are then converted to numbers relative to the true mean level for the flight and the whole operation is repeated using this new datum. All four flights are treated in this way and the resulting distributions for the mean as datum are shown in Figs.10, 16, 22 and 28, and the number of counts at each acceleration level are shown in section 4.1.

A.2 Simple peak-count method

This program, which is used to obtain the distribution of counts using the method described in section 2.2, follows that for the peak-count method described above down to *. The information between * and ** should now be replaced by the following:-

*Each peak value is found by scanning every three successive readings. If the absolute value of the second is greater than that of the first and third we have a peak and its value is noted. (Similarly, if the absolute value of the second is less than that of the first and third we have a trough.) Every peak value between successive datum crossings is noted and converted to the next lowest level number. A cumulative count of each level is then made for each peak.**

The frequency distributions are displayed in Figs.11, 17, 23 and 29 for the results using the mean as datum.

A.3 Range-restricted peak-count method

This program which is used to obtain the distribution of counts using the method described in section 2.3 follows that for the peak-count method down to * in the description in section A.1. Then, as before, each peak and trough value between datum crossings is found and converted to the next lowest and next highest level numbers respectively and these values stored in two separate arrays for peaks and troughs. In this program, however, the actual peak and trough values are also stored in two separate arrays.

The method is described for positive excursions from the datum only. For the negative case, the usual substitution of minimum for maximum and maximum for minimum is all that is necessary.

The first peak value is found and the second trough value is subtracted from it. (The datum crossing is regarded as being the first trough value.) If this absolute value is greater than or equal to 0.05 g or half the absolute value of the maximum whichever is the greater the maximum is noted. If this condition is not satisfied then the next minimum less than the one considered is subtracted from the greatest maximum preceding it. In the event of the absolute difference of the peak and trough values fulfilling the criterion a cumulative count is made of the maximum level, and the whole operation is started again with the next minimum after the maximum in question, in order to find a trough, *preceding* a peak, which also satisfies the condition.

The results (from the mean as datum) are illustrated in Figs 12, 18, 24

A.4 Simple level-crossing method

This program is used to obtain the distributions of counts using the method described in section 2.4 and is exactly analogous to the description given in section A.1 except for the information between * and **. This should be substituted by the following:-

*Each peak and trough value is found and converted to the next lowest and next highest level numbers respectively and these numbers are stored in two separate arrays for peaks and troughs. A count is then made of each positive slope crossing of the separate levels between successive minima and maxima.**

The results for data relative to the mean are shown in Figs.13, 19, 25 and 31.

A.5 Variable re-set method

This program is used to obtain the distributions of counts using the method described in section 2.5. As in the previous program every peak and trough between successive datum crossings is found and converted to the next lowest and next highest level numbers respectively. The first peak level value (i.e. the value of the first peak in terms of the maximum level crossed by it) is then considered and a record is made of all the levels crossed up to the peak, i.e. the appropriate elements of an array 'RP' are set to 1.0. The second trough level value is now considered and a count is made of all levels of positive slope which have been recorded for which this trough satisfies the threshold condition. The recording array elements for which a count has been made are now reset to 0.0. The whole operation is restarted with the next peak after the trough in question, a record being made of all levels of positive slope between the trough and peak considered. The resulting distributions for the mean as datum are shown in Figs.14, 20, 26 and 32.

A.6 Level-restricted peak-count method

This program, used to obtain the distribution of counts using the method described in section 2.6 is a combination of the counting criterion used in the range-restricted peak-count method program with the threshold condition of the variable re-set method program.

The first peak level value is found and the second trough level value examined. If the threshold condition of the peak is not satisfied by this trough then the first succeeding trough level value less than the one considered is

examined with respect to the greatest peak level value preceding it. If the trough satisfies the threshold condition for the peak then a cumulative count is made of all levels up to the peak level value.

The results for the mean as datum are illustrated in Figs.15, 21, 27 and 33.

SYMBOLS

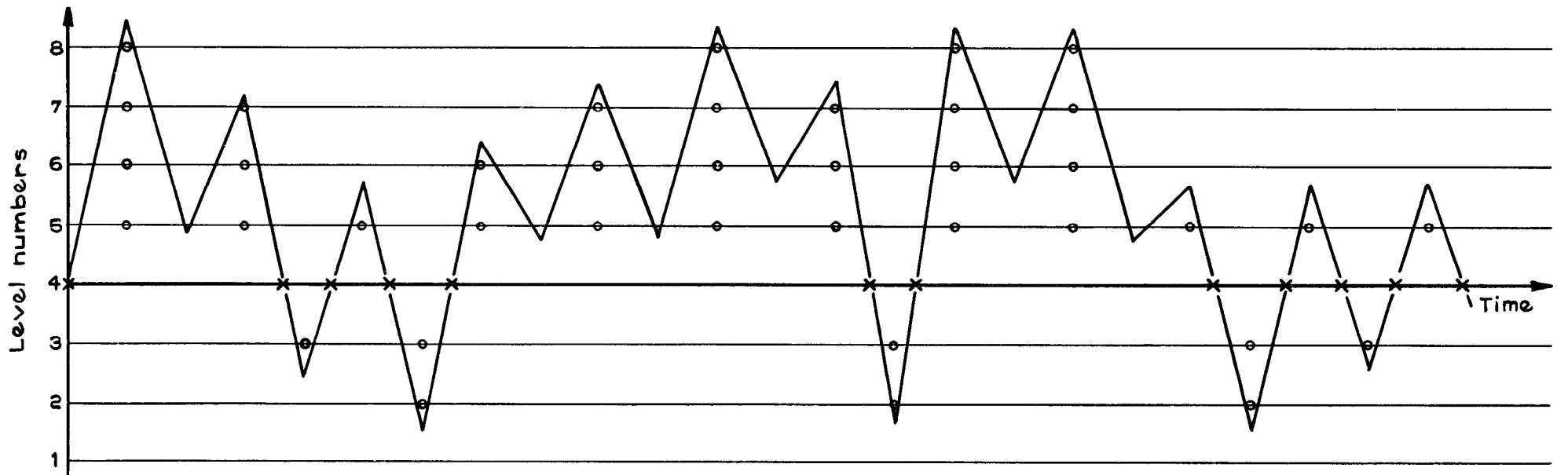
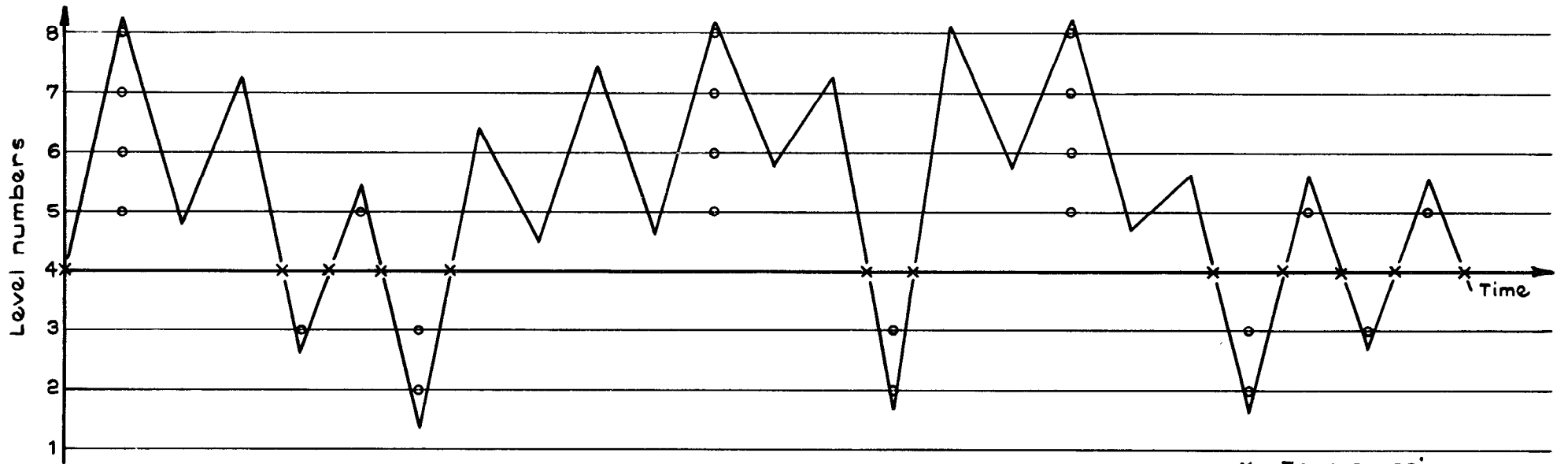
g	units of normal acceleration = 9.81 m/s^2
x_n	acceleration levels in g relative to the datum
N_n	number of counts made at level x_n
$F(x)$	piecewise continuous fit for the arithmetic mean of the cumulative frequency distributions of counts
m	number of levels at which the arithmetic mean of the distributions of counts is non-zero
A_n	} parameters for the exponential fit between the points at x_n and x_{n+1} ($n < m$)
a_n	
m'	mean acceleration in true g units
y	stress level
m''	mean stress corresponding to m'
γ	ratio of stress/acceleration
C'	constant stress in S-N curve equation
D	damage integral value
$N(y)$	number of cycles to failure in terms of stress
Q	value of the constant threshold required in the counting methods
R	value of the percentage threshold required in the counting methods
MB	number of the mean level in the counting methods programs
C	integer array of normal acceleration data points
K	number of data points per flight
I	element number in the array C
RP	'noting' array used in variable re-set method program

REFERENCES

- | <u>No.</u> | <u>Author</u> | <u>Title, etc.</u> |
|------------|--------------------------|---|
| 1 | R. Sewell | An investigation of flight loads counting methods and effects on estimated fatigue life.
NAE 1412 LTR-ST 431, October 1970 |
| 2 | G.M. van Dijk | Statistical load data processing.
NLR MP 71007 U, April 1971 |
| 3 | J. Schijve | The analysis of random load-time histories with relation to fatigue tests and life calculations.
NLR MP 201, May 1961 |
| 4 | J Schijve
F.A. Jacobs | Programme-fatigue tests on notched light alloy specimens of 2024 and 7075 materials.
Nat. Aero Res. Institute, Amsterdam Report M 2070, March 1960 |
| 5 | G. Miller | Tukey spectrum; cross spectra and power spectra.
FORTRAN PROGRAM
Computation Facility, University of California, San Diego, August 1961 |
| 6 | J. Taylor | Measurement of gust loads on aircraft.
J. Roy. Aero. Soc. 57, 78 (1953) |
| 7 | J. Taylor | Measuring fatigue.
The Aeroplane 86, 478 (1954) |
| 8 | H.M. Wells JNR | Flight recording for aircraft structural integrity.
AGARD Symp. on flight instrumentation, Paris
Sept 1965 |
| 9 | I.W. Kaynes | Gust loads on Comet aircraft.
ARC CP No.1247 (1971) |
| 10 | D.J. Eckford | RAE programme-337: An autocode sub-program for on-line graph plotting.
RAE Technical Memorandum Structures 703 (1967) |
| 11 | A.H. Chilver | The estimation of fatigue damage on structural elements.
ARC No. 16813 (1954) |

REFERENCES (concluded)

<u>No.</u>	<u>Author</u>	<u>Title, etc.</u>
12	I.W. Kaynes	A summary of the analysis of gust loads recorded by counting accelerometers on seventeen types of aircraft. AGARD R605, December 1972
13		British Aircraft Corporation Ltd (Filton Division). SST/B74S-5/2406/1, November 1970



