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A Comparison of Calculated and Measured Base Pressures of Cylindrically Based Projectiles

By

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of the Aerodynamics Division, N.P.L.

With Appendix

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A Comparison of Calculated and Measured Base Pressures of Cylindrically Based Projectiles

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W. F. Cope, M.A.,
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30th May, 1952

At the VI International Congress for Applied Mechanics a paper (Cope, 1946) was read on a method of calculating the base pressure of a cylindrically based projectile. The Proceedings of that Congress have never been published, so the paper in question has never appeared in print and is annexed to this report as an appendix to save repetition.

When the 1946 paper was written base pressure determinations whether in flight or in a supersonic wind tunnel were few and far between, and therefore it was not possible to check completely the theory put forward, and in particular the parameter (K_2) had to be evaluated from the very scanty data, nearly all at a Mach number of about 2.5, then available. This parameter is the height (in calibers) of the frustum of comparatively quiescent air following the projectile and plays an important part in fixing the value of the base pressure. At that time therefore the theory could only be regarded as tentative and it could not be said with any certainty if the mechanism assumed in the theory was correct.

With the passage of time photographs of projectiles in flight from which K_2 can be determined over a range of Mach and Reynolds numbers have become available, and as a result it seems fairly certain that K_2 is approximately equal to unity for Mach numbers greater than 2 and Reynolds numbers greater than 1.5 million. Also further measurements have been carried out both in this country and in America which between them cover Mach numbers from 1.5 to 3 and Reynolds numbers up to about 20 million. Therefore the base pressure has been calculated according to the theory of the 1946 paper for Mach numbers of 1.5, 2 and 3 and for Reynolds numbers up to 100 million, and the results compared with Bogdonoff's (1952) measurements or other appropriate data.

The results, for Mach numbers of 1.5, 2 and 3 are plotted as Figs. 1, 2 and 3, respectively. One difficulty has been that the projectiles tested have been of various lengths and it has not been possible in all cases to ascertain it. What is regarded as a typical length for the several series for the Mach number under consideration has been selected in each case and is given on the figure.

In Fig. 1 ($M = 1.5$) the measurements consist of determinations carried out through the years in the N.P.L. 11" supersonic wind tunnel on projectiles of several lengths and head shapes all in the laminar region, of American determinations in the turbulent region and of a solitary figure from firing trials. In Fig. 2 ($M = 2$) only determinations in the turbulent region are available. In both cases the agreement is as good as could be expected.

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In Fig. 3 ($M = 3$) again only measurements in the turbulent region are available and the agreement is satisfactory over the range of Reynolds number covered. But it is a great pity that Bogdonoff's (1952) measurements did not extend to lower Reynolds numbers. One is left wondering whether his curve (which is of a peculiar shape) would drop again and follow up the calculated laminar values, as apparently happens at lower Mach numbers, or whether it would diverge more and more from the calculated (laminar) value as R decreased.

The general conclusion seems to be that the formula proposed (Cope, 1946) satisfactorily predicts the base pressure for projectiles of ordinary proportions, at any rate in the turbulent region which is the region of practical importance.

