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by

P. R. Ashill

Aerodynamics Dept., R.A.E., Bedford

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A STUDY OF THE EFFECT OF THE WAKE OF THE MAIN AEROFOIL OF A FOWLER-FLAP CONFIGURATION ON THE LIFT OF THE FLAP

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SUMMARY

This Report is concerned with the influence on the lift of the flap of the wake of the main aerofoil of a wing with a plain Fowler flap. To decide the relative importance of the wake, its effect is compared with the influence of the boundary layer of the flap. It is found that, for the configurations examined in this Report, the wake effect is of secondary importance in comparison with that of the boundary layer.

Consideration is given to various methods of approximating the wake effect, including the conventional 'thin'-wake method. It is shown that, by correctly positioning the singularities of the 'thin'-wake formulation, a first-order correction to this theory for wake thickness can be rendered identically zero. An approximation for a wake which is at a 'small' height above the flap chord is examined. The indications of the present calculations are that this approximation overestimates the magnitude of the correction to the lift of the flap for the effect of the wake. A better estimate of the wake effect appears to be obtained if one neglects the distributed sources and vortices of the wake but allows for the non-zero displacement flux of the wake by a point source at the shroud trailing edge.

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* Replaces RAE Technical Report 72081 - ARC 34169

Figs.1-15.

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1 INTRODUCTION

Whilst a completely satisfactory theoretical solution is not yet available, the problem of the incompressible, viscid flow around an isolated, twodimensional aerofoil is well understood. The main features of this problem have been established in the papers of Preston[1,2], Spence[3] and Spence and Beasley[4]. A start would not yet seem to have been made, however, on a theory for the more involved case of multiple aerofoils, and, in particular, for the case of aerofoils with single-slotted flaps with which we will be concerned here. The need for such a theory is obvious, but there are many difficulties. Not the least of these is the determination of the first inviscid approximation. Fortunately, we are now able to achieve this, at least to some numerical approximation, by means of the Douglas computer programme[5] due to A.M.O. Smith.

Another difficulty is the evaluation of the effect of the wake of the main aerofoil on the flow around the flap. In the case of the single aerofoil the wake lies downstream of the aerofoil. Thus we would expect that, in this case, the wake vorticity does not exert a significant influence on, for example, the lift of the aerofoil. The relatively good results obtained with 'the boundary-layer camber-correction theory'[2], which ignores the effect of the wake on the lift, seem to support this view. The situation is, however, somewhat different for slotted-flapped aerofoils. Here the wake from the aerofoil passes just above the flap and consequently may have a significant influence on the flow around the flap. In turn, this may affect the circulation around the complete configuration. It might be conjectured that, because of this, the first inviscid approximation for the velocity distribution around the flap is rather poor. In consequence, an iterative method of calculation, which is based on the first inviscid approximation[3], may not converge. The difficulty of obtaining a solution is further complicated if the thickness of the wake is not small compared with the chord of the flap as is probably the case if (a) the flap chord is small compared with the chord of the main aerofoil or (b) the incidence of the main aerofoil is large.

A method of representing a 'thick' wake is discussed in section 3. It is shown that, for a wake of finite thickness, the influence of the vorticity, contained within the boundaries of the wake, on the external flow may be simulated by distributions of sources and vortices along the edges of the wake. In the conventional 'outer' approximation for thin wakes[6] it is assumed that these distributions may be placed on a suitable mean line such as the rear dividing streamline. Approximations such as this one appear in many areas in aerodynamics and are employed, for example, in linearized, subsonic aerofoil theory[7] and slender-body theory[8]. From the many applications of these theories it appears that their accuracy depends not only on the slenderness ratio of the aerofoil or body but also on the way the thickness varies along its length. Consequently, even though a wake may be 'thin' it does not necessarily follow that the 'thin'-wake approximation will yield accurate results. In section 3, therefore, consideration is given to the question of the accuracy of the 'thin'-wake method.

Since the aerofoil wake passes just above the flap we would expect the lift of the flap to be sensitive to the behaviour of this wake. Therefore we will be mainly concerned with the lift of the flap. In order to judge the relative importance of the aerofoil wake we will also examine the effect of the flap boundary layer and its associated wake. We will not consider the influence of the aerofoil boundary layer; it may be that, for some cases, this is an important omission; however, in the examples examined in this Report, approximate calculations have indicated that it is of secondary importance compared with the flap boundary layer.

As with the aerofoil wake it is possible to represent the effect of the vorticity of the flap boundary layer on the external flow field by distributions of sources and vortices along the edge of the layer (assuming that such an edge can be defined). In the 'outer' approximation for 'thin' boundary layers the edge of the layer is supposed to coincide with the contour of the flap. We propose to assume that this approximation will be adequate for our purposes. Apart from this approximation, we make a number of other approximations involving the geometry of the flap and the main aerofoil. Although the accuracy of some of these approximations is considered in section 2 the main justification for their use is that the object of this Report is to perform a comparative assessment of the various viscous effects as described above. Therefore absolute accuracy in the final answers for the various corrections to the lift of the flap is probably not important. On the other hand, it was hoped that the simplicity of the present method would allow some physical insight into a rather complicated flow situation.

In the calculations to be discussed here we have followed Preston[2] in employing, where possible, experimentally derived results for the development of the wake of the main aerofoil and the boundary layer of the flap. These results were obtained by Foster, Irwin and Williams[9] from their extensive experiments on a twodimensional aerofoil with a single-slotted plain (Fowler) flap (Fig.1). We adopt Preston's approach in order to remove, as far as possible, any uncertainties resulting from the use of an approximate theoretical method for completing the viscous part of the calculation. Throughout this analysis, and in conformity with the experiments of Foster, *et al.*, the flow is considered twodimensional and incompressible.

Finally, we note that, in these introductory remarks, we have regarded the wake of the main aerofoil and the flap boundary layer as separate and distinct items of the flow field. As the results of Foster, *et al.* show, this is certainly not a valid concept if the flap gap is sufficiently small compared with the flap chord. We will, however, examine configurations for which it is possible to distinguish between the two vorticity layers.

2 INFLUENCE OF FLAP BOUNDARY LAYER ON FLAP LIFT

2.1 Problem formulation

We begin the analysis of this section with the assumption that it is possible to define an edge to the flap boundary layer and its associated wake. Additionally, we assume that, for the purpose of examining the flow in the finite part of the flow field, the flap wake may be truncated at some station a large but finite distance downstream of the flap. It is convenient to divide the vorticity contained within this finite region, which we term Σ, into three components as follows:

(a) the vorticity that is there according to the first inviscid (or Kutta) approximation for the flow around the aerofoil and the flap;

(b) the additional vorticity resulting from the existence of a boundary layer on the flap together with the flap wake;

(c) an image distribution of vorticity that is required to ensure that the flap remains a streamline in the presence of the vorticity (external to Σ) of the boundary layer and the wake of the main aerofoil.

In this section we consider the velocities induced in the region external to Σ by the second of these components. We do this by subtracting from the velocities induced by the vorticity in Σ the velocities induced by the first and third vorticity components. This result can be written down in the form of an area integral over Σ. However, in Appendix A it is shown how this can be reduced to a line integration around the contour bounding Σ, namely c, by means of Green's second formula[10]. There is obtained for the stream function induced by the vorticity component (b) at a point P in the region external to Σ

$$(\Delta\psi_B)_\Sigma = \frac{1}{2\pi}\int_c \left\{ \frac{\partial(\Delta\psi_B)}{\partial n} \ln r + \binom{+}{-} \frac{\partial(\Delta\psi_B)}{\partial \ell} \tau \right\} d\ell \quad . \tag{1}$$

Here $\Delta\psi_B$ = the incremental stream function due to the presence of a boundary layer on the flap, and, by reference to Fig.2, we see that

ℓ = the distance, taken positive in the clockwise direction, around the contour c

$\underset{\sim}{r}$ = the vector joining P and the element $d\ell$ on c, the vector being taken positive in the direction away from P

$\underset{\sim}{n}$ = the normal vector outward from region Σ

τ = the included angle between the ℓ direction and the negative r direction

$\binom{+}{-}$ = positive or negative alternatives taken depending as $\underset{\sim}{r}$ passes, respectively, out of or into region Σ at the element $d\ell$.

It is interesting to consider the physical significance of equation (1). Examination of the stream functions of potential singularities[7] shows that equation (1) is an expression for the stream function of distributions of vortices and sources of local strengths $\partial(\Delta\psi_B)/\partial n$ and $\partial(\Delta\psi_B)/\partial\ell$ respectively. The role of these distributions is to provide the necessary changes in the normal and tangential velocities at the edges of the flap boundary layer and the flap wake that are usually associated with these vorticity layers.

In the conventional 'outer' approximation for thin boundary layers the contour c is assumed to coincide with the flap surface and either side of the rear dividing streamline of the flap. That is to say the sources and vortices are transferred from the edges of the boundary layer or wake to the flap surface or rear dividing streamline. This approach is particularly attractive for the case of an isolated aerofoil since the integral of equation (1) may then be evaluated without difficulty by employing conformal transformation methods (see, e.g. Ref.3). An even greater simplification avails itself if the flap is of small thickness/chord ratio and camber. In this case it appears reasonable to assume that the part of contour c immediately adjacent to the flap may be transferred to either side of the flap chord. We propose to use this approximation on the basis that, in most practical applications, (a) the flap boundary layer is thin compared with the flap chord and (b) both the thickness/chord ratio and the camber of the flap, examined by Foster, *et al.*[9], are small. The same approximation is employed in the linearized theory of thin aerofoils[11]; this theory fails near the leading edge of a round-nosed aerofoil. Weber[12] has given a simple technique for correcting the theory in this region. A similar correction might be attempted in the present case, but this has not been done for the reason that we are mainly concerned here with overall, rather than detailed, effects of the flap boundary layer.

The analysis may be simplified further by making the assumption that the rear dividing streamline lies on the downstream extension of the flap chord. This assumption is clearly justified for the case of small flap angles. For moderate flap angles it is probably justifiable on the grounds that the rear dividing streamline approximates to the flap-chord extension near to the flap. Only on this part of the rear dividing streamline (i.e. the part close to the flap trailing edge) would we expect the source and vortex strengths to be significant, an expectation that is confirmed for isolated aerofoils by the experimental results of Preston, *et al.*[13,14].

The above assumptions imply that we use the following approximations in equation (1):

$$d\ell = \begin{bmatrix} + \\ - \end{bmatrix} dx' \quad ; \qquad r = \left\{(x - x')^2 + z^2\right\}^{\frac{1}{2}} \quad ;$$

$$\partial(\;)/\partial n = \begin{bmatrix} + \\ - \end{bmatrix} \partial(\;)/\partial z' \quad ; \qquad \tau = \begin{bmatrix} 0 \\ \pi \end{bmatrix} + \begin{bmatrix} + \\ - \end{bmatrix} \tan^{-1}\left(z/(x - x')\right) \quad .$$

Here (x,z) is the rectangular cartesian coordinate system with the x axis along the flap chord and $x = 0$ at the flap leading edge (Fig.1), and the prime denotes the coordinates of the inducing element. The alternatives $\begin{bmatrix} 0 \\ \pi \end{bmatrix}$ and $\begin{bmatrix} + \\ - \end{bmatrix}$ are taken depending as the inducing element is adjacent to the upper or lower surfaces of the flap. Thus, using the fact that

$$\left(\partial(\Delta\psi_B)/\partial\ell\right)_c = d(\Delta\psi_B)_c/d\ell \quad ,$$

we may write in place of equation (1)

$$(\Delta\psi_B)_\Sigma = \frac{1}{2\pi}\int_0^\infty \Delta\gamma_{F,B} \ln\left(\left\{(x - x')^2 + z^2\right\}^{\frac{1}{2}}\right)dx' + \frac{1}{2\pi}\int_0^\infty \Delta q_{F,B} \tan^{-1}\left(\frac{z}{x - x'}\right)dx'$$
$$- \frac{1}{2}\int_0^\infty \frac{d(\Delta\psi_B)_L}{dx'}\,dx' \quad . \qquad (2)$$

Here $\Delta\gamma_{F,B}$ and $\Delta q_{F,B}$ are incremental vortex and source strengths defined by

$$\left.\begin{aligned} \Delta\gamma_{F,B} &= \left(\frac{\partial(\Delta\psi_B)}{\partial z'}\right)_U - \left(\frac{\partial(\Delta\psi_B)}{\partial z'}\right)_L \\ \Delta q_{F,B} &= -\frac{d(\Delta\psi_B)_U}{dx'} + \frac{d(\Delta\psi_B)_L}{dx'} \end{aligned}\right\} , \tag{3}$$

where suffix F,B refers to the singularities that are (a) associated with the flap boundary layer and the flap wake and (b) on the flap chord and its downstream extension. Suffixes U and L refer to the upper and lower edges of the vorticity layer Σ. It should also be noted that the large finite upper limit of the x'-wise integration, implied by the discussion leading to equation (1), has been replaced by infinity in equation (2).

In general

$$\Delta\psi_B \neq (\Delta\psi_B)_\Sigma$$

owing to the requirement that the main aerofoil is a streamline of the real flow. This implies an additional distribution of vorticity within the main aerofoil to nullify the normal component of velocity induced at the aerofoil contour by the boundary-layer vorticity within Σ. We can write instead

$$\Delta\psi_B = (\Delta\psi_B)_A + (\Delta\psi_B)_\Sigma \quad , \tag{4}$$

where suffix A refers to the vorticity within the main aerofoil.

The above-mentioned requirement implies that $\Delta\psi_B$ is invariant around the contour of the main aerofoil. Therefore, by analogy with equation (1), we may write

$$(\Delta\psi_B)_A = \frac{1}{2\pi} \int_{\text{aerofoil}} \frac{\partial(\Delta\psi_B)}{\partial\nu} \ln r \, dt \quad , \tag{5}$$

where t is the direction tangential to the aerofoil contour, taken positive in the clockwise direction, and ν is the normal outward from the aerofoil.

Observing that the main aerofoil tested by Foster, *et al.*[9] is of small thickness/chord ratio and camber we may approximate equation (5) by transferring the surface vortex distribution to the chord of the main aerofoil. Thus we obtain the result

$$(\Delta\psi_B)_A = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B} \ln\left(\{(\xi - \xi')^2 + \zeta^2\}^{\frac{1}{2}}\right) d\xi' \quad , \qquad (6)$$

where c_A is the chord of the main aerofoil and (ξ,ζ) is the rectangular, cartesian coordinate system having its origin at the leading edge of the main aerofoil, with ξ along the chord (Fig.1). Additionally

$$\Delta\gamma_{A,B} = \left(\frac{\partial(\Delta\psi_B)}{\partial\zeta'}\right)_+ - \left(\frac{\partial(\Delta\psi_B)}{\partial\zeta'}\right)_- \quad ,$$

suffixes + and −, respectively, denoting the upper and lower surfaces of the main aerofoil, and suffix A,B referring to the singularities, on the chord of the main aerofoil, that are due to the flap boundary layer and the flap wake.

Since $\Delta\psi_B(\xi,\zeta)$ is invariant around the contour of the main aerofoil and as the main aerofoil is considered to be thin and of small camber we may write

$$\Delta\psi_B(\xi,0) = C \quad , \qquad (7)$$

a constant. Therefore, upon combining equations (2), (4), (6) and (7) and by referring to the geometry of the main aerofoil and the flap shown in Fig.1, we obtain the result

$$\begin{aligned} C = {} & \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B} \ln(|\xi - \xi'|)d\xi' \\ & + \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B} \ln\Big(\{[(\xi - c_A + \tilde{\ell})\cos\beta - g\sin\beta - x']^2 \\ & \qquad + [(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta]^2\}^{\frac{1}{2}}\Big) dx' \\ & + \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B} \tan^{-1}\left(\frac{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta}{(\xi - c_A + \tilde{\ell})\cos\beta - g\sin\beta - x'}\right) dx' \\ & - \frac{1}{2}\int_0^{\infty} \frac{d(\Delta\psi_B)_L}{dx'} dx' \quad , \end{aligned} \qquad (8)$$

where g is the flap gap, $\tilde{\ell}$ the flap overlap and β is the angle between the chord of the main aerofoil and the flap chord (or simply the flap angle) as illustrated in Fig.1.

The arbitrary constant C may be removed from equation (8) simply by differentiating both sides of this expression with respect to ξ. This differentiation may be carried through the integral signs of the second and third integrals. We may also do this with the first integral provided that the resulting integral is defined according to the Cauchy principal value. Hence we find that

$$0 = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\,\frac{d\xi'}{\xi-\xi'}$$

$$+ \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B}\,\frac{(\xi - c_A + \tilde{\ell} - x'\cos\beta)dx'}{\left\{x' - (\xi - c_A + \tilde{\ell})\cos\beta + g\sin\beta\right\}^2 + \left\{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta\right\}^2}$$

$$- \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B}\,\frac{(g + x'\sin\beta)dx'}{\left\{x' - (\xi - c_A + \tilde{\ell})\cos\beta + g\sin\beta\right\}^2 + \left\{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta\right\}^2}\,. \qquad \ldots. (9)$$

Equation (9) is an integral equation in the unknowns $\Delta\gamma_{A,B}$, $\Delta\gamma_{F,B}$ and $\Delta q_{F,B}$. Hence, in order to determine $\Delta\gamma_{F,B}$, which will be required for the evaluation of the influence of the flap boundary layer on the flap lift, we require two other relationships. The first of these expressions may be obtained by noting that

$$(\Delta\psi_B)_{U,L} = (\psi - \Delta\psi_W - \psi_I)_{U,L}\ , \qquad (10)$$

where ψ is the stream function of the real flow, $\Delta\psi_W$ is the increment in stream function associated with the presence of the wake of the main aerofoil and suffix I refers to the first inviscid approximation.

Using equation (10) we are able to define the 'displacement fluxes'

$$\left.\begin{aligned} \psi_U^* &= -(\Delta\psi_B)_U = \int_0^{\delta_U} \left\{u_I(z) + \Delta u_W(z) - u(z)\right\}dz \\ \psi_L^* &= (\Delta\psi_B)_L = \int_0^{\delta_L} \left\{u_I(z) + \Delta u_W(z) - u(z)\right\}dz \end{aligned}\right\}, \tag{11}$$

where δ is the thickness of the flap boundary layer or the distance between the edge of the wake of the flap and the rear dividing streamline of the flap; and u is the x-wise velocity in the boundary layer or the wake of the flap. This definition differs from the usual definition of displacement flux[2],

$$\psi^* = \int_0^{\delta} \{u_I(z) - u(z)\}dz \quad ,$$

in recognition of the fact that the aerofoil wake effectively alters the inviscid flow in the boundary layer. For the present we will suppose that $\psi_{U,L}^*$ may be determined either from the experimental results of Foster, *et al.*[9] or by theoretical means.

Upon combining equations (3) and (11) we have

$$\Delta q_{F,B} = \frac{d}{dx'} \{\psi_U^* + \psi_L^*\} \quad . \tag{12}$$

This is the first of the relationships required to complete the solution for $\Delta\gamma_{F,B}$. It is evidently an explicit expression for $\Delta q_{F,B}$. The second relationship is derived by observing that equation (11) may be regarded as a boundary condition for $\Delta\psi_B$ at $z = \pm 0$. In particular, if we consider the boundary condition at $z = +0$ (i.e. at the edge of the boundary layer and the wake of the flap upper surface) we obtain, by combining equations (2), (4), (6) and (11) and by referring again to the geometry of Fig.1, the result

$$- \psi_U^*(x) = \frac{1}{2\pi} \int_0^{c_A} \Delta\gamma_{A,B} \ln\left(\left\{(c_A - \tilde{\ell} + x \cos\beta - \xi')^2 + (g + x \sin\beta)^2\right\}^{\frac{1}{2}}\right) d\xi'$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \Delta\gamma_{F,B} \ln(|x - x'|) dx'$$

$$+ \frac{1}{2\pi} \lim_{z\to 0} \int_0^{\infty} \Delta q_{F,B} \tan^{-1}\left(\frac{z}{x - x'}\right) dx'$$

$$- \frac{1}{2} \int_0^{\infty} \frac{d(\Delta\psi_B)_L}{dx'} dx' \quad . \tag{13}$$

Here, it will be seen, we have placed z equal to zero in the integrands of the first two integrals rather than evaluating the integrals with z non-zero and then taking the limit as z tends to zero. This is permissible since the respective integrals are evidently continuous functions of z near $z = 0$. The same is, however, not the case with the third integral; and here we have adopted the limiting procedure.

We find it convenient to differentiate both sides of equation (13) with respect to x. As with equation (8) the differentiation is carried through the integral sign. This is permissible for the first and third integrals and is allowed in the case of the second integral provided that the Cauchy principal value of the integral is understood. Thus we have

$$- \frac{d\psi_U^*}{dx} = \frac{1}{2\pi} \int_0^{c_A} \Delta\gamma_{A,B} \frac{(c_A - \tilde{\ell} - \xi') \cos\beta + g \sin\beta + x}{(c_A - \tilde{\ell} + x \cos\beta - \xi')^2 + (g + x \sin\beta)^2} d\xi'$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \Delta\gamma_{F,B} \frac{dx'}{x - x'}$$

$$- \frac{1}{2\pi} \lim_{z\to 0} \int_0^{\infty} \Delta q_{F,B} \frac{z}{(x - x')^2 + z^2} dx' \quad . \tag{14}$$

The last integral of this expression is of the type evaluated in Appendix B. Referring to equations (B-1) and (B-2) we find that, if $\Delta q_{F,B}$ is analytic in the interval of integration,

$$\frac{1}{2\pi} \lim_{z \to 0} \int_0^{\infty} \Delta q_{F,B} \frac{z}{(x - x')^2 + z^2} dx' = \frac{\Delta q_{F,B}}{2} , \qquad 0 < x \leqslant \infty .$$

Hence, by combining this expression with equations (12) and (14), we have

$$\frac{1}{2}\left(\frac{d\psi_L^*}{dx} - \frac{d\psi_U^*}{dx}\right) = \frac{1}{2\pi} \int_0^{c_A} \Delta\gamma_{A,B} \frac{(c_A - \tilde{\ell} - \xi') \cos \beta + g \sin \beta + x}{(c_A - \tilde{\ell} + x \cos \beta - \xi')^2 + (g + x \sin \beta)^2} d\xi'$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \Delta\gamma_{F,B} \frac{dx'}{x - x'} . \tag{15}$$

It is interesting to observe that, for a flap of small thickness/chord ratio and camber with a 'thin' boundary layer, equation (15) implies that the boundary layer effectively displaces the flap camber line and the rear dividing streamline of the flap in the z direction by the amount

$$\tfrac{1}{2}(\delta_U^* - \delta_L^*) ,$$

where $\delta^* = \psi^*/u_I(0)$.

In early work on isolated aerofoils[2] no attempt was made to satisfy the equivalent of equation (15) for points in the wake. Instead, it was assumed that the incremental vortex strength is zero there. Later work[4] indicated that this assumption is incorrect, in general. Since $(\psi_L^* - \psi_U^*)$ is not readily found, either experimentally or theoretically, in the wake of the flap, it is natural to try to estimate $\Delta\gamma_{F,B}$ downstream of the flap. We defer a detailed discussion on this aspect of the problem until section 3, wherein we consider the influence of the wake of the main aerofoil.

Eliminating $\Delta q_{F,B}$ from equation (9) by means of equation (12) we find that the resulting equation plus equation (15) represent two simultaneous integral equations in the unknowns $\Delta\gamma_{F,B}$ and $\Delta\gamma_{A,B}$. The problem of reducing these equations to quadratures would seem to be very difficult. In the next section, therefore, we give consideration to an approximate method of achieving a solution.

2.2 Approximate solution of integral equations

The maximum value of β tested by Foster, *et al.*[9] was 24°. While this angle is not 'small' in the accepted sense we will examine the possibility of approximating the two integral equations by placing β equal to zero therein. Thus, upon making this approximation, we obtain, in place of equation (15), the expression

$$\frac{1}{2}\left(\frac{d\psi_L^*}{dx} - \frac{d\psi_U^*}{dx}\right) = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\,\frac{c_A - \tilde{\ell} + x - \xi'}{(c_A - \tilde{\ell} + x - \xi')^2 + g^2}\,d\xi'$$

$$+ \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B}\,\frac{dx'}{x - x'} \quad . \qquad (16)$$

An inspection of the integrand of the first integral of equation (15) seems to show that the accuracy of this approximation may depend on the nature of the function $\Delta\gamma_{A,B}(\xi')$. Therefore we examine the effect of approximating the integral

$$I_1 = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\,\frac{(c_A - \tilde{\ell} - \xi')\cos\beta + g\sin\beta + x}{(c_A - \tilde{\ell} + x\cos\beta - \xi')^2 + (g + x\sin\beta)^2}\,d\xi'$$

for the two vortex distributions

$$\Delta\gamma_{A,B}^{(1)} = 1 \quad ; \qquad \Delta\gamma_{A,B}^{(2)} = 2[(c_A - \xi')/\xi']^{\frac{1}{2}} \quad .$$

These distributions will be recognized as the constant-load distribution and the flat-plate loading of thin aerofoil theory. The integration is routine in the case of the first distribution and there is obtained

$$I_1^{(1)} = -\frac{1}{2\pi}\left(\frac{\cos\beta}{2}\ln\left[\frac{\{\tilde{\ell} - x\cos\beta\}^2 + \{g + x\sin\beta\}^2}{\{\tilde{\ell} - c_A - x\cos\beta\}^2 + \{g + x\sin\beta\}^2}\right]\right.$$

$$\left. - \sin\beta\left[\tan^{-1}\left(\frac{\tilde{\ell} - x\cos\beta}{g + x\sin\beta}\right) - \tan^{-1}\left(\frac{\tilde{\ell} - c_A - x\cos\beta}{g + x\sin\beta}\right)\right]\right), \qquad (17)$$

where the raised suffix (1) refers to the first distribution. The approximate version of $I_1^{(1)}$ for small β, which is obtained by placing β equal to zero in equation (17), reads:

$$I_1^{(1)} = -\frac{1}{4\pi} \ln\left[\frac{\{\tilde{\ell} - x\}^2 + g^2}{\{\tilde{\ell} - c_A - x\}^2 + g^2}\right] , \qquad (\beta = 0) .$$

The integration required to obtain $I_1^{(2)}$ is rather more difficult; instead we note that $I_1^{(2)}$ is the velocity induced in the negative z direction at the flap chord by the distribution $\Delta\gamma_{A,B}^{(2)}$ on the slit representing the chord of the main aerofoil. Therefore we seek a complex velocity function

$$V(\xi,\zeta) = v_\xi(\xi,\zeta) - iv_\zeta(\xi,\zeta)$$

(where v_ξ and v_ζ are velocity components in the ξ and ζ directions) that (a) is regular in the region external to the aerofoil slit, (b) vanishes infinitely far from the slit, and (c) yields the correct tangential velocity at the slit

$$v_\xi(\xi,\pm 0) \equiv \pm \Delta\gamma_{A,B}^{(2)}(\xi)/2 = \pm [(c_A - \xi)/\xi]^{\frac{1}{2}} .$$

The required complex velocity is found by inspection to be given by

$$V(\xi,\zeta) = \imath \left(1 - \left\{\frac{\xi + i\zeta - c_A}{\xi + i\zeta}\right\}^{\frac{1}{2}}\right) ,$$

the positive branch of the square root being understood. Upon resolving this expression into real and imaginary parts we obtain

$$v_\xi(\xi,\zeta) = \pm\left(\frac{\{(\xi^2 + \zeta^2 - c_A\xi)^2 + c_A^2\zeta^2\}^{\frac{1}{2}} - (\xi^2 + \zeta^2 - c_A\xi)}{2(\xi^2 + \zeta^2)}\right)^{\frac{1}{2}} ;$$

$$v_\zeta(\xi,\zeta) = -\left[1 - \left(\frac{\{(\xi^2 + \zeta^2 - c_A\xi)^2 + c_A^2\zeta^2\}^{\frac{1}{2}} + (\xi^2 + \zeta^2 - c_A\xi)}{2(\xi^2 + \zeta^2)}\right)^{\frac{1}{2}}\right] .$$

Here the + and - alternatives are taken depending as ζ is positive or negative. Resolving these velocity components into the negative z direction and substituting the values of ξ and ζ appropriate to the flap chord into the resulting expression we obtain finally

$$I_1^{(2)} = \left(\left[1 - \left(\frac{\{(\xi_F^2 + \zeta_F^2 - c_A\xi_F)^2 + c_A^2\zeta_F^2\}^{\frac{1}{2}} + (\xi_F^2 + \zeta_F^2 - c_A\xi_F)}{2(\xi_F^2 + \zeta_F^2)} \right)^{\frac{1}{2}} \right] \cos\beta + \left(\frac{\{(\xi_F^2 + \zeta_F^2 - c_A\xi_F)^2 + c_A^2\zeta_F^2\}^{\frac{1}{2}} - (\xi_F^2 + \zeta_F^2 - c_A\xi_F)}{2(\xi_F^2 + \zeta_F^2)} \right)^{\frac{1}{2}} \sin\beta \right) , \qquad (18)$$

where $\xi_F = c_A - \tilde{\ell} + x\cos\beta$; $\zeta_F = -g - x\sin\beta$

and use has been made of the fact that, for the flap configurations under consideration, ζ_F is negative.

The results calculated for I_1 are plotted against x/c_A in Fig.3 for the two vortex distributions, the values of β $0^\circ(10^\circ)30^\circ$ and the two configurations $g/c_A = 0.06$, $\tilde{\ell}/c_A = 0$; $g/c_A = 0.02$, $\tilde{\ell}/c_A = 0$. These configurations are typical of configurations that have been tested by Foster, *et al.*[9]; their model also had a flap chord to main aerofoil chord ratio of approximately 0.5.

We observe that, for β less than 30°, the differences between the approximate values ($\beta = 0$) and the exact values are small, being on an average less than 5% in the interval of x/c_A between 0 and 0.8. We conclude, therefore, that the approximation leading to equation (16) would seem to be acceptable, at least according to the evidence of the present calculations.

The question naturally arises whether a similar approximation may be applied to the integral equation (9). The approximate equivalent of this expression for small β may be written as follows:

$$0 = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B} \frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B} \frac{\xi - c_A + \tilde{\ell} - x'}{\{x' - (\xi - c_A + \tilde{\ell})\}^2 + g^2}\,dx' - \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B} \frac{g}{\{x' - (\xi - c_A + \tilde{\ell})\}^2 + g^2}\,dx' \; . \qquad \dots. (19)$$

Two integrals are involved in this approximation; the first represents the downwash induced at the chord of the main aerofoil by the vortices on the x axis, and the second is the source term. We will examine the implications of the approximation to the source term later; in the meantime we examine the accuracy of the approximation to the vortex term by considering the integral

$$I_2 = \frac{1}{2\pi}\int_0^\infty \Delta\gamma_{F,B}\,\frac{(\xi - c_A + \tilde{\ell} - x'\cos\beta)dx'}{\left\{x' - (\xi - c_A + \tilde{\ell})\cos\beta + g\sin\beta\right\}^2 + \left\{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta\right\}^2}$$

and the vortex distributions

$$\Delta\gamma^{(1)}_{F,B} = \begin{cases} 1\ ; & 0 \leqslant x \leqslant c_F \\ 0\ ; & c_F < x \leqslant \infty \end{cases}\ ;$$

$$\Delta\gamma^{(2)}_{F,B} = \begin{cases} 2[(c_F - x')/x']^{\frac{1}{2}}\ ; & 0 \leqslant x \leqslant c_F \\ 0\ ; & c_F < x \leqslant \infty \end{cases}\ ,$$

where c_F is the chord of the flap. These distributions are similar to those used previously except that they are placed on the flap chord instead of the chord of the main aerofoil. Note that we have not included the possible effect of the vortices of the wake of the flap. However, on the evidence of work by Spence and Beasley[4], who gave a method for determining the strength of these vortices, this omission would not seem to be of particular consequence.

For the first distribution we obtain

$$I^{(1)}_2 = -\frac{1}{2\pi}\left[\frac{\cos\beta}{2}\ln\left(\frac{\left\{c_F - (\xi - c_A + \tilde{\ell})\cos\beta + g\sin\beta\right\}^2 + \left\{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta\right\}^2}{\left\{(\xi - c_A + \tilde{\ell})\cos\beta - g\sin\beta\right\}^2 + \left\{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta\right\}^2}\right)\right.$$
$$- \sin\beta\left(\tan^{-1}\left[\frac{c_F - (\xi - c_A + \tilde{\ell})\cos\beta + g\sin\beta}{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta}\right]\right.$$
$$\left.\left. + \tan^{-1}\left[\frac{(\xi - c_A + \tilde{\ell})\cos\beta - g\sin\beta}{(\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta}\right]\right)\right]$$

whilst for the second distribution, by employing the technique used to determine $I_1^{(2)}$, we find that

$$I_2^{(2)} = \left[1 - \left(\frac{\{(x_A^2 + z_A^2 - c_F x_A)^2 + c_F^2 z_A^2\}^{\frac{1}{2}} + (x_A^2 + z_A^2 - c_F x_A)}{2(x_A^2 + z_A^2)}\right)^{\frac{1}{2}}\right]\cos\beta$$

$$\pm\left(\frac{\{(x_A^2 + z_A^2 - c_F x_A)^2 + c_F^2 z_A^2\}^{\frac{1}{2}} - (x_A^2 + z_A^2 - c_F x_A)}{2(x_A^2 + z_A^2)}\right)^{\frac{1}{2}}\sin\beta \quad . \qquad (20)$$

Here $x_A = (\xi - c_A + \tilde{\ell})\cos\beta - g\sin\beta;$

$z_A = (\xi - c_A + \tilde{\ell})\sin\beta + g\cos\beta$

and the plus and minus alternatives are taken depending on whether z_A is greater than or less than zero, respectively.

Results for $I_2^{(1)}$ and $I_2^{(2)}$ are shown in Fig.4 where they are plotted against ξ/c_A, for $c_F/c_A = 0.5$ and for the two gap and overlap cases considered previously. Evidently, the approximate results ($\beta = 0$) are in good agreement with the exact results for β less than 30° except in a narrow region adjacent to the trailing edge of the main aerofoil. Here the error rises to as much as 50% for $\beta = 30^\circ$. The reason for this can be found by expanding $I_2^{(2)}$ in powers of β. If this straightforward, though lengthy, process is carried out it is found that, if g/c_A is small and c_F/c_A is of order unity, the error in the approximate result is $O(\beta^2, g\beta/c_A)$ except where

$$O(\xi - c_A + \tilde{\ell}) \leqslant O(g) \quad .$$

In this region the error becomes $O(\beta)$: clearly, for a β of 30°, this is considerable. On the other hand, as this error occurs only over a region of width $O(g)$ the error in the mean value of $I_2^{(2)}$ along the chord of the main aerofoil (i.e. the incidence induced at the main aerofoil by the vortex distribution $\Delta\gamma_{F,B}^{(2)}$) is $O(\beta^2, g\beta/c_A)$. The same error is found for the effective change in the camber induced by $\Delta\gamma_{F,B}^{(2)}$ at the main aerofoil. We assert, therefore, that, for small g/c_A, less than 0.1 say, and small β (less than 30°) the small-angle approximation for I_2 will be suitable in the determination of $\Delta\gamma_{A,B}$ and $\Delta\gamma_{F,B}$.