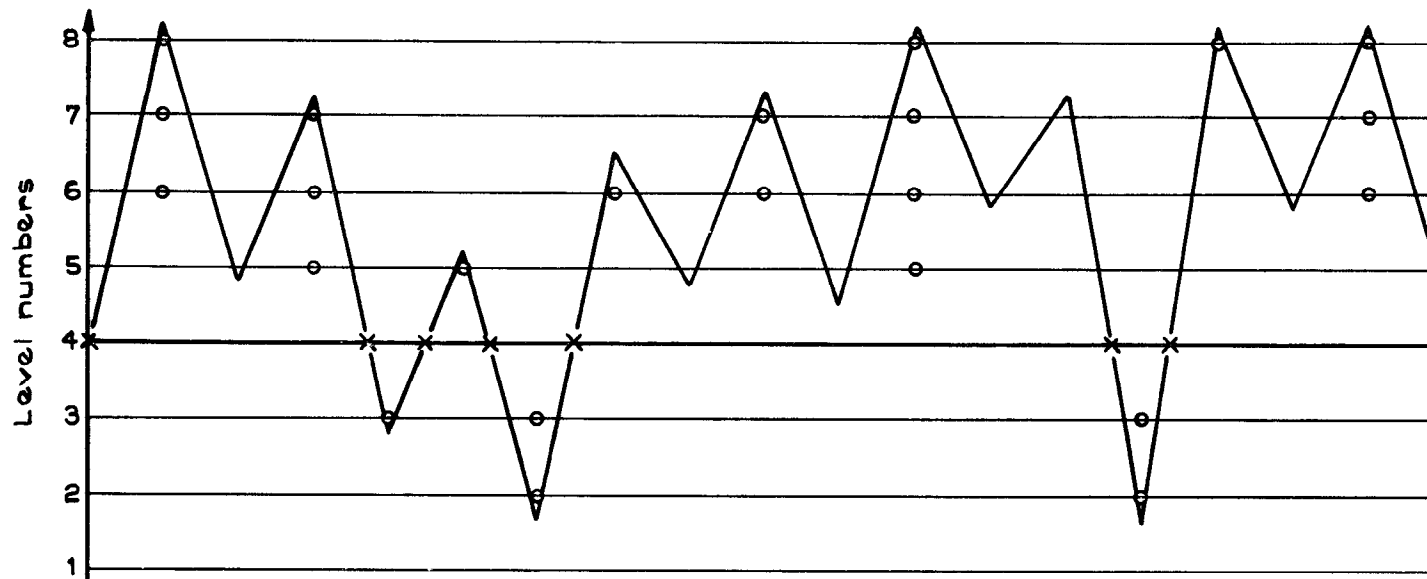


Fig.3a Variable re-set method

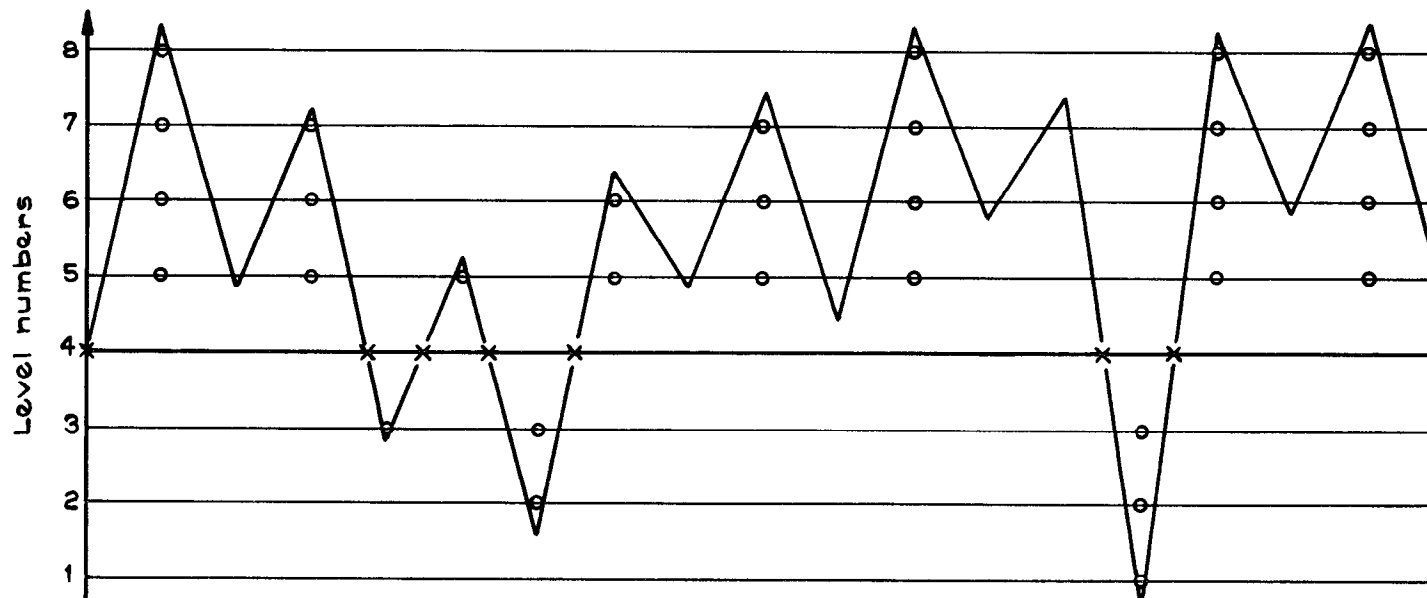


Fig.3b Level-restricted peak-count meth

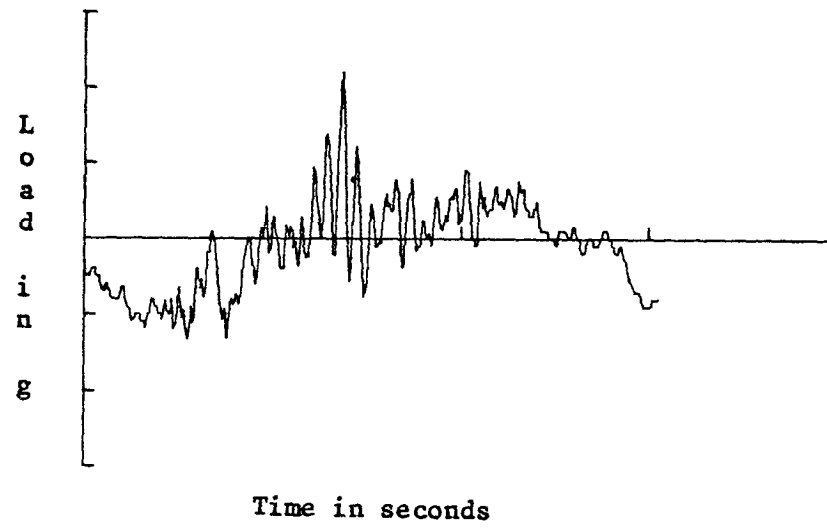


Fig.4 Load sequence of flight 10071

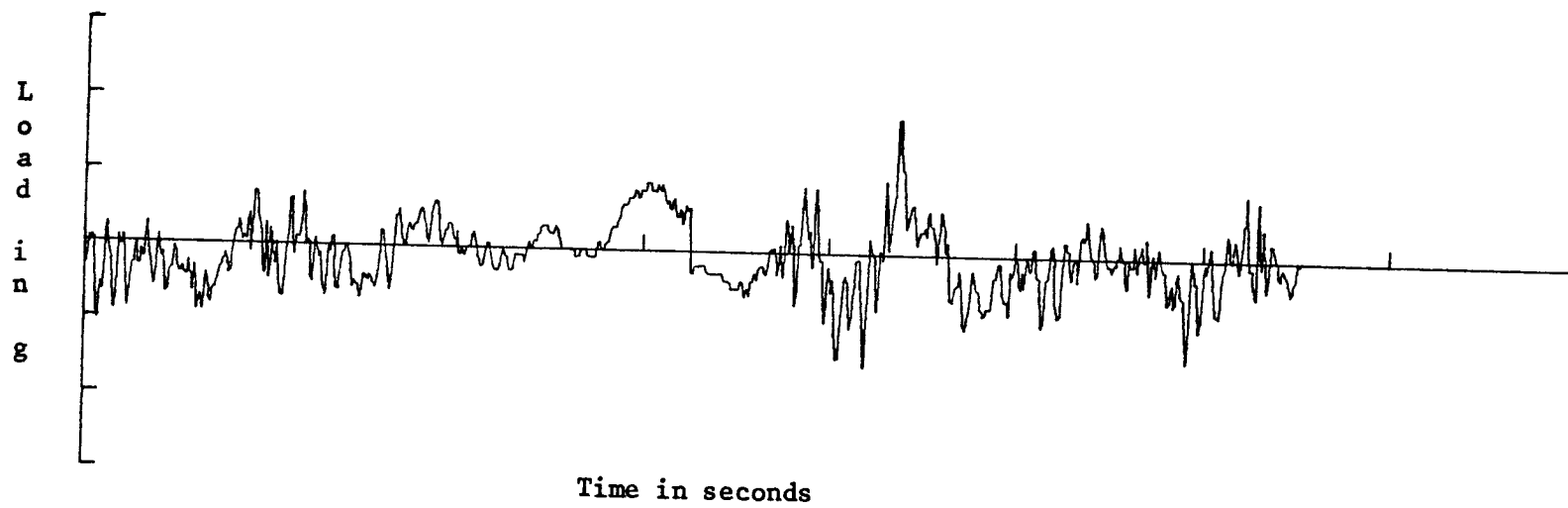


Fig.5 Load sequence of flight 10113

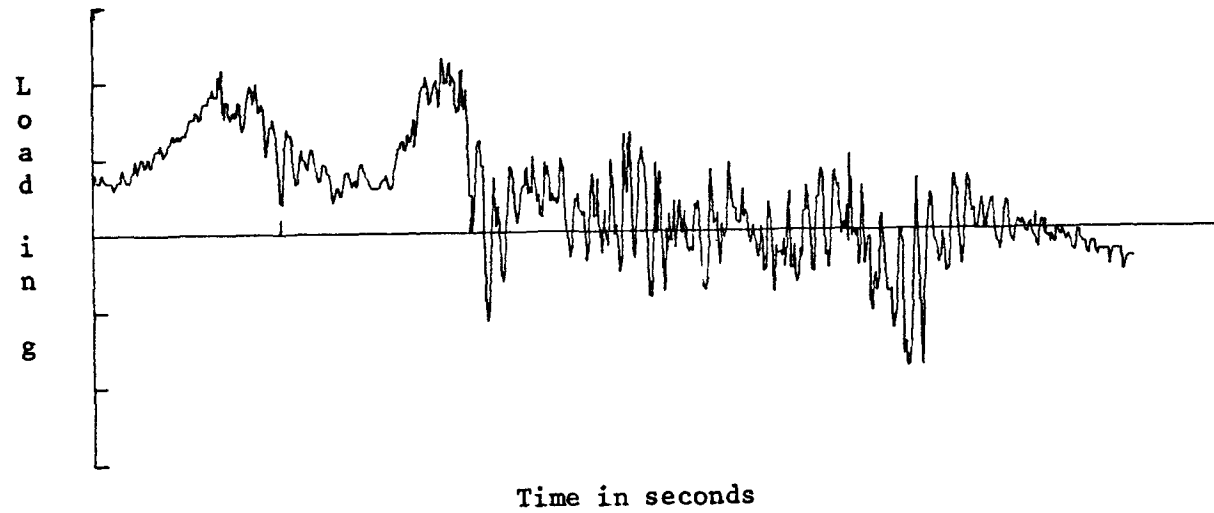


Fig.6 Load sequence of flight 10121

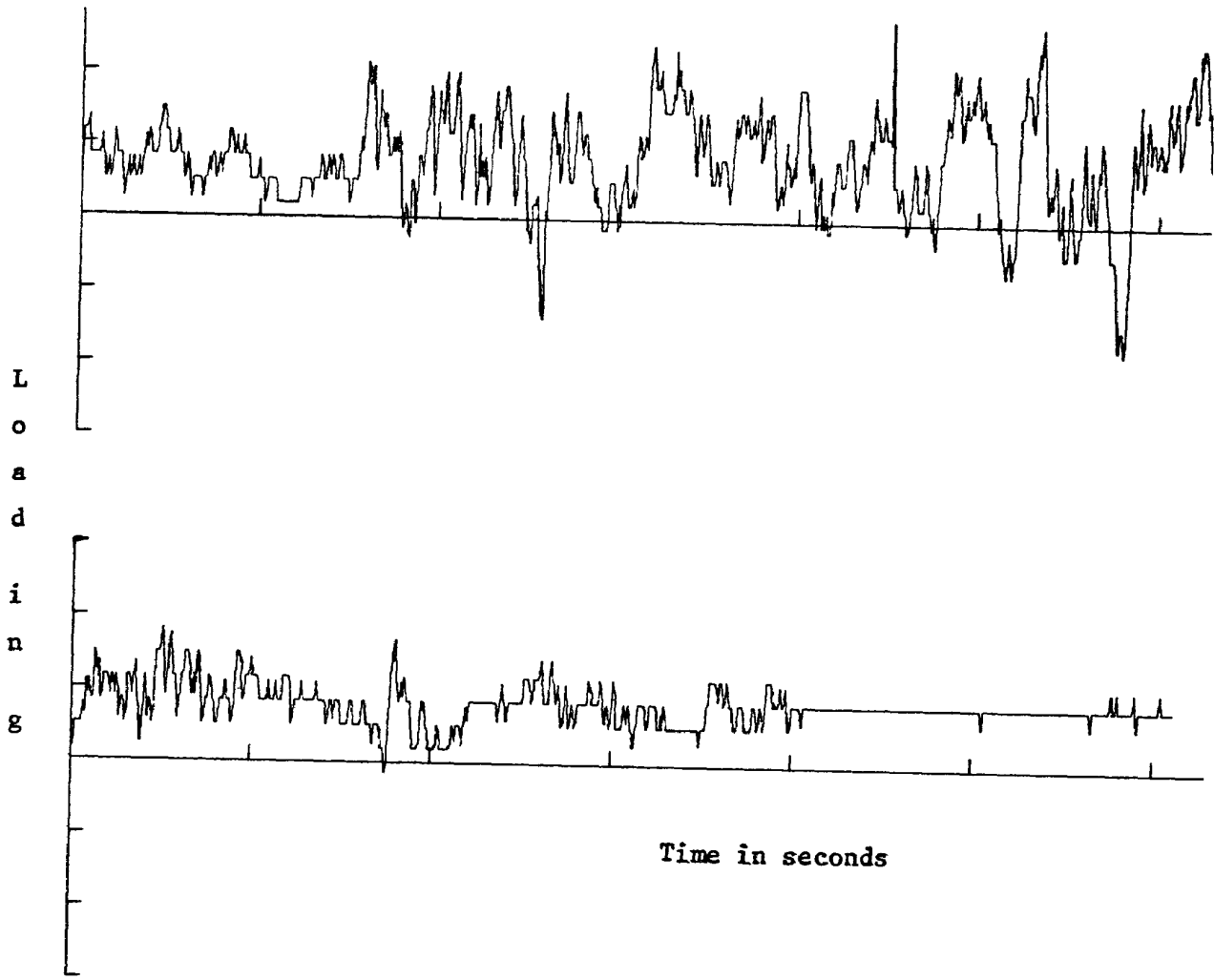


Fig.7 Load sequence of flight 20047

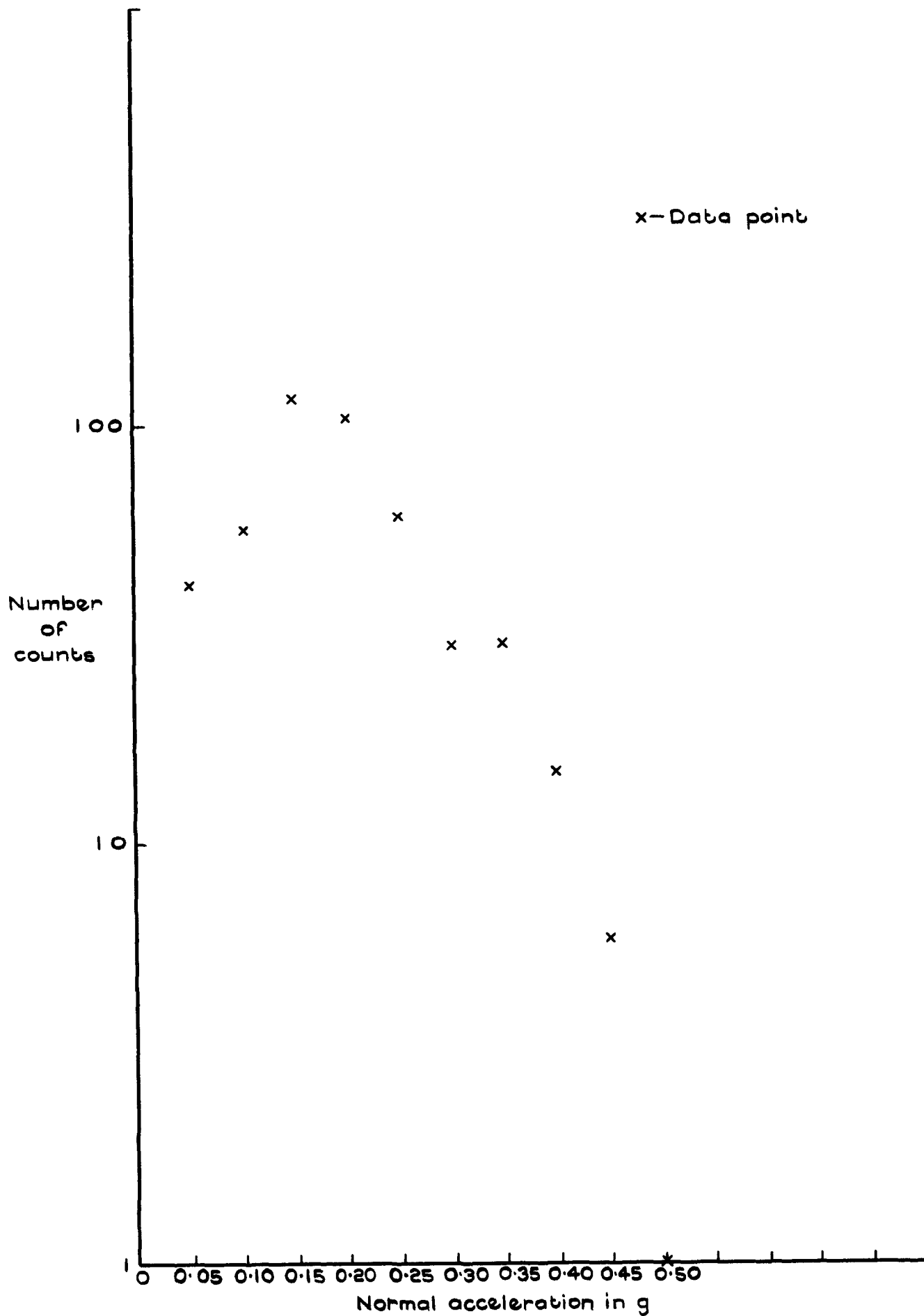


Fig. 8 Level-crossing method
Data from flight 20047 relative to 1.0g

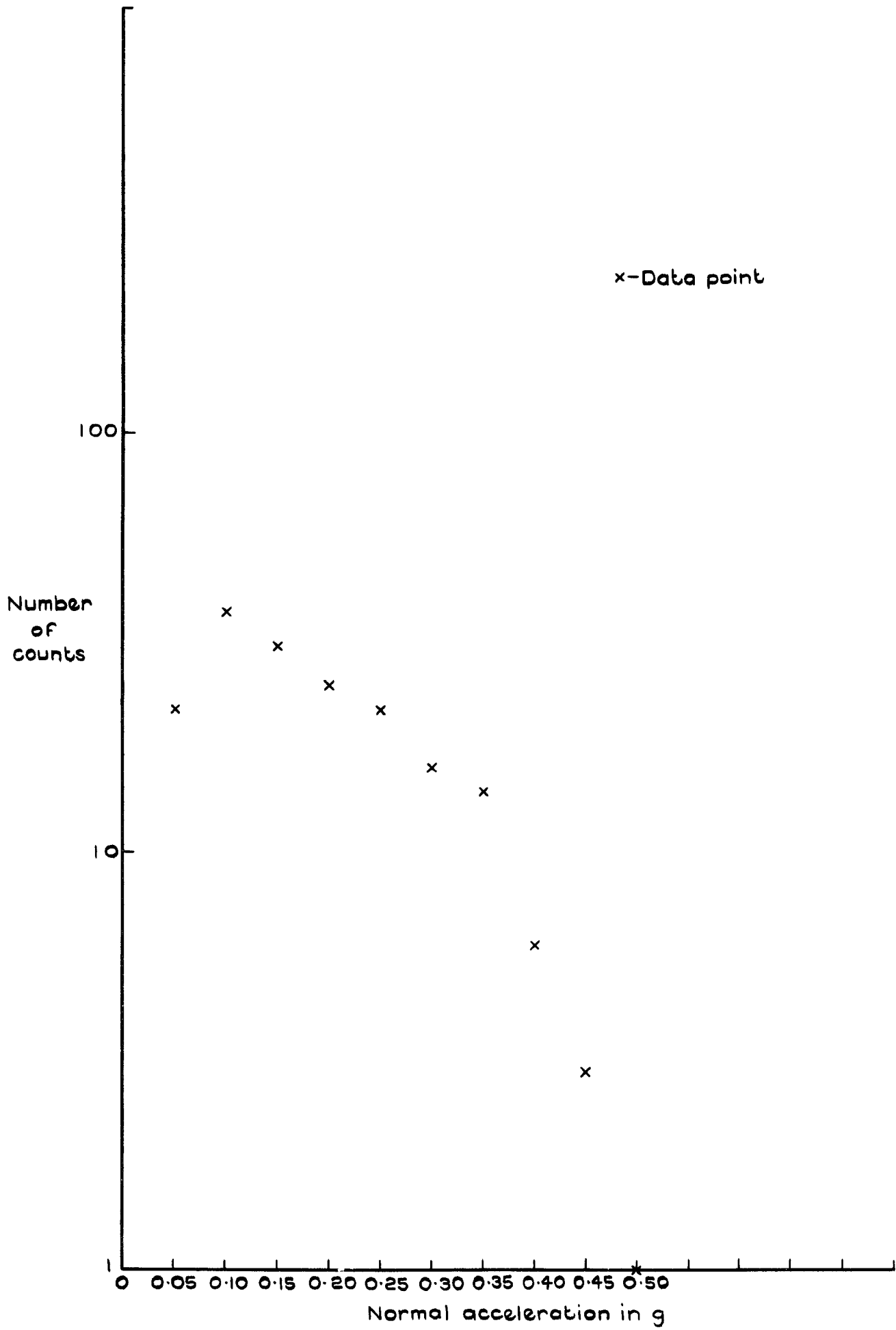


Fig.9 Variable re-set method
Data from flight 20047 relative to 1.0g