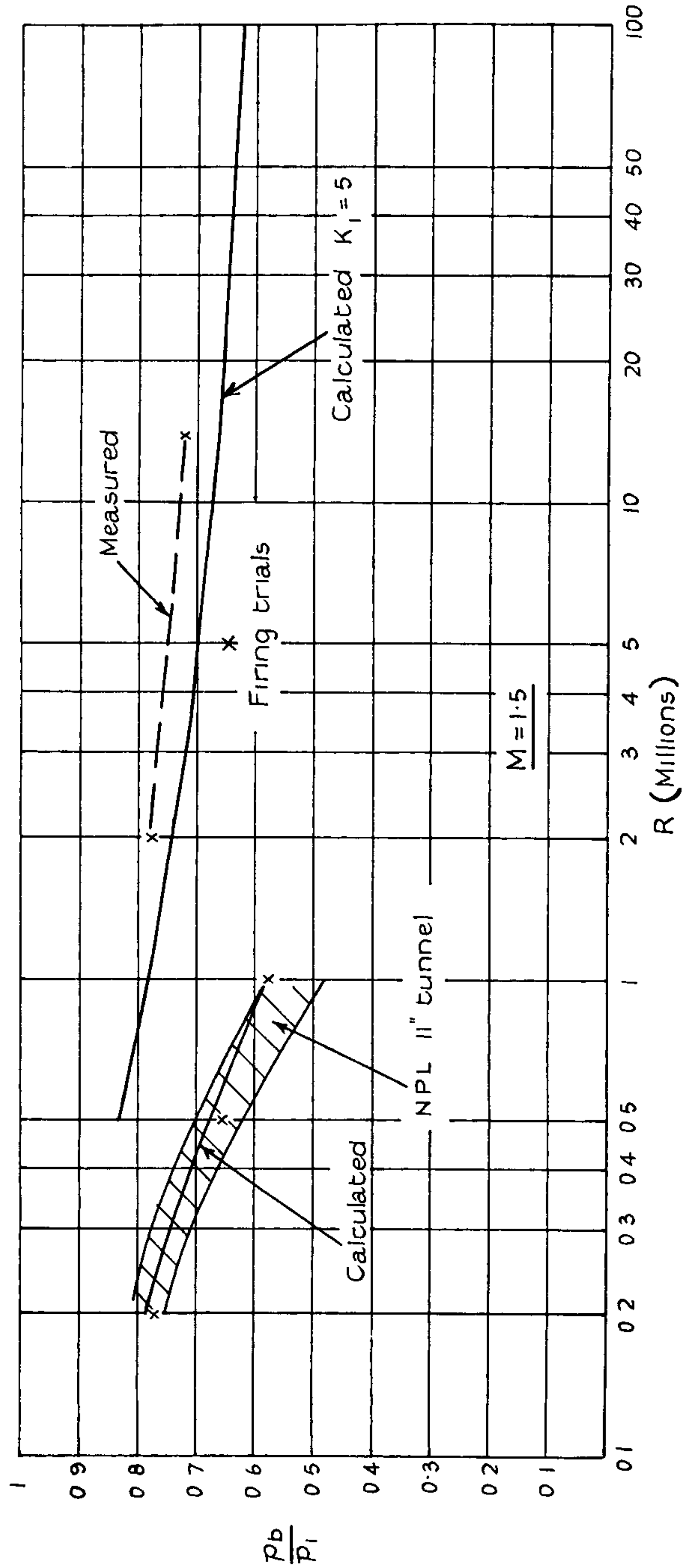
It should be noted that the formula cannot possibly be right for all lengths since it is linear in the length in calibers (K_1) and therefore the base pressure increases without limit as K_1 increases. This is obviously impossible and the most likely explanation is that the base pressure is a function of K_1 which is initially approximately linear in the latter and thereafter increases more slowly to some asymptotic finite value for large K_1. No information is known for determining this function empirically and it is not clear how it can be made to arise naturally in the analysis. In any event the whole theory is based on what is almost certainly a drastic simplification of a very complicated process and cannot be expected to do everything.

