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A Theoretical Investigation of Supersonic Jets in Subsonic Flow Fields

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A THEORETICAL INVESTIGATION OF SUPERSONIC JETS IN SUBSONIC FLOW FIELDS

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SUMMARY

A method of calculating the development of a supersonic jet exhausting into a subsonic free stream is presented.

The method of characteristics is used with the conditions on the jet boundary modified by the change in static pressure produced by the external stream. The method is particularly applied to cases where the inner boundary of the jet is a solid surface, e.g. the fan exhaust of a bypass engine.

Theory and experiment are in close agreement for the range of jet pressure ratios and free-stream Mach numbers associated with engines of high bypass ratio.

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CONTENTS

1 INTRODUCTION

A supersonic jet produced by air expanding from a choked nozzle into quiescent air or an external stream moving parallel to the jet axis gives rise to a regular pattern of expansion and shock waves. The distance between successive shocks is dependent on the overall jet pressure ratio and the free-stream Mach number.

The development of the jet can be calculated using the method of characteristics. The simplest problem is when the jet is flowing into quiescent air since the jet boundary is a constant-pressure boundary and the solution can be built up along characteristic lines progressively downstream[1,2]. The addition of a supersonic external stream does not increase the complexity of the calculation and the same basic method can be employed[3].

The supersonic jet surrounded by a subsonic external stream is an entirely different problem. The governing equations are hyperbolic and elliptic and disturbances can be propagated by the external flow in all directions. This type of mixed flow occurs quite frequently in propulsion aerodynamics and this Report deals mainly with the pressure distribution produced on the gas generator by the fan efflux of a high bypass ratio engine at cruise Mach number, Fig.1b. A method of calculating the pressure distribution on the gas generator would be useful in the design stage at least to investigate systematically a range of gas generator shapes.

Some other configurations that can be treated by the method are shown in Fig.1.

The present calculation method is based on a simple model of the flow. First the shape of the constant-pressure boundary of the jet in quiescent surroundings is calculated using the method of characteristics. Then treating this surface as a solid axisymmetric or two-dimensional body in the subsonic free stream, the pressure distribution on the surface is calculated. This pressure distribution is then applied as the outer boundary condition for the supersonic jet and a new boundary shape calculated. The process is repeated; convergence cannot be guaranteed in advance, but in practice, after five or six iterations convergent results have been obtained in all cases. Convergence here means that the pressure distribution on the inner surface does not change significantly between successive iterations implying compatibility between the subsonic pressure distribution and the jet shape.

The use of the method of characteristics does present some problems, in particular, the treatment of small patches of subsonic flow. This problem could be overcome by using the streamline-curvature technique[4] but this method requires a relaxation parameter for convergence which, as yet, can only be determined by trial and error. Another alternative would be the use of a time dependent technique[5]. The introduction of the time variable leads to a hyperbolic equation which can be solved for subsonic or supersonic flow by the method of characteristics. However, a large number of field points would be required for the supersonic jet with a subsonic free stream and the computer time would be almost prohibitive.

The general method of characteristics in two-dimensional and axisymmetric flow is briefly described in section 2. The emphasis is placed on the solution of the characteristic equations rather than their derivation. The application of the method of characteristics to a jet flowing into quiescent air is outlined in section 3 and the modifications required when a free stream is present are described in section 4. The techniques adopted for overcoming the difficulties associated with shock waves and local regions of subsonic flow are discussed in section 5.

Some comparisons between theory and experiment are presented in section 6, and some possible extensions to the method for jet/wing interference problems are outlined in section 7.

Viscosity is neglected throughout the present method. The present results show that this is not a serious omission except far downstream from the nozzle exit; in this respect the supersonic jet appears to differ significantly from the subsonic jet whose development is dominated by viscous interactions.

2 THE METHOD OF CHARACTERISTICS

The method of characteristics applied to supersonic flow fields is described in detail in a number of text books, e.g. Ref.6, section G. The derivations of the characteristic relations from the equations of continuity, momentum and energy is quite straightforward but is not repeated here, and this section is mainly concerned with the solution of the equations with particular reference to jet flows. Two-dimensional and axisymmetric flows are considered separately as there are substantial differences in the methods required to solve the equations.

The characteristic equations for potential flow are used throughout, i.e. viscosity is neglected and the entropy and total enthalpy are taken to be constant in the entire flow field.

2.1 Two-dimensional flow

The characteristic equations for two-dimensional flow can be simply expressed in terms of the velocity components u and v, in the x and y directions, along two families of lines.

Along a line belonging to the first family defined by

$$t_1 = \left(\frac{dy}{dx}\right)_1 = \tan(\theta + \mu)$$

the equation

$$du + t_2 dv = 0 \tag{1}$$

applies, and along a line belonging to the second family, defined by

$$t_2 = \left(\frac{dy}{dx}\right)_2 = \tan(\theta - \mu)$$

the corresponding relation is

$$du + t_1 dv = 0 \tag{2}$$

where θ is the direction of the streamline with respect to the x axis and μ is the Mach angle, $\mu = \sin^{-1}(1/M)$.

Transforming to polar coordinates q, θ in the hodograph plane defined by

$$u = q\cos\theta, \quad v = q\sin\theta$$

$$du = dq\cos\theta - q\sin\theta\, d\theta$$

$$dv = dq\sin\theta + q\cos\theta\, d\theta$$

equations (1) and (2) can be written

$$\frac{dq}{d\theta}\sqrt{1 - \frac{1}{M^2}} = \pm\frac{q}{M}$$

or

$$\frac{dq}{q} = \pm\tan\mu\, d\theta \tag{3}$$

where the positive sign corresponds to equation (1) and the negative sign to equation (2).

Replacing q in equation (3) by the Mach number with the relation

$$\frac{dM}{M} = \frac{dq}{q}\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

gives

$$\pm\, d\theta = \frac{\sqrt{M^2 - 1}\, dM}{M\left(1 + \frac{\gamma - 1}{2} M^2\right)}$$

which can be integrated directly

$$\mp\,\theta = \cos^{-1}\frac{1}{M} - \sqrt{\frac{\gamma + 1}{\gamma - 1}}\,\tan^{-1}\sqrt{(M^2 - 1)\left(\frac{\gamma - 1}{\gamma + 1}\right)} + \text{constant} \qquad (4)$$

or

$$\mp\,\theta = -\,\omega(M) + \text{constant} \qquad (5)$$

where $\omega(M)$ is the Prandtl-Meyer angle. The positive sign in equations (4) and (5) corresponds to equation (2), the $(\theta - \mu)$ family, and the negative sign to equation (1), the $(\theta + \mu)$ family.

Thus in two-dimensional flow there are simple relations, equations (4) or (5), between the streamline direction and the Mach number along the two families of lines which leads to a particularly easy method of solution.

Suppose that the flow conditions at two points A and B are known, where A is on a $(\theta + \mu)$ line and B is on a $(\theta - \mu)$ line, Fig.2a. The flow conditions at C where the two lines intersect can be found from equation (5).

At A

$$d_A = \omega(M_A) - \theta_A$$

and at B

$$d_B = \omega(M_B) + \theta_B$$

where d is the constant in equations (5). At C, the flow direction θ_c and the Prandtl-Meyer angle are

$$\theta_c = (d_B - d_A)/2$$

$$\omega(M_c) = (d_A + d_B)/2 \quad .$$

The Mach number can be found from the Prandtl-Meyer angle by an iterative solution of equation (4), or by the use of appropriate tables[6].

The coordinates of C are found using the equations for the characteristic lines AC and BC by taking the mean of θ and μ at A and C, B and C

$$\left.\begin{aligned} \frac{y_C - y_A}{x_C - x_A} &= \tan\left[\frac{\theta_A + \mu_A + \theta_C + \mu_C}{2}\right] \\ \frac{y_C - y_B}{x_C - x_B} &= \tan\left[\frac{\theta_B - \mu_B + \theta_C - \mu_C}{2}\right] \end{aligned}\right\} \qquad (6)$$

Characteristic lines intersecting with a solid surface of specified shape can be dealt with in a similar manner. Consider a $(\theta - \mu)$ line passing through a point B where the flow conditions are known, Fig.2b. A first approximation to the position of C is made by calculating where a straight line of slope $(\theta - \mu)_B$ intersects the surface. As the surface is a streamline; θ_C is known and M_C can be found from equation (5). The mean slope of the line BC is then used to calculate a new intersection point C' and the calculation repeated until the point of intersection does not change.

The other boundary that occurs in the calculation of jet flows is a constant or specified pressure boundary, Fig.2c. The Mach number at C is known from the pressure ratio and the streamline direction θ_C can be found from equation (5). The coordinates of C are determined from equation (6) except that the second equation is replaced by

$$\frac{y_C - y_B}{x_C - x_B} = \tan\left[\frac{\theta_C + \theta_B}{2}\right]$$

2.2 Axisymmetric flow

The characteristic equations for supersonic axisymmetric flow are more complicated than the two-dimensional equations because the streamline direction and Mach number depend on the position coordinates.

The equations corresponding to (1) and (2) are, along the line defined by

$$t_1 = \left(\frac{dr}{dx}\right)_1 = \tan(\theta + \mu)$$

$$du + t_2 dv + \frac{v}{r\left(1 - \frac{u^2}{a^2}\right)} dx = 0 \qquad (7)$$

and along the line

$$t_2 = \left(\frac{dr}{dx}\right)_2 = \tan(\theta - \mu)$$

$$du + t_1 dv + \frac{v}{r\left(1 - \frac{u^2}{a^2}\right)} dx = 0 \qquad (8)$$

where r is the radial coordinate.

Transforming to the variables q, θ as before and substituting into equations (7) and (8) gives

$$\cot\mu \frac{dq}{q} - d\theta - \frac{\sin\mu \sin\theta}{\cos(\theta + \mu)} \frac{dx}{r} = 0 \qquad (9)$$

$$\cot\mu \frac{dq}{q} + d\theta - \frac{\sin\mu \sin\theta}{\cos(\theta - \mu)} \frac{dx}{r} = 0 \ . \qquad (10)$$

These equations cannot be integrated directly as in two-dimensional flow because the radial ordinate appears in the last term. A solution is found by writing equations (9) and (10) in finite difference form and iterating for the coordinates of the point where the lines intersect and the flow conditions at this point.

As before, assume that the flow conditions are known at two points A and B on a different family of lines, Fig.3. In finite difference form, equations (9) and (10) become

$$f_2(q_C - q_A) - (\theta_C - \theta_A) - f_3(x_C - x_A) = 0 \qquad (11)$$

along the line

$$\frac{r_C - r_A}{x_C - x_A} = f_1 \qquad (12)$$

and

$$f_5(q_C - q_B) + (\theta_C - \theta_B) - f_6(x_C - x_B) = 0 \tag{13}$$

along the line

$$\frac{r_C - r_B}{x_C - x_B} = f_4 \tag{14}$$

where, as a first approximation

$$f_1 = \tan(\theta_A + \mu_A)$$

$$f_2 = \frac{\cot \mu_A}{q_A}$$

$$f_3 = \frac{\sin \theta_A \sin \mu_A}{r_A \cos(\theta_A + \mu_A)}$$

$$f_4 = \tan(\theta_B - \mu_B)$$

$$f_5 = \frac{\cot \mu_B}{q_B}$$

$$f_6 = \frac{\sin \theta_B \sin \mu_B}{r_B \cos(\theta_B - \mu_B)} .$$

The expressions for f_1 and f_4 substituted into equations (12) and (14) give a first approximation to the coordinates of C, and these substituted into equations (11) and (13) give initial values of q_C and θ_C. The solution is improved by using the conditions just calculated at C, so that

$$f_1 = \tan\left(\frac{\theta_A + \mu_A + \theta_C + \mu_C}{2}\right)$$

$$f_2 = \frac{1}{2}\left(\frac{\cot \mu_A}{q_A} + \frac{\cot \mu_C}{q_C}\right)$$

$$f_3 = \frac{1}{2}\left(\frac{\sin \theta_A \sin \mu_A}{r_A \cos (\theta_A + \mu_A)} + \frac{\sin \theta_C \sin \mu_C}{r_C \cos (\theta_C + \mu_C)}\right)$$

$$f_4 = \tan\left(\frac{\theta_B - \mu_B + \theta_C - \mu_B}{2}\right)$$

$$f_5 = \frac{1}{2}\left(\frac{\cot \mu_B}{q_B} + \frac{\cot \mu_C}{q_C}\right)$$

$$f_6 = \frac{1}{2}\left(\frac{\sin \theta_B \sin \mu_B}{r_B \cos (\theta_B - \mu_B)} + \frac{\sin \theta_C \sin \mu_C}{r_C \cos (\theta_C - \mu_C)}\right) .$$

The process is repeated using the latest calculated values at C until convergence is obtained. Normally only a few iterations are needed.

Lines intersecting with a specified surface and points on the pressure boundary can be dealt with as in two-dimensional flow using the appropriate quantities f_i.

If there is no inner boundary, the expressions for f_3 and f_6 above cannot be used since r and θ are both zero on the centreline. A limiting form of the expressions[7] is used instead,

$$\lim_{\substack{r\to 0\\ \theta\to 0}} \frac{\sin \theta \sin \mu}{r \cos (\theta \pm \mu)} = \pm\left[\frac{1}{2} \cot \mu \frac{d\omega}{dx}\right]_{r=0} .$$

3 THE SUPERSONIC JET EXHAUSTING INTO QUIESCENT AIR

The calculation of the flow produced by a two-dimensional or axisymmetric choked jet expanding into quiescent air is described in this section. In general, the same method is used when there is an external stream since only the initial expansion and the conditions along the pressure boundary are changed.

3.1 The initial expansion

The method of characteristics requires an initial line along which the flow conditions are known. This initial line is obtained by generating a Prandtl-Meyer expansion fan from the lip of the shroud, Fig.4. Six lines are chosen from the expansion fan and the Mach number at the lip on the sixth line is determined from the pressure ratio of the jet, H_j/p_0,

$$M_{1,6} = \sqrt{5\left\{\left(\frac{H_j}{p_0}\right)^{2/7} - 1\right\}} \ .$$

The streamline direction at the lip on the six lines $\theta_{1,K}$ is assumed to increase linearly and the Mach number is derived from this angle.

$$\left.\begin{aligned} \theta_{1,K} &= \frac{K}{6}\,\omega(M_{1,6}) \\ M_{1,K} &= \omega^{-1}(\theta_{1,K}) \end{aligned}\right. \qquad K = 1,2 \ldots\ldots 6$$

where ω^{-1} is the inverse Prandtl-Meyer angle.

The first line, $K = 1$ is assumed to be straight and the point where it intersects the inner surface is calculated. A cubic spline interpolation of the inner surface ordinates is used in the present method to ensure a smooth variation of the surface slope. The flow conditions at the point of intersection can then be found. Eight additional points, spaced at equal intervals are selected on the line $K = 1$ and the flow conditions are assumed to vary linearly between the shroud lip and the surface, but provision is made in the calculation for investigating the effect of a Mach number profile at the nozzle exit, see section 6.4. An initial line is therefore formed consisting of ten points where the flow conditions are known; the first point is at the shroud lip and the tenth on the inner surface or centreline of the jet.

