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Some Examples of the Application of Methods for the Prediction of Boundary-Layer Transition on Sheared Wings

by

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SOME EXAMPLES OF THE APPLICATION OF METHODS FOR THE PREDICTION OF
BOUNDARY-LAYER TRANSITION ON SHEARED WINGS

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SUMMARY

The laminar boundary layer has been calculated for the leading-edge
region of four selected aerofoils for cases where the supercritical region is
terminated by a shock wave at about 20% chord. The possibility of the boundary
layer becoming turbulent before the shock wave is then considered according to
four different criteria: leading-edge contamination, re-laminarisation, sweep
instability and Tollmien-Schlichting instability. Many simplifying assumptions
have had to be made, since the purpose of the Report is to demonstrate how the
problem might be treated, rather than to present definitive results, and how
the various mechanisms are seen in conjunction. It is concluded that much more
needs to be known before predictions can be made confidently with any degree of
precision.

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CONTENTS

1   INTRODUCTION

One of the many problems that arise in investigations of the flow past
swept wings is that of determining the state of the boundary layer. Hall[1] has
reviewed the then current knowledge of the effects of variations in Reynolds
number on the possible types of flow over a swept wing and the boundaries
between them. Here, we are concerned only with where and how transition from
the laminar to the turbulent state occurs, for a given wing shape and Reynolds
number.

There are several general features of the flow over a swept wing which
affect the problem, and which may be discussed in terms of the flow elements
sketched in Fig.9 of Ref.2. The flow along the attachment line along the
leading edge may be thought of as originating on the solid surface from a
stagnation point at the apex of the wing or at the nose of the body. Flow
separation may occur just upstream of the wing-body junction and lead to the
formation of junction vortices (as described, for example, by East and Hoxey[3])
but, in any case, the flow along the attachment line may become turbulent on its
own account by a mechanism which is commonly called 'leading-edge contamination'.
If it does, there is a possibility that the flow may revert to the laminar state.
This is commonly called 're-laminarisation'.

If the streamlines are viewed in a direction along the leading edge it is
apparent that some concavity may exist near the attachment line; thus the
possibility of an instability of the kind investigated by Görtler and Witting[4]
must be admitted although it has not been possible to treat this quantitatively
in this analysis. In planview, the streamlines downstream of the attachment
line are curved as the component of the velocity of the external flow normal to
the leading edge changes. This flow may be unstable due to cross-flow as
described, for example, by Stuart[5]. There are some indications that the actual
transition process is then fairly rapid. The curvature of the streamlines may
also be such that the flow separates, as described by Maskell and Weber[6].
Finally, the flow may become unstable in the sense of Tollmien and Schlichting;
this could occur at any point aft of the attachment line, given the right
conditions.

The present Report is concerned with the prediction of transition on
sheared wings of infinite span and consideration is given in turn to leading-
edge contamination and the probability of re-laminarisation, cross-flow
instability and the Tollmien-Schlichting type of instability. Available

criteria are employed to predict transition but the calculation of the laminar boundary layer is performed using a method recently developed by Beasley[7]. This method allows the velocity profiles to be found accurately in any direction, which is believed to be of particular significance in the context of cross-flow instability, although at present the method takes no account of effects of compressibility[*].

Four different aerofoil sections are considered which have pressure distributions fairly typical of flows that have a supersonic region terminating with a strong shock wave at about 20% chord. The free stream Reynolds number is varied from values which are representative of those that can be achieved in existing transonic wind tunnels, to values which may obtain in full-scale flight.

Of course, it must be appreciated that some of the effects observed in fully three-dimensional flows over finite wings are ignored in this analysis and, furthermore, that the omission of the effects of compressibility from the boundary-layer calculations can only be excused by the absence of a suitable method for calculating them at the present time. The methods used to predict transition are of uncertain accuracy, as discussed by Hall[1], so that the results themselves are subject to numerous and serious doubts. The main purpose of this Report in describing an attempt made to quantify the problem, is to demonstrate the various mechanisms in conjunction and thus to put them into perspective and to indicate where the main gaps in our knowledge are.

2   CASES CONSIDERED AND METHODS USED

2.1   Aerofoil sections and pressure distributions

Four different aerofoil sections were considered; they will be referred to as sections A, B, C and D respectively. The section shapes near the leading edge are shown in Fig.1 and the measured pressure distributions for zero sweepback on the upper surfaces over the forward part of the aerofoils, as used in the analysis described below, are shown in Fig.2. These are at a Mach number of 0.6 for sections A and B and a Mach number of 0.65 for sections C and D. The pressure distributions are similar in that they all have a supersonic region extending over the first 20% or so of the wing chord, but are different in detail within the supersonic regions. From these pressure distributions, velocity distributions were determined. The velocity distribution on the corresponding sheared wing, with the appropriate Mach number, was obtained by simply compounding the velocity normal to the leading edge with

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* Since this Report was initially drafted the computer program used has been extended to include compressible flow by Dr.E.H. Hirschel of DFVLR. The associated problem of extending the criteria for transition to include compressibility remains.

the component parallel to the leading edge. Sweep angles of 30° and 60° were considered; the corresponding free stream Mach numbers were 0.693 and 1.2 respectively for wings A and B, and 0.751 and 1.3 respectively for wings C and D.

2.2 Boundary-layer calculations

The velocity distributions described above were used as boundary conditions to calculate the incompressible three-dimensional laminar boundary layers on the corresponding sheared wings of infinite span at sweep angles of 30° and 60°. The method[7] is to solve the equations of momentum and of continuity for an infinite cylinder, using finite-difference substitutions and advancing step-by-step, solving the difference equations at each step by a matrix method. It can be expected that the results are much the same as would have been obtained by the method of Jaffe and Smith[8]. The boundary-layer calculations were carried out only over the first 20% of the chord length, that is to the position of the shock wave in the corresponding compressible flow. Upstream of this point, the adverse pressure gradients were small and the calculations indicated no likelihood of laminar separation.

The Reynolds number was varied between 3×10^{6} and 72×10^{6}. It is defined by

$$R = \frac{U_{\infty} c (\sec \varphi)}{\nu}$$

where U_{∞} is the free stream velocity at infinity, c the chord, ν the kinematic viscosity, and φ the angle of sweep.

2.3 Tests for instability of the laminar boundary layer

2.3.1 Leading-edge contamination and re-laminarisation

The Reynolds number based on momentum thickness at the attachment line was computed from

$$R_{\theta} = \frac{0.4V}{\sqrt{\nu \left(\frac{dU'}{ds}\right)_{a}}} \quad ,$$

(as given for example, by Cumpsty and Head[9]), where V is the component of the free stream velocity along the attachment line, U' is the component of potential flow velocity in a plane normal to the leading edge, s is the distance measured around the surface in the same plane, and the suffix a refers to

the value at the attachment line. The value of R_θ is clearly sensitive to the accuracy with which $\left(\dfrac{dU'}{ds}\right)_a$ was computed. Since, in the present exercise, this was deduced from the meagre experimental data available near the nose of the aerofoil, no great accuracy can be expected.

Experimental work by Gregory[10], Pfenninger[11], Gaster[12] and Landeryou and Trayford[13] suggests that, if the value of R_θ is below about 100, then the flow along the attachment line will have a strong tendency to remain laminar or to revert to laminar if it should have become locally turbulent for any reason. For values of R_θ significantly greater than 100, any local turbulent contamination will tend to spread along the attachment line so that, in practice, transition will occur. It was assumed here that a value of R_θ of 100 could be used as a critical value, with a range of uncertainty of from 80 to 120.

The possibility of re-laminarisation of the boundary layer following turbulent contamination at the attachment line was considered. Launder and Jones[14] have investigated the correlation between the occurrence of re-laminarisation in accelerating flows and the value of the parameter K, defined by

$$ K = \frac{\nu}{U^2} \frac{dU}{dx} , $$

where U is the local velocity at the edge of the boundary layer and x is the Cartesian co-ordinate in the flow direction. They have suggested that in two-dimensional flow a degeneration from turbulent to laminar flow might begin when K exceeds about 2×10^{-6}. But it is likely that much higher values, say in excess of 5×10^{-6}, are needed for the flow to revert effectively to laminar form. In the present exercise, it is assumed, admittedly without any direct experimental justification, that values of K of this order would be relevant if K were evaluated along the streamline. Again, it should be emphasised that the value of K is only as accurate as the velocity distribution and, since the maximum values of K occurred very close to the leading edge where the velocities were not accurately known, there was some element of uncertainty here. The calculations showed that the value of K falls off rapidly after reaching a maximum. In all cases, K was too low to suggest any possibility of re-laminarisation aft of about 1% chord. Hence, re-laminarisation had only to be considered when transition was due to turbulent contamination along the attachment line.

2.3.2 Cross-flow instability

Owen and Randall[5] have proposed that the onset of instability should be indicated when the cross-flow Reynolds number, given by

$$\chi = \frac{(v_N)_{max} \delta_c}{\nu} ,$$

exceeds a certain value. Here $(v_N)_{max}$ is the maximum value of the cross-flow velocity component and δ_c is a boundary-layer thickness not precisely defined. In the present work, δ_c was defined by

$$\delta_c = \int_0^\infty \frac{v_N}{(v_N)_{max}} dz ,$$

where z is the Cartesian co-ordinate perpendicular to the aerofoil surface. The value of χ was computed at each step in the boundary-layer calculations and should be reliable since the velocity profiles were accurately computed. But there is considerable uncertainty about the critical value itself, bearing in mind that this is only a criterion for the onset of instability and that the actual development of a turbulent flow may be influenced by other factors. Using the present methods of calculating the boundary layer and evaluating χ, an analysis of the results of an experiment on a sheared wing by Boltz, Kenyon and Allen[15] has suggested a critical value for χ of about 120, but there was some evidence that transition occurred also where the value of χ was as low as 100, or as high as 140. In the work reported here, the critical value of χ was therefore assumed to be 120, with a range of uncertainty of from 100 to 140.

2.3.3 Tollmien-Schlichting instability

The position, S_i, of the instability point was calculated using the empirical curve given in Ref.16. This is a plot of the critical value of a Reynolds number based on the momentum thickness, given by

$$R_2 = \frac{U_0 \delta_2}{\nu}$$

against a parameter λ_2, given by

$$\lambda_2 = \frac{\delta_2^2}{\nu} \frac{dU_0}{ds} ,$$

where U_0 is the local velocity at the edge of the boundary layer perpendicular to the leading edge and δ_2 is the momentum thickness. The values of R_2 and λ_2 were computed at each step in the boundary-layer calculation and the value of R_2 was compared with the critical value deduced from the curve of Ref.16 for the corresponding value of λ. Subsequently, Granville's method[17] was used to estimate the point, S_t, where transition can be expected to be completed. Granville introduced a relationship between the change in Reynolds number from instability to transition and the average value of λ_2 over that region, that is between

$$(R_2)_t - (R_2)_i$$

and

$$\bar{\lambda}_2 = - \left[\int_{S_i}^{S_t} \left(\frac{\delta_2^2}{\nu} \frac{dU_0}{ds} \right) ds \right] \bigg/ (S_t - S_i) \quad ,$$

where suffices t and i denote values at the transition point and the instability point respectively. He has deduced a relationship, as shown by the full line in Fig.3, between these two parameters, based on experimental results, and this was used here to predict the transition point in the first instance.

However, a further analysis, calculating the boundary layer by the present method, was made of the experimental results of Boltz, Kenyon and Allen[15]. This showed considerable scatter of the experimental points, as can be seen from Fig.3. A new curve was therefore drawn which, together with Granville's curve, encloses nearly all the experimental points. In the results below, two sets of values according to these two curves are given.

In the above approach, any influence of cross-flow was ignored and it is arguable whether it might have been more appropriate to have applied the criteria along the streamlines.

3   PRESENTATION OF THE RESULTS OF THE CALCULATIONS

The main results of the calculations made are given in Table 1. These are supplemented in Figs.4 and 5 by some specific results from the calculations of the cross-flow instability parameter. The results of the calculation of the critical parameters are presented in Figs.6 to 9 to show the variation with Reynolds number for the four pressure distributions considered. The three

scales used for the ordinates in these figures have been chosen in such a way that the critical values of each parameter fall on the same line. Instability leading to transition to turbulence is likely above the line whereas laminar conditions should exist below the line. The horizontal dashed lines in the figures for swept conditions indicate the possible margin of error in the critical values based on present evidence.