Bibliography

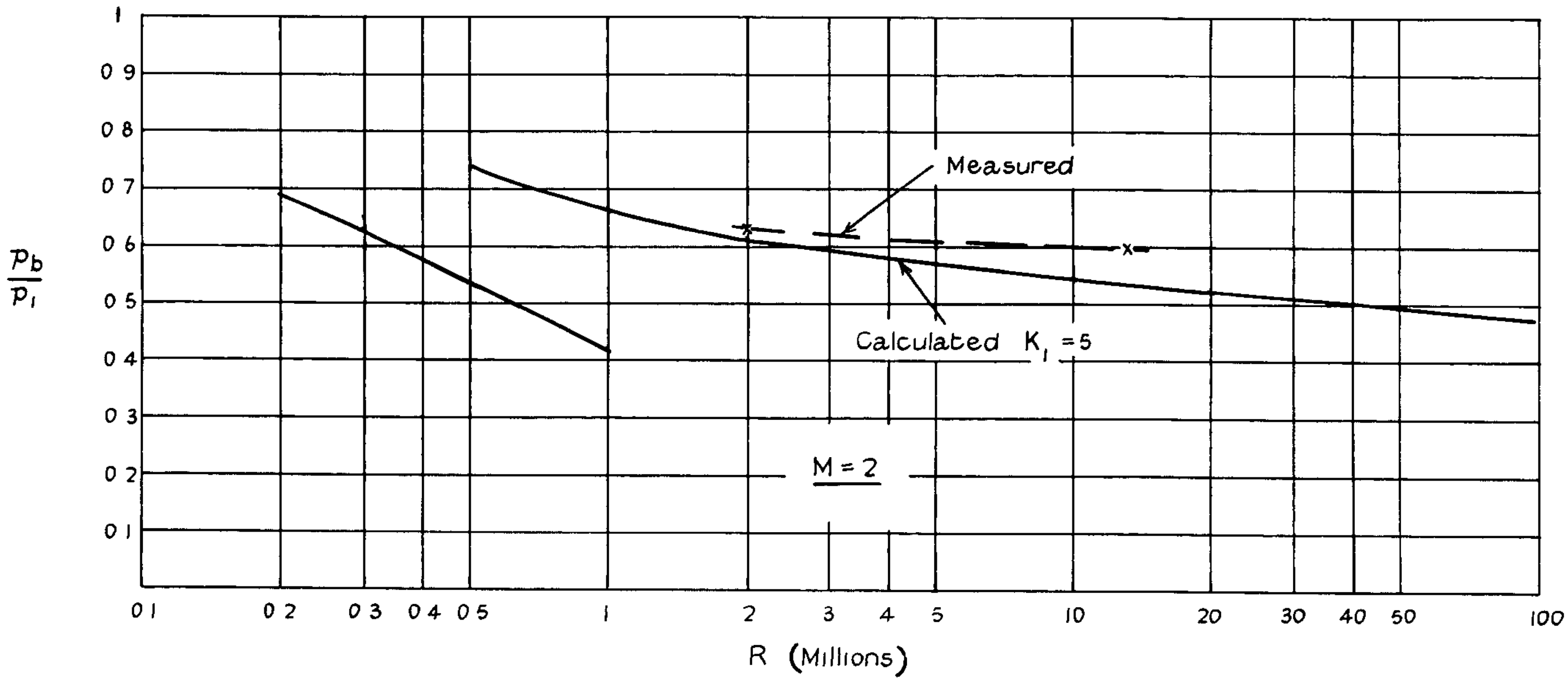
Calculation of Reynolds number effect on projectiles at supersonic speeds.- W. F. Cope, Eng. Div., 222/46. Read at VI Congress for Applied Mechanics, Paris, 1946. (Appendix to this paper).

A preliminary study of Reynolds number effects on base pressure at $M = 2.95$.- Seymour M. Bogdonoff, Journ. Ae. Sc., 1952, 19, 201-207.