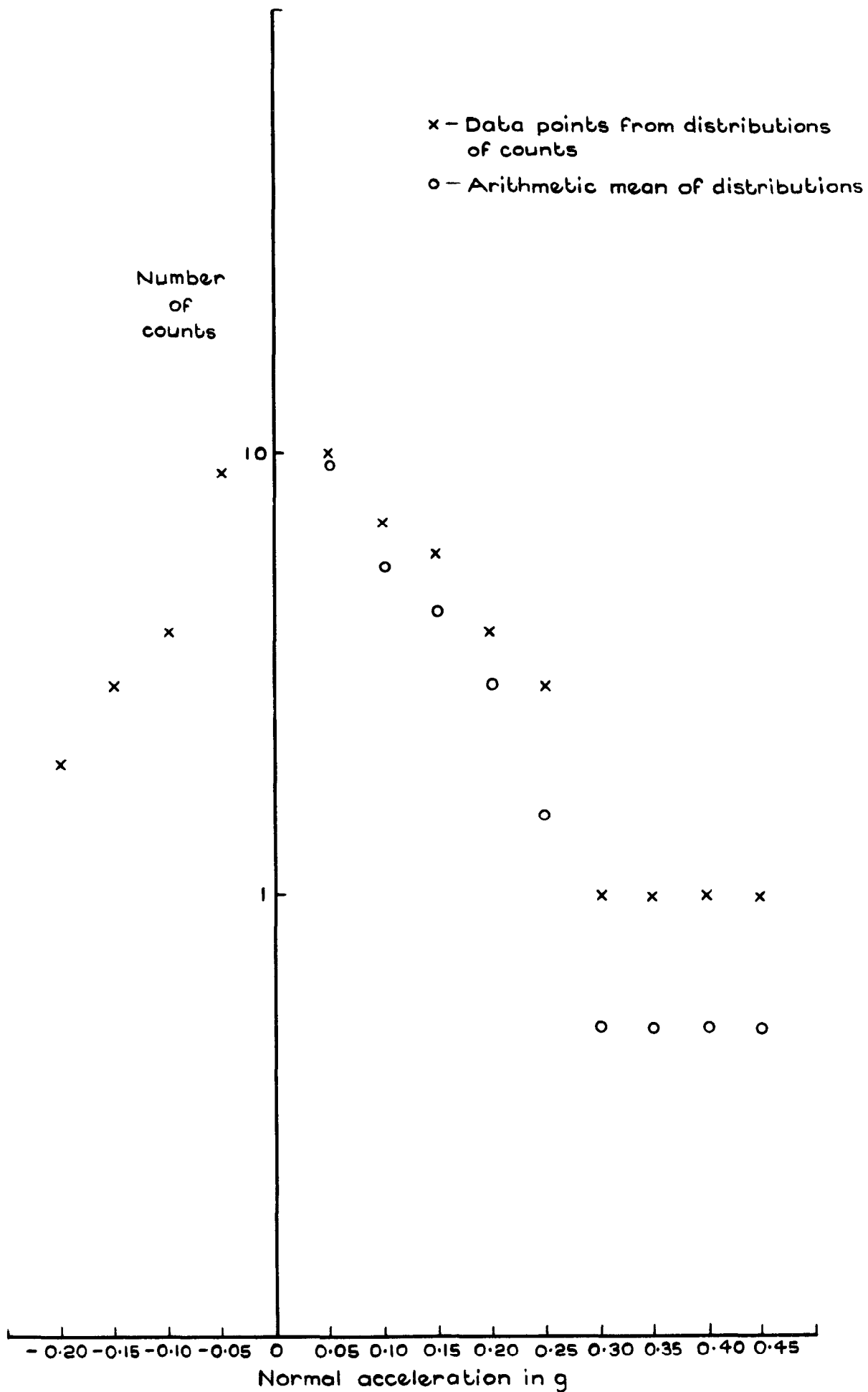


Fig.10 Peak-count method
Data from flight 10071 relative to mean

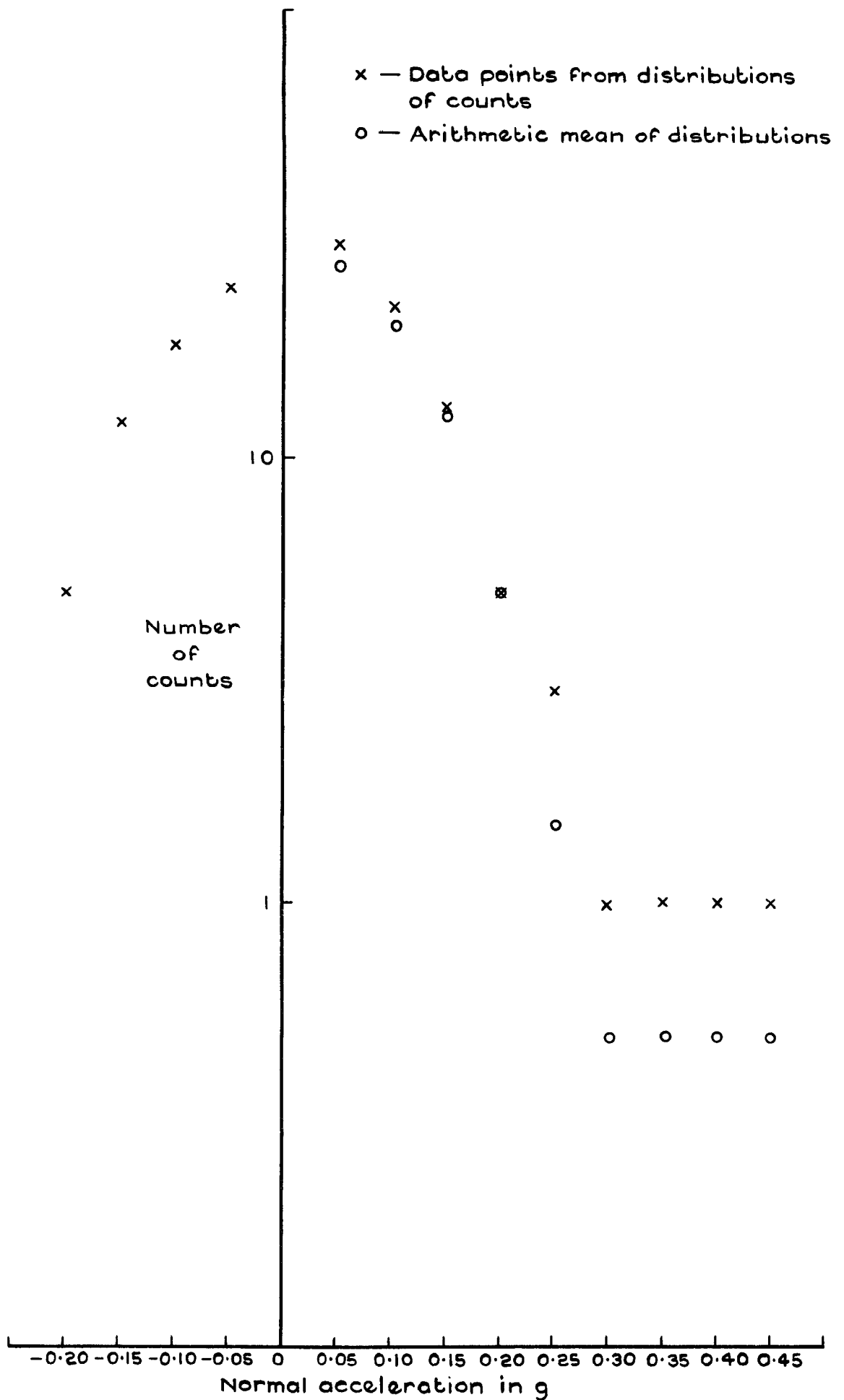


Fig.11 Simple peak-count method
 Data from flight 10071 relative to mean

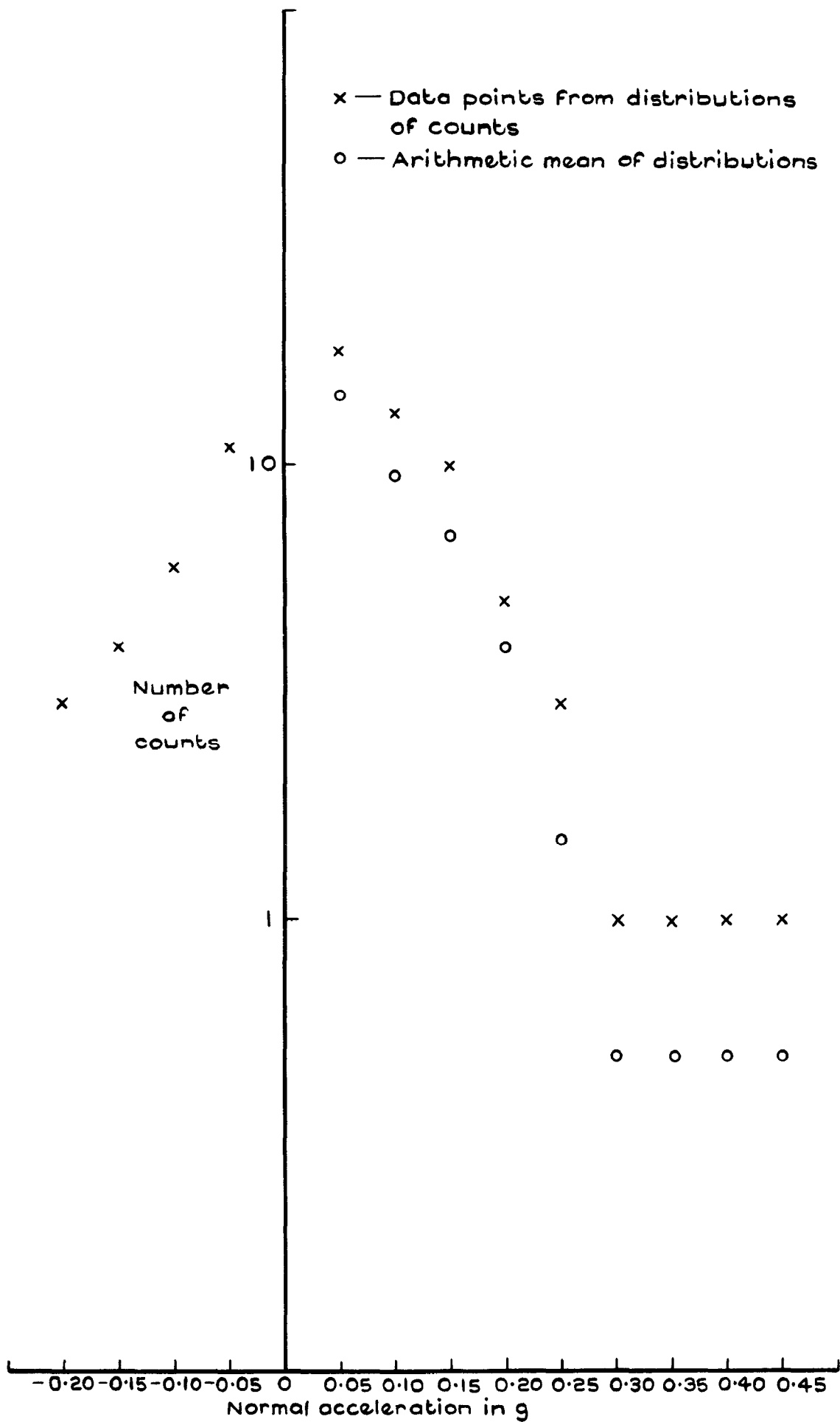


Fig.12 Range-restricted peak-count method
Data from flight 10071 relative to mean

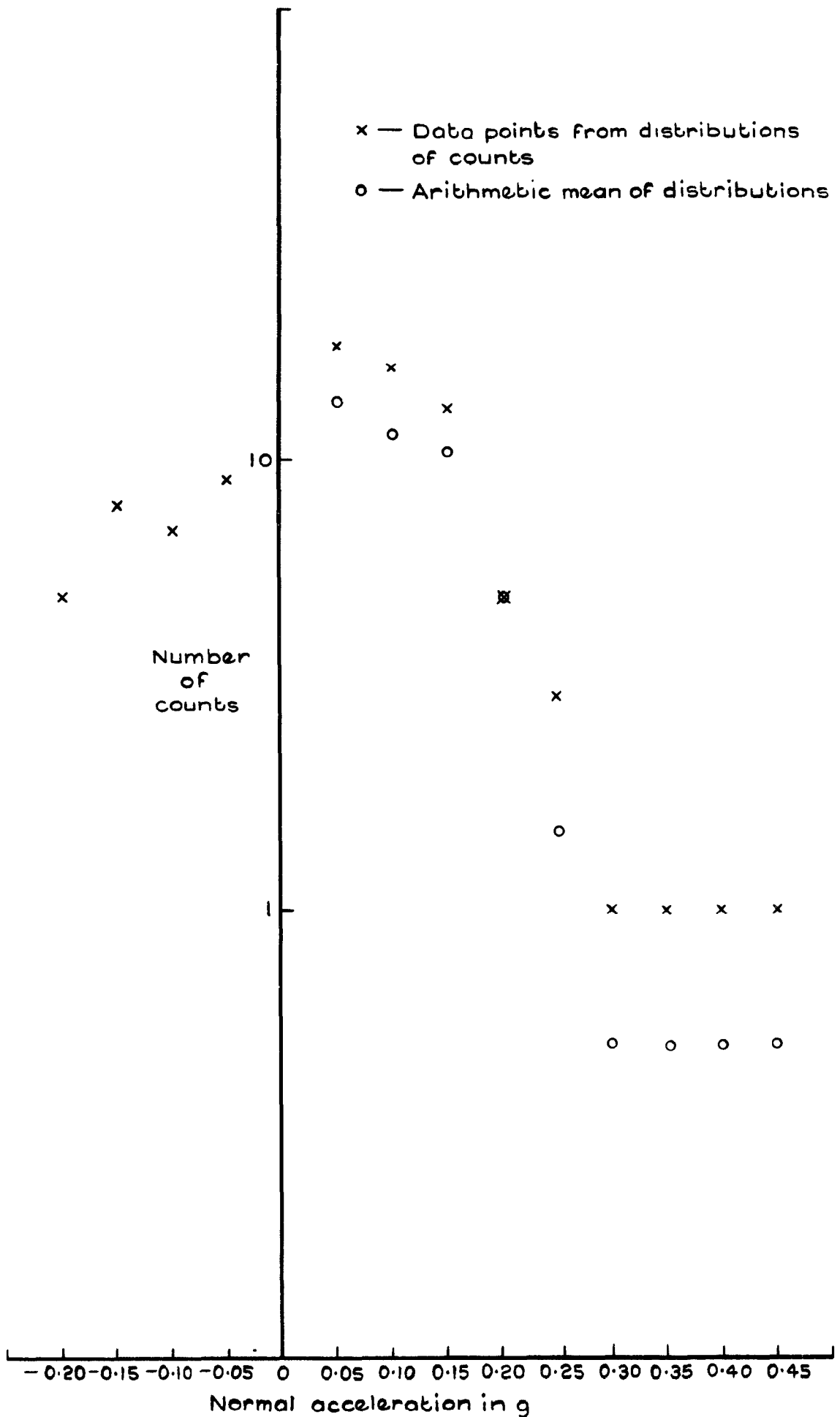


Fig.13 Simple level-crossing method
Data from flight 10071 relative to mean

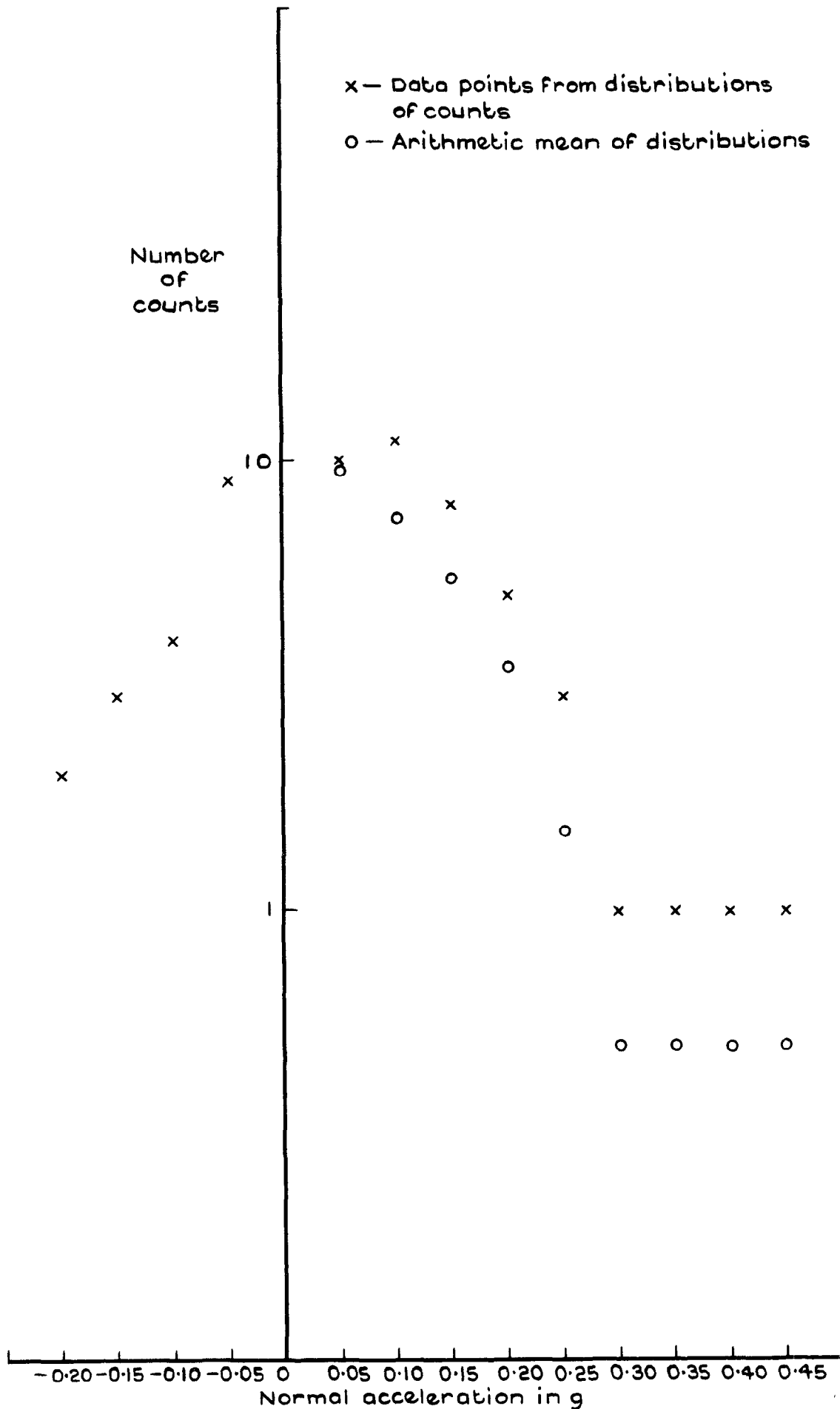


Fig.14 Variable re-set method
 Data from flight 10071 relative to mean

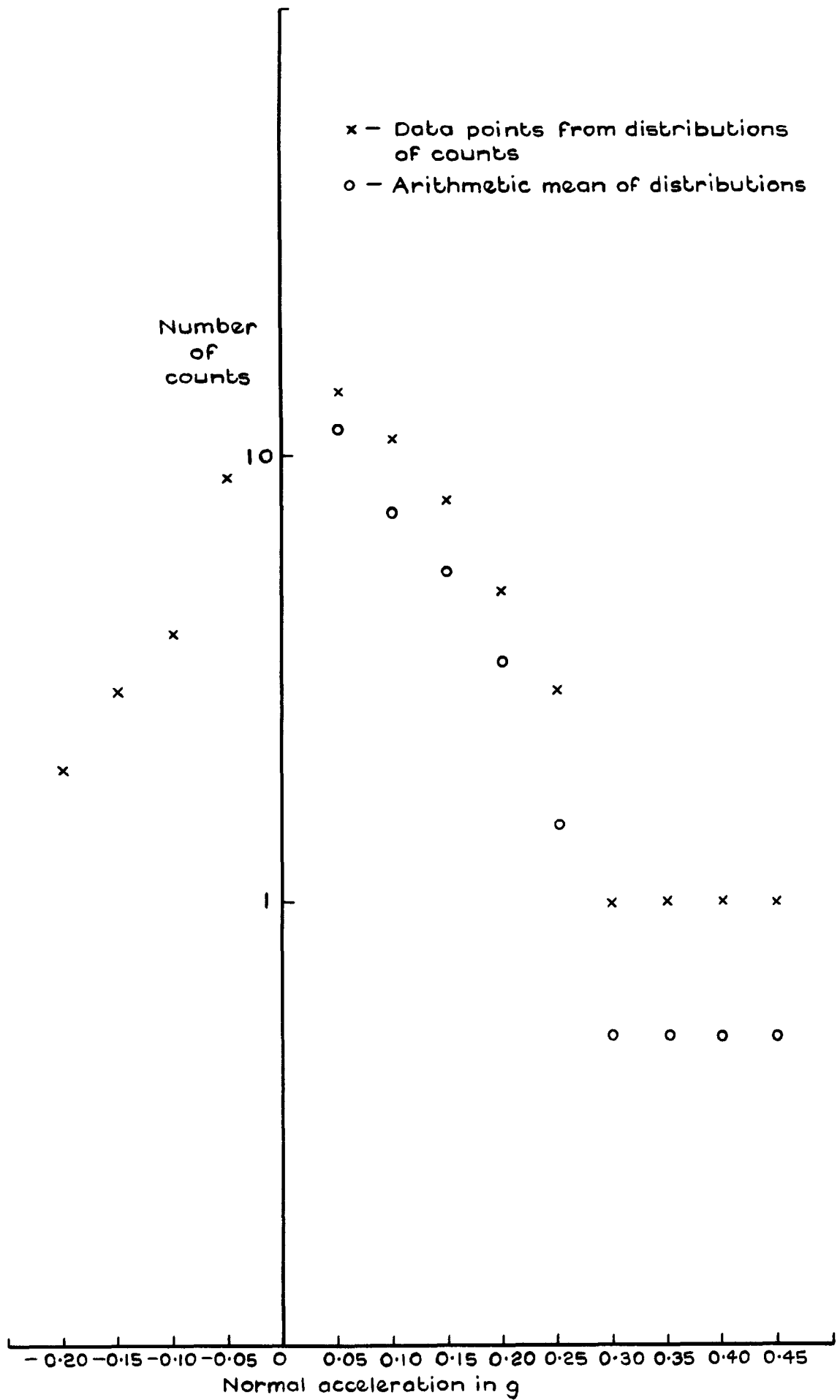


Fig.15 Level-restricted peak-count method
 Data from flight 10071 relative to mean