The calculation then proceeds along successive K lines. The flow conditions at the general point $x_{I,K}$ in the field being calculated from a $(\theta - \mu)$ line passing through $x_{I,K-1}$ and a $(\theta + \mu)$ line passing through $x_{I-1,K+1}$. K?

The number of points on a line in the initial expansion increases by one on each K line. On the last line, $K = 6$, there are fifteen points. The choice of six lines in the initial fan, and a total of ten points on the initial line is fairly arbitrary but these values have been found to give a good definition of the flow field for all geometries considered so far.

3.2 The pressure boundary and general field solution

After the initial expansion the solution is built up by a series of lines progressively downstream. The line, $K = 7$, is shown in Fig.4.

The first point on this line is on the pressure boundary. The Mach number at this point is known from the jet pressure ratio, for a jet expanding into quiescent air, or from a previous iteration when there is an external stream. The flow conditions and ordinates of this point are determined from a $(\theta + \mu)$ line passing through $K = 6$, $I = 2$ and a θ line, since the pressure boundary is a streamline, passing through $K = 6$, $I = 1$. The calculation for the other points along this line is made in exactly the same way as for a general point in the expansion fan.

4 THE SUPERSONIC JET WITH EXTERNAL FLOW

When a supersonic jet flows into a subsonic free stream the jet boundary is no longer a constant pressure boundary. The jet deflects the external flow giving rise to pressure gradients in the free stream. This leads to a considerable complication in the calculation of the jet development since the effective pressure ratio of the jet varies with the distance downstream of the nozzle exit, and an iterative method has to be used.

The shape of an initial jet pressure boundary is calculated and the pressure distribution produced by the subsonic stream flowing over this surface is then computed. This pressure distribution is then used to modify the conditions along the jet boundary and a new shape derived. This process is continued until the pressure distribution on the inner surface or the shape of the jet boundary does not change significantly between successive iterations.

The calculation will be described in three parts. The method of obtaining the shape of the initial pressure boundary is considered first. The calculation of the pressure distribution on the jet boundary is then described and the way in which this is fed back into the characteristic solution is outlined.

4.1 The initial pressure boundary

Several attempts have been made at deriving a suitable shape for the initial pressure boundary to give rapid convergence. The method now adopted as being the simplest and most satisfactory is to use the shape of the pressure boundary produced by the jet expanding into quiescent air. Convergence to the final pressure distribution then begins at the nozzle exit and the region where convergent results are obtained grows gradually downstream as the number of iterations is increased. For all configurations investigated so far, five or six iterations has produced converged results.

4.2 Calculation of the pressure distribution on the jet boundary

The number of points calculated on the jet boundary can be considerable for certain geometries and low pressure ratios, so to keep the computational time as low as possible, an interpolation is made on the shroud and jet boundary ordinates. Fifty points are selected at equal intervals from the upstream end of the shroud to the end of the jet boundary.

One problem is the discontinuity in slope at the end of the shroud, Fig.5 which exists in the inviscid calculation but is actually smoothed out by the boundary layer on the shroud in the real flow. The interpolation overcomes this problem to some extent by artificially removing this point from the calculation.

The geometry of the shroud must be included in the calculation since it can have a considerable effect on the initial development of the jet because of the shape of the pressure distribution near the end of the shroud.

The method of surface singularities[8] is used to calculate the pressure distribution with compressibility accounted for using the full Prandtl-Glauert transformation in both axisymmetric and two-dimensional flow.

From the calculated pressure distribution, the ratio of the local static pressure to the free-stream static pressure can be obtained. Another interpolation is then made to determine the modified jet pressure ratios at the axial locations of the points on the pressure boundary.

4.3 Recalculation of the characteristics solution

The modified jet pressure ratios obtained from the potential flow calculation are now used in recalculating the characteristic solution.

The method of calculating the initial expansion fan is unchanged though the effective pressure ratio used to determine the total turning angle is different because of the change in the static pressure at the shroud exit.

When the first point on the pressure boundary is reached the local jet pressure ratio corresponding to the axial locations of the first point from the previous iteration is used. Naturally, the location of the points is not quite the same but the difference becomes very small after the second iteration. The shape of the pressure boundary therefore stabilises from the shroud exit downstream.

Absolute convergence of the shape near the end of the pressure boundary is difficult to obtain but this is not too important in practice since viscous effects are becoming the dominant feature of the real flow. Six iterations have been sufficient in all but the most difficult cases so far.

5 SHOCK WAVES AND SUBSONIC PATCHES

Two problems are frequently encountered when applying the method of characteristics to jet flows; shock waves and regions of subsonic flow. The free stream adds to these difficulties because shock waves tend to be stronger as the focussing of characteristic lines caused by the curvature of the pressure boundary is aggravated by the changes in static pressure on the boundary.

5.1 Shock waves

Shock waves could be properly treated using the characteristic equations for rotational flow and the Rankine-Hugoniot relations (Ref.6, section H). This presents some formidable numerical problems especially if shock waves intersect and the use of the rotational equations could only be justified if the effects of boundary layers and mixing regions were included as well.

Shock waves are assumed to appear in the characteristic solution when two lines of the same family intersect. Different treatments are used for the two families of lines.

The intersections of two $(\theta - \mu)$ lines corresponds to a compression wave meeting an expansion wave and normally occurs very near to the inner surface. The solution adopted in the present method is to allow the lines to merge and to take the mean of the flow direction and Mach number at points on the merged lines, Fig.6a. Particularly strong shock waves which occur when an external stream is present show up as a merging of several lines with an associated rapid increase in pressure on the inner surface.

The intersection of two $(\theta + \mu)$ lines presents greater problems. Street[1] deletes the upstream characteristic and allows the other to continue with increased strength. The major disadvantage of this method is that for every intersection, one line is removed from the solution and the accuracy is reduced.

A somewhat similar approach is adopted here, but the loss of solution lines is avoided.

When the line BF intersects the line AE, Fig.6b, the point E is replaced by F and a new point G determined from characteristic lines passing through B and F; a new point is thus inserted when one is removed. Generally, further on in the calculation, the $(\theta + \mu)$ lines from F and G tend to merge and move towards the pressure boundary together, often being joined by other lines forming a plausible-looking shock wave at the jet boundary. Fig.7 shows a calculated 'shock wave' formed by four characteristic lines. This pattern was formed by a jet without an inner solid boundary at a pressure ratio of 2.4.

The techniques used to deal with shock waves are not rigorous and have been developed by trial and error; plausible results are obtained which are reasonably similar to the experimental results for shocks of moderate strength.

5.2 Local regions of subsonic flow

Local regions of subsonic flow occur quite frequently after shock waves and near the end of the inner surface. These patches are quite small in extent and it is worthwhile continuing the solution past them if possible. The present method, like Ref.1, forces the Mach number to remain slightly supersonic.

The main problem is then finding a suitable value of the flow direction since two values are possible once the Mach number has been fixed depending on which characteristic line is chosen. Considerable time has been spent investigating this problem and the method which gives the most consistent results is to use the mean of the flow direction at the origin of the characteristic lines, e.g. $\theta_C = \frac{1}{2}(\theta_A + \theta_B)$ in Fig.2a. This undoubtably introduces an error which is added to that caused by the simple treatment of shock waves, but the comparisons presented in section 6 shows that these errors are not too great for small subsonic regions.

6 COMPARISON BETWEEN THEORY AND EXPERIMENT

Two computer programs, for two-dimensional and axisymmetric flow, corresponding to the theory outlined in sections 2 to 5 were written. Each program was comprehensively tested for the jet flowing into quiescent air, and during this stage, the treatment for shock waves and subsonic patches was developed. The programs were then run with an external stream.

The main interest has been with axisymmetric flows with particular reference to the pressure distribution produced by the fan stream on the gas generator of a high bypass ratio engine operating at typical jet pressure ratios of between 2 and 3. Most of the results presented are for this type of problem. Two-dimensional jets do not occur frequently in practice and the experimental data for the two-dimensional jet with external flow used in section 6.2 was not very suitable for comparing with the theory.

A calculation for an axisymmetric jet requires about 10 to 15 minutes on an ICL 1907 computer for six iterations. A two-dimensional jet takes about half this time as the potential flow part of the calculation is simpler.

6.1 Two-dimensional jet exhausting into quiescent air

The two-dimensional jet flowing into quiescent air is the simplest problem and has been the subject of other investigations.

Experiment and theory are compared at two jet pressure ratios in Fig.8. The experimental results are from some unpublished data on flows over plug nozzles consisting of wedges with different half angles, similar to that shown in Fig.1a. The particular configuration used had a wedge angle of 11°. The agreement is quite good up to the first compression in both cases. An extensive region of subsonic flow extending almost to the pressure boundary is then predicted. Theory and experiment are in poorer agreement after this; the theoretical curve being displaced downstream. This has also been observed by Street[1] although the displacement effect of the boundary layer is included in his method. From this comparison it is assumed that the mixing at the jet boundary is more important than the boundary-layer growth along the wall.

It appears that the simple treatment of shock waves and subsonic patches is adequate up to jet pressure ratios of about three.

6.2 Two-dimensional jet with external flow

The only experimental results available for comparison with theory are those produced by Kettle *et al.*[9] on a rig representing an overwing engine installation. The flow in these tests was not very two-dimensional; the jet width is only three times its height at the nozzle and spreads sideways downstream. There is also a small rearward-facing step from the nozzle exit to the wing which cannot be represented in the theoretical model. The schlieren photographs indicate that this step caused a considerable disturbance which was reflected from the wing, near the nozzle exit, to the jet boundary where it was again reflected to the wing as a shock wave. The theoretical results, Figs.9 and 10, at two Mach numbers do not show this shock although the compression at about 80% chord which is a jet effect is quite well represented.

6.3 Axisymmetric jet exhausting into quiescent air

Two gas-generator shapes representative of high bypass ratio engines have been used for the calculations. Configuration 1 has a small jet exit height in comparison with the length of the gas generator which gives rise to a large number of compressions and expansions. The other shape, configuration 2, has a much larger exit height and a shorter gas generator. There are also significant differences in the geometries of the shroud. The shapes are shown in Fig.11, and the conditions at which the calculations have been made are summarised below.

Configuration	M	H_j/p_0	Figure
1	0	2.25	12
	0	2.5	13
	0.80	2.8	18
	0.85	2.75	19
	0.80	2.7	20
2	0	2.26	12
	0	2.54	14,21
	0.64	2.54	15,21,22
	0.85	2.55	16,21
	0.85	2.63	17
	0.64	2.54	22
	with a Mach number profile at the nozzle exit		

The experimental results were obtained from tests in the No.3 thrust measuring rig at NGTE[10].

The pressure distributions on the gas generators of the two configurations at a low pressure ratio shown in Fig.12 highlight the differences between the shapes of the gas generators. The agreement between theory and experiment is quite good, especially on configuration 1 where there are five compressions and four small regions of subsonic flow. The predicted regions of subsonic flow on configuration 2 are slightly greater. The theoretical curve is still moved downstream (as in the case of the two-dimensional jet) though the distance is less because of the lower jet velocity and presumably less significant mixing.

As the pressure ratio is increased, the shock waves move downstream and off the end of the gas generator, Figs.13 and 14, and the agreement between theory and experiment becomes worse. This trend is continued at even higher jet pressure ratios.

6.4 Axisymmetric jet with external flow

The calculation of the axisymmetric jet with an external stream becomes more difficult as the free-stream Mach number increases and it was not always possible to get a convergent solution at the end of the gas generator at Mach numbers of 0.8 and above. Configuration 1 presented more difficulties than configuration 2 and the results are presented in order of increasing difficulty.

Figs.15 and 16 show the pressure distribution on the gas generator of configuration 2 at a jet pressure ratio of about 2.5 and at free-stream Mach numbers of 0.64 and 0.85. The agreement between theory and experiment is good in both cases, perhaps better than was obtained without an external stream; this may well be because the mixing at the jet boundary is less significant due to the reduced velocity difference across it. At a higher pressure ratio, Fig.17, the agreement is still fairly good though a large region of subsonic flow is predicted at 65% to 70% of the gas generator length.

The results for configuration 1 at three fairly high jet pressure ratios are shown in Figs.18 to 20. The comparisons are quite good over most of the length of the gas generator.

At lower jet pressure ratios, a theoretical solution was not possible over the complete length of the gas generator, the calculation breaking down after the second shock. Increasing the number of iterations made no difference. The reason why the calculation would not continue is still unclear though it is probably due to an accumulation of errors introduced by the simple treatment of shock waves; the calculation always broke down after two strong shocks. The free-stream Mach numbers also has an effect since it has been possible to obtain a good solution on a shape similar to configuration 1 at a Mach number of 0.6.

The theoretical pressure distributions obtained on configuration 2 at the same pressure ratio and at three Mach numbers are shown in Fig.21 to demonstrate the effect of the external flow. The free stream not only moves the shock waves downstream but also broadens the region of low pressure following the initial expansion.

The effect of a Mach number profile across the nozzle exit can be readily investigated with the computer program; one example is shown in Fig.22. A parabolic distribution of Mach number was used with an increase of 10% at the midpoint to simulate a 'pipe flow' profile. There are significant differences in the pressure distributions along the nozzle which would have a measurable effect on the overall thrust.

7 PROPOSED EXTENSIONS TO THE METHOD

7.1 Addition of a wing

The supersonic jet in isolation has been the main topic of investigation in this Report but several extensions to the theory involving the potential flow part of the calculation are possible.

The jet situated in a typical underwing position has already been investigated experimentally[11] but a theoretical treatment would be useful in understanding some of the results.

The addition of a wing to the flow field presents no major problems, in principle, since this too could be covered with surface singularities and included in the potential flow calculation. However, for an axisymmetric jet the problem becomes truly three-dimensional. In an attempt to avoid recourse to a full vortex lattice approach, a spanwise variation of singularity strength across the wing was tried. Unfortunately this did not work.

A crude form of local linearisation of the problem was then attempted. In the calculation of the subsonic potential flow, the two-dimensional wing was represented by the appropriate distribution of singularities on the surface. An A.M.O. Smith type calculation was then made on the jet and the wing. The boundary condition that the surfaces are streamlines is only satisfied along two lines: along the generator of the jet and shroud nearest to the wing lower surface, and along a line on the wing in the plane of this generator. This therefore assumes that the velocities induced by the jet on the wing are the same across the span of the wing and the velocities induced by the wing on the jet are the same around the circumference. Such a calculation does not predict the wing pressure correctly but the change in pressure on the wing due to changing the shape of the jet by varying the pressure ratio can be obtained.