<u>FIG 1</u>

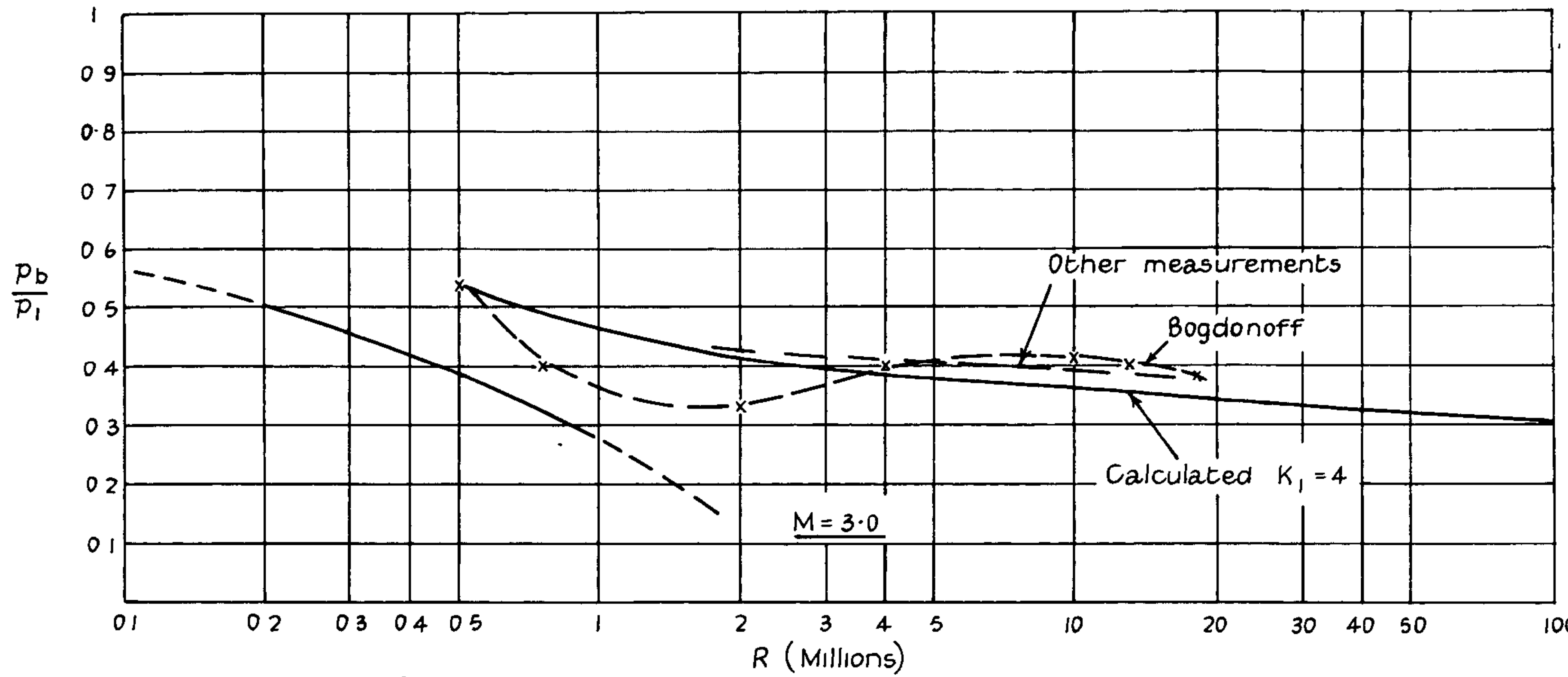


Measured
Calculated $K_1 = 5$
Firing trials
$M = 1{\cdot}5$
NPL 11" tunnel
Calculated
$\frac{P_b}{P_1}$
R (Millions)
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1
2
3
4
5
10
20
30
40
50
100



1
0 9
0 8
0 7
0 6
0 5
0 4
0 3
0 2
0 1
$\frac{p_b}{p_1}$
Measured
Calculated $K_1 = 5$
M = 2
0 1
0 2
0 3
0 4
0 5
1
2
3
4
5
10
20
30
40
50
100
R (Millions)

FIG 2



$\frac{p_b}{p_1}$
Other measurements
Bogdonoff
Calculated $K_1 = 4$
M = 3·0
R (Millions)

FIG 3

APPENDIX

Eng.Div.222/46.

NATIONAL PHYSICAL LABORATORY

Engineering Division

Calculation of Reynolds Number Effect on Projectiles at Supersonic Speeds

- By -
W. F. Cope, M.A.

July, 1946

Summary

The variation with Reynolds number of the base pressure of a cylindrically based projectile is calculated for a simplified field of flow. The calculations are compared with measured values obtained in the N.P.L. supersonic tunnel and with a value inferred from firing trials. The agreement is encouraging, but since the comparison has only been made at one Mach number, no positive conclusion about the accuracy of the simplified model can be drawn.

Introduction

In a companion paper to this Congress entitled "Experiments on Reynolds Number Effect on Projectiles at Supersonic Speeds" the author's colleague Dr. G. A. Hankins has shown that drag measurements in supersonic wind tunnel work are subject to a considerable scale effect arisirg from change of base pressure (p_b) with Reynolds number (R). The object of this paper is to describe a mechanism which accounts for this change and it is emphasised that it depends on a drastic simplification of the field of flow. Therefore, although the agreement between calculation and measurement is encouraging, it is only offered as a first and possibly crude, approximation to the solution of a complicated problem in the hope that it will stimulate discussion and lead ultimately to a complete explanation.

Analysis

Figure 1 can be regarded as typical of the field of flow near any cylindrically based projectile in flight when the Mach number is considerably greater than unity. The details of the flow pattern are very complicated but in broad outline can be simplified to that given in Figure 2. The projectile is of diameter d and length l (= K_1d); the boundary layer is δ thick at the base.