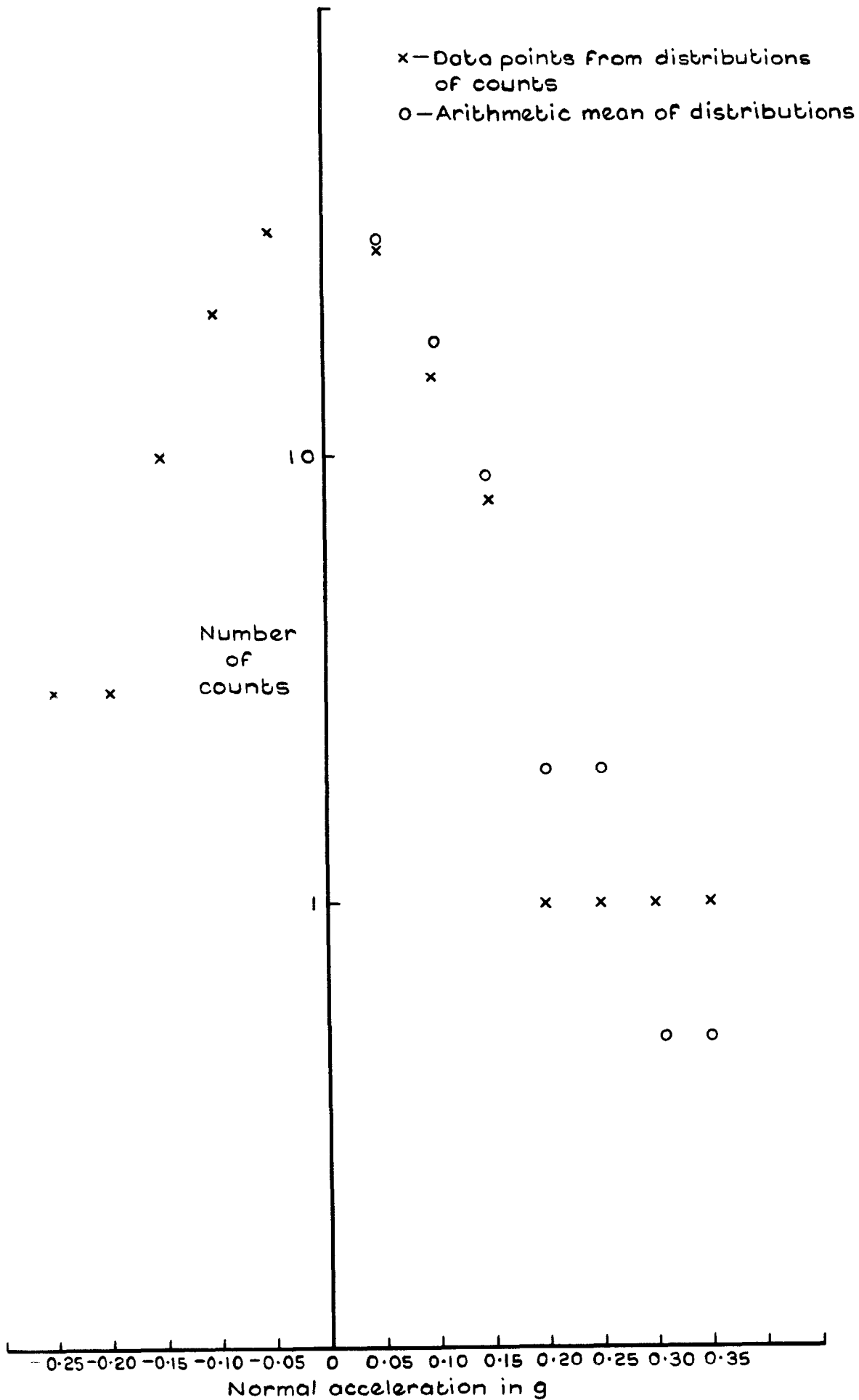


Fig.16 Peak-count method
 Data from flight 10113 relative to mean

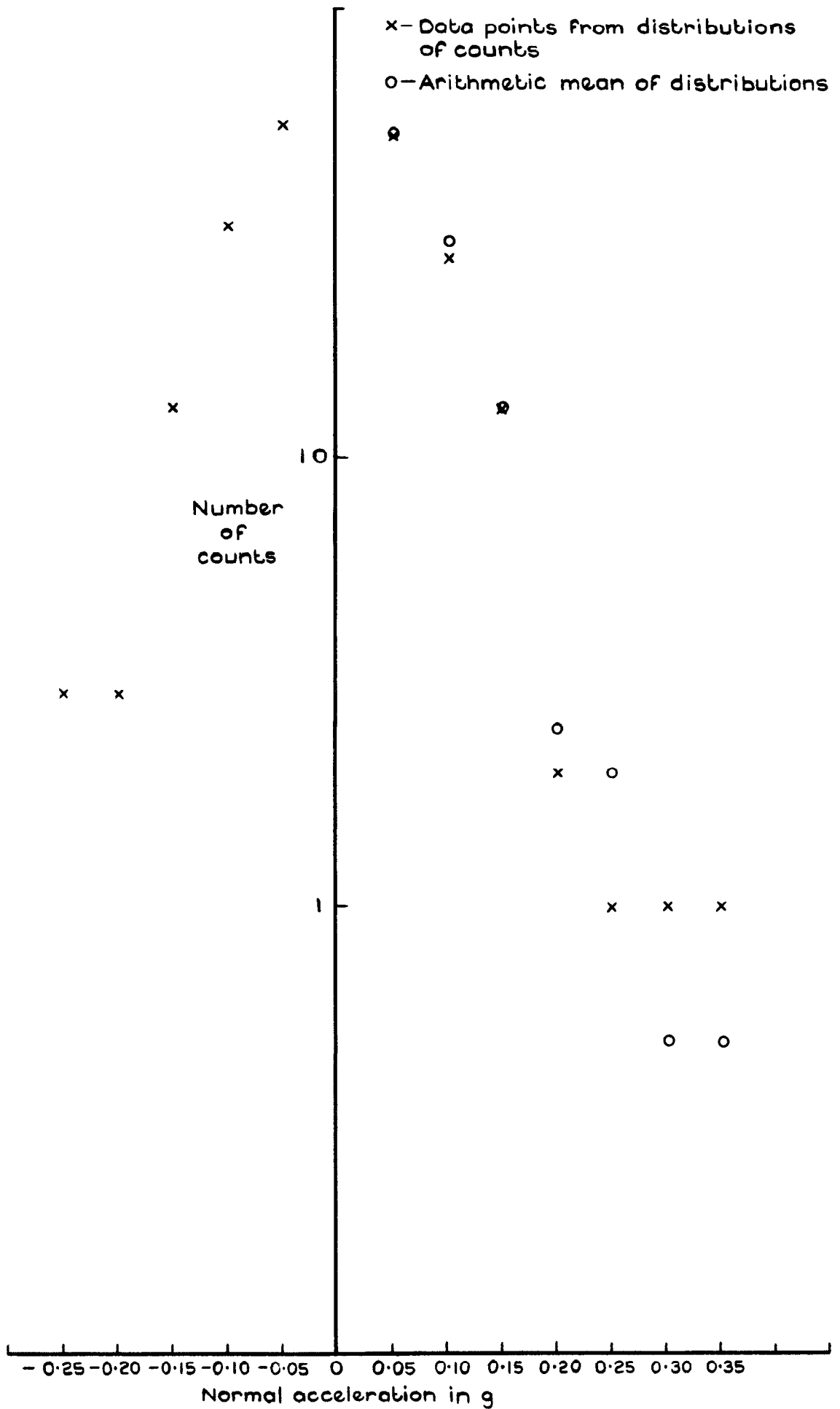


Fig.17 Simple peak-count method
Data from flight 10113 relative to mean

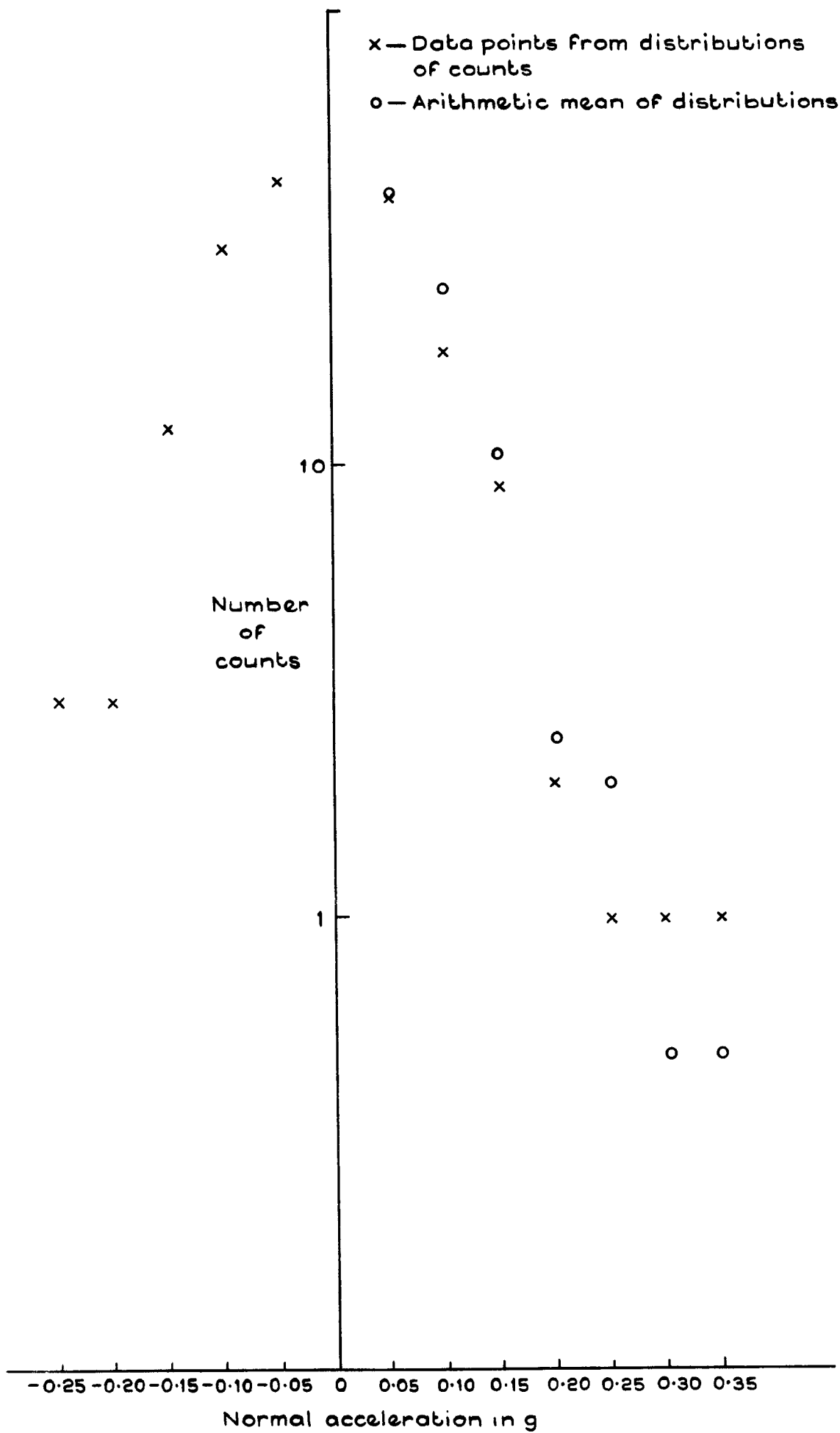


Fig.18 Range-restricted peak-count method
 Data from flight 10113 relative to mean

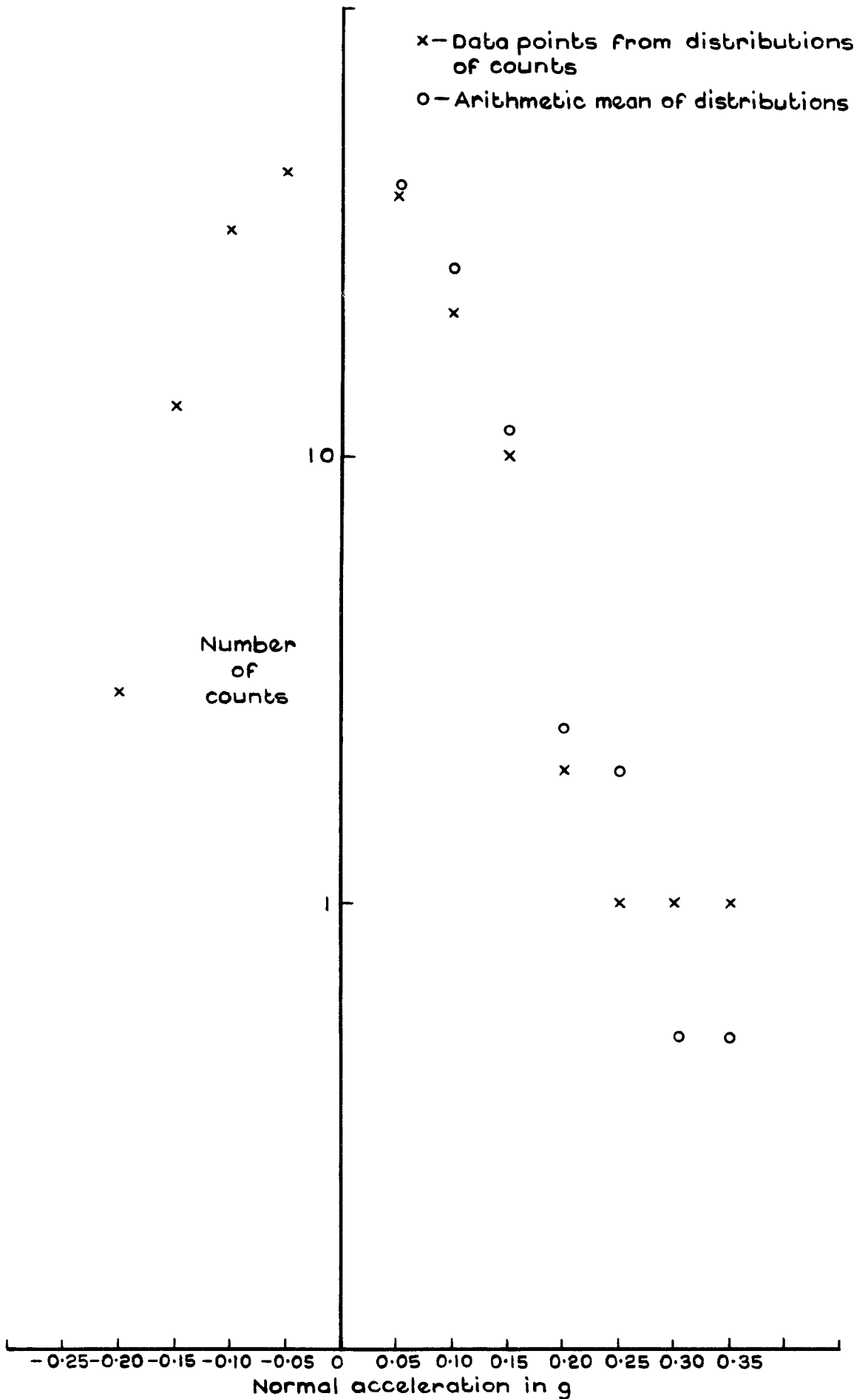


Fig.19 Simple level-crossing method
 Data from flight 10113 relative to mean

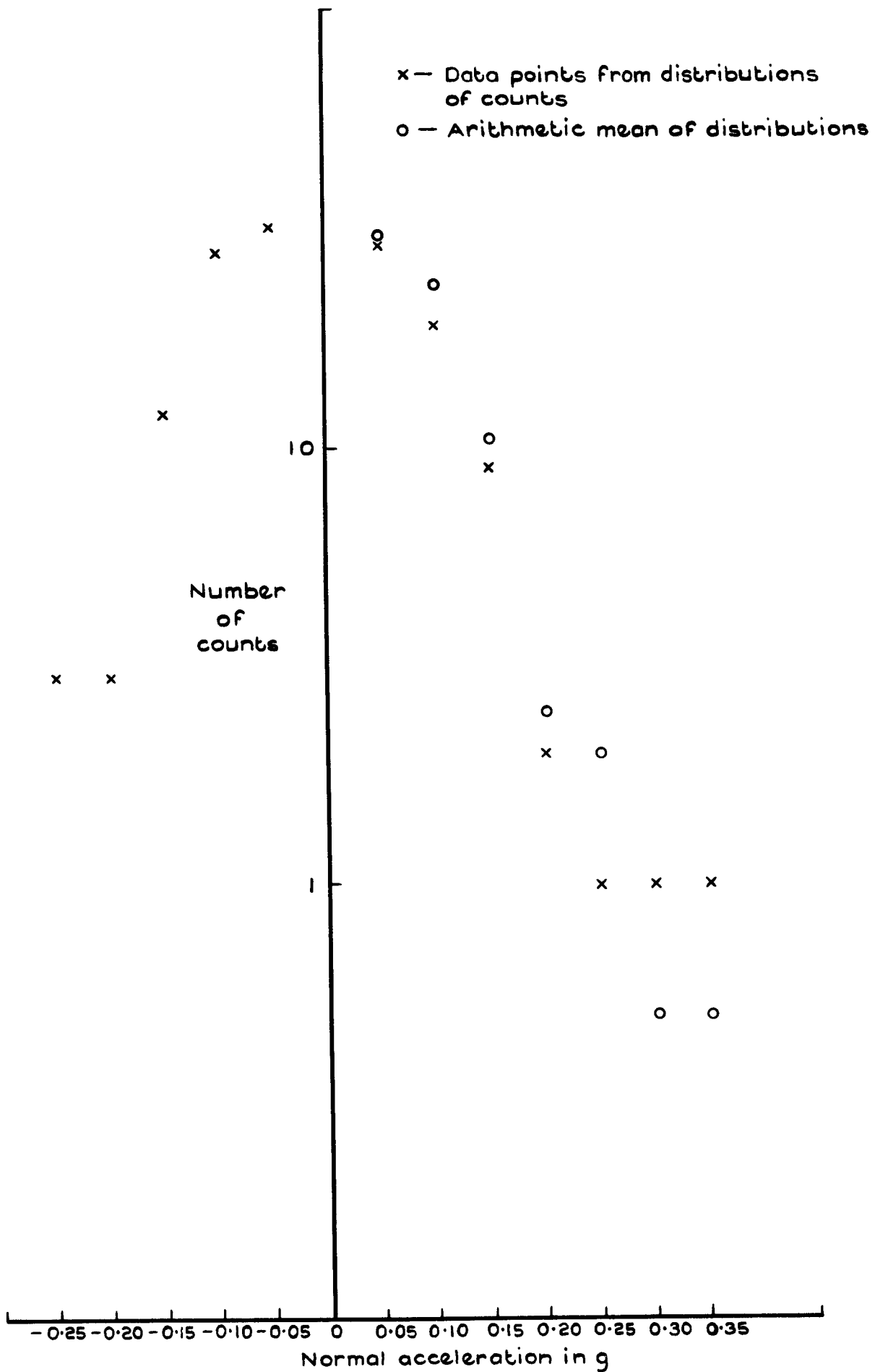


Fig. 20 Variable re-set method
 Data from flight 10113 relative to mean

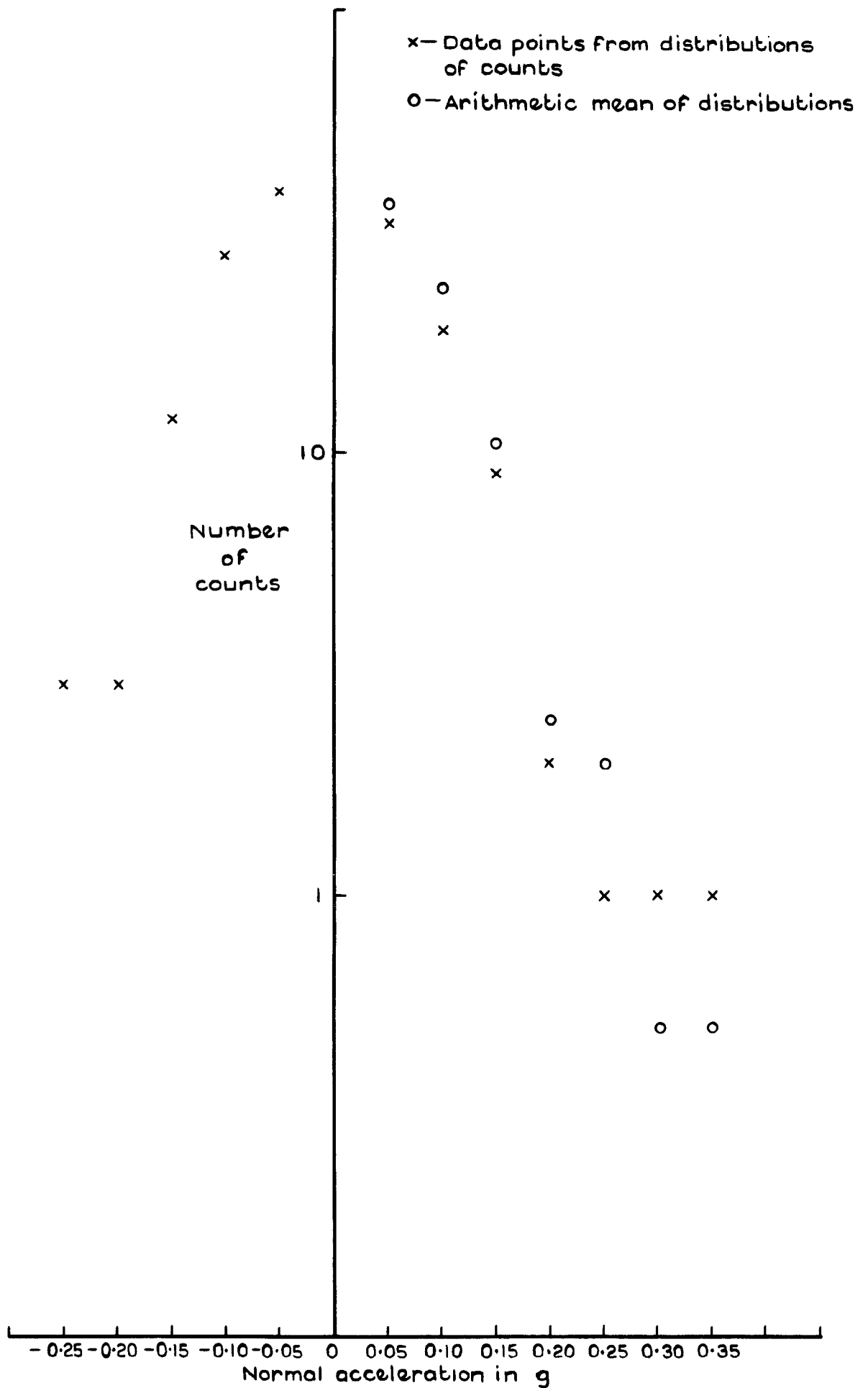


Fig.21 Level-restricted peak-count method
 Data from flight 10113 relative to mean

x — Data points from distributions
of counts
o — Arithmetic mean of distributions