Figs.6 to 9 also show for the case of zero sweep the significance of the modification to Granville's curve, mentioned in section 2 3.3, on the predicted location of the transition point. To indicate the position of the point of transition or point of instability for the examples considered the form of presentation explained in the sketch given in Fig.10 has been adopted. For sweep angles of 30° and 60° these positions are shown for the four wing sections in Figs.11 to 14.

4   SOME REMARKS ON THE EFFECTS OF SURFACE ROUGHNESS

In the analysis given in the previous section the effects of surface roughness have not directly been taken into consideration, although, of course, in many practical applications, it must be appreciated that its effect on the criteria assumed may be of great significance.

Hall[1] quotes some of the conclusions reached by Cumpsty and Head following from their study of the effects of roughness elements on the flow along the attachment line. The governing parameter for such flows is the Reynolds number at the attachment line R_{θ} = $V\theta/v$, which as mentioned above is given by

$$R_{\theta} = \frac{0.4V}{\sqrt{v\left(\frac{dU'}{ds}\right)_a}} .$$

For R_{θ} < 100, the flow remains laminar irrespective of the size of the roughness element used; in their case a wire wrapped around the leading edge A critical diameter of trip wire exists below which the flow can remain laminar up to a value of R_{θ} of at least 245. This critical diameter is given by

$$\left(\frac{Vd}{v}\right)_{crit} = 47R_{\theta}^{\frac{1}{2}} ,$$

for conical forms of excressence. However, 65 might be a more appropriate value to assume for the empirical constant in this expression when the diameter d is replaced by the height of the element. For wire diameters exceeding this

critical size, there is a transitional regime for values of R_θ in the range roughly between 100 and 150, whilst above this range the velocity profiles of the boundary layer assume a form characteristic of a fully turbulent layer.

Table 2 gives values of the critical diameters for the four wing sections under consideration. Taking, for example, sea-level conditions, a sweep of 30°, and a mean chord of 2.5 m (approximately 8 ft), then the critical diameter would be of the order of 100 μm (or 0.004 in), and at 35000 ft would be about four times this. Imperfections of this order may well be present on most operational aircraft; thus it is not unreasonable to assume that this critical roughness level is exceeded in most instances. In the case of model testing in a wind tunnel, the situation is somewhat different. For example, assuming a fifth scale model, that is a chord of 0.5 m, and a Reynolds number of 18×10^6, then the critical diameter would be about 25 μm (or 0.001 in), and imperfections of this magnitude would not normally be present.

Consideration of the conditions when transition is provoked by roughness downstream of the attachment line is more difficult. Some guidance may be provided by a crude generalisation of two-dimensional information by using a roughness Reynolds number R_k, based on height of the roughness and the resultant velocity at the top of the roughness. Experiments in two-dimensional flow appear to be insufficient to formulate reliable rules for other than zero pressure gradients, but they indicate critical values of R_k ranging from about 100 to 800, depending on the form of the roughness elements. Also, as stated in Ref.1, quoting the experimental work of Potter and Whitfield, the effects of compressibility are great when the Mach number at the roughness height reaches high subsonic or supersonic values. Nevertheless, to give an impression of the order of roughness heights which are significant in the context of the examples considered here, curves are shown in Fig.15 based on a value of R_k of 200. The curves do not indicate any high degree of sensitivity to pressure distribution for the range covered by the examples considered.

5 DISCUSSION

Even at first glance, it is clear that the results are not very definite and cannot be readily interpreted. Consider first transition following Tollmien-Schlichting instability, which may be regarded as the main criterion for unswept wings. The chordwise station where transition is supposed to be complete varies greatly from one section to another, as does the change with Reynolds number, even though the pressure distributions would not appear to

differ much. Further, the differences between the two estimates are considerable in some cases. Altogether, the uncertainties seem to be too great for engineering purposes; to narrow them down needs further work.

In all cases where the angle of sweep is high, matters appear to be more clear cut, if the present criteria are to be believed. Leading-edge contamination appears to be the dominant effect, even at relatively low Reynolds numbers, and there seems to be little likelihood of re-laminarisation in the cases considered. This would imply that the boundary layer would be turbulent right from the attachment line onwards. If leading-edge contamination is really such a powerful effect, the possibility of sweep instability need not be taken very seriously at high angles of sweep. It would be extremely useful to have adequate experimental confirmation of this fact.

Matters appear to be very complex at moderate angles of sweep. At the low Reynolds numbers of many existing wind tunnels, the boundary layer appears to be laminar over the whole of the supersonic region, except possibly in the case of section D where spanwise contamination might occur, but it might be suppressed by re-laminarisation. This is, of course, assuming that no artificial means are used to provoke transition prematurely. Conversely, at the higher Reynolds numbers considered here, the flow is likely to be turbulent right from the leading edge. So there is a Reynolds number range over which it is very difficult to forecast with certainty what type of flow to expect and which of the modes of transition would predominate. Some carefully planned tests and numerical experiments would seem to be necessary to sort out what the conditions are under which the flow will have settled down to the type expected in full-scale flight, so that subsequent changes might be smooth and monotonic, permitting confident extrapolation from tunnel to flight conditions.

The results presented also illustrate the difficulties that must be overcome if the conditions that obtain at the higher Reynolds numbers are to be simulated at lower Reynolds numbers where the boundary layer is naturally laminar over extensive regions of the surface. It is difficult to see how simple simulation devices can be expected to reproduce the desired flow in any reasonably representative manner.

The results further give an indication of a rather disturbing possibility that the flow itself can be so sensitive that it is dependent on the fine detail of the pressure distribution and profile shape. This would imply, in the first place, that these two must be known fairly completely and accurately, which

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places great demands on both the theoretical and experimental techniques. In the second place, other sections different from those considered here might yield different results and lead to quite different conclusions.

To sum up, it would appear that much more work is needed to determine the transition criteria more precisely, or determine new criteria if necessary. A larger number of representative cases needs to be investigated, and some consideration given to cases where laminar separation is a possibility. Lastly and most importantly, the work should be extended beyond the rather artificial cases of sheared wings of infinite span to include a proper treatment of fully three-dimensional wings.

Table 1

CALCULATED VALUES OF THE POSITIONS OF INSTABILITY AND TRANSITION AND OF PARAMETERS OF SWEEP INDUCED INSTABILITY OR TRANSITION

	$\varphi = 30°$						$\varphi = 60°$					
	X_i	$X_t(i)$	$X_t(ii)$	R_θ	$K \times 10^6$ (max)	X_{max}	X_i	$X_t(i)$	$X_t(ii)$	R_θ	$K \times 10^6$ (max)	X_{max}
					$R = 3 \times 10^6$							
Section A	0.036	>0.2	>0.2	55	40.0	34	0.037	>0.2	>0.2	96	10.4	46
Section B	0.032	>0.2	>0.2	42	22.6	47	0.060	>0.2	>0.2	72	3.9	65
Section C	0.044	>0.2	>0.2	49	16.6	43	0.051	>0.2	>0.2	86	4.2	56
Section D	0.024	>0.2	>0.2	78	40.5	40	0.026	>0.2	>0.2	136	7.6	54
					$R = 18 \times 10^6$							
Section A	0.028	0.089	0.175	136	6.7	84	0.031	0.174	>0.2	235	1.7	113
Section B	0.029	0.093	0.135	102	3.8	116	0.032	0.138	>0.2	178	0.6	159
Section C	0.035	0.164	>0.2	121	2.8	106	0.040	>0.2	>0.2	210	0.7	138
Section D	0.023	0.068	0.122	192	6.8	98	0.024	0.121	>0.2	333	1.3	132
					$R = 36 \times 10^6$							
Section A	0.026	0.063	0.113	192	3.3	118	0.029	0.109	>0.2	332	1.9	159
Section B	0.027	0.081	0.103	144	1.9	164	0.030	0.103	0.159	251	0.3	224
Section C	0.032	0.123	0.168	171	1.4	150	0.036	0.173	>0.2	297	0.3	195
Section D	0.023	0.054	0.077	271	3.4	139	0.024	0.077	0.183	470	0.6	187
					$R = 72 \times 10^6$							
Section A	0.023	0.052	0.077	271	1.7	167	0.028	0.076	0.143	470	0.4	225
Section B	0.025	0.085	0.085	204	1.0	232	0.028	0.087	0.121	353	0.2	317
Section C	0.027	0.089	0.127	242	0.7	212	0.034	0.154	0.179	420	0.2	275
Section D	0.022	0.044	0.061	384	1.7	196	0.023	0.061	0.096	665	0.3	265

Table 2

CRITICAL ROUGHNESS HEIGHT FOR TURBULENT FLOW AT THE ATTACHMENT LINE

Section	$R_c \times 10^{-6}$	$\varphi = 30°$		$\varphi = 60°$	
		R_θ	$\sigma_k/c \times 10^3$	R_θ	$\sigma_k/c \times 10^3$
A	3	55	0.232 (0.321)	96	0.177 (0.245)
	18	136	0.061 (0.084)	235	0.046 (0.064)
	36	192	0.036 (0.051)	332	0.028 (0.038)
	72	271	0.022 (0.030)	470	0.016 (0.022)
B	3	42	0.203 (0.281)	72	0.153 (0.211)
	18	102	0.054 (0.075)	178	0.040 (0.056)
	36	144	0.031 (0.043)	251	0.024 (0.033)
	72	204	0.019 (0.026)	355	0.014 (0.020)
C	3	49	0.219 (0.303)	86	0.168 (0.232)
	18	121	0.057 (0.079)	210	0.044 (0.060)
	36	171	0.034 (0.033)	297	0.026 (0.036)
	72	242	0.020 (0.028)	420	0.015 (0.021)
D	3	78	0.277 (0.383)	136	0.211 (0.292)
	18	192	0.072 (0.100)	333	0.055 (0.076)
	36	271	0.043 (0.060)	470	0.033 (0.045)
	72	384	0.026 (0.035)	665	0.019 (0.027)

Note: σ_k refers to the critical diameter of a wire wrapped
around the leading edge, but figures in parenthesis relate
to conical roughness elements.

SYMBOLS

a	suffix to denote value at the attachment line
c	chord length of wing, measured normal to the leading edge
C_p	pressure coefficient
i	suffix to denote value at point of instability
K	re-laminarisation parameter
R	Reynolds number, $\dfrac{U_\infty c\ (\sec \varphi)}{\nu}$
R_k	roughness Reynolds number, $\dfrac{u_k \sigma_k}{\nu}$
R_2	Reynolds number based on momentum thickness
R_θ	Reynolds number based on momentum thickness at the attachment line
S	distance measured around the surface in a plane normal to the leading edge
t	suffix to denote value at point of transition
u_k	velocity at the top of the roughness element
U	local velocity at edge of boundary layer
U_0	local velocity at edge of boundary layer perpendicular to leading edge
U_∞	free stream velocity at infinity
U'	component of the potential flow velocity in a plane normal to the leading edge
v_N	cross-flow velocity component within the boundary layer
V	component of the free stream velocity along the attachment line
x	Cartesian co-ordinate in the flow direction
X	distance measured along the wing chord from and normal to the leading edge
z	distance measured out from and normal to the wing surface
δ_c	cross-flow boundary layer thickness
σ_k	height of the roughness element
λ_2	parameter for Tollmien-Schlichting type of instability
$\bar{\lambda}_2$	average value of λ_2 over a region
ν	kinematic viscosity
φ	angle of sweep
χ	cross-flow Reynolds number
δ_c	momentum thickness