Equations (16) and (19) are much simpler than the original integral equations (9) and (15). Even so, simultaneous, analytical inversions of the simplified equations would appear not to be possible at present. However, we have already imposed the condition that g/c_A is small, which is conventionally the case, and we note that, in practice, $\tilde{\ell}/c_A$ is small[9]. Therefore we assume that we may place both g and $\tilde{\ell}$ equal to zero in equations (16) and (19). We then find that, with

$$F = \psi_L^* - \psi_U^* ,$$

$$\left.\begin{aligned} \frac{1}{2}\frac{dF}{dx} &= \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}(\xi')\,\frac{d\xi'}{c_A + x - \xi'} + \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B}(x')\,\frac{dx'}{x - x'} \\ 0 &= \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}(\xi')\,\frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B}(x')\,\frac{dx'}{\xi - c_A - x'} \end{aligned}\right\} . \quad (21)$$

Using the transformations

$$x = \xi - c_A \quad ; \qquad x' = \xi' - c_A \qquad (22)$$

in equations (21) we obtain the expressions

$$\left.\begin{aligned} \frac{1}{2}\frac{dF}{d\xi} &= \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{,B}(\xi')\,\frac{d\xi'}{\xi - \xi'} \quad ; \qquad c_A \leqslant \xi \leqslant \infty \\ 0 &= \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{,B}(\xi')\,\frac{d\xi'}{\xi - \xi'} \quad ; \qquad 0 \leqslant \xi < c_A \end{aligned}\right\} , \quad (23)$$

where $\Delta\gamma_{,B}(\xi) = \begin{cases} \Delta\gamma_{A,B}(\xi) & ; \quad 0 \leqslant \xi < c_A ; \\ \Delta\gamma_{F,B}(\xi - c_A) & ; \quad c_A \leqslant \xi \leqslant \infty . \end{cases}$

Equations (23) may be written in the more concise form

$$\frac{1}{2}\frac{dF}{d\xi} H(\xi - c_A) = \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{,B}(\xi')\,\frac{d\xi'}{\xi - \xi'} \quad ; \qquad 0 \leqslant \xi \leqslant \infty , \qquad (24)$$

with $H(\xi - c_A)$ the Heaviside unit step function defined by

$$H(\xi - c_A) = \begin{cases} 0 \; ; & \xi < c_A \\ 1 \; ; & \xi \geqslant c_A \end{cases} .$$

We observe that the combined effect of this approximation and the small-angle approximation is to remove the source integral from equation (9). This is equivalent to the assumption that the effect of the sources, of the x axis, on $v_\zeta(\xi,0)$ is negligible. Whilst it is difficult to justify this assumption formally, except for very small g/c_A, $\tilde{\ell}/c_A$ and β, calculations of the velocity fields of the source distributions for the boundary layers measured by Foster, *et al.*[9] indicate that it is well-founded. Typically, one finds that the sources contribute 0.001 to the mean value of $v_\zeta(\xi,0)/V_\infty$ along the chord of the main aerofoil, V_∞ being the speed of the uniform flow at infinity. As may be inferred from equation (24) this means that the error in $\Delta\gamma_{,B}/V_\infty$ obtained by neglecting the source integral is of the order of 0.001.

The model of the sources and vortices that is implied by equation (24) is shown in Fig.5.

To assess the likely effect of the approximation for small g/c_A on the vortex integrals we examine $I_1^{(2)}$ (equation (18)) and $I_2^{(2)}$ (equation (20)), these being the relevant integrals evaluated for the 'flat-plate' loading. Fig.6 shows plots of $I_1^{(2)}$ against x/c_A and $I_2^{(2)}$ against ξ/c_A, with, in both cases, $\tilde{\ell}/c_A$ equal to zero and β zero (corresponding to the small-angle approximation). For all the cases considered c_F/c_A is taken as 0.5. It will be seen that the effect on $I_1^{(2)}$ and $I_2^{(2)}$ of increasing g/c_A from 0.01 to 0.06 is very small except in the immediate neighbourhood of the flap gap. In this region the differences between the various curves for $I_2^{(2)}$ are large, although the differences are not as serious in the case of $I_1^{(2)}$. The reason for these large errors in the small-gap approximation for $I_2^{(2)}$ can be found by normalizing all lengths in equation (20) by c_A and then expanding this expression formally in powers of g/c_A. Upon doing this it is readily found that

$$(I_2^{(2)})_{\beta=0} \sim 1 - \left(\frac{X - c_F/c_A}{X}\right)^{\frac{1}{2}} \left(1 + \frac{1}{2}\left\{\frac{1}{(X^2 - Xc_F/c_A)} + \frac{(c_F/c_A)^2}{(X^2 - Xc_F/c_A)^2} - \frac{1}{X^2}\right\}\left(\frac{g}{c_A}\right)^2 + O[(g/c_A)^4]\right) \quad , \quad g/c_A \to 0 \quad ,$$

where $X = \xi/c_A - 1 + \tilde{\ell}/c_A$.

Thus we find that the error in the small-gap approximation for $(I_2^{(2)})_{\beta=0}$ is $O[(g/c_A)^2]$ except in a region close to the flap gap where the approximation evidently fails. This is, however, a localized failure of the approximation and, for small flap gaps, it is unlikely that it will seriously affect the validity of the method as a means of determining the correction to the lift of the flap. We also observe that the failure of the present approximation to represent the gap flow must be considered in relation to the use here of the 'thin'-aerofoil solution which also fails at the leading edge of the flap.

A similar difficulty is found with the approximation for small overlap. In this case, however, $(I_2^{(2)})_{\beta=0}$ is found to be in error by terms of order $\tilde{\ell}/c_A$ in the range of validity of the approximation. The present method is therefore restricted, it seems, to rather small $\tilde{\ell}/c_A$. On the other hand, Foster, *et al.*[9] found from both inviscid calculations and experiment that the flap lift is relatively insensitive to changes in $\tilde{\ell}/c_A$. It seems possible, therefore, that the small overlap approximation is quite accurate even for values of $\tilde{\ell}/c_A$ which may not be considered small.

It is convenient to rewrite equation (24) as

$$\frac{1}{2}\frac{dF}{d\xi}H(\xi - c_A) = \frac{1}{2\pi}\int_0^{c_E} \Delta\gamma_{,B}(\xi')\frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi}\int_{c_E}^{\infty} \Delta\gamma_{,B}(\xi')\frac{d\xi'}{\xi - \xi'} \quad , \qquad \ldots . \ (25)$$

where $c_E = c_A + c_F$.

The second integral of equation (25) represents the contribution of the wake of the flap; we propose examining the wake vortex effect, in connection with a study of the wake of the main aerofoil, in section 3. For the time being therefore we will suppose that

$$\Delta\gamma_{,B}(\xi) = 0 \quad ; \qquad c_E < \xi \leqslant \infty \ . \qquad (26)$$

The second integral of equation (25) then vanishes and the resulting integral equation may be inverted to give the result[15]

$$\Delta\gamma_{,B}(\xi) = \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \frac{dF}{d\xi'} H(\xi' - c_A) \frac{d\xi'}{\xi' - \xi} + \frac{B}{\xi^{\frac{1}{2}}(c_E - \xi)^{\frac{1}{2}}} ,$$

where B is an arbitrary constant. Hence it appears that the solution for $\Delta\gamma_{,B}$ is not unique. Nevertheless, uniqueness can be assured, firstly, by inferring from equation (3) that $\Delta\gamma_{F,B}$ is a finite quantity, referring, as it does, to velocities induced in the flow field by the flap boundary layer. Secondly, we observe that, according to Weber[15], provided that $dF/d\xi$ is analytic in the interval $c_A \leqslant \xi \leqslant c_E$, the integral term in the above expression is identically zero for $\xi = c_E$. Physical considerations suggest that $dF/d\xi$ satisfies this proviso; hence the two conditions demand that B is zero. Therefore we have

$$\Delta\gamma_{,B}(\xi) = \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \frac{dF}{d\xi'} H(\xi' - c_A) \frac{d\xi'}{\xi' - \xi} . \qquad (27)$$

2.3 Effect of flap boundary layer on flap lift

The lift on the flap may be written as the line integral

$$L_F = \cos(\beta + \alpha) \int_{\text{flap}} p dx , \qquad (28)$$

where suffix flap refers to integration around the contour of the flap in the anti-clockwise direction and α is the incidence of the main aerofoil.

According to Prandtl's boundary-layer theory[11] the rise in static pressure across a boundary layer on an essentially uncurved surface is of second order in δ/c_F compared with ρV_∞^2. Consequently, for a sufficiently 'thin' boundary layer this pressure rise can be neglected in comparison with the change in static pressure at the edge of the boundary layer due to the vorticity of the boundary layer, which change is $\rho V_\infty^2 O(\delta/c_F)$. Near to the trailing edge of the flap the flow is curved, and a significant variation in the static pressure across the boundary layer might be expected there. However, measurements made by Foster[16] indicate that, for the configurations to be examined here (section 5), this change in static pressure is, in fact, slight compared with the boundary-layer effect mentioned above. Consequently, we may write in place of equation (28)

$$L_F = \cos(\beta + \alpha) \int_0^{c_F} (p_L - p_U)\,dx \quad ,$$

$$= \tfrac{1}{2}\rho \cos(\beta + \alpha) \int_0^{c_F} \left\{ \left(\frac{\partial\psi}{\partial z}\right)_U^2 + \left(\frac{\partial\psi}{\partial x}\right)_U^2 - \left(\frac{\partial\psi}{\partial z}\right)_L^2 - \left(\frac{\partial\psi}{\partial x}\right)_L^2 \right\} dx \quad , \qquad (29)$$

from Bernoulli's equation.

In the 'thin'-aerofoil theory, on which the present method is based, it is usual to neglect the terms $(\partial\psi/\partial x)_{U,L}^2$ in comparison with $(\partial\psi/\partial z)_{U,L}^2$ and to replace $(\partial\psi/\partial z)_{U,L}$ by $(\partial\psi/\partial z)(x,\pm 0)$. Weber[12] shows that these approximations fail near the leading edge of an isolated 'thin' aerofoil and she gives a factor for correcting the 'thin'-aerofoil result. A similar factor, which took into account the proximity of the main aerofoil, could perhaps be devised for the present case. This is, however, a localized effect which should not significantly affect the correction to the flap lift due to either the flap boundary layer or the wake of the main aerofoil. Hence, using these approximations in equation (29) and noting that

$$\psi = \psi_I + \Delta\psi_W + \Delta\psi_B \quad ,$$

we obtain an expression which, after expansion, becomes

$$\begin{aligned} L_F = \tfrac{1}{2}\rho \cos(\beta + \alpha) \int_0^{c_F} \Bigg\{ & \left(\frac{\partial\psi_I}{\partial z}\right)^2 (x,+0) + 2\,\frac{\partial\psi_I}{\partial z}(x,+0)\left(\frac{\partial(\Delta\psi_B)}{\partial z}(x,+0) + \frac{\partial(\Delta\psi_W)}{\partial z}(x,+0)\right) \\ & + 2\,\frac{\partial(\Delta\psi_B)}{\partial z}(x,+0)\,\frac{\partial(\Delta\psi_W)}{\partial z}(x,+0) + \left(\frac{\partial(\Delta\psi_B)}{\partial z}\right)^2 (x,+0) + \left(\frac{\partial(\Delta\psi_W)}{\partial z}\right)^2 (x,+0) \\ & - \left(\frac{\partial\psi_I}{\partial z}\right)^2 (x,-0) - 2\,\frac{\partial\psi_I}{\partial z}(x,-0)\left(\frac{\partial(\Delta\psi_B)}{\partial z}(x,-0) + \frac{\partial(\Delta\psi_W)}{\partial z}(x,-0)\right) \\ & - 2\,\frac{\partial(\Delta\psi_B)}{\partial z}(x,-0)\,\frac{\partial(\Delta\psi_W)}{\partial z}(x,-0) - \left(\frac{\partial(\Delta\psi_B)}{\partial z}\right)^2 (x,-0) - \left(\frac{\partial(\Delta\psi_W)}{\partial z}\right)^2 (x,-0) \Bigg\} dx \;. \end{aligned}$$

.... (30)

Thus, ignoring the squares and products of the correction terms, which we anticipate are small compared with the first-order correction terms, we obtain for the correction to the flap lift due to the flap boundary layer

$$\Delta L_{F,B} = \rho \cos (\beta + \alpha) \int_0^{c_F} \left\{ \frac{\partial \psi_I}{\partial z} (x,+0) \frac{\partial (\Delta\psi_B)}{\partial z} (x,+0) - \frac{\partial \psi_I}{\partial z} (x,-0) \frac{\partial (\Delta\psi_B)}{\partial z} (x,-0) \right\} dx \quad . \qquad \ldots . (31)$$

To evaluate this expression we need to know $\Delta\psi_B$; this we find by employing equations (2), (4) and (6) and by referring to the geometry of Fig.1. Thus we have

$$\Delta\psi_B(x,z) = \frac{1}{2\pi} \int_0^{c_A} \Delta\gamma_{A,B} \ln \left(\left\{ (c_A - \tilde{\ell} + x \cos \beta + z \sin \beta - \xi')^2 + (g + x \sin \beta - z \cos \beta)^2 \right\}^{\frac{1}{2}} \right) d\xi'$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \Delta\gamma_{F,B} \ln \left(\left\{ (x - x')^2 + z^2 \right\}^{\frac{1}{2}} \right) dx'$$

$$+ \frac{1}{2\pi} \int_0^{\infty} \Delta q_{F,B} \tan^{-1} \left(\frac{z}{x - x'} \right) dx'$$

$$- \frac{1}{2} \int_0^{\infty} \frac{d(\Delta\psi_B)_L}{dx'} dx' \quad .$$

Differentiating this expression with respect to z, and noting that the differentiation may be carried through the integral signs, we find that

$$\frac{\partial(\Delta\psi_B)}{\partial z}(x,z) = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\left(\frac{\sin\beta\,(c_A - \tilde{\ell} + x\cos\beta + z\sin\beta - \xi') - \cos\beta\,(g + x\sin\beta - z\cos\beta)}{(c_A - \tilde{\ell} + x\cos\beta + z\sin\beta - \xi')^2 + (g + x\sin\beta - z\cos\beta)^2}\right)d\xi'$$

$$+ \frac{1}{2\pi}\int_0^{\infty} \Delta\gamma_{F,B}\,\frac{z}{(x - x')^2 + z^2}\,dx'$$

$$+ \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B}\,\frac{x - x'}{(x - x')^2 + z^2}\,dx' \quad .$$

Hence, by referring to equations (B-1) and (B-2), we obtain the expression

$$\frac{\partial(\Delta\psi_B)}{\partial z}(x,\pm 0) = \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\,\frac{\sin\beta\,(c_A - \tilde{\ell} + x\cos\beta - \xi') - \cos\beta\,(g + x\sin\beta)}{(c_A - \tilde{\ell} + x\cos\beta - \xi')^2 + (g + x\sin\beta)^2}\,d\xi'$$

$$\pm \frac{\Delta\gamma_{F,B}}{2} + \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B}\,\frac{dx'}{x - x'} \quad , \qquad 0 < x \leqslant \infty \quad . \tag{32}$$

As before, we might attempt to approximate this expression for small β, g/c_A and $\tilde{\ell}/c_A$. We note that, as β and g/c_A tend to zero, the first term on the right-hand side vanishes. On the other hand, for a β of greater than 20°, the error obtained in the flap lift by neglecting this term could be significant. Therefore, whilst placing g and $\tilde{\ell}$ equal to zero, as before, we retain terms of order β; so that we have

$$\frac{\partial(\Delta\psi_B)}{\partial z}(x,\pm 0) = \frac{\beta}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}\,\frac{c_A - \xi'}{(c_A + x - \xi')^2}\,d\xi' \pm \frac{\Delta\gamma_{F,B}}{2}$$

$$+ \frac{1}{2\pi}\int_0^{\infty} \Delta q_{F,B}\,\frac{dx'}{x - x'} \quad . \tag{33}$$

Combining equation (33) with equation (31) we are then in a position to obtain the correction of the flap boundary layer to the flap lift coefficient

$$\Delta C_{L_{F,B}} = \frac{\Delta L_{F,B}}{\frac{1}{2}\rho V_\infty^2 c_0} ,$$

where c_0 is the basic chord of the configuration. Since $\Delta\gamma_{A,B}$ and $\Delta\gamma_{F,B}$ are known from equation (27) and $\Delta q_{F,B}$ can be determined from equation (12) it follows that, provided $(\partial\psi_I/\partial z)(x,\pm 0)$ are known, $\Delta C_{L_{F,B}}$ may be evaluated. In fact, to the order of our approximation $(\partial\psi_I/\partial z)(x,\pm 0)$ are the speeds of the flows at the upper and lower surfaces of the flap according to the first inviscid approximation. These speeds can be obtained from the Douglas numerical method[5]; and this has been done for the configurations to be studied here by Foster[16].

3 INFLUENCE OF WAKE OF MAIN AEROFOIL

In this section we consider the influence of the wake of the main aerofoil on the lift of the flap. The aim is to provide information that will help us to answer two questions. Firstly, how large is this effect in comparison with that due to the flap boundary layer? Secondly, is the conventional 'outer' approximation[6] for a 'thin' wake adequate as a means of representing this type of wake? Clearly, the answer to the first question will decide how much emphasis needs to be placed on the second question; since, if the wake effect is small compared with other viscous effects, an approximate representation of the wake of the main aerofoil may be acceptable.

The method employed in this section to satisfy the boundary conditions of the flap and the main aerofoil is essentially the same as used in section 2.2 for the flap boundary layer. Consequently, inasmuch as the same approximations are used in the study of the wake as were used in dealing with the flap boundary layer, the answer to the first question should not be significantly affected by the fact that we use an approximate method.

With the vorticity distribution within the wake presumed known (in this case from the experimental results of Foster, *et al.*[9]) we are able to derive the velocity field induced by the wake in the region external to the wake. This expression, which is derived without any restrictions being placed on wake thickness, is then approximated for 'small' wake thickness. As well, and consistent with the small-gap approximation given previously, use is made of the fact that the wake of the main aerofoil is close to the flap upper surface. In section 3.2 a first-order correction for non-zero wake thickness is derived with the intention of providing an answer to the second question. Finally, the results obtained for the induced velocities, including the contributions from the images of the wake vorticity within the main aerofoil and the flap, are used to determine the correction to the flap lift.

The stream function induced by the vorticity within the region occupied by the wake of the main aerofoil, which region we call Γ, at a point P external to Γ is derived in Appendix A, namely

$$(\Delta\psi_W)_\Gamma = \frac{1}{2\pi}\int_k \left\{ \frac{\partial(\Delta\psi)}{\partial n} \ln r + \binom{+}{-} \frac{\partial(\Delta\psi)}{\partial \ell} \tau \right\} d\ell \quad , \tag{34}$$

where $\Delta\psi = \Delta\psi_W + \Delta\psi_B$.

The meaning of the notation employed in equation (34) is the same as for equation (1), except that we have distinguished the contour bounding the wake of the main aerofoil with the title k (Fig.7).

The downwash induced by the wake at P is obtained by differentiating equation (34) with respect to x. To do this we will need the following results:

$$r = \left\{(x - x')^2 + (z - z')^2\right\}^{\frac{1}{2}} \quad ;$$

$$\tau = \tan^{-1}\left(\frac{z - z'}{x - x'}\right) - \tan^{-1}\left(\frac{dz_k}{dx}(x',z')\right)$$

for $\underset{\sim}{r}$ entering Γ at the element $d\ell$; and

$$\tau = \pi - \tan^{-1}\left(\frac{z - z'}{x - x'}\right) + \tan^{-1}\left(\frac{dz_k}{dx}(x',z')\right)$$

for $\underset{\sim}{r}$ leaving Γ at $d\ell$. Here $z_k = z_k(x)$ is the equation defining contour k. Therefore, recalling that the plus or minus alternatives of equation (34) are taken depending as $\underset{\sim}{r}$ passes out of or into region Γ at the element $d\ell$, respectively, we find that

$$\frac{\partial(\Delta\psi_W)_\Gamma}{\partial x}(x,z) = \frac{1}{2\pi}\int_k \left\{ \frac{\partial(\Delta\psi)}{\partial n} \frac{x - x'}{(x-x')^2 + (z-z')^2} + \frac{\partial(\Delta\psi)}{\partial \ell} \frac{z - z'}{(x-x')^2 + (z-z')^2} \right\} d\ell \quad .$$

.... (35)

It will be seen that the differentiation has been performed under the integral sign, an operation that is permitted for a point P external to Γ. Similarly we find that

$$\frac{\partial(\Delta\psi_W)_\Gamma}{\partial z}(x,z) = \frac{1}{2\pi}\int_k \left\{\frac{\partial(\Delta\psi)}{\partial n}\,\frac{z - z'}{(x-x')^2 + (z-z')^2} - \frac{\partial(\Delta\psi)}{\partial \ell}\,\frac{x - x'}{(x-x')^2 + (z-z')^2}\right\} d\ell \quad . \quad \ldots\ (36)$$

for the x-wise velocity induced at P by the vorticity within Γ.

3.1 Approximation for thin wakes

It is convenient to rewrite equation (35) as

$$-\frac{\partial(\Delta\psi_W)_\Gamma}{\partial x}(x,z) = w_i(x,z) = (w_i)_+ + (w_i)_- + (w_i)_\ell \quad . \qquad (37)$$

Here, with AB the leading edge of the wake (Fig.7),

$$(w_i)_+ = -\frac{1}{2\pi}\int_{A\infty}\left(\frac{\partial(\Delta\psi)}{\partial n}\,\frac{d\ell}{dx'}\right)_+ \frac{x - x'}{(x - x')^2 + (z - z')^2}\,dx'$$

$$-\frac{1}{2\pi}\int_{A\infty}\frac{d(\Delta\psi)_+}{dx'}\,\frac{z - z'}{(x - x')^2 + (z - z')^2}\,dx' \quad ; \qquad (38a)$$

$$(w_i)_- = -\frac{1}{2\pi}\int_{\infty B}\left(\frac{\partial(\Delta\psi)}{\partial n}\,\frac{d\ell}{dx'}\right)_- \frac{x - x'}{(x - x')^2 + (z - z')^2}\,dx'$$

$$-\frac{1}{2\pi}\int_{\infty B}\frac{d(\Delta\psi)_-}{dx'}\,\frac{z - z'}{(x - x')^2 + (z - z')^2}\,dx' \quad ; \qquad (38b)$$

$$(w_i)_\ell = -\frac{1}{2\pi}\int_{BA}\left(\frac{\partial(\Delta\psi)}{\partial n}\right)_\ell \frac{x - x'}{(x - x')^2 + (z - z')^2}\,d\ell$$

$$-\frac{1}{2\pi}\int_{BA}\frac{d(\Delta\psi)_\ell}{d\ell}\,\frac{z - z'}{(x - x')^2 + (z - z')^2}\,d\ell \quad . \qquad (38c)$$

The suffixes +, - and ℓ refer respectively to the upper edge, the lower edge and the leading edge of the wake of the main aerofoil.

The leading edge of the wake, BA, is defined, more or less arbitrarily, as the straight line, drawn normal to the bisector of the shroud trailing-edge angle. Shrouds are usually cusped in shape[9]; we would expect, therefore, that both the real flow and the flow of the first inviscid approximation would be sensibly normal to BA at the leading edge of the wake. This implies that $(\partial(\Delta\psi)/\partial n)_\ell$ is very small compared with V_∞. Consequently we assume that we may neglect the first term on the right-hand side of equation (38c) and thus obtain instead

$$(w_i)_\ell = -\frac{1}{2\pi}\int_{BA} \frac{d(\Delta\psi)_\ell}{d\ell}\,\frac{z - z'}{(x - x')^2 + (z - z')^2}\,d\ell \quad . \qquad (39)$$

In the 'thin'-wake approximation x' and z' are replaced in equations (38a) and (38b) by x_W and z_W, the x and z ordinates of the rear dividing streamline of the main aerofoil. In equation (39) x' and z' are replaced by x_T and z_T, the x and z ordinates of the shroud trailing edge. With these approximations we have, using equations (37), (38a), (38b) and (39), and performing the routine integration in the approximated version of equation (39),

$$\begin{aligned} w_i(x,z) = &-\frac{1}{2\pi}\int_{x_T}^{\infty} \gamma_W(x_W)\,\frac{x - x_W}{(x - x_W)^2 + (z - z_W)^2}\,dx_W \\ &+\frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^*}{dx_W}(x_W)\,\frac{z - z_W}{(x - x_W)^2 + (z - z_W)^2}\,dx_W \\ &+\frac{1}{2\pi}\,\psi_W^*(x_T)\,\frac{z - z_T}{(x - x_T)^2 + (z - z_T)^2} \quad , \end{aligned} \qquad (40)$$

where

$$\gamma_W = \left(\frac{\partial(\Delta\psi)}{\partial n}\,\frac{d\ell}{dx}\right)_+ - \left(\frac{\partial(\Delta\psi)}{\partial n}\,\frac{d\ell}{dx}\right)_- \qquad (41)$$

is 'the wake vortex strength' per unit x-wise length, and

$$\psi^*_W = \Delta\psi_- - \Delta\psi_+ \tag{42}$$

is 'the displacement flux' of the wake.

The physical significance of the terms in equation (40) may be described as follows: the first term on the right-hand side is the upwash induced by the vortices required in the simulation of the wake of the main aerofoil. These vortices exist as a consequence of the reduced momentum of the air in the wake being turned through an angle[4]. In early work on viscid aerofoil theory[1,2,3] this effect was disregarded on the basis of a conjecture by G.I. Taylor that the circulation around any simply closed circuit cutting the wake in two places at right angles is zero. Subsequently, Spence and Beasley[4], using an analysis derived from the jet-flap theory, showed that, in general, this conjecture is not correct. However, they indicated that, for an isolated aerofoil, the effect is of secondary importance to the boundary-layer displacement effect. In the case of the slotted flap the vortex effect may be of rather more significance in view of (a) the proximity of the wake of the main aerofoil to the flap upper surface and (b) the relatively large turning angle involved in the flow above the flap.

The second term on the right-hand side of equation (40) is due to the source distribution representing the growth of displacement flux along the wake of the main aerofoil. As with the wake vortices this may be a particularly significant effect owing not only to the closeness of the aerofoil wake to the flap but also to the adverse pressure gradient induced by the flap at the wake. This may result in a relatively rapid growth in the displacement flux of the wake.

Finally, the last term in equation (40) arises from an isolated source that is situated at the shroud trailing edge. The presence of this singularity is a direct result of our neglect of the boundary layer of the main aerofoil. The source provides the step in displacement flux at the shroud trailing edge that is necessary to yield a non-zero displacement flux in the wake of the main aerofoil. With the inclusion of the boundary layer of the main aerofoil the point source is replaced by a distribution of sources along the edge of the boundary layer. Consideration of the continuity of $\Delta\psi$ around the edge of the boundary layer shows that the integrated strength of these sources is equal to the strength of the point source. We may therefore regard the point source as an approximation to the distributed sources of the boundary layer of the main aerofoil. Also associated with the existence of a boundary layer on the main aerofoil is an additional distribution of vortices on the chord of the main aerofoil. These are required to cater for the effective change in the camber of the main aerofoil that is caused by the aerofoil boundary layer (see section 2.1 in connexion with the flap boundary layer). We propose to neglect this effect on the basis of the observation that this correction to the camber is small compared with the corresponding change in the effective camber of the flap, at least for the configurations studied here. Should it be considered necessary the present analysis can be modified fairly easily to include this effect simply by amending equation (27) to allow for the correction to the effective camber of the main aerofoil.

Whilst neglecting the additional vortices due to the boundary layer of the main aerofoil we retain the point source so as to ensure the correct value of the displacement flux in the wake. In fact, as we shall see, it appears that, in the cases considered, this effect is of secondary importance compared with the effect of distributed vortices and sources of the wake in the determination of the correction to the flap lift.

A similar analysis applied to

$$u_i(x,z) = \frac{\partial(\Delta\psi_W)_\Gamma}{\partial z}(x,z)$$

gives the result

$$\begin{aligned} u_i(x,z) = {} & \frac{1}{2\pi}\int_{x_T}^{\infty} \gamma_W(x_W)\,\frac{z - z_W}{(x - x_W)^2 + (z - z_W)^2}\,dx_W \\ & + \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^*}{dx_W}\,\frac{x - x_W}{(x - x_W)^2 + (z - z_W)^2}\,dx_W \\ & + \frac{1}{2\pi}\,\psi_W^*(x_T)\,\frac{x - x_T}{(x - x_T)^2 + (z - z_T)^2} \quad . \end{aligned} \tag{43}$$

In section 2 we employed the assumption that the flap gap is small compared with the flap chord. This implies that z_W is small compared with c_F

except far downstream where the wake is remote and consequently has little effect on the flow around either the flap or the main aerofoil. Therefore, supposing that all lengths in equation (40) are normalized with respect to c_F, we replace z_W by ε, a small parameter, and we then take the limit as ε tends to zero. In consequence, we obtain, for points on the flap chord and its downstream extension, the result

$$\begin{aligned} w_i(x,0) = &- \frac{1}{2\pi} \lim_{\varepsilon\to 0} \int_{x_T}^{\infty} \gamma_W(x_W) \frac{x - x_W}{(x - x_W)^2 + \varepsilon^2} dx_W \\ &- \frac{1}{2\pi} \lim_{\varepsilon\to 0} \int_{x_T}^{\infty} \frac{d\psi_W^*}{dx_W} \frac{\varepsilon}{(x - x_W)^2 + \varepsilon^2} dx_W \\ &- \frac{1}{2\pi} \psi_W^*(x_T) \frac{z_T}{(x - x_T)^2 + z_T^2} \quad . \end{aligned}$$

In the case of the first integral the limit may be taken through the integral sign provided that the integral is interpreted according to the Cauchy principal value. The second integral may be evaluated by employing equations (B-1) and (B-2). Thus we have finally

$$w_1(x,0) = \frac{1}{2\pi} \int_{x_T}^{\infty} \gamma_W(x_W) \frac{dx_W}{x_W - x} - \tfrac{1}{2}H(x - x_T) \frac{d\psi_W^*}{dx} - \frac{1}{2\pi} \psi_W^*(x_T) \frac{z_T}{(x - x_T)^2 + z_T^2} \quad . \qquad \text{.... (44)}$$

It will be seen that, in this approximation, the vortices are effectively transferred to the flap chord and its downstream extension. We note, as well, that the upwash induced by the distributed sources depends only on the local strength of the sources. This result, which is a consequence of the assumption that the distributed sources lie just above the flap chord, seems likely to be an accurate approximation only if ψ_W^* varies slowly with x; hence it may be a questionable approximation close to the singular points of the first inviscid approximation such as the shroud trailing edge and the trailing edge of the flap. Finally, it should be remarked that we have refrained from approximating the point source term for small z_T (i.e. for 'small' flap gap); the reason for this is that this approximation evidently fails for $x = x_T$.

On applying the same approximation to u_i (equation (43)) we obtain, for points on the flap chord, the expression

$$u_i(x,0) = -\tfrac{1}{2}H(x - x_T)\gamma_W(x) + \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi^*_W}{dx_W}(x_W)\,\frac{dx_W}{x - x_W}$$

$$+ \frac{1}{2\pi}\,\psi^*_W(x_T)\,\frac{x - x_T}{(x - x_T)^2 + z_T^2} \quad . \tag{45}$$

Consistent with the 'thin'-wake approximation and with the assumption that the wake lies just above the flap chord is the approximation

$$\frac{\partial(\ )}{\partial n}\frac{d\ell}{dx} = \frac{\partial(\ )}{\partial z} \quad . \tag{46}$$

Therefore we may write in place of equation (41) the approximate result

$$\gamma_W = \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_+ - \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_- \quad . \tag{47}$$

By using approximation (46) it is shown in Appendix C that, for a 'thin' wake that is close to the flap chord,

$$\gamma_W = \kappa_W\bar{U}(\delta^*_W + \theta_W) \quad . \tag{48}$$

Here κ_W = the weighted mean curvature of the streamlines of the wake, as defined in Appendix C, the centre of curvature being taken below the wake;

δ^*_W = the wake displacement thickness;

θ_W = the wake momentum thickness; and

$$\bar{U} = \frac{1}{2}\left\{\left(\frac{\partial\psi}{\partial z}\right)_+ + \left(\frac{\partial\psi}{\partial z}\right)_-\right\} ,$$

that is the mean of the x-wise velocities at the upper and lower edges of the wake.

Equation (48) is due essentially to Spence and Beasley[4], who, however, used the approximation $\bar{U} = V_\infty$ and employed a different method in their derivation of the relationship.

To ensure that the flap remains a streamline in the presence of the wake of the main aerofoil we will need a distribution of vorticity within the flap. With the assumption of section 2 that the flap is of small thickness-chord ratio and camber this distribution is replaced by a distribution of vortices on the flap chord. Likewise, the boundary condition that the main aerofoil is a streamline is maintained by a distribution of vortices on the aerofoil chord. Consistent with the analysis leading to equation (24) we assume that, for small flap gaps, overlaps and flap angles, these two distributions may be combined into one distribution that is placed on the ξ axis. This distribution is defined by

$$\Delta\gamma_{,W}(\xi) = \begin{cases} \Delta\gamma_{A,W}(\xi) \; ; & 0 \leqslant \xi < c_A \\ \Delta\gamma_{F,W}(\xi) \; ; & c_A \leqslant \xi \leqslant c_E \end{cases} .$$

Thus by employing equation (44) for the upwash induced by the wake of the main aerofoil at the flap chord and noting that, for small flap angles and overlaps,

$$x = \xi - c_A \quad , \tag{49}$$

we find, by using the fact that

$$\xi_T = c_A \quad ,$$

that the flap remains a streamline under the action of the wake provided that

$$\frac{1}{2\pi}\int_{c_A}^{\infty} \gamma_W(x_W)\,\frac{d\xi_W}{\xi_W - \xi} - \tfrac{1}{2}H(\xi - c_A)\,\frac{d\psi_W^*}{d\xi}(x) - \frac{1}{2\pi}\,\psi_W^*(x_T)\,\frac{z_T}{(\xi - c_A)^2 + z_T^2}$$

$$= \frac{1}{2\pi}\int_0^{c_E} \Delta\gamma_{,W}(\xi')\,\frac{d\xi'}{\xi - \xi'} \quad , \qquad c_A \leqslant \xi \leqslant c_E \quad . \tag{50}$$

To satisfy the condition that the boundary of the main aerofoil is a streamline of the real flow we require information on the velocity induced in the ζ direction at the chord of the main aerofoil by the wake. For small flap angles this velocity is approximately equal to $w_i(x,z_T)$ in the interval

$0 \leqslant \xi < c_A$. According to the analysis of section 2.2 the error in this (small angle) approximation is likely to be small for flap angles less than 30° except possibly in a narrow region adjacent to the trailing edge of the main aerofoil.

With $v_{\zeta i}(\xi,\zeta)$ the velocity induced by the wake of the main aerofoil in the ζ direction we find, by reference to the 'thin'-wake approximation, equation (40), that

$$v_{\zeta i}(\xi,0) = w_i(x,z_T) = -\frac{1}{2\pi}\int_0^{\infty} \gamma_W(x_W) \frac{x - x_W}{(x - x_W)^2 + (z_T - z_W)^2}\, dx_W$$

$$+ \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^*}{dx_W}(x_W) \frac{z_T - z_W}{(x - x_W)^2 + (z_T - z_W)^2}\, dx_W \;, \begin{cases} x < x_T \\ \xi < c_A \end{cases} . \quad (51)$$

Again for small flap angles we would expect that $|z_T - z_W|$ is small compared with c_F. Therefore, supposing all lengths in equation (51) are scaled with respect to c_F, we replace $z_T - z_W$ by ε and take the limit as ε tends to zero. Referring to equations (B-1) and (B-2) we find that

$$v_{\zeta i}(\xi,0) = \frac{1}{2\pi}\int_{x_T}^{\infty} \gamma_W(x_W) \frac{dx_W}{x_W - x} \;, \qquad \xi < c_A \;, \quad (52)$$

the Cauchy principal value of the integral being taken. This expression implies that only the wake vortices contribute directly to the velocity normal to the chord of the main aerofoil at the aerofoil chord. This normal velocity is nullified by the vortex distribution $\Delta\gamma_{,W}(\xi)$, thus ensuring that the boundary of the main aerofoil is a streamline of the real flow. Hence we have, upon employing equation (49) and noting that $\xi_T = c_A$,

$$\frac{1}{2\pi}\int_{c_A}^{\infty} \gamma_W(x_W) \frac{d\xi_W}{\xi_W - \xi} = \frac{1}{2\pi}\int_0^{c_E} \Delta\gamma_{,W}(\xi') \frac{d\xi'}{\xi - \xi'} \;, \qquad 0 \leqslant \xi < c_A \;. \quad (53)$$

where $s_{\mu\nu}^{(4)}$, a Weber coefficient, is defined in Ref.15, and ξ_μ and ξ_ν are pivotal points defined by

$$\theta_\mu(\xi_\mu) = \frac{\mu\pi}{N} .$$

Strictly, Weber's method is only applicable if $G(\xi)$ is continuous and differentiable. As is evident in equation (95) this condition is not satisfied in our case, there being a finite discontinuity in $dG/d\xi$ at $\xi = c_A$. This type of discontinuity is met in the linearized theory of aerofoils with plain flaps[18], and it evidently results in a logarithmic type of singularity in $\Delta\gamma_{,B}$ at the point of discontinuity. Since this is a weak singularity it seems unlikely that the failure to represent the discontinuity will cause serious errors in the lift of the flap. To check the validity of this assertion we have performed some calculations for the distribution

$$F(\xi) = \xi - c_A \tag{99}$$

which gives a finite discontinuity in $dG/d\xi$ at $\xi = c_A$.