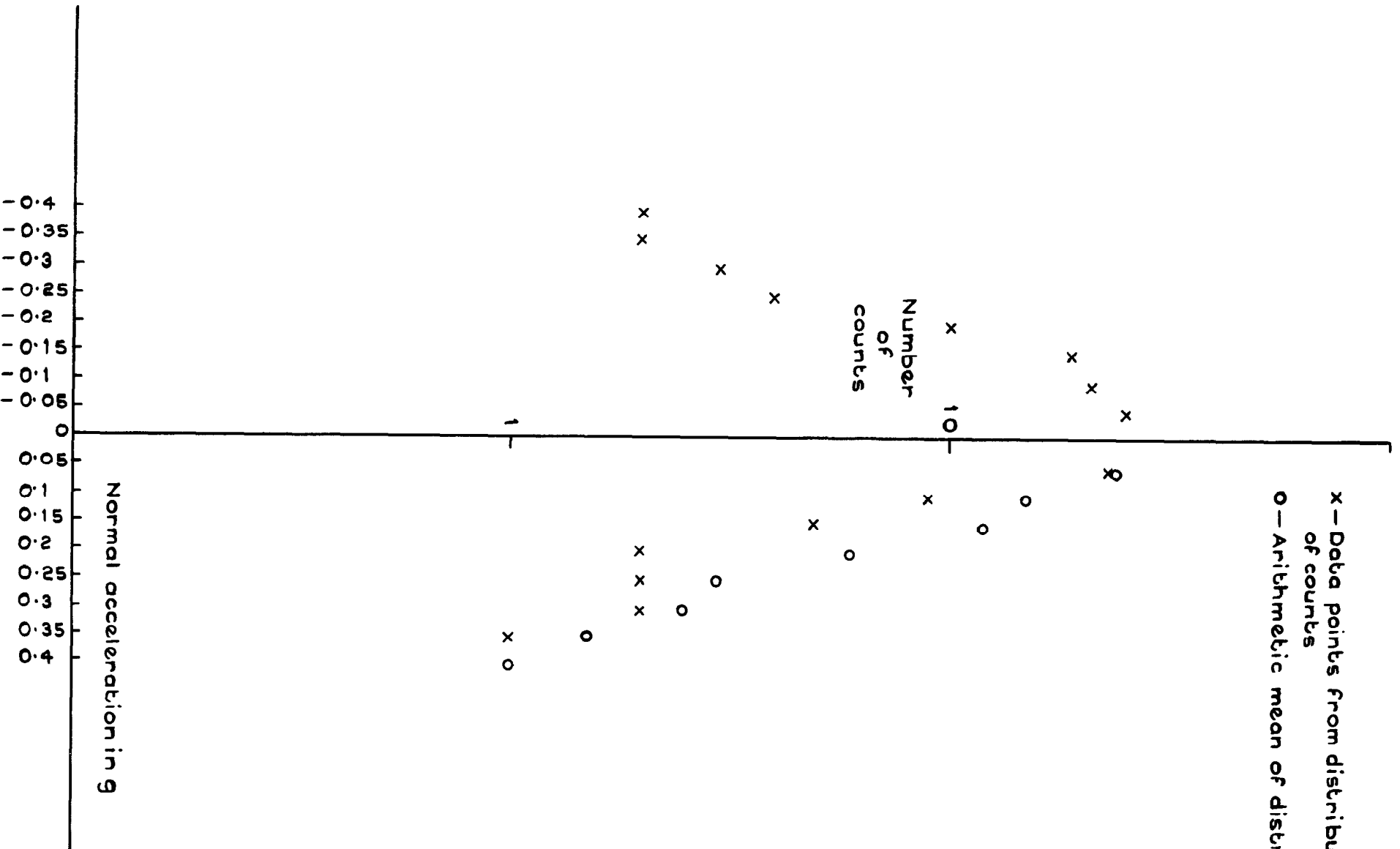


Fig. 22 Peak-count method
Data from flight IO121 relative to mean

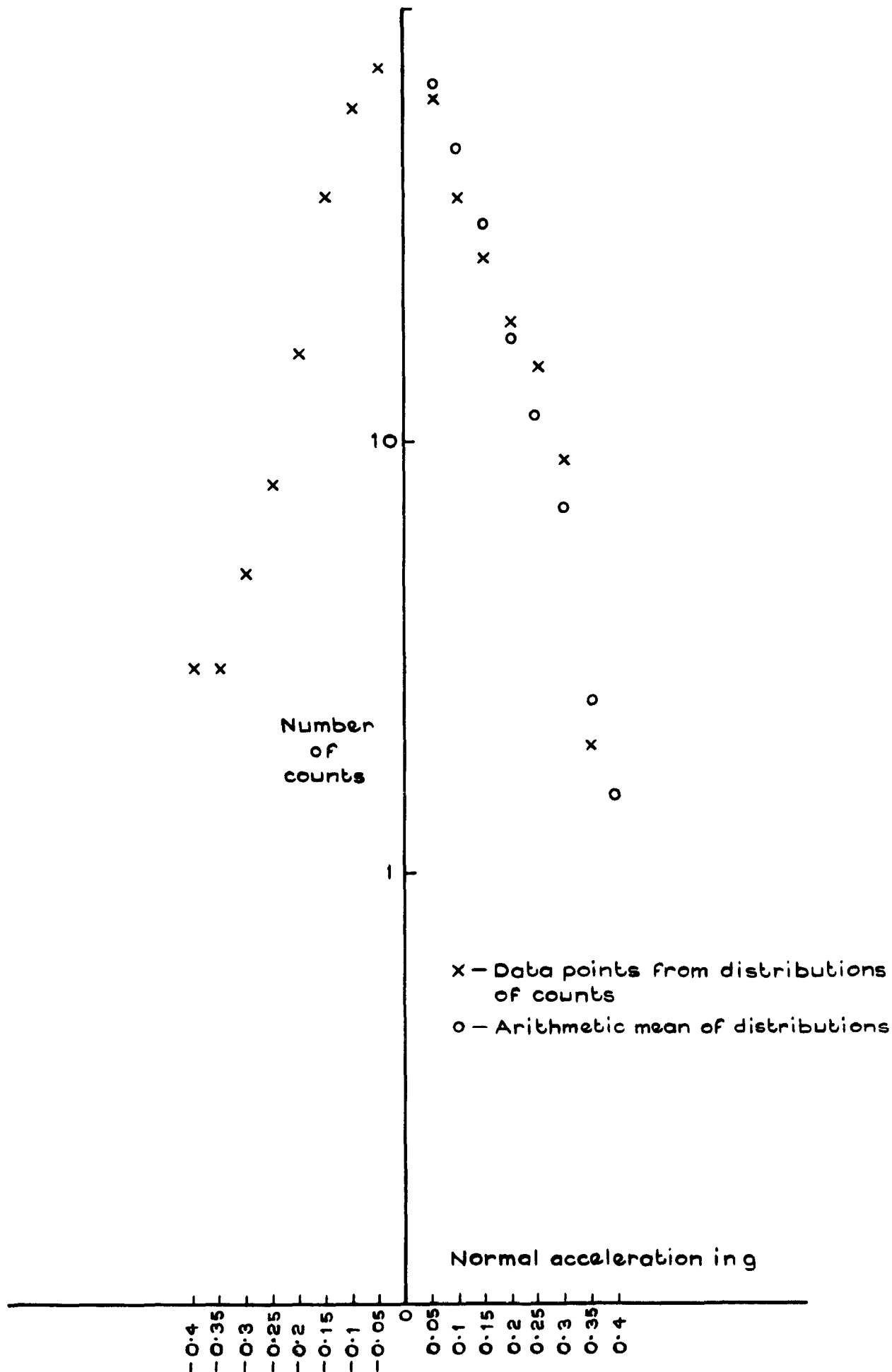


Fig. 23 Simple peak-count method
Data from flight 10121 relative to mean

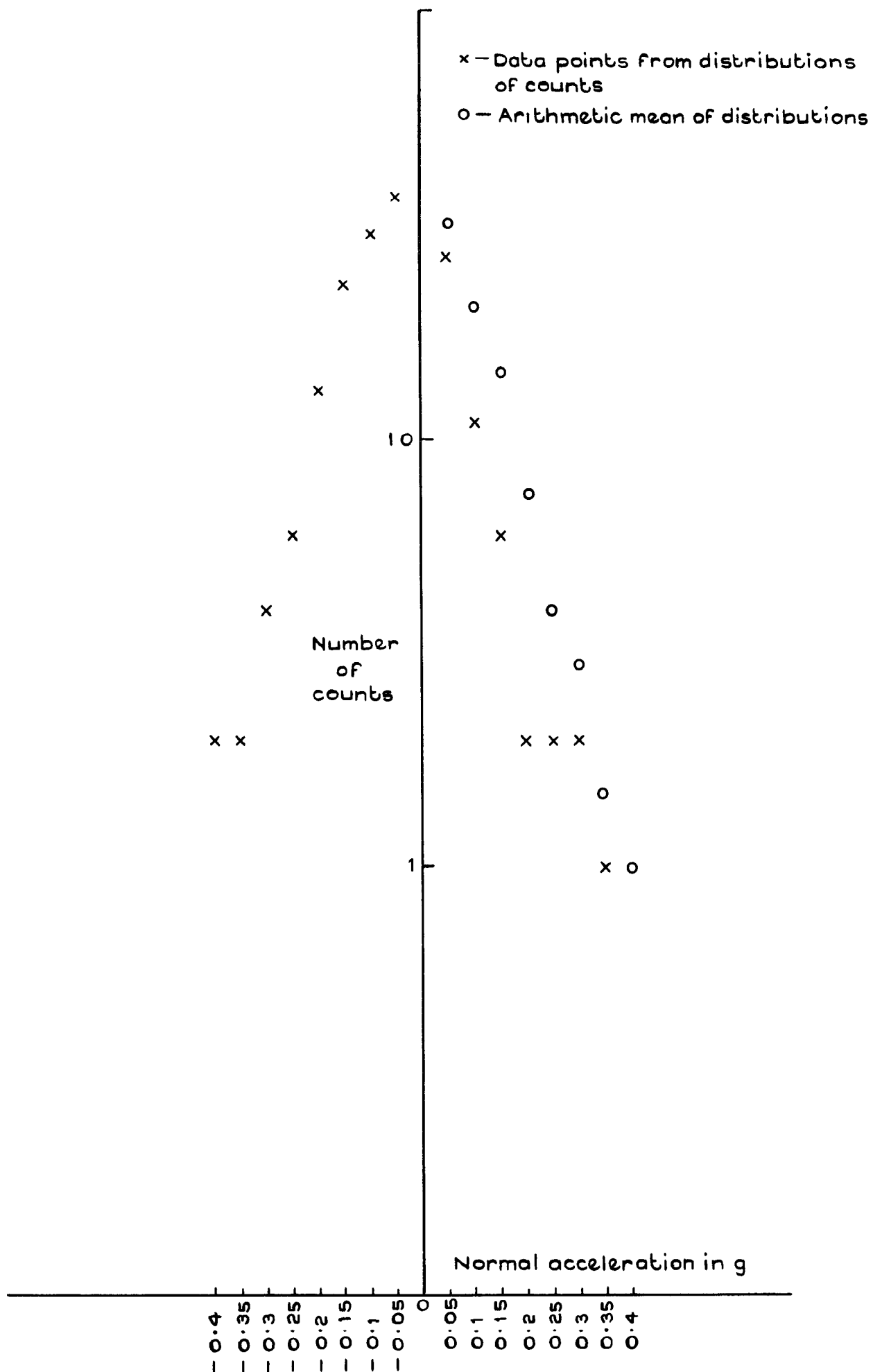


Fig.24 Range-restricted peak-count method
Data from flight 10121 relative to mean