The increment in pressure coefficient ΔC_{p_j} used in Ref.11 was a simple measurement of the jet effect. It was found by measuring the difference between the wing lower-surface pressures when the jet was blown at a specified pressure ratio and when the jet was blown at a pressure equal to the free-stream total pressure simulating a free flow nacelle. The shape of the jet used in the calculations for the latter case was the same as the 'equivalent solid body' used by Kurn in some earlier tests[12]. The difference in the wing pressure coefficients obtained from the calculations is compared with the measured values in Fig.23b. The agreement is quite good.

The pressure distribution on the gas generator was not measured in Kurn's experiments but the predicted results are also shown in Fig.23a. The trend is similar to that observed on the schlieren photographs, the presence of the wing suppresses the shock formation over much of the gas generator length on the side adjacent to the wing.

The good agreement may be fortuitous in this case as the method breaks down if the jet is moved too close to the wing and a full three-dimensional calculation of the potential flow is required.

7.2 Viscous effects

The boundary layer on the inner wall could be treated in a similar manner to Street[1] but this seems hardly worth doing unless the mixing layer at the edge of the jet can also be included.

8 CONCLUSIONS

A method of calculating the development of a supersonic jet flowing into a subsonic external stream has been developed. Quite good agreement has been obtained on a limited range of geometries at jet pressure ratios of 2.2 to 3. The model of the flow neglects viscous interactions but the results suggests that the omission of the mixing region is less important when there is a subsonic free stream.

The method has been primarily used to predict the pressure distributions on gas generators of high bypass ratio engines and although the pressure distribution is not accurate enough for estimating drag forces it could be useful in initial design studies.

The effect of changing the velocity profile at the nozzle exit can also be investigated by the present method. Significant changes in the pressure distribution can be produced by a small change in the velocity profile.

Acknowledgment

The author wishes to thank Mr. A. Seed of NGTE for providing most of the experimental data and for permission to quote the results.

SYMBOLS

a	speed of sound
c	wing chord length
f_i	functions of θ, μ, x, r (see section 2.2)
H	total pressure
ℓ	gas generator length
M	Mach number
p	static pressure
q	total velocity
r	radial ordinate
t_1	$\tan(\theta + \mu)$
t_2	$\tan(\theta - \mu)$
u	velocity component in axial direction
v	velocity component in vertical or radial direction
x	axial ordinate
y	vertical ordinate
ΔC_{p_j}	incremental change in pressure coefficient due to jet stream
θ	streamline direction
μ	Mach angle
ω	Prandtl Meyer-angle

<u>subscripts</u>

I	number of point on a characteristic
j	refers to jet conditions
K	number of $(\theta - \mu)$ lines
0	free-stream conditions

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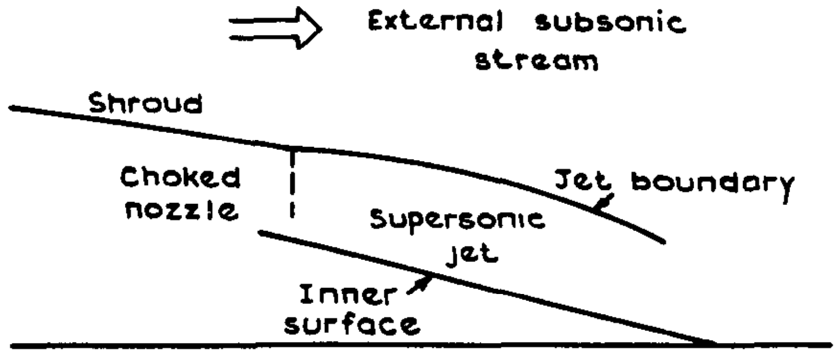
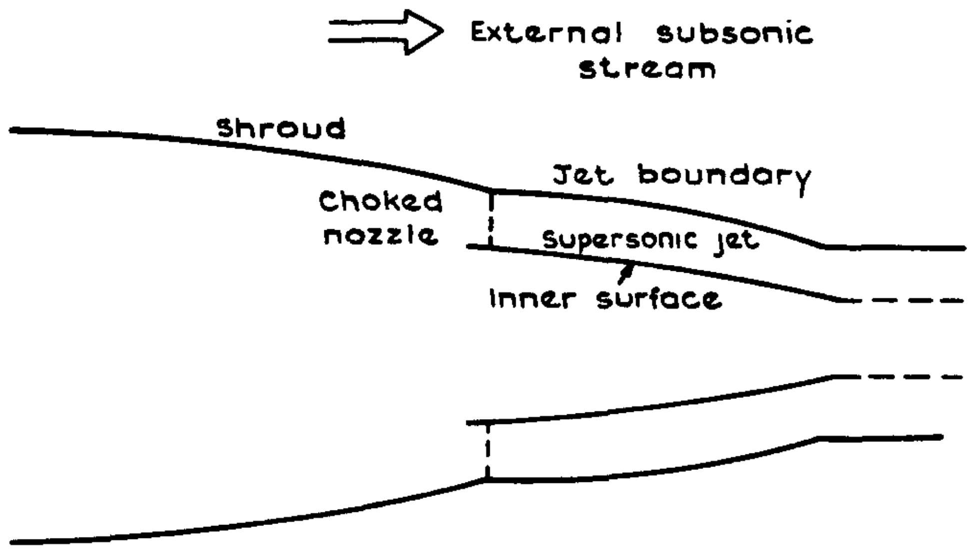
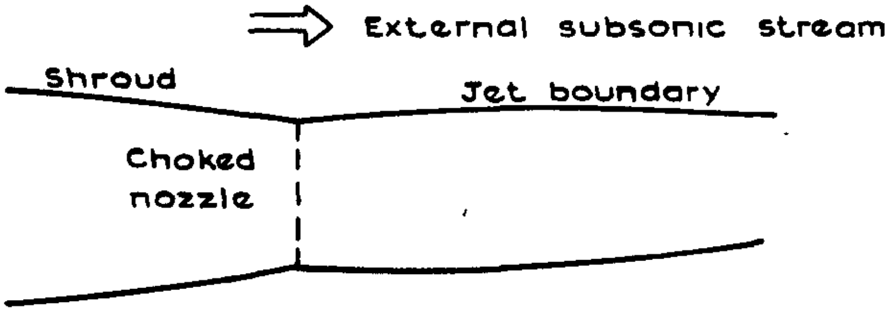


a Two-dimensional flow

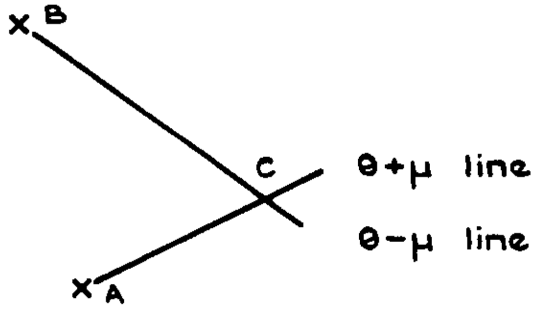


b Axisymmetric flow with inner surface

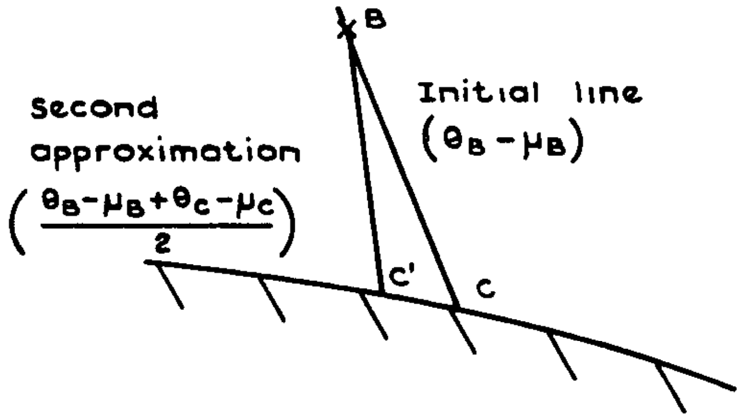


c Axisymmetric flow without inner surface

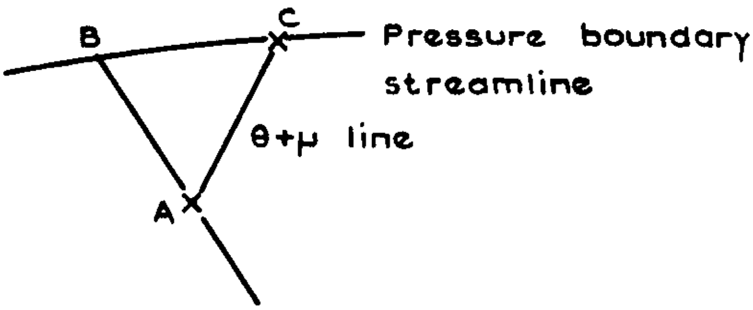
Fig. 1 a-c Typical configurations and terminology