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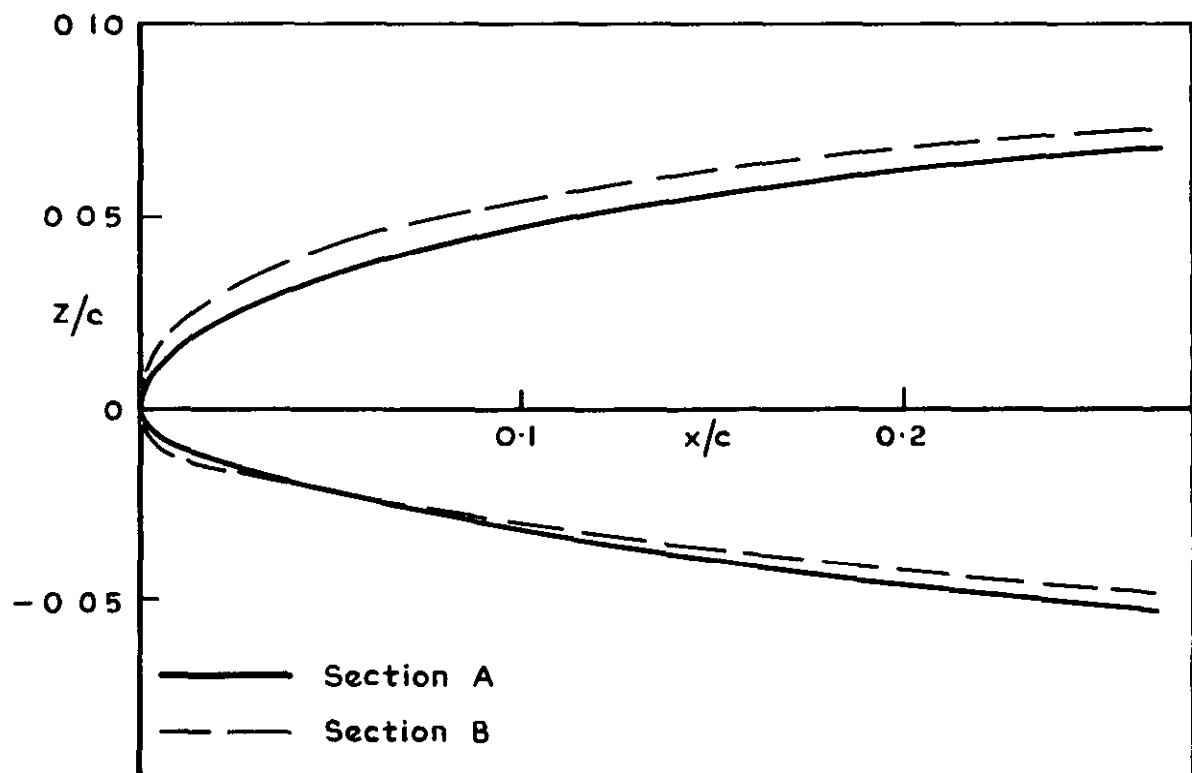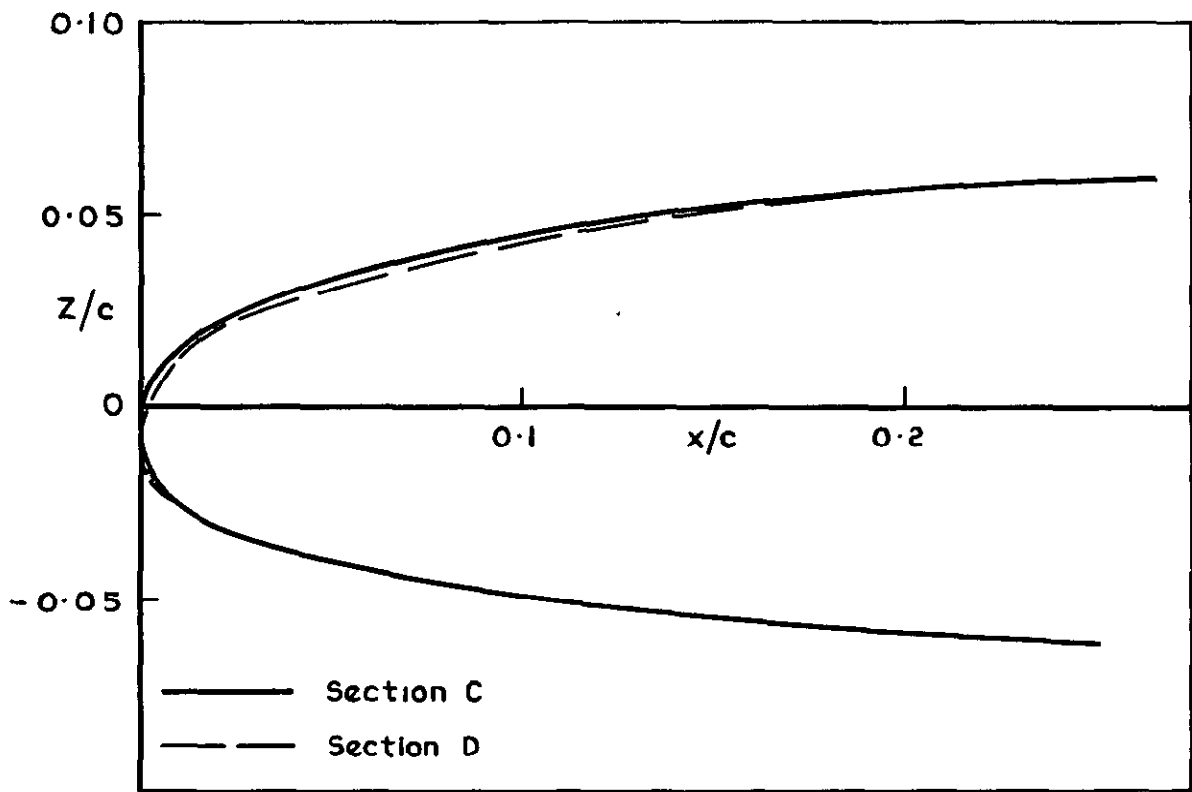
Fig. I Shapes of the aerofoil sections near the leading edge
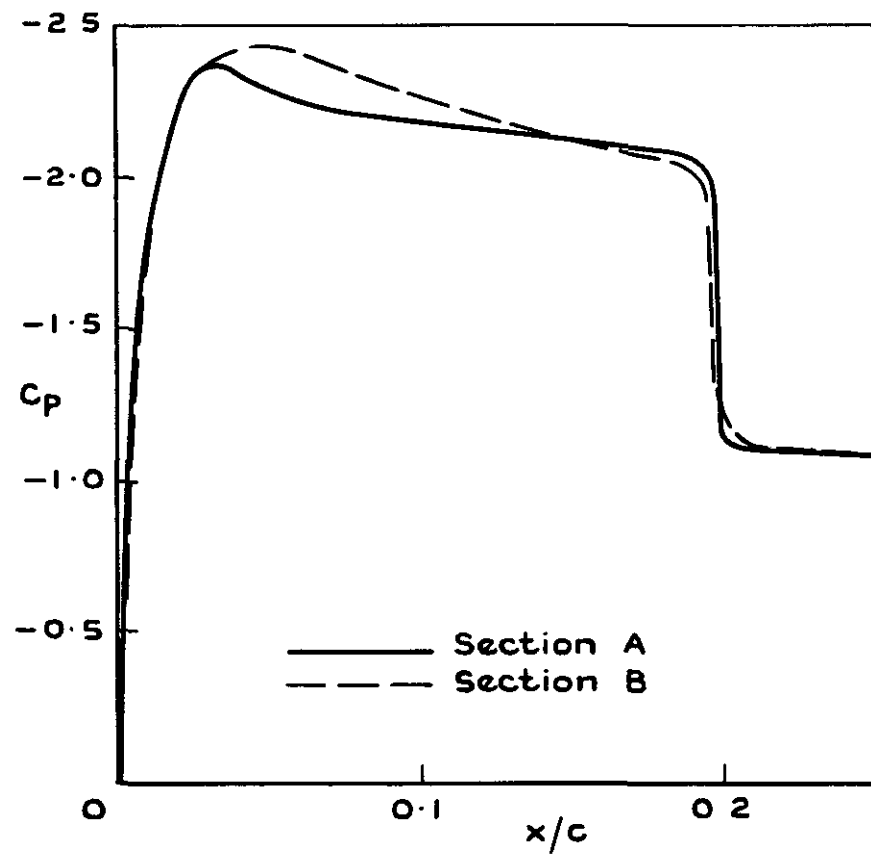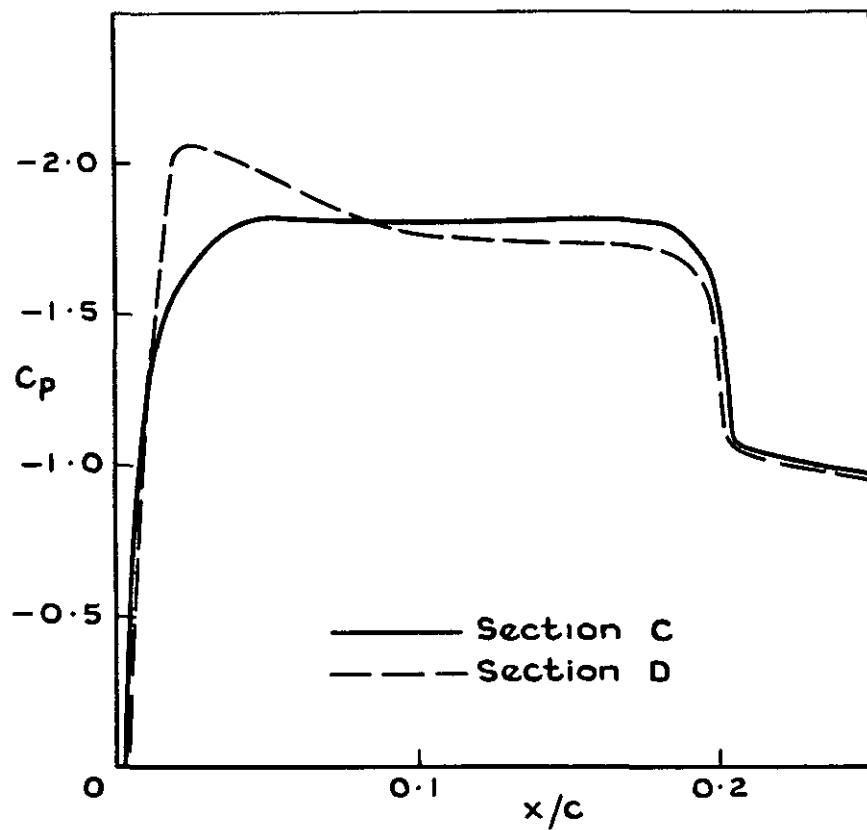
Fig. 2 Pressure distributions over the forward part of the aerofoils at zero sweepback
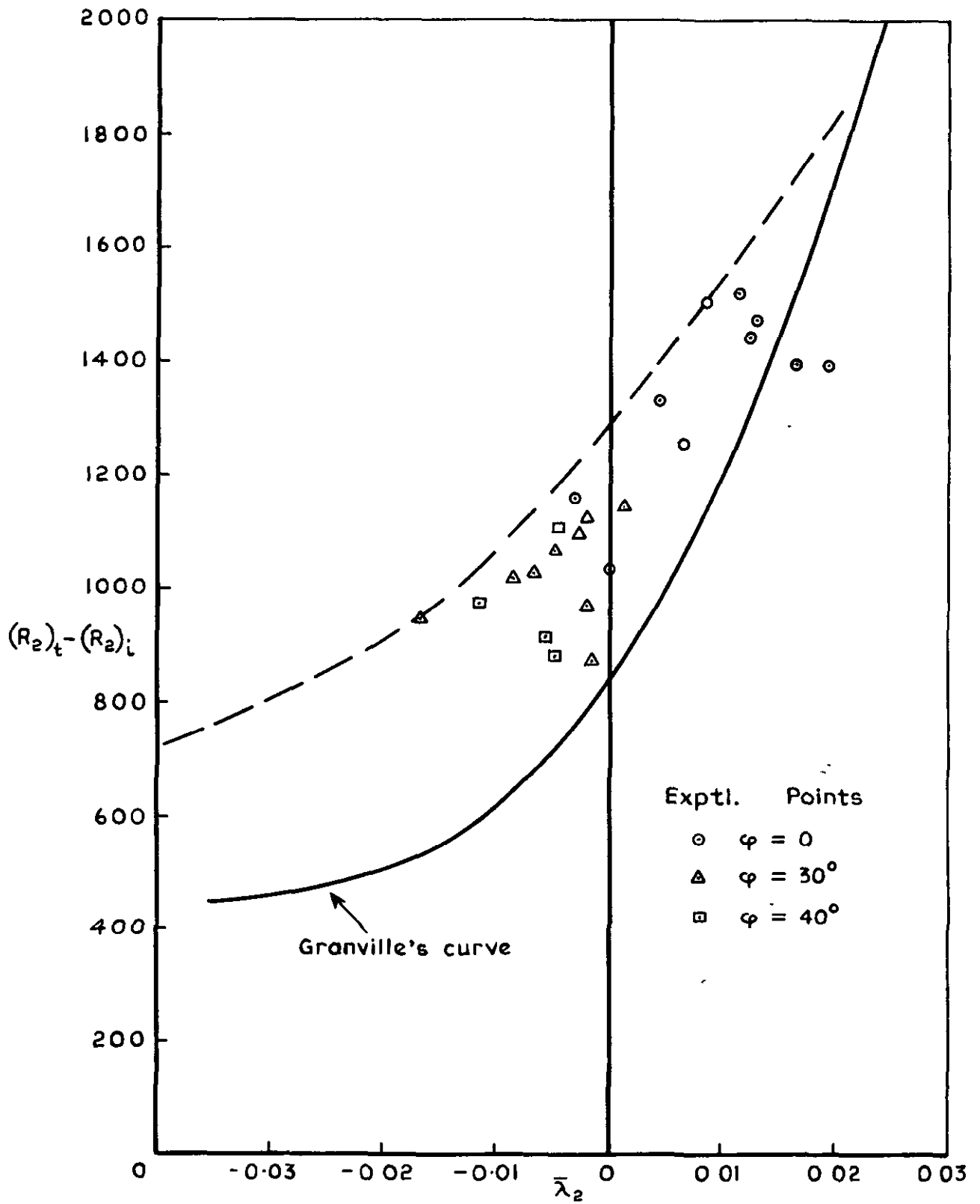
Fig. 3 Values of Granville's transition parameter deduced from experimental swept wing data compared with Granville's curve