The second integral of equation (94) is readily evaluated in this case[18] and it is found that the corresponding vortex strength is given by

$$\Delta\gamma_{,B}(\xi) = \frac{\chi}{\pi}\tan\frac{\theta}{2} + \frac{1}{\pi}\ln\left|\frac{\sin(\theta+\chi)/2}{\sin(\theta-\chi)/2}\right| , \tag{100}$$

with

$$\cos\chi = \frac{2c_A}{c_E} - 1 .$$

The approximate summation equivalent of equation (100) is derived from equation (98) as follows:

$$\Delta\gamma_{,B}(\xi_\nu) = \left(\frac{c_E - c_A}{c_E}\right)\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(4)}\left\{(\xi_\mu - c_A)H(\xi_\mu - c_A) - (c_E - c_A)\xi_\mu/c_E\right\} . \tag{101}$$

Results, calculated by employing the 'exact' equation (100) and the approximate equation (101) for $\chi = \pi/3$ and with $N = 32$, are exhibited in Table 1 for various values of ν. Generally, the agreement between the two sets

Here the first term on the right-hand side of equation (56) is the inversion of the first term on the right-hand side of equation (55); the second is the inversion of the last term in equation (55); and the third is the inversion of the second term on the left-hand side of equation (54).

In the absence of a detailed viscous solution for the flow around the flap in the presence of the wake we assume that the combined flow of the first inviscid approximation and the wake flow satisfies the Kutta condition of smooth flow at the trailing edge of the flap. Since the first inviscid approximation satisfies the Kutta condition this implies that

$$\Delta\gamma_{,W}(c_E) = 0 \quad . \qquad (57)$$

Weber[15] has considered the implications of this condition for equations similar to equation (54). It appears that, if the left-hand side of equation (54) is discontinuous at a finite number of positions and an analytic function between, in the interval $0 \leqslant \xi \leqslant c_E$, condition (57) is satisfied. This requirement is satisfied by the second term on the left-hand side of equation (54). In the case of the first term, however, there is a logarithmic singularity at $\xi = c_E$ resulting from the discontinuity in γ_W at the flap trailing edge. This discontinuity is due to the inclusion of the vortices of the flap wake with the vortices of the wake of the main aerofoil downstream of the flap trailing edge. We may overcome this difficulty by fairing the two distributions of γ_W upstream and downstream of $\xi = c_E$ smoothly into one another. Alternatively we could relax condition (57) and permit finite values of $\Delta\gamma_{,W}(c_E)$, which the logarithmic singularity implies. Regardless of which method is chosen, the fact that γ_W is smooth and continuous elsewhere, ensures that $\Delta\gamma_{,W}(c_E) < \infty$. Hence the arbitrary constant B in equation (56) is required to be zero.

Spence[17] has considered integrals like the first one on the right-hand side of equation (56); he shows that

$$\frac{1}{\pi^2}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \left(\int_{c_E}^{\infty} \gamma_W(x_W)\, \frac{d\xi_W}{\xi_W - \xi}\right) \frac{d\xi'}{\xi' - \xi}$$

$$= \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi_W}{\xi_W - c_E}\right)^{\frac{1}{2}} \gamma_W(x_W)\, \frac{d\xi_W}{\xi_W - \xi} \quad . \qquad (58)$$

For the sake of completeness, the analysis leading to this result is given in Appendix D.

Hence, using equation (58) and recalling that $B = 0$, we have in place of equation (56)

$$\Delta\gamma_{,W}(\xi) = - H(\xi - c_A)\gamma_W(x)$$

$$+ \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi_W}{\xi_W - c_E}\right)^{\frac{1}{2}} \gamma_W(x_W) \frac{d\xi_W}{\xi_W - \xi}$$

$$- \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} H(\xi' - c_A) \left\{\frac{d\psi_W^*}{d\xi'}(x')\right.$$

$$\left. + \frac{1}{\pi}\psi_W^*(x_T)\frac{z_T}{(\xi' - c_A)^2 + z_T^2}\right\} \frac{d\xi'}{\xi' - \xi} , \qquad 0 \leqslant \xi \leqslant c_E \quad . \qquad (59)$$

3.2 Effect of wake thickness

In this section we derive a first-order correction to the results given in section 3.1 for the effect of non-zero wake thickness as well as the influence of the non-zero distance of the wake from the flap chord. As previously we suppose that the flap angle, β, is small (say less than 30^o) and consequently we assume that BA is normal to the flap chord (see Fig.5). Hence, by using this approximation together with equations (37), (38a and b) and (39), we may write for the upwash induced by the wake of the main aerofoil at a line parallel to, and a distance ε below, the flap chord.

$$w_i(x,-\varepsilon) = -\frac{1}{2\pi}\int_{x_T}^{\infty}\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_+ \frac{x - x'}{(x-x')^2 + (\varepsilon + z'_+)^2}\,dx'$$

$$+ \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{d(\Delta\psi)_+}{dx'}\,\frac{\varepsilon + z'_+}{(x-x')^2 + (\varepsilon + z'_+)^2}\,dx'$$

$$+ \frac{1}{2\pi}\int_{x_T}^{\infty}\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_- \frac{x - x'}{(x-x')^2 + (\varepsilon + z'_-)^2}\,dx'$$

$$- \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{d(\Delta\psi)_-}{dx'}\,\frac{\varepsilon + z'_-}{(x-x')^2 + (\varepsilon + z'_-)^2}\,dx'$$

$$+ \frac{1}{2\pi}\int_{z_A}^{z_B}\frac{d(\Delta\psi)_\ell}{dz'}\,\frac{\varepsilon + z'}{(x-x_T)^2 + (\varepsilon + z')^2}\,dz' \quad , \qquad (60)$$

where suffixes A and B refer to the points A and B.

It is reasonable to expect that, if the wake width and the flap gap are small compared with the flap chord, z'_+ and z'_- will be small in comparison with c_F. Likewise, on segment AB, $z' - z_T$ will be small compared with c_F. Therefore, supposing the dimensions in equation (60) are referred to the flap chord and expanding the first four integrals in powers of z'_+ or z'_- and the last integral in powers of $z' - z_T$, we have, to first order in these quantities

$$
\begin{aligned}
w_i(x,-\varepsilon) = &-\frac{1}{2\pi}\int_{x_T}^{\infty}\left\{\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_+ - \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_-\right\}\frac{x-x'}{(x-x')^2+\varepsilon^2}\,dx' \\
&+\frac{1}{2\pi}\int_{x_T}^{\infty}\left\{\frac{d(\Delta\psi)_+}{dx'}-\frac{d(\Delta\psi)_-}{dx'}\right\}\frac{\varepsilon}{(x-x')^2+\varepsilon^2}\,dx' \\
&+\frac{1}{2\pi}\int_{z_A}^{z_B}\frac{d(\Delta\psi)_\ell}{dz'}\,\frac{\varepsilon+z_T}{(x-x_T)^2+(\varepsilon+z_T)^2}\,dz' \\
&+\frac{1}{\pi}\int_{x_T}^{\infty}\left\{\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_+ z'_+ - \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_- z'_-\right\}\frac{\varepsilon(x-x')}{\{(x-x')^2+\varepsilon^2\}^2}\,dx' \\
&+\frac{1}{2\pi}\int_{x_T}^{\infty}\left\{\frac{d(\Delta\psi)_+}{dx'}z'_+ - \frac{d(\Delta\psi)_-}{dx'}z'_-\right\}\left(\frac{1}{(x-x')^2+\varepsilon^2}-\frac{2\varepsilon^2}{\{(x-x')^2+\varepsilon^2\}^2}\right)dx' \\
&+\frac{1}{2\pi}\int_{z_A}^{z_B}\frac{d(\Delta\psi)_\ell}{dz'}\,(z'-z_T)\left(\frac{1}{(x-x_T)^2+(\varepsilon+z_T)^2}-\frac{2(\varepsilon+z_T)^2}{\{(x-x_T)^2+(\varepsilon+z_T)^2\}^2}\right)dz'.
\end{aligned}
$$

.... (61)

Here it will be observed we have not approximated the last integral of equation (60) for small z_T, as might be thought necessary for consistency with the approximations to the first four integrals. The reason for this is that we wish to recover the approximation of section 3.1, wherein the point source at the leading edge of the wake was placed at the shroud trailing edge rather than on the flap chord.

It is convenient to express the right-hand side of equation (61) as

$$
w_i(x,-\varepsilon) = w_i^{(1)}(x,-\varepsilon) + \Delta w_i(x,-\varepsilon) \quad ,
$$

where $w_i^{(1)}(x,-\varepsilon)$ is a first approximation to $w_i(x,-\varepsilon)$ for 'small' wake width and flap gap represented by the first three integrals of equation (61), and $\Delta w_i(x,-\varepsilon)$ is a first-order correction term for non-zero wake width and flap gap given by the last three integrals of the same equation.

Considering firstly $w_i^{(1)}$; by employing equations (41) and (42) it is readily found that

$$w_i^{(1)}(x,-\varepsilon) = -\frac{1}{2\pi}\int_{x_T}^{\infty} \gamma_W(x')\,\frac{x-x'}{(x-x')^2+\varepsilon^2}\,dx' - \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^*}{dx'}\,\frac{\varepsilon}{(x-x')^2+\varepsilon^2}\,dx'$$

$$-\frac{1}{2\pi}\,\psi_W^*(x_T)\,\frac{\varepsilon+z_T}{(x-x_T)^2+(\varepsilon+z_T)^2} \quad . \qquad (62)$$

The corresponding upwash at the flap chord is obtained by taking the limit of the right-hand side of equation (62) as ε tends to zero. Note that we approach the flap chord from below the flap to ensure that, in the limit, the wake lies above the flap chord. Using equations (B-1) and (B-2) we find that

$$w_i^{(1)}(x,0) = \frac{1}{2\pi}\int_{x_T}^{\infty} \gamma_W(x')\,\frac{dx'}{x'-x} - \tfrac{1}{2}H(x-x_T)\,\frac{d\psi_W^*}{dx} - \frac{1}{2\pi}\,\psi_W^*(x_T)\,\frac{z_T}{(x-x_T)^2+z_T^2} \quad .$$

Evidently, this expression is in exact agreement with the approximation for $w_i(x,0)$ that was derived in section 3.1, namely equation (44). Therefore, in correcting the results of section 3.1 for non-zero wake width and distance of the wake from the flap chord, we need only examine $\Delta w_i(x,-\varepsilon)$, which is given by

$$\Delta w_i(x,-\varepsilon) = \frac{1}{\pi}\int_{x_T}^{\infty} P(x')\,\frac{\varepsilon(x-x')}{\left\{(x-x')^2+\varepsilon^2\right\}^2}\,dx'$$

$$+\frac{1}{2\pi}\int_{x_T}^{\infty} Q(x')\left(\frac{1}{(x-x')^2+\varepsilon^2} - \frac{2\varepsilon^2}{\left\{(x-x')^2+\varepsilon^2\right\}^2}\right)dx'$$

$$+\frac{1}{2\pi}\left(\int_{z_A}^{z_B} \frac{d(\Delta\psi)_\ell}{dz'}\,(z'-z_T)\,dz'\right)\left(\frac{1}{(x-x_T)^2+(\varepsilon+z_T)^2} - \frac{2(\varepsilon+z_T)^2}{\left\{(x-x_T)^2+(\varepsilon+z_T)^2\right\}^2}\right) ,$$

.... (63)

where

$$\left.\begin{aligned} P &= \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_+ z'_+ - \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx'}\right)_- z'_- \\ Q &= \frac{d(\Delta\psi)_+}{dx'} z'_+ - \frac{d(\Delta\psi)_-}{dx'} z'_- \end{aligned}\right\} . \tag{64}$$

Equation (63) may be recast into a more suitable form by carrying out the integration of the first two integrals by parts. Thus the first integral becomes

$$\frac{1}{\pi}\int_{x_T}^{\infty} P(x')\frac{\varepsilon(x - x')}{\{(x - x')^2 + \varepsilon^2\}^2}\,dx' = \frac{1}{2\pi}\left[P(x')\frac{\varepsilon}{(x - x')^2 + \varepsilon^2}\right]_{x_T}^{\infty} - \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dP}{dx'}\frac{\varepsilon}{(x - x')^2 + \varepsilon^2}\,dx' . \tag{65}$$

The evaluation of the right-hand side of equation (65) is complicated by the problem of determining the upper limit of the first term. This involves the determination of the limit of $P(x)$ as x tends to infinity. This is considered in Appendix E where it is argued that

$$\lim_{x\to\infty}(P(x)) = 0 .$$

Thus, combining this result with equation (65), we find that

$$\frac{1}{\pi}\int_{x_T}^{\infty} P(x')\frac{\varepsilon(x - x')}{\{(x - x')^2 + \varepsilon^2\}^2}\,dx' = -\frac{1}{2\pi}P(x_T)\frac{\varepsilon}{(x - x_T)^2 + \varepsilon^2} - \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dP}{dx'}\frac{\varepsilon}{(x - x')^2 + \varepsilon^2}\,dx' . \tag{66}$$

The second integral of equation (63) may be integrated by parts to give the result

$$\frac{1}{2\pi}\int_{x_T}^{\infty} Q(x')\left(\frac{1}{(x-x')^2+\varepsilon^2}-\frac{2\varepsilon^2}{\{(x-x')^2+\varepsilon^2\}^2}\right)dx' = -\frac{1}{2\pi}\left[Q(x')\,\frac{x'-x}{(x'-x)^2+\varepsilon^2}\right]_{x_T}^{\infty}$$

$$+\frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dQ}{dx'}\,\frac{x'-x}{(x'-x)^2+\varepsilon^2}\,dx' \; . \qquad \ldots\ldots (67)$$

As with equation (65) we are faced with the problem in equation (67) of evaluating the upper limit of the first term. However, arguments are given in Appendix E in favour of the result

$$\lim_{x\to\infty}(Q(x)) = 0 \; .$$

Therefore we have in place of equation (67)

$$\frac{1}{2\pi}\int_{x_T}^{\infty} Q(x')\left(\frac{1}{(x-x')^2+\varepsilon^2}-\frac{2\varepsilon^2}{\{(x-x')^2+\varepsilon^2\}^2}\right)dx' = \frac{1}{2\pi}\,Q(x_T)\,\frac{x_T-x}{(x_T-x)^2+\varepsilon^2}$$

$$+\frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dQ}{dx'}\,\frac{x'-x}{(x'-x)^2+\varepsilon^2}\,dx' \; . \qquad \ldots\ldots (68)$$

The final result for $\Delta w_i(x,-\varepsilon)$ is obtained by combining equations (63), (66) and (68). The corresponding expression for $\Delta w_i(x,0)$ is then found by taking the limit of $\Delta w_i(x,-\varepsilon)$ as ε tends to zero. Referring to equations (B-1) and (B-2) we find that

$$\Delta w_i(x,0) = -\tfrac{1}{2}H(x-x_T)\,\frac{dP}{dx}+\frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dQ}{dx'}\,\frac{dx'}{x'-x}+\frac{1}{2\pi}\,Q(x_T)\,\frac{1}{x_T-x}$$

$$+\frac{1}{2\pi}\left(\int_{z_A}^{z_B}\frac{d(\Delta\psi)_\ell}{dz'}\,(z'-z_T)\,dz'\right)\frac{(x-x_T)^2-z_T^2}{\{(x-x_T)^2+z_T^2\}^2} \; . \qquad (69)$$

Here it is possible to identify two types of terms. The first two on the right-hand side may be identified as first-order corrections for non-zero wake thickness and flap gap. The last two, on the other hand, which are due to a point vortex on the flap chord at $x = x_T$ and a doublet at the shroud trailing edge, exist as a consequence of the neglect of the boundary layer of the main aerofoil. In a more complete representation, which included the boundary layer of the main aerofoil, these isolated singularities would be replaced by continuous distributions of sources and vortices on the chord of the main aerofoil. A similar point has already been made in section 3.1 in connexion with the first approximation for $w_i(x,0)$. In this approximation the displacement effect of the boundary layer of the main aerofoil is simulated by a point source at the shroud trailing edge instead of the more usual source distribution on the aerofoil chord. Since we are primarily concerned with the thickness effect of the aerofoil wake we will ignore the last two terms of equation (69), so that we have

$$\Delta w_i(x,0) = -\tfrac{1}{2}H(x - x_T)\frac{dP}{dx} + \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{dQ}{dx'}\frac{dx'}{x' - x} \quad . \tag{70}$$

An alternative way of deriving equation (70) is to note that, consistent with the neglect of the thickness effect of the boundary layer of the main aerofoil, it is permissible to assume that $z_A = z_B = 0$. This implies that

$$Q(x_T) = 0 \quad ,$$

as may be verified by examining equation (64); thus the vortex at $(x_T,0)$ is of zero strength. Additionally, equation (69) shows that this assumption leads to the doublet at the shroud trailing edge being of zero strength.

An examination of equation (66) shows that the first term on the right-hand side of equation (70) is due to a source distribution placed just above the flap upper surface. The last term of equation (70) arises from a vortex distribution on the flap chord and its downstream extension. The fact that distributions of this type appear in the first approximation for $w_i(x,0)$ suggests that it will be convenient in the second approximation to define effective vortex and source strengths. Thus, after correcting the first approximation for $w_i(x,0)$, namely equation (44), by using equation (70), we find that to a second approximation

$$w_i(x,0) = \frac{1}{2\pi}\int_{x_T}^{\infty} \bar{\gamma}_W \frac{dx'}{x' - x} - \tfrac{1}{2}H(x - x_T)\frac{d\bar{\psi}^*_W}{dx} - \frac{1}{2\pi}\psi^*_W(x_T)\frac{z_T}{(x - x_T)^2 + z_T^2} \quad , \quad (71)$$

where

$$\bar{\gamma}_W = \gamma_W + \frac{dQ}{dx'} \quad ; \qquad \frac{d\bar{\psi}^*_W}{dx} = \frac{d\psi^*_W}{dx} + \frac{dP}{dx} \qquad (72)$$

are the effective vortex and source strengths per unit x-wise length, of the second approximation. Here, we note, the strength of the isolated source is $\psi^*_W(x_T)$ and not $\bar{\psi}^*_W(x_T)$ as might have been expected. This can be explained by our use of the small-gap approximation $(z_T \ll c_F)$ to arrive at equation (69). This approximation evidently fails to include the contribution of a point source, of strength $P(x_T)$, that is situated at the shroud trailing edge. However, we have previously neglected the effect of the non-zero thickness of the boundary layer of the main aerofoil; consequently, it is consistent to assume that $P(x_T)$ is zero. As may be inferred from equation (72) this implies that we may write $\psi^*_W(x_T)$ in place of $\bar{\psi}^*_W(x_T)$.

Substituting the effective vortex and source strengths for γ_W and $d\psi^*_W/dx$ in equation (54) we are able to define a second approximation to $\Delta\gamma_{,W}(\xi)$, $\overline{\Delta\gamma}_{,W}(\xi)$, such that

$$\frac{1}{2\pi}\int_{c_A}^{\infty} \bar{\gamma}_W \frac{d\xi_W}{\xi_W - \xi} - \tfrac{1}{2}H(\xi - c_A)\left\{\frac{d\bar{\psi}^*_W}{d\xi} + \frac{1}{\pi}\psi^*_W(x_T)\frac{z_T}{(\xi - c_A)^2 + z_T^2}\right\}$$
$$= \frac{1}{2\pi}\int_0^{c_E} \overline{\Delta\gamma}_{,W}(\xi')\frac{d\xi'}{\xi - \xi'} \quad , \qquad 0 \leqslant \xi \leqslant c_E \quad . \quad (73)$$

By following the analysis between equations (54) and (59) we are able to write the solution of equation (73) as

$$\overline{\Delta\gamma}_{,W}(\xi) = - H(\xi - c_A)\bar{\gamma}_W$$

$$+ \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi_W}{\xi_W - c_E}\right)^{\frac{1}{2}} \bar{\gamma}_W \frac{d\xi_W}{\xi_W - \xi}$$

$$- \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} H(\xi' - c_A) \left\{\frac{d\bar{\psi}^*_W}{d\xi'}(x')\right.$$

$$\left. + \frac{1}{\pi} \psi^*_W(x_T) \frac{z_T}{(\xi' - c_A)^2 + z_T^2}\right\} \frac{d\xi'}{\xi' - \xi} \quad , \quad 0 \leqslant \xi \leqslant c_E \quad . \tag{74}$$

The effective vortex and source strength concept may also be used to determine the second approximation to $u_i(x,0)$. Thus instead of equation (45) we have the expression

$$u_i(x,0) = - \tfrac{1}{2}H(x - x_T)\bar{\gamma}_W + \frac{1}{2\pi} \int_{x_T}^{\infty} \frac{d\bar{\psi}^*_W}{dx_W} \frac{dx_W}{x - x_W}$$

$$+ \frac{1}{2\pi} \psi^*_W(x_T) \frac{x - x_T}{(x - x_T)^2 + z_T^2} \quad . \tag{75}$$

It is necessary to make two further observations regarding the corrections given in equations (72). The first concerns the fact that these corrections allow for the non-zero distance of the lower edge of the aerofoil wake from the flap chord as well as non-zero wake thickness. It is desirable, therefore, to distinguish between the two effects.

In the second observation we note that the correction terms of equation (72), dP/dx and dQ/dx, are not known until $\Delta\psi$ has been determined. Evidently, therefore, equation (74) is implicit in character. On the other hand, as Δw_i is a first-order correction for 'small'-wake thickness and flap gap, it seems reasonable to evaluate P and Q by using the approximation

$$\Delta\psi = \Delta\psi^{(1)} = \Delta\psi_W^{(1)} + \Delta\psi_B \quad , \tag{76}$$

where suffix (1) refers to the first approximation for 'small'-wake thickness and flap gap of section 3.1. Additionally, since, in the 'small'-gap approximation, the wake is assumed to lie just above the flap chord, we may use equation (46) to write

$$\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx}\right)_- = \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_- ,$$

$$= \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) \quad , \qquad (77a)$$

for a 'thin' wake situated just above the flap chord. Similarly,

$$\left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx}\right)_+ = \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_+ ,$$

$$= \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) + \gamma_W^{(1)} \quad . \qquad (77b)$$

Here we have used equation (47) to arrive at equation (77b) via equation (77a) and, for consistency, we have replaced γ_W by

$$\gamma_W^{(1)} = \kappa_W \bar{U}(\delta_W^* + \theta_W) \qquad (78)$$

the first approximation to γ_W for a 'thin' wake and 'small' flap gap (see equation (48)). Likewise, referring to equation (76), we have

$$\frac{d(\Delta\psi)_-}{dx} = \frac{\partial(\Delta\psi^{(1)})}{\partial x}(x,+0) \quad ; \qquad (79a)$$

$$\frac{d(\Delta\psi)_+}{dx} = \frac{\partial(\Delta\psi^{(1)})}{\partial x}(x,+0) - \frac{d\psi_W^*}{dx} \quad , \qquad (79b)$$

where equation (79b) is derived by combining equations (42) and (79a).

Equations (79a and b) may be written in a more suitable form by observing that, consistent with the approximations of section 2.1, we may write

$$\frac{\partial(\Delta\psi_B)}{\partial x}(x,+0) = \frac{d(\Delta\psi_B)_U}{dx} ,$$

$$= -\frac{d\psi_U^*}{dx} , \qquad (80)$$

from equation (11). Therefore equations (76) and (80) may be used to give the expression

$$\frac{\partial(\Delta\psi^{(1)})}{\partial x}(x,+0) = \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{d\psi_U^*}{dx} . \qquad (81)$$

Hence equations (64), (79a and b) and (81) may be combined to give the result

$$Q = -\frac{d\psi_W^*}{dx} z_+ + \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0)(z_+ - z_-) - \frac{d\psi_U^*}{dx}(z_+ - z_-) .$$

Therefore, by reference to equations (72), we find that

$$\bar{\gamma}_W = \gamma_W - \frac{d}{dx}\left(\frac{d\psi_W^*}{dx} z_+\right) + \frac{d}{dx}\left(\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0)(z_+ - z_-)\right)$$

$$- \frac{d}{dx}\left(\frac{d\psi_U^*}{dx}(z_+ - z_-)\right) . \qquad (82)$$

Using equation (82) it is shown in Appendix C that, for the wake of the main aerofoil,

$$\bar{\gamma}_W = \kappa_W \bar{U}(\delta_W^* + \theta_W) - \frac{1}{2}\frac{d^2\psi_W^*}{dx^2}(z_+ + z_-) . \qquad (83)$$

Here, it will be seen, the first term on the right-hand side is the 'thin'-wake approximation for 'small' flap gaps $\gamma_W^{(1)}$. As in section 3.1, downstream of the flap trailing edge we include the vortex strength of the flap wake in the expression for $\bar{\gamma}_W$. The vortex strength of the flap wake is calculated by using equation (48) which is valid for a 'thin' wake that either lies close to the x axis or has a slowly-varying displacement flux (see Appendix C). The flap wake and the wake of the main aerofoil are in close proximity; hence, as in the method of section 3.1, it is reasonable to combine this vortex strength with the first term of equation (83). Specifically, for points downstream of the flap trailing edge, we write

$$\bar{\gamma}_W = \left|\kappa_W \bar{U}(\delta^*_W + \theta_W)\right|_{\left(\substack{\text{aerofoil} + \\ \text{flap}}\right)} - \left|\frac{1}{2}\frac{d^2\psi^*_W}{dx^2}(z_+ + z_-)\right|_{\text{aerofoil}} \quad , \quad x \geqslant c_F \quad . \qquad (84)$$

It is relevant to note that, if the wake of the main aerofoil is 'thin', so that

$$z_+ = z_- = z_W$$

equation (83) becomes

$$\bar{\gamma}_W = \kappa_W \bar{U}(\delta^*_W + \theta_W) - \frac{d^2\psi^*_W}{dx^2} z_W \quad .$$

Strictly speaking, therefore, the thickness correction to $\bar{\gamma}_W$ is

$$-\frac{d^2\psi^*_W}{dx^2}\left(\tfrac{1}{2}(z_+ + z_-) - z_W\right) \quad .$$

Consequently, if we assume that the rear dividing streamline of the main aerofoil coincides with the mean line of the wake there is no first-order thickness correction. It should be remarked here that the rear dividing streamline represents but one possible line on which the singularities of the 'thin'-wake theory could be placed. An equally reasonable suggestion is the mean line of the wake. Indeed, the indications of the present work are that, for the purpose of representing the vortex effect, the mean line is the optimum.

Using equations (64), (76) and (77a and b) we may write

$$P = \gamma_W^{(1)} z_+ + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0)(z_+ - z_-) \quad .$$

Thus it may be inferred from equations (72) that

$$\bar{\psi}^*_W = \psi^*_W + \gamma_W^{(1)} z_+ + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0)(z_+ - z_-) \quad . \qquad (85)$$

Equation (85) entails the determination of $(\partial(\Delta\psi^{(1)})/\partial z)(x,+0)$; an expression which permits this is given in section 3.3 in connexion with the derivation of the effect of the wake on the flap lift. However, in Appendix F, by using equation (85), we derive an expression which does not require the evaluation of this velocity, namely

$$\bar{\psi}^*_W = \psi^{*\prime}_W + \frac{\gamma^{(1)}_W}{2}(z_+ + z_-) \quad , \tag{86}$$

where $\psi^{*\prime}_W = \int_W (\bar{U} - u)dz$

is a pseudo displacement flux. We prefer to use equation (86), rather than equation (85), for the following reason: it is usual in 'thin'-wake analyses[1,2,3] to assume that the true displacement flux is equal to the pseudo displacement flux, which is more easily calculated or deduced from experiment. This assumption may be justified for a 'thin' wake by comparing equations (85) and (86). Therefore, in calculations based on the 'thin'-wake method of section 3.1, we will use the result

$$\psi^*_W = \psi^{*\prime}_W \quad . \tag{87}$$

Consequently the last term on the right-hand side of equation (86) may be regarded as a correction to the results of section 3.1 for non-zero wake thickness and non-zero distance of the aerofoil wake from the flap. This correction is evidently much simpler to evaluate than the correction term of equation (85).

If the wake is supposed 'thin' equation (86) becomes

$$\bar{\psi}^*_W = \psi^{*\prime}_W + \gamma^{(1)}_W z_W \quad . \tag{88}$$

Hence the thickness correction to $\psi^{*\prime}_W$ is

$$\gamma^{(1)}_W \left\{ \frac{(z_+ + z_-)}{2} - z_W \right\} \quad .$$

This implies that, as with the vortex effect, the thickness correction may be ignored if, in the 'thin'-wake formulation, the singularities are placed on the mean line of the wake.

3.3 Effect of wake on flap lift

The lift acting on the flap is derived in section 2.3 and given in equation (30). Neglecting the squares and products of the correction terms in this equation it is possible to infer that

$$\Delta L_{F,W} \simeq \rho \cos(\beta+\alpha) \int_0^{c_F} \left(\frac{\partial\psi_I}{\partial z}(x,+0) \frac{\partial(\Delta\psi_W)}{\partial z}(x,+0) - \frac{\partial\psi_I}{\partial z}(x,-0) \frac{\partial(\Delta\psi_W)}{\partial z}(x,-0) \right) dx .$$

Since $(\partial(\Delta\psi_W)/\partial z)(x,\pm 0)$ is small compared with $(\partial\psi_I/\partial z)(x,\pm 0)$ only if the wake of the main aerofoil is 'thin', in general, this equation may be considered to be an approximation for a 'thin' wake. Therefore a consistent first approximation for the increment in the lift of the flap due to a 'thin' wake may be written as

$$\Delta L^{(1)}_{F,W} = \rho \cos(\beta+\alpha) \int_0^{c_F} \left(\frac{\partial\psi_I}{\partial z}(x,+0) \frac{\partial(\Delta\psi^{(1)}_W)}{\partial z}(x,+0) - \frac{\partial\psi_I}{\partial z}(x,-0) \frac{\partial(\Delta\psi^{(1)}_W)}{\partial z}(x,-0) \right) dx . \quad \text{.... (89)}$$

To evaluate this expression we require to know $(\partial(\Delta\psi^{(1)}_W)/\partial z)(x,\pm 0)$. This may be found by noting that the total x-wise velocity in the flow either side of $z = 0$, arising from the presence of the wake of the main aerofoil, consists of:

(a) the velocity induced by the vorticity within the aerofoil wake;

(b) the component due to the vortices required on the flap chord to satisfy the condition that the flap is a streamline of the wake flow;

(c) the x-wise velocity induced by the vortices needed on the chord of the main aerofoil to ensure that the aerofoil is also a streamline of the wake flow.

If the distance of the wake from the flap is small compared with the flap chord, contribution (a) is given by equation (45); contributions (b) and (c) may be obtained in a similar manner to that used to derive the contributions of $\Delta\gamma_{A,B}$ and $\Delta\gamma_{F,B}$ to the x-wise velocity at $z = \pm 0$ (equation (32)). Thus we have for small z_W/c_F

$$\frac{\partial(\Delta\psi_W^{(1)})}{\partial z}(x,\pm 0) = -\tfrac{1}{2}H(x - x_T)\gamma_W^{(1)} + \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^{*\prime}}{dx_W}\frac{dx_W}{x - x_W}$$

$$+ \frac{1}{2\pi}\psi_W^{*\prime}(x_T)\frac{x - x_T}{(x - x_T)^2 + z_T^2}$$

$$+ \frac{1}{2\pi}\int_0^{c_A} \Delta\gamma_{A,W}\frac{\sin\beta\,(c_A - \tilde{\ell} + x\cos\beta - \xi') - \cos\beta\,(g + x\sin\beta)}{(c_A - \tilde{\ell} + x\cos\beta - \xi')^2 + (g + x\sin\beta)^2}\,d\xi'$$

$$\pm \frac{\Delta\gamma_{F,W}}{2}, \qquad 0 \leqslant x \leqslant c_F \quad . \qquad (90)$$

Here we have used equation (87) to replace ψ_W^* by $\psi_W^{*\prime}$ and, consistent with the use of the 'thin'-wake approximation, we have replaced γ_W by $\gamma_W^{(1)}$.

Approximating the second integral of equation (90) in the manner used in approximating the similar integral in equation (32) (i.e. the integrand is expanded to order β, and g and $\tilde{\ell}$ are placed equal to zero) we have

$$\frac{\partial(\Delta\psi_W^{(1)})}{\partial z}(x,\pm 0) = -\tfrac{1}{2}H(x - x_T)\gamma_W^{(1)} + \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\psi_W^{*\prime}}{dx_W}\frac{dx_W}{x - x_W}$$

$$+ \frac{1}{2\pi}\psi_W^{*\prime}(x_T)\frac{x - x_T}{(x - x_T)^2 + z_T^2} + \frac{\beta}{2\pi}\int_0^{c_A} \Delta\gamma_{A,W}\frac{c_A - \xi'}{(c_A + x - \xi')^2}\,d\xi'$$

$$\pm \frac{\Delta\gamma_{F,W}}{2}, \qquad 0 \leqslant x \leqslant c_F \quad . \qquad (91)$$

To include the first-order corrections of section 3.2 for non-zero $(z_+ + z_-)/2$ we use equation (75) instead of equation (45) to obtain contribution (a) and replace $\Delta\gamma_{,W}$ by $\overline{\Delta\gamma}_{,W}$. We then have in place of equation (91)

$$\frac{\partial(\Delta\psi_W)}{\partial z}(x,\pm 0) = -\tfrac{1}{2}H(x - x_T)\bar{\gamma}_W + \frac{1}{2\pi}\int_{x_T}^{\infty}\frac{d\bar{\psi}_W^*}{dx_W}\frac{dx_W}{x - x_W}$$

$$+ \frac{1}{2\pi}\psi_W^{*\prime}(x_T)\frac{x - x_T}{(x - x_T)^2 + z_T^2} + \frac{\beta}{2\pi}\int_0^{c_A}\overline{\Delta\gamma}_{A,W}\frac{c_A - \xi'}{(c_A + x - \xi')^2}\,d\xi'$$

$$\pm \frac{\overline{\Delta\gamma}_{F,W}}{2}\ , \qquad 0 \leqslant x \leqslant c_F \quad . \qquad (92)$$

However, to proceed to higher-order approximations for $\Delta L_{F,W}$, including the effect of non-zero wake thickness, it may not be sufficient to replace the correction terms in equation (90) by their higher-order equivalents. The reason for this is that the terms neglected in deriving equation (89) may be of the same order of magnitude as the correction to $(\partial(\Delta\psi_W)/\partial z)(x,\pm 0)$ for the effect of non-zero thickness of the aerofoil wake. Thus, when including the effect of non-zero wake thickness in $(\partial(\Delta\psi_W)/\partial z)(x,\pm 0)$ we will retain the squares and products of the correction terms, containing this velocity correction, in the expression for $\Delta L_{F,W}$. By reference to equation (30) we see that this implies that we use instead of equation (89) the expression

$$\Delta L_{F,W}$$

$$= \rho\cos(\beta + \alpha)\int_0^{c_F}\Bigg(\left\{\frac{\partial\psi_I}{\partial z}(x,+0) + \frac{1}{2}\frac{\partial(\Delta\psi_W)}{\partial z}(x,+0) + \frac{\partial(\Delta\psi_B)}{\partial z}(x,+0)\right\}\frac{\partial(\Delta\psi_W)}{\partial z}(x,+0)$$

$$- \left\{\frac{\partial\psi_I}{\partial z}(x,-0) + \frac{1}{2}\frac{\partial(\Delta\psi_W)}{\partial z}(x,-0)\right.$$

$$\left. + \frac{\partial(\Delta\psi_B)}{\partial z}(x,-0)\right\}\frac{\partial(\Delta\psi_W)}{\partial z}(x,-0)\Bigg)\,dx \quad . \qquad (93)$$

It will be noted here that $\Delta L_{F,W}$ now contains a term associated with the flap boundary layer as well as the wake of the main aerofoil. For convenience, however, we have supposed this to be part of the wake correction.

4 CALCULATION PROCEDURE

4.1 Determination of $\Delta\gamma_{,B}$ and $\Delta\gamma_{,W}$

The vortex strengths $\Delta\gamma_{,B}$ and $\Delta\gamma_{,W}$ are, respectively, given by equations (27) and (59). Dealing with the first of these we find it is convenient to rewrite equation (27) as

$$\Delta\gamma_{,B}(\xi) = \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \frac{F(c_E)}{c_E} \frac{d\xi'}{\xi' - \xi}$$

$$+ \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \frac{dG}{d\xi'} \frac{d\xi'}{\xi' - \xi} , \qquad (94)$$

where

$$\frac{dG}{d\xi} = \frac{dF}{d\xi} H(\xi - c_A) - \frac{F(c_E)}{c_E} . \qquad (95)$$

Integrating equation (95) we obtain, with $G(0) = 0$,

$$G(\xi) = F(\xi)H(\xi - c_A) - \frac{F(c_E)\xi}{c_E} . \qquad (96)$$

Here we have used the fact that the displacement flux, ψ^*, vanishes at the attachment point of the flap, which point is assumed to be at $\xi = c_A$ (a reasonable assumption for 'small' flap gaps and overlaps). In consequence, $F = \psi^*_L - \psi^*_U$ also vanishes at this point.