a General field point



b Intersection with specified surface



c Pressure boundary point

Fig. 2 a–c Possible intersection of characteristic lines

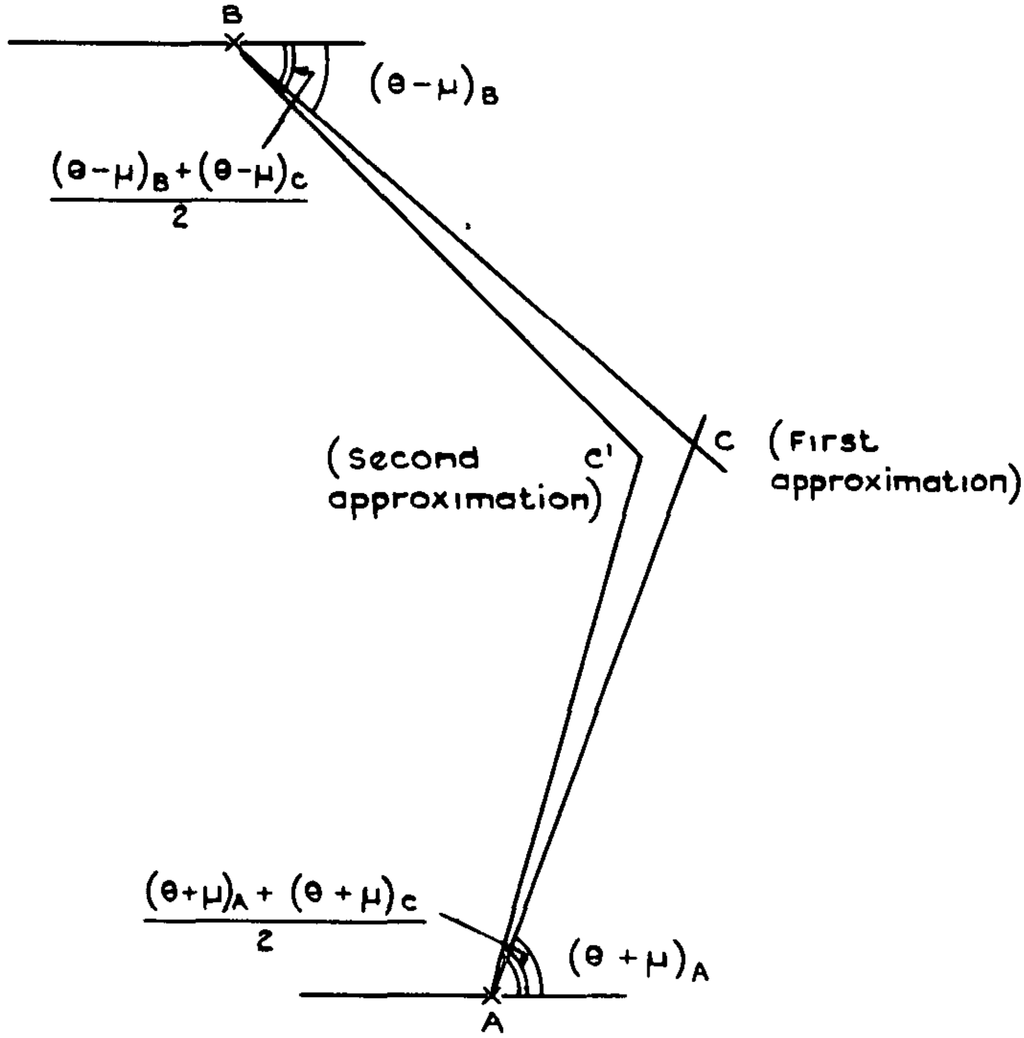


Fig. 3 Intersection of characteristic lines for axisymmetric flow

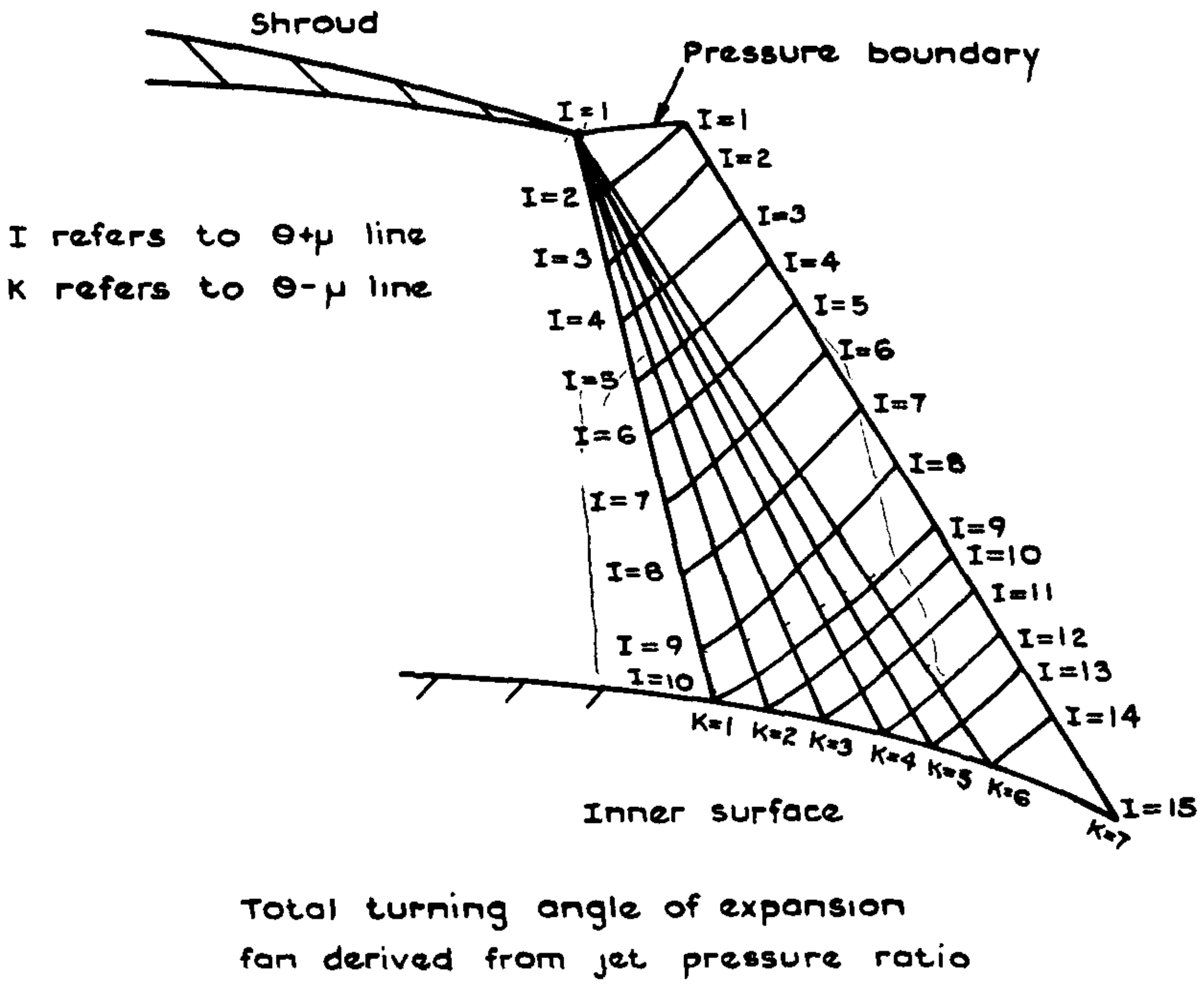


Fig. 4 Initial expansion from shroud exit and first line of general calculation

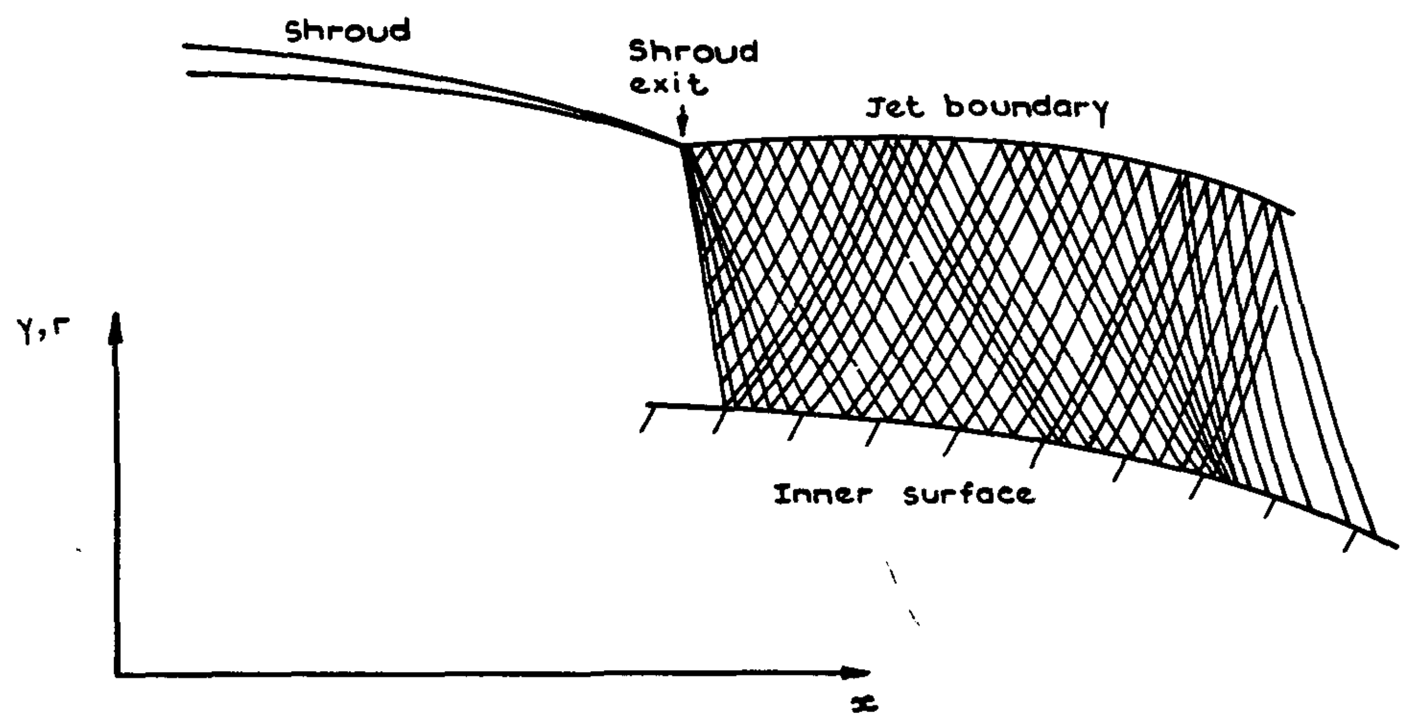
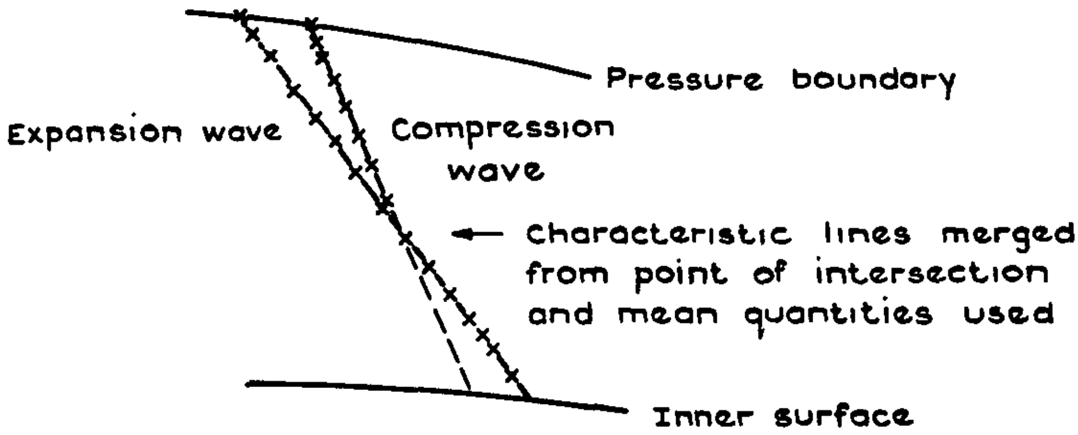
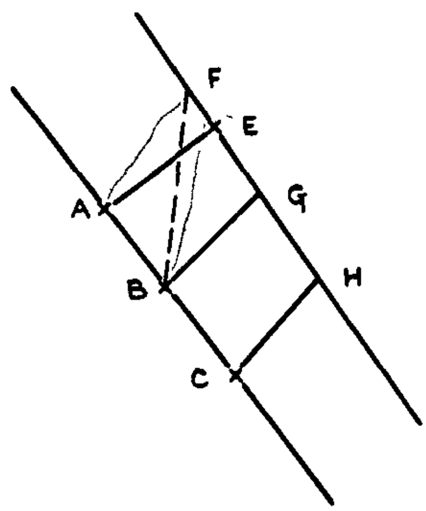


Fig. 5 Geometry of the configuration



a Intersection of two $\theta-\mu$ lines



Point E replaced by F and a new line BG inserted

b Intersection of two $\theta+\mu$ lines

Fig.6 a & b Intersection of characteristic lines

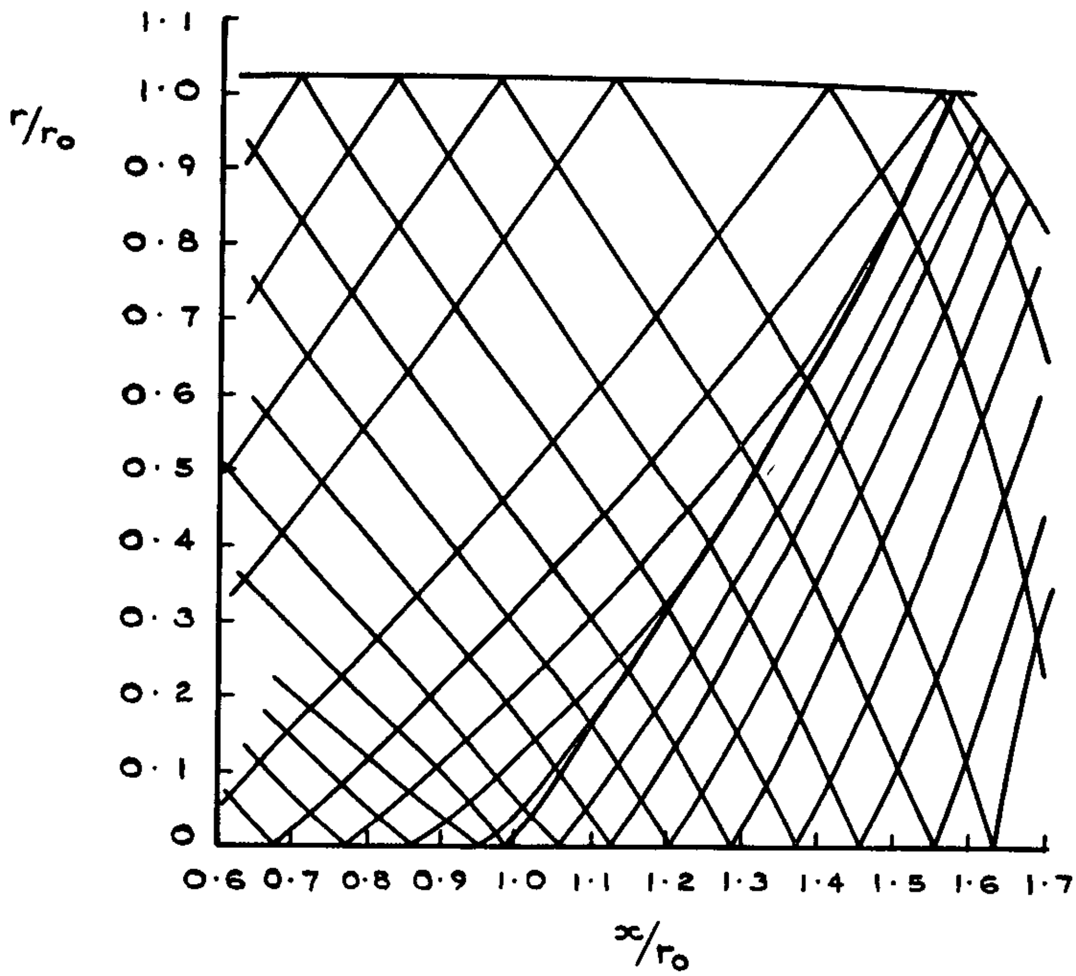


Fig.7 Calculated "shock wave"

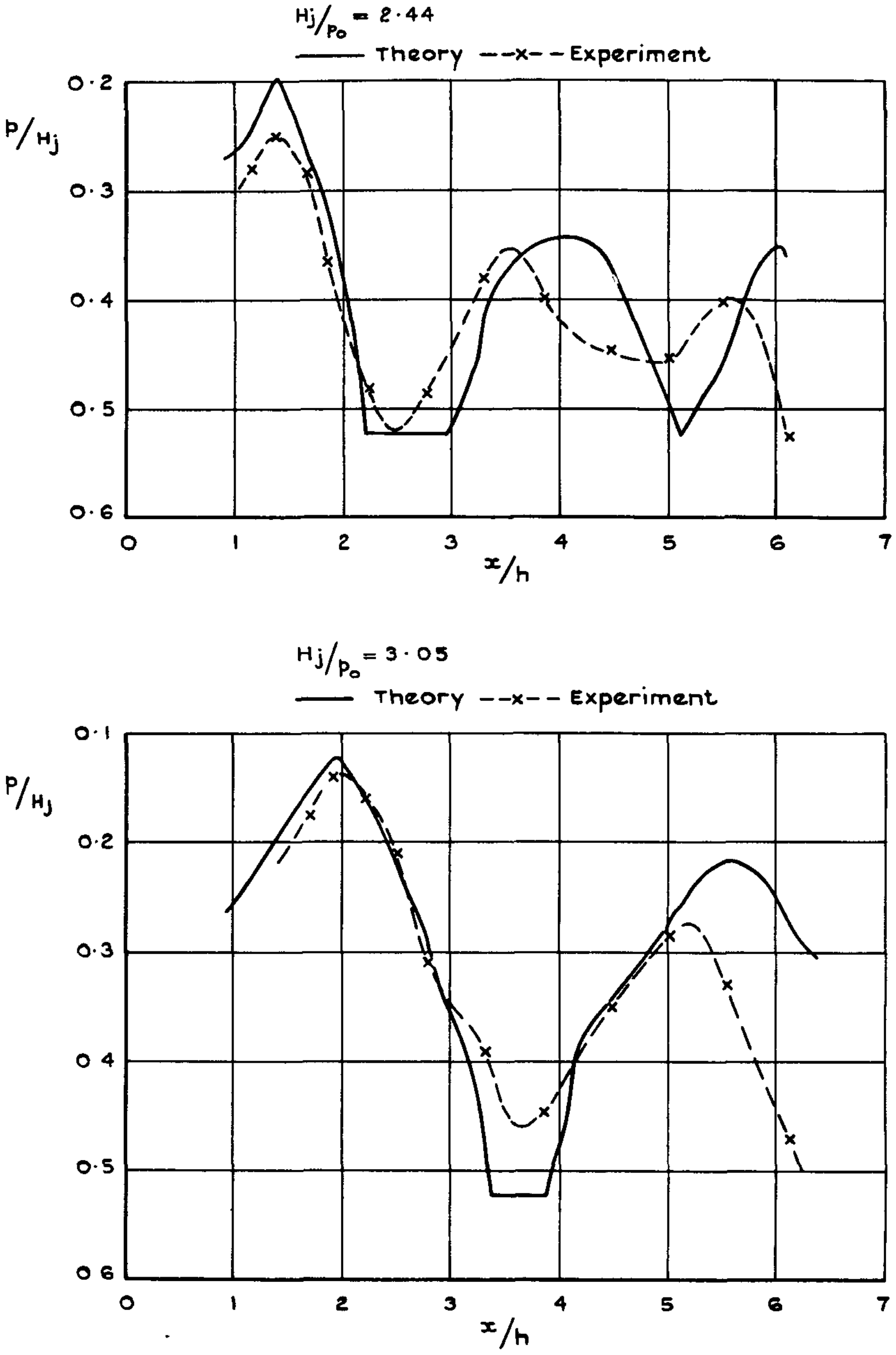


Fig. 8 Comparison between theory and experiment; two dimensional jet; no external flow