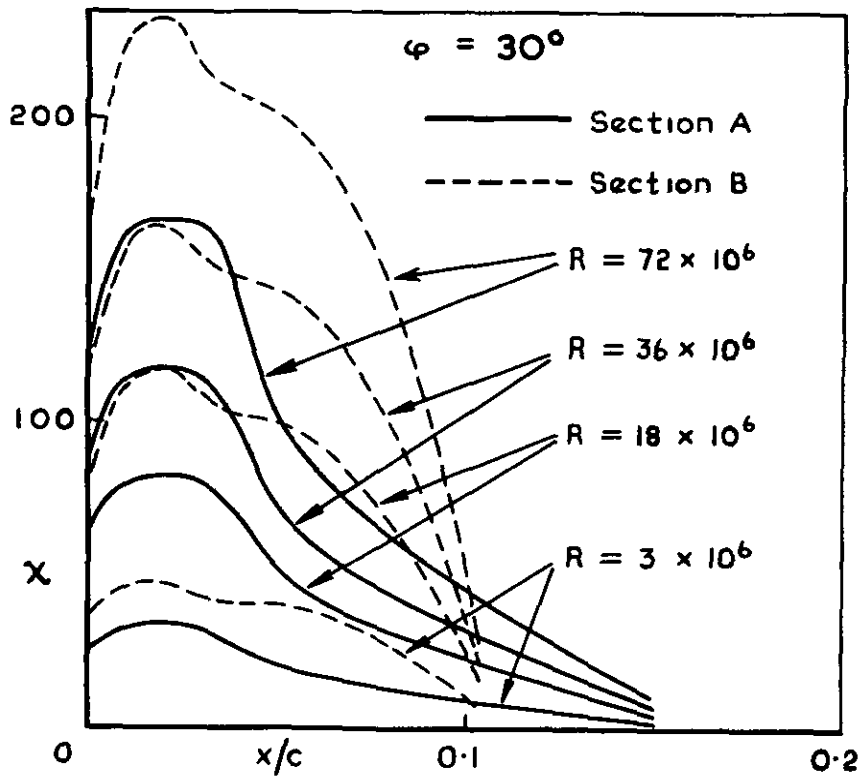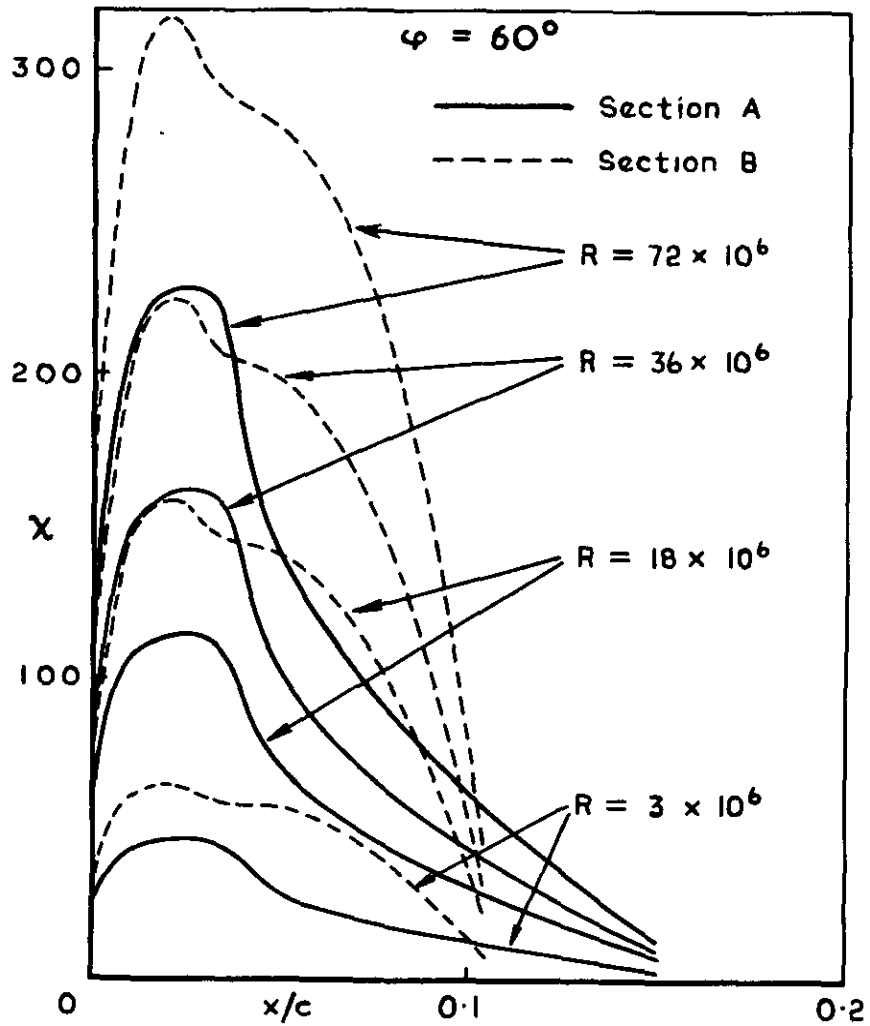
Fig. 4 Variation of cross-flow Reynolds number with
chordwise position, sections A & B
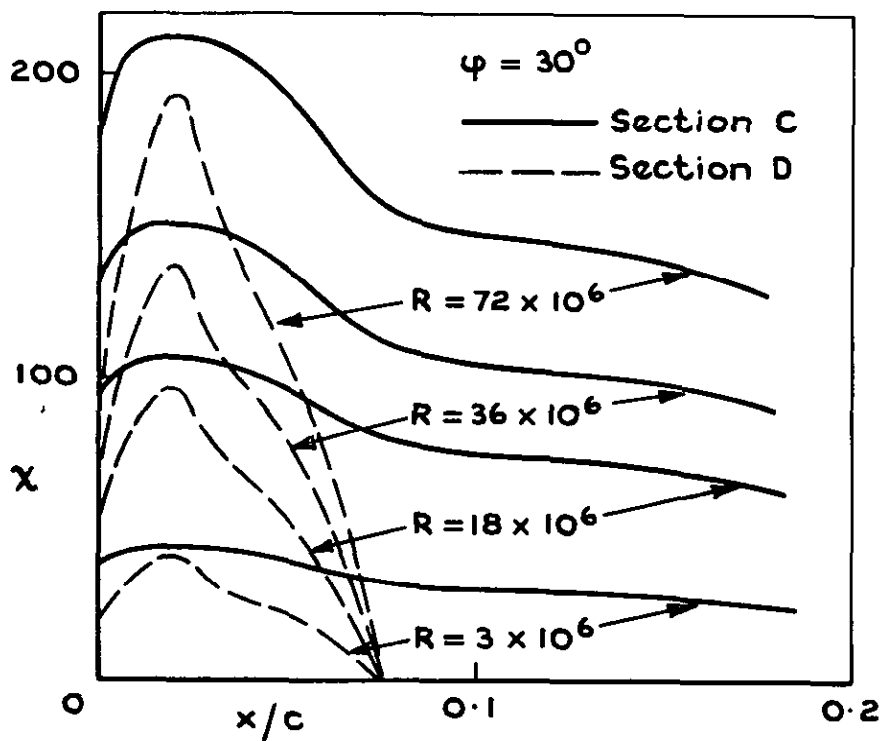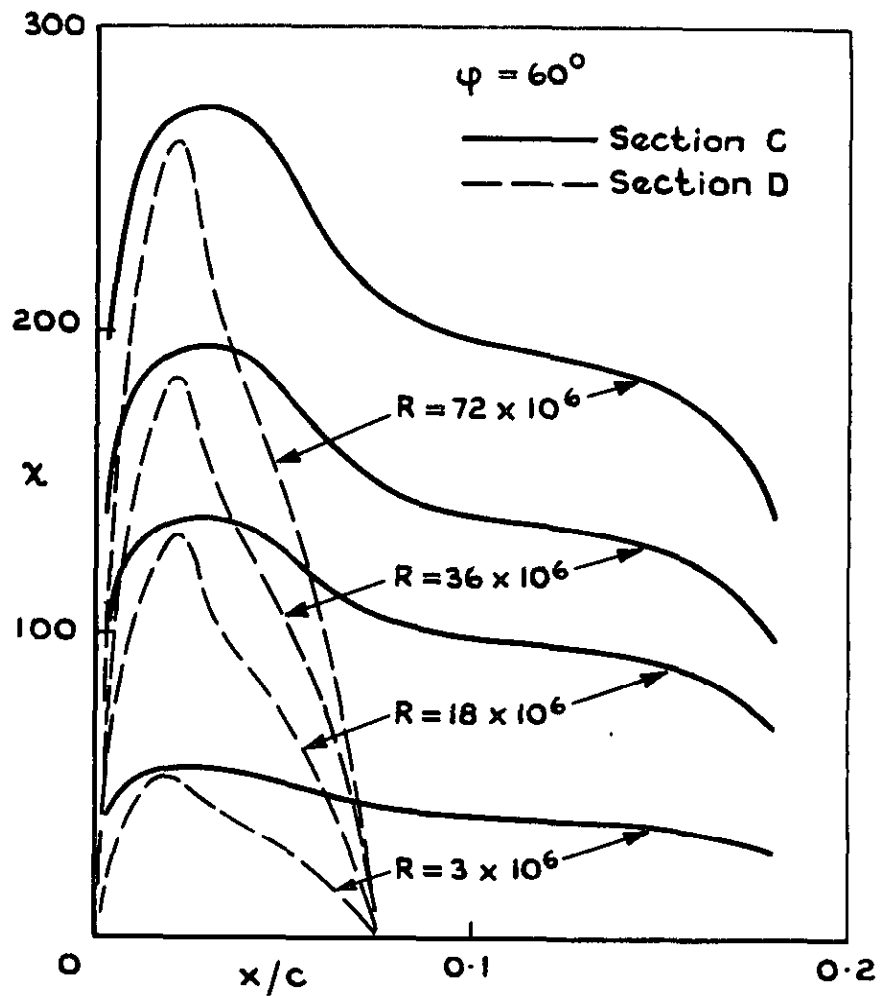
Fig. 5 Variation of cross-flow Reynolds number with chordwise position; sections C and D
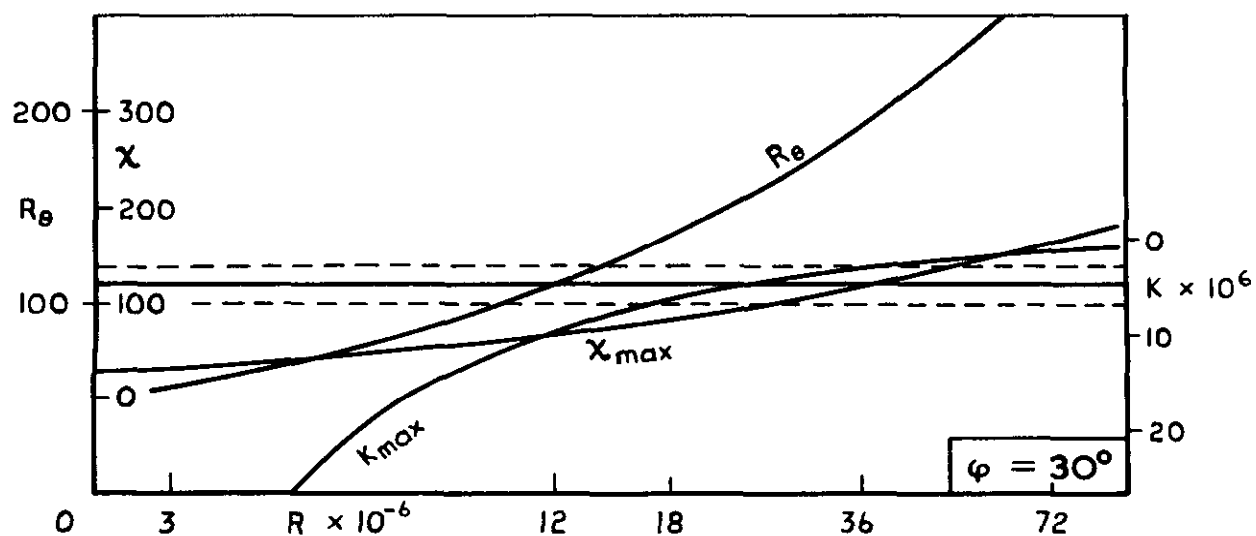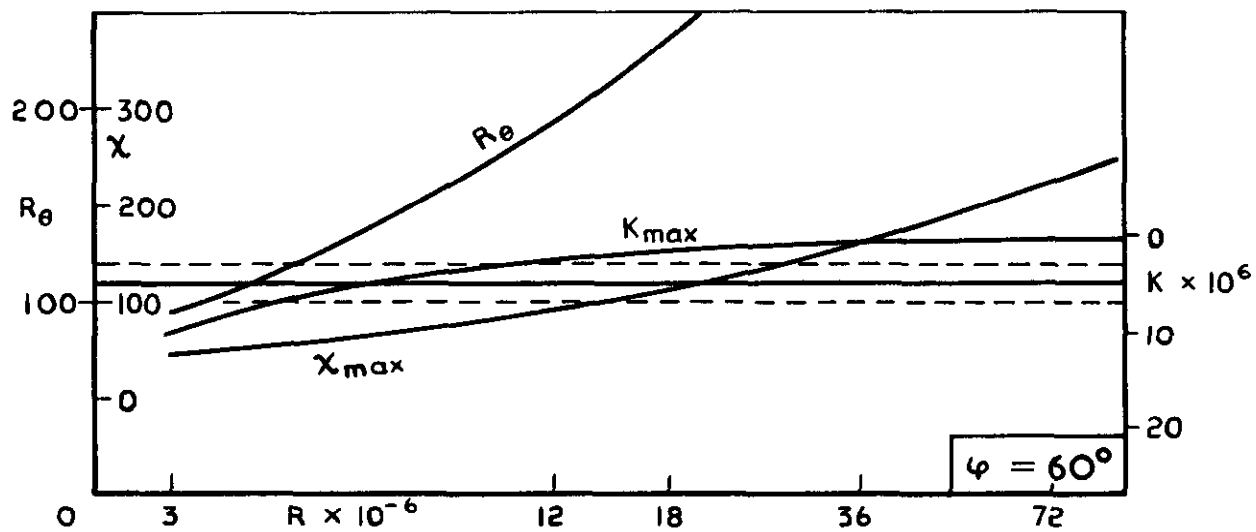
Parameters of sweep – induced transition
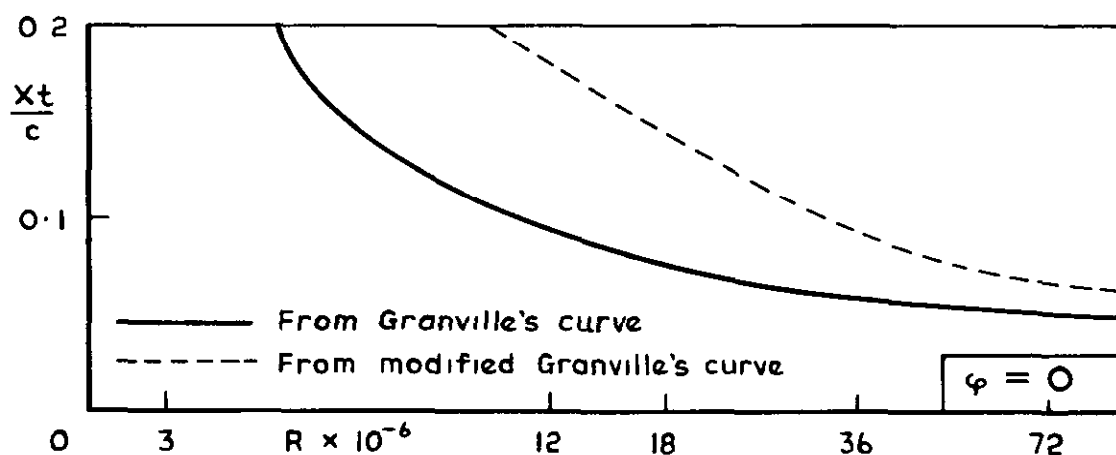
Position of transition following instability of
Tollmien – Schlichting type