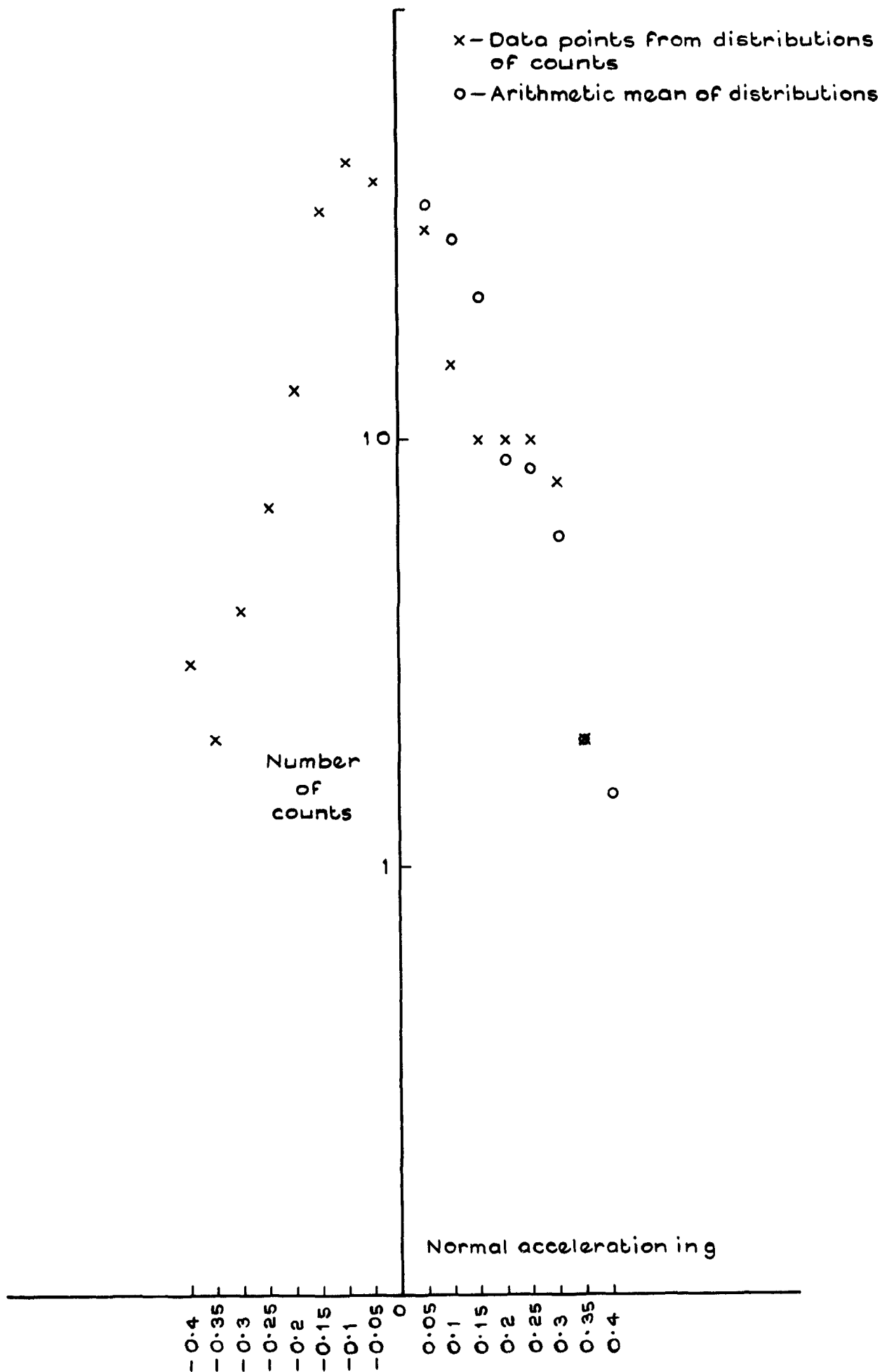


Fig.25 Simple level-crossing method
 Data from flight 10121 relative to mean

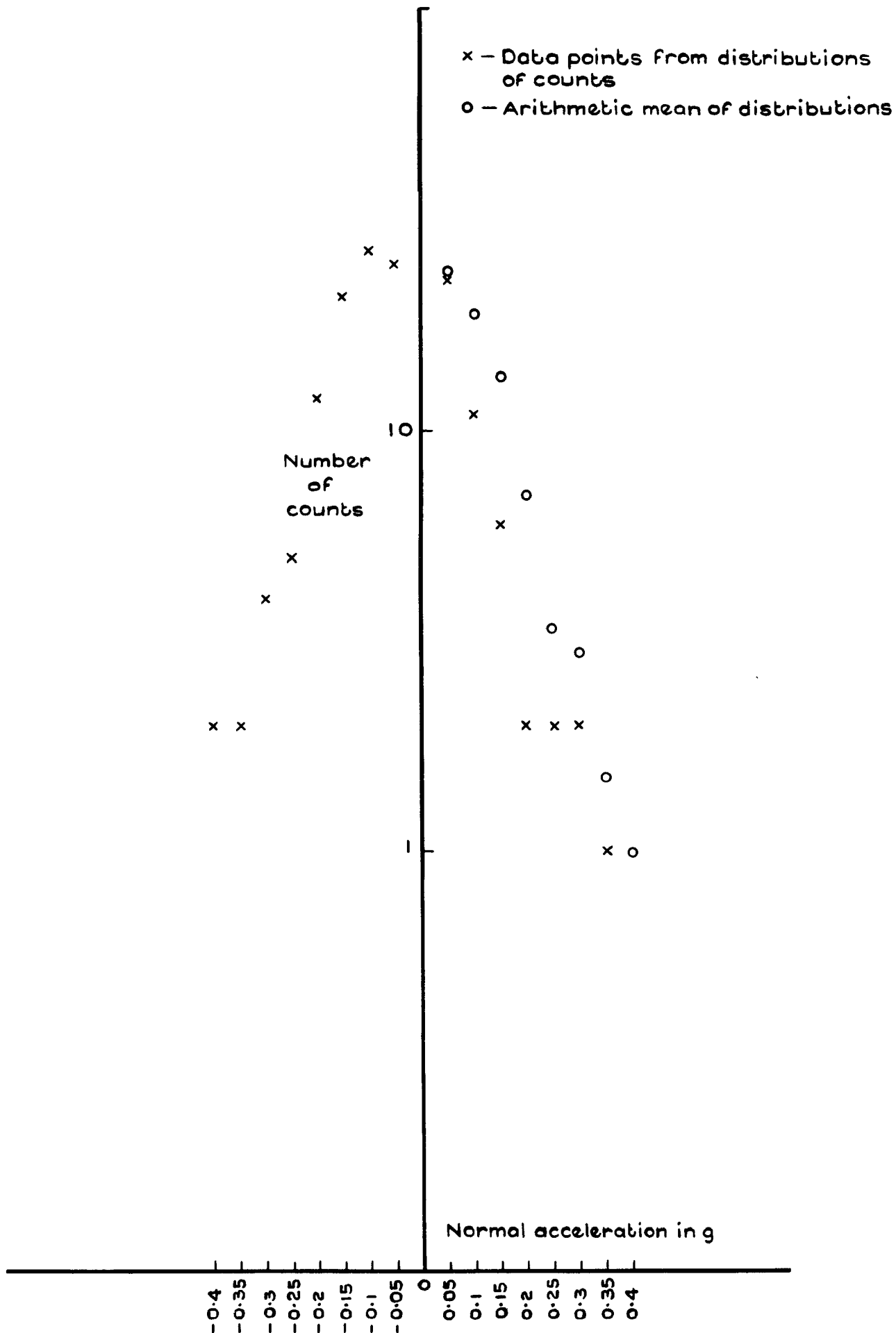


Fig.26 Variable re-set method
 Data from flight IO121 relative to mean

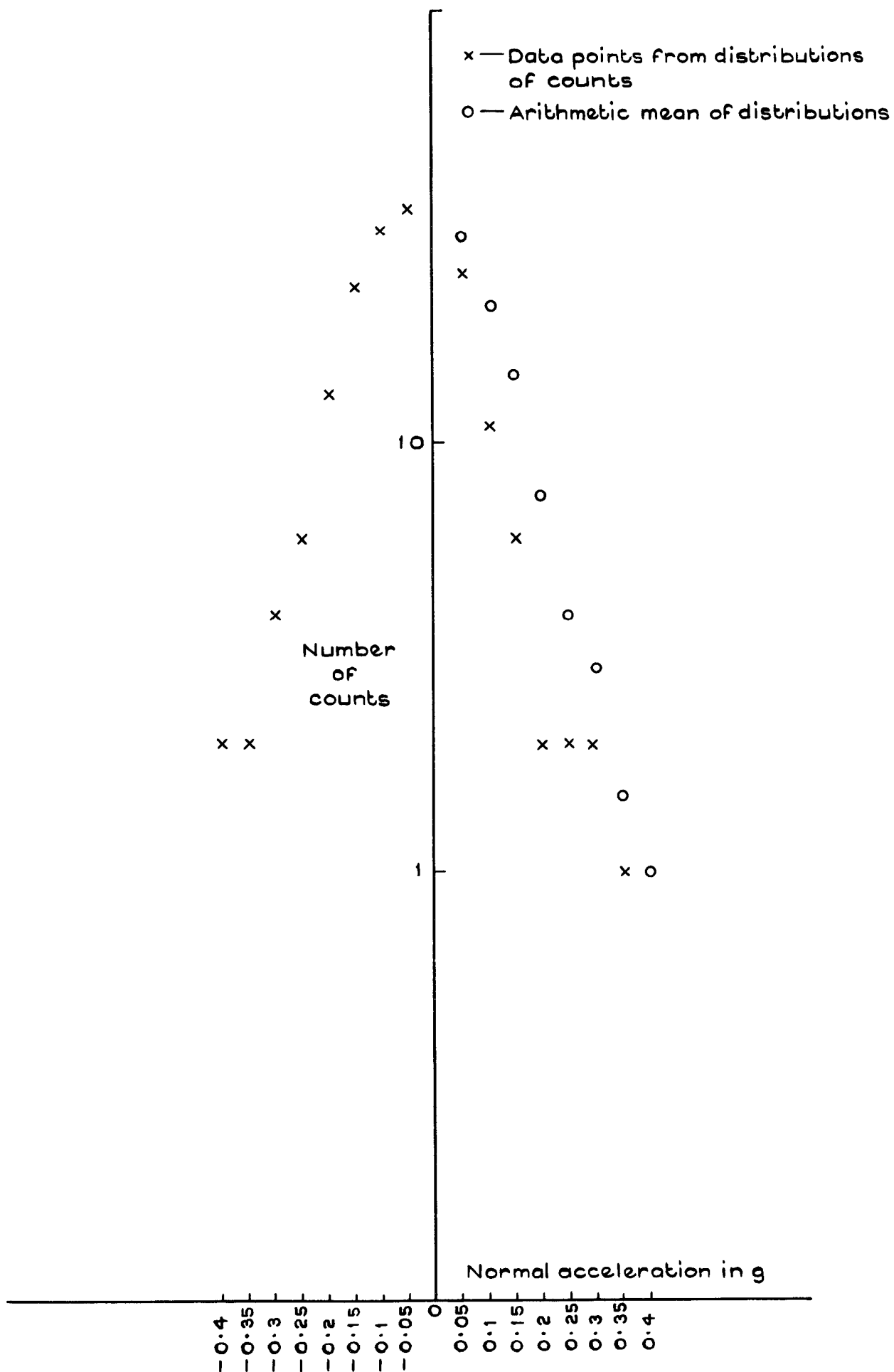


Fig. 27 Level-restricted peak-count method
Data from flight 10121 relative to mean

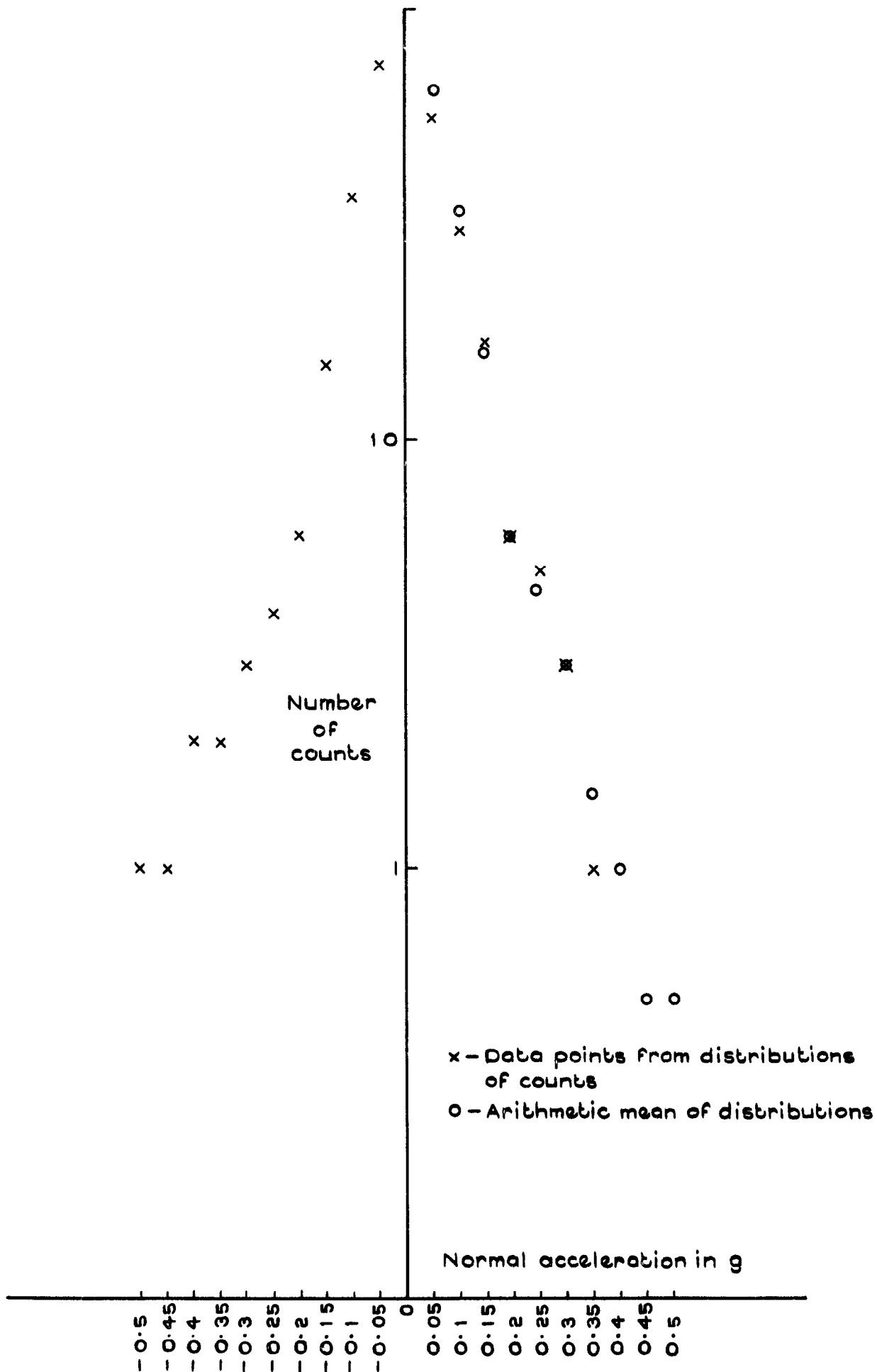


Fig. 28 Peak-count method
Data from flight 20047 relative to mean

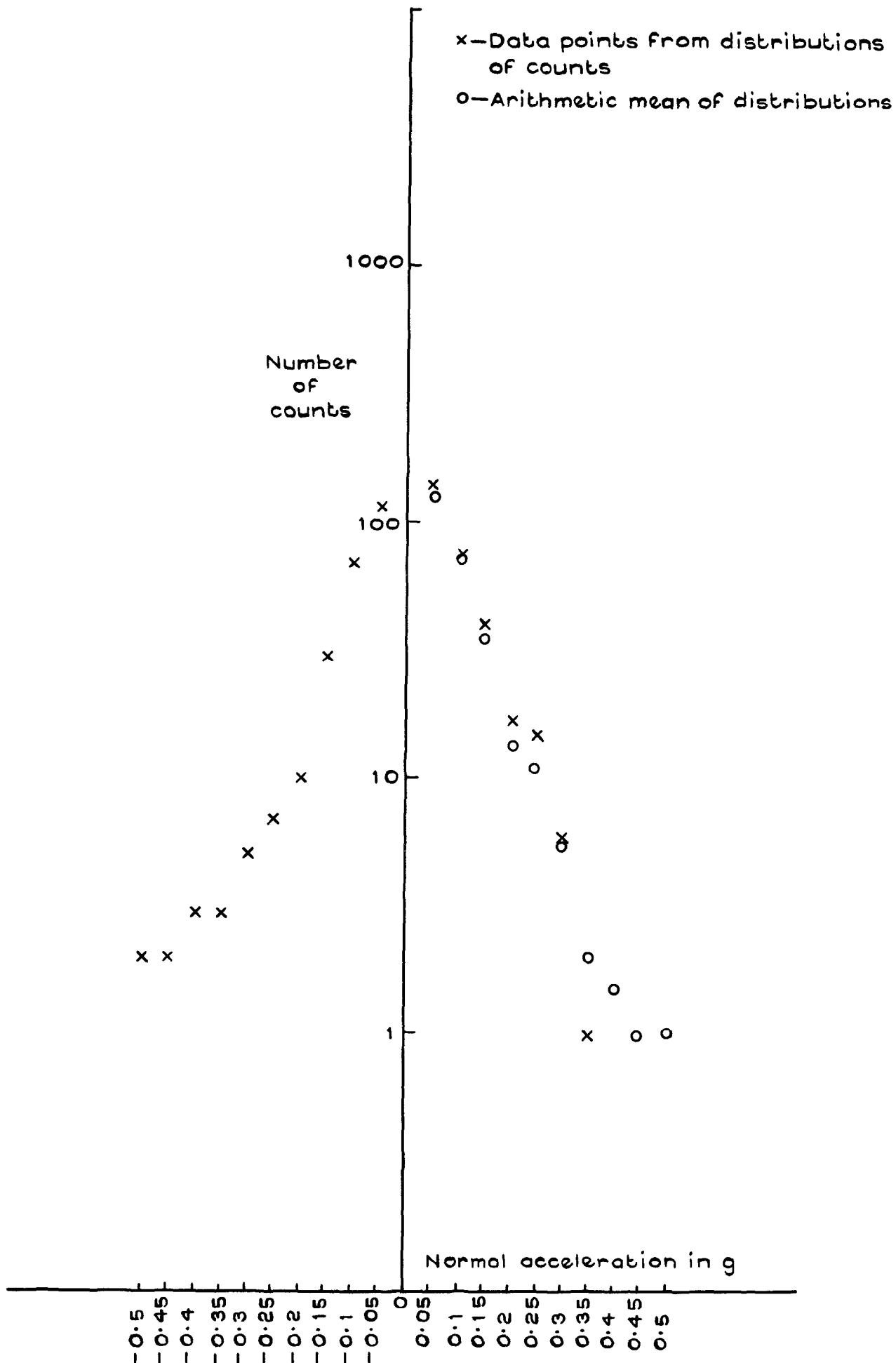


Fig.29 Simple peak-count method
 Data from flight 20047 relative to mean

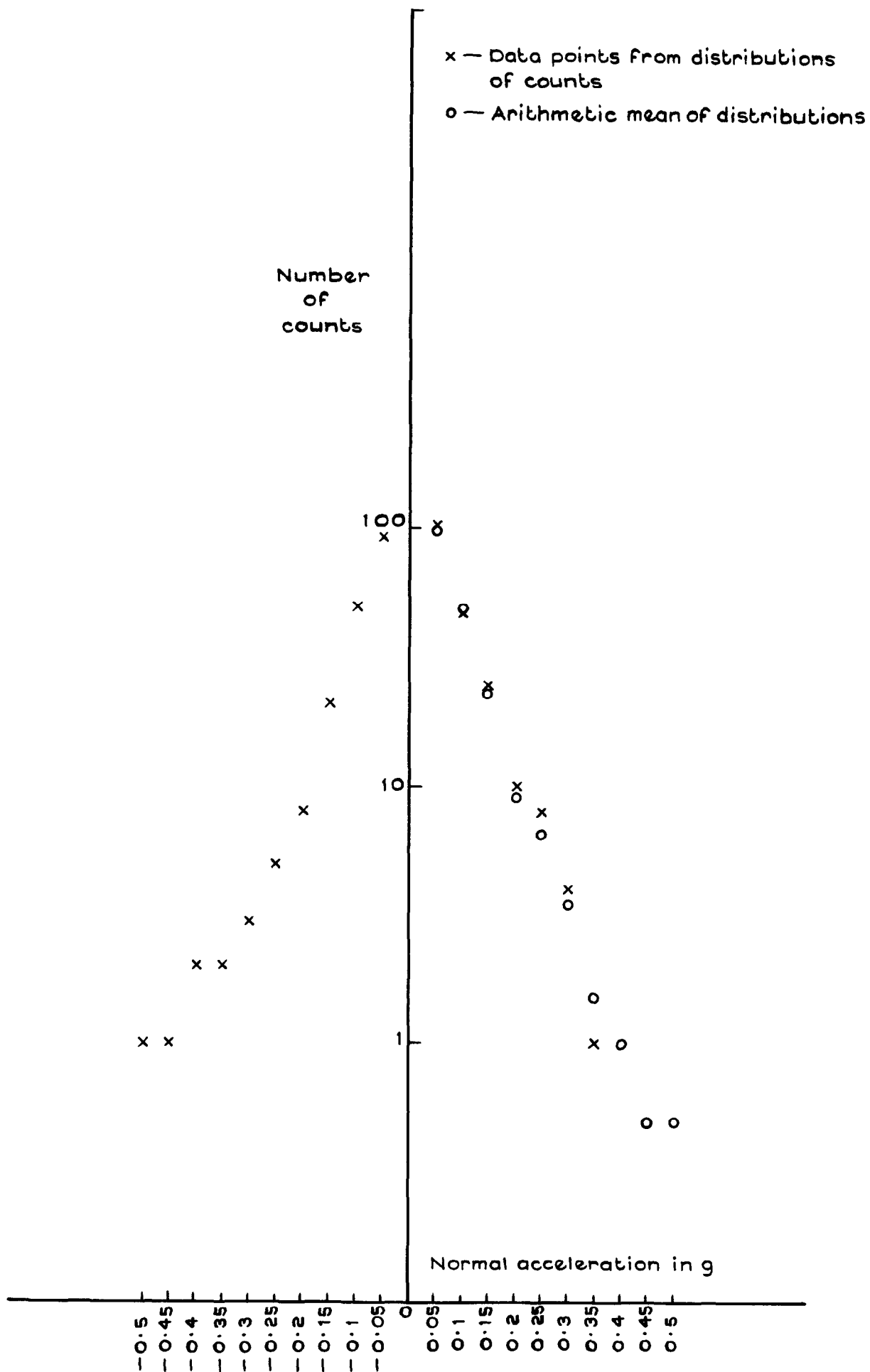


Fig.30 Range-restricted peak-count method
Data from flight 20047 relative to mean