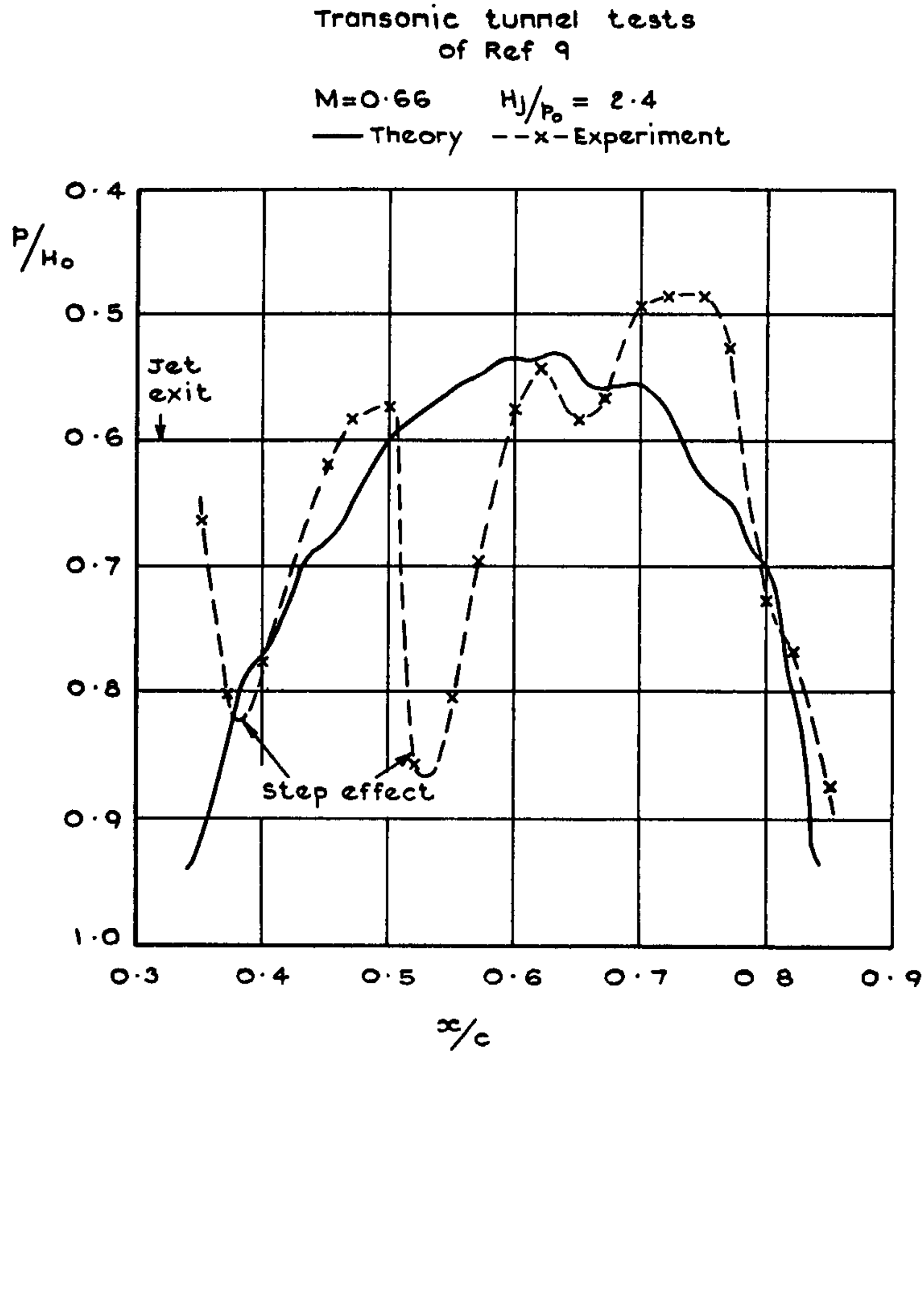


Fig. 9 Comparison between theory and experiment; two dimensional jet; with external flow

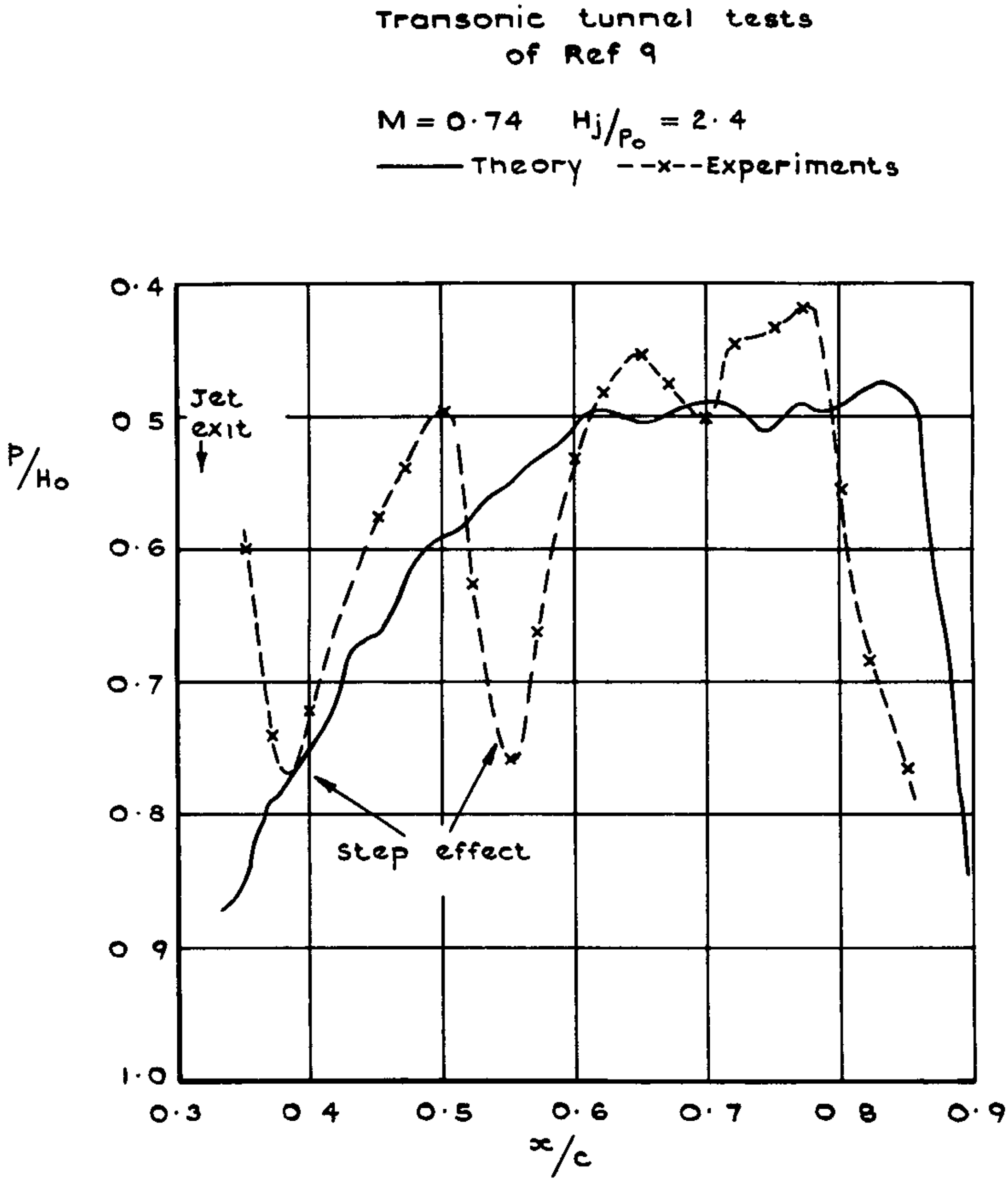


Fig.10 Comparison between theory and experiment; two dimensional jet; with external flow

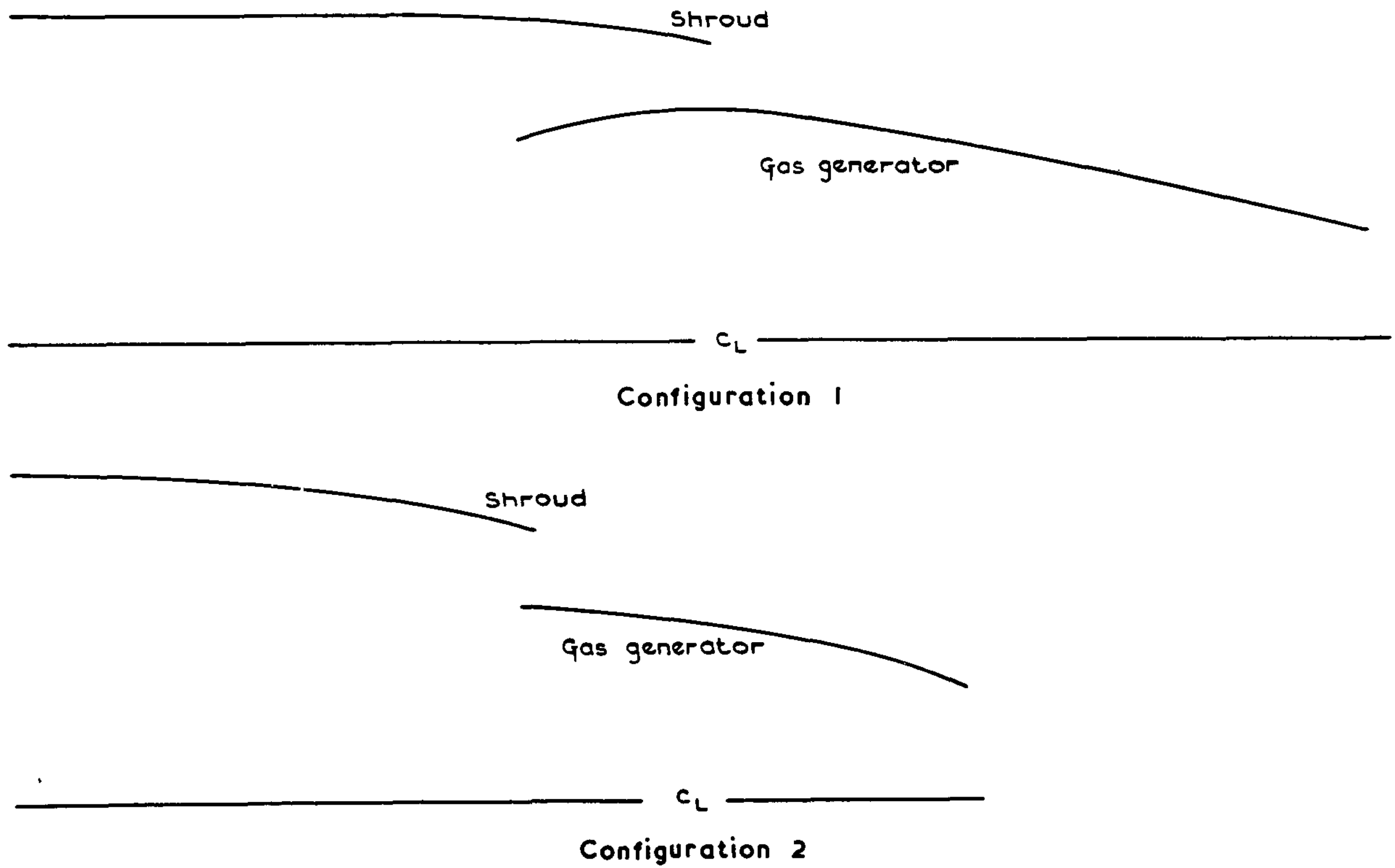
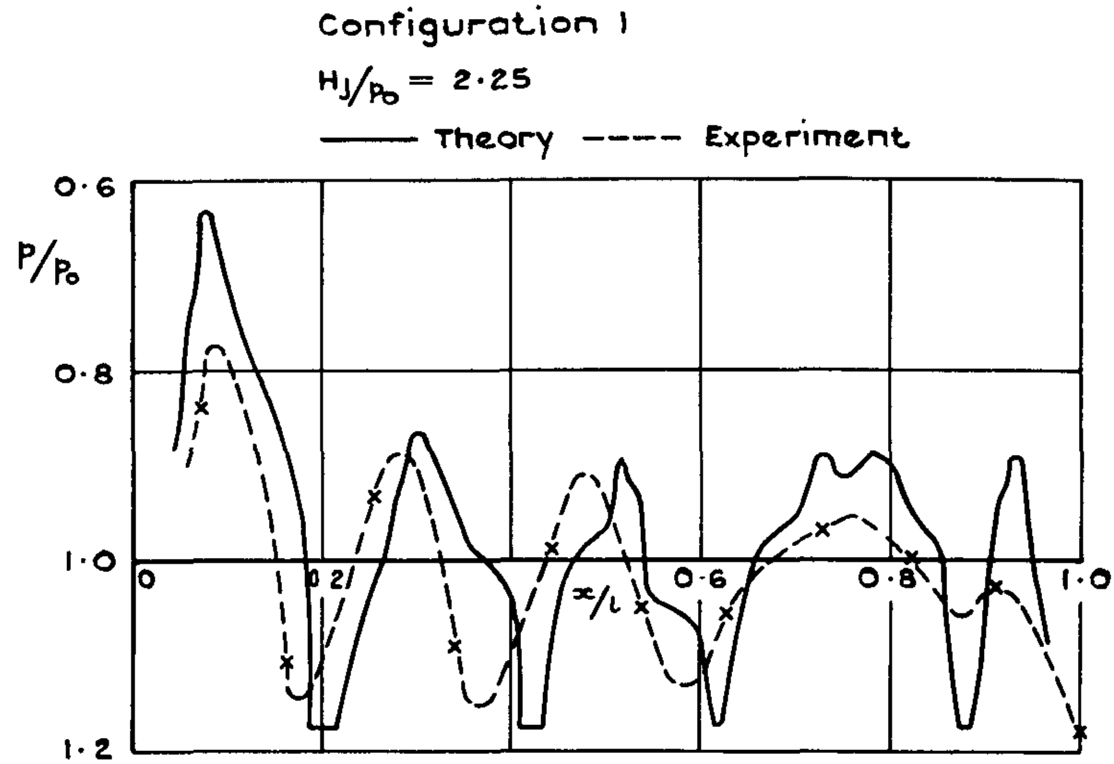