The first integral in equation (94) is standard in aerofoil theory and is evaluated without difficulty with the aid of the trigonometric substitution

$$\cos\theta' = \frac{2\xi'}{c_E} - 1 . \qquad (97)$$

The second integral is evaluated numerically by means of the method that has been extensively applied by Weber[15] to integrals of this type for functions $G(\xi)$ satisfying the requirement

$$G(0) = G(c_E) = 0 .$$

Hence, noting from equation (96) that our function $G(\xi)$ does indeed satisfy this requirement, we obtain the result

$$\Delta\gamma_{,B}(\xi_\nu) = \frac{F(c_E)}{c_E}\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s^{(4)}_{\mu\nu} G(\xi_\mu) , \qquad (98)$$

where $s_{\mu\nu}^{(4)}$, a Weber coefficient, is defined in Ref.15, and ξ_μ and ξ_ν are pivotal points defined by

$$\theta_\mu(\xi_\mu) = \frac{\mu\pi}{N} .$$

Strictly, Weber's method is only applicable if $G(\xi)$ is continuous and differentiable. As is evident in equation (95) this condition is not satisfied in our case, there being a finite discontinuity in $dG/d\xi$ at $\xi = c_A$. This type of discontinuity is met in the linearized theory of aerofoils with plain flaps[18], and it evidently results in a logarithmic type of singularity in $\Delta\gamma_{,B}$ at the point of discontinuity. Since this is a weak singularity it seems unlikely that the failure to represent the discontinuity will cause serious errors in the lift of the flap. To check the validity of this assertion we have performed some calculations for the distribution

$$F(\xi) = \xi - c_A \tag{99}$$

which gives a finite discontinuity in $dG/d\xi$ at $\xi = c_A$.

The second integral of equation (94) is readily evaluated in this case[18] and it is found that the corresponding vortex strength is given by

$$\Delta\gamma_{,B}(\xi) = \frac{\chi}{\pi} \tan\frac{\theta}{2} + \frac{1}{\pi} \ln\left|\frac{\sin\,(\theta + \chi)/2}{\sin\,(\theta - \chi)/2}\right| , \tag{100}$$

with

$$\cos\chi = \frac{2c_A}{c_E} - 1 .$$

The approximate summation equivalent of equation (100) is derived from equation (98) as follows:

$$\Delta\gamma_{,B}(\xi_\nu) = \left(\frac{c_E - c_A}{c_E}\right)\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(4)}\left\{(\xi_\mu - c_A)H(\xi_\mu - c_A) - (c_E - c_A)\xi_\mu/c_E\right\}. \quad \ldots\ (101)$$

Results, calculated by employing the 'exact' equation (100) and the approximate equation (101) for $\chi = \pi/3$ and with $N = 32$, are exhibited in Table 1 for various values of ν. Generally, the agreement between the two sets of results is seen to be good, with the possible exception of the point $\nu = 11$ at which the error in the approximate result is nearly 7%. This error is, however, considerably greater than the mean error, of less than 1%, of the other values. The reason for the comparatively large error at the point $\nu = 11$ appears to be that this particular point is close to the discontinuity in $dG/d\xi$ at $\theta = \pi/3$. Here the approximation can be expected to fail. Nevertheless, in view of the apparent accuracy of the approximate solution elsewhere, we feel confident to use it, at least for a value of N no less than 32.

Two integrals appear in the expression for $\Delta\gamma_{,W}$, namely equation (59). The first of these integrals is basically similar to the first integral in equation (29) of Ref.18 for the strength of the vortices on the chord of an aerofoil with a jet-augmented flap. The only difference between the two integrals is that γ_W of equation (59) is replaced by the jet vortex strength

$$\gamma_J = -\kappa_J V_\infty c_E C_J/2$$

in Ref.18. Here $C_J = J/\frac{1}{2}\rho V_\infty^2 c_E$ is the jet-momentum coefficient.

In Appendix G the analogy between the jet sheet of a blown flap and the wake (first suggested by Spence and Beasley[4]) is used to evaluate the first integral of equation (59) approximately. It is shown that

$$\frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi_W}{\xi_W - c_E}\right)^{\frac{1}{2}} \gamma_W \frac{d\xi_W}{\xi_W - \xi}$$

$$= 2V_\infty \left(\frac{c_E}{\xi}\right)^{\frac{3}{2}} \left[\alpha\left\{B_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} B_n X^n\right\} + \beta\left\{D_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} D_n X^n\right\}\right] , \qquad (102)$$

where $X = \dfrac{1 - (1 - \xi/c_E)^{\frac{1}{2}}}{1 + (1 - \xi/c_E)^{\frac{1}{2}}}$.

The coefficients B_n and D_n are solutions of M linear, algebraic equations[18], which are quoted in Appendix G. There it is shown that, for small

$C_D = D/\frac{1}{2}\rho V_\infty^2 c_0$, which implies small δ_W^*/c_E and θ_W/c_E, these equations may be written as

$$\left.\begin{aligned} \sum_{n=0}^{M-1} b_{mn} D_n &= -\frac{c_0}{c_E}\frac{C_D}{2} f_m(\chi) \\ \sum_{n=0}^{M-1} b_{mn} B_n &= -\frac{c_0}{c_E}\frac{C_D}{2} f_m(\pi) \end{aligned}\right\} , \quad m = 0,1,2,\ldots M-1 \ . \qquad (103)$$

Here the functions b_{mn} and $f_m(\chi)$, which are defined in Appendix G, are independent of C_D. Consequently, for small C_D, B_n and D_n are linearly dependent on C_D.

Solutions of equations (103) for $M = 3$ are given in Table 2 for a value of c_F/c_E typical of that tested by Foster, *et al.*[9], namely 0.31 ($\chi = 1.18$ rad). The indication of calculations performed by Spence[17] for various M are that, with $M = 3$, the coefficients B_n and D_n $(n < 3)$ should be estimated with adequate accuracy. Furthermore, he shows that these coefficients converge rapidly in the case of a large value of M. Therefore, noting that X is significantly less than unity over the major part of the flap chord, we conclude that the terms $D_n X^n$ and $B_n X^n$ $(n > 2)$ in equation (102) are likely to give only a small contribution to the flap lift and may reasonably be ignored.

The second integral of equation (59) may be evaluated by following the method used in obtaining equation (98) from equation (27). Hence we find that this integral, which we will term I_3, may be written as

$$I_3 = \frac{L(c_E)}{c_E}\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(4)}\left\{L(\xi_\mu) - L(c_E)\xi_\mu/c_E\right\} , \qquad (104)$$

where $L(\xi)$ is an integral of the equation

$$\frac{dL}{d\xi} = -\left(\frac{d\psi_W^*}{d\xi}(x) + \frac{1}{\pi}\psi_W^*(x_T)\frac{z_T}{(\xi - c_A)^2 + z_T^2}\right)H(\xi - c_A)$$

satisfying the condition $L(0) = 0$. This integral is found to be given by

$$L(\xi) = -\left\{\psi_W^{*\prime}(x) - \psi_W^{*\prime}(x_T) + \frac{1}{\pi}\psi_W^{*\prime}(x_T)\tan^{-1}\left(\frac{\xi - c_A}{z_T}\right)\right\} H(\xi - c_A) , \quad (105)$$

where we have replaced ψ_W^* by $\psi_W^{*\prime}$ in accordance with equation (87).

Consequently, employing the truncated version of equation (102) and replacing the second integral of equation (59) by the right-hand side of equation (104), we are able to rewrite equation (59) as

$$\begin{aligned}\Delta\gamma_{,W}(\xi_\nu) = & - H(\xi_\nu - c_A)\gamma_W^{(1)} \\ & + 2V_\infty\left(\frac{c_E}{\xi_\nu}\right)^{\frac{3}{2}}\left[\alpha\left\{B_0\left(\frac{2X_\nu}{1+X_\nu}\right) + B_1X_\nu + B_2X_\nu^2\right\}\right. \\ & \left. + \beta\left\{D_0\left(\frac{2X_\nu}{1+X_\nu}\right) + D_1X_\nu + D_2X_\nu^2\right\}\right] \\ & + \frac{L(c_E)}{c_E}\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(4)}\left\{L(\xi_\mu) - L(c_E)\xi_\mu/c_E\right\} \ . \qquad (106)\end{aligned}$$

A similar process is used to derive $\overline{\Delta\gamma}_{,W}(\xi_\nu)$ from equation (74). By comparing equations (59), (74) and (106) we find that we may write

$$\begin{aligned}\overline{\Delta\gamma}_{,W}(\xi_\nu) = & - H(\xi_\nu - c_A)\bar{\gamma}_W \\ & + 2V_\infty\left(\frac{c_E}{\xi_\nu}\right)^{\frac{3}{2}}\left[\alpha\left\{\bar{B}_0\left(\frac{2X_\nu}{1+X_\nu}\right) + \bar{B}_1X_\nu + \bar{B}_2X_\nu^2\right\}\right. \\ & \left. + \beta\left\{\bar{D}_0\left(\frac{2X_\nu}{1+X_\nu}\right) + \bar{D}_1X_\nu + \bar{D}_2X_\nu^2\right\}\right] \\ & + \frac{\bar{L}(c_E)}{c_E}\left(\frac{c_E - \xi_\nu}{\xi_\nu}\right)^{\frac{1}{2}} + \frac{1}{c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(4)}\left\{\bar{L}(\xi_\mu) - \bar{L}(c_E)\xi_\mu/c_E\right\} \ , \qquad (107)\end{aligned}$$

where

$$\bar{L}(\xi) = \left\{\bar{\psi}_W^*(x) - \bar{\psi}_W^*(x_T) + \frac{1}{\pi}\psi_W^{*\prime}(x_T)\tan^{-1}\left(\frac{\xi - c_A}{z_T}\right)\right\}H(\xi - c_A) \ , \qquad (108)$$

and it is shown in Appendix G, with the aid of the previously-mentioned jet-flap analogy, that $\bar{D}_n$ and $\bar{B}_n$ are solutions of the linear equations

$$\left.\begin{aligned}\sum_{n=0}^{M-1} b_{mn}\bar{D}_n &= -\frac{c_0}{c_E}\frac{\bar{C}_D}{2} f_m(\chi)\\ \sum_{n=0}^{M-1} b_{mn}\bar{B}_n &= -\frac{c_0}{c_E}\frac{\bar{C}_D}{2} f_m(\pi)\end{aligned}\right\} \quad , \quad m = 0,1,2,\ldots M-1 \quad . \tag{109}$$

Here, with the aid of equations (G-19) and (87), we have that

$$\bar{C}_D = C_D - \left(\left|\left\{\frac{d\psi_U^*}{dx} + \frac{1}{2}\frac{d\psi_W^{*\prime}}{dx}\right\}(\delta_W^* + \theta_W) + \frac{1}{2}\frac{d\psi_W^{*\prime}}{dx}(z_+ + z_-)\right|_{x=c_F}\right) \Big/ V_\infty c_0(\beta + \alpha) \quad .$$

Noting from equations (103) and (109) that

$$\bar{D}_n = \frac{D_n\bar{C}_D}{C_D} \quad ; \quad \bar{B}_n = \frac{B_n\bar{C}_D}{C_D} \quad ,$$

we observe that it is possible to derive $\bar{D}_n$ and $\bar{B}_n$ from the solutions for D_n and B_n.

4.2 Evaluation of corrections to speeds at edges of flap boundary layer

The correction to the speed of the flow at the edges of the flap boundary layer due to the flap boundary layer is given by equation (33). The first integral in this equation may be written in a more suitable form by using the trigonometric substitution (97); thus, referring to this integral as I_4, we have

$$\begin{aligned} I_4 &\equiv \frac{\beta}{2\pi}\int_0^{c_A} \Delta\gamma_{A,B}(\xi')\,\frac{c_A - \xi'}{(c_A + x - \xi')^2}\,d\xi' \quad ,\\ &= \frac{\beta}{2\pi}\int_\chi^\pi \Delta\gamma_{,B}(\xi')\,\frac{(\cos\chi - \cos\theta')\sin\theta'}{(\cos\theta - \cos\theta')^2}\,d\theta' \quad , \end{aligned} \tag{110}$$

where $\cos\theta = 2(c_A + x)/c_E - 1$.

Consistent with equation (98) we disregard the possibility of a logarithmic singularity in $\Delta\gamma_{,B}$ at $\theta = \chi$. Thus, with the exception of a simple pole at $\xi' = c_A$ that exists when $x = 0$, the integrand of I_4 is bounded. The singularity may be dealt with separately by rewriting equation (110) as

$$I_4 = \frac{\beta}{2\pi}\left(\int_{\chi}^{\pi} \Delta\gamma_{,B}(c_A)\,\frac{(\cos\chi - \cos\theta')\sin\theta'}{(\cos\theta - \cos\theta')^2}\,d\theta' + \int_{\chi}^{\pi}\left\{\Delta\gamma_{,B}(\xi') - \Delta\gamma_{,B}(c_A)\right\}\frac{(\cos\chi - \cos\theta')\sin\theta'}{(\cos\theta - \cos\theta')^2}\,d\theta'\right) , \quad (111)$$

where the first integral, which contains the singularity, may be evaluated explicitely and the second integral has a bounded integrand.

Evaluating the second integral of equation (111) by means of the trapezium rule we have, subject to two assumptions to be given shortly,

$$I_4 = \frac{\beta}{2\pi}\left(\Delta\gamma_{,B}(\xi_m)\left\{\ln\left(\frac{c_A + x}{x}\right) + \frac{x}{c_A + x} - 1\right\} + \frac{\pi}{N}\left(\sum_{\mu=m+1}^{N-1}\left\{\Delta\gamma_{,B}(\xi_\mu) - \Delta\gamma_{,B}(\xi_m)\right\}\frac{(\cos\chi - \cos\theta_\mu)\sin\theta_\mu}{(\cos\theta - \cos\theta_\mu)^2} + \frac{\Delta\gamma_{,B}(\xi_{N-1})\sin\theta_{N-1}(\cos\chi + 1)}{2(\cos\theta + 1)^2}\right)\right) . \quad (112)$$

Here m is the integral part of $(1 + N\chi/\pi)$ and we have made use of the two assumptions, firstly that, for sufficiently large N,

$$\Delta\gamma_{,B}(\xi_{N-1})\sin\theta_{N-1} = \lim_{\theta'\to\pi}\left(\Delta\gamma_{,B}(\xi')\sin\theta'\right) ,$$

and, secondly,

$$\Delta\gamma_{,B}(c_A) = \Delta\gamma_{,B}(\xi_m) .$$

The error in I_4 resulting from the use of the first of these assumptions is readily found to be $O[(\pi/N)^3]$ for distributions which behave like $\tan(\theta/2)$ near $\theta = \pi$. For $N = 32$ this error is evidently negligible. The second assumption can be justified by the observation that, in all the cases examined in this Report, $\Delta\gamma_{,B}(\xi)$ varies slowly near $\xi = c_A$.

We have not attempted to assess the accuracy of the approximate integration scheme used in evaluating I_4; however, the contribution of I_4 to the correction to the flap lift is found to be small - no more, in fact, than 10% of the total correction due to the flap boundary layer. We consider therefore that the present integration scheme with $N = 32$ is adequate for our purposes.

The second integral in equation (33) requiring evaluation is given by

$$I_5 = \frac{1}{2\pi} \int_0^{\infty} \Delta q_{F,B} \frac{dx'}{x - x'} \quad . \tag{113}$$

It is convenient to rewrite equation (113) by employing transformations (22) and dividing the integration into two parts from 0 to c_E and from c_E to ∞, as follows:

$$I_5 = \frac{1}{2\pi} \int_0^{c_E} \Delta q_{F,B} H(\xi' - c_A) \frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi} \int_{c_E}^{\infty} \Delta q_{F,B} \frac{d\xi'}{\xi - \xi'} \quad . \tag{114}$$

The first of the integrals in equation (114),

$$I_6 = \frac{1}{2\pi} \int_0^{c_E} \Delta q_{F,B} H(\xi' - c_A) \frac{d\xi'}{\xi - \xi'} \quad , \tag{115}$$

may be written in the form

$$I_6 = \frac{1}{2\pi} \int_0^{c_E} \frac{E(c_E)}{c_E} \frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi} \int_0^{c_E} \frac{d}{d\xi'} \left(E(\xi') - \frac{E(c_E)}{c_E} \xi' \right) \frac{d\xi'}{\xi - \xi'} \quad , \tag{116}$$

where

$$E(\xi) = \int_0^{\xi} \Delta q_{F,B} H(\xi - c_A) d\xi$$

$$= (\psi_U^* + \psi_L^*) H(\xi - c_A) \tag{117}$$

from equations (12) and (22). Here we have supposed, as before, that ψ_U^* and ψ_L^* vanish at the leading edge of the flap, which point is assumed to be at $\xi = c_A$.

The second of the integrals in equation (116) may be evaluated numerically by the method Weber[12] has used for determining the velocity distributions of aerofoil thickness distributions in a potential flow. Hence, evaluating the first integral in equation (116) explicitely and integrating the second integral numerically by Weber's method we have the result

$$I_6 = \frac{1}{2\pi}\frac{E(c_E)}{c_E}\ln\left(\frac{\xi_\nu}{c_E - \xi_\nu}\right) + \frac{1}{2c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(1)}\left\{E(\xi_\mu) - \frac{E(c_E)}{c_E}\xi_\mu\right\} , \quad (118)$$

where $s_{\mu\nu}^{(1)}$ is defined in Ref.12.

The accuracy of Weber's method for smooth and continuous distributions of $E(\xi)$ is undoubted, at least, for values of $N \geqslant 16$. However, we note from equation (117) that, in the present case, $E(\xi)$ has a discontinuous first slope at $\xi = c_A$, in general. It is appropriate, therefore, to consider the accuracy of a distribution of this type, such as

$$E(\xi) = (\xi - c_A)H(\xi - c_A) . \quad (119)$$

Substituting $E(\xi)$ from equation (119) into equation (116) and performing the routine integration we find that, for this distribution,

$$I_6 = \frac{1}{2\pi}\ln\left|\frac{\xi - c_A}{c_E - \xi}\right| . \quad (120)$$

Results for I_6 that have been computed by using the exact result (120) and the approximate expression (118), with $N = 32$, are shown in Table 3 for $\chi = \pi/3$. It will be noticed that the agreement between the two sets of results is satisfactory for ν either less than 6 or greater than 12; but in the neighbourhood of the discontinuity in $dE/d\xi$ at $\theta = \pi/3$ the agreement is not good, the error being as much as 12% for $\nu = 11$. An improvement in accuracy in this region might be obtained by removing the discontinuity in the form of a distribution similar to that given in equation (119). This has not been done, however, since it was found that, in each of the cases to be examined in section 5, the slope of $E(\xi)$ at $\xi = c_A +$ is very small compared with $E(c_E)/c_E$. This suggests that the present test of the accuracy of equation (118) is rather severe. Additionally, it is found that the contribution to the lift of the flap associated with I_6 is small compared with that associated with $\Delta\gamma_{,B}$. It was decided, therefore, to retain approximation (118) with $N = 32$.

In the absence of either experimental results or a reliable theory for the behaviour of $\Delta q_{F,B}$ downstream of the flap trailing edge we have assumed that it has the form

$$\Delta q_{F,B} = -\Big(E(c_E) - E(\infty)\Big) H\Big((1+\lambda)c_E - \xi\Big)/\lambda c_E \quad ; \qquad \xi > c_E \quad , \qquad (121)$$

where λ is an arbitrary parameter.

The model of the displacement flux implied by equation (121) is illustrated in Fig.8, where it is compared with a plausible suggestion for the actual distribution. It will be seen that our model correctly represents the total change in displacement flux between the trailing edge of the flap and infinity.

In section 5 it is shown that the effect, on the lift of the flap, of varying λ is insignificant. The implication of this is that the flap lift is insensitive to detailed changes in the shape of the distribution of displacement flux in the wake; and thus it is considered that the approximation (121) is acceptable for our purposes.

Substituting $\Delta q_{F,B}$ from equation (121) into the second integral of equation (114), performing the integration and combining the resulting expression with equations (116) and (118) we have

$$I_5 = \frac{1}{2\pi}\frac{E(c_E)}{c_E}\ln\left(\frac{\xi_\nu}{c_E - \xi_\nu}\right) + \frac{1}{2c_E}\sum_{\mu=1}^{N-1} s_{\mu\nu}^{(1)}\left\{E(\xi_\mu) - \frac{E(c_E)}{c_E}\xi_\mu\right\}$$
$$+ \frac{1}{2\pi}\frac{E(c_E) - E(\infty)}{\lambda c_E}\ln\left(\frac{(1+\lambda)c_E - \xi_\nu}{c_E - \xi_\nu}\right) . \qquad (122)$$

This completes the description of the methods used to evaluate the integrals of equation (33). It only remains to note that we may write in place of equation (33)

$$\frac{\partial(\Delta\psi_B)}{\partial z}(x,\pm 0) = I_4 \pm \frac{\Delta\gamma_{F,B}}{2} + I_5 \quad , \qquad (123)$$

with I_4 given by equation (112), I_5 given by equation (122) and $\Delta\gamma_{F,B}$ ($= \Delta\gamma_{,B}$ for $c_A \leqslant \xi \leqslant c_E$) obtainable from equation (98).

A similar procedure is used to determine the correction to the speeds at the edges of the flap boundary layer due to the wake of the main aerofoil. Noting that in our approximation

$$\left(\frac{\partial(\Delta\psi_W)}{\partial z}\right)_{U,L} = \frac{\partial(\Delta\psi_W)}{\partial z}(x,\pm 0) \quad ,$$

we find that these speeds are given by equation (92), which includes a first-order correction for wake thickness as well as non-zero displacement of the wake from the flap chord. In the last-named expression there are two integrals similar to those considered previously. The first,

$$I_7 = \frac{1}{2\pi}\int_{x_T}^{\infty} \frac{d\bar{\psi}^*_W}{dx'}\frac{dx'}{x - x'} \tag{124}$$

(in which we have replaced x_W by x' for convenience), may be approximated in the same manner as for the integral I_5; that is to say we first rewrite equation (124), with the aid of equations (22), as

$$I_7 = \frac{1}{2\pi}\int_0^{c_E} \frac{d\bar{\psi}^*_W}{d\xi'} H(\xi' - c_A)\frac{d\xi'}{\xi - \xi'} + \frac{1}{2\pi}\int_{c_E}^{\infty} \frac{d\bar{\psi}^*_W}{d\xi'}\frac{d\xi'}{\xi - \xi'} \quad . \tag{125}$$

Here we have used the fact that $\xi_T = c_A$. The first of these integrals is evaluated by using the method employed in the evaluation of the integral I_6 (equation (115)). In the reduction of the second integral we follow the method used in the evaluation of the second integral of I_5. That is to say we assume that

$$\frac{d\bar{\psi}^*_W}{d\xi} = -\frac{\left\{\bar{\psi}^*_W(c_E) - \bar{\psi}^*_W(\infty)\right\}}{\lambda c_E} H\Big((1+\lambda)c_E - \xi\Big) \quad ; \quad \xi > c_E \quad . \tag{126}$$

This equation may be written in a more suitable form by using a result given in Appendix E, namely that P vanishes far downstream. Consequently, as may be inferred from equations (72),

$$\bar{\psi}^*_W(\infty) = \psi^*_W(\infty) \quad . \tag{127}$$

As with the approximation to $\Delta q_{F,B}$ for points downstream of the flap trailing edge we find that the variation in flap lift due to changes in the parameter λ in equation (126) is negligible (see section 5).

Using equation (127) to eliminate $\bar{\psi}^*_W(\infty)$ from equation (126) and substituting the right-hand side of the resulting expression for $d\bar{\psi}^*_W/d\xi$ in the last integral of equation (125) we obtain finally

$$I_7 = \frac{1}{2\pi}\frac{\bar{\psi}^*_W(c_E)}{c_E}\ln\left(\frac{\xi_\nu}{c_E - \xi_\nu}\right) + \frac{1}{2c_E}\sum_{\mu=1}^{N-1} s^{(1)}_{\mu\nu}\left\{\bar{\psi}^*_W(\xi_\mu)H(\xi_\mu - c_A) - \frac{\bar{\psi}^*_W(c_E)}{c_E}\xi_\mu\right\}$$

$$+ \frac{1}{2\pi}\frac{\bar{\psi}^*_W(c_E) - \psi^*_W(\infty)}{\lambda c_E}\ln\left(\frac{(1+\lambda)c_E - \xi_\nu}{c_E - \xi_\nu}\right) . \quad (128)$$

The second integral in equation (92),

$$I_8 = \frac{\beta}{2\pi}\int_0^{c_A} \overline{\Delta\gamma}_{A,W}\frac{c_A - \xi'}{(c_A + x - \xi')^2}\,d\xi' , \quad (129)$$

differs from I_4 (equation (110)) only in having in its integrand $\overline{\Delta\gamma}_{A,W}$ instead of $\Delta\gamma_{A,B}$. Therefore, using the approximation that was employed in evaluating I_4, we obtain

$$I_8 = \frac{\beta}{2\pi}\left(\overline{\Delta\gamma}_{,W}(\xi_m)\left\{\ln\left(\frac{c_A + x}{x}\right) + \frac{x}{c_A + x} - 1\right\}\right.$$

$$+ \frac{\pi}{N}\left(\sum_{\mu=m+1}^{N-1}\left\{\overline{\Delta\gamma}_{,W}(\xi_\mu) - \overline{\Delta\gamma}_{,W}(\xi_m)\right\}\frac{(\cos\chi - \cos\theta_\mu)\sin\theta_\mu}{(\cos\theta - \cos\theta_\mu)^2}\right.$$

$$\left.\left. + \frac{\overline{\Delta\gamma}_{,W}(\xi_{N-1})\sin\theta_{N-1}(\cos\chi + 1)}{2(\cos\theta + 1)^2}\right)\right) . \quad (130)$$

Thus, by referring to equations (92), (124) and (129), we are able to write for the corrections to the speeds at the edges of the flap boundary layer due to the wake of the main aerofoil

$$\frac{\partial(\Delta\psi_W)}{\partial z}(x,\pm 0) = -\tfrac{1}{2}H(x - x_T)\bar{\gamma}_W + I_7 + \frac{1}{2\pi}\bar{\psi}^{*\prime}_W(x_T)\frac{x - x_T}{(x - x_T)^2 + z_T^2} + I_8 \pm \frac{\overline{\Delta\gamma}_{F,W}}{2} \quad , \qquad (131)$$

with I_7 and I_8 given by equations (128) and (130), and $\overline{\Delta\gamma}_{F,W}$ given by equation (107). The corresponding expression for a 'thin' wake at a 'small' distance from the flap chord is obtained simply by replacing $\bar{\gamma}_W$ by $\gamma_W^{(1)}$, $\bar{\psi}^*_W$ by $\psi^{*\prime}_W$ and $\overline{\Delta\gamma}_{F,W}$ by $\Delta\gamma_{F,W}$.

4.3 Calculation of corrections to flap lift

The correction to the lift of the flap due to the flap boundary layer is given by equation (31). To evaluate the integral in this expression we first employ transformations (22) and the trigonometric substitution (97). Hence we find that

$$\Delta C_{L_{F,B}} \equiv \Delta L_{F,B}\Big/\tfrac{1}{2}\rho V_\infty^2 c_0 = \cos(\beta + \alpha)\int_0^\chi J(\theta)\sin\theta\, d\theta \quad , \qquad (132)$$

where

$$J(\theta) = \frac{c_E}{c_0 V_\infty^2}\left(\frac{\partial\psi_I}{\partial z}(x,+0)\frac{\partial(\Delta\psi_B)}{\partial z}(x,+0) - \frac{\partial\psi_I}{\partial z}(x,-0)\frac{\partial(\Delta\psi_B)}{\partial z}(x,-0)\right) \quad , \qquad (133)$$

$(\partial(\Delta\psi_B)/\partial z)(x,\pm 0)$ being obtained from equation (123).

Performing the integration of equation (132) approximately with the aid of the trapezium rule and noting that, for sufficiently large N,

$$J(\theta_{m-1}) \simeq J(\chi)$$

we obtain the result

$$\Delta C_{L_{F,B}} = \cos(\beta + \alpha)\left(\frac{\pi}{N}\sum_{\mu=1}^{m-2} J(\theta_\mu)\sin\theta_\mu + \frac{\pi}{2N}J(\theta_{m-1})\sin\theta_{m-1} + \left(\chi - \frac{\pi(m - 1)}{N}\right)J(\theta_{m-1})\sin\theta_{m-1}\right) \quad . \qquad (134)$$

Here, it may be recalled, m is the entire or integral part of $(1 + N\chi/\pi)$.

To test the accuracy of equation (134) we have considered the hypothetical case having

$$\left.\begin{array}{l} \frac{\partial\psi_I}{\partial z}(x,+0) = \frac{\partial\psi_I}{\partial z}(x,-0) = V_\infty = 1 \\ I_4 = I_5 = 0 \end{array}\right\} . \tag{135}$$

Hence, combining equations (123), (132), (133) and (135), we have in this case

$$\Delta C_{L_{F,B}} = \cos(\beta+\alpha)\frac{c_E}{c_0}\int_0^\chi \Delta\gamma_{F,B}\sin\theta\, d\theta . \tag{136}$$

In particular, for the $F(\xi)$ distribution of equation (99) we may use equation (100) to eliminate $\Delta\gamma_{F,B}$ from equation (136) to yield

$$\Delta C_{L_{F,B}} = \cos(\beta+\alpha)\frac{c_E}{c_0}\int_0^\chi \left\{\frac{\chi}{\pi}\tan\frac{\theta}{2} + \frac{1}{\pi}\ln\left|\frac{\sin(\theta+\chi)/2}{\sin(\theta-\chi)/2}\right|\right\}\sin\theta\, d\theta .$$

Spence[18] has evaluated the integral of this equation explicitely in his work on blown flaps. We find that it is possible to infer from his equation (11) that

$$\Delta C_{L_{F,B}} = \cos(\beta+\alpha)\frac{c_E}{c_0}\frac{\chi^2}{\pi} . \tag{137}$$

Using equations (123), (132), (133) and (135) we may write for the approximate equivalent to equation (136)

$$\Delta C_{L_{F,B}} = \cos(\beta+\alpha)\frac{c_E}{c_0}\left(\frac{\pi}{N}\sum_{\mu=1}^{m-2}\Delta\gamma_{,B}(\xi_\mu)\sin\theta_\mu + \frac{\pi}{2N}\Delta\gamma_{,B}(\xi_{m-1})\sin\theta_{m-1} + \left(\chi - \frac{\pi(m-1)}{N}\right)\Delta\gamma_{,B}(\xi_{m-1})\sin\theta_{m-1}\right) . \tag{138}$$

To compare the approximate result with the exact result (137) it is only necessary to substitute the $\Delta\gamma_{,B}$ given by equation (100) into equation (138) and to perform the summation. However, in the computer programme, which has been written to evaluate $\Delta C_{L_{F,B}}$ equation (138) employs the approximation (101) for $\Delta\gamma_{,B}$. We have previously shown, however, that the errors in approximation (101) are small except possibly close to $\xi = c_A$. Hence the results obtained by this procedure should enable us to judge the accuracy of equation (134) compared with equation (132). In fact, with $\chi = \pi/3$, we obtain the following results for $\Delta C_{L_{F,B}} c_0/c_E \cos(\beta + \alpha)$:

Exact, Eqn.(137)	Approximate, Eqn.(138) with Eqn.(101); N = 32
0.349	0.340

The error in the approximate result is evidently small, and this test appears to justify the use of approximation (134) with N = 32.

The increment in the lift of the flap due to the wake of the main aerofoil, including second-order correction terms to allow for the non-zero thickness of the wake, is given by equation (93). Rewriting this expression in coefficient form and employing transformations (22) and (97) we find that

$$\Delta C_{L_{F,W}} \equiv \frac{L_{F,W}}{\frac{1}{2}\rho V_\infty^2 c_0} = \cos(\beta + \alpha) \int_0^\chi K(\theta) \sin\theta \, d\theta \quad , \qquad (139)$$

where

$$K(\theta) = \frac{c_E}{c_0 V_\infty^2}\left(\left\{\frac{\partial\psi_I}{\partial z}(x,+0) + \frac{1}{2}\frac{\partial(\Delta\psi_W)}{\partial z}(x,+0) + \frac{\partial(\Delta\psi_B)}{\partial z}(x,+0)\right\}\frac{\partial(\Delta\psi_W)}{\partial z}(x,+0)\right.$$

$$\left. - \left\{\frac{\partial\psi_I}{\partial z}(x,-0) + \frac{1}{2}\frac{\partial(\Delta\psi_W)}{\partial z}(x,-0) + \frac{\partial(\Delta\psi_B)}{\partial z}(x,-0)\right\}\frac{\partial(\Delta\psi_W)}{\partial z}(x,-0)\right) .$$

Using the approximation that was employed to derive equation (134) from equation (132) we have in place of equation (139)

$$\Delta C_{L_{F,W}} = \cos(\beta + \alpha)\left(\frac{\pi}{N}\sum_{\mu=1}^{m-2} K(\theta_\mu)\sin\theta_\mu + \frac{\pi}{2N}K(\theta_{m-1})\sin\theta_{m-1} + \left(\chi - \frac{\pi(m-1)}{N}\right)K(\theta_{m-1})\sin\theta_{m-1}\right) . \qquad (140)$$

The accuracy of this approximation has already been considered in relation to equation (132).

Comparing equations (90) and (93) we see that, for a sufficiently 'thin' wake, we may replace equation (140) by the expression

$$\Delta C^{(1)}_{L_{F,W}} = \cos(\beta + \alpha)\left(\frac{\pi}{N}\sum_{\mu=1}^{m-2} K^{(1)}(\theta_\mu)\sin\theta_\mu + \frac{\pi}{2N}K^{(1)}(\theta_{m-1})\sin\theta_{m-1} + \left(\chi - \frac{\pi(m-1)}{N}\right)K^{(1)}(\theta_{m-1})\sin\theta_{m-1}\right) ,$$

where

$$K^{(1)}(\theta) = \frac{c_E}{c_0 V_\infty^2}\left(\frac{\partial\psi_I}{\partial z}(x,+0)\,\frac{\partial(\Delta\psi_W^{(1)})}{\partial z}(x,+0) - \frac{\partial\psi_I}{\partial z}(x,-0)\,\frac{\partial(\Delta\psi_W^{(1)})}{\partial z}(x,-0)\right) .$$

4.4 Evaluation of corrections to overall lift

Whilst our main interest is in the lift of the flap we have also estimated the corrections to the overall lift arising from the flap boundary layer and the wake of the main aerofoil. Employing the lift-circulation theorem[11] for viscous flows we have for the increment in lift due to the various viscous effects

$$\Delta L = \rho V_\infty \Delta\Gamma . \qquad (141)$$

Here $\Delta\Gamma$ is the increment in circulation, due to the viscous effects, around a simply-closed contour which surrounds the main aerofoil, the flap and their associated boundary layers, and cuts the wake at right angles far downstream of the flap.

It will be recalled that we have regarded the wakes of the flap and the main aerofoil as one for the purpose of representing the vortex effect of the wakes downstream of the flap trailing edge. Hence we write

$$\Delta\Gamma = \Delta\Gamma_{,B} + \Delta\Gamma_{,WS} + \Delta\Gamma_{,WV} \quad , \tag{142}$$

where

$$\Delta\Gamma_{,B} = \int_0^{c_E} \Delta\gamma_{,B} d\xi \tag{143}$$

is the increment in circulation due to the flap boundary layer; $\Delta\Gamma_{,WS}$ is the increment due to the sources representing the displacement effect of the wake of the main aerofoil; and $\Delta\Gamma_{W,V}$ is the increment due to the vortices of the wakes of both the flap and the main aerofoil. Note that we have not included an increment due to the sources of the wake of the flap. In our approximation this is zero owing to the fact that these sources are assumed to be on the line $\zeta = 0$. Hence they induce zero downwash at the chords of the flap and the main aerofoil and consequently do not influence Γ.