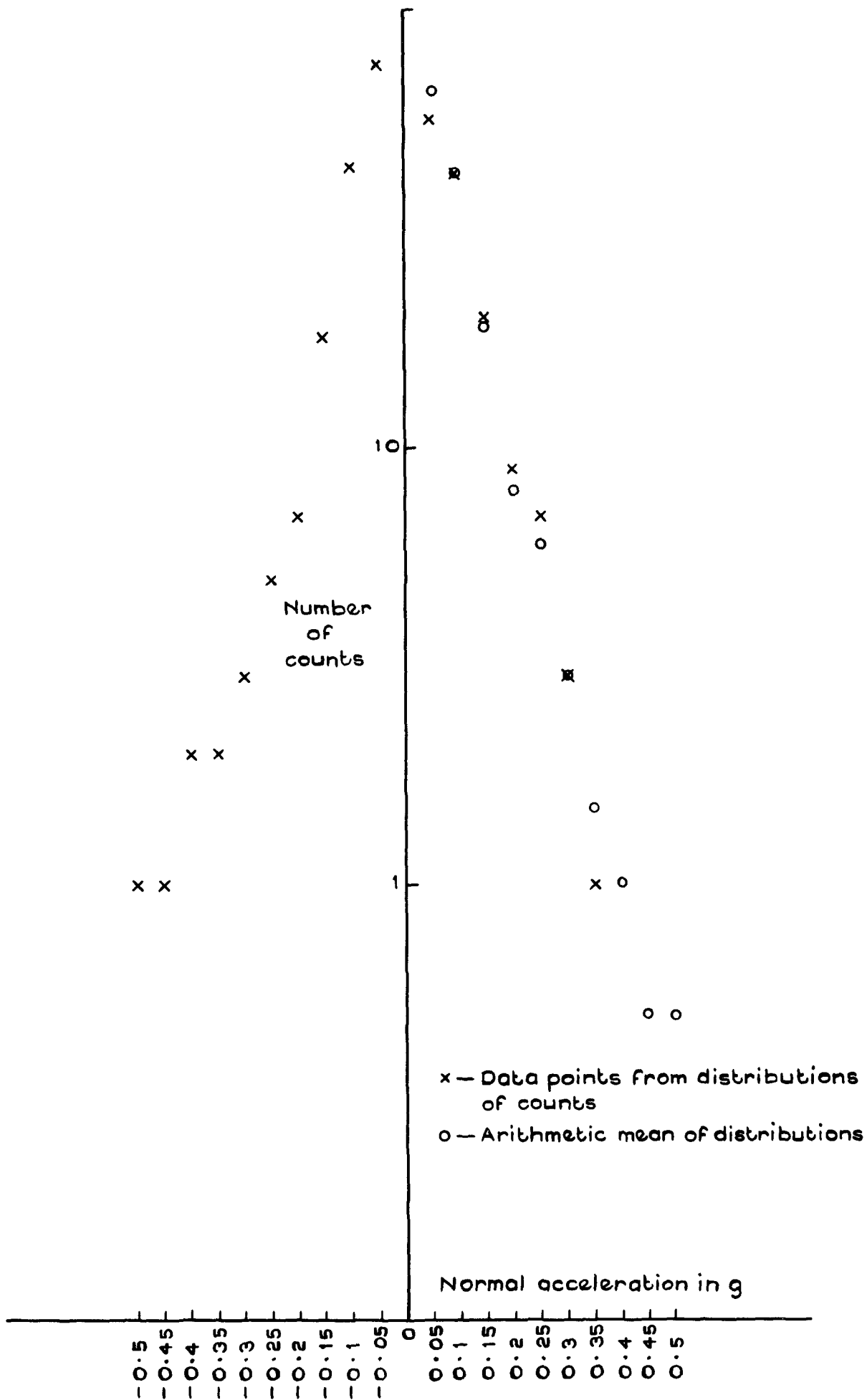
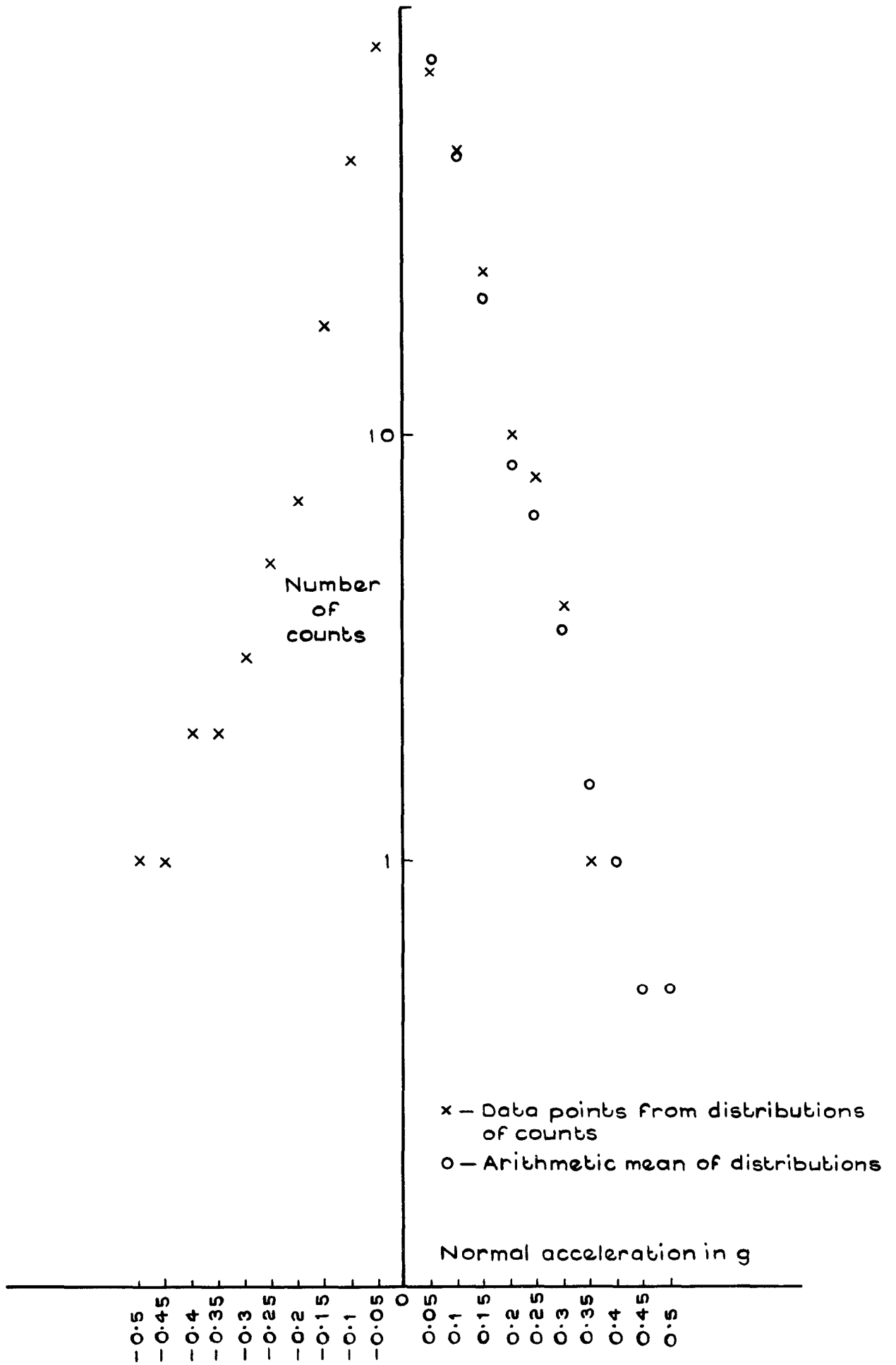