Fig. 6 Parameters of sweep-induced transition and the estimated
position of transition following viscous instability; section A
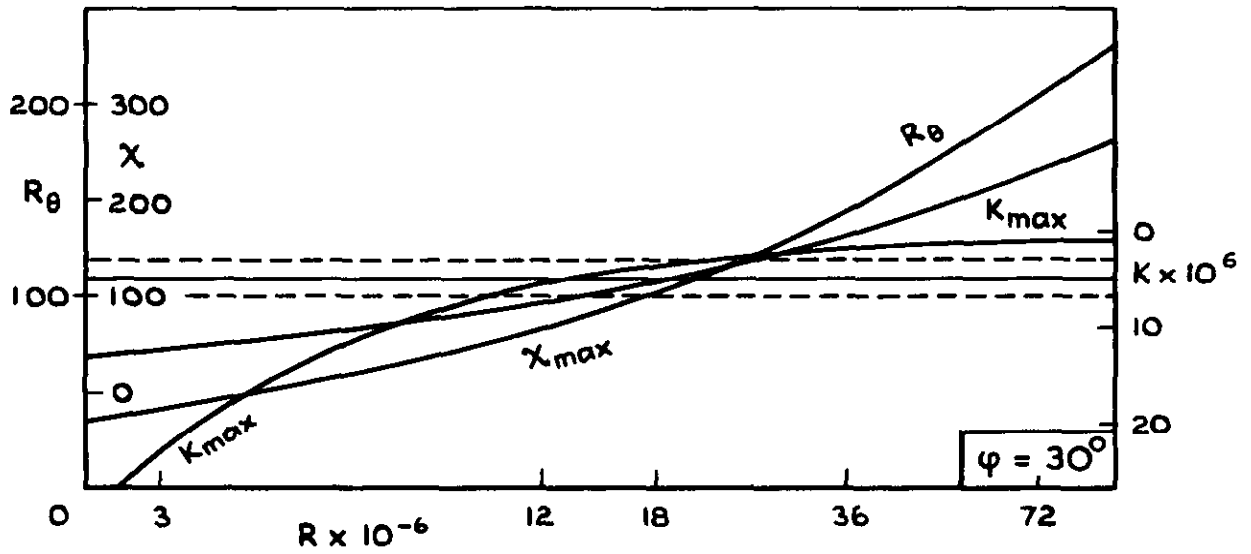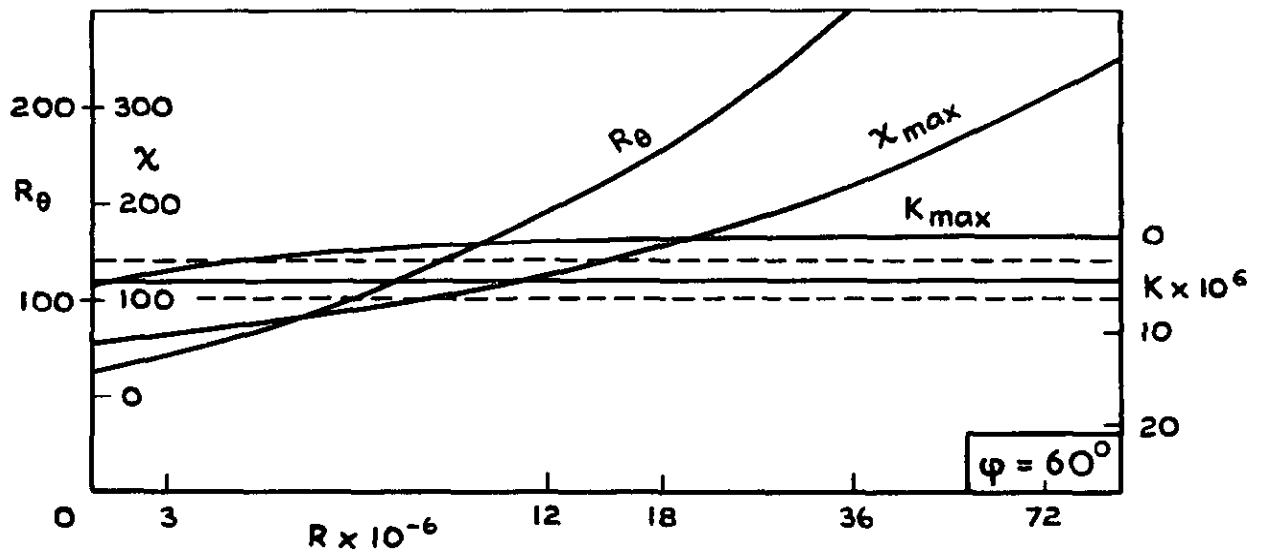
Parameters of sweep-induced transition
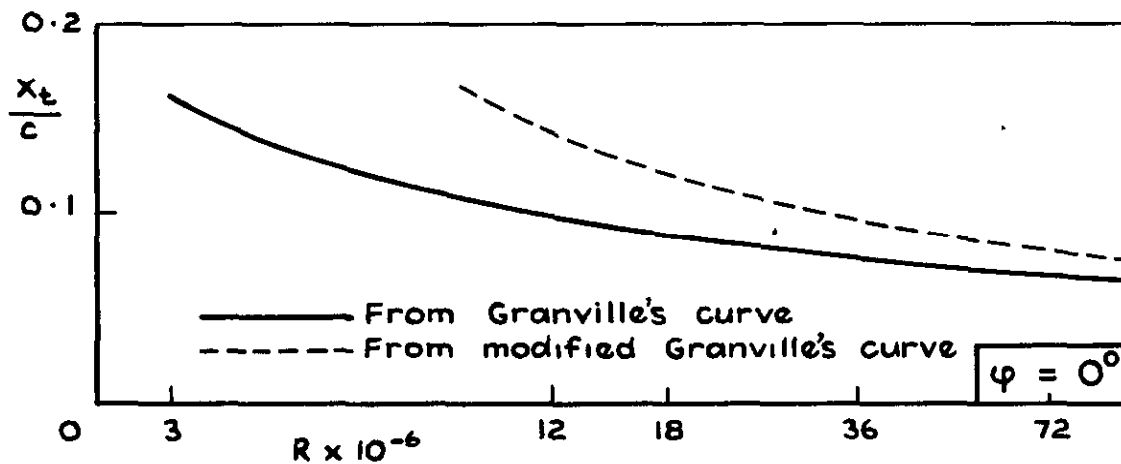
Position of transition following instability of
the Tollmien–Schlichting type