Fig. 11 Axisymmetric shapes



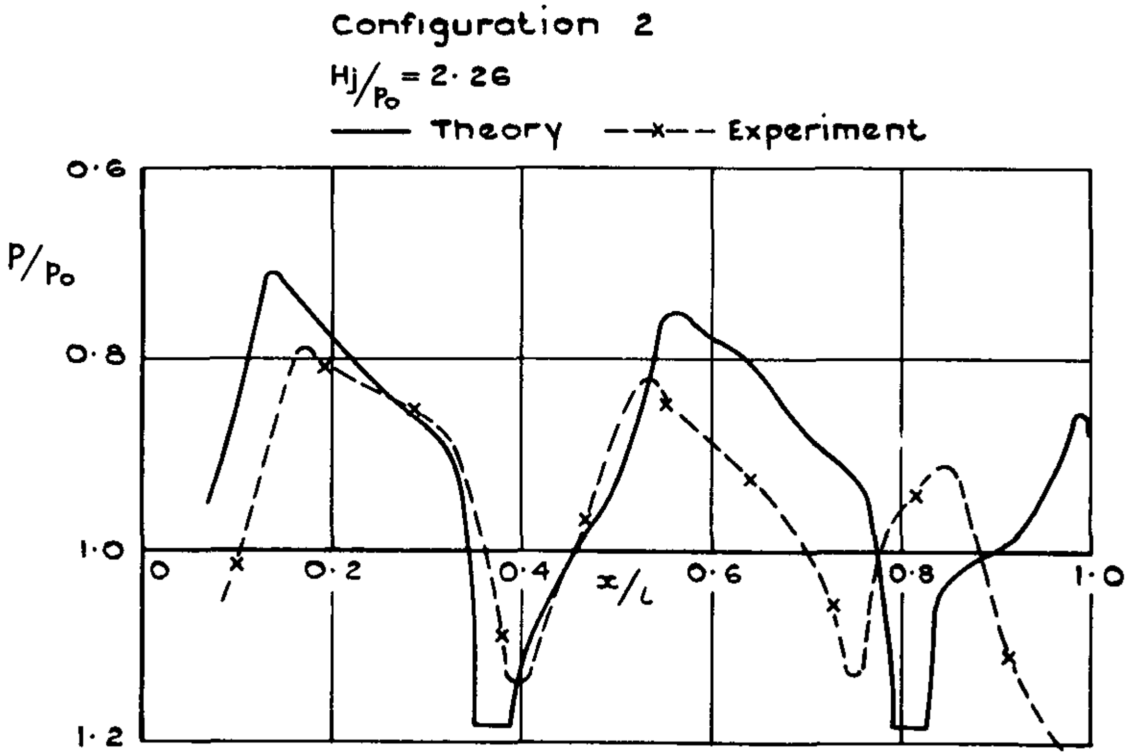


Fig. 12 Comparison between theory and experiment; axisymmetric jet; no external flow

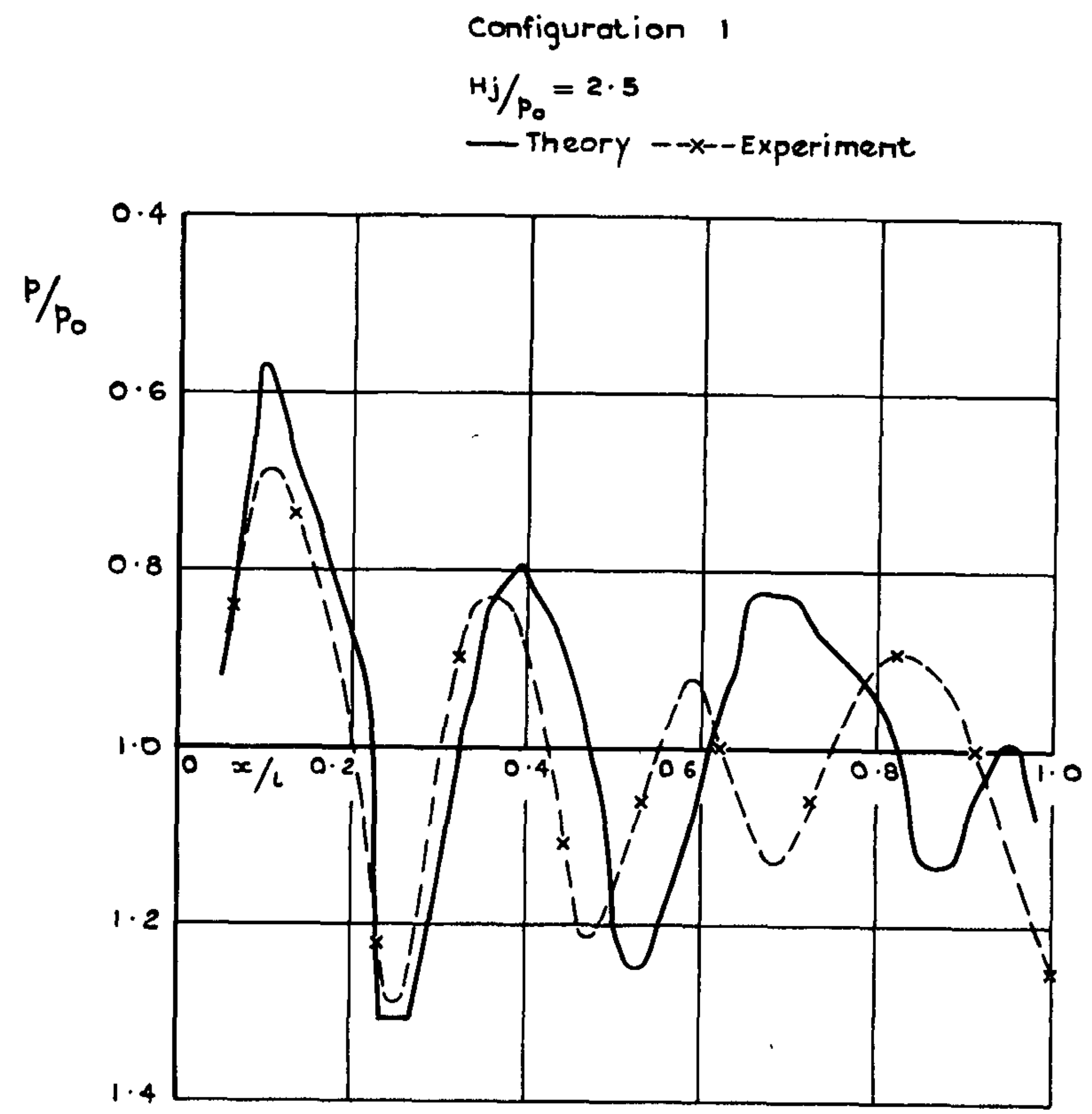


Fig. 13 Comparison between theory and experiment ; axisymmetric jet ; no external flow

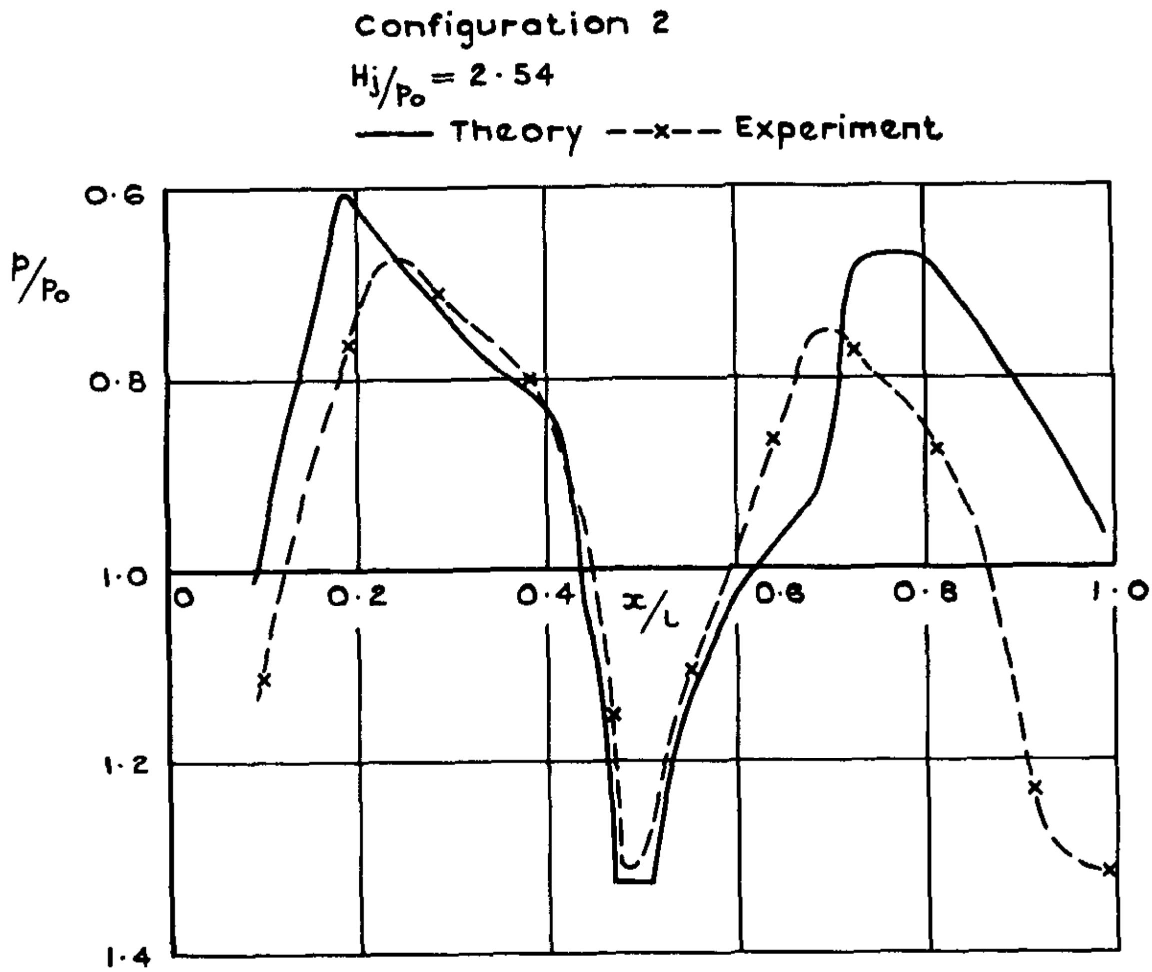


Fig. 14 Comparison between theory and experiment; axisymmetric jet; no external flow

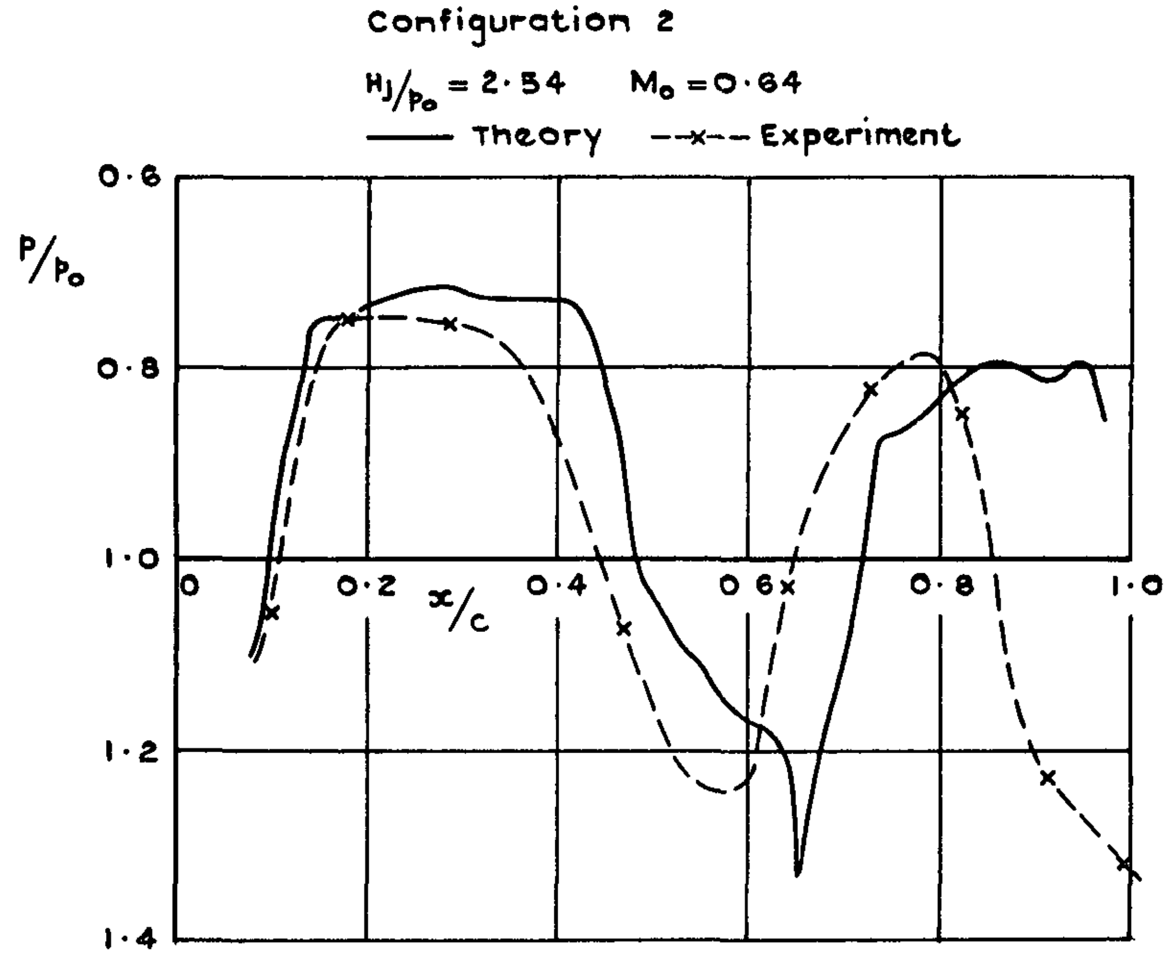


Fig. 15 Comparison between theory and experiment ; axisymmetric jet ; with external flow