Immediately behind the base the wake "necks down" to form a frustum of a cone ABB'A', the height h(= K_2d) of the cone is of the same order as d and the angle (ϕ) which the side makes with the axis of the projectile is comparatively small. From near BB' a shock wave emerges whose angle is comparable with the nose wave. Behind BB' the wake proper becomes approximately of constant width and changes markedly in appearance. Along AB or A'B' the edge is clearly defined and nearly straight: behind BB' the edge is much more irregular and appears to be bounded by vortices regularly arranged. In short, it looks very like a subsonic wake. Immediately behind the base there is a region of still air; it is not always visible in photographs, but its presence has been verified by wind tunnel measurements.

As the Mach number approaches unity from above this simplification becomes less accurate. The principla changes are that the edges AB, A'B' become less clearly defined and that the shock wave starts further from the neck and is more obviously generated by the concurrence of wavelets emerging from AB, A'B'.

The general idea underlying the present mechanism is that AB and A'B' are the trace of the dividing surface between the boundary layer and the main stream. This surface may be regarded as a vortex sheet across which there is a velocity but not a pressure difference; but all that is essential to the arguement is that the two flows do not mix (if at all) until after they have passed BB'. The boundary layer expands from the annulus AA' to fill the circle BB'; the main stream turns through an angle ϕ . At BB' a compression occurs which results in the shock waves from the neck and the wake further downstream. Broadly speaking this picture amounts to saying that the projectile "streamlines" itself by adding the frustum ABB'A' of still air to its (cylindrical) base and that the pressure on BB' is p_b . The angle (ϕ) of the streamlining

/is

is fixed by the fact that changing it has opposite effects on p_b as calculated from the flow in the main stream and from the boundary layer respectively. Increasing ϕ means that the main stream expands more so decreasing p_b (for a given $p_A \approx p_1$) but the boundary layer expands less so increasing p_b, so the proper value of ϕ and hence of p_b is that which yields the same value of p_b for both methods of calculation. Obviously p_b will be a function of δ and therefore of R and since δ is usually set proportional to some inverse power of R, p_b will decrease as R increases so long as the state of the boundary layer is unchanged. It should be noticed that the mechanism yields a change of p_b although the state and separation point of the boundary layer are unchanged and therefore differs essentially from the change of form drag (typified by the sphere) in which a change of state of the layer alters the position of the point of separation.

In translating the above ideas into symbols for comparison with measurements two alternative assumptions on the flow in the boundary layer are possible. That the expansion from AA' to BB' is at constant total energy or that it is at constant entropy. Actual calculation shows the numerical consequence is a difference of at most 10%, but the former assumption has been selected as it seems probable that it corresponds better with the facts.

The equations of continuity and of energy, and the geometry of Figure 2 yield

$$\frac{p_b}{p_1} = \frac{\rho_B T_B}{\rho_A T_A} \cdot \frac{p_A}{p_1}$$

$$= \left\{ \frac{p_A}{p_1} \; \frac{1 + \frac{\gamma-1}{2} M^2_A}{1 + \frac{\gamma-1}{2} M_B^2} \; \frac{U_A}{U_B} \right\} \left(1 - \frac{\delta_c}{\delta} \right) \frac{\delta}{l} \, \frac{4K_1}{(1 - 2K_2 \tan \phi)^2}$$

The value of the expression in the curly brackets is fixed by the value of K_1 but for lengths ($K_1 = 4$) used in the calculations in this paper does not differ from unity by more than a few per cent and can be regarded as a correcting factor.

$1 - \frac{\delta_c}{\delta}$ and $\frac{\delta}{l}$ are known functions of M once the velocity distirbution in the boundary layer is known. Unfortunately very little has been published about supersonic boundary layers so that the calculations have had to be based on incompressible flow distributions. A sinusoidal distribution has been assumed for a laminar layer and a "1/7th power law" for a turbulent layer.

K_2 has been obtained by measurement of photographs of projectiles in flight and in wind tunnels. The available information is not as abundant as could be desired, but enough exists to justify that, for $2 < M < 3$, $K_2 = 1$ $R \gg 1\frac{1}{2}$ million increasing to $K_2 = 1\frac{1}{2}$ $R = \frac{1}{2}$ million.