Fig. 7 Parameters of sweep-induced transition and the estimated
position of transition following viscous instability; section B
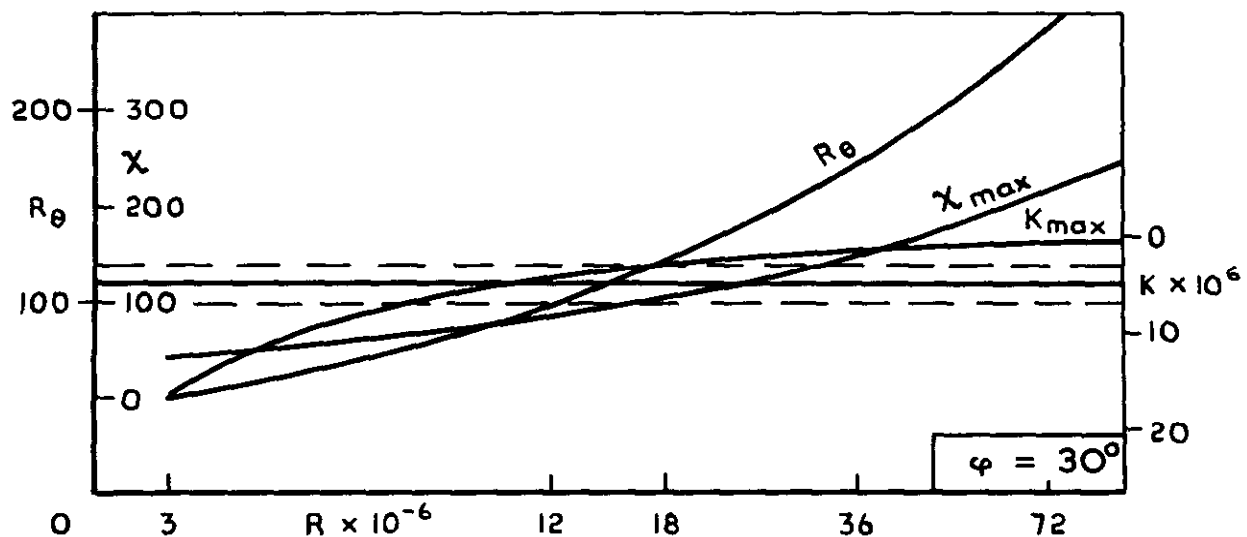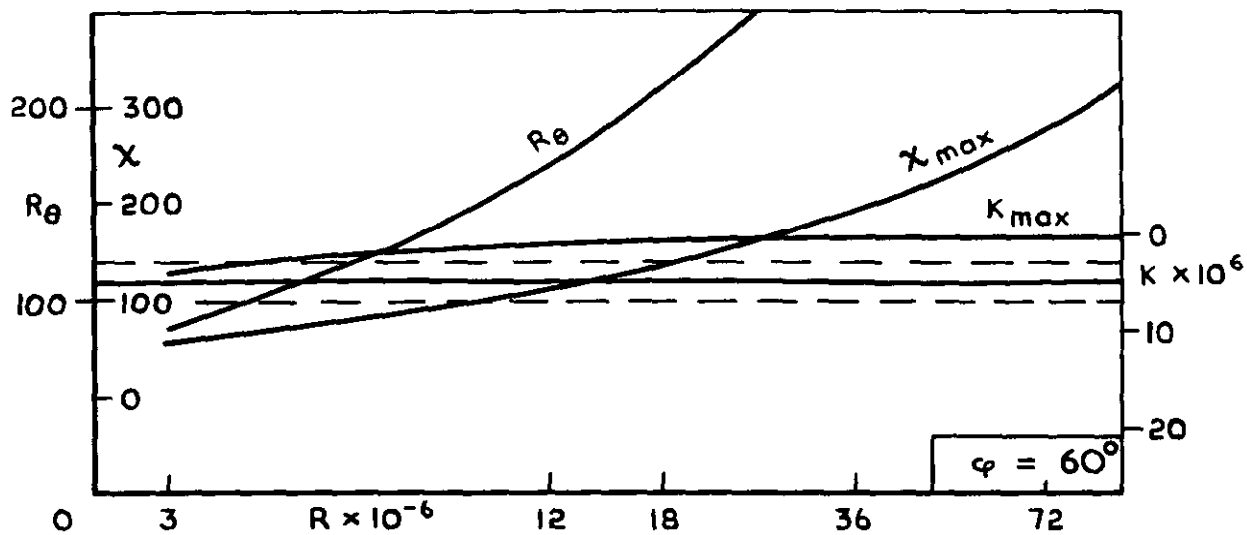
Parameters of sweep – induced transition
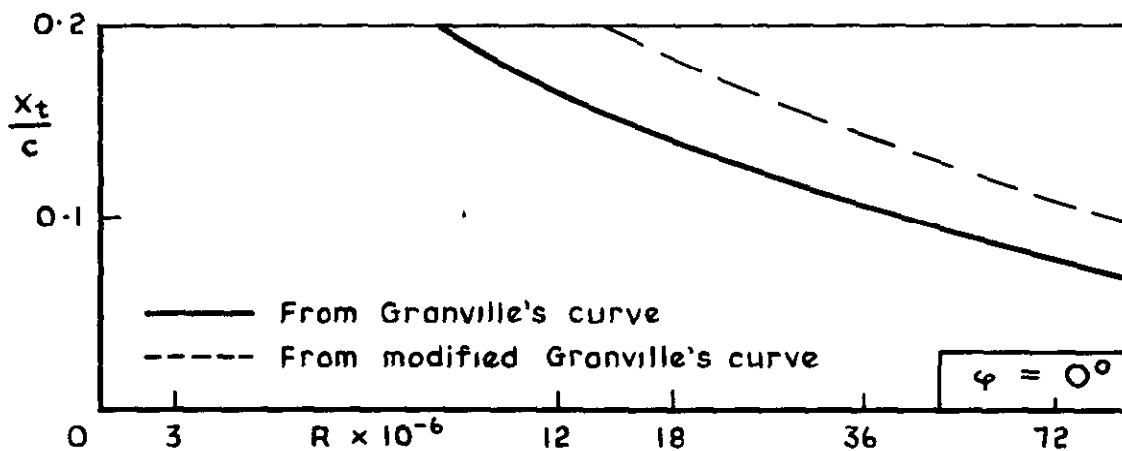
Position of transition following instability of
Tollmien – Schlichting type