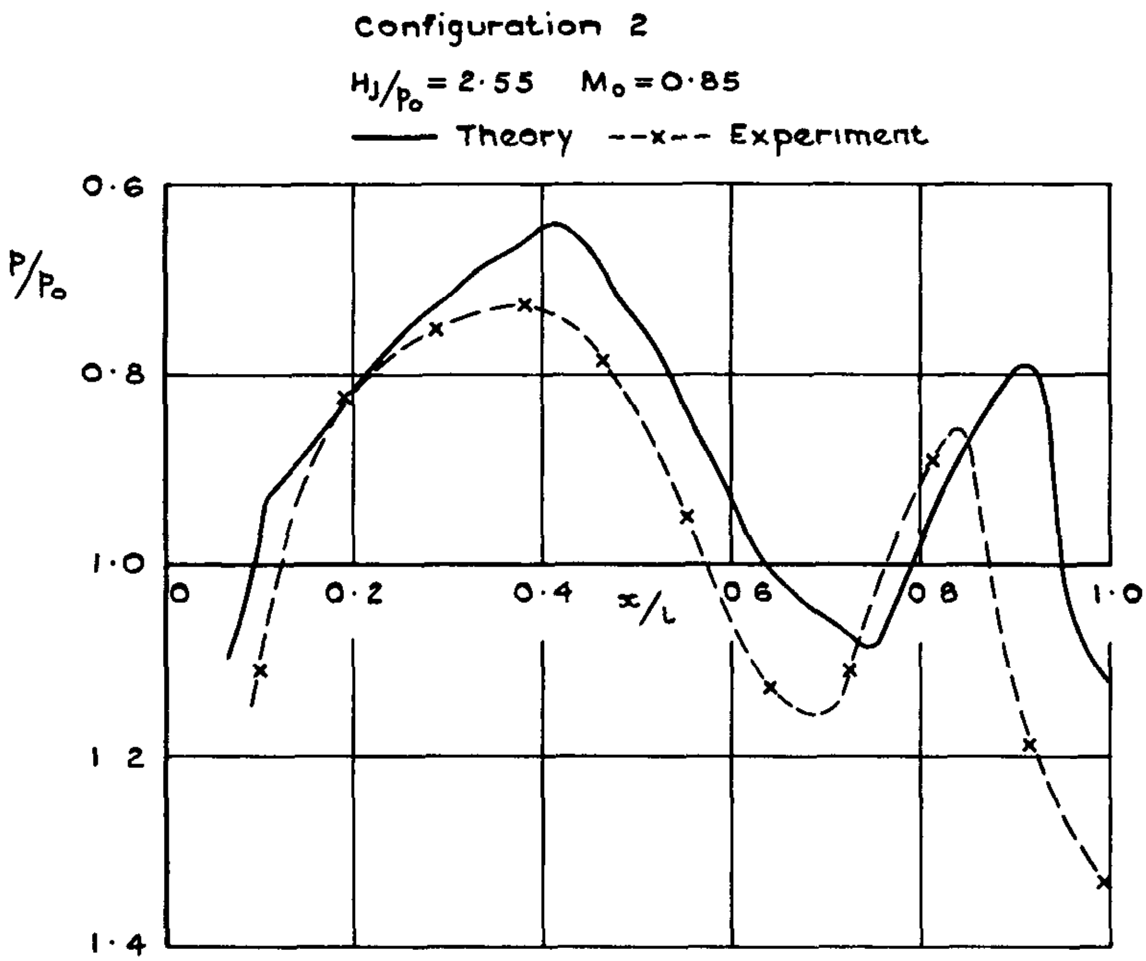


Fig. 16 Comparison between theory and experiment; axisymmetric jet; with external flow

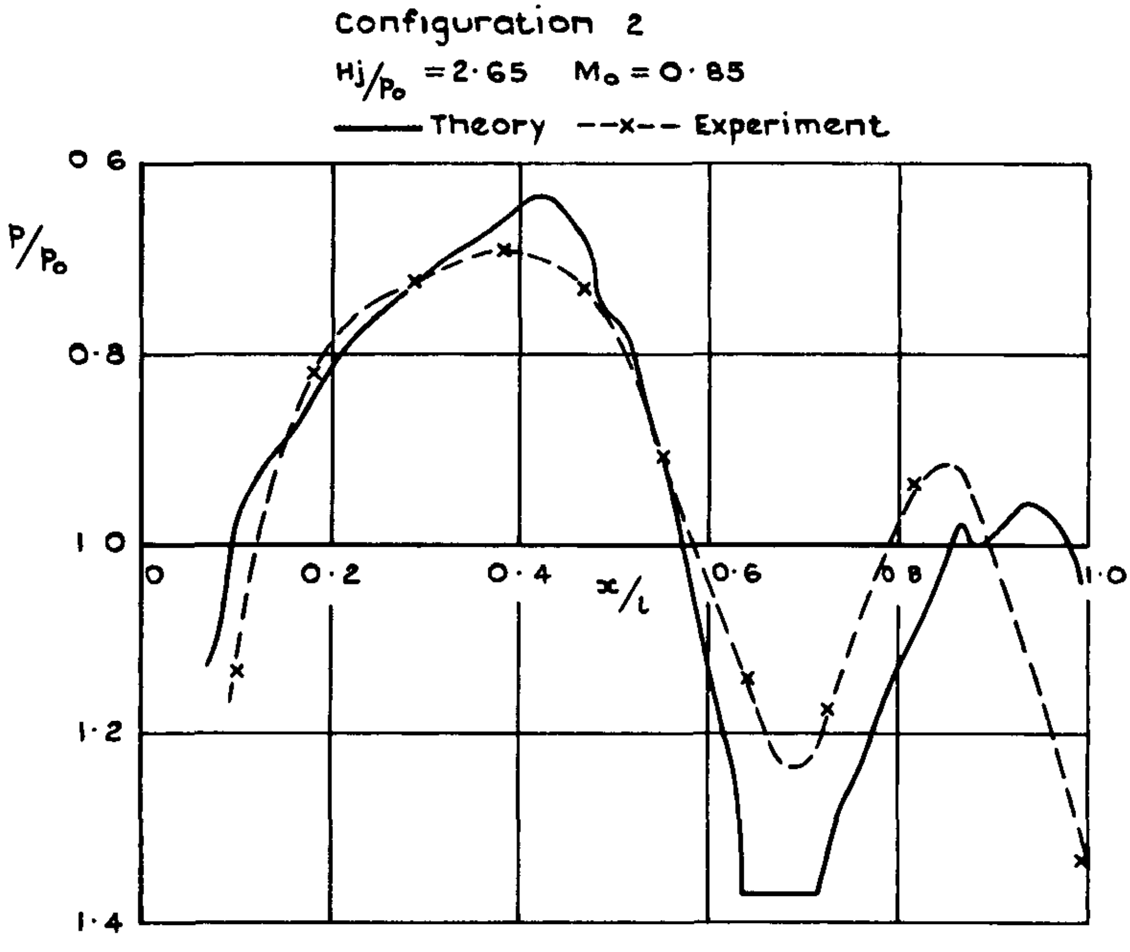


Fig. 17 Comparison between theory and experiment; axisymmetric jet; with external flow

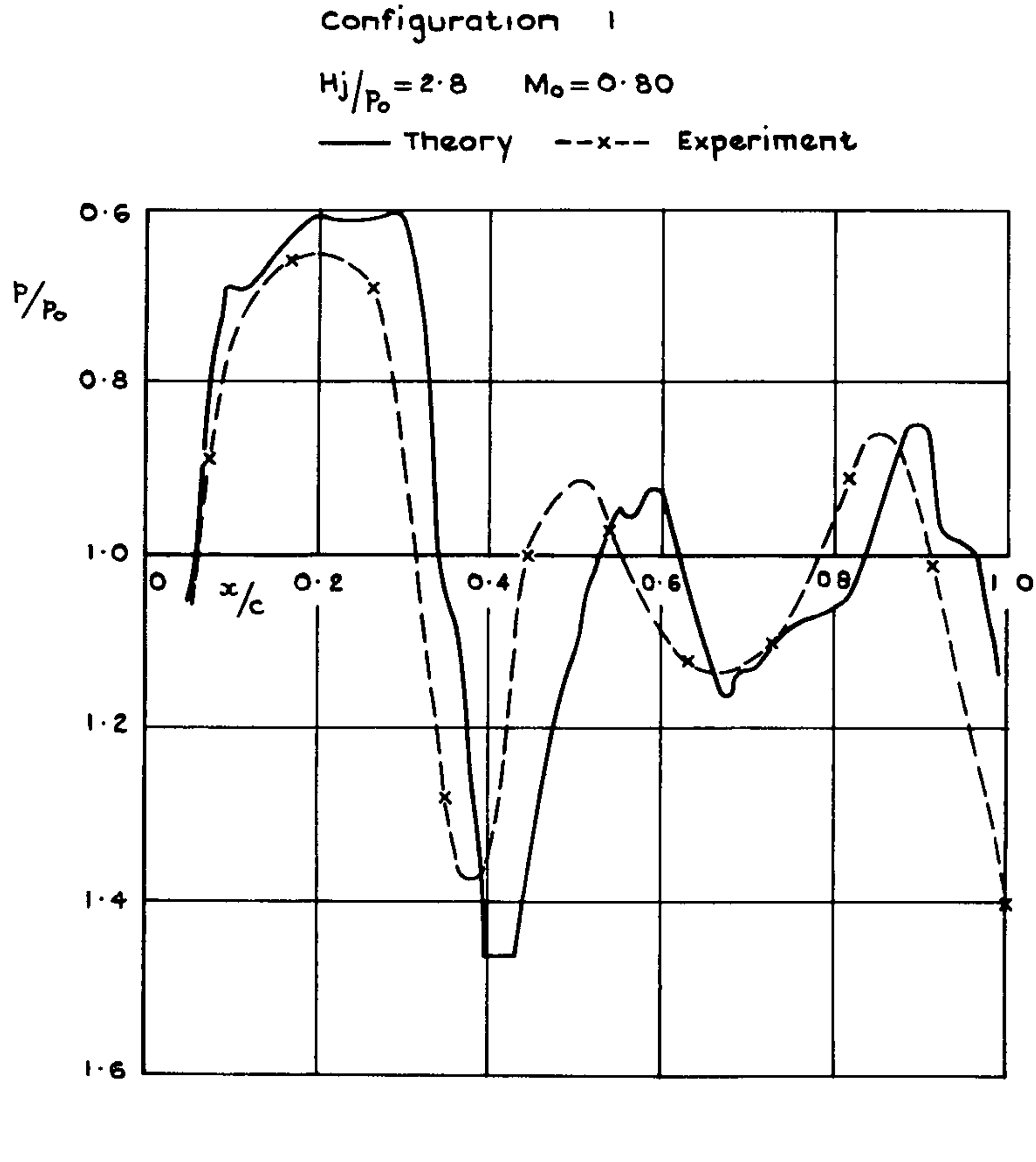


Fig. 18 Comparison between theory and experiment; axisymmetric jet; with external flow

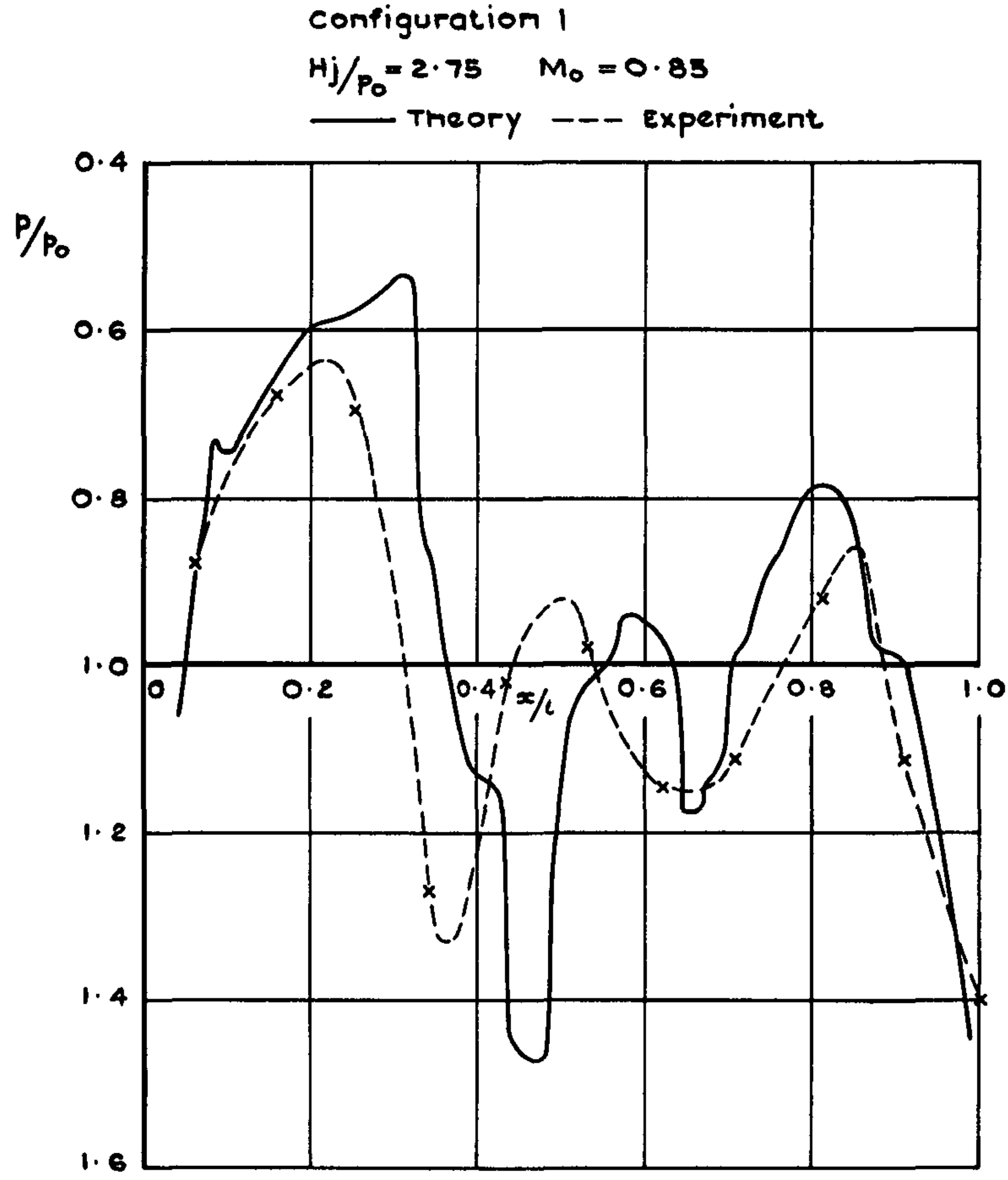


Fig. 19 Comparison between theory and experiment; axisymmetric jet ; with external flow