/The

The analysis thus yields an expression for p_b/p_1 in terms of ϕ and of quantities which can be regarded as known in a form which is very suitable for computation. Similarly consideration of the flow in the main stream on the basis of the Meyer-Prandtl expansion yields an expression for p_b/p_1 in terms of ϕ which is equally suitable for computation. The method of obtaining numerical results, therefore, is to plot curves of p_b/p_1 for the boundary layer and for the main stream against ϕ and to read off the intersection. This has been done for $M = 2.4$ over the whole range of R (from $\frac{1}{4}$ million to 100 million) likely to be attained in practice and the results plotted as Figure 3. The mean line of the measurements described in the companion paper are plotted as well as a value inferred from firing trials.*

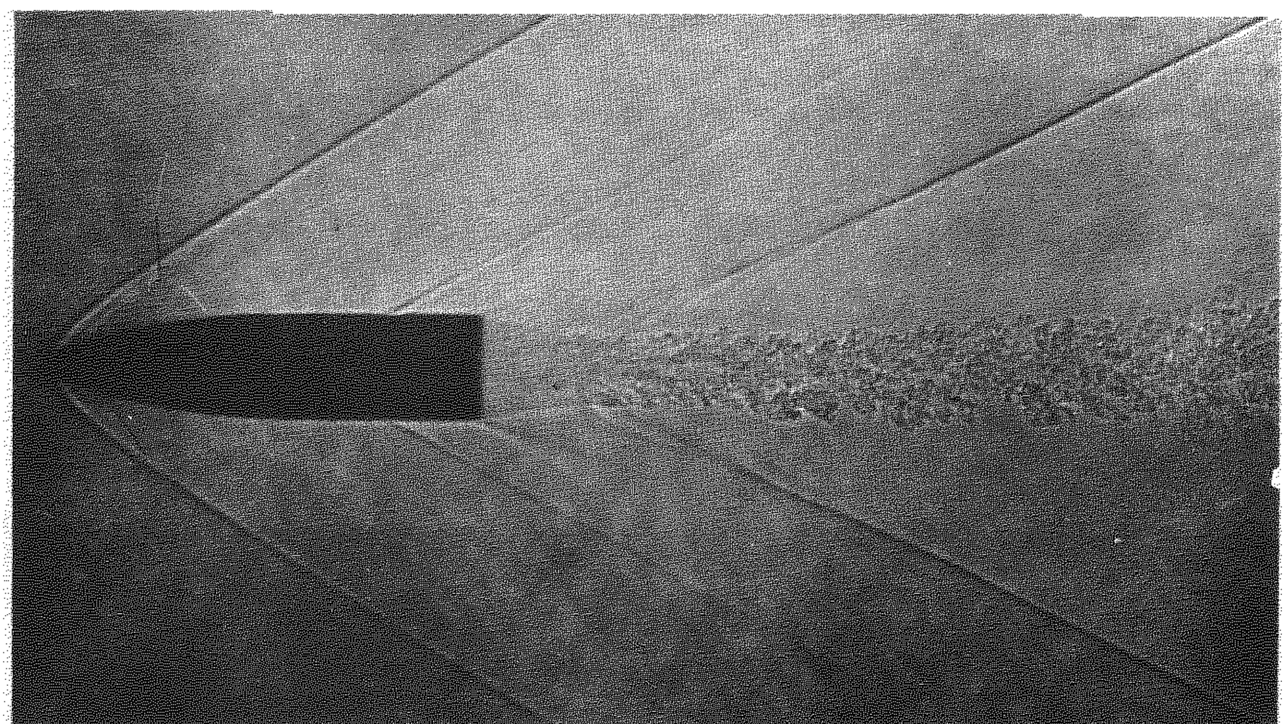
The agreement is encouraging but, as stated earlier, a more thorough comparison at several Mach numbers is necessary before any final verdict can be given. In particular more knowledge of the supersonic boundary layer would enable the causes of any discrepancy to be more confidently located.

Acknowledgment

The work was carried out in the Engineering Division of the National Physical Laboratory on behalf of the Chief Scientific Officer, British Ministry of Supply, by whose permission the paper is published.