Fig.8 Parameters of sweep – induced transition and
the estimated position of transition following
viscous instability; section C
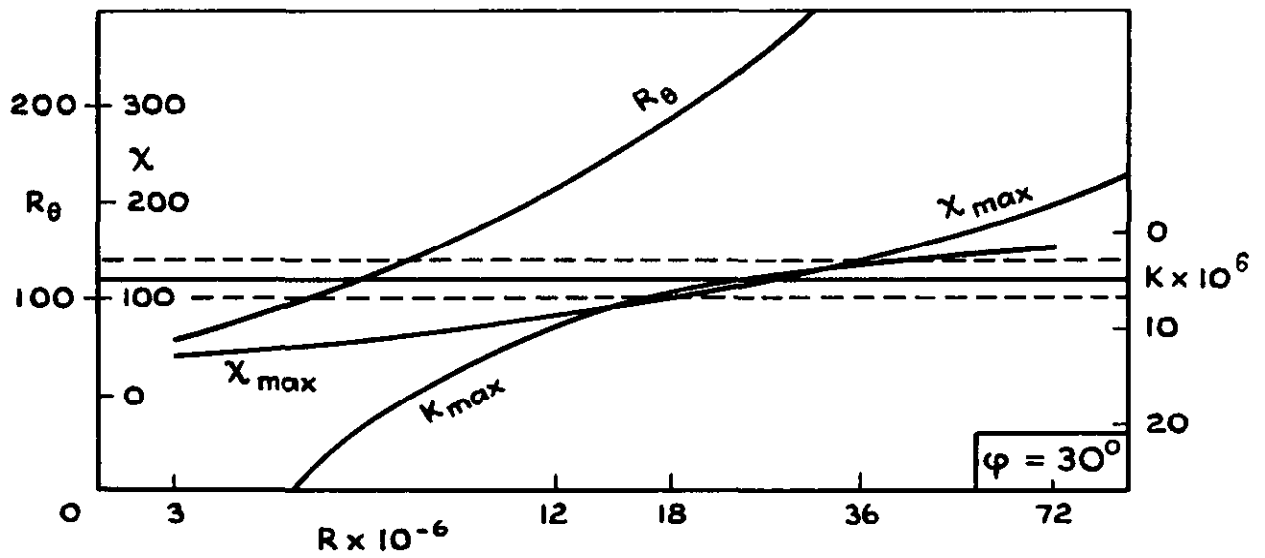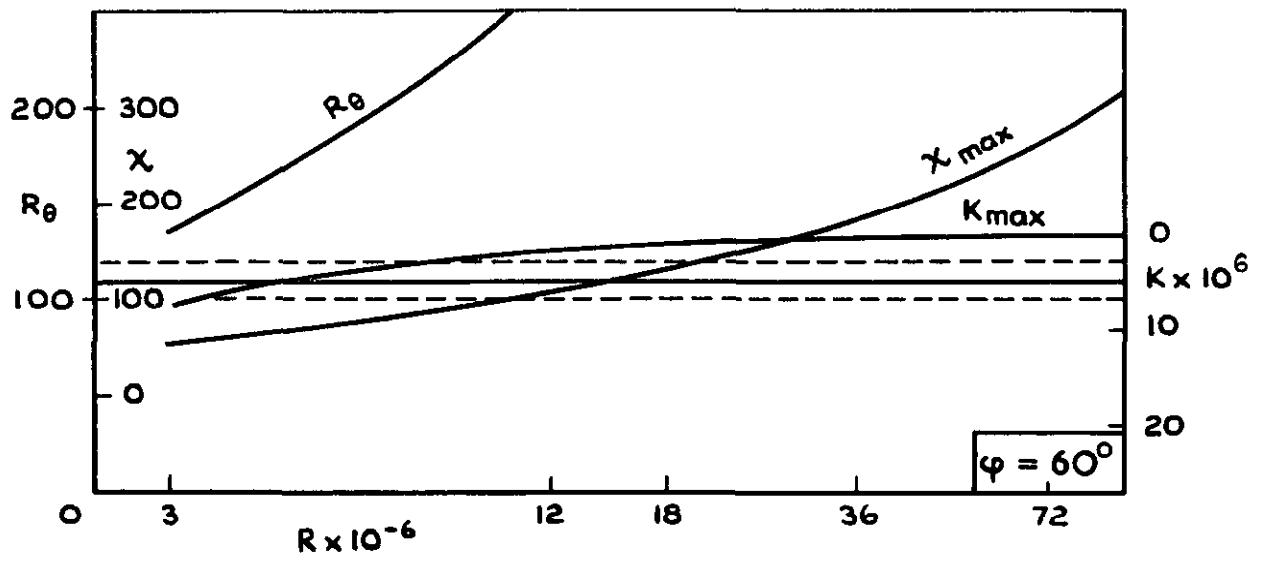
Parameters of sweep-induced transition
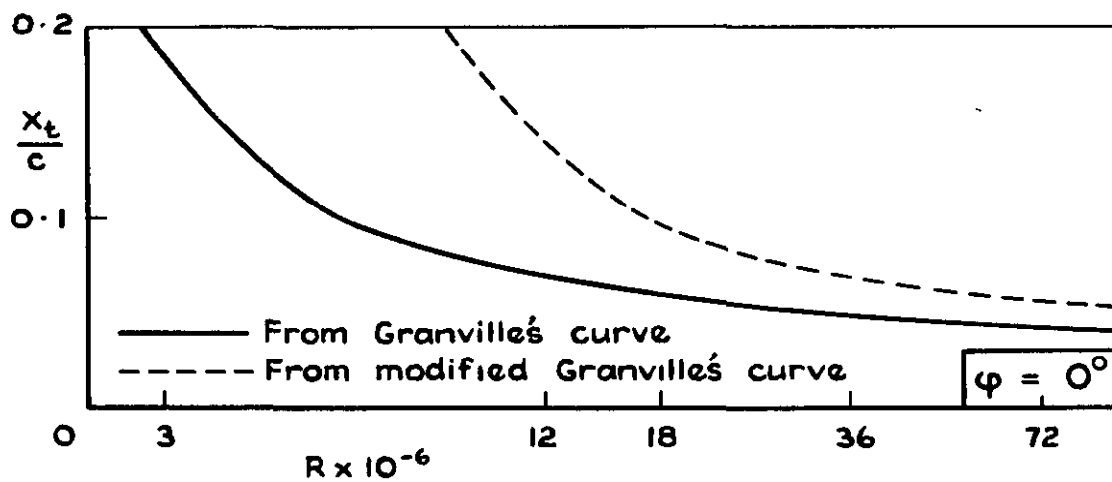
Position of transition following instability of
the Tollmien – Schlichting type

Fig.9   Parameters of sweep-induced transition and the estimated
position of transition following viscous instability; section D