The increment in lift coefficient caused by the flap boundary layer may be written, by reference to equations (141), (142) and (143), as

$$\Delta C_{L_{,B}} \equiv \frac{\Delta L_{,B}}{\frac{1}{2}\rho V_\infty^2 c_0} = \frac{2}{V_\infty c_0} \int_0^{c_E} \Delta\gamma_{,B} d\xi \quad .$$

Employing transformations (22) and (97) we may rewrite this expression as follows:

$$\Delta C_{L_{,B}} = \frac{c_E}{c_0} \int_0^{\pi} \frac{\Delta\gamma_{,B}}{V_\infty} \sin\theta \, d\theta \quad . \tag{144}$$

This integral may be evaluated by using the trapezium rule viz:

$$\Delta C_{L_{,B}} = \frac{c_E}{c_0} \frac{\pi}{N} \left(\sum_{\mu=1}^{N-1} \frac{\Delta\gamma_{,B}(\xi_\mu)}{V_\infty} \sin\theta_\mu + \frac{1}{2} \frac{\Delta\gamma_{,B}(\xi_{N-1})}{V_\infty} \sin\theta_{N-1} \right) . \tag{145}$$

Here, as in the derivation of equation (112), we have assumed that

$$\Delta\gamma_{,B}(\xi_{N-1}) \sin \theta_{N-1} = \lim_{\theta \to \pi} (\Delta\gamma_{,B}(\xi) \sin \theta) ,$$

which results in an error of $O((\pi/N)^3)$ for a $\Delta\gamma_{,B}$ that behaves like $\tan(\theta/2)$ near $\theta = \pi$, as noted before.

The accuracy of equation (145) may be assessed by considering again the $F(\xi)$ distribution of equation (99). Upon replacing $\Delta\gamma_{,B}$ in equation (144) by the vortex distribution corresponding to this $F(\xi)$ distribution, i.e. from equation (100), we have, with $V_\infty = 1$

$$\Delta C_{L_{,B}} = \frac{c_E}{c_0} \int_0^\pi \left\{ \frac{\chi}{\pi} \tan \frac{\theta}{2} + \frac{1}{\pi} \ln \left| \frac{\sin (\theta + \chi)/2}{\sin (\theta - \chi)/2} \right| \right\} \sin \theta \, d\theta .$$

This integration has been performed by Spence[18], and it is readily inferred from his equations (8), (9) and (10) that

$$\Delta C_{L_{,B}} = \frac{c_E}{c_0} (\chi + \sin \chi) . \tag{146}$$

To determine $\Delta C_{L_{,B}}$ from equation (145) we have used the approximate summation version of equation (100), namely equation (101), the N in both summations being 32. Using this procedure, we find that, for $\chi = \pi/3$, $\Delta C_{L_{,B}} c_0/c_E$ is 1.913 which, to four significant figures, is in agreement with the exact result. It would appear, therefore, that the approximate formula (145) is suitable for our purposes.

For the purpose of estimating the correction due to the wakes we propose to use the 'thin'-wake approximation and, as in section 3.1, we suppose that the distance of the wake of the main aerofoil from the flap is small compared with the flap chord. Hence to obtain

$$\Delta\Gamma_{,WS} = \int_0^{c_E} \Delta\gamma_{,WS} \, d\xi$$

we refer to equation (59) for $\Delta\gamma_{,W}$ and note that $\Delta\gamma_{,WS}$ is given by the last integral term of that equation. This term has previously been called I_3. Therefore, using the method employed in the derivation of equation (145) from equation (144) and replacing $\Delta\gamma_{,WS}$ by I_3 we have the result

$$\Delta C_{L_{,WS}} = \frac{c_E}{c_0}\frac{\pi}{N}\left(\sum_{\mu=1}^{N-1} I_3(\xi_\mu)\sin\theta_\mu + \tfrac{1}{2}I_3(\xi_{N-1})\sin\theta_{N-1}\right) , \qquad (147)$$

where I_3 is defined as an approximate sum in equation (104).

The wake vortices are comprised of (a) a distribution of vortices just above the flap, representing the vortex effect of that part of the wake of the main aerofoil which is immediately above the flap and (b) a distribution representing the combined aerofoil-flap wake downstream of the flap trailing edge. As is evident in equation (59) the former distribution requires an equal and opposite distribution on the flap chord. Therefore, in practice, the net contribution of these two distributions to $\Delta\Gamma_{,WV}$ can be expected to be very small or negligible. As remarked before, the distribution (b) is analogous to the distribution of vortices along the jet sheet of a blown flap. According to Spence[18] the increment in lift coefficient of a blown flap due to these vortices is given by

$$\Delta C_L = \frac{c_E}{c_0} 4\pi(D_0\beta + B_0\alpha) \quad ,$$

where, it should be noted, ΔC_L is based on c_0 instead of the c_E that Spence uses. Therefore if, as previously, we employ the analogy between the wake vortices and the vortices of the jet sheet of a blown flap we obtain for the increment in lift coefficient due to the wake vortices

$$\Delta C_{L_{,WV}} = \frac{c_E}{c_0} 4\pi(D_0\beta + B_0\alpha) \quad ,$$

where D_0 and B_0 are coefficients in the linear equations (103). Referring to the solutions of these equations for $M = 3$ in Table 2 we find that it is possible to write

$$\Delta C_{L_{,WV}} = - 4\pi C_D(0.52\beta + 0.46\alpha) \quad .$$

Finally, we note that the total increment in lift coefficient due to the various viscous effects is given by

$$\Delta C_L = \Delta C_{L,B} + \Delta C_{L,WS} + \Delta C_{L,WV} .$$

5 RESULTS OF CALCULATIONS

The calculations have been performed for a number of Fowler-flap configurations. These configurations were tested by Foster, *et al.*[9] who used the RAE 3ft (0.91m) chord twodimensional model. Throughout these tests the Reynolds number based on c_0 was 3.8×10^6. Each of these configurations have in common the flap angle $\beta = 24^\circ$ and the overlap $\tilde{\ell}/c_0 = 0.042$. Three flap gaps ($g/c_0 = 0.020$, 0.025 and 0.040) are considered and two values of α, -5° and 3°, are examined.

Where possible, the calculations have been based on results derived from velocity surveys conducted above the flap in the boundary layer of the flap and the wake of the main aerofoil. The object of this is to ensure that we use nominally correct values for the quantities associated with the viscous part of the flow field. By so doing we consider we will be better able to judge:

(i) the relative importance of the wake of the main aerofoil as compared with the boundary layer of the flap; and

(ii) the significance of the thickness effect of the wake of the main aerofoil.

Throughout the calculations the integer N in the summation formulae of section 4 was kept constant at 32, this value appearing to give accurate values for the corrections, as indicated before.

5.1 Effect of flap boundary layer

Foster[16] *has derived from velocity surveys the distribution of the dis-*placement thickness

$$\delta^*_U = \int_0^{\delta_U} \left(1 - \frac{u(z)}{U}\right) dz$$

along the flap upper surface, U being the velocity at the edge of the boundary layer. These results are illustrated graphically in Fig.9 for the various configurations.

Associated with the displacement thickness δ_U^* is a pseudo displacement flux

$$\psi_U^{*\prime} = U\delta_U^* = \int_0^{\delta_U} \left(U - u(z)\right) dz \quad .$$

This differs from the displacement flux ψ_U^* (equation (11)) by the amount

$$\int_0^{\delta_U} \left(U - u_I(z) - \Delta u_W(z)\right) dz \quad .$$

This term is invariably assumed to be negligible in the case of isolated wings[3]; and hence ψ_U^* is assumed to be equal to $\psi_U^{*\prime}$. In the present case, we assert that it is consistent with our assumption that the flap boundary layer is 'thin' to neglect this term in comparison with ψ_U^*. This may be proved by examining the first-order thickness correction to the approximate equation (15) for a 'thin' boundary layer. The terms neglected by using this equation include terms like $\left(\partial(\Delta\psi_B)/\partial z\right)_U z_U$ which is of the same order as the difference between ψ_U^* and $\psi_U^{*\prime}$ shown above. Consequently we replace ψ_U^* by $\psi_U^{*\prime}$ in the calculation of the boundary-layer effect.

No corresponding velocity surveys have so far been made below the flap; consequently it has been necessary to estimate the displacement-thickness distribution along the flap lower surface. The boundary layer on this surface was assumed initially to be laminar; it was then calculated by using a computer programme which is based on Thwaites method[11] for laminar layers and which embodies the transition criterion of Crabtree[19]. The experimental lower-surface pressure distribution was used as input data. Insofar as the results obtained by using this programme indicate that transition to a turbulent layer does not take place, our initial assumption of laminar flow appears to be justified. The results derived for the displacement thickness are shown in Fig.10. Evidently, the displacement thickness of the lower-surface layer is very much less than that of the corresponding chordwise position on the upper surface. As with ψ_U^* we replace ψ_L^* by $\psi_L^{*\prime}$.

The results calculated for the correction to the flap lift due to the flap boundary layer are shown in Table 4 under the column labelled $\Delta C_{L_{F,B}}$. We see that there is a significant increase in $-\Delta C_{L_{F,B}}$ upon altering g/c_0 from 0.02 to 0.04, as might have been anticipated from the distributions of displacement thickness given in Fig.8. On the other hand, an increase in α from -5° to 3° results in only a small change in $\Delta C_{L_{F,B}}$.

All the results for $\Delta C_{L_{F,B}}$ were derived with the parameter λ of equation (121) set equal to 0.04. This was considered to be a reasonable value from observations of the behaviour of the displacement flux in the wakes of isolated aerofoils[13,14]. Some calculations have, however, been performed on the effect of λ on $\Delta C_{L_{F,B}}$; the results of these calculations are shown in Fig.11 for the case $g/c_0 = 0.02$, $\alpha = -5^\circ$. It will be seen that $\Delta C_{L_{F,B}}$ varies very slowly with λ.

Table 4 also includes the corrections to the overall lift due to the flap boundary layer, $\Delta C_{L,B}$. As with the correction to the flap lift, the largest correction is obtained with the largest gap for both values of α.

5.2 Influence of wake of main aerofoil

5.2.1 'Thin'-wake calculations

As noted in section 3.2 we may replace ψ_W^* by $\psi_W^{*\prime}$ for a sufficiently 'thin' wake. Results for $\psi_W^{*\prime}/V_\infty c_0$ deduced from velocity surveys performed by Foster[16] are shown in Fig.12.

For a 'thin' wake the vortex strength $\gamma_W = \gamma_W^{(1)}$ may be estimated by using equation (48). However, since this involves the determination of the curvature of the wake, it is found preferable to use instead an alternative result. This is derived in Appendix C and given as equation (C-28). Using this expression and referring to Foster's wake-survey results we have been able to calculate the results for $\gamma_W^{(1)}/V_\infty$, shown in Fig.13, for the part of the wake immediately above the flap. Note that, without exception, $\gamma_W^{(1)}$ changes sign at some point along the flap. The reason for this may be found by examining equation (48). We see that, since $\bar{U}$, δ_W^* and θ_W are all positive, the only quantity in which the change in sign can occur is κ_W, i.e. the curvature of the wake. An examination of the shape of the wake from Foster's results seems to suggest that this indeed happens. Shortly after leaving the main aerofoil, the wake attempts to follow the flap upper surface, thus ensuring that κ_W is positive. Almost immediately after this, however, the wake begins to bend towards the direction of the main stream, and, in consequence, κ_W becomes negative.

The correction $\Delta C^{(1)}_{L_{F,W}}$ is calculated using the method of section 3.1. This method, it will be recalled, is based on the assumption that the wake is not only 'thin' but it is situated at a 'small' distance from the flap. The results for $\Delta C^{(1)}_{L_{F,W}}$ so obtained are shown in Table 4. Comparing these results with those for $\Delta C_{L_{F,B}}$ we see that, for $\alpha = -5^{o}$, the effect of the wake on the lift of the flap is small compared with that due to the boundary layer of the flap. Not surprisingly, perhaps, the wake effect increases in magnitude upon increasing the incidence to 3^{o}; it is, nevertheless, smaller, in magnitude, than the corresponding correction for the flap boundary layer.

Throughout the calculations of $\Delta C^{(1)}_{L_{F,W}}$ the parameter λ of equation (126) was held constant at 0.04. However, as in the case of $\Delta C_{L_{F,B}}$, the influence of λ on $\Delta C^{(1)}_{L_{F,W}}$ is found to be insignificant.

Some comments are appropriate on the relative significance of the various terms in $\Delta C^{(1)}_{L_{F,W}}$. We may divide the various contributions into those due to:

(i) the point source at the shroud trailing edge;

(ii) the distributed sources of the aerofoil wake;

(iii) the wake vortices including the vortices of the combined wake downstream of the flap trailing edge.

In all the cases considered, we find that the largest contribution to $-\Delta C^{(1)}_{L_{F,W}}$ comes from the distributed sources, contributions (i) and (iii) being relatively small in magnitude. This may be illustrated by reference to the case $g/c_0 = 0.02$, $\alpha = -5^{o}$ for which a breakdown of the various contributions may be given as follows:

Contribution	$\Delta C_{L_{F,W}}$
(i)	+0.006
(ii)	-0.023
(iii)	+0.001

The result of adding the corrections $\Delta C_{L_{F,B}}$ and $\Delta C^{(1)}_{L_{F,W}}$ to $\left(C_{L_F}\right)_I$, the flap-lift coefficient of the first inviscid approximation, is shown in Table 4, where the corrected coefficient is referred to as $C^{(1)}_{L_F}$. Comparing this coefficient with the flap-lift coefficient derived from experimental pressure distributions[16], C_{L_F}, we see that $C^{(1)}_{L_F}$ is, in all cases, lower than C_{L_F}. It may be that this is due to our neglect of the thickness of the wake and the non-zero distance of the wake from the flap. We will consider this aspect in section 5.2.2.

Table 4 also contains results for $\Delta C_{L,WS}$ and $\Delta C_{L,WV}$; these are used in conjunction with $\Delta C_{L,B}$ to correct $(C_L)_I$, the corrected result being referred to as $C^{(1)}_L$. Comparing this coefficient with the result for the overall-lift coefficient derived from experimental pressure distributions we see that the present method overestimates the correction, generally.

5.2.2 Effect of wake thickness and distance of wake from flap

We recall from section 3.2 that, if the singularities in the 'thin'-wake formulation are placed on the mean line of the wake, the first-order corrections to the vortex and source strengths of the wake for non-zero wake thickness are both zero. We suppose therefore that these singularities are placed on this line. Consequently, the corrections to $\psi^{*\prime}_W$ and $\gamma^{(1)}_W$ given in section 3.2 *apply exclusively to the effect of non-zero distance of the wake from the flap chord.* The results calculated for $\Delta C_{L_{F,W}}$ are shown below for the case $g/c_0 = 0.02$. They are compared there with the results for $\Delta C^{(1)}_{L_{F,W}}$, $\Delta C_{L_{F,B}}$ and C_{L_F}.

α^o	$\Delta C_{L_{F,W}}$	$\Delta C^{(1)}_{L_{F,W}}$	$\Delta C_{L_{F,B}}$	C_{L_F}
-5	+0.005	-0.017	-0.103	0.543
3	+0.024	-0.049	-0.117	0.520

In the calculation of $\Delta C_{L_{F,W}}$ we have used the second-order expression for $\Delta L_{F,W}$, equation (93). However, almost identical results are obtained if one neglects the products and squares of the velocity corrections as is done in

$\Delta C_{L_{F,W}}^{(1)}$. We may conclude from the above results, therefore, that the correction to $\Delta C_{L_{F,W}}$ for the non-zero distance of the aerofoil wake from the flap is of the same order as $\Delta C_{L_{F,W}}^{(1)}$. Indeed, we see that the result of applying this correction is to change the sign of $\Delta C_{L_{F,W}}$ from negative to positive, although, in the process, the magnitude of this coefficient is noticeably reduced. Hence it is apparent that the solution for $\Delta C_{L_{F,W}}$ is sensitive to the choice of position of the line on which the singularities are placed.

Insofar as $\Delta C_{L_{F,W}}$ and $\Delta C_{L_{F,W}}^{(1)}$ are small in magnitude compared with C_{L_F} the change in $\Delta C_{L_{F,W}}$ due to the non-zero distance of the aerofoil wake is not particularly significant. Of more significance, perhaps, is the effect of the distance correction on the flap pressure distribution insofar as this affects the development of the boundary layer on the flap upper surface. We have therefore examined the pressure distributions, corresponding to the two methods, on the surface of the flap for the case $\alpha = -5^o$, $g/c_0 = 0.02$. The two distributions are derived as follows: To the upper and lower surface velocity distribution of the first inviscid approximation we add $\left(\partial(\Delta\psi)/\partial z\right)(x,\pm 0)$. For a sufficiently 'thin' boundary layer this procedure yields the velocity distribution at the edge of the boundary layer. Consequently we may use Bernoulli's equation to obtain the static-pressure distribution there. In the case of the first-order method for a 'thin' wake and 'small' flap gap we neglect the squares of $\left(\partial(\Delta\psi)/\partial z\right)(x,\pm 0)/V_\infty$. On the other hand, for the theory further corrected in a first-order way for wake thickness and distance of the wake from the flap (which reduces to a correction for the distance of the wake from the flap if the singularities in the 'thin'-wake method are supposed to be on the mean line of the wake) we include the products and squares of the velocity terms containing $\left(\partial(\Delta\psi)/\partial z\right)(x,\pm 0)$ but neglect the term $\left\{\left(\partial(\Delta\psi_B)/\partial z\right)(x,\pm 0)\right\}^2/V_\infty^2$. Hence, assuming that the static pressure does not vary across the flap boundary layer, we arrive at the surface-pressure distribution. These two approaches are consistent with the corresponding methods used to derive the corrections for the flap lift.

The results of these calculations are shown in Fig.14, where they are plotted in coefficient form, viz:

$$C_p = (p - p_\infty)/\tfrac{1}{2}\rho V_\infty^2 ,$$

and are compared with the pressure distributions of the first inviscid approximation and experiment. Evidently, the change in pressure gradient associated with the correction for non-zero distance of the wake from the flap and wake thickness is small compared with the pressure gradient of the experimental pressure distribution. Although this remains to be proved quantitatively, it seems likely, therefore, that the development of the boundary layer on the flap will not be materially altered by including the thickness-distance correction in the wake contribution. Taking this point of view a stage further we infer from the results given above for $g/c_0 = 0.02$ that the effect of the wake on the lift of the flap, after correction for wake thickness and distance of the wake from the flap, is small. It becomes even smaller if one excludes from the wake correction the contribution of the point source at the shroud trailing edge $\left(\text{e.g. with } g/c_0 = 0.02,\ \alpha = -5^{\circ}\ \Delta C_{L_{F,W}} \text{ is then equal to } -0.001\right)$, implying therefore that the effect of the distributed sources and vortices of the wake on the flap lift is, in fact, very small. A quite adequate approximation for estimating C_{L_F} and also the displacement thickness of the flap boundary layer may therefore be obtained by neglecting the distributed sources and vortices of the wake altogether. The flap-lift coefficient so derived is shown in Table 4 as $C_{L_F}^{(2)}$; and we observe that this coefficient is in much better agreement with C_{L_F} than is $C_{L_F}^{(1)}$. To avoid overcrowding we have not shown the pressure distribution achieved by this method of correction; but we find that it lies very close to curve (b).

Fig.15 shows pressure distributions for the case $g/c_0 = 0.04$, $\alpha = -5^{\circ}$. Again we see that the difference between the distribution containing a first-order correction for the wake effect and the distribution with a further correction for wake thickness and distance from the flap is small. We notice, however, that the curve obtained by neglecting the distributed sources and vortices of the wake (but retaining the effect of the flap boundary layer and the point source) is noticeably different from the other two corrected curves in the region $0.1 < x/c_0 < 0.3$ on the upper surface. However, it is open to question whether this discrepancy would seriously influence the boundary layer on the flap upper surface, since the differences between the pressure gradients of the three cited curves do not seem to be significant.

The agreement between the corrected pressure distributions and the experimental distributions of Figs.14 and 15 cannot be described as good. It is outside the scope of the present investigation to examine, in detail, the reasons for this. We note, however, that the major part of the viscous correction is due to the flap boundary layer for the present flap configurations. Future work on this subject might therefore be concerned with possible higher-order approximations to the velocity field of the vorticity layer comprising the boundary layer of the flap. Since the flow separates from the upper surface of the flap[9] for both the cases exhibited in Figs.14 and 15, a method that is based on the notion of a 'thin' boundary layer, such as the present one, may be inadequate. To illustrate this we note in Fig.15 that, downstream of the separation point at $x/c_0 = 0.29$, the load on the flap is significantly under-estimated by the present method. In this respect it is appropriate to remark that work is currently in hand at the RAE on the determination of a suitable theoretical model for multiple aerofoils having regions of separated flow.

6 CONCLUDING REMARKS

The present investigation shows that the effect of the wake on the lift of the flap is of secondary importance compared with the effect of the flap boundary layer, at least for the flap configurations examined.

Consideration has been given in this Report to the question of how the neglect of the thickness of the wake in the 'thin'-wake theory might influence the results for the flap lift and the pressure distribution of the flap. It is shown that, provided (a) the sources and vortices of the wake are placed on the mean line of the wake, and (b) the displacement flux ψ_W^* is replaced by the pseudo displacement flux

$$\psi_W^{*\prime} = \int_W (\bar{U} - u)dz \quad ,$$

the first-order correction to the 'thin'-wake theory for wake thickness vanishes.

As a first approximation for flap gaps that are small compared with the flap chord, it is reasonable to place the singularities of the wake on the upper surface of the flap, or, if the flap is of small thickness - chord ratio and camber, on the flap chord. The indications of the present calculations are that this approximation overestimates the magnitude of the correction to the lift of the flap for the effect of the wake.

Comparison between pressure distributions calculated by using the various approximations discussed here and the experimental pressure distributions suggest that there is scope for improvement in the present theory. Since the boundary layer contributes the major part of the reduction in the lift of the flap, future work should be directed towards examining the accuracy of the 'thin' boundary-layer approximation.

Finally, it is appropriate to comment on the type of flow that has been considered in this Report. The configurations examined here were chosen because with them it is possible to distinguish between the wake of the main aerofoil and the boundary layer of the flap upper surface. With this type of flow it is possible to envisage a calculation procedure by which the viscous or 'inner' solutions for the wake and the boundary layer can be determined virtually independently of each other, the effect of the one on the other being regarded as a change in the 'outer' velocity distribution. Two of the gap cases studied here, namely $g/c_0 = 0.02$ and 0.025, are close to the gap giving optimum lift and are thus of practical significance. However, Foster, *et al.*[9] remark that, to establish the optimum gap and to calculate the lift for off-design cases, it will be necessary to examine flows in which the boundary layer and the wake cannot be separated. For such flows the viscous part of the solution poses some severe problems; but, in principle, the flow induced in the irrotational part of the flow field by the combined vorticity layer can be determined in much the same manner as was used in sections 2 and 3.

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Appendix A

DERIVATION OF THE FLOW FIELD INDUCED BY AN ARBITRARY VORTICITY DISTRIBUTION

Consider the finite region Σ, which represents the region bounded by the contour c defining the edges of the flap boundary layer and the wake of the flap. This region is illustrated in Fig.2.

Green's second formula[10] for the scalars u and v in the region Σ may be written as

$$\int_c \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right) d\ell = \int_\Sigma (u\nabla^2 v - v\nabla^2 u) dS \quad , \tag{A-1}$$

where the line integration around c is performed in the clockwise sense, $\underset{\sim}{n}$ is the normal vector outward from Σ, $d\ell$ is an element of length, and dS is an element of area.

Firstly, identify u with ψ, the stream function of the (real) flow around the main aerofoil and the flap. Outside Σ, ψ satisfies Poisson's equation[10]

$$\nabla^2 \psi = \eta \quad ,$$

with η the vorticity of the flow within Σ. Here the sign convention is such that η is taken positive if the rotation of the vorticity is clockwise. Hence equation (A-1) may be written as

$$\int_c \left(\psi \frac{\partial v}{\partial n} - v \frac{\partial \psi}{\partial n} \right) d\ell = \int_\Sigma (\psi\nabla^2 v - v\eta) dS \quad . \tag{A-2}$$

Secondly, we observe that the stream function induced at a point P by an elementary vortex, which is of strength ηdS and situated at a point E within Σ, is given by[7]

$$d(\psi)_\Sigma = \eta dS \ln (r)/2\pi \quad , \tag{A-3}$$

where $\underset{\sim}{r}$ is the vector, with origin at P, in the direction PE. Therefore, making the substitution

$$v = - \ln (r)/2\pi$$

for v in equation (A-2), and referring to equation (A-3), we find that the stream function induced at P by the vorticity in Σ is

$$(\psi)_\Sigma = \frac{1}{2\pi} \int_c \left\{ \frac{\partial \psi}{\partial n} \ln r - \psi \frac{\partial}{\partial n} (\ln r) \right\} d\ell + \frac{1}{2\pi} \int_\Sigma \psi \nabla^2 (\ln r) dS \quad . \quad \text{(A-4)}$$

For points P external to the region Σ, equation (A-4) may be reduced further by noting that, in this case,

$$\nabla^2 (\ln r) = 0 \quad .$$

Therefore we have, in the region external to Σ,

$$(\psi)_\Sigma = \frac{1}{2\pi} \int_c \left\{ \frac{\partial \psi}{\partial n} \ln r - \psi \frac{\partial}{\partial n} (\ln r) \right\} d\ell \quad . \quad \text{(A-5)}$$

That is to say, the expression for the induced stream function in the region external to Σ has been reduced from an area integration, as implied by equation (A-3), to a line integration.

Equation (A-5) may be written in an alternative form by noting from simple geometrical and trigonometrical considerations (see Fig.2) that

$$\left. \begin{aligned} \frac{\partial}{\partial n} (\ln r) &= \frac{1}{r} \frac{\partial r}{\partial n} = \pm \frac{\sin \tau}{r} \\ d\ell &= r d\tau / \sin \tau \end{aligned} \right\} \quad . \quad \text{(A-6)}$$

Here τ is the included angle between the ℓ direction and the negative r direction. Further, the positive or negative signs are to be taken depending on whether the vector $\underset{\sim}{r}$ passes, respectively, out of or into Σ at the element $d\ell$. Thus equations (A-5) and (A-6) may be combined to read

$$(\psi)_\Sigma = \frac{1}{2\pi} \int_c \left\{ \frac{\partial \psi}{\partial n} \ln r - \binom{+}{-} \psi \frac{\partial \tau}{\partial \ell} \right\} d\ell \quad . \quad \text{(A-7)}$$

Partial integration of the second term of the integrand of equation (A-7) allows us to write

$$(\psi)_\Sigma = \frac{1}{2\pi} \int_c \left\{ \frac{\partial\psi}{\partial n} \ln r + \left(\substack{+\\-}\right) \frac{\partial\psi}{\partial \ell} \tau \right\} d\ell \quad . \qquad \text{(A-8)}$$

It is convenient to write

$$\psi = \psi_I + \Delta\psi_B + \Delta\psi_W \quad , \qquad \text{(A-9)}$$

where suffix I refers to the first inviscid approximation, $\Delta\psi_B$ is the incremental stream function due to the existence of a boundary layer on the flap, and $\Delta\psi_W$ is the additional stream function resulting from the presence of the wake of the main aerofoil. Associated with each of these components of the stream function is a certain vorticity distribution which will be presumed to be known (e.g. from inviscid-flow calculations and boundary-layer calculations). It follows, therefore, from equation (A-3) that

$$(\psi)_\Sigma = (\psi_I)_\Sigma + (\Delta\psi_B)_\Sigma + (\Delta\psi_W)_\Sigma \quad . \qquad \text{(A-10)}$$

Consequently, upon comparing equations (A-8), (A-9) and (A-10) we have

$$(\Delta\psi_B)_\Sigma = \frac{1}{2\pi} \int_c \left\{ \frac{\partial(\Delta\psi_B)}{\partial n} \ln r + \left(\substack{+\\-}\right) \frac{\partial(\Delta\psi_B)}{\partial \ell} \tau \right\} d\ell \quad . \qquad (1)$$

A similar analysis may be constructed to determine the stream function induced by the vorticity of the wake of the main aerofoil. Calling the region enclosed by the wake Γ and the contour, defining the edges of the wake, k (Fig.7) we have, by analogy with equation (1), the result

$$(\Delta\psi_W)_\Gamma = \frac{1}{2\pi} \int_k \left\{ \frac{\partial(\Delta\psi_W)}{\partial n} \ln r + \left(\substack{+\\-}\right) \frac{\partial(\Delta\psi_W)}{\partial \ell} \tau \right\} d\ell \quad . \qquad \text{(A-11)}$$

External to the region Σ and to the main aerofoil the flow $\Delta\psi_B$ is irrotational. Therefore, neglecting the fact that the wake of the main aerofoil tends to merge with the wake of the flap downstream of the flap trailing edge, we have

$$(\Delta\psi_B)_\Gamma = 0$$

and, in consequence,

$$0 = \frac{1}{2\pi} \int_k \left\{ \frac{\partial(\Delta\psi_B)}{\partial n} \ln r + \binom{+}{-} \frac{\partial(\Delta\psi_B)}{\partial \ell} \tau \right\} d\ell \quad .$$

Hence, combining this expression with equation (A-11) we obtain the result

$$(\Delta\psi_W)_\Gamma = \frac{1}{2\pi} \int_k \left\{ \frac{\partial(\Delta\psi)}{\partial n} \ln r + \binom{+}{-} \frac{\partial(\Delta\psi)}{\partial \ell} \tau \right\} d\ell \quad , \tag{34}$$

where $\Delta\psi = \Delta\psi_W + \Delta\psi_B$.

Appendix B

THE EVALUATION OF THE LIMIT OF AN INTEGRAL

We wish to evaluate the limit

$$I_9 = \frac{1}{2\pi} \lim_{z\to 0} \int_0^\infty f(x') \frac{z}{(x - x')^2 + z^2} dx' \qquad \text{(B-1)}$$

for all x. Provided that f(x) is analytic in the interval $0 \leqslant x \leqslant \infty$ we may use Taylor's series to write

$$f(x') = f(x) + (x' - x)(df/dx)(x) + (x' - x)^2(d^2f/dx^2)(x)/2! + \ldots \ .$$

Therefore, in place of equation (B-1), we have

$$I_9 = \frac{1}{2\pi} \lim_{z\to 0} \left(f(x) \int_0^\infty \frac{z}{(x - x')^2 + z^2} dx' + \frac{df}{dx}(x) \int_0^\infty \frac{(x' - x)z}{(x - x')^2 + z^2} dx' \right.$$
$$\left. + \frac{1}{2!} \frac{d^2f}{dx^2}(x) \int_0^\infty \frac{(x' - x)^2 z}{(x' - x)^2 + z^2} dx' + \ldots \right) .$$

Performing the indicated integration we find that all but the first term vanish in the limit as z tends to zero, and we are left with

$$I_9 = \frac{f(x)}{2\pi} \lim_{z\to 0} \left(\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{z}\right) \right) ,$$

which becomes, on taking the limit,

$$I_9 = \begin{cases} 0 , & x < 0 \\ f(x)/4 , & x = 0 \\ f(x)/2 , & x > 0 \ . \end{cases} \qquad \text{(B-2)}$$

Appendix C

THE STRENGTH OF THE VORTICES SIMULATING THE WAKE OF THE MAIN AEROFOIL

The strength of the wake vortices per unit x-wise length is given by equation (41), viz:

$$\gamma_W = \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx}\right)_+ - \left(\frac{\partial(\Delta\psi)}{\partial n}\frac{d\ell}{dx}\right)_- \quad . \tag{41}$$

For a wake having edges that are substantially parallel to the flap chord we may use the approximation

$$(d\ell)_\pm = \pm\, dx \quad . \tag{C-1}$$

Examination of wake-survey data taken by Foster[16] indicates that the error in γ_W, resulting from the use of this approximation, is negligible. Therefore we may write in place of equation (41)

$$\gamma_W = \left(\frac{\partial(\Delta\psi)}{\partial n}\right)_+ + \left(\frac{\partial(\Delta\psi)}{\partial n}\right)_- \quad ,$$

$$= \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_+ - \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_- - \left(\frac{\partial(\Delta\psi)}{\partial x}\right)_+ \frac{dz_+}{dx} + \left(\frac{\partial(\Delta\psi)}{\partial x}\right)_- \frac{dz_-}{dx} \quad . \tag{C-2}$$

To determine the first two terms on the right-hand side of equation (C-2) we examine the z component of the Navier-Stokes equations[11], viz.:

$$w\frac{\partial w}{\partial z} + u\frac{\partial w}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right) \quad . \tag{C-3}$$

Using the equation of continuity

$$\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} = 0 \tag{C-4}$$

we are able to recast equation (C-3) in the form

$$\frac{\partial p}{\partial z} = -\rho u^2\frac{\partial(w/u)}{\partial x} + \mu\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right) \quad . \tag{C-5}$$

The requirement that the flow is tangential to a streamline may be expressed as

$$\frac{dz_s}{dx}(x;\psi) = \frac{w}{u} , \qquad \text{(C-6)}$$

where suffix s refers to the streamline. Therefore, combining equations (C-5) and (C-6), we have

$$\frac{\partial p}{\partial z} = -\rho u^2 \frac{\partial}{\partial x}\left(\frac{dz_s}{dx}(x;\psi)\right) + \mu\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right) ,$$

$$= -\rho u^2 \left\{\frac{d^2 z_s}{dx^2}(x;\psi) - \frac{\partial}{\partial z}\left(\frac{dz_s}{dx}(x;\psi)\right)\frac{dz_s}{dx}(x;\psi)\right\} + \mu\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2}\right) . \qquad \text{(C-7)}$$

Consequently, integrating equation (C-7) across the wake, we have

$$p_+ - p_- = -\rho \int_W u^2 \frac{d^2 z_s}{dx^2}(x;\psi)dz + \rho \int_W u^2 \frac{\partial}{\partial z}\left(\frac{dz_s}{dx}(x;\psi)\right)\frac{dz_s}{dx}(x;\psi)dz$$

$$+ \mu \left[\frac{\partial w}{\partial z}\right]_-^+ + \mu \int_W \frac{\partial^2 w}{\partial x^2} dz . \qquad \text{(C-8)}$$

The indications of wake surveys[16] are that, in the wake,

$$\frac{c_0}{V_\infty}\frac{\partial u}{\partial x} = 0(1) . \qquad \text{(C-9)}$$

Therefore, by using equation (C-4), we find that the third term on the right-hand side of equation (C-8) is $0(\nu/V_\infty c_0)$ compared with ρV_∞^2. Therefore, under the conditions of the experiments of Foster, *et al.*[9], in which $V_\infty c_0/\nu$ was 3.8×10^6, the cited term may be considered to be negligible.

If

$$\delta_W = \int_W dz$$

is the thickness of the wake we may infer from equations (C-4) and (C-9) that

$$w = O[V_\infty \delta_W / c_0] \ .$$

Hence, regarding x/c_F as being of $O(1)$, we deduce that the last term of equation (C-8) is $O[\nu(\delta_W/c_F)^2/V_\infty c_0]$ compared with ρV_∞^2. Therefore, noting that $\delta_W/c_F \ll 1$, we conclude that the last term, like the previous one considered, is negligible. Consequently, we may disregard the last two terms of equation (C-8) to obtain instead

$$p_+ - p_- = -\rho \int_W u^2 \frac{d^2 z_s}{dx^2}(x;\psi)dz + \rho \int_W u^2 \frac{\partial}{\partial z}\left(\frac{dz_s}{dx}(x;\psi)\right)\frac{dz_s}{dx}(x;\psi)dz \ . \qquad \text{(C-10)}$$

In other words, for the purpose of deriving the rise in static pressure across the wake, the flow may be supposed inviscid.