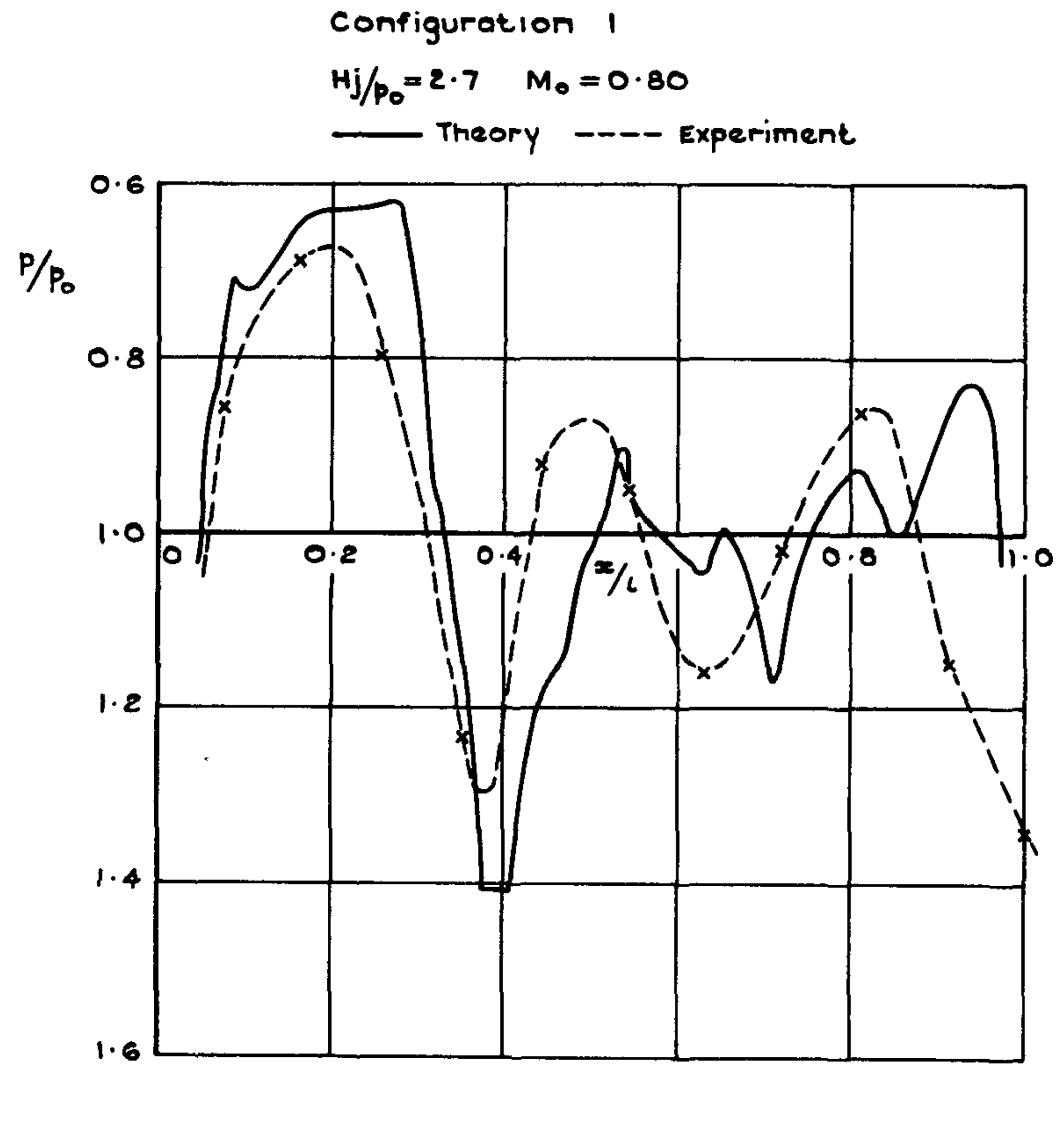


Fig. 20 Comparison between theory and experiment; axisymmetric jet; with external flow

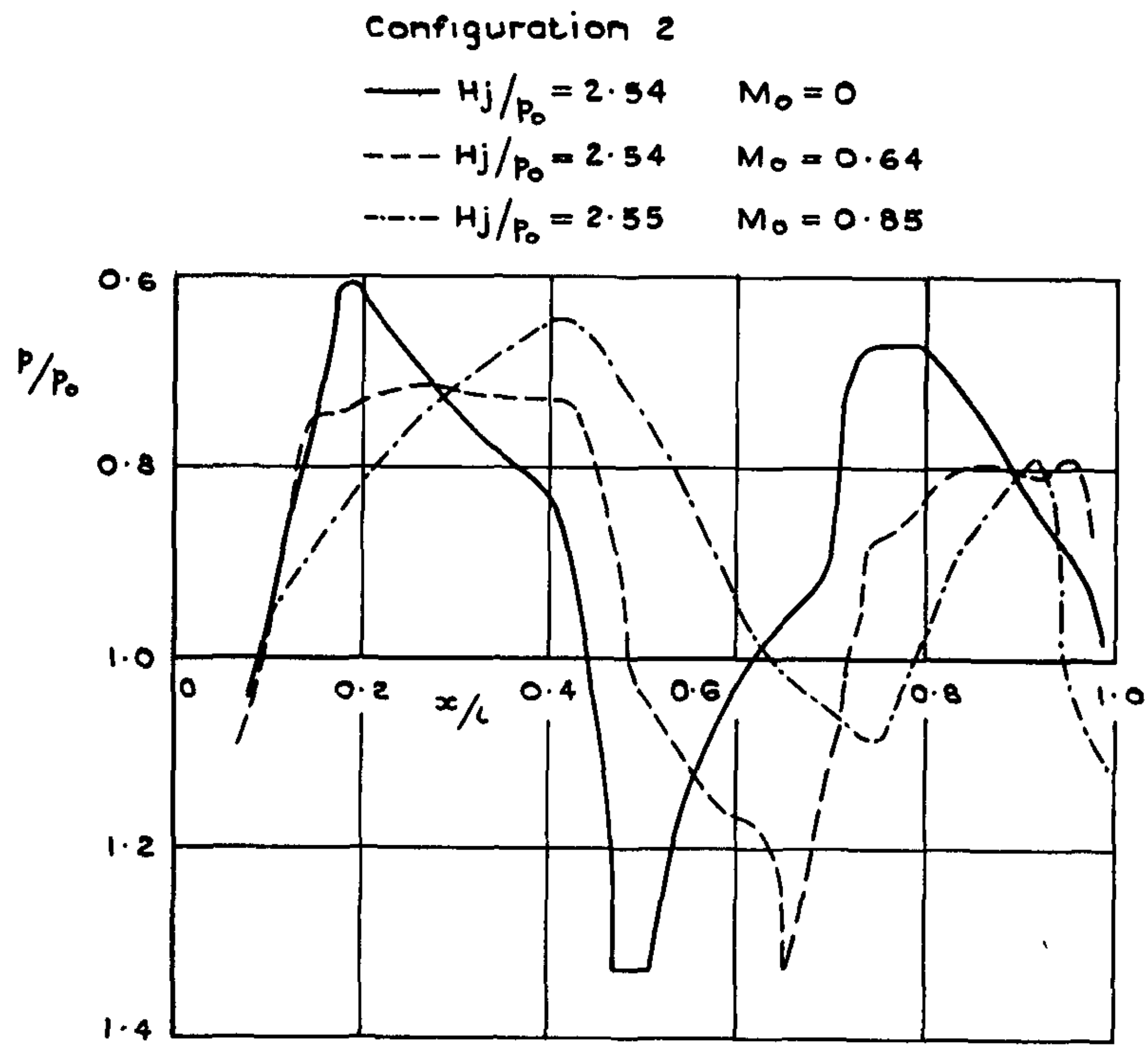


Fig.21 The effect of Mach number on the gas generator pressure distribution

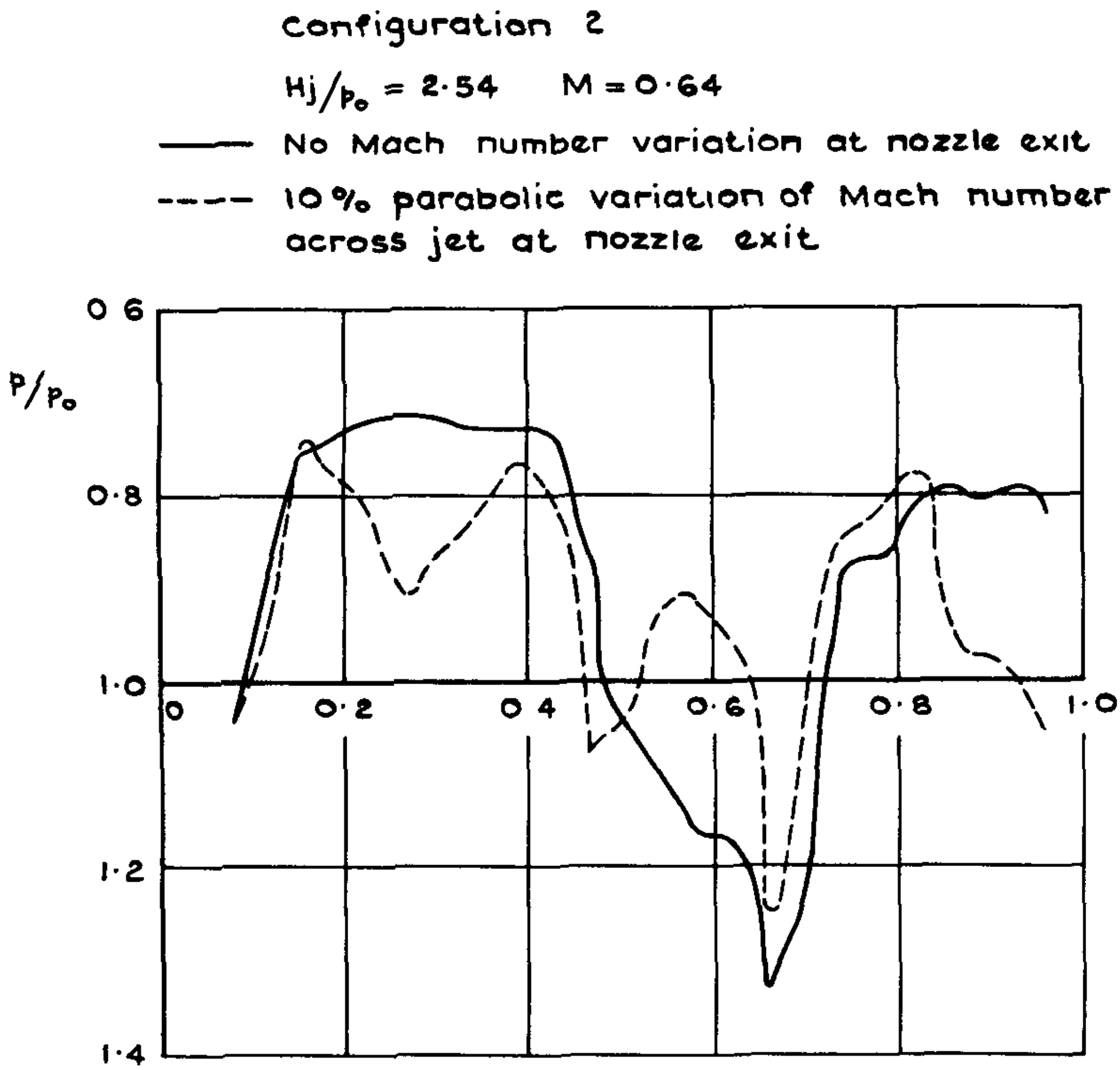
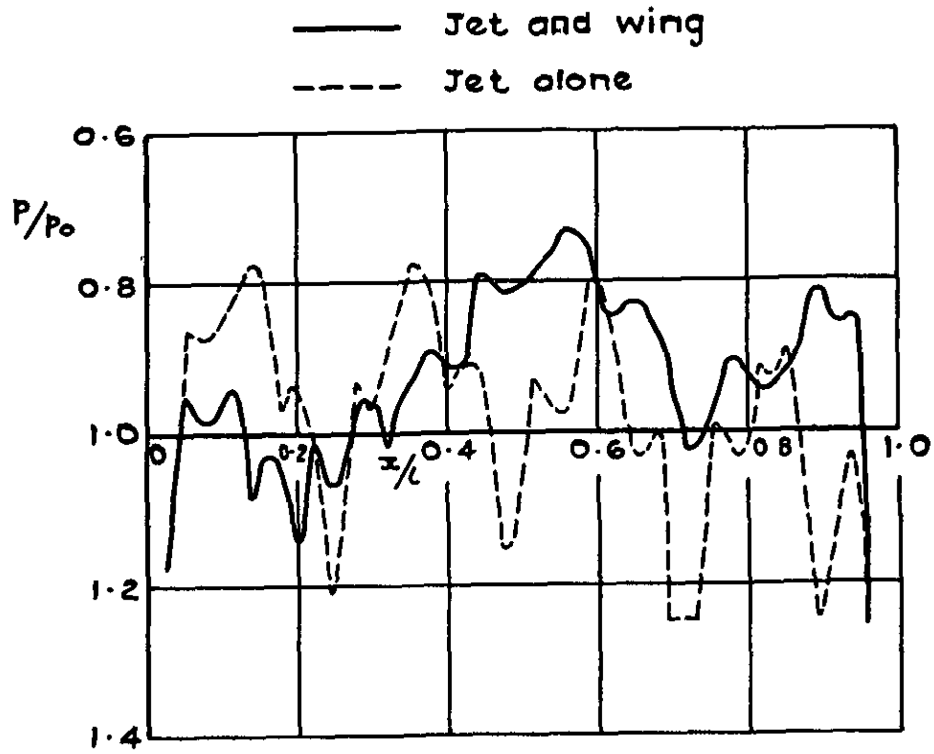
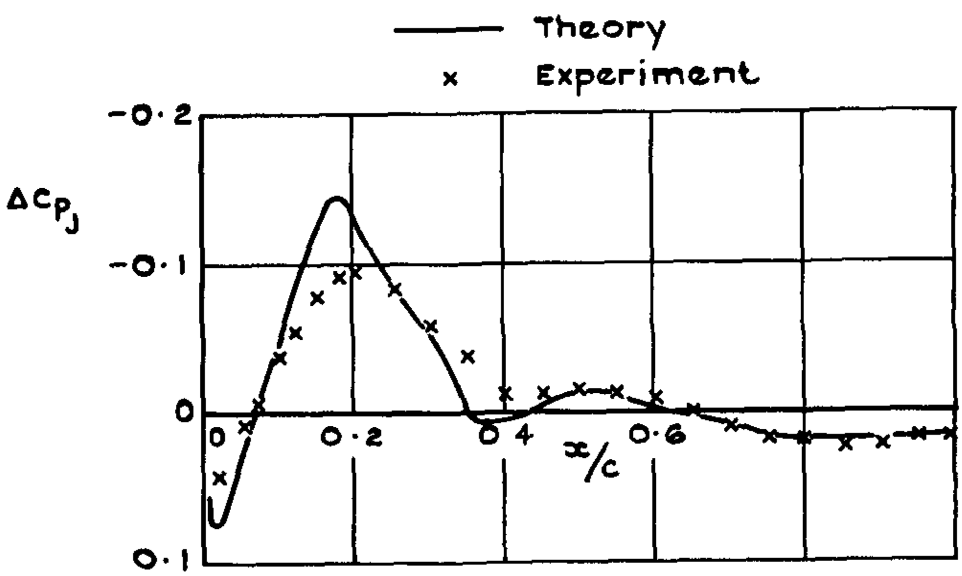


Fig. 22 Effect of varying Mach number distribution across nozzle exit



a Gas generator pressure distribution



b Incremental change in wing pressure distribution

Fig. 23 a & b Jet wing interference

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ERRATA

NASA Technical Report R-6

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and Mildred J. Woodling
1959

Page 108:

In the key at the top of figure 20, a solid line should appear opposite the word "Experiment."

Page 116:

The line keys above figure 23 should be corrected as follows:

———————— Characteristic boundary

\- - - - - - - Circular-arc, satisfying δ_j and $\left(\frac{y}{r_j}\right)_{max}$

——— - ——— Circular-arc, $\frac{\rho_o}{d_j}$

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The method of characteristics is used with the conditions on the jet boundary modified by the change in static pressure produced by the external stream. The method is particularly applied to cases where the inner boundary of the jet is a solid surface, e.g. the fan exhaust of a bypass engine.

Theory and experiment are in close agreement for the range of jet pressure ratios and free-stream Mach numbers associated with engines of high bypass ratio.

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