Fig.32 Variable re-set method
Data from flight 20047 relative to mean



**Fig. 33 Level-restricted peak-count method
Data from flight 20047 relative to mean**

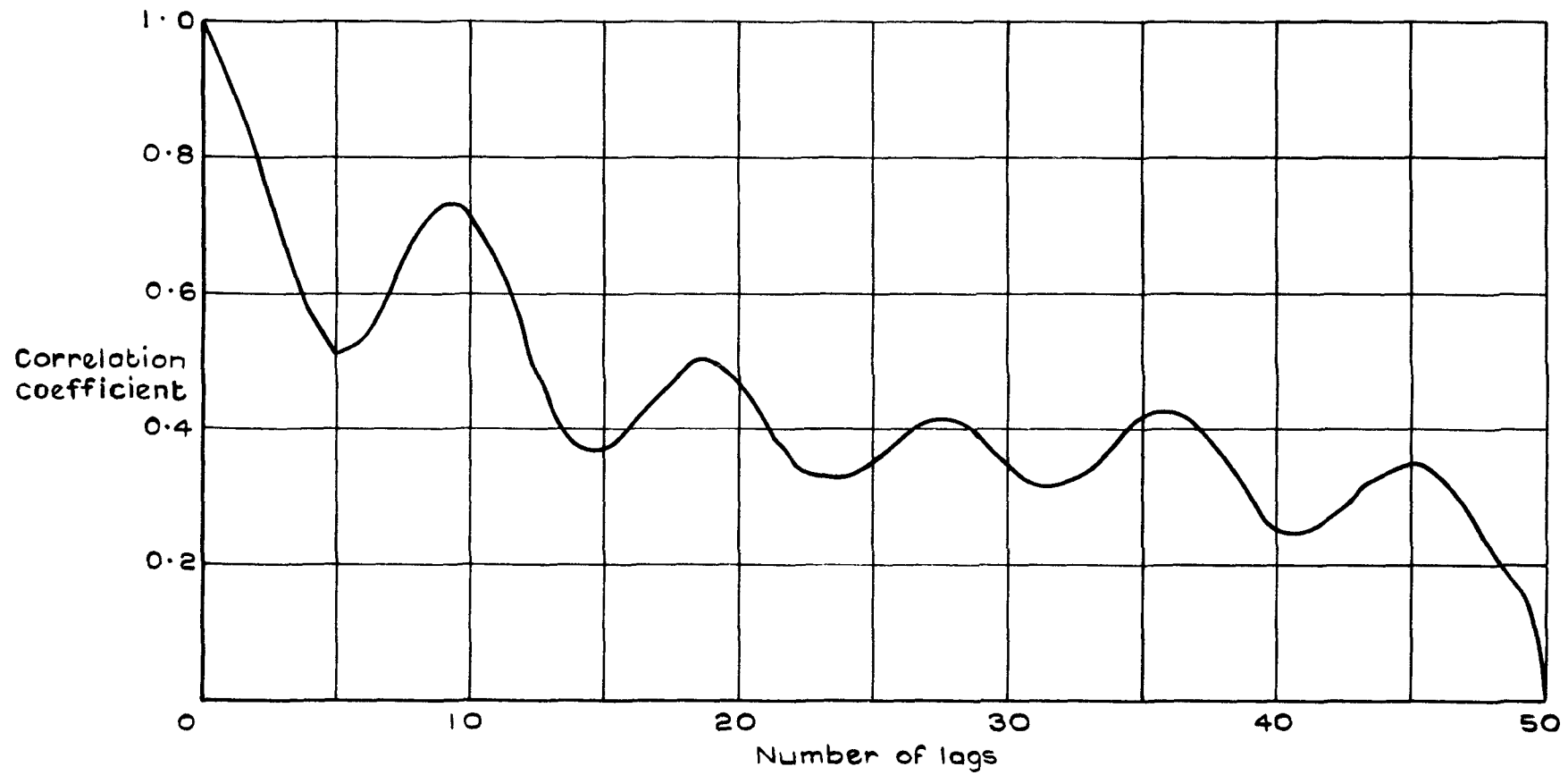


Fig.34 Correlogram of flight 10071

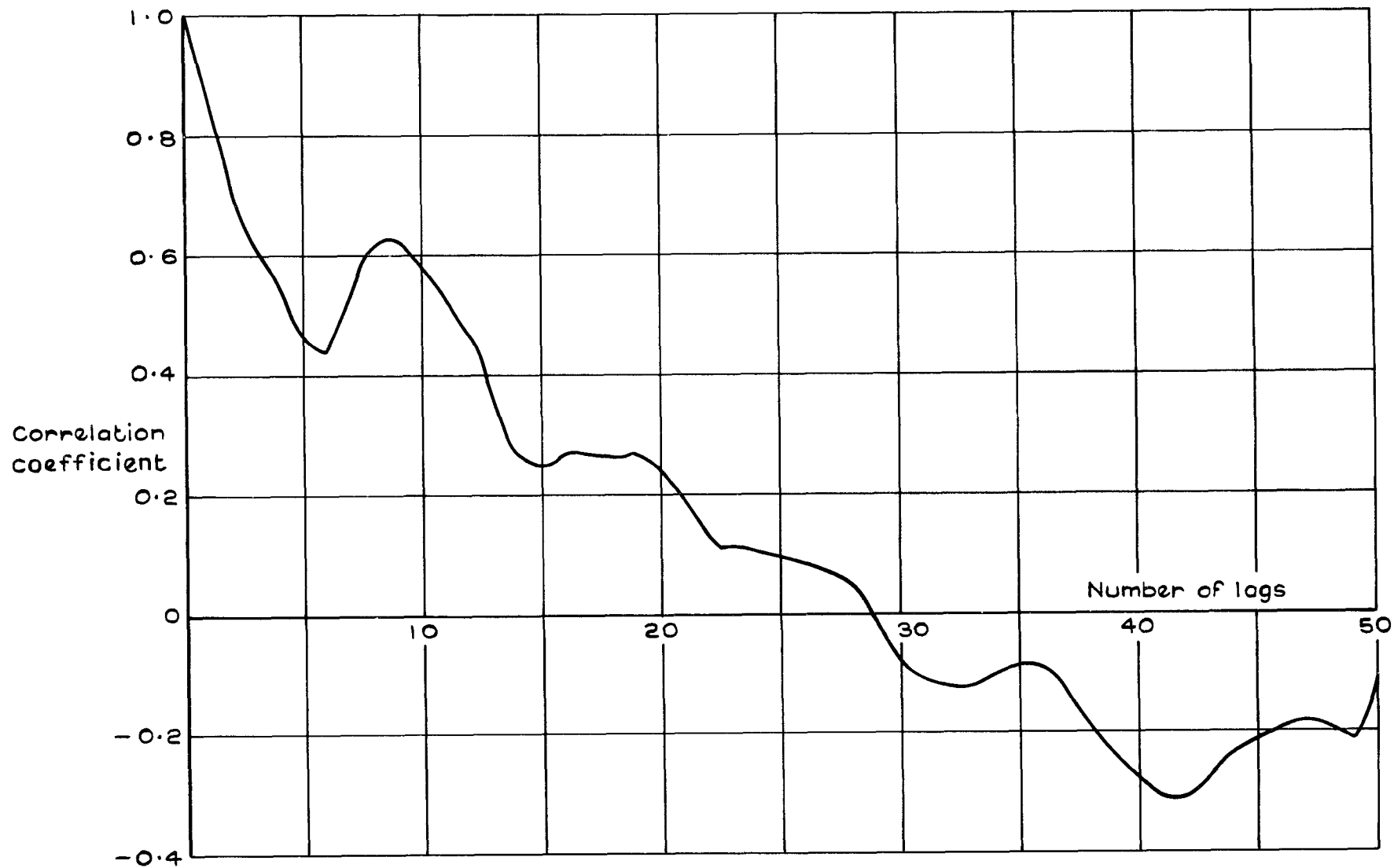


Fig. 35 Correlogram of flight 10113

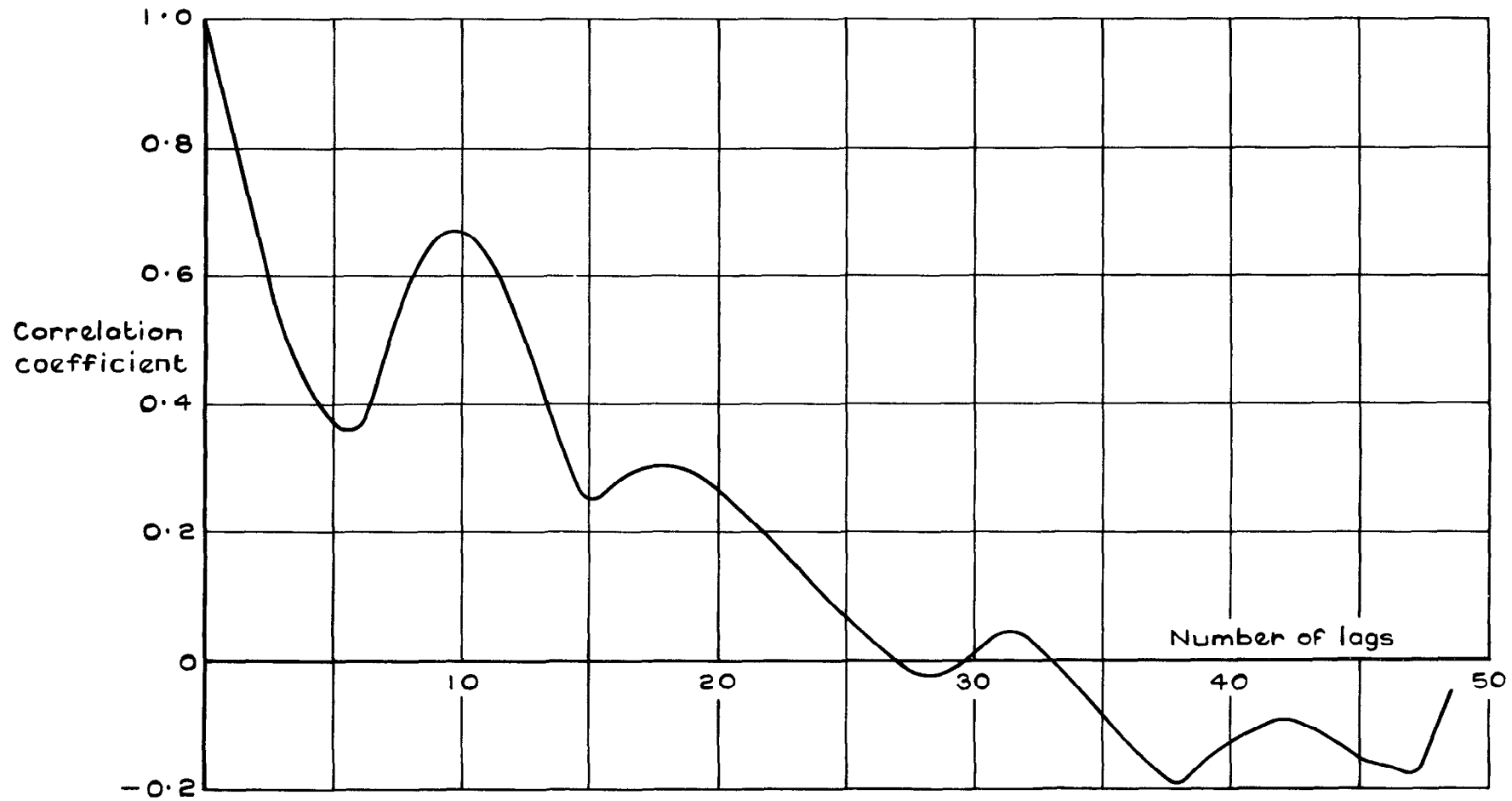


Fig.36 Correlogram of flight 10121

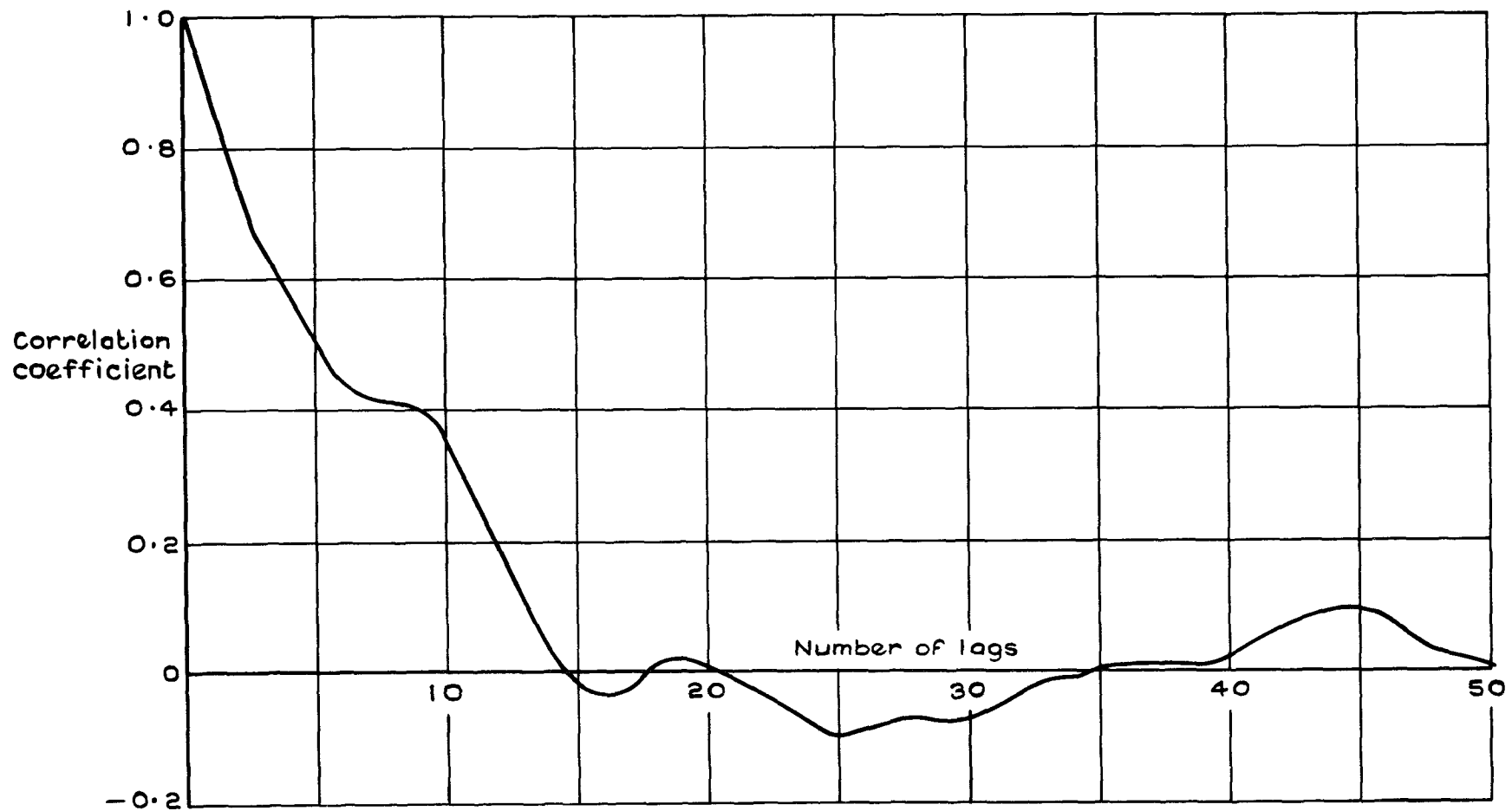


Fig.37 Correlogram of flight 20047

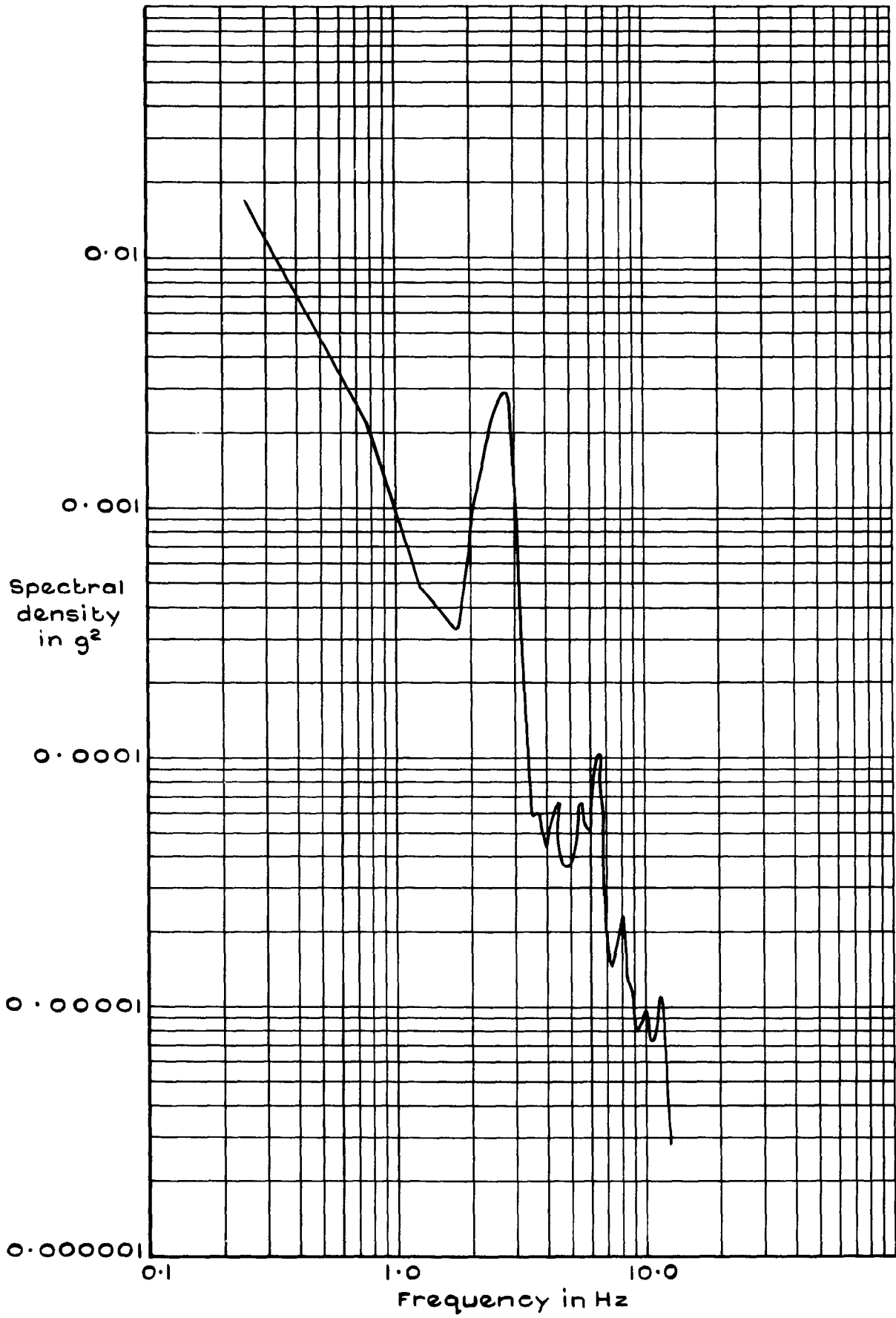


Fig.38 Power spectrum Flight 10071

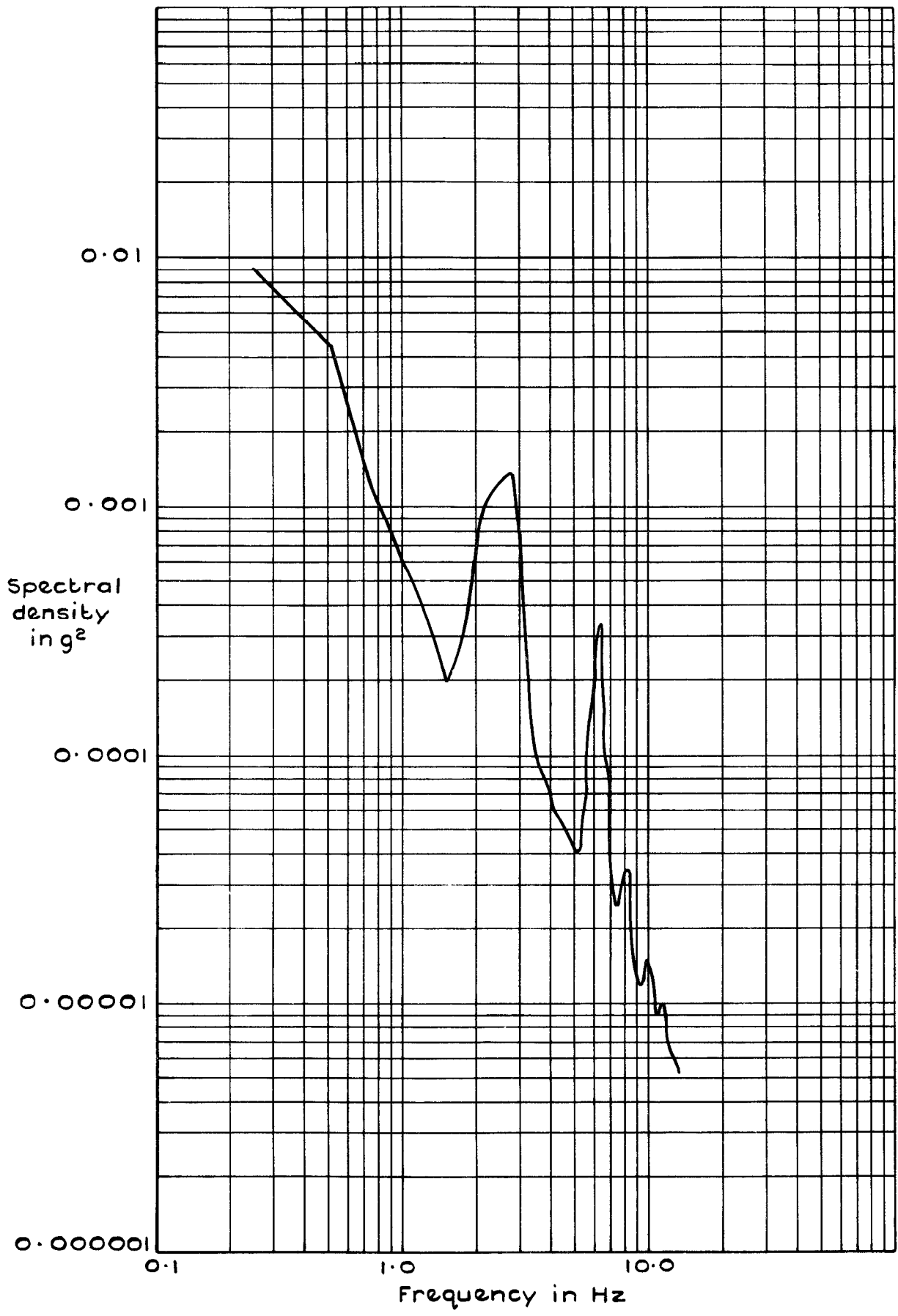


Fig.39 Power spectrum Flight 10113

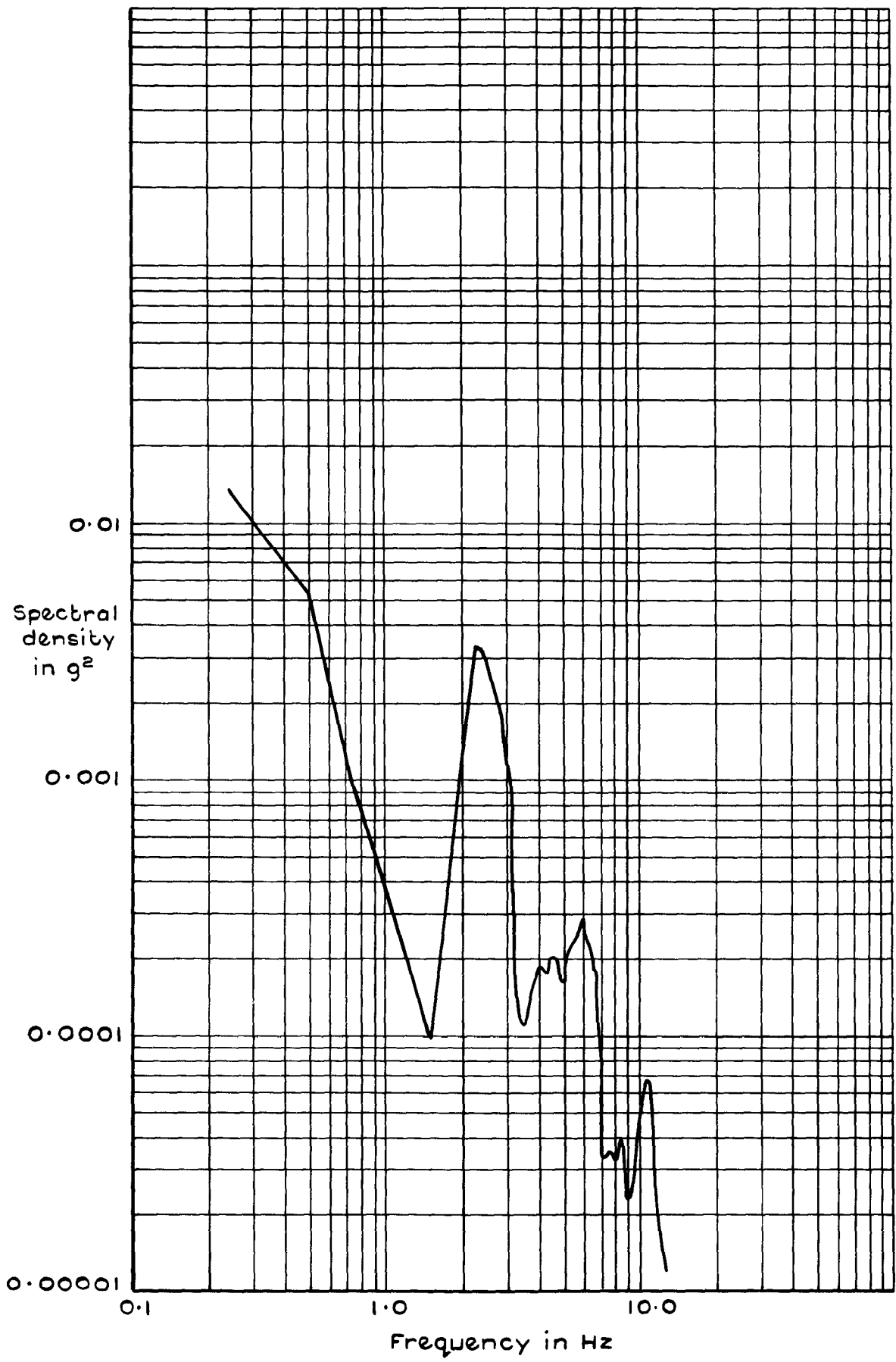


Fig.40 Power spectrum Flight 10121

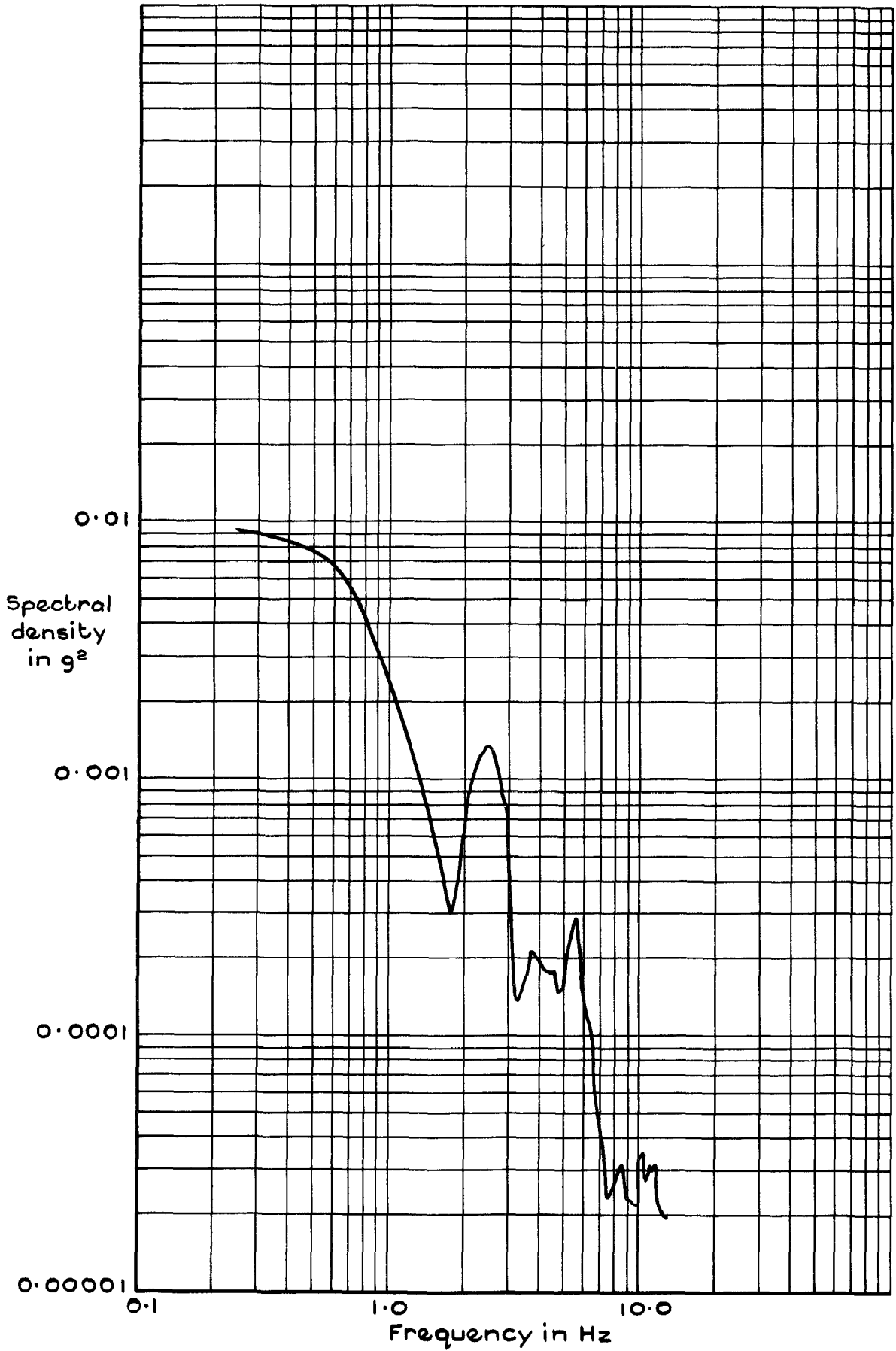


Fig. 41 Power spectrum Flight 20047

© *Crown copyright* 1974

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
49 High Holborn, London WC1V 6HB
13a Castle Street, Edinburgh EH2 3AR
41 The Hayes, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
Southey House, Wine Street, Bristol BS1 2BQ
258 Broad Street, Birmingham B1 2HE
80 Chichester Street, Belfast BT1 4JY
or through booksellers

ARC CP No.1304
December 1973

533.6.048.5 :
551.551 :
539.431

Goodwillie, Ann G.

**A COMPARISON OF FLIGHT LOADS COUNTING
METHODS AND THEIR EFFECTS ON FATIGUE LIFE
ESTIMATES USING DATA FROM CONCORDE**

Four load-time histories of turbulence encountered by Concorde are analysed by six different peak-counting and level-crossing counting procedures using the ICL 1904A computing facilities. The frequency distributions obtained are then used to find fatigue life ratios using the Miner-Palmgren hypothesis of cumulative damage.

The results show that those methods which consider every level crossed or every peak encountered give shorter life estimates than those which disregard all minor intermediate load fluctuations. Those which depend upon threshold conditions occupy an intermediate position. Correlation and spectral analyses are also conducted for each flight.

Cut here

ARC CP No.1304
December 1973

533.6.048.5 :
551.551 :
539.431

Goodwillie, Ann G.

**A COMPARISON OF FLIGHT LOADS COUNTING
METHODS AND THEIR EFFECTS ON FATIGUE LIFE
ESTIMATES USING DATA FROM CONCORDE**

Four load-time histories of turbulence encountered by Concorde are analysed by six different peak-counting and level-crossing counting procedures using the ICL 1904A computing facilities. The frequency distributions obtained are then used to find fatigue life ratios using the Miner-Palmgren hypothesis of cumulative damage.

The results show that those methods which consider every level crossed or every peak encountered give shorter life estimates than those which disregard all minor intermediate load fluctuations. Those which depend upon threshold conditions occupy an intermediate position. Correlation and spectral analyses are also conducted for each flight.

DETACHABLE ABSTRACT CARDS

ARC CP No.1304
December 1973

533.6.048.5 :
551.551 :
539.431

Goodwillie, Ann G.

**A COMPARISON OF FLIGHT LOADS COUNTING
METHODS AND THEIR EFFECTS ON FATIGUE LIFE
ESTIMATES USING DATA FROM CONCORDE**

Four load-time histories of turbulence encountered by Concorde are analysed by six different peak-counting and level-crossing counting procedures using the ICL 1904A computing facilities. The frequency distributions obtained are then used to find fatigue life ratios using the Miner-Palmgren hypothesis of cumulative damage.

The results show that those methods which consider every level crossed or every peak encountered give shorter life estimates than those which disregard all minor intermediate load fluctuations. Those which depend upon threshold conditions occupy an intermediate position. Correlation and spectral analyses are also conducted for each flight.

Cut here

DETACHABLE ABSTRACT CARDS