Fig.10 Schematic sketch of the movement with Reynolds
number of the predicted point of transition or of instability
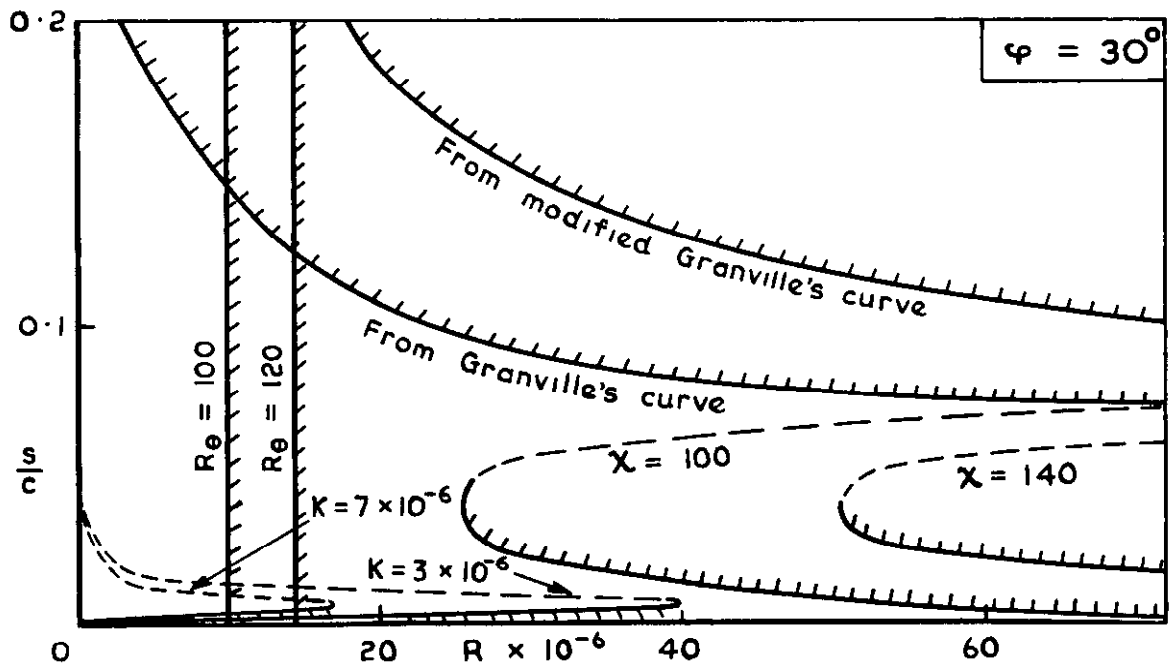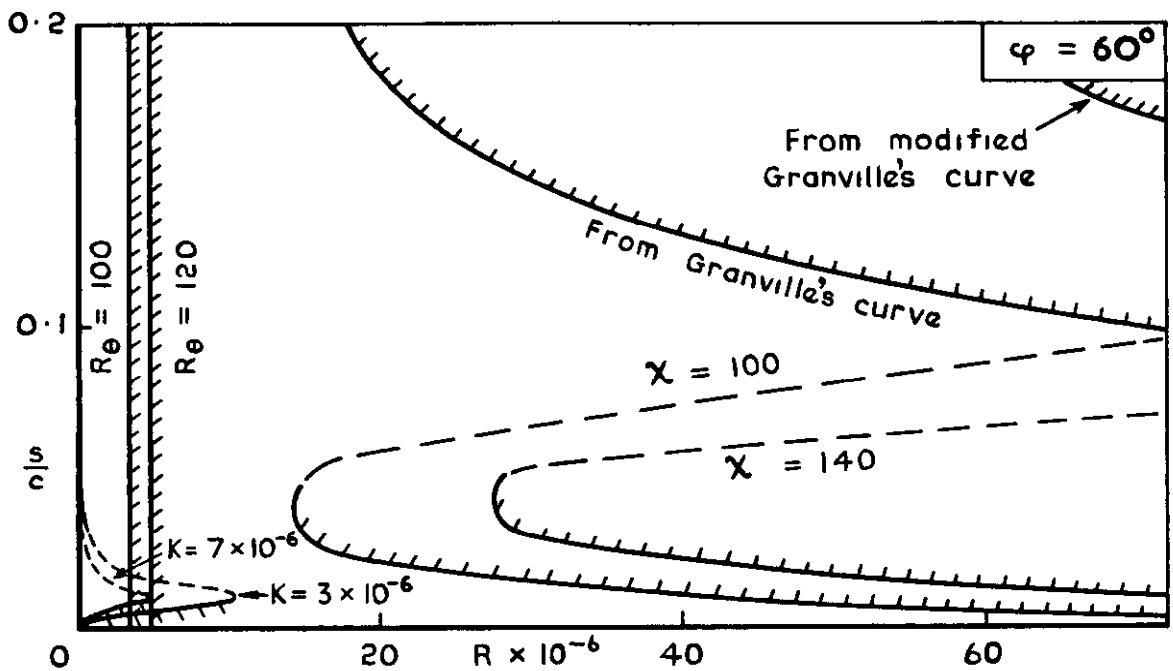
Fig.II The movement with Reynolds number of the
predicted point of transition or of instability; section A
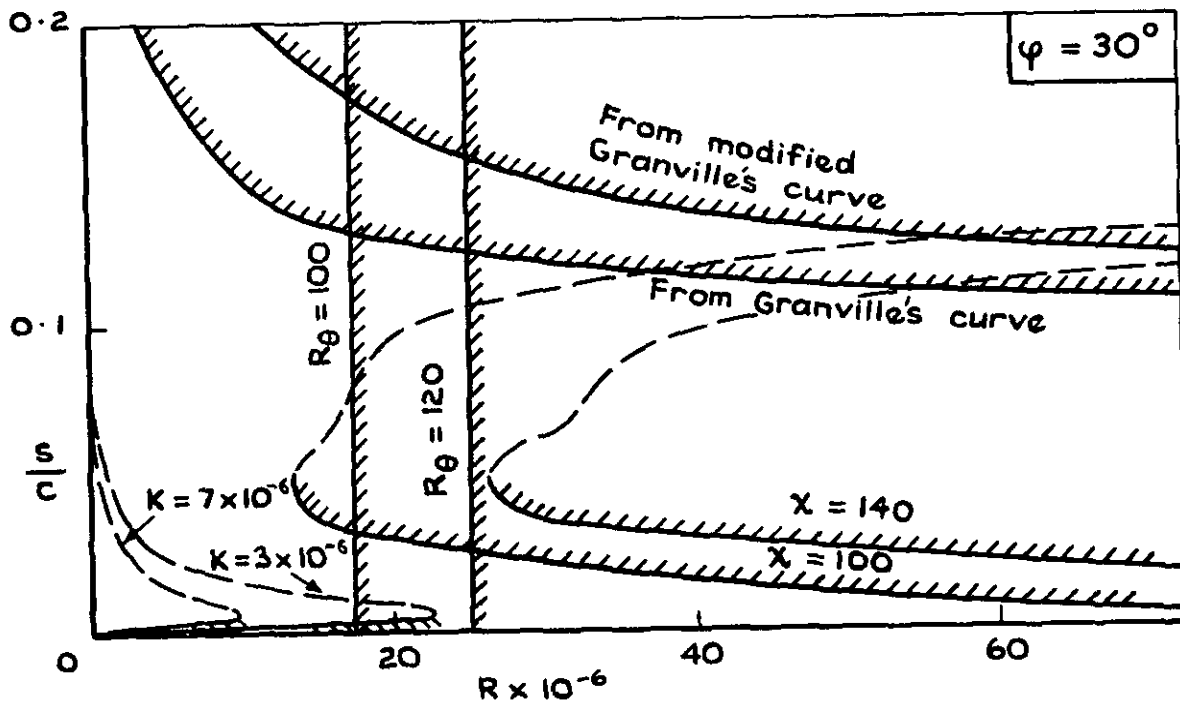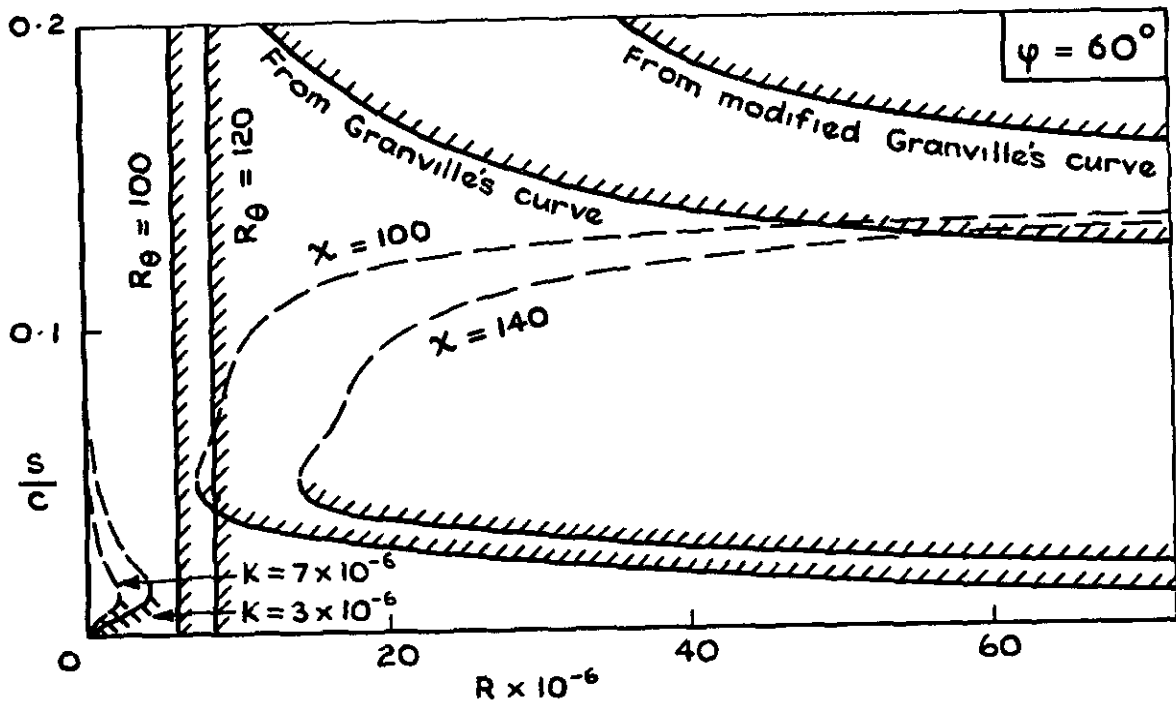
Fig. 12 The movement with Reynolds number of the predicted point of transition or of instability; section B
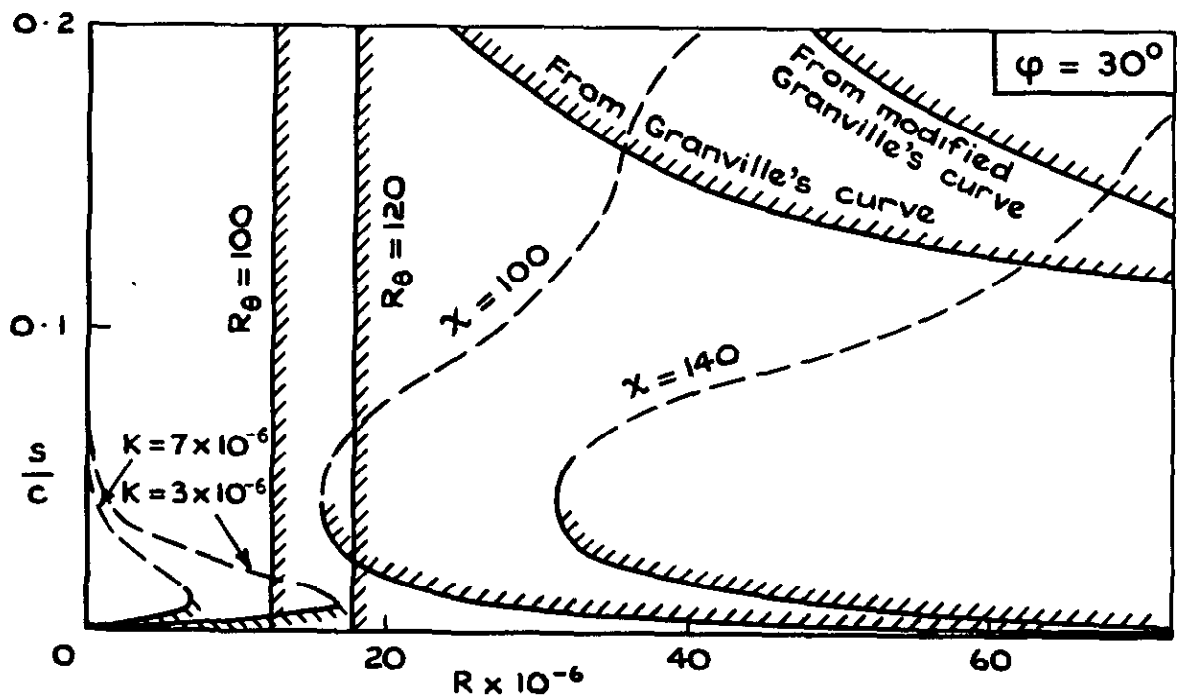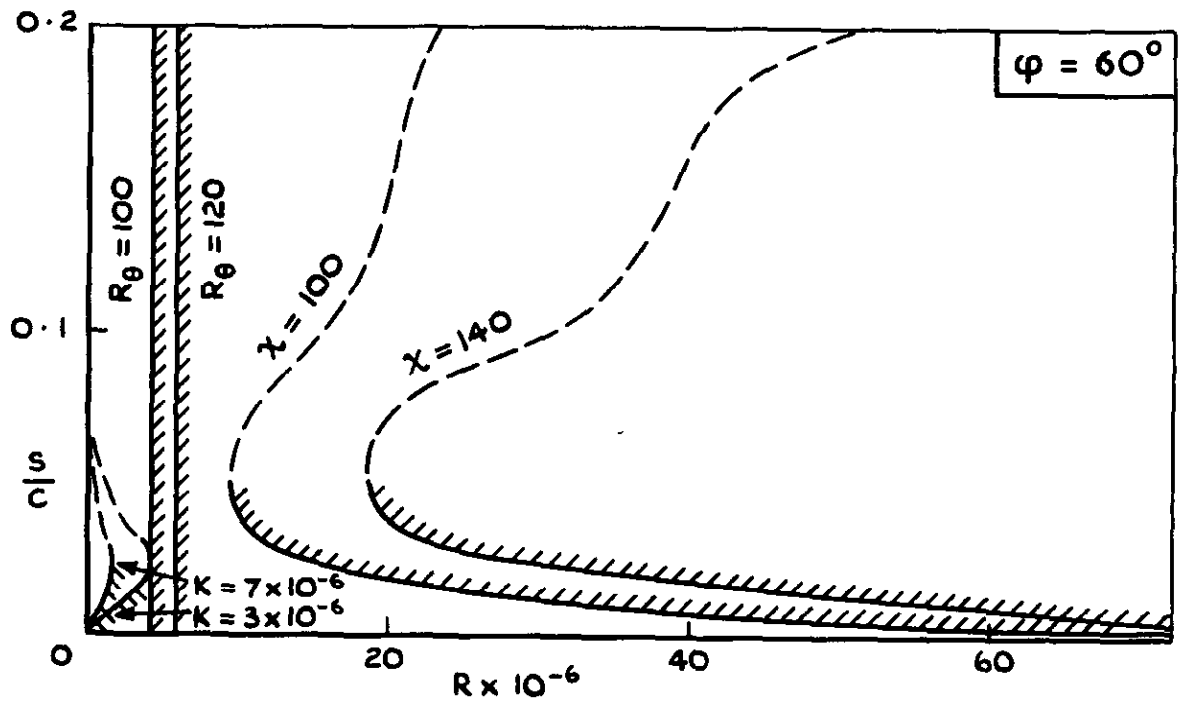
Fig. 13 The movement with Reynolds number of the predicted
point of transition or of instability; section C
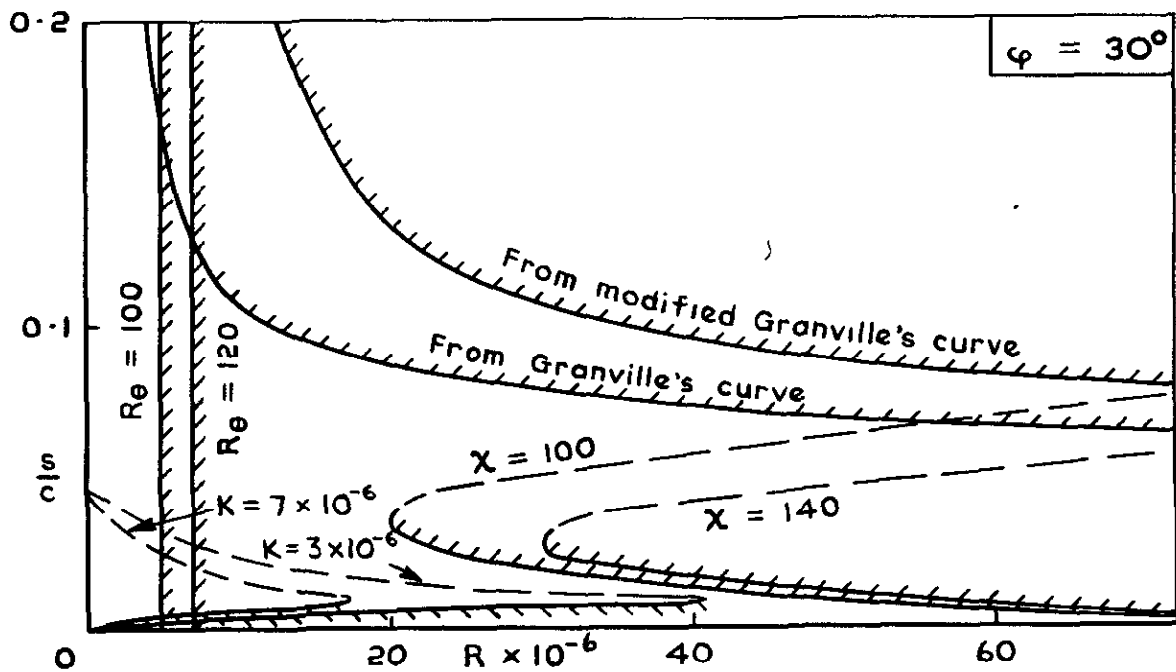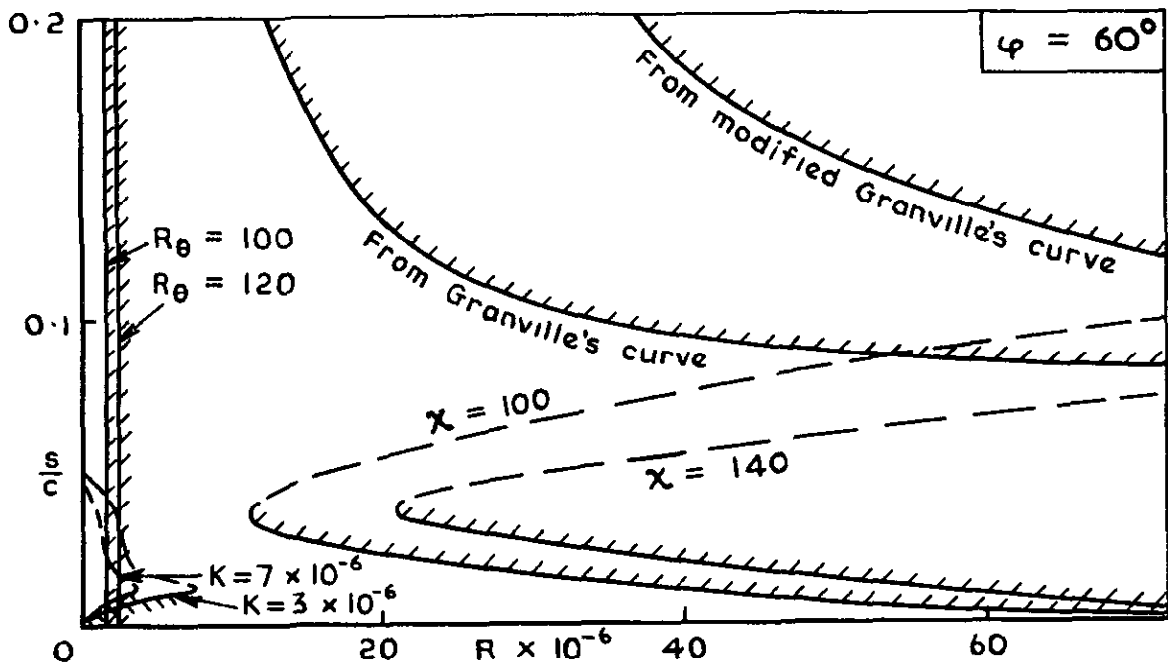
Fig.14 The movement with Reynolds number of the
predicted point of transition or of instability; section D
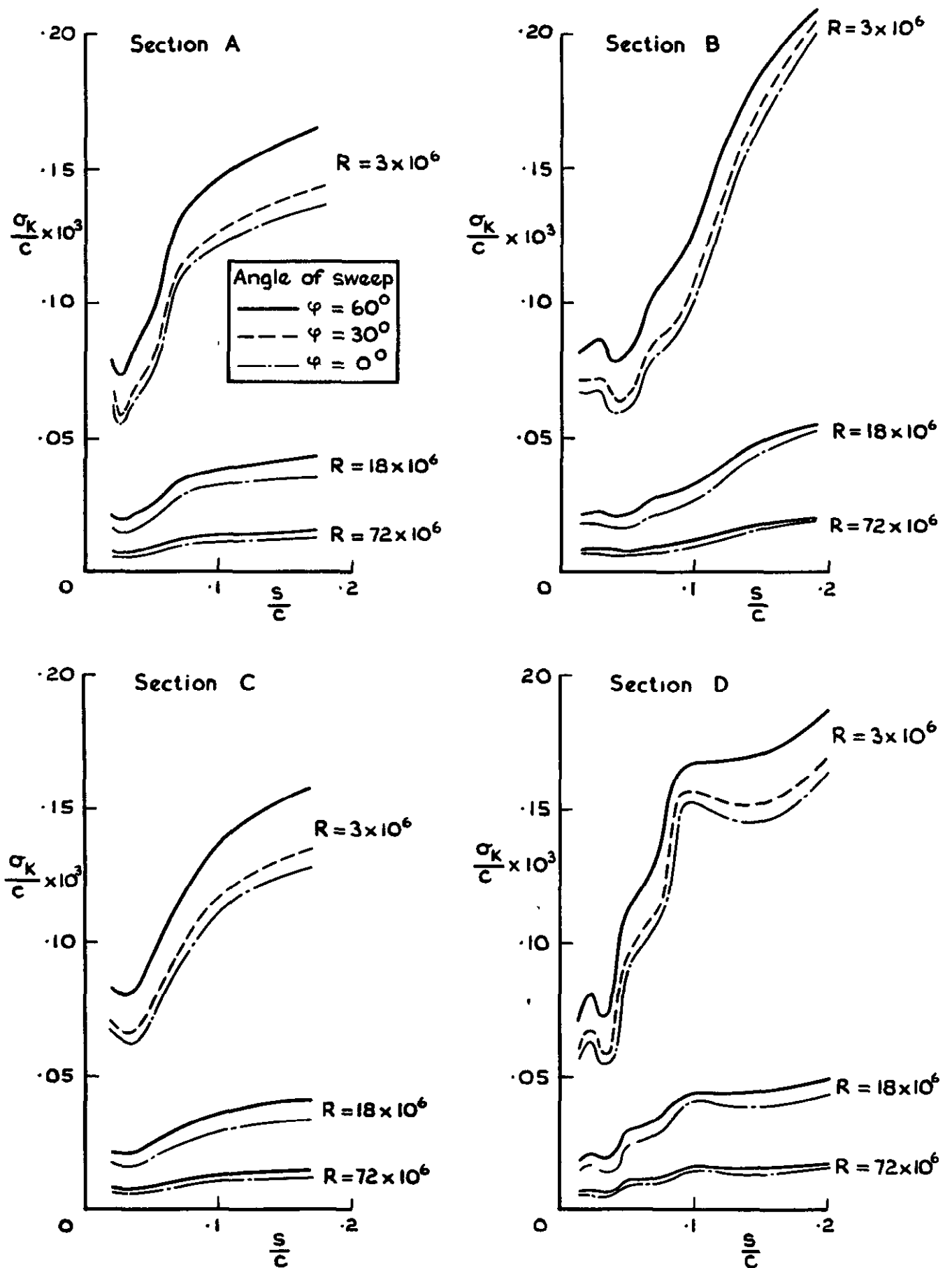
Fig.15 Variation of critical roughness height with chordwise position and Reynolds number

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Treadgold, D. A
Beasley, J A

SOME EXAMPLES OF THE APPLICATION OF METHODS
FOR THE PREDICTION OF BOUNDARY-LAYER
TRANSITION ON SHEARED WINGS

The laminar boundary layer has been calculated for the leading-edge region of four
selected aerofoils for cases where the supercritical region is terminated by a shock wave at
about 20% chord   The possibility of the boundary layer becoming turbulent before the
shock wave is then considered according to four different criteria   leading-edge contamina-
tion, re-laminarisation, sweep instability and Tollmien-Schlichting instability   Many sim-
plifying assumptions have had to be made, since the purpose of the Report is to demon-
strate how the problem might be treated, rather than to present definitive results, and how
the various mechanisms are seen in conjunction   It is concluded that much more needs to
be known before predictions can be made confidently with any degree of precision.

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DETACHABLE ABSTRACT CARDS   **DETACHABLE ABSTRACT CARDS**

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