Examining the orders of magnitude of the two terms on the right-hand side of equation (C-10) we find that the first of these is $O\left(\frac{d^2 z_s}{dx^2}\delta_W\right)$ compared with ρV_∞^2, whilst the second is $O\left[\left(\frac{dz_s}{dx}\right)^2\right]$ in comparison with ρV_∞^2. Results obtained experimentally by Foster[16] indicate that, typically, dz_s/dx is of the order of 0.01 in the wake above the flap, whilst $\frac{d^2 z_s}{dx^2}\delta_W$ reaches a value of the order of 0.1 near the flap trailing edge. This implies that, for the region above the flap, the last term in equation (C-10) is very small compared with the first term on the right-hand side. Further downstream, we would expect the situation to be somewhat different. However, since it is probably true that the vortex strengths of the wake of the main aerofoil and of the flap wake reach their respective maximum values near the trailing edge of the flap, we will disregard the last term of equation (C-10). Similarly, noting from simple geometrical considerations that the curvature of each streamline is given by

$$\kappa = -\frac{d^2 z_s}{dx^2}(x;\psi)\left\{1 + \left(\frac{dz_s}{dx}\right)^2 (x;\psi)\right\}^{-\frac{3}{2}}$$

(with the centre of curvature taken below the streamline) it is consistent with the last approximation to replace $d^2 z_s/dx^2$ in equation (C-10) by $-\kappa$. Therefore the rise in static pressure across the wake becomes finally

$$p_+ - p_- = \rho \int_W u^2 \kappa dz \quad . \tag{C-11}$$

Outside the wake, in the irrotational part of the flow field, the total head is constant. Hence, using Bernoulli's equation, we have in place of equation (C-11)

$$-\tfrac{1}{2}\rho \left\{ \left(\frac{\partial\psi}{\partial z}\right)_+^2 + \left(\frac{\partial\psi}{\partial x}\right)_+^2 - \left(\frac{\partial\psi}{\partial z}\right)_-^2 - \left(\frac{\partial\psi}{\partial x}\right)_-^2 \right\} = \rho \int_W u^2 \kappa dz \quad . \tag{C-12}$$

In consequence of the observation that $(dz_s/dx)^2 \ll 1$ in the region where the vortex strength is most significant we may disregard the terms $(\partial\psi/\partial x)_\pm^2$ in equation (C-12) compared with $(\partial\psi/\partial z)_\pm^2$. Therefore equation (C-12) may be rewritten as

$$-\tfrac{1}{2}\rho \left\{ \left(\frac{\partial\psi}{\partial z}\right)_+^2 - \left(\frac{\partial\psi}{\partial z}\right)_-^2 \right\} = \rho \int_W u^2 \kappa dz \quad .$$

In turn, we may rewrite this as

$$\left(\frac{\partial\psi}{\partial z}\right)_+ - \left(\frac{\partial\psi}{\partial z}\right)_- = -\frac{\kappa_W}{\bar{U}} \int_W u^2 dz \quad , \tag{C-13}$$

where $\bar{U} = \tfrac{1}{2}\left\{ \left(\frac{\partial\psi}{\partial z}\right)_+ + \left(\frac{\partial\psi}{\partial z}\right)_- \right\}$,

and

$$\kappa_W = \int_W u^2 \kappa dz \Big/ \int_W u^2 dz \tag{C-14}$$

is the weighted mean curvature of the streamlines of the wake.

Defining the wake displacement and momentum thicknesses as

$$\left.\begin{aligned} \delta^*_W &= \int_W \left(1 - \frac{u}{\bar{U}}\right) dz \\ \theta_W &= \int_W \frac{u}{\bar{U}} \left(1 - \frac{u}{\bar{U}}\right) dz \end{aligned}\right\} \tag{C-15}$$

we may rewrite equation (C-13) in the following manner:

$$\left(\frac{\partial\psi}{\partial z}\right)_+ - \left(\frac{\partial\psi}{\partial z}\right)_- = \kappa_W \bar{U}\left(\delta^*_W + \theta_W - \delta_W\right) . \tag{C-16}$$

In the first inviscid approximation the streamline slopes and curvatures are slightly different from those of the real flow. However, the approximations leading from equation (C-10) to equation (C-16) would appear to be equally valid in the case of the Kutta approximation. Therefore noting from equations (C-15) that, in the first inviscid approximation, provided $\kappa_W \delta_W$ is small,

$$\theta_W = \delta^*_W = 0 ,$$

we may use equation (C-16) to write

$$\left(\frac{\partial\psi_I}{\partial z}\right)_+ - \left(\frac{\partial\psi_I}{\partial z}\right)_- = -(\kappa_W)_I \bar{U}_I \delta_W . \tag{C-17}$$

Consequently, recalling that

$$\Delta\psi = \psi - \psi_I ,$$

we may combine equations (C-2), (C-16) and (C-17) to obtain the result

$$\gamma_W = \kappa_W \bar{U}(\delta^*_W + \theta_W) - \left(\kappa_W \bar{U} - (\kappa_W)_I \bar{U}_I\right)\delta_W - \left(\frac{\partial(\Delta\psi)}{\partial x}\right)_+ \frac{dz_+}{dx} + \left(\frac{\partial(\Delta\psi)}{\partial x}\right)_- \frac{dz_-}{dx} . \tag{C-18}$$

If the wake of the main aerofoil and the boundary layer of the flap are both 'thin' we would expect that

$$\kappa_W \bar{U} - (\kappa_W)_I \bar{U}_I \ll \kappa_W \bar{U}$$

as a result of the fact that κ_W and $\bar{U}$ are close to their corresponding inviscid values in this case. Additionally, for a 'thin' wake at a 'small' distance above the flap chord, the last two terms of equation (C-18) are small compared with the incremental velocities associated with the 'thin'-wake approximation. Hence, as a first approximation for a 'thin' wake at a small distance above a flap with a 'thin' boundary layer, we write

$$\gamma_W = \kappa_W \bar{U}(\delta_W^* + \theta_W) \quad . \tag{48}$$

This result is similar to an expression derived by Spence and Beasley[4] for the wake of an isolated aerofoil, the only difference being that these authors used the approximation $\bar{U} = V_\infty$. In the region of the trailing edge of the flap, where γ_W/V_∞ generally attains its greatest magnitude (at least for the part of the wake above the flap) this seems to be a good approximation.

As we shall see later we may also confidently use equation (48) to calculate the vortex effect of the (thin) wake of the flap.

With a correction for the effect of wake thickness and the non-zero distance of the wake from the flap the strength, per unit x-wise length, of the vortices simulating the wake of the main aerofoil may, by reference to equation (82), be written as

$$\bar{\gamma}_W = \gamma_W - \frac{d}{dx}\left(\frac{d\psi_W^*}{dx} z_+\right) + \frac{d}{dx}\left(\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0)(z_+ - z_-)\right) - \frac{d}{dx}\left(\frac{d\psi_U^*}{dx}(z_+ - z_-)\right) \quad . \tag{82}$$

The last term in this equation is rather curious, since it seems to imply that, even if the wake did not exist, so that γ_W and ψ_W^* were both zero, there would, nevertheless, be a distribution of vortices above the flap. This apparent contradiction may be resolved by using equation (C-18) for γ_W instead of equation (48).

The second term on the right-hand side of equation (C-18), $-\left(\kappa_W\bar{U} - (\kappa_W)_I\bar{U}_I\right)\delta_W$, may be interpreted as the increase in the jump in x-wise velocity across the wake resulting from:

(a) the flow induced at the wake by the vorticity of the flap boundary layer;

(b) the effect of non-zero wake thickness on the velocities induced at the edges of the wake by the vorticity, within and exterior to the wake, associated with the wake.

Contribution (a) is found by noting from equations (11) that the vorticity of the flap boundary layer induces an increment in stream function at the upper edge of the flap boundary layer or flap wake

$$(\Delta\psi_B)_U = -\psi_U^*(x) .$$

Since the flap is supposed to be of small thickness-chord ratio and camber and the flap boundary layer is considered to be 'thin' we may rewrite this expression as

$$\Delta\psi_B(x,0) = -\psi_U^*(x) .$$

We have also assumed the wake to be at a 'small' distance above the flap chord. Hence, in the region occupied by the wake,

$$\Delta\psi_B(x,z) = -\psi_U^*(x) \qquad \text{(C-19)}$$

to a good approximation. Therefore, using the fact that, in the region external to Σ and to the main aerofoil, the flow induced by the vorticity of the flap boundary layer is irrotational, we have in this region

$$\frac{\partial^2(\Delta\psi_B)}{\partial z^2} + \frac{\partial^2(\Delta\psi_B)}{\partial x^2} = 0 . \qquad \text{(C-20)}$$

Hence, upon combining equations (C-19) and (C-20), we have for the region of the wake

$$\frac{\partial^2(\Delta\psi_B)}{\partial z^2} = \frac{d^2\psi_U^*}{dx^2} .$$

Thus, by integrating this equation with respect to z across the wake, we find that

$$\left(\frac{\partial(\Delta\psi_B)}{\partial z}\right)_+ - \left(\frac{\partial(\Delta\psi_B)}{\partial z}\right)_- = \frac{d^2\psi_U^*}{dx^2}(z_+ - z_-) \quad . \tag{C-21}$$

We determine contribution (b) by examining the downwash induced at the wake by the vorticity resulting from the presence of the wake. For a 'thin' wake and a sufficiently 'small' flap gap, the downwash, caused by the existence of the wake, just below the wake may be written as

$$\left(\frac{\partial(\Delta\psi_W)}{\partial x}\right)_- = \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) \quad .$$

Therefore, after allowing for the presence of the wake-source distribution (of linear strength $d\psi_W^*/dx$) we find that the downwash induced at the wake itself by the vorticity identified with the wake is given by

$$\left(\frac{\partial(\Delta\psi_W)}{\partial x}\right)_{wake} = \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi_W^*}{dx} \quad . \tag{C-22}$$

To obtain contribution (b) we have to subtract from this expression the downwash induced at the wake by the vorticity in that portion of the wake a distance of $O(\delta_W)$ either side of the x-wise position under consideration. This is the 'near field' or 'inner' vorticity which determines the value of γ_W in the approximation for a 'thin' wake and a 'thin' flap boundary layer. The remainder is therefore irrotational in the inner region; and, for sufficiently small δ_W/c_0, this remainder may be regarded as the Cauchy principal value of the downwash induced at the wake by the vorticity associated with the wake. We may write therefore

$$\left(\frac{\partial(\Delta\psi_W)_b}{\partial x}\right)_{wake} = \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi_W^*}{dx} \quad , \tag{C-23}$$

with the Cauchy principal value of the right-hand side being understood, and suffix b referring to contribution (b).

In consequence of the irrotationality of the flow (b) we have for the region of the wake

$$\frac{\partial^2(\Delta\psi_W)_b}{\partial z^2} = -\frac{\partial^2(\Delta\psi_W)_b}{\partial x^2} ,$$

$$= -\frac{d}{dx}\left(\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi_W^*}{dx}\right) ,$$

from equation (C-23). Hence, upon integrating this equation across the wake, we have

$$\left(\frac{\partial(\Delta\psi_W)_b}{\partial z}\right)_+ - \left(\frac{\partial(\Delta\psi_W)_b}{\partial z}\right)_- = -\frac{d}{dx}\left(\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi_W^*}{dx}\right)(z_+ - z_-) . \quad \text{(C-24)}$$

Therefore, combining the two contributions (a) and (b) given by equations (C-21) and (C-24), we may rewrite the second term on the right-hand side of equation (C-18) as

$$-\left(\kappa_W\bar{U} - (\kappa_W)_I\bar{U}_I\right)\delta_W = \frac{d^2\psi_U^*}{dx^2}(z_+ - z_-) - \frac{d}{dx}\left(\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi_W^*}{dx}\right)(z_+ - z_-) .$$

.... (C-25)

It only remains to determine the last two terms of equation (C-18). For 'small' wake thickness and flap gap

$$\left(\frac{\partial(\Delta\psi)}{\partial x}\right)_- = \frac{\partial(\Delta\psi^{(1)})}{\partial x}(x,+0) ,$$

$$= \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{d\psi_U^*}{dx} \quad \text{(C-26)}$$

from equation (81). Similarly, referring to equation (42), we find that

$$\left(\frac{\partial(\Delta\psi)}{\partial x}\right)_+ = \frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0) - \frac{d\psi_U^*}{dx} - \frac{d\psi_W^*}{dx} . \quad \text{(C-27)}$$

Therefore, combining equations (C-18), (82), (C-25), (C-26) and (C-27), we obtain finally

$$\bar{\gamma}_W = \kappa_W\bar{U}(\delta_W^* + \theta_W) - \frac{1}{2}\frac{d^2\psi_W^*}{dx^2}(z_+ + z_-) . \quad (83)$$

A similar analysis could be used to determine the second order vortex strength of the wake of the flap (including a first-order correction for wake thickness and distance of the wake from the x axis). We find, in fact, that, if the suffix W is supposed to refer to the flap wake, an expression that is identical to equation (83) is obtained. However, we have assumed that the flap wake is 'thin'. Therefore, the term $(z_+ + z_-)/2$ in equation (83) may be supposed identical to the ordinate of the rear dividing streamline of the flap. Various experimental results for isolated aerofoils[13,14] indicate that $(d^2\psi^*_W/dx^2)(c_0/V_\infty)$ differs significantly from zero only close to the flap trailing edge, where the ordinate of the rear dividing streamline is small compared with c_0. Therefore it seems reasonable to neglect the last term of equation (83) in the case of the flap wake, implying, therefore, that we may estimate the vortex strength of the flap wake by using equation (48).

We observe that in equations (48) and (83) we are faced with the difficulty of having to evaluate κ_W to determine the vortex strength. This may be obviated, however, by using equation (C-16) to eliminate κ_W from equations (48) and (83). Thus, for example, we have in place of equation (48)

$$\gamma_W = -\left\{\left(\frac{\partial\psi}{\partial z}\right)_+ - \left(\frac{\partial\psi}{\partial z}\right)_-\right\}\left(\frac{\delta^*_W + \theta_W}{\delta_W - \delta^*_W - \theta_W}\right) . \tag{C-28}$$

Appendix D

MANIPULATION OF THE WAKE VORTEX INTEGRAL

We use here a method given by Spence[18] for transforming the double integral

$$I_{10} = \frac{1}{\pi^2}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \left(\int_{c_E}^{\infty} \gamma_W \frac{d\xi_W}{\xi_W - \xi'} \right) \frac{d\xi'}{\xi' - \xi} \qquad \text{(D-1)}$$

into a single integral.

Spence observes that, since ξ is in $(0, c_E)$ there is no residual at the singularity of the integrand at $\xi' = \xi_W = c_E$. Consequently we may interchange the order of the integration in equation (D-1) and by using the fact that

$$\frac{1}{(\xi_W - \xi')(\xi' - \xi)} = \frac{1}{\xi_W - \xi}\left(\frac{1}{\xi' - \xi} - \frac{1}{\xi' - \xi_W}\right)$$

we may rewrite I_{10} as

$$I_{10} = \frac{1}{\pi^2}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \frac{\gamma_W}{\xi_W - \xi} \left(\int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \left(\frac{1}{\xi' - \xi} - \frac{1}{\xi' - \xi_W}\right) d\xi' \right) d\xi_W$$

The integration with respect to ξ' may be performed by using the following identities given by Spence

$$\frac{1}{\pi}\int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \frac{d\xi'}{\xi' - \xi} = \begin{cases} 1 , & 0 \leqslant \xi < c_E \\ 1 - \left(\dfrac{\xi}{\xi - c_E}\right)^{\frac{1}{2}} , & c_E \leqslant \xi \leqslant \infty . \end{cases}$$

Therefore we have finally

$$I_{10} \equiv \frac{1}{\pi^2}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_0^{c_E} \left(\frac{\xi'}{c_E - \xi'}\right)^{\frac{1}{2}} \left(\int_{c_E}^{\infty} \gamma_W \frac{d\xi_W}{\xi_W - \xi'} \right) \frac{d\xi'}{\xi' - \xi} ,$$

$$= \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi_W}{\xi_W - c_E}\right)^{\frac{1}{2}} \gamma_W \frac{d\xi_W}{\xi_W - \xi} .$$

Appendix E

THE LIMIT OF P AND Q INFINITELY FAR DOWNSTREAM

Downstream of the flap trailing edge the wakes of the main aerofoil and the flap merge; hence it is no longer possible to distinguish the separate effects of either wake. Since, however, we have supposed the wake of the flap to be 'thin' and since we are concerned here with the thickness effect of the aerofoil wake we will consider the merged wake to be made up entirely of the wake of the main aerofoil.

According to the mixing-length theory[19] a turbulent, twodimensional wake grows like $x^{\frac{1}{2}}$ far downstream. Therefore we have, for small α and β,

$$z_+ - z_- \sim ax^{\frac{1}{2}} \quad , \qquad x \to \infty \quad , \qquad \text{(E-1)}$$

where a is a finite, non-zero constant.

To determine the asymptotic behaviour of z_+ and z_- separately we require another relationship between z_+, z_- and x. Unfortunately, there appears to be little theoretical or experimental information available on the far-downstream behaviour of curved, turbulent wakes. Those results which are available from experiments on isolated aerofoils[13,14] appear to indicate that the wake tends to become symmetrically disposed with respect to the rear dividing streamline far downstream. We will assume, therefore, that, in the present case, the combined wake is symmetrical with regard to the rear dividing streamline of the main aerofoil for sufficiently large x. Hence, by considering the behaviour of this streamline far downstream, we find that

$$z_+ + z_- \sim b(\alpha + \beta)x \quad , \qquad x \to \infty \quad , \qquad \text{(E-2)}$$

with b a finite, non-zero constant and α and β supposed small, as before. Hence, combining equations (E-1) and (E-2), we obtain the result

$$z_+ \quad , \quad z_- \propto x^{\frac{1}{2}} \quad , \qquad x \to \infty \quad . \qquad \text{(E-3)}$$

In order to determine the asymptotic behaviour of P and Q it only remains to examine the velocity corrections $\partial(\Delta\psi)/\partial z$ and $\partial(\Delta\psi)/\partial x$ in the far field. This we do by observing (see Appendix A) that the correction to the irrotational part of the flow field for the effects of viscosity can be obtained by placing suitable distributions of sources and vortices along the edges of the boundary layers and the wakes. Sufficiently far downstream the effect of these distributions may be represented by a point source and a point vortex placed close to the flap. For convenience, we position these singularities at the leading edge of the flap. Hence we are able to deduce that

$$\left.\begin{aligned} \frac{\partial(\Delta\psi)}{\partial z} &\sim K_1 \frac{z}{x^2+z^2} + K_2 \frac{x}{x^2+z^2} \\ \frac{\partial(\Delta\psi)}{\partial x} &\sim K_1 \frac{x}{x^2+z^2} - K_2 \frac{z}{x^2+z^2} \end{aligned}\right\} , \qquad x \to \infty , \qquad \text{(E-4)}$$

where K_1 and K_2 are constants that are proportional to the vortex and source strengths, respectively. Therefore, noting that

$$P = \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_+ z_+ - \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_- z_- \qquad (64)$$

it is possible to infer from equations (E-3) and (E-4) that

$$\lim_{x\to\infty} (P) = 0 .$$

Similarly, recalling that

$$Q = \frac{d(\Delta\psi)_+}{dx} z_+ - \frac{d(\Delta\psi)_-}{dx} z_- \qquad (64)$$

and employing the result

$$\frac{d(\Delta\psi)_\pm}{dx} = \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_\pm \frac{dz_\pm}{dx} + \left(\frac{\partial(\Delta\psi)}{\partial x}\right)_\pm$$

it is readily inferred from equations (E-3) and (E-4) that

$$\lim_{x\to\infty} (Q) = 0 .$$

Appendix F

DERIVATION OF THE EFFECTIVE DISPLACEMENT FLUX $\bar{\psi}^*_W$

The effective displacement flux, $\bar{\psi}^*_W$, may be written, by reference to equation (85), as

$$\bar{\psi}^*_W = \psi^*_W + \gamma_W^{(1)} z_+ + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0)(z_+ - z_-) \quad , \tag{85}$$

$$= \psi^*_W + \frac{\gamma_W^{(1)}}{2}(z_+ + z_-) + \left\{\frac{\gamma_W^{(1)}}{2} + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0)\right\}(z_+ - z_-) \quad . \tag{F-1}$$

The first term on the right-hand side of equation (F-1),

$$\psi^*_W = \int_W \{u_I(z) - u(z)\} dz \quad ,$$

may be written as

$$\psi^*_W = \psi^{*\prime}_W - \left\{\bar{U}(z_+ - z_-) - \int_W u_I(z) dz\right\} \quad , \tag{F-2}$$

where $\psi^{*\prime}_W = \int_W \{\bar{U} - u(z)\} dz$

is referred to as the pseudo displacement flux.

Suppose firstly that all velocities in equations (F-1) and (F-2) are normalized with respect to V_∞ and all lengths with respect to c_0. Thus, as will be seen later, the last term of equation (F-2) is of the same order as the last term in equation (F-1). Consequently, if we define

$$\Delta\bar{U} = \bar{U} - \left(\int_W u_I(z) dz\right) \Big/ (z_+ - z_-) \quad , \tag{F-3}$$

we are able to write in place of equation (F-2)

$$\psi^*_W = \psi^{*\prime}_W - \Delta\bar{U}(z_+ - z_-) \quad , \tag{F-4}$$

where $\Delta\bar{U}$ is of the same order as the curly-bracketed term in equation (F-1). Hence the error in $\bar{\psi}_W^*$ resulting from the use of the theory of section 3.1 (for a 'thin' wake and a 'small' flap gap) to determine $\Delta\bar{U}$ is no greater than the errors in $\bar{\psi}_W^*$ implicit in the use of $\gamma_W^{(1)}$ and $\partial(\Delta\psi^{(1)})/\partial z$ in place of γ_W and $\partial(\Delta\psi)/\partial z$ in equation (85). In the following, therefore, we derive $\Delta\bar{U}$ on the basis of these approximations.

The x-wise velocity at the lower edge of the wake

$$\left(\frac{\partial\psi}{\partial z}\right)_- = u_I(z_-) + \left(\frac{\partial(\Delta\psi)}{\partial z}\right)_- ,$$

$$= u_I(z_-) + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) , \qquad \text{(F-5)}$$

for a 'thin' wake and 'small' flap gap. Similarly, we have for the upper edge of the wake

$$\left(\frac{\partial\psi}{\partial z}\right)_+ = u_I(z_+) + \left(\frac{\partial(\Delta\psi^{(1)})}{\partial z}\right)_+ ,$$

$$= u_I(z_+) + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) + \gamma_W^{(1)} , \qquad \text{(F-6)}$$

from equations (77).

To determine the relationship between $u_I(z_+)$ and $u_I(z_-)$ we use Taylor's theorem

$$u_I(z) = u_I(z_-) + \frac{\partial u_I}{\partial z}(z_-)(z - z_-) + \frac{1}{2}\frac{\partial^2 u_I}{\partial z^2}(z_-)(z - z_-) + 0\left(u_I(z_-)(z - z_-)^3\right) ,$$

.... (F-7)

wherein we have assumed that the curvature of the streamlines of the first inviscid approximation are such that the z derivatives of $u_I(z)$ in the wake exist and are, at most, of order $u_I(z_-)$. Hence replacing z by z_+ in equation (F-7) and combining the resulting expression with equations (F-5) and (F-6) we find that

$$\bar{U} = \frac{1}{2}\left\{\left(\frac{\partial\psi}{\partial z}\right)_+ + \left(\frac{\partial\psi}{\partial z}\right)_-\right\} = u_I(z_-) + \frac{1}{2}\frac{\partial u_I}{\partial z}(z_-)(z_+ - z_-)$$

$$+ \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) + \frac{\gamma_W^{(1)}}{2} + 0\left(u_I(z_-)(z_+ - z_-)^2\right) \quad . \quad \text{(F-8)}$$

Equation (F-7) may be used to derive the expression

$$\int_W u_I(z)dz = u_I(z_-)(z_+ - z_-) + \frac{1}{2}\frac{\partial u_I}{\partial z}(z_-)(z_+ - z_-)^2 + 0\left(u_I(z_-)(z_+ - z_-)^3\right). \quad \text{(F-9)}$$

Therefore we may use equations (F-8) and (F-9) to replace equation (F-3) by the expression

$$\Delta\bar{U} = \frac{\gamma_W^{(1)}}{2} + \frac{\partial(\Delta\psi^{(1)})}{\partial z}(x,+0) + 0\left(u_I(z_-)(z_+ - z_-)^2\right) \quad . \quad \text{(F-10)}$$

The last term of equation (F-10) may be neglected on the basis that it is, at most, of order $(z_+ - z_-)$ compared with the term

$$\frac{\gamma_W^{(1)}}{2} + \frac{\partial(\Delta\psi_W^{(1)})}{\partial z}(x,+0)$$

which comprises the contribution of the wake to the first two terms of the last-named equation. Hence, as asserted before, $\Delta\bar{U}$ is of the same order as the curly-bracketed term of equation (F-1). Therefore, upon disregarding the last term of equation (F-10) and combining the modified expression with equations (F-1) and (F-4), we have finally that

$$\bar{\psi}_W^* = \psi_W^{*\prime} + \frac{\gamma_W^{(1)}}{2}(z_+ + z_-) \quad . \quad (86)$$

Appendix G

THE ANALOGY BETWEEN THE WAKE DOWNSTREAM OF THE FLAP AND THE JET SHEET OF A BLOWN FLAP

For 'small' wake thickness and 'small' flap gap the linear strength of the vortex distribution of the combined aerofoil-flap wake is given by

$$\gamma_W = \kappa_W \bar{U}(\delta^*_W + \theta_W) \quad , \tag{48}$$

provided it is understood that the suffix W refers to the combined wake.

Spence and Beasley[4] remark that δ^*_W and θ_W are close to their asymptotic values far downstream except near to the trailing edge of the flap. They therefore suggest the use of the approximation

$$\delta^*_W = \theta_W = (\theta_W)_\infty = C_D c_0/2 \quad , \tag{G-1}$$

where $C_D = D/\frac{1}{2}\rho V_\infty^2 c_0$

is the drag coefficient of the configuration. Similarly, the indications of pressure distributions[9] are that $\bar{U}$ is nearly equal to V_∞ even close to the flap trailing edge. Therefore, in place of equation (48), we may write

$$\gamma_W = \kappa_W V_\infty c_0 C_D \quad . \tag{G-2}$$

The strength of the vortex distribution of the jet sheet of a jet flap or blown flap is given by[18]

$$\gamma_J = -\kappa_J V_\infty c_E C_J/2 \quad , \tag{G-3}$$

with

$$C_J = J/\tfrac{1}{2}\rho V_\infty^2 c_E$$

the jet momentum coefficient, which, following convention, we base on c_E.

We see that, for the vortex distributions of the two systems to be identical, equations (G-2) and (G-3) require that

$$\kappa_W c_0 C_D = -\kappa_J c_E C_J/2 \quad . \tag{G-4}$$

Now, for sufficiently small C_J and C_D, we expect the curvature of the jet or the wake to be approximately equal to the curvature of the rear dividing streamline of the flap as predicted by the first inviscid approximation in the case of no flap blowing. In consequence, for small C_D and C_J, we have as an approximation

$$\kappa_W = \kappa_J \quad .$$

It follows, therefore, from equation (G-4) that the two cases are analogous, at least approximately, provided that

$$C_J = -2c_0 C_D/c_E \quad . \qquad \text{(G-5)}$$

Spence[17,18] shows that the vortex distribution, needed on the chord of an aerofoil with a blown flap to nullify the downwash induced by the vortices of the jet sheet, is given by

$$\begin{aligned}\Delta\gamma_{,J}(\xi) &= \frac{1}{\pi}\left(\frac{c_E - \xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty} \left(\frac{\xi'}{\xi' - c_E}\right)^{\frac{1}{2}} \gamma_J \frac{d\xi'}{\xi' - \xi} \\ &= 2V_\infty \left(\frac{c_E}{\xi}\right)^{\frac{3}{2}} \left[\alpha \left\{B_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} B_n X^n\right\}\right. \\ &\qquad \left. + \beta \left\{D_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} D_n X^n\right\}\right] \quad . \qquad \text{(G-6)}\end{aligned}$$

Here $X = \dfrac{1 - (1 - \xi/c_E)^{\frac{1}{2}}}{1 + (1 - \xi/c_E)^{\frac{1}{2}}}$

and B_n, D_n are solutions of the linear equations[18]

$$\left.\begin{aligned}\sum_{n=0}^{M-1}\left(a_{mn} + \frac{4}{C_J} b_{mn}\right) D_n &= f_m(\chi) \\ \sum_{n=0}^{M-1}\left(a_{mn} + \frac{4}{C_J} b_{mn}\right) B_n &= f_m(\pi)\end{aligned}\right\} \; , \quad m = 0,1,2,\ldots M-1 \quad . \qquad \text{(G-7)}$$

The coefficients a_{mn} and b_{mn} and the function $f_m(\chi)$ are defined as follows:

$$\left.\begin{aligned} a_{m0} &= \sin\phi_m \quad ; \quad a_{mn} = (1+\cos\phi_m)\sin n\,\phi_m, \qquad n > 0 \quad ; \\ b_{mn} &= \frac{4}{4n^2-1}\left(\cos n\,\phi_m + 2n\tan\frac{\phi_m}{2}\sin n\,\phi_m\right) \quad ; \\ f_m(\chi) &= \frac{2\chi}{\pi}\tan\frac{\phi_m}{2} - \frac{4}{\pi}\sec\frac{\phi_m}{2}\tan^{-1}\left(\tan\frac{\chi}{2}\Big/\sin\frac{\phi_m}{2}\right), \end{aligned}\right\} \qquad \text{(G-8)}$$

where $\phi_m = m\pi/M$, $m = 0,1,2,\ldots M-1$.

Inspection of equations (G-7) and (G-8) reveals that a valid approximation to equations (G-7) for small C_J is obtained by neglecting the terms a_{mn}, viz:

$$\left.\begin{aligned} \sum_{n=0}^{M-1} b_{mn} D_n &= \frac{C_J}{4} f_m(\chi) \\ \sum_{n=0}^{M-1} b_{mn} B_n &= \frac{C_J}{4} f_m(\pi) \end{aligned}\right\}, \qquad m = 0,1,2,\ldots M-1 \ . \qquad \text{(G-9)}$$

Equations (G-5), (G-6) and (G-9) allow us to write down the analogous relationships

$$\frac{1}{\pi}\left(\frac{c_E-\xi}{\xi}\right)^{\frac{1}{2}} \int_{c_E}^{\infty}\left(\frac{\xi'}{\xi'-c_E}\right)^{\frac{1}{2}} \gamma_W \frac{d\xi'}{\xi'-\xi}$$

$$= 2V_\infty\left(\frac{c_E}{\xi}\right)^{\frac{3}{2}}\left[\alpha\left\{B_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} B_n X^n\right\} + \beta\left\{D_0\left(\frac{2X}{1+X}\right) + \sum_{n=1}^{\infty} D_n X^n\right\}\right], \qquad (102)$$

with

$$\left.\begin{aligned} \sum_{n=0}^{M-1} b_{mn} D_n &= -\frac{c_0}{c_E}\frac{C_D}{2} f_m(\chi) \\ \sum_{n=0}^{M-1} b_{mn} B_n &= -\frac{c_0}{c_E}\frac{C_D}{2} f_m(\pi) \end{aligned}\right\}, \qquad m = 0,1,2,\ldots M-1 \ . \qquad (103)$$

These results may be extended to allow for the non-zero thickness of the wake of the main aerofoil as well as the non-zero distance of this wake from the x-axis. With this allowance the vortex strength becomes

$$\bar{\gamma}_W = \left| \kappa_W \bar{U}(\delta^*_W + \theta_W) \right|_{\substack{\text{aerofoil}\\ +\text{ flap}}} - \left| \frac{1}{2} \frac{d^2\psi^*_W}{dx^2} (z_+ + z_-) \right|_{\text{aerofoil}} , \quad x \geqslant c_F . \tag{84}$$

An examination of Spence's[18] analysis of the blown flap shows that the approximation leading to equations (103), namely the neglect of the terms a_{mn} of equation (G-7), is equivalent to the assumption that

$$\kappa_W = (\kappa_W)_I ,$$

which is consistent with the assumption of section 3.1 that the combined wake is 'thin'. It is not sufficient, therefore, in the calculation of $\bar{\gamma}_W$, to determine the first term on the right-hand side of equation (84) by the method given above. To be consistent with this equation we have to allow for the change in the curvature of the combined wake due to the effect of viscosity. The appropriate change in the curvature of the wake of the main aerofoil may be inferred from equation (C-25). Thus using, as before, the approximation

$$\bar{U} = \bar{U}_I = V_\infty \tag{G-10}$$

for stations downstream of the flap trailing edge, and employing the fact that

$$\delta_W = z_+ - z_-$$

we may rewrite equation (C-25) as

$$\left| \kappa_W - (\kappa_W)_I \right|_{\text{aerofoil}} = - \frac{1}{V_\infty} \frac{d^2\psi^*_U}{dx^2} + \frac{1}{V_\infty} \left| \frac{d}{dx} \left(\frac{\partial (\Delta\psi_W^{(1)})}{\partial x} (x,+0) - \frac{1}{2} \frac{d\psi^*_W}{dx} \right) \right|_{\text{aerofoil}} . \quad \text{.... (G-11)}$$

Since the flap wake is supposed 'thin' (section 2.1), it is probable that no extra accuracy in the vortex strength of the flap-wake component of the combined wake can be expected by incorporating the viscous correction to

$|(\kappa_W)_I|_{flap}$. Therefore we apply the curvature correction only to the aerofoil constituent of the combined wake. Consequently, we have, by employing equations (84), (G-10) and (G-11)

$$\bar{\gamma}_W = \left|(\kappa_W)_I V_\infty(\delta^*_W + \theta_W)\right|_{\substack{aerofoil\\+flap}}$$
$$- \left\{\frac{d^2\psi^*_U}{dx^2} - \left|\frac{d}{dx}\left(\frac{\partial(\Delta\psi^{(1)}_W)}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi^*_W}{dx}\right)\right|_{aerofoil}\right\}\left|\delta^*_W + \theta_W\right|_{aerofoil}$$
$$- \left|\frac{1}{2}\frac{d^2\psi^*_W}{dx^2}(z_+ + z_-)\right|_{aerofoil} . \qquad \text{(G-12)}$$

The effect of the first term on the right-hand side of equation (G-12) on the vortex distributions of the main aerofoil and the flap having already been considered we examine the effect of the last two terms. Thus the increment in circulation, that is identified with these terms, around the combined aerofoil-flap wake is given by

$$\Delta\Gamma_W = -\int_{c_F}^{\infty}\left\{\frac{d^2\psi^*_U}{dx^2} - \frac{d}{dx}\left(\frac{\partial(\Delta\psi^{(1)}_W)}{\partial x}(x,+0) - \frac{1}{2}\frac{d\psi^*_W}{dx}\right)\right\}(\delta^*_W + \theta_W)dx$$
$$- \frac{1}{2}\int_{c_F}^{\infty}\frac{d^2\psi^*_W}{dx^2}(z_+ + z_-)dx , \qquad \text{(G-13)}$$

where it is to be understood hereafter that suffix W refers to the wake of the main aerofoil.

Experience with isolated aerofoils[13,14] indicates that the first and second derivatives of ψ^*_U and ψ^*_W decay rapidly downstream of the flap trailing edge. On intuitive grounds we would expect that the same is true of $\Delta\psi^{(1)}_W$. Therefore a reasonable approximation to the integrals of equation (G-13) can probably be obtained by replacing $z_+ + z_-$ and $\delta^*_W + \theta_W$ by their respective values at $x = c_F$. If we do this we find that

$$\Delta\Gamma_W = \left|\left\{\frac{d\psi_U^*}{dx} + \frac{1}{2}\frac{d\psi_W^*}{dx}\right\}(\delta_W^* + \theta_W)\right|_{x=c_F}$$

$$+ \frac{1}{2}\left|\frac{d\psi_W^*}{dx}(z_+ + z_-)\right|_{x=c_F} \quad . \tag{G-14}$$

Here we have used the fact that the flap is a streamline of the flow induced by the vorticity of the wake of the main aerofoil, so that

$$\left|\frac{\partial(\Delta\psi_W^{(1)})}{\partial x}(x,+0)\right|_{x=c_F} = 0 \quad .$$

Further, we have made use of the property of ψ_U^*, ψ_W^* and $\Delta\psi_W^{(1)}$ noted above, namely that the first derivatives of ψ_U^*, ψ_W^* and $\Delta\psi_W^{(1)}$ tend to zero rapidly downstream of the flap trailing edge.