*It is of interest, though not strictly relevant, to note that this value agrees quite well with that obtained (0.41) on the Karman-Moore method which takes no account of changes in R. (Trans. A.S.M.E. APM 54-27-303 1932).

FIG. 1



Mark VII 0·303" rifle bullet

Nominal M. V. 2,400 ft. per sec.

$M \approx 2 \cdot 15$

FIG. 2

CALCULATION OF REYNOLDS NUMBER EFFECT ON PROJECTILES AT SUPERSONIC SPEEDS

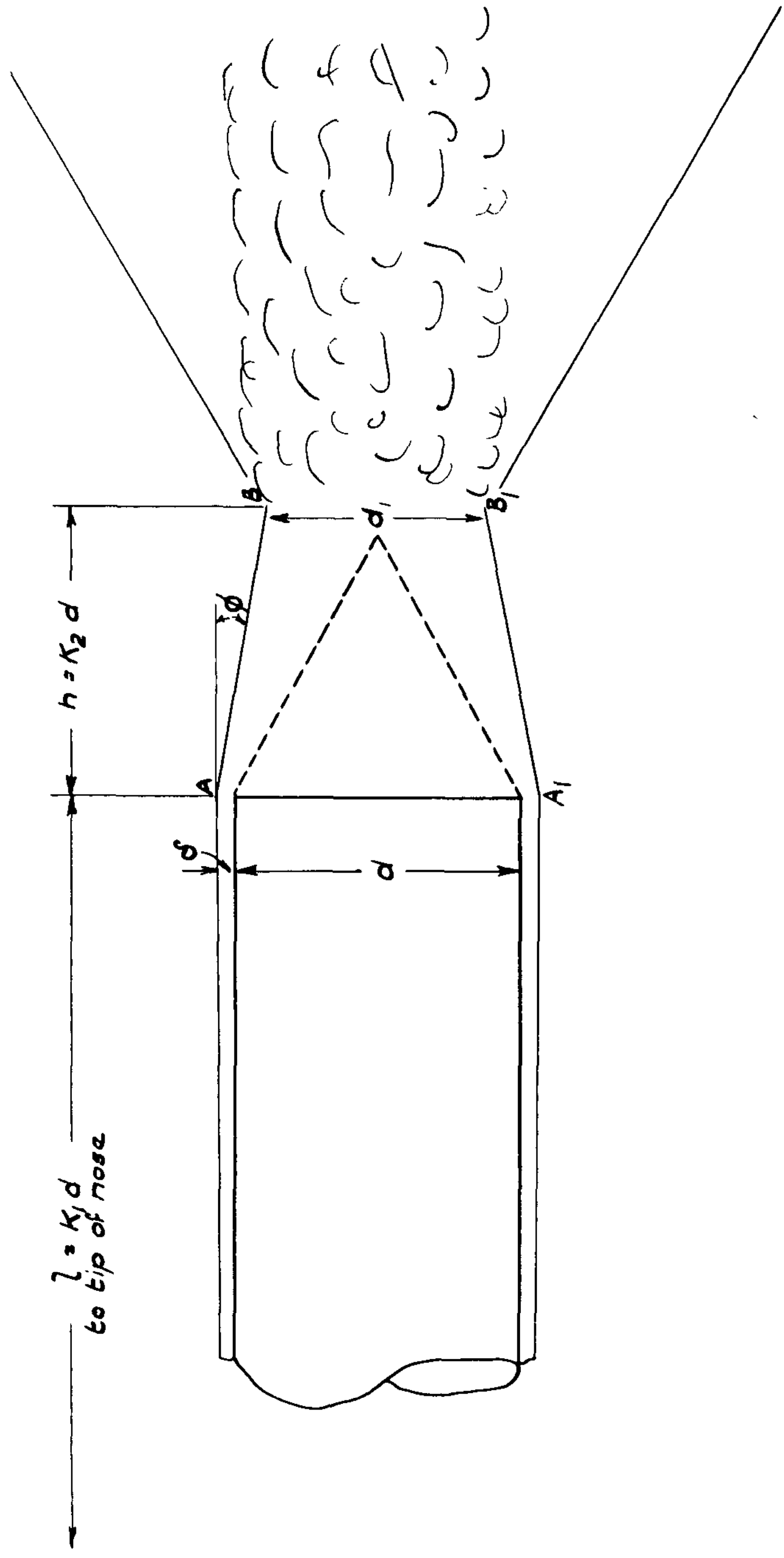
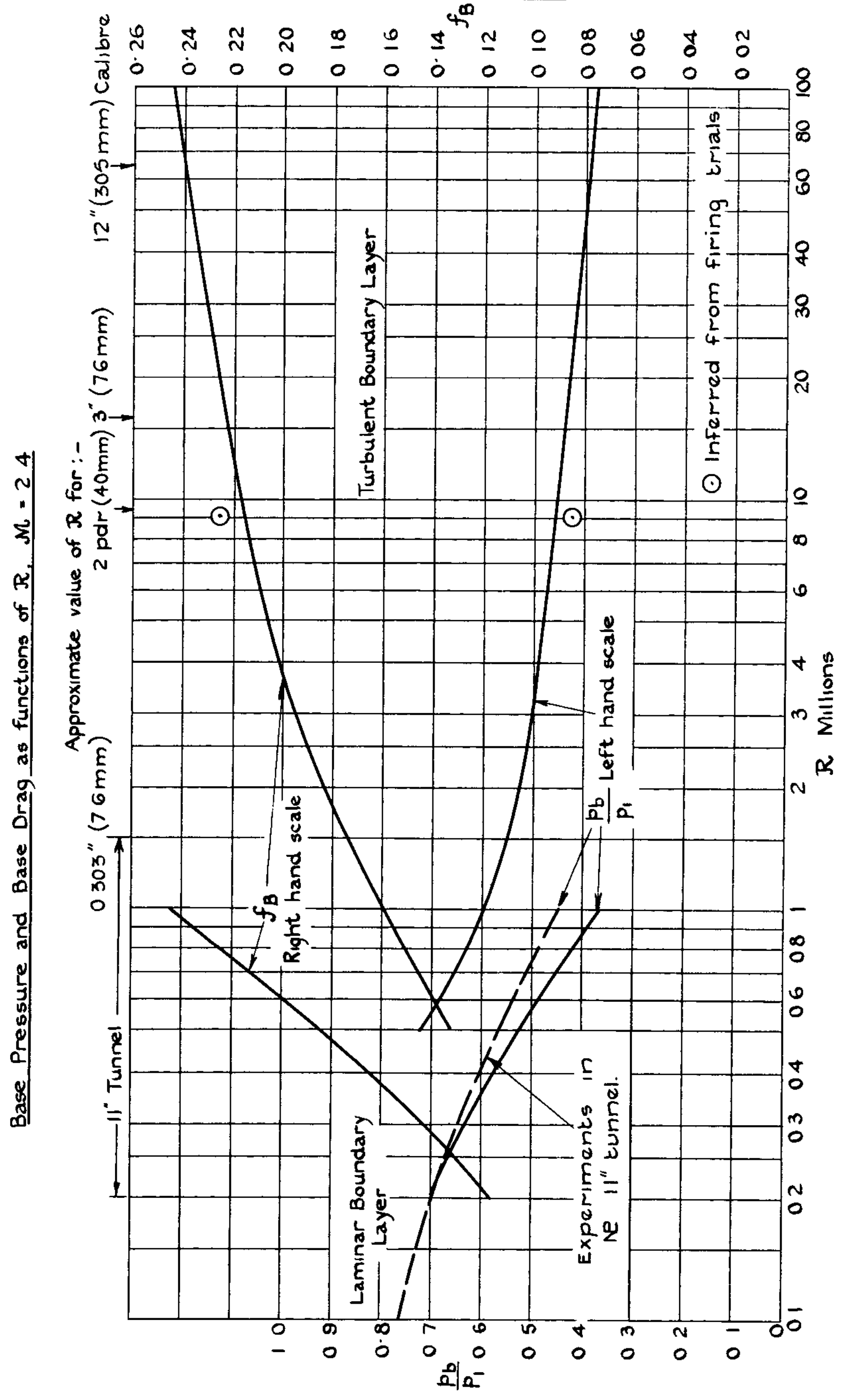


Diagram of flow near base of projectile

FIG 3

CALCULATION OF REYNOLDS NUMBER EFFECT ON PROJECTILES AT SUPERSONIC SPEEDS



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