Insofar as the combined wake is downstream of the flap it seems reasonable to suppose that the lift of the flap will be influenced mainly by the overall circulation around the combined wake without particular regard to the detailed distribution of circulation in the wake. Therefore we assume that we can estimate the change in lift of the flap associated with the last term of equation (84) by equating $\Delta\Gamma_W$ to the increment in wake circulation resulting from an increase in C_D, ΔC_D say. The increment in wake circulation resulting from an increase in C_D is found by integrating equation (G-2) (with C_D replaced with ΔC_D) along the wake. Thus we find that

$$\Delta\Gamma_W = \int_{c_F}^{\infty} \kappa_W V_\infty c_0 \Delta C_D dx \quad . \tag{G-15}$$

For a sufficiently 'thin' wake and a 'small' flap gap κ_W may be assumed equal to the curvature of the rear dividing streamline of the main aerofoil. Hence we may write from simple geometrical considerations

$$\kappa_W = -\frac{d^2 z_W}{dx^2}\left\{1 + \left(\frac{dz_W}{dx}\right)^2\right\}^{-\frac{3}{2}} \quad ,$$

$$\approx -\frac{d^2 z_W}{dx^2} \quad . \tag{G-16}$$

Therefore, combining equations (G-15) and (G-16), we have

$$\Delta\Gamma_W = - V_\infty c_0 \Delta C_D \left| \frac{dz_W}{dx} \right|_{c_F}^{\infty} ,$$

$$= - V_\infty c_0 \Delta C_D (\beta + \alpha) \qquad \text{(G-17)}$$

for small g/c_0, β and α. Therefore, upon combining equations (G-14) and (G-17), we find that

$$\Delta C_D = -\left(\left| \left\{ \frac{d\psi_U^*}{dx} + \frac{1}{2}\frac{d\psi_W^*}{dx} \right\} (\delta_W^* + \theta_W) + \frac{1}{2}\frac{d\psi_W^*}{dx}(z_+ + z_-) \right|_{x=c_F} \right) \Big/ V_\infty c_0 (\beta + \alpha) \quad . \text{(G-18)}$$

Consequently we may define an effective or equivalent drag coefficient, which includes a correction for the non-zero thickness and non-zero distance from the flap of the wake of the main aerofoil, namely

$$\bar{C}_D = C_D + \Delta C_D ,$$

$$= C_D - \left(\left| \left\{ \frac{d\psi_U^*}{dx} + \frac{1}{2}\frac{d\psi_W^*}{dx} \right\} (\delta_W^* + \theta_W) + \frac{1}{2}\frac{d\psi_W^*}{dx}(z_+ + z_-) \right|_{x=c_F} \right) \Big/ V_\infty c_0 (\beta + \alpha) ,$$

.... (G-19)

from equation (G-18). The corresponding coefficients $\bar{B}_n$ and $\bar{D}_n$, which are used in place of B_n and D_n in equation (102) when allowance is made for the thickness and distance effects, are obtained simply by replacing C_D by $\bar{C}_D$ in equations (103) (see equations (109)).

The change in the lift of the flap associated with term ΔC_D is found to be very small for the configurations studied in section 5. Hence the errors in C_{L_F} resulting from the approximations used in obtaining equation (G-18) are considered to be of no significance.

Table 1

COMPARISON BETWEEN EXACT AND APPROXIMATE VALUES OF $\Delta\gamma'_{,B}(\xi)$ FOR THE $F(\xi)$ DISTRIBUTION OF EQUATION (99); $\chi = \pi/3$

ν	$\Delta\gamma_{,B}$	
	Exact equation (100)	Approximate equation (101), N = 32
4	0.300	0.293
8	0.728	0.712
11	1.47	1.57
12	1.04	1.04
16	0.752	0.751
20	0.760	0.757
24	0.960	0.960
28	1.74	1.75

Table 2

SOLUTIONS OF EQUATIONS (103) FOR $M = 3$; $c_F/c_E = 0.31$

n	$2D_n c_E/c_0 C_D$	$2B_n c_E/c_0 C_D$
0	-1.03	-0.92
1	-1.46	-1.18
2	-0.65	-0.45

Table 3

COMPARISON BETWEEN EXACT AND APPROXIMATE VALUES OF I_6 (EQUATION (116)) FOR THE $E(\xi)$ DISTRIBUTION OF EQUATION (119); $\chi = \pi/3$

ν	I_6	
	Exact equation (120)	Approximate equation (118), N = 32
2	0.513	0.510
4	0.273	0.270
6	0.108	0.101
8	-0.0550	-0.0600
10	-0.331	-0.367
11	-0.464	-0.518
12	-0.264	-0.255
16	-0.110	-0.110
20	-0.0714	-0.0713
24	-0.0551	-0.0552
28	-0.0479	-0.0479

Table 4

RESULTS FOR THE CORRECTIONS TO FLAP LIFT AND OVERALL LIFT

α^o	g/c_0	$\Delta C_{L_{F,B}}$	$\Delta C_{L,B}$	$\Delta C^{(1)}_{L_{F,W}}$	$C^{(1)}_{L_F}$	$C^{(2)}_{L_F}$	C_{L_F}	$\Delta C_{L,WS}$	$\Delta C_{L,WV}$	$C^{(1)}_L$	Measured C_L
-5	0.020	-0.103	-0.293	-0.017	0.522	0.546	0.543	-0.075	-0.063	2.20	2.23
-5	0.025	-0.103	-0.292	-0.021	0.540	0.566	0.550	-0.063	-0.057	2.20	2.20
-5	0.040	-0.158	-0.448	-0.027	0.536	0.565	0.567	-0.044	-0.067	2.01	2.07
+3	0.020	-0.117	-0.343	-0.049	0.464	0.523	0.520	-0.184	-0.125	3.04	3.25
+3	0.025	-0.100	-0.250	-0.053	0.515	0.577	0.519	-0.180	-0.126	3.09	3.16
+3	0.040	-0.216	-0.605	-0.065	0.448	0.517	0.510	-0.128	-0.149	2.70	2.93

NB:- $\Delta C_{L_{F,B}}$ = increment of lift coefficient of flap due to flap boundary layer

$\Delta C_{L_{F,W}}$ = increment of lift coefficient of flap due to wake of main aerofoil

C_{L_F} = flap lift coefficient

$\Delta C_{L,WS}$ = increment of overall-lift coefficient due to sources of wake of main aerofoil

$\Delta C_{L,WV}$ = increment of overall-lift coefficient due to vortices of combined aerofoil-flap wake

C_L = overall-lift coefficient

suffix (1) refers to first approximation for 'thin' wake and 'small' flap gap

suffix (2) refers to approximation obtained by neglecting distributed sources and vortices of wake of main aerofoil.

SYMBOLS

a_{mn}, b_{mn}	coefficients defined in Appendix G
B_n, D_n	solutions of linear equations (103)
B	arbitrary constant
c	contour bounding region Σ (Fig.2)
C	arbitrary constant
c_A	chord of main aerofoil
C_D	drag coefficient; $= D/\frac{1}{2}\rho V_\infty^2 c_0$
c_E	extended chord of wing with high-lift devices deployed
c_F	chord of flap
C_J	jet-momentum coefficient based on c_E; $= J/\frac{1}{2}\rho V_\infty^2 c_E$
C_L	lift coefficient; $= L/\frac{1}{2}\rho V_\infty^2 c_0$
C_p	static-pressure coefficient; $= (p - p_\infty)/\frac{1}{2}\rho V_\infty^2$
c_0	chord of basic wing
E	function defined in equation (117)
F	$= \psi_L^* - \psi_U^*$
$f_m(\chi)$	function defined in equations (G-8)
g	flap gap
G	function defined in equation (96)
H	Heaviside unit step function
$I_1, I_2, \ldots I_9$	various integrals defined in text
J	jet momentum flux
$K(\theta)$	function defined in respect of equation (139) in section 4.3
k	contour bounding region Γ (Fig.7)
ℓ	distance, taken positive in clockwise direction, around contour c or contour k
$\tilde{\ell}$	flap overlap
L	lift
$L(\xi)$	function defined in equation (105)
m	integer
M	integer
N	even integer
n	integer
$\underset{\sim}{n}$	normal vector outward from regions Σ or Γ

SYMBOLS (continued)

p	static pressure
P	field point external to Σ or Γ
P,Q	quantities defined in equations (64)
q	source strength per unit length
$\underset{\sim}{r}$	vector joining P and element $d\ell$, the vector being taken positive in direction away from P
$s_{\mu\nu}^{(1)}, s_{\mu\nu}^{(4)}$	Weber coefficients[12,15]
t	distance, taken positive in clockwise direction, around contour of main aerofoil
u, w	velocity components in x and z directions
U	x-wise velocity at edge of flap boundary layer
$\bar{U}$	mean of x-wise velocities at upper and lower edges of wake
v_ξ, v_ζ	velocity components in ξ and ζ directions
V	complex velocity; $= v_\xi - iv_\zeta$
V_∞	free-stream speed
(x,z)	rectangular cartesian coordinate system, x axis along flap chord, $x = 0$ at flap leading edge (Fig.1)
X	$= \dfrac{1 - (1 - \xi/c_E)^{\frac{1}{2}}}{1 + (1 - \xi/c_A)^{\frac{1}{2}}}$
$\mathcal{X}$	$= \xi/c_A - 1 + \tilde{\ell}/c_A$
α	angle between chord of main aerofoil and incident flow
β	angle between chord of flap and chord of main aerofoil (Fig.1)
γ	vortex strength per unit length
Γ	circulation (section 4.4)
Γ	region occupied by wake of main aerofoil (Fig.7)
δ	boundary-layer thickness
δ_W	wake thickness
δ^*	displacement thickness
Δ	incremental part of
ε	small parameter
η	vorticity, taken positive when rotation is in clockwise sense
θ_W	wake momentum thickness
θ	$= \cos^{-1}(2\xi/c_E - 1)$
θ_μ	$= \mu\pi/N$

SYMBOLS (continued)

κ	streamline curvature
κ_W	weighted mean curvature of streamlines of wake, defined in equation (C-14)
λ	parameter used in equations (121) and (126)
μ	index of inducing point for Weber summation (section 4)
μ	coefficient of viscosity (Appendix C)
ν	normal outward from main aerofoil (section 2.1)
ν	kinematic viscosity (Appendix C)
(ξ,ζ)	rectangular cartesian coordinate system, ξ axis along chord of main aerofoil, $\xi = 0$ at leading edge of main aerofoil (Fig.1)
ρ	density
Σ	region bounded by edges of flap boundary layer and flap wake (Fig.2)
τ	included angle between ℓ direction and negative r direction
χ	$= \cos^{-1}(2c_A/c_E - 1)$
ψ	stream function
ψ^*_U, ψ^*_L	'displacement fluxes' of flap boundary layer, defined in equation (11)
$\psi^{*\prime}$	pseudo displacement flux of flap boundary layer, $= \int_0^\delta (U - u)dz$
ψ^*_W	displacement flux of wake, defined in equation (42)
$\psi^{*\prime}_W$	pseudo displacement flux of wake $= \int_W (\bar{U} - u)dz$

SUFFIXES

A	refers to main aerofoil, except for z_A which is z ordinate of point A (Fig.7)
B	refers to effect of vorticity identified with flap boundary layer, except for z_B which denotes z ordinate of point B (Fig.7)
,B	due to flap boundary layer
c	refers to contour c
F	refers to flap
I	refers to first inviscid approximation
i	refers to velocities induced by wake of main aerofoil
J	refers to jet of blown flap
k	refers to contour k

SYMBOLS (concluded)

ℓ	refers to leading edge of wake of main aerofoil
s	refers to streamline of wake
T	refers to shroud trailing edge
U, L	refer, respectively, to upper and lower edges of region Σ
W	refers to effect of vorticity identified with wake of main aerofoil or simply to aerofoil wake
,W	due to wake of main aerofoil
,WS	due to sources of wake of main aerofoil
,WV	due to vortices of combined aerofoil-flap wake
Γ	refers to region Γ
μ	refers to inducing point
ν	refers to pivotal point
+, -	refer, respectively, to upper and lower edges of wake of main aerofoil
∞	refers to conditions at infinity
(1), (2)	refer, respectively, to constant load and 'flat-plate' vortex distributions (section 2.2)
(1)	refers to first approximation for 'thin' wake and 'small' flap gap (sections 3, 4 and 5)
(2)	refers to approximation obtained by neglecting distributed sources and vortices of wake of main aerofoil
$\bar{}$	denotes first approximation above corrected for non-zero wake thickness and distance of wake from flap, except in case of $\bar{U}$ which is defined above

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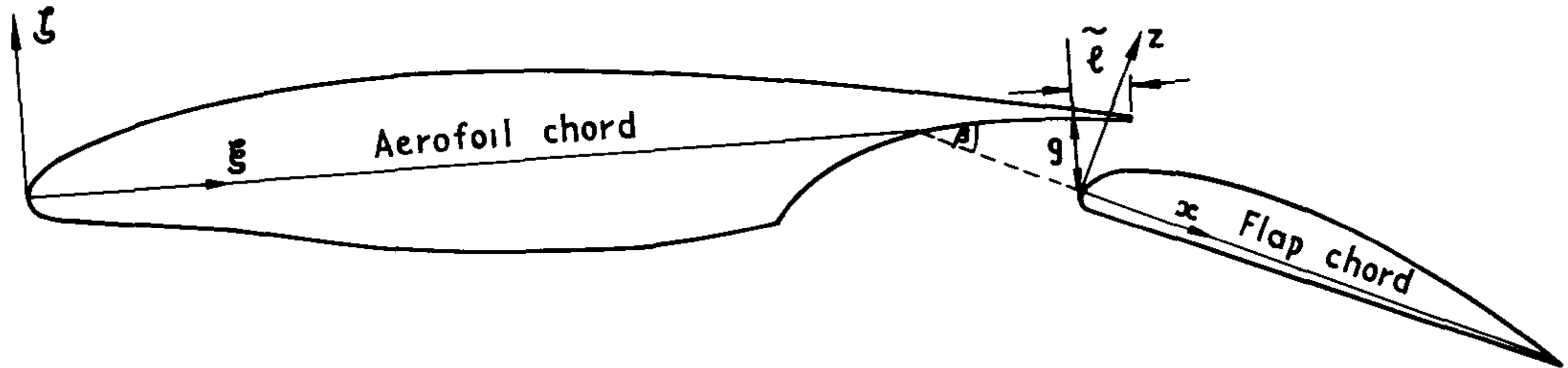


Fig1 Coordinate systems and notation for aerofoil with Fowler flap

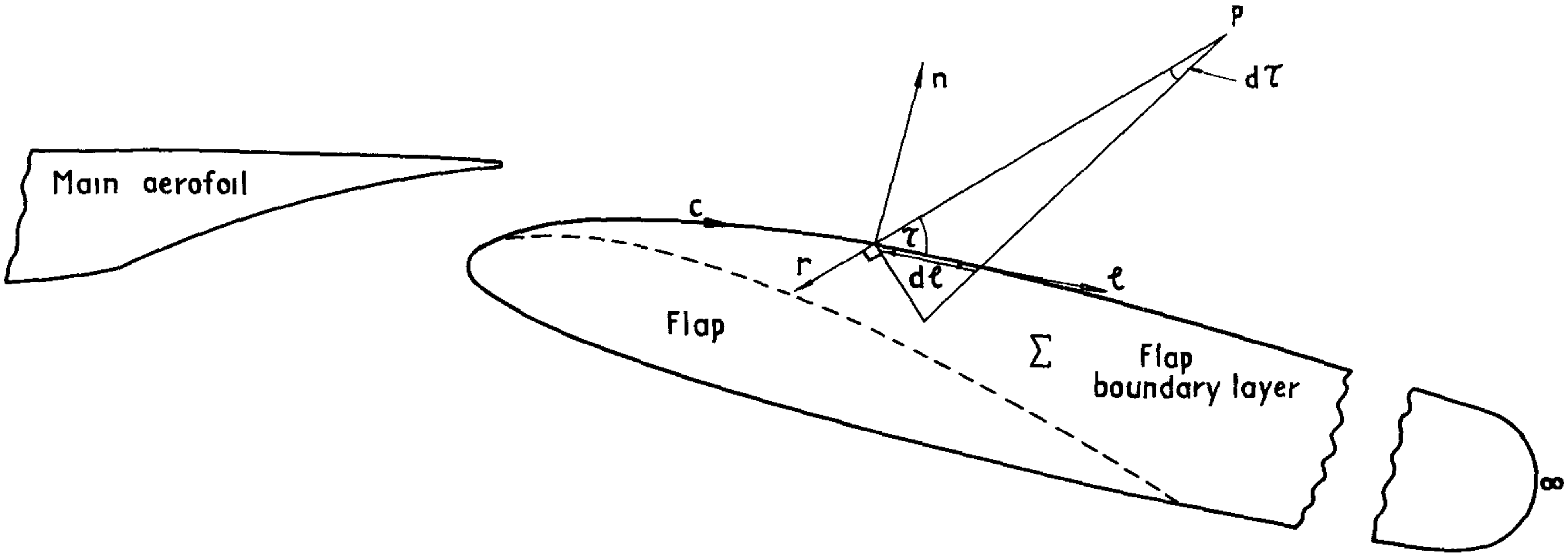


Fig.2 Definition of notation and geometry for analysis of flow field induced by vorticity within region Σ

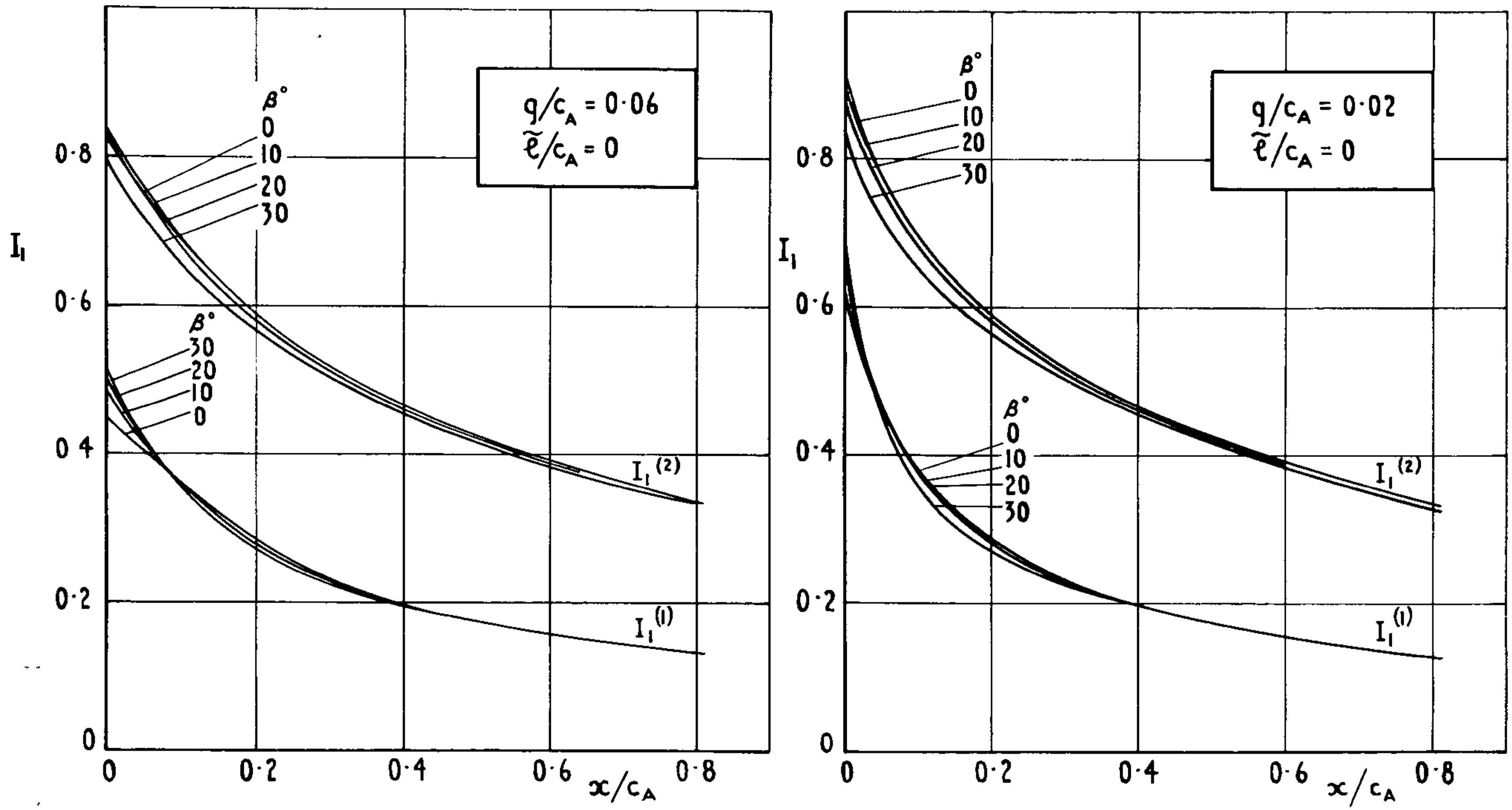
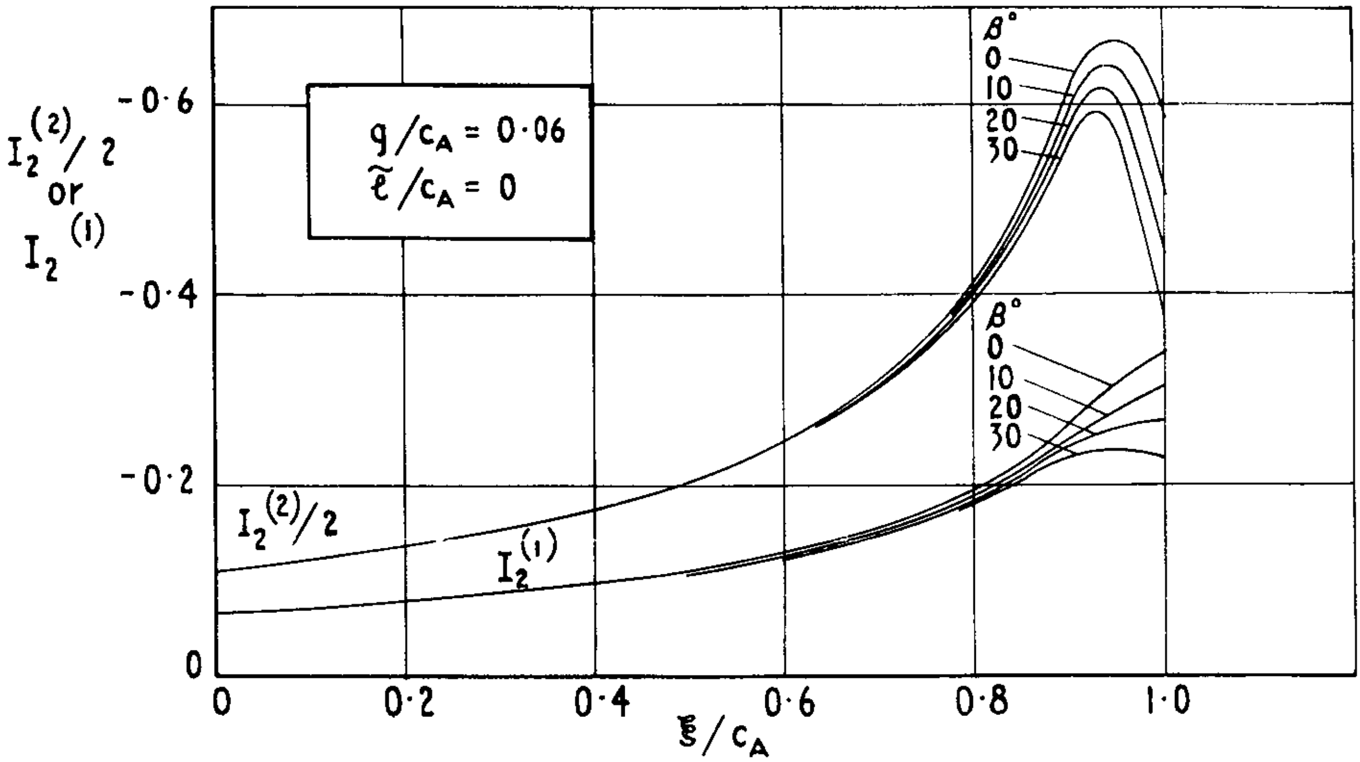


Fig.3 Variation of I_1 with distance along flap chord



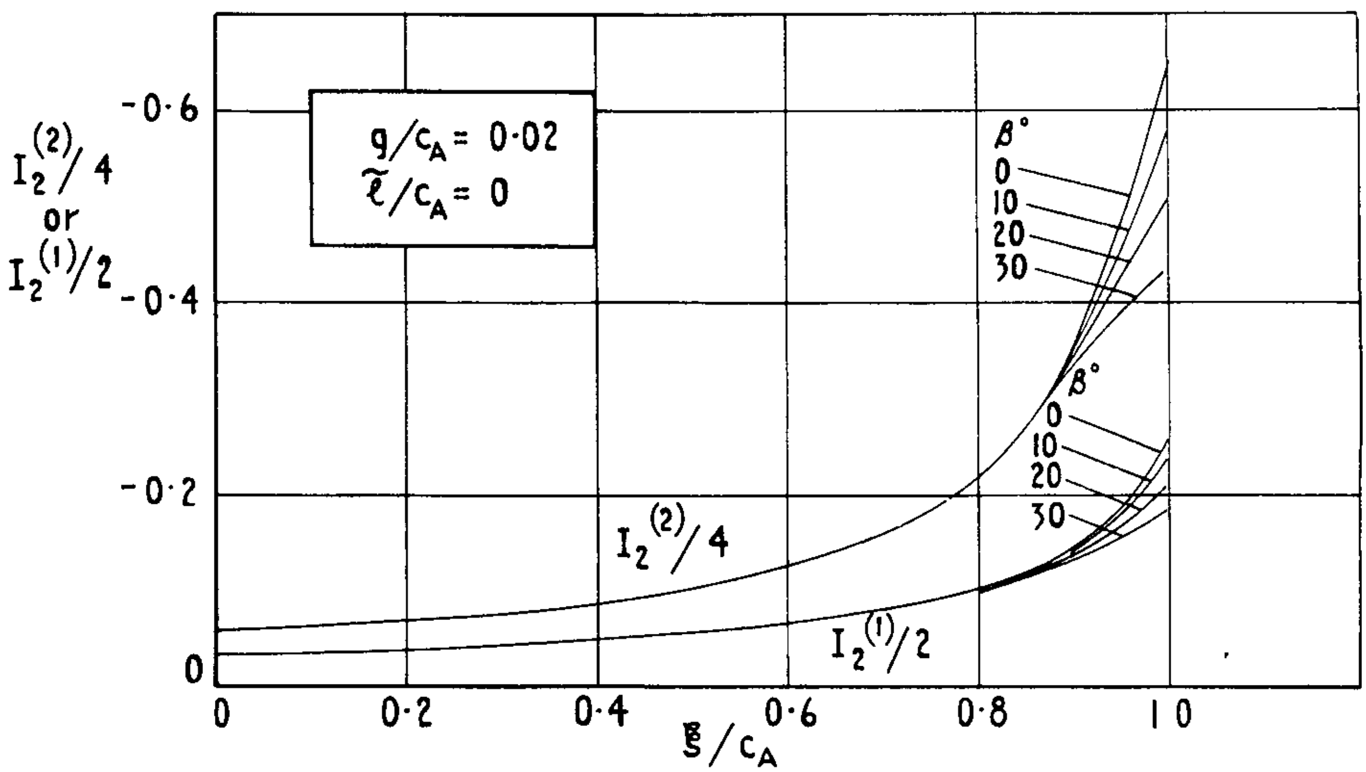


Fig. 4 Variation of I_2 with distance along the chord of the main aerofoil, $c_F / c_A = 0{\cdot}5$

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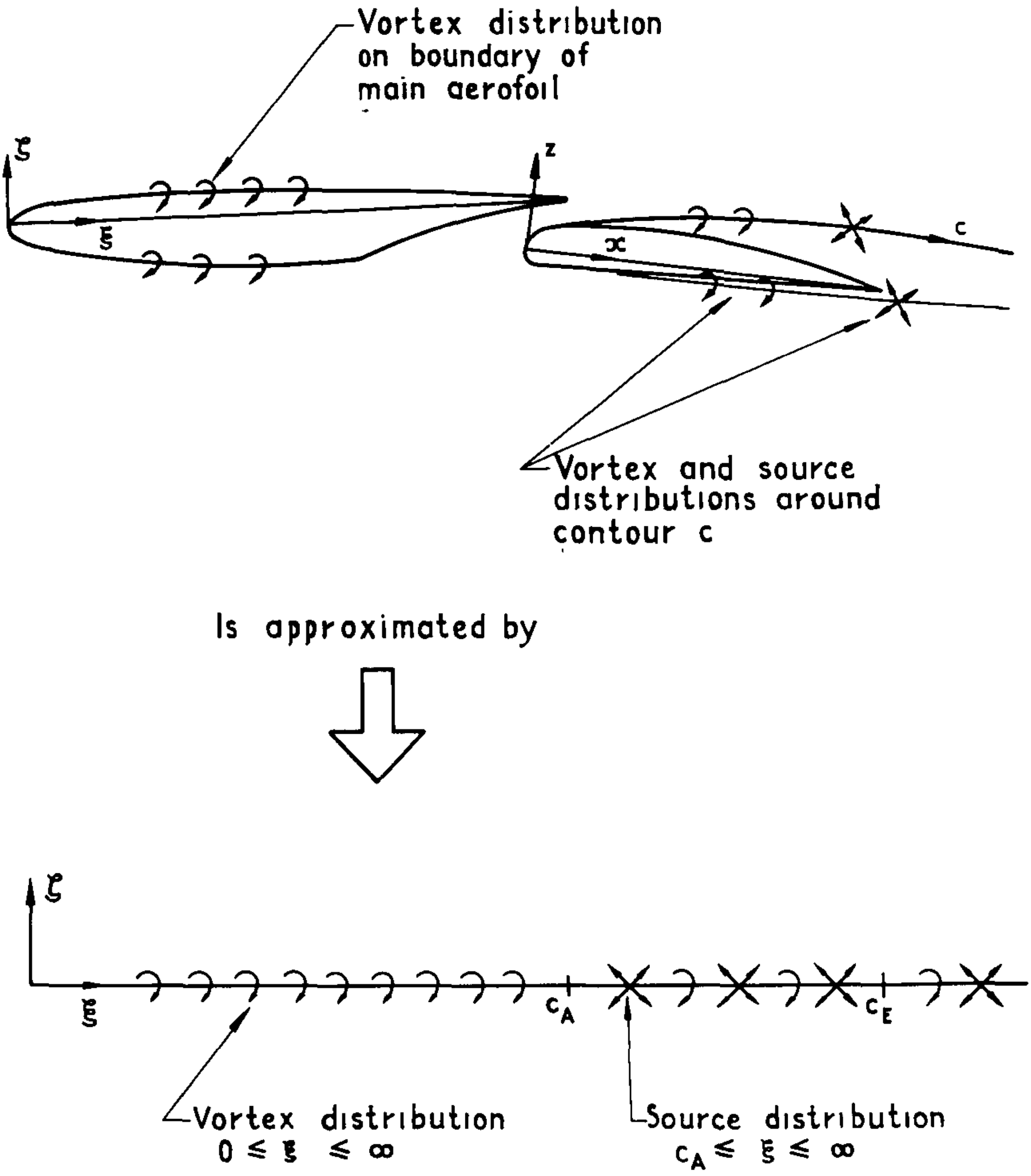

Fig 5 Approximation of vortex and source distributions associated with flap boundary layer

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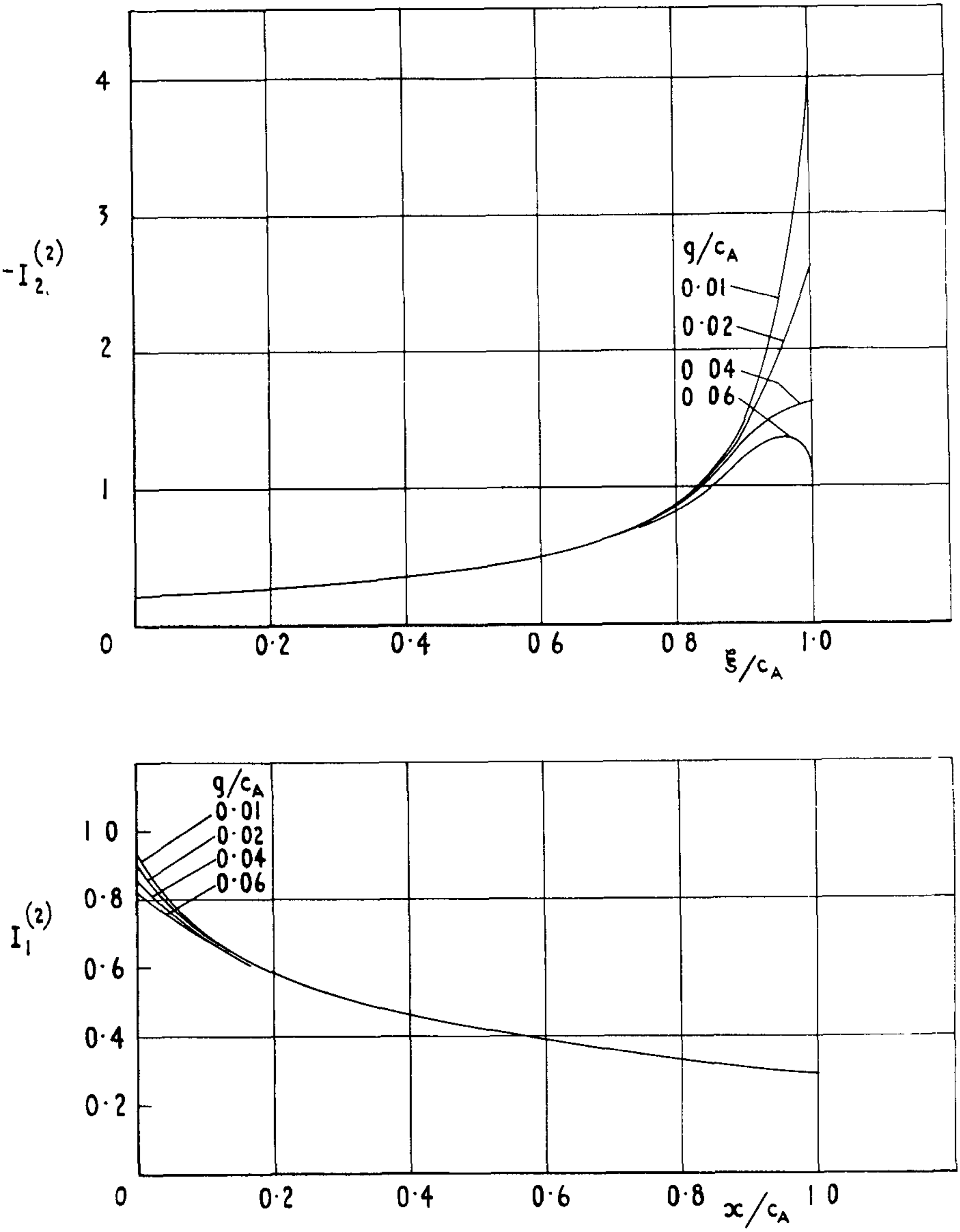

Fig.6 Effect of flap gap on $I_2^{(2)}$ and $I_1^{(2)}$, $\tilde{\ell}/c_A=0$.
$c_F/c_A=0\ 5$; $\beta=0$

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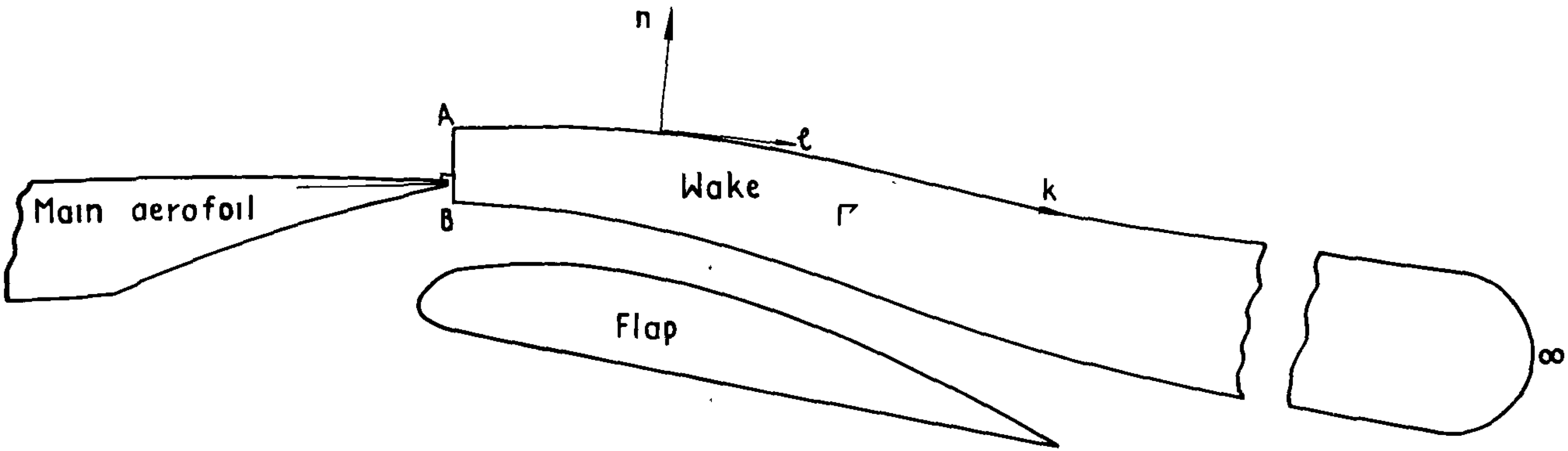


Fig.7 Definition of notation for analysis of flow field induced by vorticity of wake of main aerofoil

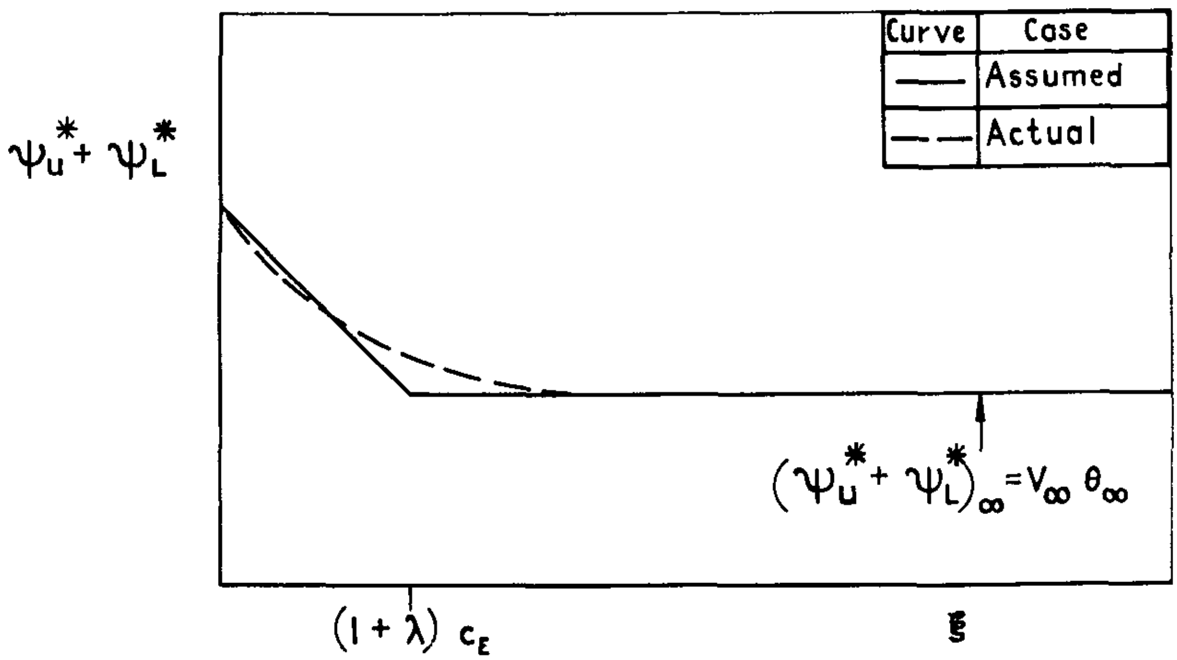


Fig 8 Model of displacement flux in wake of flap, implied by equ^n (121)

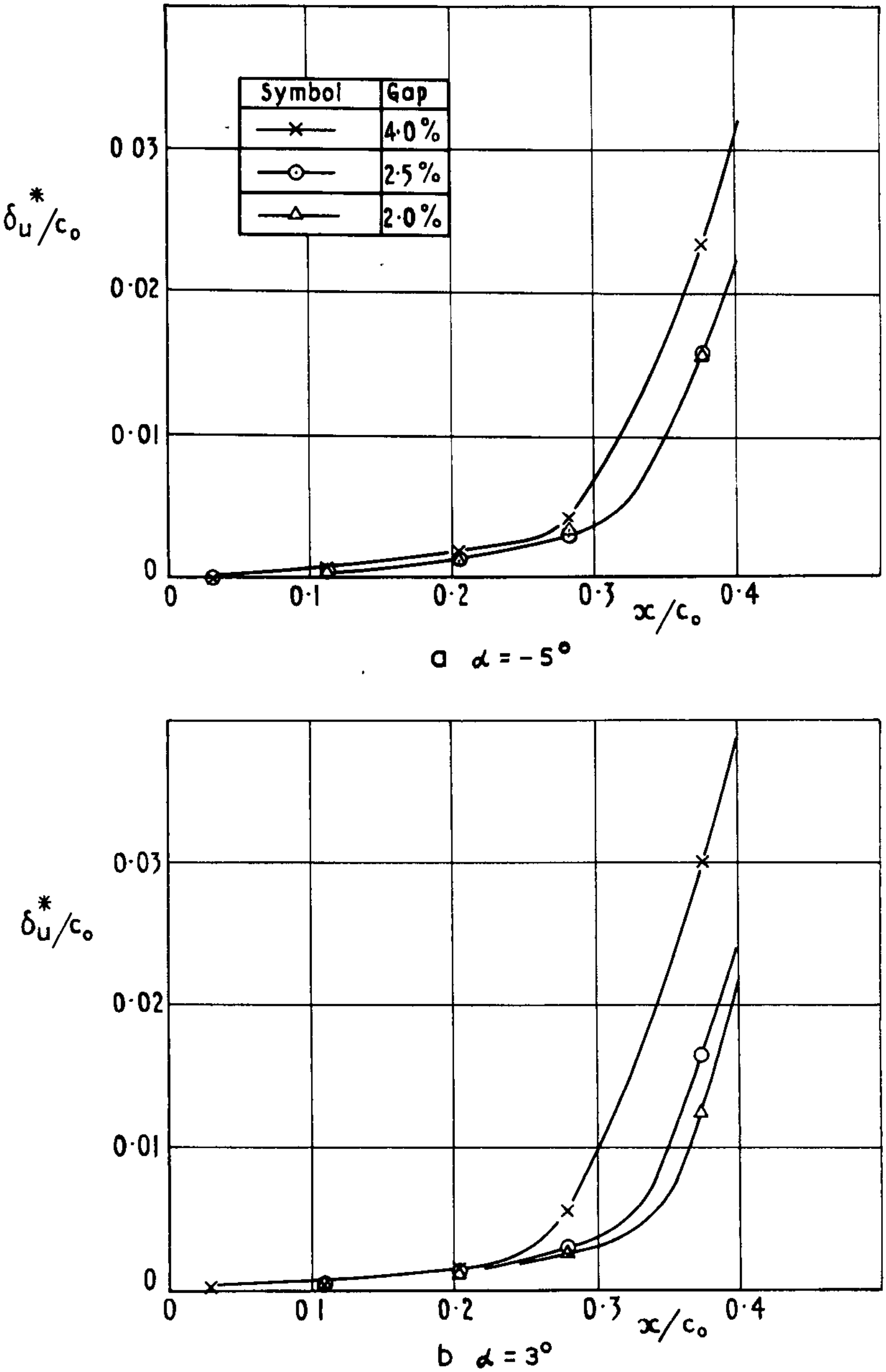
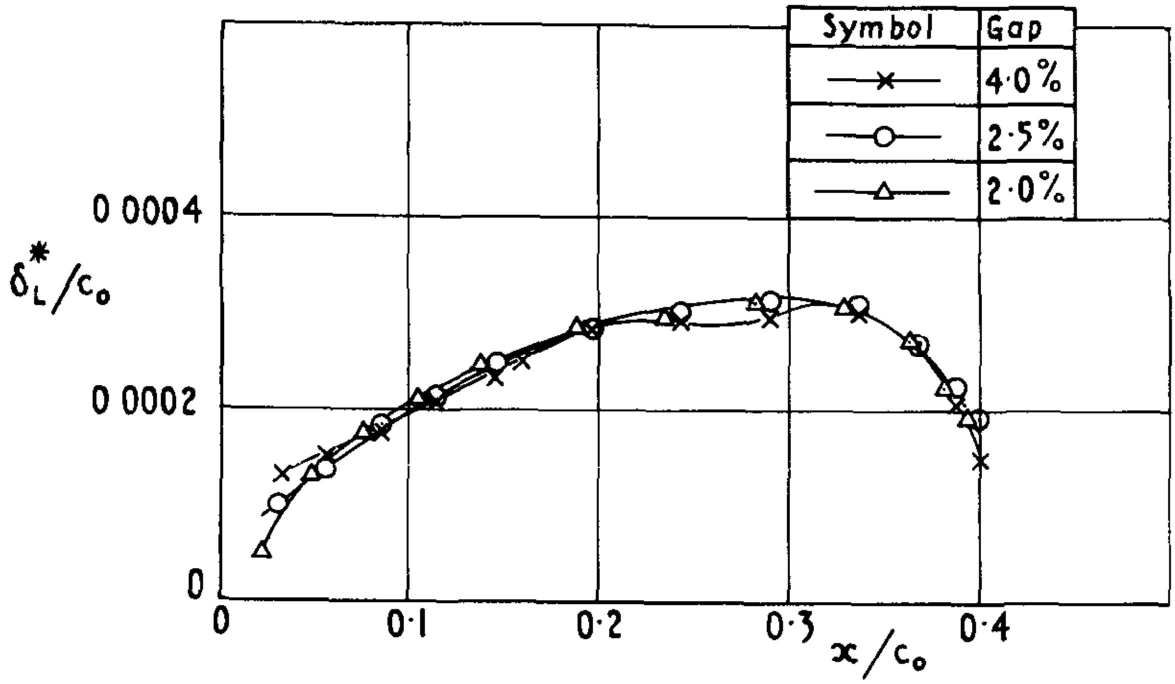
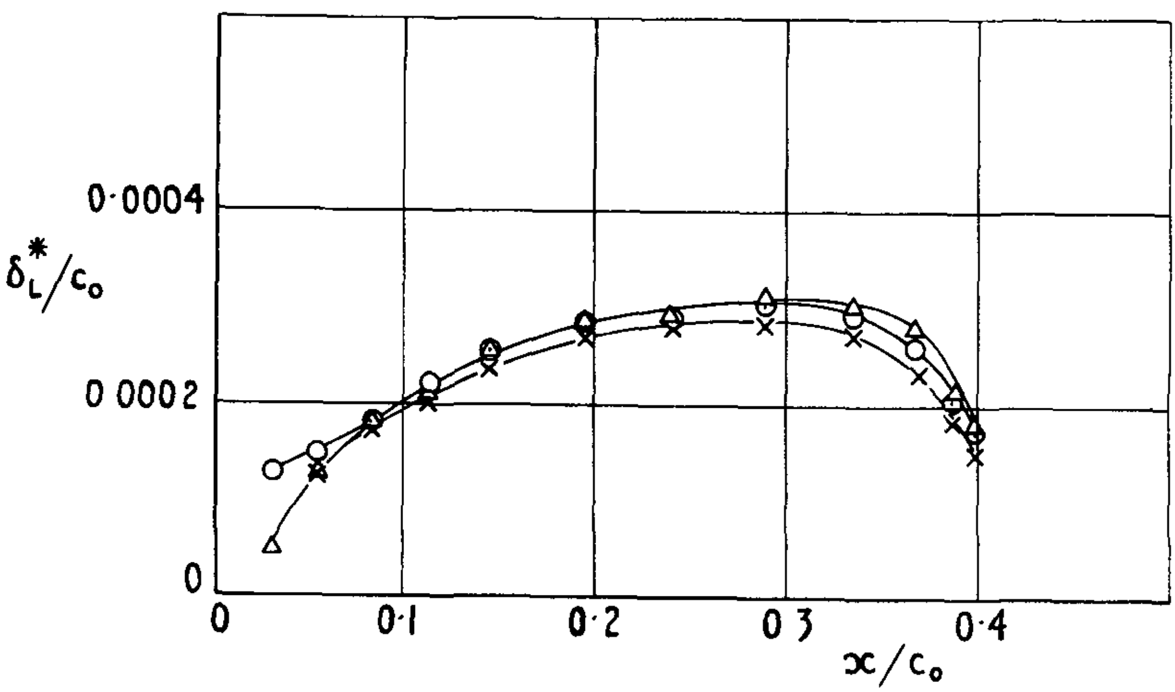


Fig.9 a & b Boundary-layer displacement thickness measured on flap upper surface



a $\alpha = -5°$



b $\alpha = 3°$

Fig.10 a & b Boundary-layer displacement thickness calculated for flap lower surface

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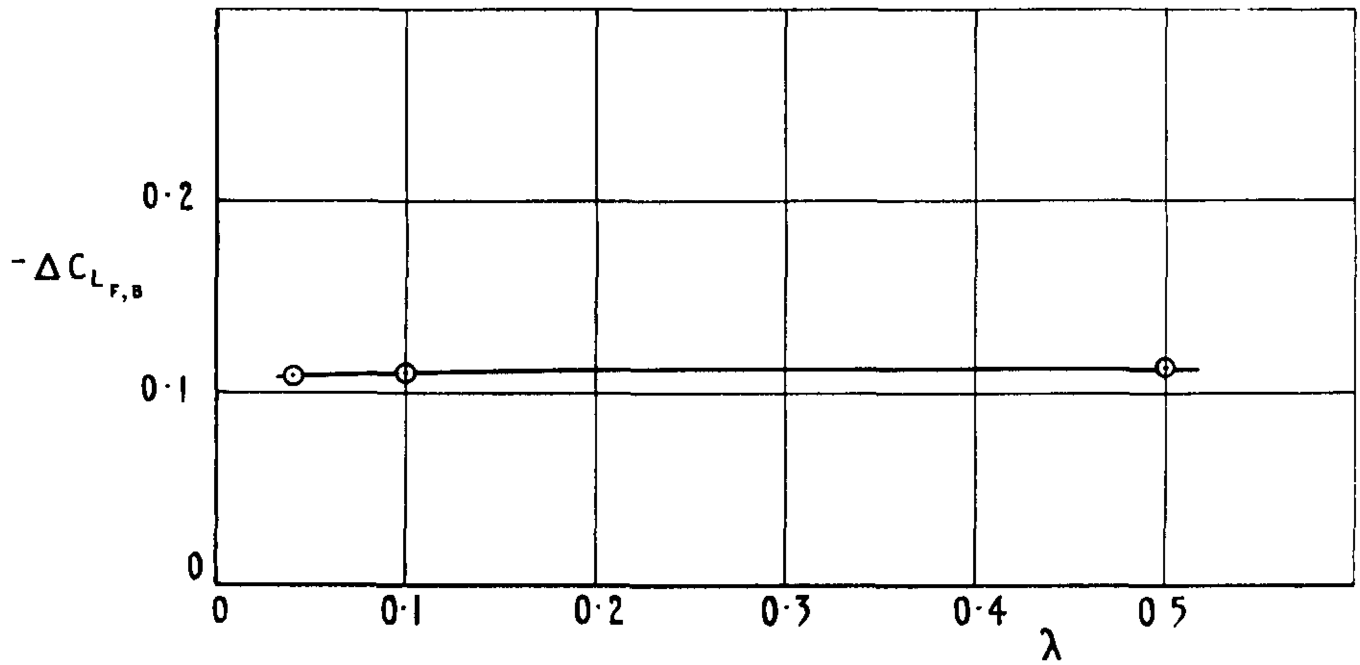
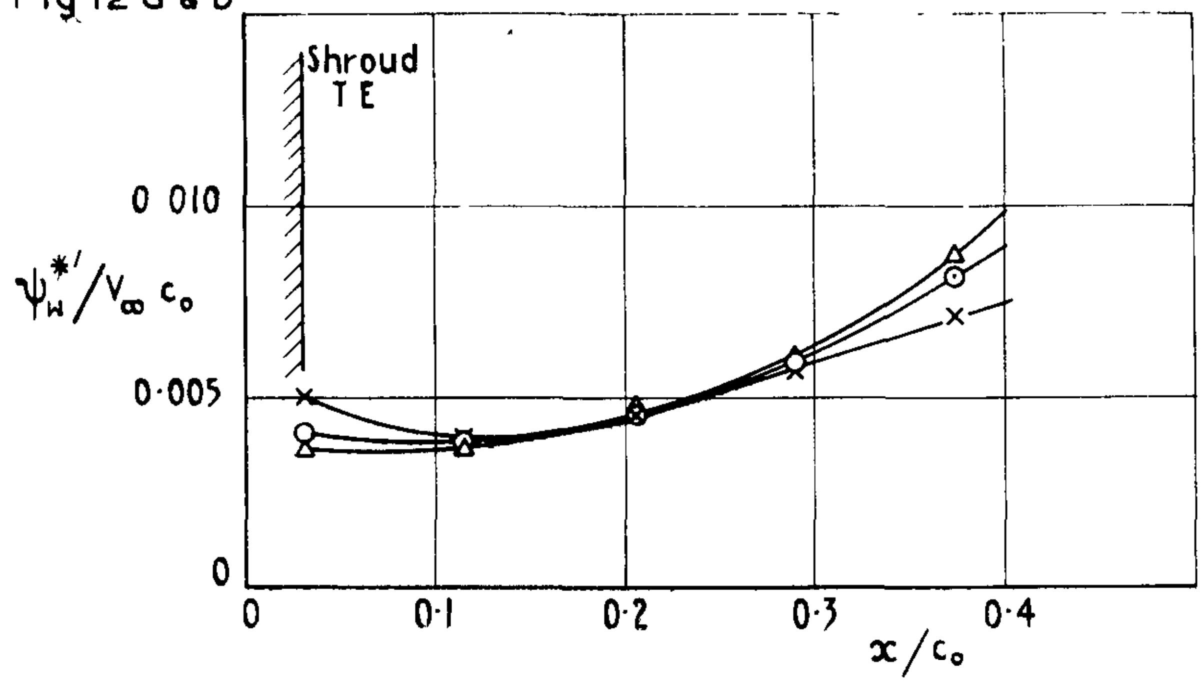
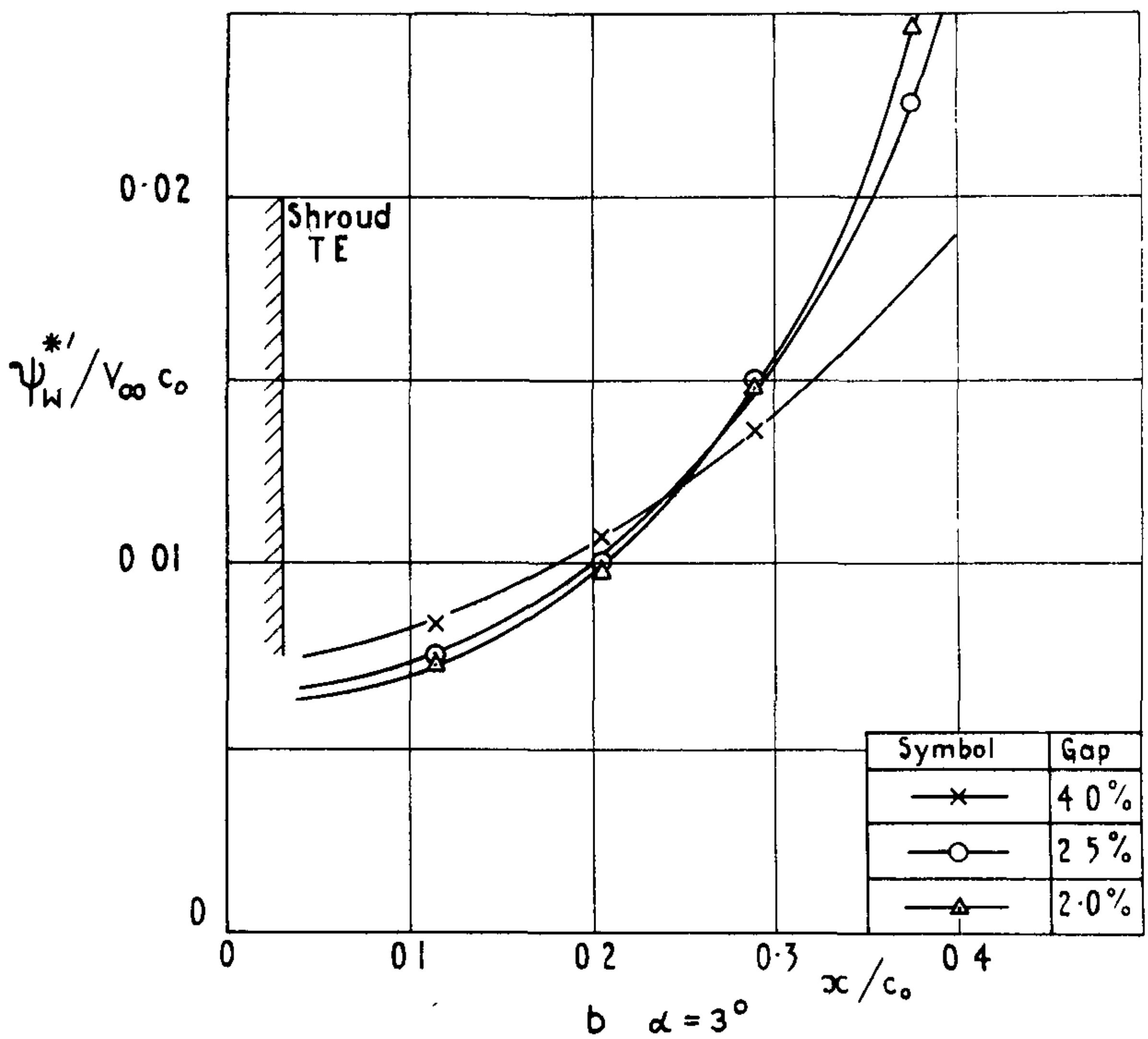


Fig.11 Influence of parameter λ on correction to flap lift due to flap boundary layer; $g/c_0 = 0{\cdot}02$ $\alpha = -5^\circ$

Fig 12 a & b

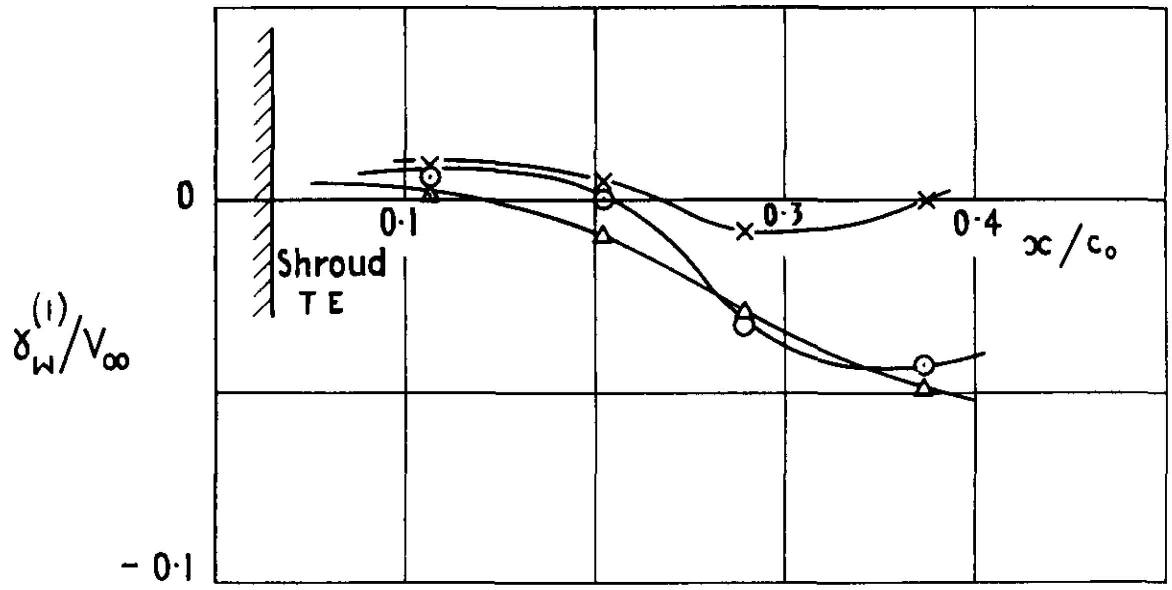


a $\alpha = -5°$

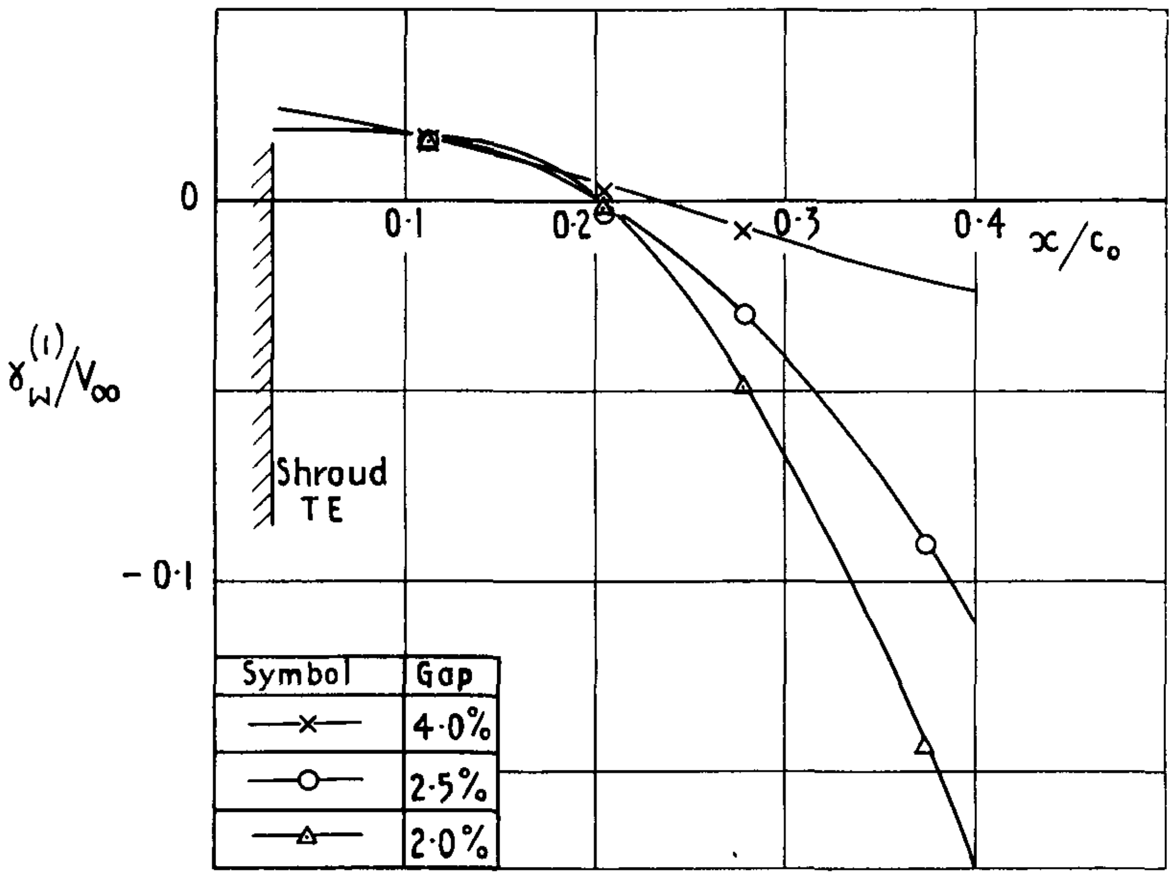


b $\alpha = 3°$

Fig. 12 a & b Measured displacement flux of aerofoil wake



a $\alpha = -5°$



b $\alpha = 3°$

Fig. 13 a & b Vortex strength of aerofoil wake deduced from measured velocity profiles

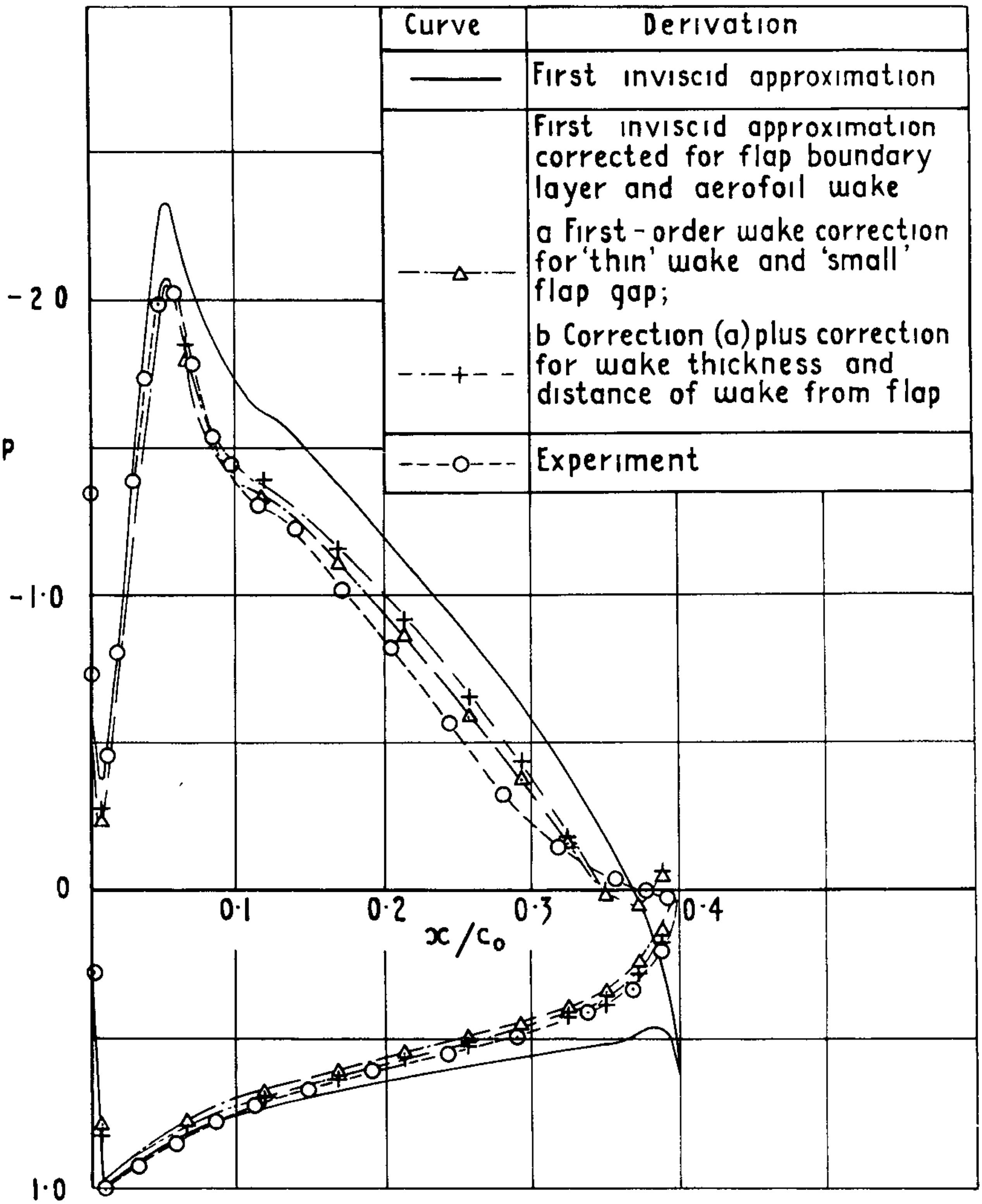


Fig 14 Pressure distributions on flap $g/c_o = 0.02$, $\alpha = -5°$

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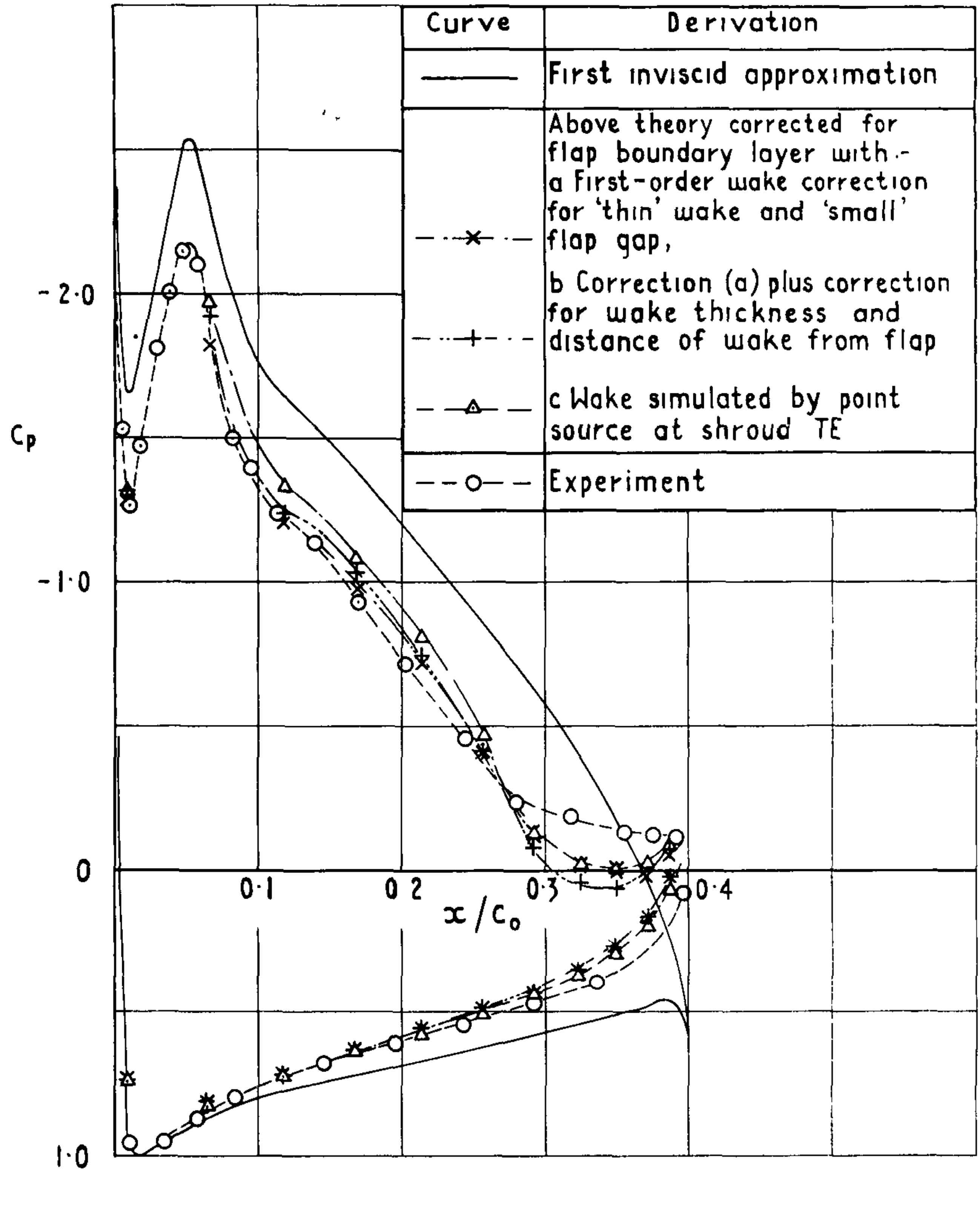

Fig. 15 Pressure distributions on flap $g/c_0 = 0{\cdot}04$, $\alpha = -5°$

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Ashill, P R

A STUDY OF THE EFFECT OF THE WAKE OF THE MAIN AEROFOIL OF A FOWLER-FLAP CONFIGURATION ON THE LIFT OF THE FLAP

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Consideration is given to various methods of approximating the wake effect, including the conventional 'thin'-wake method It is shown that, by correctly positioning the singularities of the 'thin'-wake formulation, a first-order correction to this theory for wake thickness can be rendered identically zero. An approximation for a wake which is at a 'small' height above the flap chord is examined The indications of the present calculations are that this approximation overestimates the magnitude of the correction to the lift of the flap for the effect of the wake A better estimate of the wake effect appears to be obtained if one neglects the distributed sources and vortices of the wake but allows for the non-zero displacement flux of the wake by a point source at the shroud trailing edge

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Ashill, P R

A STUDY OF THE EFFECT OF THE WAKE OF THE MAIN AEROFOIL OF A FOWLER-FLAP CONFIGURATION ON THE LIFT OF THE FLAP

This Report is concerned with the influence on the lift of the flap of the wake of the main aerofoil of a wing with a plain Fowler flap To decide the relative importance of the wake its effect is compared with the influence of the boundary layer of the flap It is found that for the configurations examined in this Report, the wake effect is of secondary importance in comparison with that of the boundary layer

Consideration is given to various methods of approximating the wake effect, including the conventional 'thin'-wake method It is shown that, by correctly positioning the singularities of the 'thin'-wake formulation, a first-order correction to this theory for wake thickness can be rendered identically zero An approximation for a wake which is at a 'small' height above the flap chord is examined The indications of the present calculations are that this approximation overestimates the magnitude of the correction to the lift of the flap for the effect of the wake A better estimate of the wake effect appears to be obtained if one neglects the distributed sources and vortices of the wake but allows for the non-zero displacement flux of the wake by a point source at the shroud trailing edge

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